THE EFFECTS OF EXPLORATORY LEARNING ENVIRONMENTS ON STUDENTS' MATHEMATICS ACHIEVEMENT

A Dissertation

by

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ABSTRACT

The objective of this dissertation was to advance the knowledge about mathematics instruction regarding the use of exploratory graphical embodiments in Pre-K to College levels. More specifically, the study sought to find out which graphical representations generate the highest learning effect sizes as well as which teaching method is the most supportive when graphical representations are applied.

The dissertation is organized into three coherent research studies that correspond to different schooling levels. The primary method of data analysis in this study was meta-analysis supported by synthesis of qualitative and comparative studies. A total of 73 primary studies (N = 9055) from 22 countries conducted over the past 13 years met the inclusion criteria. Out of this pool, 45 studies (N = 7293) were meta-analyzed. The remaining 28 studies (N = 1762) of qualitative or mixed method designs where scrutinized for common themes. The results support the proposed hypothesis that visualization aids mathematics learning. At the primary level, the mean effect size for using exploratory environment was ES = 0.53 (SE = 0.05, 95% CI: 0.42-0.63), the mean effect size for using computerized programs at the grade levels 1-8 was ES = 0.60 (SE = 0.03, 95% CI: 0.53-0.66), and the results of applying congruent research techniques at the high school and college levels revealed an effect size of ES = 0.69 (SE = 0.05, 95% CI: 0.59–0.79).

At each of the teaching level, implementing an exploratory environment generated a moderate effect size when compared to traditional teaching methods. These findings support a need for a broader implementation of exploratory learning media to mathematics school practice and provide evidence to formulate a theoretical instructional framework.

DEDICATION

To my family and my parents

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I would like to express my gratitude to those who provided me with opportunity, guidance, and time during this dissertation completion. Much gratitude is dedicated to Dr. Yeping Li for chairing my committee and providing assistance along my journey by challenging and inspiring me to pursue high-quality research. I would also like to thank Dr. Victor Willson, who sparked my interest in meta-analytic research techniques and who has always been available to provide guidance. I would also like to extend my thanks to my committee members, Dr. Scott Slough and Dr. Robert Gustafson, for their suggestions during the process of formulation of the dissertation.

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NOMENCLATURE

ES	Effect size
SE	Standard error
CI	Confidence interval
SS	Sample size
df	Degree of freedom
QE	Quasi-experimental
R	Randomized
MM	Mixed methods
QUAL	Qualitative
ECE	Exploratory computerized environment
SC	Student centered
MEA	Model eliciting activity
Q	Test of homogeneity of effect sizes

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CHAPTER I

INTRODUCTION

Visualization and Mathematics Teaching

There have been multiple research studies conducted in the area of enhancing mathematics teaching and learning through visualization (e.g., Gershman & Sakamoto, 1981; Niss, 2010; Podolefsky, Perkins, & Adams, 2010; Thomas & Hooper, 1991). Although the majority of scholars supported the idea that visualization—often displayed by programmed computer software—helps students learn math concepts, they often explored visual media as tools supporting tutoring or enhancing the process of graphing functions or drawing geometrical objects. Despite proven advantages of using visualization in these capacities, there exists a potential for applying graphical representations to immerse learners in scientific exploratory environments that allow conceptualization of math ideas and their deeper understanding. Although some elements of scientific inquiry, such as analysis and measurements, have received significant attention in the newly developed common core standards (Porter, McMaken, Hwang, & Yang, 2011), the process of inquiry design in mathematics classes, for example, the process of mathematical modeling or problem solving, seems to be left out of these discussions. Research by Grouws and Cebulla (2000) suggests that students who develop math conceptual understanding are able to perform successfully on problem solving requiring a task transfer; thus, it is hypothesized that by enriching math curriculum via elements of scientific inquiry and mathematical explorations, the process of conceptualization of abstract math ideas will be enacted. Consequently, shifting from procedural to conceptual teaching and learning methods in mathematics could be initiated. The three coherent articles in this dissertation will attempt to seek answers to these hypotheses. Supported by the statistical apparatus of meta-analytic research, these manuscripts will aim to quantify the effects of using these visualization techniques in school practice and highlight moderators that increase students' learning effects.

Purpose, Problem Statement, and Inquiry Method

The purpose of the dissertation is bifocal; it is to advance the knowledge about mathematics instruction regarding the use of various graphical embodiments to enhance the process of mathematics reasoning and also to propose an instructional method that will help improve students' achievement when exploratory environments are used. The following are the research questions that will guide the study: (a) what type of visualizations or scientific embodiments contextualizes the mathematical concepts and theorems most effectively? (b) do contexts presented by computerized simulations help students develop the skills of math knowledge transfer to real life situations? (c) does situating the instruction in mathematical modeling, help students conceptualize abstract mathematical ideas? What are the constructs that improve/increase these skills? As a method of inquiry, meta-analysis will be employed. Depending on the availability of primary research studies, an inclusion of research situated in naturalistic paradigms will also be considered. The inclusion of synthesis of qualitative studies is intended to enrich and to strengthen the general research findings.

Structure of the Dissertation

This dissertation comprises three articles that employ meta-analysis as the primary research method. Research conducted globally over the past 12 years will be considered and located studies will be referenced with an asterisk. Constituted by these three study foci, the dissertation will be solidified through a common theme—how and why exploratory environments enhance understanding of mathematic concepts—and will reveal which research constructs should lead to further investigations in this domain.

The first article will aim at analyzing the effect of various forms of graphical representations on students' math learning in Pre-K through Grade 5. The purpose of this study is to determine effect sizes of using representations in math classes and to inquire about students' learning from using representations. This study will investigate the effects of using regular static blackboard drawings as well as a manipulative and knowing why some means of visualization work better than others. In addition to providing quantifiable results, this study examines potential mediators in the learning effects. Emphasis will also be placed on identifying the type of instructional support and its impact on student learning. The study will conclude with recommendations and conclusions intended to depict avenues for further research.

The second article will focus on analyzing effectiveness of visual representations, delivered through computer programs— on students' math achievement— in Grades 1 through Grade 8. The purpose of this article is to contribute to this body of research by examining exploratory computerized environments (ECEs) used to support the process of word problems solving and explorations. The themes of this study will reflect on the notion that any form of visualization—whether static or dynamic—can be converted into digital format and be delivered through a computer screen.

The goal of the third manuscript will be to extend inferences about using graphical representations in mathematics to high school and college levels, and to seek quantifications of these constructs that support exploratory learning. Since an emerging view on math knowledge acquisition is that exploratory environments in the form of computer-simulated environments promise to be very effective (Jong, 1991), synthesis of research in this area will be of particular interest.

The proposed three study format aimed at researching different domains of mathematical cognition is internally linked. Its inductive sequencing, from comprising and analyzing general visualization techniques to more detailed analysis of their applications, is purposeful. It is intended to lead to providing recommendations on what type of visualization increases students' achievement and how to organize math lesson cycles and develop their content so that students' understanding of math concepts as mediated through the quality of their problem-solving techniques is improved. All three studies follow different yet domain-coherent theoretical frameworks that guide the studies and direct examination of their constructs.

The following chart flow (see Figure 1) illustrates the sequencing of the manuscripts and the structure of the dissertation.

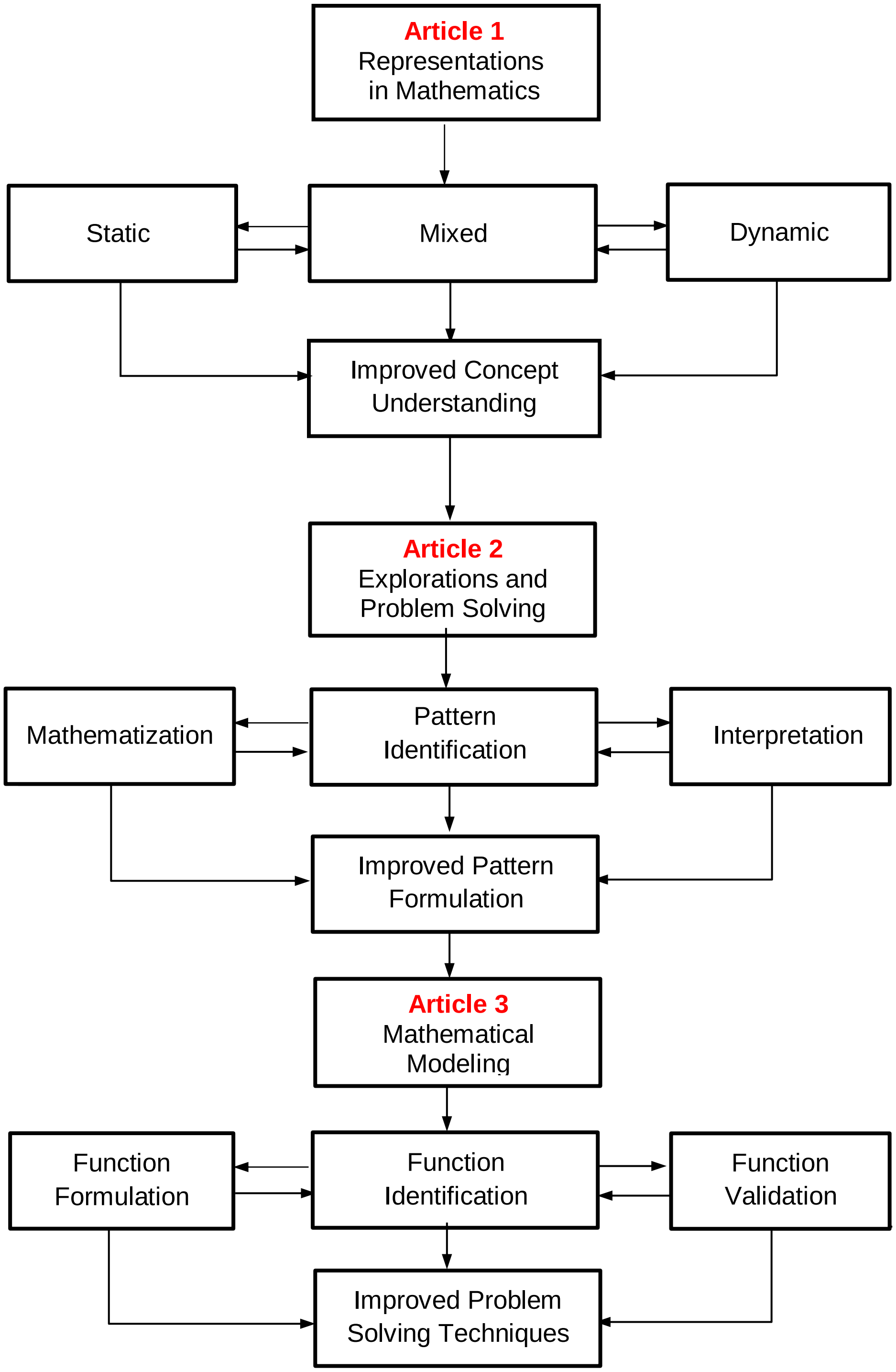

Figure. 1. Adopted model for the article path sequencing

Constituted by the three components, the dissertation will solidify through a common theme—how and why exploratory environments enhance understanding of mathematics concepts—and will attempt to reveal constructs that should be propagated in school practice and lead further investigations in this domain. The benefits for the learners, and teachers, and the curriculum policy makers of identifying such contracts are far-reaching; they are to: (a) help with organizing instructions, and (b) help the learners more effectively comprehend abstract math concepts and apply these concepts to other disciplines. The theoretical framework accompanying each article will guide the synthesis of previous research findings and support the formulation of the current study research objectives. Due to its substantial reference to math curriculum content design, this study might be of interest to mathematics teachers, curriculum developers, software math programmers, and other stakeholders who are involved in mathematics curriculum development and who are concerned about improving school mathematics teaching and learning.

CHAPTER II

THE EFFECTS OF USING REPRESENTATIONS IN ELEMENTARY MATHEMATICS: META-ANALYSIS AND SYNTHESIS OF REASERCH

Introduction

This study provides a synthesis of global research conducted over the past 12 years on using representations to support the learning of mathematics concepts in Pre-K through Grade 5. The purpose of this study was to determine a general effect size of using representations in Pre-K through Grade 5 math classes and to learn about students' progress in learning from using representations. A total of 22 primary studies encompassing 2448 subjects were analyzed. In order to reflect more accurately on the research objectives, the pool of studies was divided into a meta-analysis of 13 primary research studies, which provided necessary statistical quantities to calculate the mean effect size, and 9 comparative studies, which were used to formulate general conclusions about how students progress and advance their skills in applying representations. The weighted mean effect size for the 13 primary studies (13 effect sizes) was reported to be ES = 0.53 (SE = 0.05). A 95% confidence interval around the overall mean—C_{lower} = 0.42 and C_{upper} = 0.63— proved its statistical significance and its relative precision. The calculated effect size signifies strong, robust support for use of representations in Pre-K through Grade 5 mathematics classes. The findings of 9 comparative studies enhanced the study findings by shedding more light on the conceptual interpretations of the representations and teachers' role during lessons where representations are implemented.

Discussion of some of the representations, called schemata, and their conceptual alignment with high school math and science equivalent structures concludes this study.

Multiple researchers (e.g., Hoffler & Leutner, 2007; Mayer & Anderson, 1992) have determined that people learn more deeply from words supported by graphics than from words alone. This finding corresponds to the modern view on mathematical learning, which claims that utilizing multiple representations that make connections between abstract, graphical, symbolic, and verbal descriptions of mathematical relationships during teaching and learning will empower and simultaneously help students develop a deeper understanding of mathematical relationships and concepts (Kaput, 1989; National Council of Teachers of Mathematics [NCTM], 2000; Porzio, 1999).

Representations, especially their graphical forms, can also be perceived as learning experiences that are transmitted to the learner by pictorial media (Clark & Mayer, 2011). As such, they help the learner identify meaningful pieces of information and link the information with the learner's prior experience. Although the constructs of using diverse forms of representations to enhance the development of mathematical concepts and problem-solving techniques at the elementary school level has been widely researched (e.g., Jitendra, Star, Rodriguez, Lindell, & Someki, 2011; Van Oers, 1998; Weber-Russell & LeBlanc, 2004), a formal meta-analysis in this domain could not be located using standard library search engines. Since students' early experiences with the content of mathematics have a tremendous impact on their further engagement and success in the subject (Dienes, 1971), this study emerged to fill in the gap and to contribute to enriching the research on using representations in elementary school mathematics.

Theoretical Background

Representations and Constructivist Learning Theory

The strengths of the knowledge delivered by means of representations are supported by one of the leading learning theories, the constructivist theory, which is strongly advocated by Kant, Dewey, and von Glasserfield (Merrill, 1991). Dienes (1971) concluded that students' learning of mathematics concepts can be optimized by using a variety of representations. Clark and Mayer (2011) suggested that knowledge acquisition is based on the following principles of learning: (a) dual channel—people have separate channels for processing visual/pictorial material and auditory/verbal material; (b) limited capacity—people can actively process only a few pieces of information in each channel at one time; and (c) active processing—learning occurs when people engage in appropriate cognitive processing such as attending to relevant material and organizing the material into a coherent structure. The principles of learning reflect on how knowledge is stored and retrieved from learners' memory. Shepard (1967) showed that memory created by pictures is retained much longer by a learner than the memory of spoken words. Shepard's findings support the theory of human cognitive architecture (Paas, Renkl, & Sweller, 2003), which states that the most crucial structures affecting the rate of information processing are working memory and long-term memory. Human working memory has a limited capacity as oppose to long-term memory whose capacity is unlimited (Kintch, 1998). In order for the information to be stored in a learner's long-term memory, it needs to be processed initially through its working stage. Being presented with a dose of complex information, the learner might feel overwhelmed, which can result in the information not being fully processed. This state will consequently block the information from reaching the learner's long-term memory, and prevent it from being learned and accumulated. The primary goal of using representations is to convert the information to a visual form and to transmit it to the learner's visual channel. The cognitive weight of the information is not reduced though, but it is converted to a different and an easier accessible format. This process, according to human cognitive architecture (Shepard, 1967), reduces the need for high working memory capacity and allows the information to be accumulated in the learner's long-term memory. Thus, the virtue of using representations lies in their capacity to present the knowledge in conveyable graphical embodiments supported by verbal elaborations rather than vice versa. Such knowledge presentation creates appealing conditions for not only being accumulated, but also longer retained, and accessible for a further usage. Thus developing effective visual representations plays a significant role in learners' rate of math knowledge acquisition and their potential to apply the knowledge in other subjects.

Representations in Mathematics

Broadly defined representations are passive entities. Due to learner's active engagement, they are transformed into active semiotic resources (Thomas, Yoon, & Dreyfus, 2009) and can be stored in a learner's long-term memory. Knowledge externalized by graphics can be easily accessible for analysis and can be readily exhibited and communicated (Ozgun-Koca, 1998; Zhang, 1997). Representations as a means by which individuals make sense of situations (Kaput & Roschelle, 1997) can be expressed in forms of combinations of written information on paper, physical objects, or a carefully constructed arrangement of thoughts. Monk (2003) noticed that representations can be used to explore aspects of a context that might otherwise not be apparent to a learner; thus they amplify properties of mathematical structures not easily imaginable. In the process of knowledge accumulation, representations are converted into internal images. Mediated by the level of entry into learners' memory system, Kaput (1989) categorized representations as external or internal. Both types of representations are interrelated in the sense that the meaning of internal representations stored in a learner's long-term memory strongly depends on the learner's perception of its external counterpart. The following sections will provide more details on specificities of each type of representation along with elaboration on their mutual relations.

External representations encompass physically embodied, observable configurations—such as pictures, concrete materials, tables, equations, diagrams, and drawings of one-, two-, or three-dimensional figures (Kaput, 1989)—or various forms of schemata (Jitendra, Griffin, McGoey, Gardill, Bhat, & Riley, 1998). All of these embodiments can be provided in the forms of drawings, or digitalized by computer programs. They can also be generated by the instructor as he/she introduces the representations to the learners. External representations encompass also dynamic graphics which are generated with the help of technology, for example, graphing calculators or computer-based simulations (Goldin & Shteingold, 2001).

The advantage of external dynamic representations is rooted in the feature that they can explicate more clearly the dependence between isolated variables. Being able to observe how a change of one variable affects the change of the other can help with mathematization of their mutual relations. Modern technology provides multiple advantages of exploring these features (Blum, Galbraith, Henn, & Niss, 2007). Being able to produce representations plays also an important role in deriving new theories; major scientists made their discoveries by carefully selecting representations and analyzing their properties, or by inventing new representations (Cheng, 1999). Thus one can hypothesize that having the ability to convert verbal information into its different format; diagram, graph or algebraic function can help with other subjects' understanding that utilizes quantification processes.

Developing students' skills of converting verbal contexts into external graphical representations and using the representations to contextualize math concepts has proven to be beneficial for the students. While learning or constructing mathematical structures involves not only manipulating mathematical symbols but also identifying relationships and interpreting the relationships, graphical representations, especially their dynamic embodiments have a great potential to help students with learning these processes. According to National Research Council (2000), representation can also encompass clarification of problems, deduction of consequences, and development of appropriate tools. Thus exploring these capacities and making them available to mathematics school learning seem to be a worthy undertaking. As Eisenberg and Dreyfus (1991) noted, students might end up with an incorrect solution if their algebraic skills are not strong even though their reasoning might be correct. However, if the learner possesses the skills of graphically solving the problem or support its solution process, the graphed representation might serve as a backup or a way of solution verification. Being exposed to mathematical representations, the learners "acquire a set of tools that significantly expand their capacity to model and interpret physical, social, and mathematical phenomena" (NCTM, 2000, p. 4). Identifying meaningful representations that are attractive and conveyable to elementary mathematics students appears to have a profound impact on their mathematics education and interest in studying this subject.

Internal representations encompass mental images corresponding to internal formulations of what human beings perceive through their senses. Internal representations cannot be directly observed. They are defined as the knowledge stored in learner's long-term memory (Zhang, 1997). Internal representations are formulated based on one's interaction with the environment (external representations) and are altered throughout a lifespan. In the process of learning, external representations prompt the emergence of internal representations. Being able to formulate concepts' internal representations through the process of understanding their external embodiments and retrieve the mental pictures plays an essential role in communicating messages in mathematics. The extent to which the internal representations solidify determines the rate learner understands the concept or idea. In this vein, Hiebert and Carpenter (1992) maintained that there exists a strong relationship between external and internal representations created by learners, and that the strength of linkage these representations determines students understanding. Internal representations of the knowledge accumulated through experiencing visual representations is more efficiently stored and retrieved from a learner's long-term memory. Von Glasersfeld (1990) suggested that the environment is construed by one's internal representations, while Perkins and Unger (1994) stated, "Mental maps or mental models or other sorts of mental representations mediate what we would call understanding performances" (p. 4). Projected through this postulates, the concept of internal representations asserts knowledge considered as a body dependent on a learner's own experiences. Enabling these experiences by engaging and intellectually stimulating learners through carefully designed learning environments deems to be a significant factor in nurturing effective learning.

Challenges of Inducing Representations in Pre-K through Grade 5

The effect of using representations is not new to mathematics education community. However it has recently attracted more attention due being supported by constructivist learning theorem that leads contemporary research in education (Cuoco, 2001). By treating mathematical concepts as objects, thus by embodying them with observable representations, a construction of mental pictures in the minds of the students can evoke (Dubinski, 1991). Such constructed mental pictures are stored in students' long term memory and are being available for retrieval. Research (Zaskis & Liljedahl, 2004) suggests that one of the ways to induce the process of converting concepts to objects is act on them or to manipulate on them. Thus having students construct a representation, for example, of a ratio of an area of an inscribed circle in a square to the area of the square, should help students with a proportionality constant formulation. Researchers (e.g., Sfard, 1991) concluded that the process of transferring abstract mathematical concepts into their mental images is challenging for the learner and also for the instructor who is to guide the learner with the transferring processes. What are the challenges faced by elementary school children to embody mathematical structures into visual representations? The following is a discussion of some of them.

Algebraic equations and their conceptualization are frequently investigated in K-5 mathematics research. Swafford and Langrall (2000) noted that students generally can make use of various representations and they can identify patterns between isolated variables, but they cannot find consistency along a larger set of variables along with generalizing the patterns and converting them into algebraic forms. Bruner (1961) suggested three learning phases for problem solving: (a) the enactive, (b) the iconic and (c) the symbolic with representations serving as a mediator between there three levels of learning. Dreyfus (1991) suggested another learning phases with representations: (a) using one representation, (b) using more than one representation, (c) making links between parallel representations, and (d) integrating the representations. Although the inductive way of utilizing representations proposed by Dreyfus (1991) should lead to generalization, not much is said about principle identification that would direct the learner to selecting a correct mathematical representation. More recently English and Walters (2005) proposed introducing mathematical modeling to elementary math school learning as a way to support students' skills of problem solving techniques. They proposed a shift of attention from representations to conceptual analysis of the variables of the problem and then searching for representations that would mathematize formulated dependence. In a congruent view, Terwel, van Oers, van Dijk, & van den

Eeden (2009) claimed that representations appear to be more general, overarching concepts from cognitive psychology, while model is a more domain - specific term. This position is aligned with our perspective on the relation between a representation and a model. Representations, at the elementary school level, encompass general structures used in mathematics thus ratio, rate, percent or newly developed schemata for problem solving. Models depict — using mathematical representations—real quantifiable events. Viewed through this prism, pinpointing and understanding the principle embedded in a given problem act as catalysts of selecting correct representation which consequently provides a gateway for students' correct model formulation. According to Swafford et al., (2000), the emphasis in the curriculum at the prealgebra level should be on developing and linking multiple representations to generalize problem situations. They concluded that the lack of generalization skills is rooted in instruction focusing on reaching only the initial stages of problem analysis and leaving the process of generalization for the students to formulate.

The process of symbolically expressing problem patterns is difficult and it might be out of students' reach if a prior learning of such techniques did not take place. A similar conclusion was formulated by Deliyianni, Monoyiou, Elia, Georgiou, & Zannettou (2009) who observed that first graders restricted themselves to providing unique solutions even though the questions required a general patter formulation. They further suggested that seeking unique solutions to a problem is students' habit based on their previous experience, thus a place in the math curriculum should be found and an effort should be made to create such learning environments that would broaden students' perspective on problem solving by including more tasks on generalization. Other researchers (e.g., Kieren, 1984; Lesh & Harel, 2003) have shown that elementary school children bring powerful intuitions and sense-making tool, yet mediating these intuitions with abstract math concepts to embody these concepts into representations is a challenge still facing the math research community. The benefits of identifying and using the constructs that enable learners to effectively transfer abstract mathematics concepts into meaningful internal representations are limitless. It seems that a design of sound instructional techniques to support the processes is still being researched and developed.

The formulated theoretical framework will guide the synthesis of previous research findings and support the current study research objectives.

Synthesis of Prior Research

As the constructivist theory strongly supports the use of representations in the learning process, several research studies have been undertaken to explore the effects of using representations on students' math concepts understanding. These results converge with contemporary theories of cognitive load and multimedia learning principles developed by Clark and Mayer (2011) and have practical implications for math instructional designs. A meta-analysis of 35 independent experimental studies conducted by Haas (2005) shed light on using representations as a means of supporting teaching methods at the secondary school level. Haas concluded that math instruction supported by multiple representations, manipulatives, and models produced a high (ES = 0.75) effect size. Schemas, which are defined as generalized representations that link two or more concepts (Gick & Holyoak, 1983), are frequently being researched at the Pre-K through fifth-grade level. For example, Jitendra and colleagues (1998) found that having students of Grades 2-6 categorize problems and then having them solve the problems by using schemas produced a positive medium size learning effect (ES = 0.45). The virtue of using representations embodied by schemas is that they are easily converted by learners into internal representations, and, as such, they can be stored in long-term memory and allow treating diverse elements of information in terms of larger, more general units (Kalyuga, 2006). According to Pape and Tchoshanov (2001), schematic representations also lead to enhanced student problem-solving performance.

Another group of researchers investigated whether representations should be provided to students or if the students should be the producers of representations (e.g., De Bock, Verschaffel, Janssens, Van Dooren, & Claes, 2003; Rosenshine, Meister, & Chapman, 1996). These scholars concluded that if representations are provided, their forms must be sufficiently informative and detailed to be transferrable by students into mathematical algorithms. They also emphasized that having students construct their own representations benefits the learners the most. The importance of possessing the ability to transfer a given context (e.g., a story problem) into a representation was highlighted by Jonassen (2003), who claimed that successful problem solving requires the comprehension of relevant textual information along with the capacity to visualize that data and transfer the data into a conceptual model. Following Riley, Greeno, & Heller (1983) developing students' abilities to identify the matching representation that helps with problem conceptual understanding should emerge as a priority of elementary mathematics teaching.

Representations are also used to support the introduction of new mathematics concepts. For example, several studies (e.g., Corwin, Russell, & Tierney, 1990; Tzur, 1999) were conducted on the development of students' notations of fractional parts of areas, called *fair sharing*, which provided a meaningful representation of dividing a whole into parts that were then easily comprehended by elementary students. Hiebert (1988) noted that students' understanding of new ideas strongly depends on the degree to which the learners are engaged in investigating the relations between new representations and the representations whose understanding is already mastered. A study conducted by Ross and Willson (2012) not only supported the claim that math students learn more effectively when instruction focuses on using representations, but moreover, they proved that the most effective strategies for building representations are these rooted in constructivist learning theorem. The range of using representations in Pre-K through fifth grade is wide; thus, synthesizing the experimental research findings and identifying the most effective strategies manifests as a worthy undertaking.

Research Methods

Based on prior research, a hypothesis for this study emerged, suggesting that using representations in mathematics classes helps students comprehend abstract concepts and enhances the skills of the concepts' applications. Understanding the degree to which representations help learners with understanding the different mathematics entities as compared to traditional methods of instruction emerged as the main objective of this study.

Research Questions

The following research questions guided this study:

1. What are the magnitude and direction of the learning effect sizes of using representations in Pre-K through fifth-grade mathematics when compared to traditional teaching methods?
2. Do the magnitudes of effect sizes of applying representations in Pre-K through Grade 5 differ across the main modes of their classroom induction—concept introduction and problem solving?
3. What are the possible moderators that affect students' achievements when representations are used?

Meta – analysis with its quantitative methods providing means to computing the mean effect size as well as of applying subgroup moderator analysis will be used to quantify the research findings. Yet, there are other questions of a qualitative nature that the current study will also attempt to answer though the analysis of the available comparative studies.

1. How can the learner be assisted with making a connection between abstract mathematical symbolism and its embodied representation? How the complexity of representation should evolved as students' progress with their schooling?
2. What are the main mathematical domains, at elementary level, where representations are used? Are these representations induced in a manner consistent with definitions that they use in high school math classes?

It is hoped that the answers to these questions will advance the knowledge of using representations and help students improve their math understanding.

Data Collection Procedure

This synthesis sought to encompass the past 12 years of global research on using representations in Pre-K through fifth-grade mathematics, with student groups ranging in age from 3 to 12, in both public and private schools, with a minimum sample size of 15 participants. The primary intention of this undertaking was to analyze only peer-reviewed experimental research that included treatment and control groups with associated quantifications, as described by Lipsey and Wilson (2001). However, because researchers found a valuable pool of comparative studies, the study scope was expanded to include a synthesis of findings of these studies as well. This modification was employed to enhance the general inferences and strengthen the study generalizability. In the process of collecting the applicable research, ERIC (Ebsco), Educational Full Text (Wilson), Professional Development Collection, and ProQuest Educational Journals, as well as Science Direct, Google Scholar, and other resources available through the university library were used to identify relevant studies published between January 1, 2000, and December 31, 2012. In the process of extracting the relevant literature, researchers searched for the following terms: *graphical representations, mathematics education, primary, students, and research*. In order to broaden the search criteria, we also used the terms *graphics, visualization*, and *problem solving.* Such defined criteria returned 131 papers, out of which 13 satisfied the conditions for meta-analysis (13 effect sizes) and 9 represented comparative studies that analyzed the effects of using representations in the functions of group age or group levels.

Coding Study Features

The main construct under investigation was the learning effect of using representations in Pre-K through fifth-grade mathematics classes. While some of the characteristics, for example, year of study conduct, locale or type of research design were extracted to support the study reliability, some, like grade level or intervention type, were extracted to seek possible mediators. Following are the descriptions of these features. These features were further aggregated to apply a subgroup moderator analysis.

Grade. This variable described the grade level of the group under investigation and referred to groups ranging from kindergarten to Grade 5.

Descriptive parameters. Descriptive parameters encompassed the locale where the studies were conducted, the date of publication, and the sample size representing the total number of subjects under investigation in experimental and control groups.

Publication bias. All studies included in this synthesis were peer-reviewed and published as journal articles, thus no additional category for publication was created.

Group assignment. This categorization refers to the mode that was used to select and assign research participants to treatment and controlled groups. Two main groups were identified: (a) randomized, where the participants were randomly selected and assigned to the treatment and control group; and (b) quasi-experimental, where the participants were assigned by administrator selection. This categorization is aligned with Shadish, Cook, and Campbell (2002) definitions of group assignment.

Type of research design. Two types of research design were synthesized in this study: a pretest-posttest experimental study with a control group, and comparative studies. The two types of research were analyzed separately.

Intervention. The intervention (treatment approach) was classified into four categories reflecting the type of representations used in Pre-K through fifth-grade mathematics as defined by Swing, Stoiber, and Peterson (1988) and Xin and Jitendra (1999): (a) pictorial (e.g., diagramming); (b) concrete (e.g., manipulatives); (c) mapping instruction (e.g., schema based); and (d) other (e.g., storytelling, key word).

Output assessment measure. This variable described assessment instrument and indicates whether the assessment was developed by the researcher or was standardized.

Data Analysis

General Study Characteristics

The summaries of the studies characteristics extracted from the pool of experimental pretest-posttest studies is presented in the Table1.

Table 1

Tabularization of Experimental Pretest-Posttest Studies Features

Authors	Date	Locale	RD	SS	GL	Intervention Representation Type
Alibali, Phillips, & Fischer	2009	USA	QE	91	4th (38) 3rd (53)	Pictorial
Van Oers	2010	The Netherlands	QE	239	4th	Pictorial
Poland, Van Oers, & Terwel	2009	The Netherlands	QE	54	2nd	Schemata based

Table 1 *continued*

Authors	Date	Locale	RD	SS	GL	Intervention Representation Type
Xin, Zhang, Park, Tom, Whipple, & Si	2011	USA	QE	27	4th	Schemata based
Booth & Siegler	2008	USA	R	52	1st	Pictorial
Csikos, Szitányi, & Kelemen	2012	Hungary	QE	244	3rd	Pictorial
Gamo, Sander, & Richard	2010	France	QE	261	4th/ 5th	Schemata based
Terwel, Van Oers, Van Dijk, & Van den Eeden	2009	The Netherlands	R	238	5th	Pictorial
Casey, Erkut, Ceder, & Young	2008	USA	QE	76	Pre-K	Other (storytelling)
Jitendra, Griffin, Haria, Leh, Adams, & Kaduvettoor	2007	USA	QE	88	3rd	Schemata based
Fuchs, Fuchs, Finelli, Courney, & Hamlett	2004	USA	R	436	3rd	Schemata based
Saxe, Taylor, McIntosh, & Gearhart	2005	USA	QE	84	4th and 5th	Pictorial
Fujimura	2001	Japan	R	51	4th	Concrete

Note. SS = sample size, GL = grade level, RD = research design, QE = quasi-experimental, R = randomized

The data revealed that there is substantial diversity in the representations used in elementary mathematics classes that ranges from schemas supporting problem solving to storytelling supporting operations on fractions. Majority of the studies 9 (69%) were quasi-experimental and 4 (31%) were randomized. Grade wise, a dominated group of 6 studies was represented by fourth grade. Because in K-12 math education, problem solving dominates the math learning objectives, how representations help students improve their problem solving techniques emerged as a possible subgroup to be analyzed. Table 2 summarizes the main features of the comparative studies whose findings were also synthesized in this research.

Table 2

Tabularization of Comparative Studies

Authors	Date	Locale	RD	SS	Grade	Cognitive Domain/Strategy Applied
Deliyianni, Monoyiou, Elia, Georgiou, & Zannettou	2009	Cyprus	QE	38 34	K 1st	Addition and subtraction; compared kindergarteners and Grade 1 students' representations on problem solving.
Castle & Needham	2007	USA	QE	16	1st	Measurements; investigated students' change of analyzing objects' dimensions given by different representations.
Coquin-Viennot & Moreau	2007	France	R	44 46	3rd 4th	Solving problems; compared mathematical models to qualitative representations.
Coquin-Viennot & Moreau	2003	France	QE	91	3rd 5th	Arithmetic; compared how students choose between qualitative analysis and schema models while solving arithmetic problems.
Yuzawa, Bart, Yuzawa, & Junko	2005	Japan	QE	69	Pre-K 1st	Geometry; investigated how children compared figures areas given by their various relative positions.

Table 2 *continued*

Authors	Date	Locale	RD	SS	Grade	Cognitive Domain/Strategy Applied
David & Tomaz	2012	Brazil	QE	25	5th	Geometry; examined how various representations become subjects of an activity.
Rittle-Johnson, Siegler, & Alibali	2001	USA	R	74	6th 7th	Decimals; reviewed how correct problem representations (number line) mediate relations between conceptual and procedural knowledge.
McNeil & Alibali	2004	USA	R	70	4th	Equation evaluation; looked at how students distinguish different representations in which the equation sign is used.
Moseley & Okamoto	2008	USA	QE	91	4th	Fractions and decimals; examined how students understand rational number representations.

Note. RD = research design, QE = quasi - experimental, R= randomized, SS = sample size.

The data analysis in this study consisted of three parts. First, descriptive analysis was applied to the entire pool of 22 studies to identify general trends. Next, a meta-analysis of 13 experimental pretest-posttest studies was conducted. The third and final part of the analysis examined the findings of the 9 comparative studies.

Descriptive Analysis

The pool of experimental studies generated data collected from 1,941 elementary school students (see Table 1), while the comparative studies collected data from 507 elementary students (see Table 2). The majority of the studies (15, or 68%) were supported by a quasi-experimental design, and 7 (or 32%) were randomized.

Cognitive domains ranged from the theory of solving equations, to algorithms for operations on fractions, and geometry shape analysis.

The data were further categorized the data by the year of research conduct and the locale where the studies were conducted. Table 3 displays the frequencies of the studies by publication date, showing the number of studies conducted globally between January 2000 and December 2012 and published in peer-reviewed journals in the English language. A substantial number of these studies (15, or 68%) were conducted within the past 5 years. This result signifies increasing interests in using representations in mathematics teaching and learning on a global scale.

Table 3

Descriptive Analysis of Date of Study Publication

Year	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012
Study Count	2	0	1	2	2	0	3	3	4	2	1	2
Percent	9%	0%	5%	9%	9%	0%	14%	14%	18%	9%	5%	9%

Table 4 illustrates the locales where the studies were conducted. As the table indicates, the idea of inducing representations for embodying mathematics concepts and processes in Pre-K through Grade 5 has a multinational range. However, the majority of the studies were conducted in North America (50%), followed by European countries (36%).

Table 4

Descriptive Analysis of the Reviewed Studies by Locale

Country	Cyprus	Brazil	France	Hungary	Japan	The Netherlands	USA
Study Count	1	1	3	1	2	3	11
Percent	5%	5%	13%	5%	9%	13%	50%

Meta-Analysis of Pretest-Posttest Experimental Studies

The Mean Effect Size and Significance

Quantitative inferential analysis in the form of a meta-analysis was performed on pretest-posttest experimental studies. In order for the meta-analytic methods to be applied, the responses for the experimental studies were standardized, and the accuracy of the effect sizes was then improved by applying Hedges formula, which eliminated sampling bias (Lipsey & Wilson, 2001). The overall weighted mean effect size for the 13 primary studies (13 effect sizes) was reported to have a magnitude of 0.53 (SE = 0.05) and positive direction. A 95% confidence interval around the overall mean—C_{lower} = 0.42 and C_{upper} = 0.63—which does not include zero, proved its statistical significance and its relative precision (Hunter & Schmidt, 1990). According to Lipsey and Wilson (2001), the effect of 0.53 is reported as being of a medium size. Its magnitude along with its positive direction indicated that the score of an average student in the experimental groups was 0.53 of standard deviation above the score of an average student in the control groups. By incorporating the U_3 Effect Size Matrix (Cooper, 2010), the average pupil who was taught mathematics structures using representations scored higher on unit tests than 70% of students who were taught by traditional methods. Thus, it can be deduced that using representations in the teaching of mathematics, as a medium supporting instruction, has a profound impact on students' math concept understanding when compared to conventional methods of teaching. Therefore, contextualizing math ideas and letting students embed math operations in contexts meaningful to them has a positive effect on storing the ideas in their long-term memory. The following table provides summaries of individual effect sizes of the meta-analyzed studies along with confidence intervals and qualitative research findings.

Table 5

Effect Sizes of Using Representations in Pre-K through Grade 5

Study (First Author)	ES	SE	95% CI		Research Findings	Source of Assessment
			Lower	Upper		
Alibali (2009)	0.92	0.22	0.19	1.05	Strategy of representing the process of equalizing equations improved problem representation techniques.	Researcher designed
Van Oers (2010)	0.23	0.13	0.36	0.89	Children improved fraction understanding when they were allowed to construct own representations guided by the teacher.	Researcher designed
Poland (2009)	1.22	0.29	0.04	1.22	Introducing dynamic schematizing improved understanding of the concept of process during problem solving.	Researcher-created schematizing test
Xin (2011)	0.60	0.39	-0.19	1.44	Conceptual representations helped students learn the process of problem solving.	Used textbook items adopted by the districts Cronbach's alpha = 0.70

Table 5 *continued*

Study (First Author)	ES	SE	95% CI		Research Findings	Source of Assessment
			Lower	Upper		
Booth (2008)	0.20	0.28	0.05	1.19	Providing accurate visual representations of the magnitudes of addends and sums increased children's computational skills.	Wide Range Achievement Test–Expanded (WRAT–Expanded)
Csikos (2012)	0.62	0.13	0.36	0.88	Presenting word problems with different types of visualization (e.g., arrows) improved techniques of problem solving.	Test items adopted from National Core Curriculum, Cronbach's alpha = 0.83
Gamo (2010)	0.61	0.14	0.34	0.91	Mapping data into graphical representations helped students with problems involving fractions.	Researcher designed
Terwel (2009)	0.41	0.13	0.36	0.88	Having students learn to design representations helped them bring more model-based knowledge to the structure of mathematics problems.	Researcher developed criteria, Cronbach's alpha = 0.76.
Casey (2008)	2.00	0.31	0.38	2.63	Representing geometry concepts in a story context improved math knowledge retention.	Used Kaufman-Assessment Battery for Children (K-ABC; Kaufman & Kaufman, 1983)
Jitendra (2007)	1.36	0.22	-0.12	1.07	Addition and subtraction; used graphics to support multiple representations.	Used Pennsylvania System of School Assessment math test
Fuchs (2004)	0.22	0.19	0.26	0.99	Applied schema for problem solving improved students' algorithmic outcomes.	Researcher developed
Saxe (2005)	0.33	0.22	0.18	1.07	Percent: representing fraction with standard part-to-whole representations.	Researcher developed

Table 5 *continued*

Study (First Author)	ES	SE	95% CI		Research Findings	Source of Assessment
			Lower	Upper		
Fujimura (2001)	0.71	0.29	0.05	1.20	Highlighting the idea physical units in setting the proportions improved students' conceptual understanding.	Researcher developed; interrater agreement 97% (N = 76)

Note. ES = effect size, SE = standard error.

Calculated confidence intervals (CIs) for each effect size revealed that nine of the effect sizes fell within 95% confidence intervals. Homogeneity of the studies was verified by calculating the Q value and evaluating its statistical significance; Q = 40.86, *df* =12, p < 0.001 showed that the variability across the effect sizes was greater than expected from the sampling error. The researchers used Statistical Package for the Social Sciences (SPSS) software to visualize the position of the effect sizes as well as the confidence intervals for each study around the computed overall mean effect size of the pool of studies. Some of the means (see Figure 2) showed to be outside of the area of the funnel graph that was earlier anticipated by a statistical significance of the Q value.

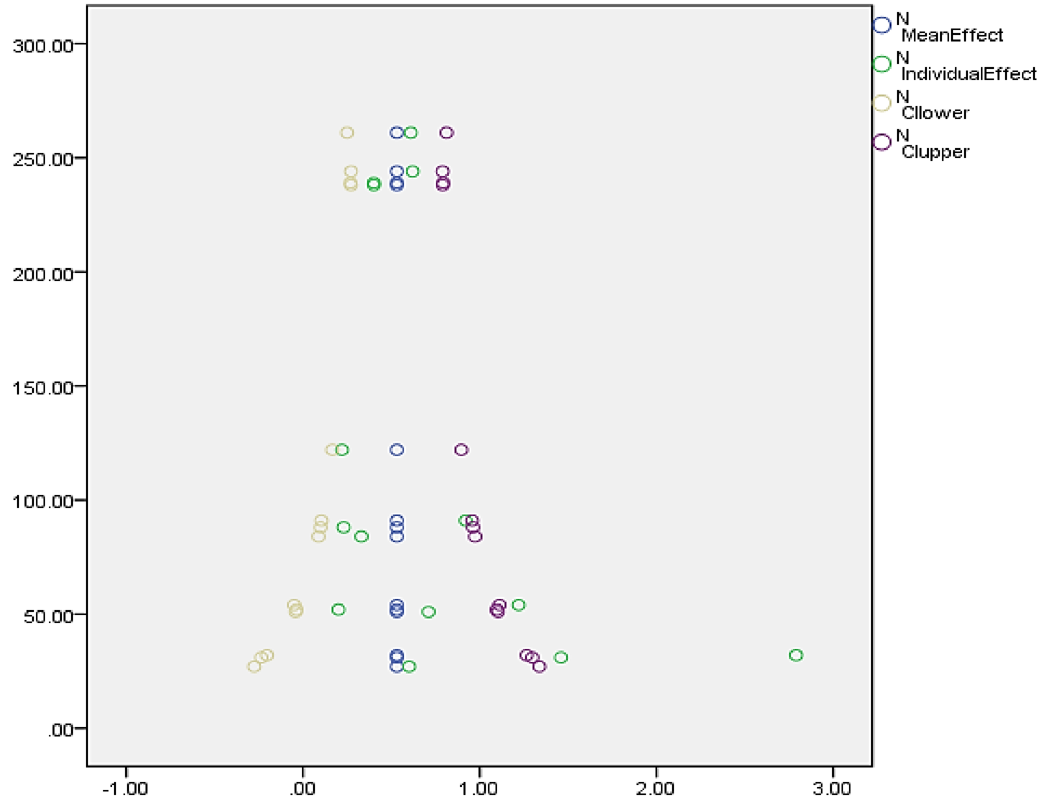

Figure 2. Funnel graph for the pretest-posttest experimental studies.

The individual effect sizes of some of the studies showed to be outside of the confidence intervals indicating a lack of homogeneity of distributions within the pool. This was also depicted by the significant *p*-value ($p < 0.001$). As the purpose of a meta-analytic study is to compute effect size (Willson, 1983), the lack of homogeneity does not undermine the validity of the calculated mean effect; rather, it explicates the characteristics of the studies, indicating that some of them originated from different distributions.

The highest learning effect size (ES = 2.00) was generated in a study conducted with kindergarten pupils who were exploring the creation of verbal representations of geometry concepts (Casey et al., 2008). This study revealed that immersing math concepts in an environment that students can relate to their experiences and fantasies and letting students explore the links makes the math concepts tangible and results in them being easily stored in their long-term memories. Another study with a high effect size (ES = 1.22), conducted by Poland et al. (2009), investigated the impact of dynamic representations on kindergarten students' math achievement. Dynamic representations provided more opportunities for having the learners explore their structures, thus generating a higher engagement factor and consequently higher learning effects. A positive learning effect of students' explorations was also advocated by Lesh and Harel (2003), who concluded that such situated learning enhances the processes of mathematical modeling that play a vital role in developing students' scientific curiosity and their problem-solving skills in high school and college.

Analysis of Moderator Effects

Experimental pretest-posttest studies were aggregated into subgroups to provide opportunities for computing possible moderators that reflected the research objectives. Where applicable, the levels within the subgroups were contrasted and inferences on differences were made. The following criteria were applied to formulate subgroups and calculate their relative effect sizes.

Treatment length. The treatment length classification followed a partition established by Xin and Jitendra (1999): (a) short—less than 1 week; (b) intermediate—between 1 week and 1 month; and (c) long—more than 1 month.

Mode of representation induction in the lesson cycle. This category followed operational roles of representations and contained two levels: concept introduction and problem solving.

Grade level. Large range of grades was comprised into two levels according to standard classification (NCTM, 2000). The lower group level encompassed all students from Pre-K to Grade 3, and the upper level included Grades 4 and 5.

Content standards. This subgroup reflected general standards mediated in the studies: number and operations, proportions, and geometry. The summary of the weighted effect sizes is presented in Table 6.

Table 6

Summary of Subgroups' Weighted Effect Sizes

Variable and Class	N	ES	SE	95 % CI	
				Lower	Upper
Grade Level					
Lower: Pre-K through 3	6	0.60	0.08	0.45	0.76
Upper: 4-5	7	0.47	0.07	0.33	0.60
Representation Type					
Pictorial	6	0.45	0.06	0.32	0.57
Schemata based	5	0.49	0.09	0.31	0.67
Concrete	1	0.71	0.29	0.05	1.20
Other	1	2.00	0.31	1.38	2.63

Table 6 *continued*

Variable and Class	N	ES	SE	95 % CI	
				Lower	Upper
Treatment Length					
Short	5	0.46	0.07	0.31	0.61
Intermediate	4	0.53	0.10	0.33	0.72
Long	4	0.60	0.10	0.40	0.80
Content Standard					
Numbers and operations	10	0.45	0.06	0.34	0.56
Geometry	2	1.61	0.22	0.17	0.24
Ratio and proportions	1	0.71	0.29	0.05	0.20
Mode of Induction in the Lesson					
Concept introduction	7	0.68	0.07	0.54	0.82
Concept applications	6	0.49	0.08	0.34	0.64

Note N = number of participants, ES = effect size, SE = standard error.

The subgroup effect sizes provided a basis for answering more research questions. When compared by grade level, the effect of using representations was higher in Pre-K through Grade 3 than in Grades 4-5. This conclusion might be rooted in the fact that as students progress with learning math concepts, they learn more abstract semantics that might be difficult to embody in representations, for instance, the idea of dividing. Students can observe the initial and the final stage of the process, but the diversity of the means of dividing that is embodied by the syntax of division along with the various rational number representations might not be fully comprehended by young learners and not fully diversified by teachers. As Mosely and colleagues (2008) noted, teachers' preparation and flexibility to deliver the content plays a significant role in student achievement.

When mediated by the type of representation, *concrete* and *others* produced the highest effect size; yet, their significance as a result of meta-analytic procedures could not be fully apprehended since each subgroup was represented by a single primary study. When pictorial representations (ES = 0.49) and schemata-based representations (ES = 0.45) were contrasted, schemata-based representations showed a higher impact on student learning, which supports scholastic research (e.g., Jitendra et al., 2007; Terwel et al., 2009; Xin et al., 2011). Schemata-based representations are often embedded to support the process of problem solving. Used in this regard, they do help students with solving word problems. An interesting, linear relation was observed when effect sizes were contrasted with treatment lengths. It became apparent from this comparison that the longer the treatment, the higher the effect size (ES = 0.46 for short treatments, ES = 0.53 for intermediate, and ES = 0.60 for long). This result provides support for applying representations in classes on a daily basis. In regards to content standards, geometry concept representations received the higher effect size (ES = 1.61). This result reflects the visual nature of content of this branch of mathematics, which by virtue is rooted in representations. The concluding subgroup provided an answer to how representations help with concept understanding and concept applications. It is apparent that representations help more with concept introduction (ES = 0.69) than problem solving (ES = 0.49). As was shown in the comparative studies, for example, Coquin-Viennot and Moreau (2003), once introduced to representations, students apply them successfully in new situations. Thus, one could conclude that supporting concept introduction with representations builds a strong network of impulses in students' long-term memory.

Synthesis of Comparative Research Findings

In order to provide a more complete picture of how representations affect math knowledge acquisition in PreK-5, an analysis of comparative studies conducted between 2000 and 2012 was included. In all of the studies, the construct under investigation was the effect of using representations on students' conceptual understanding of mathematics ideas and their computational skills in Pre-K through fifth-grade math classes. Table 7 summarizes the qualitative research findings of this pool of studies.

Table 7

Synthesis of Comparative Study Findings

Study (First Author)	Research Findings/Recommendations
Deliyianni (2009)	First-grade students have the ability to obey and apply the didactical contract rule to supply their graphical representations to solve problems.
Castle (2007)	More emphasis should be given to meanings of measurements along with conservations on numbers and length.
Coquin-Viennot (2007)	More emphasis should be given to teaching students to identify correct mathematical representation of problem modeling.
Coquin-Viennot (2003)	Students chose schemas to solve problems if the schemas are available to them. Yet, teachers should avoid moving too quickly from the text to problem models.
Yuzawa (2005)	Children should learn diverse ways of comparing areas (e.g., adjusting sizes). This will improve their problem-solving strategies and appreciation for math sophistication.
David (2012)	Representing (drawing) figures should be the subject of class activities. Letting students explore diverse ways of area calculations increases their motivations.

Table 7 *continued*

Study (First Author)	Research Findings/Recommendations
Rittle-Johnson (2001)	Experimentally manipulating students' correct problem representations improves their procedural knowledge and conceptual knowledge that develop iteratively.
McNeil (2004)	Correctly encoded problem structure representation (equation or evaluation) affects students' choice of applied action to solve the problem.
Moseley (2008)	More emphasis should be given to providing students with multiple representations of rational numbers.

The objectives of the majority of the comparative studies were to determine how students' ability to use representations helped them with understanding of math concepts when compared across various grades or ability levels. Cognitively, the studies could be categorized into two major groups: those that investigated the development of geometry (e.g., Castle & Needham, 2007; David & Tomaz, 2012), and those that investigated the use of schemas in problem solving (e.g., Deliyianni et al., 2009). Implementing representations to enhance teaching of these two domains along with a discussion of instructional support emerged as themes for a further discussion that follows.

Schemata and Solving Problems

Several researchers concluded that once children are exposed to certain representations— for instance, schematic representations to solve problems—they retain those methods and apply the schemas in their next math courses (Coquin-Viennot & Moreau, 2003). Thus, one can infer that the schematic representations are accessible to children's realities and that possessing the internal representations of problem-solving schemata seems to appeal to students. Yet, as some scholars noted (e.g., Castle & Needham, 2007), this idea cannot be overemphasized: children also need some *working space* to analyze problems and devise their own ways to solve the problems with the *support* of provided schemata. Thus, schemata should be perceived as suggestions for mathematization of certain patterns, not as fixed formulas to use. It seems that more research should focus on the type of inquiry methods that students should apply to determine the principles embedded in a given word problem.

Carpenter and Moser (1984) proposed four semantic categories for arithmetical operations: *change, combine, compare,* and *equalize.* It is apparent that applying these schemas to model story problems allows certain flexibility. For example, in some cases *equalize* can be perceived as *compare*, or *compare* can include *combine*. There can be cases when two or more schemata can be used in succession. For example, in order to *compare* items or properties, students might need to *combine* them first. Thus, certain degree of flexibility in applying these schemas should be allowed. However, it seems that the primary meaning of each schema should be consistently executed to allow solidifications of these meanings in the learners' long-term memories.

Jitendra and colleagues (2007) proposed the following word problem: *Music Mania sold 56 CDs last week. It sold 29 fewer CDs last week than this week. How many CDs did it sell this week?* This problem was intended to support the schema of *compare.* There is a merit of using *compare* in this problem, but is the schema *compare* the most representative to mathematize the process of selling the CDs? Since the problem involves two events happening at two different time instants referring to congruent objects, then can the learner be directed to finding rather the difference?

Thus, would *change* better describe the process and elicit its solution? It seems that referring students to *compare* gears their thinking toward the output of the problem, not toward the process or principle that was the cause of reaching the process output. By directing students' attention to the problem output, the phase of problem analysis is significantly reduced. Considering the definition of change as *Change = Final value – Initial value,* and solving for *Change,* one will receive *Change = This week's sales - Last week's sales.* Substituting the given values results in *29 = This week's sales – 56*, which leads further to *This week's sales* to be 85 CD. With the implementation of *change,* the representation involved negative numbers that perhaps were not intended in Jitendra's study. Thus to further discuss applicability of this problem to Grade 3 math curriculum, the problem needs to be redesigned. Another example, discussed by Marshall (1995), illustrates how the schema *change* is proposed to be induced (see Figure 3). The idea of using *change* is proposed to solve the following word problem: *Jane had 4 video games. Then her mother gave her 3 video games for her birthday. Jane now has 7 video games.*

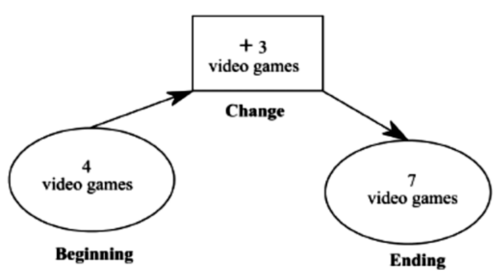

Figure 3. Example representing *change* proposed by Marshall (1995).

Change in quantity values is concluded by subtracting the initial value from the final value: *Change = Final Value – Initial Value*. This standard definition of quantity change is applied not only in mathematics, for example, to constitute part of computing the instantaneous or average rate of change (e.g., Stewart, 2006) but also in physics to compute a change of objects' temperature or an object's velocity (e.g., Giancoli, 2005). This equation can be rearranged to *Beginning + Change = Ending* to reflect Marshal's idea but the rearranged form is not aligned with the fundamental principle that the schemata of *change* supports. If the schema of *change* were to be used, then the diagram should be redesigned to reflect the difference in the quantity magnitudes.

These two examples were brought up to signify a need for verifying interdisciplinary consistency of the schemata interpretations and their adherence to the principle definitions. It is understood that the equations symbolizing the schemata can be rearranged and used in multiple ways. It also seems that with every algorithm done on them, the interpretations take different meanings, for example, $x + y = 5$ represents *compare,* whereas $x - 5 = -y$ would rather represent *change*. Thus, what stage is being used will depend on individual perception, yet general foundations for problem analysis must remain consistent. Perhaps establishing fewer such schemata and letting students manipulate them to reflect on a given problem would benefit the learner more? As it was mentioned, applications of *change, combine, compare,* and *equalize* are very fundamental in sciences, thus understanding their core meanings might have a profound impact on students' success not only in problem solving at elementary level but also at higher levels beyond math classroom boundary.

Providing students with well-designed problems that will correspond with their level of experiences would help with inducing the schemata's core meanings. Furthermore, zooming into the processes of students' reasoning and learning how they interpret the schemata would help with correlating schemata meanings with students' thinking processes and identify areas of strengthening the understandings. Lee and Ginsburg (2009) have proven that removing acquired misconceptions is a more complicated task than learning new concepts. It is hoped that the suggestions would prevent the misconceptions to occur.

Representations of Geometry Concepts

Another construct examined in the comparative studies was representations of geometry concepts. Yuzawa et al. (2005) pointed out that geometry investigations should be organized deductively: "educators should pay attention first to children learning the general shape and strategy and then progress into more detailed representational analysis" (p. 251). The idea of having students explore geometrical concepts as a means of effective learning was also investigated by Castle and Needham (2007), who highlighted a positive effect of letting students explore learned methods and techniques in and outside of school using diverse representations. David and Tomaz (2012) highlighted the positive effect of exploration on student achievement and concluded that letting students explore different paths of solving problems via producing diverse figure representations engages the students in the learning process, which in turn generates their higher intellectual effort to understand and solve problems.

These inferences bring to light the teacher's role in guiding new learners in their discoveries of mathematical relations and patterns.

Using exploratory environments in which representations can be applied as a medium of learning is yet another suggestion for organizing productive learning that emerged from this pool of research.

The Effects of Teacher Support

Deliyianni and colleagues (2009) investigated the impact of using different problem representations (informational pictures, decorative pictures, verbal forms, and number lines) on students' ability to solve problems in different grade levels. They noted that children in all age groups selected decorative pictures and verbal descriptions to work on and that the usage of these representations yielded the highest learning effect size; in contrast, informative representations, rather formal in form, recorded the lowest effect size. The rates proportionally increased with students' age. Fraction representation techniques in the function of experimental manipulation were investigated by Rittle-Johnson et al. (2001). They concluded that teaching concept understanding is not sufficient for the domain understanding and that a substantive procedural knowledge along with teacher's support must be delivered to students. Regardless of student age, "Children who received representational support made greater gains in procedural knowledge" (Rittle-Johnson et al., 2001, p. 360). David and Tomaz (2012) also noted that the teacher needs to take the role of a guider during such activities and redirect students' thinking if needed. It is important that the teacher support and enhance students' selection of the most suitable representation to solve a given problem; letting students explore the representations without guidance might not generate desirable results. Another important factor for the study analysis is the content selection and wording used to formulate the problems. Mathematics is to develop students' concise thinking, but to achieve that it needs to reflect on daily life problems whose contents need to be adequate to students' experiences. Solving artificially created problems might be discouraging for students and consequently it may disconnect mathematics from reality.

Study Limitations and General Recommendations

The findings of this study support the study's hypothesis: representations help Pre-K through fifth-grade students learn and apply abstract math concepts, especially when such representations are applied to supporting new concept understanding and students' problem-solving skills. However, certain limitations and recommendations emerged from this study, as discussed below.

Threats to Research Validity

The main parameter limiting the study finding was a lower-than-expected pool of primary studies to be meta-analyzed. Still, the authors believe that the inclusion of comparative studies enhanced the study inferences. The validity of the study computations was supported by double research data coding at the initial and concluding stages of the study process. Any potential discrepancies were resolved. Although strictly specified, the literature search was undertaken with broader conceptual definitions in mind that allowed for, as suggested by Cooper (2010), adjustment of the definitions and strengthening of the literature relevance. Thus, as the initial literature search revealed that representations in Pre-K through Grade 5 are often used to support problem solving, the term *problem solving* was then used to locate more studies.

Recommendations for Further Research

Schemata-based representations and their application emerged as the main type of representations to support problem solving. According to Owen and Sweller (1985), a schema is a cognitive general structure that allows the problem solver to categorize the problem and then apply certain tools to solve it. A moderate effect size (ES = 0.49) indicates that this learning strategy helps students understand underlying math ideas in given word problems and solve them. Hiebert and Carpenter (1992) posited that in the process of developing the schemas, students' domain conceptual understanding consists of a complex network of concepts. Furthermore, the networks constitute the model that will be called an internal representation of the domain embodied by an external representation. The learners can be provided with the representations, or the representations can be derived by the learners under a teachers' guidance, and then the conceptual networks can be developed.

Cheng (1999) proposed four learning stages that can lead the learner to developing concept understanding through using external representation: domain, external representation, concept, and internal network of concepts. In the process of moving from one stage to another to reach the internal network, the learner is immersed in four processes: observation, modeling, acquisition, and integration. With the exception of the studies conducted by Rittle-Johnson and collegues (2001) and Terwel and colleagues (2009), the majority of the gathered pool of studies did not explicate on these processes, focusing instead on applying fixed models without discussing and possibly conditioning them.

As Perkins and Unger (1994) noted, having the right representation does not suffice for having an understanding. To confirm an understanding, one needs to be able to “put this representation through its paces, explaining and predicting novel cases” (p. 45). Thus, to have an understanding of a representation is to be in a state of readiness, taking the representation as a point of departure in the solution process, not as an unquestionable formula. Terwel and colleagues (2009) proved that having students explore and modify given representations produced the highest effect size. We would support this success by inducted math modeling phase that allowed the students to explore and adopt representation to given real scenarios. While this interpretation is hypothetical, the particular finding needs further investigation to be legitimately explained.

Having students develop principles of representations by identifying commonalities due to applications and then having them apply such representations to model other contexts beyond the math classroom would be a valid pursuit for future studies. Other question worth further investigation is: If students are to be placed in the role of mathematicians applying the schemata to solve other problems, should the processes be organized inductively, as suggested by Nunokawa (2005), or deductively? How do students perceive these inquires? Are these inquiring rooted in virtues of mathematical representations or are they rather content-domain related?

Related questions generated by this study and suggested for examination in future studies are the following: How does the use of representations continue as students' progress in math education? How are representations, especially schemata-based representations that dominate problem solving in Pre-K through Grade 5, linked to higher-level math classes?

After generating a positive effect size in a study with students with learning disabilities, Fush and colleagues (2004) suggested using schemata more extensively for problem solving at the high school level, especially targeting students will learning disabilities. Having high school students derive processes of transitioning from proportion to a linear or rational function or from percent rate to an exponential function seem like valuable research topics to explore further.

Another conclusion calls for expanding the idea of using schemata to sciences and other subjects in a consistent manner that will carry out their general principles. This transition will help students broaden the meanings and consequently built a stronger image of these schemata in students' long term memories. Do students experience applying similar representations in their science classes? It seems that applying, for example, the schema of *equalize* to verifying the law of conservation of mass or energy would enrich the spectrum of the schema applications and induce more contextual meaning. Should these main avenues of knowledge acquisition depend on the nature of the representation (schemata or pictorial) or their general purpose? Further research in these regards is needed, and we believe that this paper will provide some prompts for its initiations.

CHAPTER III

USING COMPUTERS TO SUPPORT EXPLORATORY LEARNING ENVIRONMENTS IN GRADE 1-8 MATHEMATICS: A META-ANALYSIS OF RESEARCH

Introduction

The process of solving word problems is difficult for students; thus, mathematics educators have made multiple attempts to seek ways of making this process more accessible to students. The purpose of the study was to contribute to this research by examining the effect size statistic of utilizing exploratory computerized environments (ECEs) to support the process of word problems solving and explorations in Grade 1-8 mathematics. The findings of 19 experimental pretest and posttest studies (19 primary effect sizes) published in peer-reviewed journals between January 1, 2000 and January 31, 2013 revealed that exploratory computerized environments produced a moderate (ES = 0.60) effect size (SE = 0.03) when compared to traditional methods of instruction. A 95% confidence interval around the overall mean—C_{lower} = 0.53 and C_{upper} = 0.66—proved its statistical significance along with its relative precision. A further moderator analysis revealed differences among the effects of students' achievement between problem solving and ECEs favoring the latter. Discussion of these results and their potential impact on improving students' mathematical problem solving skills along with implications for further research is also undertaken in this study.

Advancement in capabilities of varied technologies caused that school practice problem solving, traditionally difficult for students, has become a domain of a particular interest. Whereas researchers have examined the use and impact of computers on presenting the content of word problems to the learners, comparatively little research has focused on using computers as a means to have the learners explore dependence of a given problem variables in the attempt to mathematize and solve it. While interest in improving students' problem-solving solving skills has have a wide range, the rate of the progress in this domain has not been satisfactory (Kim & Hannafin, 2011). High interactivity of contemporary computer programs that allow for dynamizing problem contents and consequently inducing and exercising elements of scientific processes such as isolating parameters of a system and depicting the system changes that result from varying the parameters are not utilized in fully in mathematics classroom yet. Although some elements of scientific inquiry, such as measurements and data analysis, have received a substantial attention in the newly developed common core standards (Porter, McMaken, Hwang, & Yang, 2011), the process of inquiry organization in mathematics classes, for example, the process of mathematical explorations or problem solving, are not discussed. Research (Grouws & Cebulla, 2000) suggests that students who develop math conceptual understanding are able to perform successfully on problem solving, even these requiring a task transfer. Capabilities of modern technology open multiple opportunities for applying mathematical structures to quantify system changes and simultaneously help with understanding of math concept applications. It is hypothesized that by enriching mathematics problem solving processes by phases of scientific inquiry such as; hypothesis stating, analysis and model formulation, the process of conceptualization of abstract math ideas and their applications in real world will be more accessible to students. Consequently, the shift from procedural to conceptual teaching methods in mathematics might be initiated. The current research in mathematics education encompasses many aspects of using technology, yet exploratory computerized environments (ECEs) focusing especially on supporting mathematical explorations, problem solving, and mathematical modeling has many commonalities with scientific discovery and scientific inquiry. From the three avenues: explorations, problem solving and mathematical modeling, the processes of explorations dominate the elementary and middle school math curricula at the current research. A formal process of explorations—mathematical modeling—is more frequently applied at the high school and college levels (Blum & Booker, 1998; English, 2004). Although, problem solving can integrate scientific methods, this idea does not mediate in the current research. Historically, a major contribution to the field of problem solving was done by Polya's (1957) who codified four stages of problem solving processes as understanding the problem, devising a plan, carrying out the plan, and looking back. Bransford and Stein (1984) extended Polya's approach by developing 5-stage problem solving model which encompassed identifying problem, defining goals, exploring possible stages, anticipating outcomes, and looking back and learning. To varying extents, these stages represent integral elements of contemporary problem solving methods (Kim & Hannafin, 2011). As in science classes, technology rich inquiry has proven to help students with problem-solving techniques (e.g., see Reid, Zhang, & Chen, 2003; Stern, Barnea, & Shauli, 2008) searching for ways of inducing congruent ideas to a math classroom appeared to be promising undertaking worth of examining.

The purpose of this study was to synthesize, using meta-analytic techniques, current research on applying such learning environments, often called exploratory (Remillard & Bryans, 2004), in school practice at the elementary and middle school levels.

Technology as a Means of Supporting Explorations and Word Problem Solving

The following section provides a theoretical framework that has guided this study. It summarizes the advantages of utilizing technology in mathematics school practice focusing on how technology is used to enhance explorations and word problems solutions. Since the far reaching goal of this study is to search for means of improving students' math achievement on problem solving, this section also discusses the role of competencies associated with explorations and word problems solving in math learning. According to the National Council of Teachers of Mathematics (NCTM; 2000), "Technology is essential in learning mathematics" (p. 3). Applying technology to enhance students' problem-solving skills is an intermediate area of interest. A problem's setup and information component expressed in word format are often difficult for students to comprehend, analyze, and solve. Such presented problems also have a low motivational factor, which consequently affects the degree to which a learner engages in finding the solution. The advancement of multimedia technology has opened new possibilities for dynamically expressing a problem's contents and extending its analysis. The process can now be externalized and magnified through digital constructions, showing more explicit properties and structures that were previously silent. Several researchers, for instance, Chen (2010) and Merrill and Gilbert (2008), have found that students' word-problem-solving skills can be significantly enhanced through the integration of computer technologies. Embodied by tangible representations, such presented problem-solving scenarios are more realistic and are thus more meaningful to students.

While the engaging factors of computerized environments on students' motivation have been widely documented (see, for example, Lewis, Stoney, & Wild, 1998), their interactive features that enable the learner to hypothesize, make predictions, and verify those predictions have not yet been meta-analyzed at the elementary and middle school levels. This study sought to examine these areas and identify moderators that contribute to increasing the learning effects. As a result of this undertaking, we hope to formulate conditions for learning environment design that will advance students analytic skills and consequently improve their problem solving techniques at the elementary, middle school level and beyond. We hope that through our research findings, the math research community will be encouraged to support curricula whose notion is to propagate the idea of unified math-science problem solving techniques.

The Role of Explorations and Word Problems in Math Learning

Explorations. The processes of explorations, data interpretation, and validation are closely related to the level of mathematical modeling that has traditionally been reserved for secondary schools (Blum & Booker, 1998). However, a recent study (English & Watters, 2004) shows that young children are capable of analyzing situations beyond of those involving simple processes of counts and measures. Furthermore, researchers (e.g., Lai & White, 2012; English, 2004) recommend that children have more exposure to situations where they explore informal notions of rate, or where they quantify information, transform quantities, and deal with quantities that cannot be seen. Flum and Kaplan (2006) claimed that explorations engage the learner with the environment through definite actions of gathering and investigating information. By inducing the use of terms that are central to scientific inquiry, like *observe*, *identify*, and *analyze* (Slough & Rupley, 2010), explorations promote the transfer of knowledge, problem-solving skills, and scientific reasoning (Kuhn, 2007). Furthermore, Schwarz and White (2005) advocate that learning about the nature of scientific models and engaging the learners in the process of creating and testing models should be a central focus of science education. Thus, enhancing these processes in mathematics classes by the development of modeling processes and knowledge acquired through these processes may simultaneously facilitate the learning of science.

Explorations can be externalized in various forms. One of these forms, computerized simulations, offers great promise for providing a rich medium for learning. Grouws and Cebulla (2000) suggested that students who develop scientific inquiry are able to successfully solve problems; thus, it is hypothesized that by enriching math curriculum via elements of such inquiry, presented, for example, by mathematical explorations, students' problem-solving skills can be strengthened. Consequently, a shift from procedural to conceptual teaching and learning methods in mathematics might be initiated. These shifts posit certain challenges.

At the elementary school level, manipulatives have been extensively used to help build conceptual understanding of abstract ideas (Jitendra et al., 2007), as they are often replicates of real manipulatives accessed through computer software or the Internet. Research (Kaput, 1991; Kieran & Hillel, 1990) has proven their positive impact on students' math achievement. Since manipulatives are restricted to geometrical objects, exploratory learning environments provide a far richer context for inducing mathematical ideas. As a result, their applications in math classrooms have gained momentum over the past decades (Neves, Silva, & Teodoro, 2011); thus, there is a need for a more systematic way of using these environments.

The process of explorations usually concludes with a formulation of a mathematical model. As such, multifaceted cognitive goals are achieved by learners while they undertake such activities. Bleich, Ledford, Hawley, Polly, and Orrill (2006) concluded that such activities expand students' views of mathematics by integrating mathematics with other disciplines, especially sciences, and engage students in the process of mathematization of real phenomena. In addition to being able to express a situation using mathematical symbols, explorations help students develop problem-solving skills (NCTM, 2000). Viewed through this prism, interactive exploratory learning environments dominate the previously applied drill-and-practice computer applications in school mathematics classroom.

Word problems. Situations carrying open questions that challenge learners intellectually (Blum & Niss, 1991) are called word problems or story problems. The general structures of word problems are centered on three components: (a) a setup component, which provides the content (for instance, place or story problem); (b) an information component, which provides data to derive a mathematical model; and (c) a question component, which is the main task directed to the solver (Gerofsky, 2004). A setup component of a word problem can be externalized by a static diagram, short video, computer simulation, or physical demonstration. With the exception of static diagrams, all of these means, though not yet commonly used in mathematics classes (Kim & Hannafin, 2011), assist with the visualization of problem scenarios and thus help with identifying patterns and formulating their symbolic description. Word problem solving is one area of mathematics that is particularly difficult because it requires students to analyze content, transfer it into mathematical representations, and map it into learned mathematical structures. Therefore, it requires not only a retrieval of a particular problem-solving model from learners' long-term memory but also the need to create a novel solution (Zheng, Swanson, & Marcoulides, 2011).

According to Polya (1957), solving word problems requires that the solvers immerse themselves in certain phases during which they are to analyze the problem, organize the facts, devise a plan, find the solutions, and validate the results for reasonableness. Among these phases, the phase of exploration, which leads the solver to a model formulation and validation, is of the highest importance (Arthur & Nance, 2007). Once the model is validated, it can be used for forecasts, decisions, or actions determined by the problem-question component. Francisco and Maher (2005) suggested that the stage of modeling must exist in the problem-solving process for authentic mathematical problem solving to occur. A similar conclusion was reached by

Gravemeijer and Doorman (1999), who claimed that "the role of context problems and of symbolizing and modeling are tightly interwoven" (p. 112). Linking the mathematical apparatus with the problem information component, mathematical modeling appears to be one of the critical skills for student success in solving word problems. The forms of the mathematical models depend on the problem content and on elementary and middle school levels; they are often externalized by geometrical objects, ratios, and proportions (NCTM, 2000). Learners' skills of applying mathematical structures to investigate the world outside of the classroom is of highest importance in students' general mathematical deposition because it develops students' confidence in their own ability to think mathematically (Schifter & Fosnot, 1993). For these reasons, the skill of applying math tools is a predominant requirement of mathematics teaching (NCTM, 2000).

Over the past 30 years, the research on teaching and learning math applications has undergone modifications reflecting research advancements in this area, one of which is a change in the instructional approach to problem solving: from teaching problem solving, to teaching *via* problem solving (Lester et al., 1994). Some of the main elements of teaching via problem solving include (a) providing students with enough information to let them establish the background of the problem, (b) encouraging students to make generalizations about the rules or concepts, and (c) reducing teachers' role to providing guidance during the solution process (Evan & Lappan, 1994). According to more recent research about the cognitive process of problem solving, Yimer and Ellerton (2009) proposed an inclusion of a prelude phase, called *engagement,* whose role is to increase students' motivation and, consequently, their success rate.

Despite these changes, problem solving processes still require more research, especially in the area of linking these processes to the process of scientific inquiry that students exercise in their science classes.

Synthesis of Findings of Prior Meta-Analytic Research

The study of problem-solving methods in the domain of mathematics education has been frequently undertaken by researchers and has especially influenced mathematical practices during the past 30 years (Santos-Trigo, 2007). As "problem solving refers to the entire process of dealing with a problem in attempting to solve it" (Blum & Niss, 1991, p. 38), the process challenges learners (Schoenfeld, 1992) because it encompasses several stages such as analysis, pattern extraction, model formulation, and verification, which often are not explicitly elaborated on for the learner. In addition to applying an adequate mathematical apparatus, the solver needs to uncover the principle embedded in the given problem (Jonassen, 1997) neglected in the current research that is often of a science or other content domain.

Computer programs have been recognized as highly powerful tools for the numerical and graphical treatment of mathematical applications and models that assist learners with the problem-solving process (Blum & Niss, 1991). Tall (1986) provided an insightful analysis on how computers can be used for testing mathematical concepts. He claimed that "computer programs can show not only examples of concepts, but also, through dynamic actions; they can show examples of mathematical processes" (p. 5). He questioned the formal approaches to mathematical representations used in textbooks, calling them inaccessible to students, and suggested instead using computer programs to show the dynamics of the processes. Following Skemp's (1971) findings, Tall claimed that building concepts on cognitive principles instead of on the principles of logic teaches students mathematical processes and mathematical thinking.

Computer programs used to support problem solving were one of the moderators in a meta-analysis on methods of instructional improvement in algebra undertaken by Rakes, Valentine, McGatha, and Ronau (2010). Using 82 relevant studies from 1968 and 2008, these researchers extracted five categories, of which two contained technology and computers as a medium supporting instruction and learning. Contrasting procedural and conceptual understanding of mathematics ideas, these scholars found that conceptual understanding as a separate construct, appearing initially in research in 1985, produced the highest effect size when enhanced by computer programs. The timeline of this finding corresponded with the emergence of mathematical explorations, which also exemplify math conceptual understanding. In addition, Rakes et al. found that technology tools including calculators, computer programs, and java applets produced a moderate 0.30 effect size when compared to traditional methods of instruction. Another systematic review of using computer technology and its effects on K-12 students' learning in math classes between 1990 and 2006 was undertaken by Li and Ma (2010). Analyzing the effects of tutorials, communication media, exploratory environments, tools, and programming language, they concluded that exploratory environments, characterized by the constructivist approach, produced the highest (ES = 1.32) learning effect size. Li and Ma did not compute the effects of computer technology on math cognitive domains and type of learning objectives, suggesting a need for another review that would focus on “the nature of the use of technology” (p. 235) on student achievement.

Prior literature has provided many insightful conclusions about the effectiveness of exploratory computer programs on math students’ achievement. However, it has also led to many research questions on how the content delivery methods or problem-solving settings presented by the computer programs will yield the highest learning effect sizes. It seems that the high capability of exploratory computerized environments to provide opportunities for enriched dynamic visualization demands more detailed research in order to better understand how to direct students’ attention to embedded mathematical structures and help them uncover the underlying mathematization of their principles.

As the above literature synthesis illustrates, several concerns regarding the improvement of students’ problem-solving skills in mathematics are still unresolved. For instance, Artzt and Armour-Thomas (1992) concluded that students’ difficulties with problem solving are often attributed to their failure to initiate active monitoring and regulation of their own cognitive processes. It seems that presented with problem content, students face uncertainties about how to proceed through the phases of the solution stages that will lead to the mathematical model formulation. As several potential ways of improving students’ initiation of active monitoring have been already researched (e.g. see Grouws, & Cebulla, 2000; Kapa, 2007), by undertaking this study we hope to uncover moderators that have been silent in the previous research. We are especially interested in learning whether extending the exploration stage of the solution process and guiding students through the phases of the inquiry could materialize as a construct of addressing this issue worth of a further investigation? The effect of such organized support might reduce the working memory capability needs and consequently allow students to attempt to solve the problem without being overwhelmed at the start. In addition, Hart (1996) concluded that students find word problems difficult because they lack motivation. Presenting word problems in an engaging format might increase learners' motivation factor and drive them to solve the problems. Furthermore, providing some guidance during the solution process might improve their productivity and decision-making (Stillman & Galbraith, 1998). Finally, Blum and Niss (1991) expressed their concern that the implementation of ready-made software in applied problem solving may put an unintentional emphasis on routine and recipe-like procedures that neglect essential phases, such as critically analyzing and comparing models. Closely examining how this concern is resolved in newly developed math software was an additional focus of this meta-analysis.

Problem solving mediates with multiple external factors. It seems that the use of ECEs to promote problem solving is a promising avenue, but more research-driven actions are needed. It is hoped that this study's findings will generate directions for such actions.

Research Methods

A literature review can take several venues, for example, narrative, quantitative, or meta-analytic. This study took the form of the latter, using the systematic approach proposed by Glass, (1976) called *meta-analysis*, which can further be described as an analysis of the analyses. A statistical meta-analysis integrates empirical studies, investigating the same outcome described as a mean effect size statistic. Thus meta-analytic techniques were selected for this study because they provide tools to assess effect size considering a pool of studies as a set of outputs collected within prescribed criteria. There are two main advantages of such investigations: (a) a large number of studies that vary substantially can be integrated, and (b) the integration is not influenced by the interpretation or use of the findings by the reviewers (Gijbels, Dochy, Van den Bossche, & Segers, 2005).

The main objective of this study was to assess the impact of computerized exploratory environments on students' mathematics achievement is Grades 1-8.

Key Term Descriptions

Exploratory computerized environment. This is defined as a medium of learning that engages the learner with the environment through definite actions of gathering and investigating information (Flum & Kaplan, 2006). The medium can be displayed on the computer screen or iPod and provided via software or the Internet.

Student achievement in mathematics. Student achievement represents the outcome measure in this study and is defined as scores on solving various mathematical problems presented in various mathematical structures, such as equations, ratios, proportions, and formulas, measured by students' performance on standardized or researcher- or teacher-developed tests expressed as a ratio or percent. Student achievement scores are further expressed as effect size computed using mean posttest scores of experimental and control groups and coupled standard deviation or other statistic parameters as defined by Lipsey and Wilson (2001).

Research Questions

The research questions formulated for this study are divided into two groups; main and supplementary. As the main question reflect on the accumulated literature, supplementary questions are introduced to seek answers to additional inquiries whose goal is to enrich the study objective

Main Research Question

1. What are the magnitude and direction of the effect sizes of using computerized exploratory environments to support the process of problem solving and explorations as compared to conventional learning methods?

Supplementary Research Questions

1. Are the effect sizes of student achievement depending on grade levels or mathematics content domain?
2. Are the effect sizes of student achievement different when problem solving is contrasted with explorations?
3. How does the type of instructional support (teacher guided or student centered) affect student achievement when computers are used?

While the answer to the main question will be based on the interpretation of the magnitude and direction of the computed mean effect size statistic, the answers to the additional research questions will be based on applied moderator analysis and interpretation of computed moderator effects made available through applying rigorous meta-analytic techniques.

Data Collection Criteria and Procedures

Several criteria for literature inclusion in this study were established before the search was initiated. This synthesis intended to analyze and summarize the research published between January 1, 2000, and January 31, 2013, on using computerized programs to support student explorations in elementary and middle school mathematics classes in either public or private schools. The minimum sample size established in this meta-analysis was 10 participants. The study included only experimental research that provided pretest-posttest mean results, standard deviation (SD), F-ratios, t-statistics, or other quantifications necessary for meta-analysis. In the process of collecting the applicable research, ERIC (Ebsco), Educational Full Text (Wilson), Professional Development Collection, and ProQuest Educational Journals, as well as Science Direct, Google Scholar, and other resources available through the university library, were used to identify relevant studies. In the process of extracting the relevant literature, we used the following terms: *explorations*, *simulations*, *computers in mathematics, mathematics education, problem solving, exploratory environment, and student achievement*. This search returned 238 articles, out of which 14 satisfied the established criteria. In order to expand the pool, a further search, undertaken with broader conceptual definitions, including *dynamic investigations, techniques of problem solving*, *and computerized animations and learning*, was conducted. These modifications, which allowed for the adjustment of the contexts and strengthening the relevance of the literature (Cooper, 2010), returned 31 studies. The additional search extracted a number of studies that although very informative (e.g., Chen & Liu, 2007; Harter &Ku, 2008), could not be meta-analyzed because the instrument of computers was used in both the control and experimental groups. After additional scrutiny, five studies were added to the original pool, resulting in 19 primary studies and 19 corresponding effect sizes.

The validity of the study was supported by a double research data rating at the initial and at the concluding stages of the study. Any potential discrepancies were resolved.

Coding Study Features

The coding process was conducted in a two-phase mode reflecting the two-stage analysis. During the first phase, general characteristics of the studies, such as research authors, sample sizes, study dates, research design type, and pretest-posttest scores, were extracted to describe the study features. During the second phase, additional scrutiny took place to more accurately reflect on the stated research questions and seek possible mediators of the effect sizes. Majority of the coding futures, for instance study authors, date of study publication, locale or research design type are utilized to support the study validity. The formulation of other coding, such as grade level, instrumentation or learning type was enacted to apply moderator analysis that will lead to answering additional research questions.

Date of study publication. Despite the fact that computer programs as a medium supporting learning were introduced into education several decades ago (Joyce, Weil, & Calhoun, 2009), a rapid increase in this field occurred around the year 2000, which was selected as the initial timeframe for the search.

Descriptive parameters. Descriptive parameters encompassed the following: the grade level of the group under investigation, the locale where the studies were conducted, the sample size representing the number of participants in experimental and control groups, the date of the study publication, and the time span of the research expressed in a common week metric.

Inferential parameters. Posttests mean scores of experimental and control groups and their corresponding standard deviations were extracted to compute study effect sizes. If these were not provided, F-ratios or t-statistics were recorded. Although most of the studies reported more than one effect size, for example, Kong (2007) and Guven (2012), who also reported on students' change of attitude toward computers, this study focused only on student achievement, thus reporting one effect size per study.

The research authors. A complete list of authors involved in the study completion was compiled in the first tabularization. As the analysis of the study progressed, each research study was labeled by the first author and the year of conduct.

Publication bias. All studies included in this meta-analysis were peer-reviewed and published as journal articles; thus, no additional category in the summaries was created to distinguish the publication mode of the studies. By embracing the research selection in the criteria, publication bias was expected to be reduced.

Group assignment. This categorization was supported by the way the research participants were assigned to treatment and control groups as defined by Shadish, Cook, and Campbell (2002). During the coding process, two main categories emerged: (a) randomized, where the participants were randomly selected and assigned to the treatment and control group; and (b) quasi-experimental, where the participants were assigned by the researchers.

Type of research design. Only experimental studies that provided pretest-posttest means or other statistic parameters representing the means were utilized in this study.

Instrumentation. Two main modes of using a computer as an instrument to promote students' mathematical knowledge acquisition were identified: (a) as a medium supporting problem solving, and (b) as a medium supporting explorations. Additional descriptions of the instrumentations were included if they were provided by the primary researchers.

Type of learning setting. The purpose of implementing this construct was to learn about the effect of the type of instructional support. Two sub-categories were identified: (a) teacher-guided support, where the teacher served as a source of support during student explorations or problem solving; or (b) student-centered support, where support was provided on the computer screen by the software.

Data Analyses

Homogeneity Verification and Summary of Data Characteristics

The data analysis in this study was initially performed using SPSS 21 with verification of homogeneity of the study pool as suggested by Hedges (1992). A standardized mean difference effect size was calculated using posttest means on experimental and control groups as suggested by Lipsey and Wilson (2001). The individual effect sizes were then weighted, and an overall weighted mean effect size of the study pool was calculated. The homogeneity statistics ($Q_T = 117.52$, with $d_f = 18$, $p < 0.01$) showed that the set of effect sizes varied statistically significantly; thus, a random-effect model was adopted for the data analysis. The following funnel plot visualizes the pool mean effect size and displays the confidence intervals and the individual effect sizes. In order to improve the clarity of the graph, the vertical axis scale was compressed by taking a natural logarithm of each sample size.

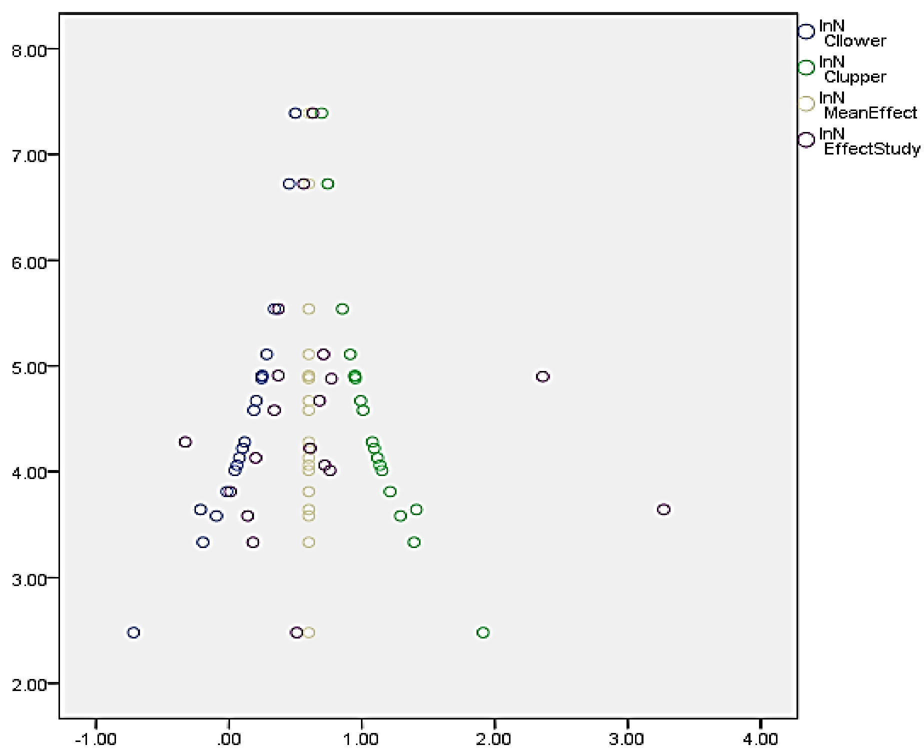

Figure 4. Funnel plot for the data.

The funnel plot (see Figure 4) shows evident outliers in the gathered data (e.g., Huang et al., 2012; Erbas et al., 2011). However, due to the complex study design and valuable research findings, both of these studies were included in the meta-analysis and their contributed through their weighted effect sizes to the overall effect size.

Table 8 summarizes the extracted general characteristics of the studies. The studies were further aggregated into classes to reflect the objectives of the research questions.

Table 8

General Characteristics of the Studies' Features

Authors	Date	Locale	RD	SS	Grade Level	RTL (in ws)	Treatment Approach	Learning Setting
Pilli & Aksu	2013	Cyprus	R	55	4th	12	Explor	TG
Kong	2007	Hong Kong	QE	72	4th	5	Explor	TG
Hwang & Hu	2013	Taiwan	R	58	5th	8	PS	SC
Lai & White	2012	USA	QE	12	6th 7th	1	Explor	SC
Chang, Sung, & Lin	2006	Taiwan	QE	132	5th	6	PS	SC
Erbas & Yenmez	2011	Turkey	QE	134	6th	2	Explor	TG
Roschelle, Shechtman, Tatar, Hegedus, Empson, Knudsen, & Gallagher	2010	USA	R	1621	7th	40	Explor	SC
Roschelle et al.	2010	USA	R	825	8th	80	Explor	SC

Table 8 *continued*

Authors	Date	Locale	RD	SS	Grade Level	RTL (in ws)	Treatment Approach	Learning Setting
Kapa	2007	Israel	R	107	8th	8	PS	SC
Papadopoulosa & Dagdilelis	2008	Greece	QE	98	5th 6th	4	PS	SC
Eid	2004	Kuwait	QE	62	5th	1	PS	SC
Huang, Liu, & Chang	2012	Taiwan	QE	28	2nd 3rd	1	PS	SC
Lan, Sung, Tan, Lin, & Chang	2010	Taiwan	R	28	4th	4	PS	SC
Van Loon-Hillen, van Goga, & Gruwel	2012	The Nether-lands	QE	45	4th	3	PS	TG
Guven	2012	Turkey	QE	68	8th	40	Explor	SC
Chen & Liu	2007	Taiwan	QE	165	4th	4	PS	TG
Ku & Sullivan	2002	Taiwan	QE	136	4th	1	PS	SC
Suh & Moyer-Packenham	2007	USA	QE	36	3th	1	PS	SC
Panaoura	2012	Cyprus	QE	255	5th	8	PS	SC

Note. R = Randomized, QE = Quasi-Experimental, RD = research design, Explor = Explorations, SS = sample size, RTL = research time length, ws = weeks, SC = student centered, TG = teacher guided, PS = problem solving.

The majority of the studies (12, or 63%) were conducted quasi-experimentally, while the remaining seven (37%) were randomized. The study duration was expressed in a common (weeks) metric scale, although some of the studies reported the duration in months or by semesters. The duration of experimental treatment usually lasted for one unit lesson (45 minutes) with a frequency of application twice a week. The highest sample size of 1,621 students was reported for a study conducted by Roschelle et al. (2010), and the lowest sample size of 12 participants was reported by Lai and White (2012). While analyzing the pool from a grade-level point of view, students whose primary level was Grade 4 (42%) dominated the pool. Since this study focused on gathering research on exploratory environments provided by computer programs or the Internet, the examined studies were aggregated by their focus on either supporting problem solving or explorations in mathematics. The study-highlighted characteristics were further aggregated.

Descriptive Analysis

The analysis of the data was organized deductively. It began with a synthesis of the general features of the studies, encompassed by a descriptive analysis, and then moved to an examination of the differences of the effect sizes mediated by the type of instrumentation, cognitive domain, study duration, grade level, and content domain.

The research pool generated data collected from 3,682 elementary and middle school students. The average sample size was 202 participants. Applied descriptive analysis provided information about the frequencies of the studies per year (see Figure 5) and the locale distribution where the studies were conducted (see Figure 6).

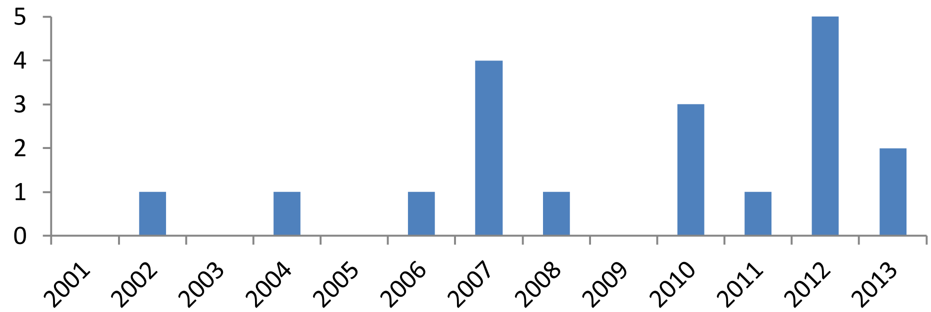

Figure 5. Distribution of studies per date of publication.

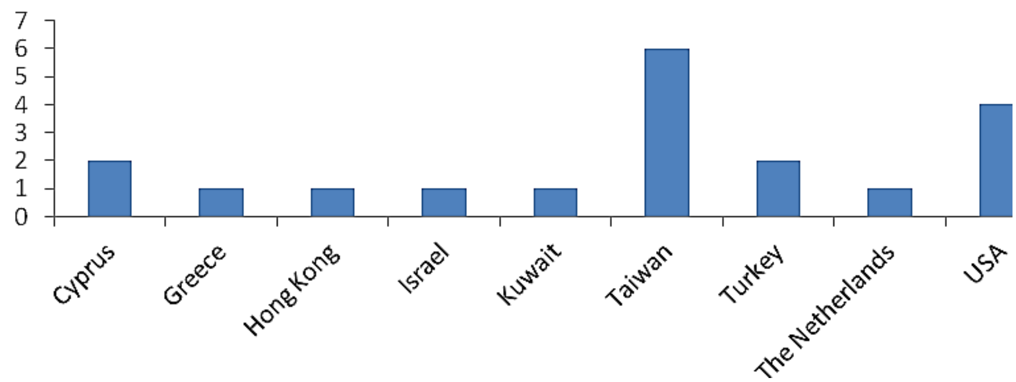

Figure 6. Distribution of studies per locale.

The majority of the studies (14, or 74%) were conducted within the past 5 years, which indicates a growing interest is using computerized programs to support the learning of mathematics. In terms of research location, Taiwan dominated the pool with six studies (32%), followed by the United States with three studies (10%). It is to note that applying and investigating the effects of ECEs in mathematics classrooms has accumulated a global interest.

Inferential Analysis

Quantitative inferential analysis was performed on the primary studies to find the weighted effect size of each study and the mean weighted effect size of the study pools. The mean effect size for the 19 primary studies (19 effect sizes) was reported to have a magnitude of 0.60 (SE = 0.03) and in the positive direction. A 95% confidence interval around the overall mean—C_{lower} = 0.53 and C_{upper} = 0.66—supported its statistical significance and its relative precision as defined by Hunter and Schmidt (1990). The magnitude of the mean effect size statistics was ES = 0.60. Such effect magnitude along with a positive direction is described by Lipsey and Wilson (2001) as being of a medium size. When applied to school practice, it indicated that the score of an average student in the experimental groups, who learned using ECEs, was 0.60 of standard deviation above the score of an average student in the control groups, who was taught using traditional methods of instruction. Closer examination of the computed effect size and incorporation of the U_3 Effect Size Matrix (Cooper, 2010) led to the conclusion that the average pupil who learned mathematical structures using exploratory environments scored higher on unit tests than 70% of students who learned the same concepts using traditional textbook materials. It can thus be deduced that using exploratory environments as a medium of support in the teaching of mathematics has a profound impact on students' math concept understanding when compared to conventional methods of teaching. Table 9 provides a summary of the individual effect sizes of the meta-analyzed studies along with their confidence intervals, qualitative research findings, and the computer programs used as the instruments.

The table also contains reliability of measures used to compute the individual mean scores, expressed by indicating whether the test was researcher developed or standardized. For studies where it was available, Cronbach's alpha (α) is also listed. Finally, the table contains additional information provided by the primary researchers that distinguish the given study within the pool.

Table 9

Effect Sizes of Using ECE in Grade 1-8 Mathematics

Study (First Author)	ES	SE	95% CI		Reliability of Measure	Program Used, Research Findings, Research Specifications
			Lower	Upper		
Pilli (2013)	0.76	0.24	0.05	1.09	Researcher developed, Cronbach's α = 0.9	Used Frizbi Math 4. Explored arithmetic operations.
Kong (2007)	-0.33	0.27	0.12	1.15	Teacher developed	Used Graphical Partitioning Model (GPM). Interface not appealing. Fraction operations were explored.
Hwang (2013)	0.72	0.59	0.07	1.93	Researcher developed	Used virtual manipulative and 3D objects. Investigated the effect of peer learning.
Lai (2012)	0.51	0.18	0.71	0.97	California Math Standard Test	Used NeoGeo. Interactive environment helped make the applications meaningful. Investigated a peer effect.
Chang (2007)	0.77	0.18	0.26	0.96	Researcher developed	Used schemata-developed problem solving. Provided teachers guidance, and helped with stage understanding.
Erbas (2011)	2.36	0.05	0.26	0.71	Researcher developed	Used dynamic geometry environment (DGE). Dynamic environment contextualized scenarios well.

Table 9 *continued*

Roschelle (2010)	0.63	0.07	0.51	0.75	Researcher developed	Used SimCalcMathWorlds. Explored the concepts of change. Investigated the effect of teachers' professional skills.
Kapa (2007)	0.68	0.20	0.20	1.00	Used Ministry of Education guidance	Used three-stage problem solving and open-ended scenarios.
Papadopoulosa (2008)	0.34	0.21	0.22	1.02	Researcher developed	Used computers to help explore hypotheses and verify the solutions.
Eid (2004)	0.20	0.16	0.19	0.92	Standardized	Contrasted students' performance using computerized scenarios and traditional representations.
Huang (2012)	3.27	0.26	0.29	1.13	Researcher designed	Used onscreen presented solutions to walk students through the course of thinking.
Lan (2010)	0.18	0.40	0.09	1.42	CEA assessment	Used Group Scribbles (GS) platform that enhances collaboration. Developed stages of problem solving.
Van Loon-Hillen (2012)	-0.01	0.39	0.20	1.40	Researcher developed	Worked examples to help with following procedures.
Guven (2012)	0.61	0.32	0.19	1.22	Researcher developed	Used dynamic geometry software (DGS). Developed four stages of difficulty: recognition, analysis, deductive, and rigorous.
Chen (2007)	0.71	0.34	0.01	1.30	Teacher developed	Incorporated personal contexts that helped students relate math concepts with their experience.
Ku (2012)	0.23	0.18	0.26	0.96	Teacher developed	Used personalized context to help students with math concept understanding.
Suh (2007)	0.14	0.34	0.09	1.30	Researcher developed	Incorporated principle of balance scale to model linear equations.
Panaoura (2012)	0.37	0.13	0.34	0.85	Researcher developed	Incorporated explorations to problem solving.

Note. ES = effect size, SE = standard error.

The majority of the studies (16, or 84%) used researcher- or teacher-developed evaluation instruments, and only one (Pilli & Aksu, 2013) reported a Cronbach's α-coefficient of reliability measure. In addition, the majority of the studies (17, or 89%) reported positive effect sizes when an exploratory environment was used as a medium of learning. Only two studies- one conducted by Van Loon-Hillen et al. (2012) and one conducted by Kong (2007)- reported negative effect sizes favoring traditional instruction, illustrating that exploratory environments cannot replace good teaching and that some concepts, like operations on fractions (Kong, 2007), require the instructor to deliver the concept and its stages and to suggest ways of overcoming obstacles students may face. Exploratory environments seemed to produce high effect sizes in cases where students applied math concepts in practice (e.g., Chang et al., 2006; Guven, 2012; Roschelle et al., 2010), but not when they first learned the concepts. The highest effect sizes were reported by Huang and colleagues (2012; ES = 3.27), who investigated the effect of embedded support during the process of problem solving, and Erbas and Yenmez (2011; ES = 2.36), who investigated the effect of open-ended explorations on students' mathematical achievements. Although an influx of onscreen instructional support might work well in many school settings, we believe that elements of mathematical modeling induced in the study by Erbas and Yenmez (2011) support more accurately the objectives of this study.

Analysis of Moderator Effects

Just as a mean effect size provides certain evidences for study findings' potential for duplication, subgroup moderator effects allow for uncovering potential mediators that maximize the effect. Since student mathematical achievement was the main outcome measured in this study, during the process of moderator formulation, attention was paid to extracting the study features that mediated achievement with using ECEs. We anticipated that through identifying such features, an optimum classroom setting would emerge. In addition, we realized that the effect of ECEs is strongly influenced by the degree of interactivity of the educational program used and the applied scaffolding necessary to have students assimilate tasks presented in context; however, such extractions from the studies were not feasible. Thus, a set of five moderators was identified: grade level, instrumentation, treatment duration, content domain, and type of learning setting. This categorization resulted in 12 subgroups whose effects were individually computed. The mathematical calculations associated with this part of the analysis were performed following Cooper (2010), who suggested giving more weight to effect sizes with larger sample populations (w = inverse of the variance in the effect calculations). Along with calculating subgroup effects, researchers computed their corresponding confidence intervals and standard errors, which helped determine the statistical significance of the subgroup effects. Table 10 displays the effect sizes according to the formulated moderators and the subgroups (levels). In order to provide a common metric for the subgroup effects magnitudes comparisons, the effect sizes were weighted by the sample sizes.

Table 10

Summary of Subgroups' Weighted Effect Sizes

Variable and Class	N	ES	SE	95 % CI	
				Lower	Upper
Grade Level					
Lower elementary: 1 through 3	2	0.91	0.58	-0.23	2.06
Upper elementary: 4-5	11	0.41	0.07	0.27	0.54
Middle school: 6-8	6	0.65	0.04	0.58	0.73
Instrumentation					
Problem solving	11	0.52	0.07	0.39	0.66
Explorations	8	0.61	0.04	0.54	0.69
Treatment Duration					
Short	4	0.36	0.14	0.08	0.63
Intermediate	7	0.65	0.09	0.48	0.83
Long	8	0.66	0.04	0.60	0.75
Content Domain					
Geometry	7	0.73	0.09	0.55	0.91
Arithmetic and algebra	12	0.54	0.04	0.47	0.61
Learning Type					
Teacher centered	6	0.69	0.09	0.52	0.87
Student centered	13	0.55	0.04	0.47	0.62

Note. N = sample size, ES = effect size, SE = standard error.

The grouping into levels and its analysis provided a more insightful picture about the effects of ECEs on the achievement of students in Grade 1-8 mathematics classes and helped answer research questions of this study. Detailed discussions reflecting these questions follow.

Are the effect sizes of student achievement different across grade levels? A block of *Grade Level* was created to answer this question. Following NCTM (2000), three subgroup levels were formulated: lower elementary, which included Grades 1-3;

upper elementary, which included Grades 4 and 5; and middle school, which encompassed Grades 6-8. The computed effect size showed differences across grade levels, with lower elementary producing the highest effect size (ES = 0.91), which according to Lipsey and Wilson (2001) can be classified as large. This result can be attributed to the fact that students at the lower elementary school level often use manipulatives to support their understanding of math concepts (Jitendra et al., 2007); thus, these students' transition to ECEs occurs rather naturally, resulting in the highest score gain. The effect sizes in the other grades showed a moderate magnitude. A larger pool of studies would help conclude whether the effect size distribution is common.

Are the effect sizes of student achievement different when problem solving is compared to explorations? The moderator category *Instrumentation* was established to discover whether ECEs affect student achievement differently through supporting problem solving and exploration. This was apparently one of the most important questions in this research. As explorations often led students to pattern formulations (Panaoura, 2012; Suh & Moyer-Packenham, 2007), problem solving was usually constructed within defined stages, gearing students' thought processes toward finding numerical answers to the stated problems (Chen & Liu, 2007; Hwang & Hu, 2013). When mediated, learning supported by explorations produced a higher effect size of ES = 0.61 as opposed to ES = 0.51 for problem solving. This result generates several conclusions and some further research questions. First, it can be concluded that the processes of explorations appear to resonate better with students' prior experiences, which consequently contribute to students' higher motivation to immerse in the inquiry processes as opposed to problem solving.

The research shows that problem solving is an integrated part of any math curriculum perceived globally, and efforts to help students understand the solution processes are multidimensional, ranging from creating schemas (Kapa, 2007) to inducing personalization (Chen & Liu, 2007; Ku & Sullivan, 2012). However, attempts at helping students learn the processes of problem solving by embedding explorations in some of the transitioning stages are nonexistent. The research on problem solving gravitates toward creating cognitive support rather. For example, by using onscreen help, showing worked-out solutions that students can follow (Van Loon-Hillen et al., 2007), or varying the segmentation of the stages of the solution process. As shown by calculated effect sizes, all of these attempts seem to produce desirable positive results, however by focusing on simplifying the mechanics of the problem solving processes, the attempts shift the focus and diminish the scientific principles the problems *intertwine*. As word problems usually embody scientific scenarios, the scientific inquiry processes that would uncover the underlying principles and then direct the learner to find a particular solution are not emphasized in the accumulated pool of research on problem solving, with the exception of the research by Roschelle et al. (2010) and Panaoura (2007). English (2004) advocated for wider implementations of mathematical modeling, which includes explorations, in elementary math curriculum. We support that idea.

Are the effect sizes dependent on the mathematics content domain? Two mathematics content domains were examined in this study: geometry and algebra. Geometry, traditionally dominated by visualization, showed a higher effect size (ES = 0.73) compared to algebra (ES = 0.54). As geometric objects can also be externalized by their real embodiments, it seems that more effort should be placed on visualization of other, more abstract, mathematical structures- such as equations and functions. A potential to enhance the teaching of algebraic structures via exploratory environments seems to exist and need to be further explored. Embodying these structures using context-driven scenarios deems to be a challenge, as reflected in finding only seven (37%) such studies. We realize that these two subgroups did not reflect the entire spectrum of the content domain, and further for detailed classifications, such as applications of ratios, proportions, or solving linear equations, could have been built in. Furthermore, ECEs could have been applied to any grade and mathematics domain. Due to lack of available research, a more detailed categorization did not emerge.

How does the type of instructional support (teacher guided or student centered) affect student achievement when computers are used? There were two main categories of instructional support provided to the students in the study pool: student-center support, provided on the computer screen, or teacher-centered support, provided by the teacher. Student-centered studies (13, or 68%) dominated the pool, but teacher-centered support produced a higher effect (ES = 0.69) than student-centered support (programmed tips provided by the computer; ES = 0.55). This result stresses the importance of the teacher's role in developing students' understanding of mathematics structures and helping them apply math concepts to solve problems. Programmed tips are important, but the instructors' expertise and support from a live person appear to have a higher impact on student success. Certainly, an effective teacher "transfers the knowledge development and justification responsibilities to students" (Li & Li, 2009, p. 275). Further research contrasting these two modes of instructional support would likely shed more light on cause of their differences.

Based upon linking the levels with the highest effects, it appears that month-long (at minimum) explorations in mathematics classes, guided by the teacher, produce the highest learning effect. Of course, variations of the setting are possible based on individual student needs. This type of learning organization, according to the findings of this meta-analysis, would result in increasing students' mathematical achievements.

In addition to computer moderators that reflected the stated research questions, the effect of treatment length was also included in the moderator analysis. There is a noticeable effect of treatment duration on student math achievement. This conclusion corresponds to an inference reached by Xin, Jitendra and Deatline-Buchman (1999), who in their meta-analysis also proved that longer treatment results in higher student achievement. The learner needs to be acquainted with the mechanics of the new learning medium; thus, it is important that the first contact with an ECE be absorbed into a learner's working memory. More frequent exposure to the new environment allows for more focus on task-driven objectives related to the content analysis, which results in learning more from the medium. However, as Guven (2012) and Roschelle and colleagues (2010) found out, there is an achievement saturation level, which perhaps suggests that ECEs need to mediate with other factors, for instance, with different learning goals not necessarily related to content knowledge, such as analysis, synthesis, or evaluation (Anderson & Krathwohl, 2001), in order to further promote learning.

General Conclusions and Study Limitations

Though several meta-analyses have been conducted on the effects of technology on student achievement, this study sought to examine the effects of exploratory environments on students' understanding of math concepts. Although the study found a moderate positive effect size (ES = 0.60) associated with ECE use, this does not diminish the importance of good teaching. Christmann, Badgett, and Lucking (1997), as well as Clark (1994), found that using computers purely as a method of instruction does not improve students' math understanding. Hence, as computers have been used in mathematics classrooms for several decades now, the question regarding what extent to which they can impact the teaching and learning of mathematics seems to remain unanswered. This meta-analysis of up-to-date literature allowed for formulations of some inferences based on implementations of technology, but many new questions emerged, such as the following: How do exploratory environments help students with the transfer of math concepts to new situations? If ECEs embed scientific principles, then how using them helps students with understanding these principles in mathematics classes? How to assure that the methods of quantitative scientific modeling that students apply in their physics, biology, or chemistry classes are coherent with these used in mathematics? As models and the process of modeling are fundamental aspects in science

(Schwarz, & White 2005) should there be a general modeling cycle designed for all subjects?

Mathematics provides tools for scientific phenomena quantifications, thus unifications of the techniques of modeling seem to benefit the transition of knowledge between mathematics and science, and consequently affect the learners' perception of mathematics as a subject of a high applicability range. It seems that a more detailed research studies in this domain are worthy of consideration and common availability of computerized exploratory environments will be very helpful in organizing such studies.

The Impact of ECE on Students' Problem Solving Techniques

Problem-solving techniques are developed on the basis of understanding the context through identifying the principles of the system's behavior. However, it is a highly intertwined process that might include verbal and syntactic processing, special representations storage and retrieval in short-and long-term memory, algorithmic learning and its most complex element —conceptual understanding (Goldin, 1992). Computerized programs offering basis for investigations display a great potential for improving problem conceptual understanding, yet this study shows that this area is not fully explored yet and taking a full advantage of such learning environments bears as a possible extension of this research. Enriching the problem analysis through explorations to focus the learners' attention on its underpinning principles and then formulate patterns and generalize the patterns using mathematical apparatus, emerges to be an approach worthy of further investigations. Higher student achievement on explorations (ES = 0.61) when compared to problem solving (ES = 0.51) also encourages the need for a search of moderators that were silent in the accumulated study pool. Is the act of allowing students some flexibility to explore a given scenario, formulate a problem, and have them hypothesize, test, and prove or disprove their hypothesis a possible moderator affecting the learning effects? If yes, then to what extent can this type of learning be applied to problem solving? Although, the contrast between traditional static methods of teaching problem solving and the support of computerized simulations shows that dynamic ECEs produce a higher learning effects (ES = 0.60), certain stages of problem solving processes need to be elaborated and delivered by the teacher. Thus, in a congruent vein, teachers' role and their offered support should also be investigated. Finally, another possible research topic emerging from this study is whether the appealing format of ECEs is what dominates students' engagements and consequently impacts their persistence to stay on tasks, or whether being in control of the scenario's parameters and having the opportunity of manipulating its variables is the dominant factor.

Limitations and Suggestions for Future Research

There are certain limitations of this meta-analytic research, primarily because this study could not be conducted in an experimental fashion where ECEs constituted instrumentation provided by computer programs and a direct contrast between two different modes of learning was exploited. Furthermore, the limited count of studies available to be meta-analyzed also affected the study generalizability. Although sensitivity to smaller sample sizes was restored by the process of weighing, the impact of the mean effect would validate the replication of the findings more significantly by being computed over a larger study pool.

In addition, it was not possible to evaluate the designs of the interventions through the lenses of the multimedia principles defined by Clark and Mayer (2011). Such a moderator, if possible to compute, would shed more light into ECEs design effectiveness and help identify the most optimal. We were especially interested in examining the magnitude of the exploratory effects on improving students' problem solving skills and inversely the presence and effects of the problem-solving phases exercised in the explorations on students' achievements. However, we encountered limited research findings for extracting these features from the accumulated studies. Thus the effect of scientific empirical methods on building theoretical mathematical models that was intended to examine could not be completely furnished. *We realized that both types of interventions—ECE's supporting problem solving and explorations*— contain common features, and their effects on students' problem solving skills not just their problem solving performance— as measured by testing —should be the study main objective. Further research focusing primarily on extracting these features in a systematic way surfaced as an extension of this study.

Another factor limiting the study findings involved the widely varied methods that have been used to assess student achievement, ranging from traditional multiple-choice exams mostly locally developed to new assessment techniques such as standardize-based assessments. Although some of these studies (e.g., Pilli et al., 2013) reported a Cronbach's alpha reliability coefficient, most did not, thereby decreasing the reliability of the measuring instrument.

Furthermore, although we initially planned to investigate similar issues mediating mathematical modeling and problem solving, we encountered very limited research data on mathematical modeling; thus, we had to give up on this idea and instead focus our attention on explorations and problem solving. Thus, the effect of mathematical modeling on students' problem-solving techniques was not investigated as initially planned. We would like to encourage researchers to engage in studies on the effects of exploratory environments on mathematical modeling and problem solving as a unified three-strand research area. This meta-analysis, to a certain extent, exposed a narrow focus of the existing primary studies on the effects of exploratory environments of problem solving in mathematics education. We suggest creating more constructs that would help quantify students' problem-solving techniques in the function of their mathematical modeling skills with exploratory computerized environments and as a medium for such. There seems to also be a need to evaluate how learners link concepts with principles due to given conditions and how they initiate applications of the procedures that they select.

CHAPTER IV

THE EFFECTS OF MATHEMATICAL MODELING ON STUDENTS' ACHIEVEMENT: MIXED-METHODS RESEARCH SYNTHESIS

Introduction

This study intended to broaden the research findings about the learning effects of applying mathematical modeling at the high school and college levels as well as to contribute to improving the design of modeling activities. Such formulated orientation called for synthesizing not only quantitative but also qualitative research that led to undertaking two separate yet objective wise coherent lines of research: meta-analytic, which was applied to quantify the experimental research, and qualitative, which was grounded in evaluating the subjects' behavior. A total of 32 research studies from 16 countries encompassing the past 12 years (from January 1, 2000 to December 31, 2012) met the inclusion criteria. The results of meta-analytic techniques that included 13 primary research studies and 1,670 subjects revealed a positive moderate magnitude effect size of ES = 0.69 (SE = 0.05, 95% CI: 0.59–0.79). A subgroup analysis displayed differences of the effect sizes due to different modeling designs, grade levels, and content domains. Qualitative research encompassing 19 primary studies and 1,256 subjects allowed for emergence of a grounded theory that embodied a proposal of an integrated math modeling cycle situated in a scientific inquiry framework. Several venues for further research that came forth during the study are also discussed.

The modeling processes constitute central methods of science knowledge acquisition (Schwarz & White, 2005). Scientific modeling provides methods for analyzing data, formulating theories—often expressed in symbolic mathematical forms—and testing those theories. As such, learning by the processes of modeling plays a vital role in developing students' skills in both science classes (Wells, Hastens, & Swackhamer, 1995) and mathematics classes, especially during problem-solving (Lesh & Harel, 2003). One of the many advantages of modeling activities as compared to problem-solving is shifting the learning focus from finding solutions to enhancing skills of developing the solution processes through transforming and interpreting information, constructing models, and verifying the models (Lim, Tso, & Lin, 2009).

Mathematical modeling, traditionally a core part of engineering courses (Diefes-Dux & Salim, 2012), became an important learning method in emerging new academic fields that integrate the contents of various subjects such as biophysics or bioengineering. The interest in integrating all branches of science, technology, engineering, and mathematics (STEM) education has increased very rapidly lately (Ferrini-Mundy & Gucler, 2009). Modeling viewed as an interdisciplinary activity is now also being implemented in earlier schooling levels (English & Sriraman, 2010). The development of students' modeling skills starting at the earlier educational levels can have a profound impact on their success in engineering, medicine, and other college programs that aim at graduating professionals who engage successfully in problem-solving processes (Diefes-Dux, Zawojewski, & Hjalmarson, 2010).

The purpose of this study was to synthesize the current quantitative and qualitative research findings on mathematical modeling at the high school and college levels as well as identify its strengths and weaknesses and search for ways of advancing the knowledge about the design of modeling activities that will increase students' learning. Through identifying modeling features that produce high learning effects, we hoped to formulate suggestions for improving students' performance on such tasks, focusing particularly on a high school mathematical curriculum, which provides educational foundations for students' college success (Hoyt & Sorensen, 2001). By uncovering how students engage and perceive modeling tasks, we hoped to reach conclusions for strengthening the phases of the modeling processes and consequently better prepare students for their college programs and professional jobs.

The process of mathematical modeling, defined as an activity of finding quantifiable patterns of a phenomenon and its generalizations (Lesh & Harel, 2003), was first introduced into a mathematics classrooms about four decades ago (Pollak, 1978). Its ultimate goal was to bridge the gap between reasoning in a mathematics class and reasoning about a situation in the real world (Blum, Galbraith, Henn, & Niss, 2007). Research shows that through immersing into the modeling cycle, learners develop the skills of scientific reasoning, which are essential in broadly defined problem-solving processes. Such situated, mathematical modeling recently emerged as one of the most often researched methods of mathematics learning in education (Schwarz & White, 2005).

During the past decades, substantial research (e.g., Gravemeijer, 1997; Schoenfeld, 1992) has proved that the process of problem-solving is disconnected from mathematical modeling, meaning that without considering the factual relationship between real-world situations and mathematical operations, students mimic the process of solving word problems (Reusser & Stebler, 1997). Specifically, students tend to use superficial key word methods rather than analyze embedded mathematical structures in the attempt to solve the problems (Schoenfeld, 1992). As a result, in the current math curriculum setting, exercising problem-solving processes does not generate the skills that it intends, and a substantial research body (e.g., Klymczuk & Zverkova, 2001; Lesh & Zawojewski, 2007; Schoenfeld, 1992) has proven this deficiency. In light of these findings, the following ultimate question arises: What phases of mathematical modeling help students improve their problem-solving understanding and enrich their techniques of formulating solution designs? Viewed through this prism, other questions can be generated: Should mathematical modeling constitute a separate type of classroom activity, or should it instead be considered as a phase of problem-solving processes? If both of these two competencies—modeling and problem-solving—are integrated, which one should dominate learning inquiry in mathematics classes? Should modeling be considered a subset of problem-solving, as shown in Figure 7A or should problem-solving be a subset of modeling, as shown in Figure 7B?

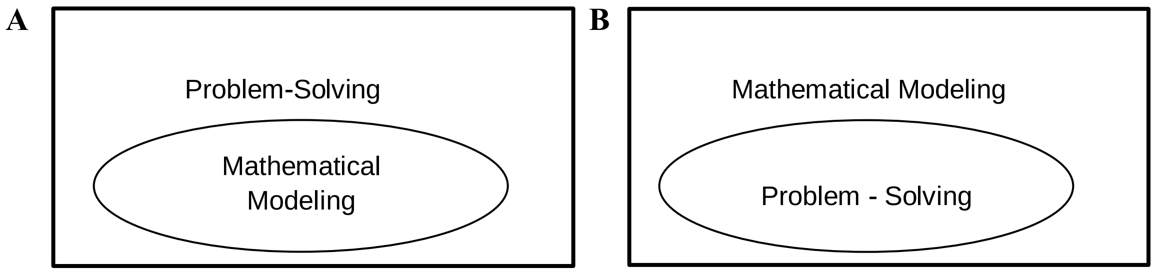

Figure 7. Current relations between math modeling and problem-solving.

The systematic research review undertaken in this study is poised to reflect on this relation and will attempt to propose a diagrammatic learning process that will comprise both mathematical modeling and problem-solving.

Theoretical Background and Synthesis of the Prior Research

Mathematical modeling is defined as an activity of finding patterns, generalizing the patters, and expressing the patters using mathematical apparatus. In these processes, mathematical models are elicited. Such elicited models represent simplified but accurate representation of some aspect of the real world (Winsberg, 2003). The models can take various forms, ranging from physical objects (e.g., solids or plane figures) to mathematized statistical models, differential equations, or mathematical functions, all of which describe algebraic dependences of the system variables.

Mathematical modeling utilizing real scenarios, phenomena, or data that can be provided or gathered through experimentation is often classified as an exploratory type of learning (Thomas & Young, 2011). Conducting experiments and gathering data are often difficult in mathematics classrooms that are not traditionally designed for that purpose. Since computerized experiments can substitute for real experiments in science classes (Podolefsky, Perkins, & Adams, 2010), their adoption for enhancing mathematical modeling has become more tangible in contemporary math classrooms. One of the purposes of this study was to identify and statistically quantify the effect sizes of using various learning media that can support mathematical modeling, focusing especially on the learning effects produced by computerized simulations.

The mathematical modeling process usually concludes with a formulation of a mathematical representation. As such, multifaceted cognitive goals are achieved by learners undertaking modeling activities. Bleich, Ledford, Hawley, Polly, and Orrill (2006) claimed that such activities (a) expand students' views of mathematics by integrating mathematics with other disciplines, especially sciences; and (b) engage students in the process of mathematization of real phenomena. Understanding how models develop has a great potential to inform the field of mathematical education, much like research on the development of other math ideas, such as rational numbers or proportional reasoning (Lesh & Zawojewski, 2007). In this view, being able to express a situation using mathematical representations helps students develop problem-solving skills (National Council of Teachers of Mathematics [NCTM], 2000). A study by Lingefjärd (2005) concluded that after working on modeling activities that are usually supplied by real embodiments, students handled word problems with embedded visual representations better than those taught by conventional methods. A study conducted by McBride and Silverman (1991) revealed that mathematical modeling used during integrated lessons increased students' achievement in all subjects whose content was utilized. Another advantage of exercising modeling is improving students' affective skills. By analyzing the processes of a phenomenon, students critically validate its stages, which in turn provide them with a contextual reference to problem-solving.

Mathematical modeling as a process of mathematizing real phenomena has been frequently researched, and its structural phases have undergone several modifications. As a result, multiple theoretical designs have emerged to organize mathematical modeling activities. Their structures are presented in the next paragraph, and a short analysis accompanies each of them. The purpose of the following section is to shed light on the historical perspective on modeling and identify conceptual trends in which the designs are evolving.

Review of Existing Modeling Cycles

One of the precursors of mathematical modeling designs was proposed by Pollak (1978) and is displayed in Figure 8.

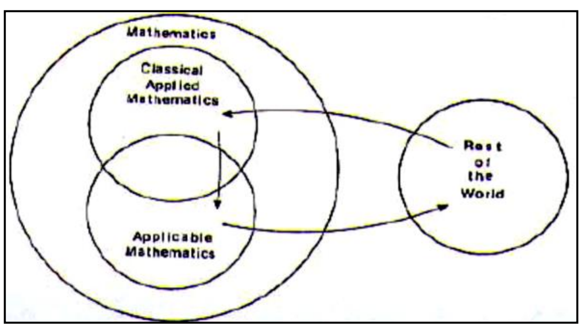

Figure 8. A prototype of the modeling cycle (Pollak, 1978).

This design focused more on amplifying the domains encompassing modeling stages than the processes linking these stages, which are silent in this model.

Mathematics is depicted as a distinct academia separated from the rest of the world. The schema does not underline the initial phase of the process, nor does it elaborate on the final stage. Mathematics is divided into two sections—*classical applied* and *applicable mathematics*—with an intersection of these domains indicating common features. As presented, the applications of the model to school practice seem to have been limited. The method of inquiry was not specified and did not resemble scientific processes per se. Another, more detailed cycle for modeling activity design was developed by Blum (1996) and is illustrated by Figure 9.

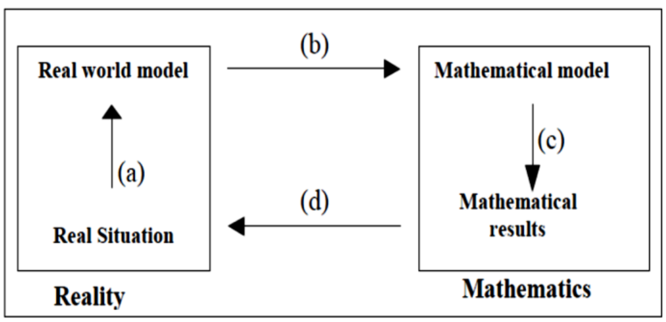

Figure 9. Modeling cycle (Blum, 1996).

This cycle consists of two equal chambers called *Reality* and *Mathematics,* which consist of building blocks defined as *real situation*, *real world model, mathematical model,* and *mathematical results*. This cycle appears to be better balanced, weighting equally the aspects of reality and the models that are to reflect on the reality. The initial stage of the process is labeled as *real situation*. By moving through the modeling cycle, the modelers are to return to the *real situation* by validating their answers. Although the designated stages are labeled, the processes that link the stages are only ordered by (a), (b), (c), and (d), with no further elaboration.

Blum and Leiss (2007) developed a more detailed schema for modeling that included not only the stages of modeling but also short descriptions of the processes linking the stages (see Figure 10).

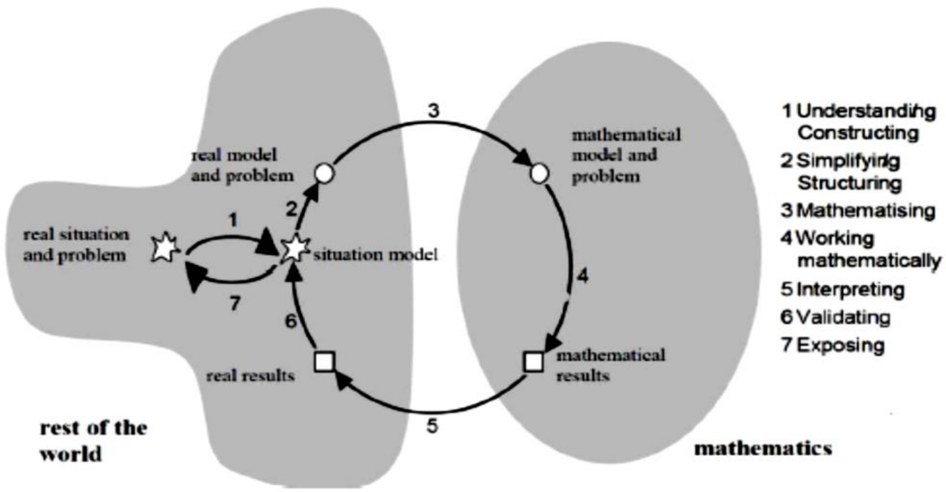

Figure 10. Modeling cycle (Blum & Leiss, 2007).

In their proposal, Blum et al. (2006) identified the following main phases of the modeling cycle: real situation; situation model; mathematical model, which leads the learners to generate mathematical results; and real results. They highlighted the phase of *situation model,* suggesting that during this phase the learner immerses in the process of understanding the task that will affect the correctness of the next phases of the cycle. Situation model that is meant to emerge as a quantitative structure independent from the text of the given problem is perceived by some researchers as a phase of solving word problems (e.g., Nesher, Hershkovitz, & Novotna, 2003). In this vein, Ferri (2006) posited the question of whether formulation of the situation model follows understanding of the real model. The interaction between real and mathematical worlds is depicted through the processes of mathematization and interpretation. While the process of mathematization helps to express given elements of reality in symbolic forms, during the process of validation the modeler returns to the given real problem and contrasts its mathematical description with its real parameters. As this cycle provides many details about the phases, the separation of mathematics from the real world highlighted in this model seem to dilute the main idea of modeling activities that are about integrating mathematics with "the rest of the world" rather than separating these two.

More recently, Lim et al. (2009; see Figure 11) proposed yet a more detailed modeling cycle in which the tasks at certain stages are further elicited.

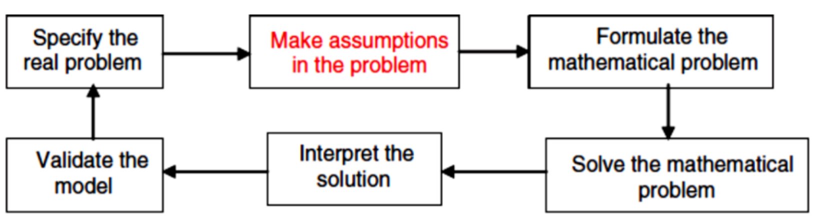

Figure 11. Modeling cycle (Lim et al., 2009).

According to this model, the participants initiate the process by specifying the problem given in the scenario, then they isolate important features of the model by making assumptions, and then they move to formulating and solving the problem. A stage of derived model validation is also present. Lim and his colleagues (2009) especially highlighted the phase of making assumptions in their modeling cycle, which is a precursor to model formulations.

More diverse modeling cycles can be found in the literature (e.g., Berry & Davies, 1996; Geiger, 2011; Lesh & Lehrer, 2003; Yoon, Dreyfus, & Thomas, 2010). They share some common features; for example, they begin from a real situation and conclude with mathematical form of the situation. Yet, as Perrent and Zwaneveld (2012) noticed, the problem that students and teachers are confronted with during the modeling exercises is a lack of uniformity about the essence of tasks of these processes that cause obstacles in organizing such activities in school practice where the students are expected to express, test, and revise their own current ways of thinking (Yoon et al., 2010). Are the methods of mathematics defined widely as pattern seeking and conjecture formulations (Devlin, 1996) sufficient to lead the learners through modeling processes typical for a scientific inquiry? None of the models proposes an adoption of some of the phases of scientific modeling that students learn in their science classes. Situation, model, and analysis of the model appear to be crucial elements of the scientific modeling with the model as the central element of the process. As real situations at the high school and college levels often involve physics, chemistry, or biology concepts, referring to these concepts in the modeling and comparing them against their mathematical counterparts appeared to us as an interesting task. For a brief reference, we would like to discuss the main elements of scientific modeling using Hestenes (1995) modeling design (see Figure 12). While science modeling can take two forms; qualitative and quantitative, the phase of mathematical modeling is not isolated in the proposed cycle.

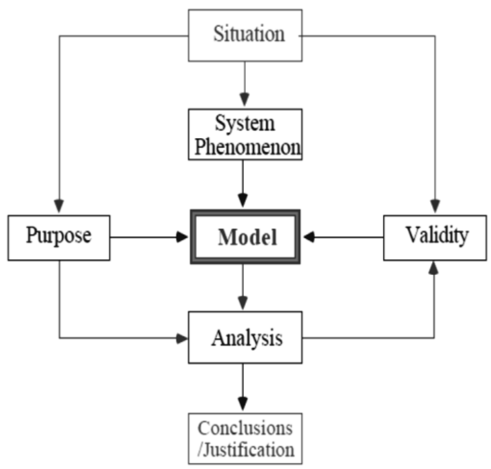

Figure 12. Science modeling cycle (Hestenes, 1995).

Situation, model, and model analysis appear to be critical elements of scientific modeling with the model being the central element of the process. This cycle highlights an embedded *system phenomenon* not mentioned in the discussed mathematical modeling processes. The idea of extracting a *system phenomenon* or *principle* gained more attention in this study, and its role will be further discussed. Felder and Brent (2004) stated that following a scientific inquiry process, learners exercise inductive reasoning, which is a precursor to students' natural curiosity and their intellectual development. Thus, the degree to which scientific inquiry is present in math modeling activities will also be discussed in this paper.

The Concept of Model - Eliciting Activities (MEAs)

Mathematical modeling cycles provide a general framework for a structural design of mathematical modeling activities. Lesh and Kelly (2000) went further and developed six principles for modeling activities' context design. Several research studies located on mathematical modeling have been centered on MEAs (e.g., Inversen & Larson, 2006; Yoon et al., 2010; Yu & Chang, 2011). It seems that this framework crystallizes as a main structure for modeling activities' context design. Each of the principles invokes certain questions that when satisfied, guarantee that the given activity can be classified as an MEA. A discussion of the principles along with the following invoking questions follows: (1) the reality principle—does the given situation appear realistic to the learners and is it built on the learners' prior experience? (2) the model construction principle—does the situation generate a need for inducing mathematical tools? (3) the self-assessment principle—does the activity involve assessment of the developed model? (4) the construct documentation principle—does the activity make the students document their thought processes? (5) the model share-ability and reusability principle—can the elicited mathematical model be used to solve other similar problems? (6) the simplicity principle—is the content of the problem and the mathematical tools used to solve it in the range of students' abilities and possessed knowledge? Coupled with the mathematical modeling cycle, the six principles for MEAs enrich the design framework and clearly distinguish it from typical problem-solving frequently applied in mathematics classroom. What are the differences between problem-solving and MEA?

Some researchers (e.g., Mousoulides, Christou, & Sriraman, 2008; Yoon et al., 2010) defined problem-solving as repetition of procedures, whereas modeling activities, according to these researchers, tend to be context-rich problems that do not assume that students have already learned the procedure for solving the problem. Yet, this distinction between modeling and problem-solving seems to contradict the simplicity principle, which assumes that students immersed in modeling activities possess necessary skills and knowledge and thus the procedures to enact and test derived models.

Models, the product of modeling, can take various forms in mathematics. Representing simplified but accurate aspects of the real world, the models can be formulated using physical objects or nonphysical abstract mathematical forms expressed symbolically. In the process of mathematical modeling, a system under investigation, as well as its variables, must be defined using mathematical rules. The goal of immersing math students in the process of modeling is to have them view given phenomenon through the rules of mathematics representations. Suitable rules and their corresponding mathematical embodiments are identified through observing the system, identifying related parameters, formulating patterns, and constructing a symbolic representation of the patterns. Such defined models consist of a set of constraints that are embedded in the model selection. A formulated model can be further used to predict the behavior of the system in new circumstances. Viewed through this lens, the process of mathematical modeling places itself as a precursor to developing learners' problem-solving techniques. This vision is also supported by Lesh and Kelly's (2000) multi-tiered teaching experiments, which suggest conducting research on problem-solving as modeling.

Computer simulations as the means of enhancing modeling. Along with technological advances, education multimedia technologies create interactive learning environments whose goal is to enhance teaching and learning. Software in the forms of tutorials, simulations, games, and graphing and computational programs are created to help learners make knowledge more accessible and consequently increase their rate of the knowledge acquisition. Exploratory learning environments engage the learner with the environment through definite actions of gathering and investigating information (Flum & Kaplan, 2006). They also promote transfer of knowledge, inquiry, problem-solving skills, and scientific reasoning (Kuhn, 2007). Exploratory learning environments can be externalized in various forms, such as real experimentation, provided data, scenarios provided on video or other multimedia, and scenarios provided by computers. Computerized simulations offer great promise for providing a rich medium for such learning. Determining the effect size of using technology to enhance the process of mathematical modeling constituted one of the research subgroup moderators of this study.

Math modeling as an essential skill in engineering, science, and technology. Mathematical modeling, whose essence is to bridge mathematics with an outside world, is particularly important in engineering, science, and technology where transitions between real-word problems and the models are the substance of the disciplines (Crouch & Haines, 2004). Research shows that how students perceive mathematical modeling is greatly affected by their previous experiences with this type of learning. If problem solving is perceived as applying procedures dictated by the teacher, the students will carry on this notion through their further math education (Christiansen, 2001), and likely they will perceive modeling as an activity of following teacher-dictated procedures. Stenberg (1997) claimed that students' difficulties are due to being new to mathematical modeling activities entering their undergraduate college programs. The phase that is especially difficult is formulating sets of hypotheses, identifying the variables, and testing the derived model. Making modeling more accessible at a high school level seems to be one of the actions toward a better preparation of these students for their undergraduate college modeling activities. During the process of analysis of the accumulated research, we were also interested in extracting the contracts and expectations of undergraduate engineering, science, and technology programs that are essential in those disciplines and linking them with high school modeling designs. We concluded that learning the general conceptual approach to modeling in undergraduate programs would help us to reflect on high school modeling activities and more accurately formulate recommendations for their design. The following description of mathematical modeling for in undergraduate programs is proposed by Crouch and Haines (2004): "Mathematical modeling involves moving from a real-world situation to a model, working with that model and using it to understand and develop or solve real-world problems" (p. 197). The mathematical modeling appears as a phase that is embedded in world problems. Bearing this, we will be interested in examining how the current research on mathematical modeling links students thinking processes to problem solving. The following cyclic process of mathematical modeling at undergraduate levels is proposed by Berry and Davies (1996): real-word problem statement; formulating a model; solving mathematics; interpreting outcomes; evaluating solution; refrying model; real-world problem statement. According to Crouch and Haines (2004), the transition from formulated model back to real-world problem is the most difficult; thus, more attention should be given to having learners analyze and further use the developed model. It is to note that in the proposed mathematical modeling cycles (e.g., Blum & Leiss, 2007; Pollak, 1978), the role of scientific methods is diminished, whereas at the undergraduate level, they provide the basis for conveying mathematical meaning. The scientific methods are not separated from mathematical modeling, but they constitute an integrated part of the cycle. Searching for ways to reinstall this link at a high school level seems to be an important factor in connecting these two phases of schooling.

Synthesis of Prior Research

Although mathematical modeling was implemented in mathematics education about four decades ago, its contribution to mathematics education research has gained momentum recently. This section will synthesize major findings from prior qualitative and quantitative studies on effectiveness of mathematical modeling lessons, focusing on using computer programs as a medium for such activities.

Past Research Major Findings

In supporting the need for this study and reflecting on previous research, we searched for meta-analyses and other types of research syntheses on mathematical modeling in education using ERIC (Ebsco), Educational Full Text (Wilson), Professional Development Collection, and ProQuest Educational Journals, as well as Science Direct and Google Scholar. Although several meta-analytic research studies targeting various aspects of conceptualization of math ideas were located and their discussion follows, a meta-analysis specifically targeting research on mathematical modeling was not found. Syntheses of qualitative research on mathematical modeling were not found either. The cited research will then, at large, refer to several constructs that aim at investigating various processes or stages of mathematical modeling activities.

The effect of using computer simulations as an instructional strategy on students' early math knowledge development was meta-analyzed by Dekkers and Donatti (1981). The findings gathered from 93 empirical studies "did not support the contention that simulation activities cause an increase of students' cognitive development (d = - 0.075) when compared with other teaching strategies" (Dekkers & Donatti, 1981, p. 425). In light of these findings, these researchers suggested that "attention should be given to reporting details of methodology employed" (p. 426). The lack of promising results was associated with inadequate teaching methods that simulations were supposed to support. Sequentially arranged, the following will summarize findings conducted by Fey (1989). While discussing the capabilities of producing dynamics graphs, which are essential tools of math modeling, Fey uncovered that technology is not helping students with graph interpretation, as was expected. Consequently, Fey suggested developing projects that will address these difficulties and conducting research that will investigate eliminations of these difficulties. He also noticed a need for a change in teachers' perception regarding graph introduction—from teaching students "*how to* produce a graph to focusing more on explanations and elaboration on *what the graph is saying*" (p. 250). Another advantage of using computers in math education is their capability of creating micro-worlds that allow students to make changes in their environments. Being able to manipulate system variables sustains the phase of model verification; thus, investigating computer capabilities in these regards contributes to strengthening the design of modeling activities. Thomas and Hooper (1991) advocated for a more precise definition of computer simulations and claimed that the lack of such hinders precision and questions validity of research aimed at quantification of simulation effectiveness. Kaput and Thomson (1994) elaborated on the pitfalls of the research pertaining to the role of technology in learning in general, stating that research studies played "less emphasis on controlled comparisons" (p. 677), thereby generating more research questions than providing answers. Yet, Kaput and his colleague underlined technological interactivity as a significant advantage to enabling students to experience active learning. They also claimed that an obstacle of injecting the meaning into procedures that students previously "automatizied meaninglessly" (p. 679) had not yet been overcome and that there was a need for attracting more researchers and curriculum developers to address this issue. Quantification of effect learning sizes when computer simulations were compared to traditional methods of instruction was presented by Lee (1999), who meta-analyzed 19 empirical studies and concluded that they produced a moderate (ES = 0.54) learning effect size. Lee pointed out that "specific guidance in simulations helps students to perform better" (p. 81). In the light of this finding, he advocated a need for placing more emphasis to guidance design. A meta-analytic study conducted by Kulik (2003) who located six research studies published after 1990 on the effectiveness of computerized exploratory environments in secondary schools revealed an effect size of

0.32. Kulik did not elaborate on how these simulations were embedded in the lesson cycles or discussed the design of instructional support. He did, however, question the validity of some of the research procedures and evaluation instruments that quantified the research. A substantial meta-analysis including studies conducted after 1990 on the effectiveness of using computer technology in mathematics classrooms was conducted by Li and Ma (2010) who extracted a total of 85 independent effect sizes from 46 primary studies representing all grades from elementary to senior secondary school. These researchers computed the effect sizes for various types of technology used, such as communication media, tutorials, simulations and the like. The overall effect size of ES = 0.28 proved the statistical significance and supported the claim that using technology in math classes improves students' achievements. The effect of using simulations (ES = 1.32) outpaced the effectiveness of tutorials (ES = 0.68) and communication media (ES = 0.39). The researchers also reported that "using technology in school settings where teachers practiced constructivist approach to teaching produced larger effects on mathematics achievement" (Li & Ma, 2010, p. 233) when compared to traditional teaching methods. They further concluded that learning through technology does require a context to produce desired learning outcomes. Yet, suggestions on the forms of the contexts and how the contexts should be executed were not discussed.

Identified Areas of Concern in the Prior Research

As an emerging method of mathematical knowledge acquisition, modeling still faces unresolved issues that prevent the process of design of its conceptual framework from solidifying. One such issue involves the stage of verification. Zbiek and Conner

(2006) suggested that there should be multiple opportunities for learners to verify derived models. Yet, in pen-and-pencil problems, verification might be lacking a reality aspect that modeling is centered on. Providing students with some format of real experiments or easily accessible computerized simulations during which isolated variables can be manipulated manifests itself as suggestion worthy of further investigation. Zbiek and Conner (2006) further reflected on the process of assessing students' competency in math modeling and asked if the skills of mathematical modeling should be included as one of the math assessment items. Bleich and colleagues (2006) expressed concerns about inadequate teacher methodological preparation in inducing graphical representations during the modeling processes of motion. A similar conclusion was reached by Sokolowski and Gonzalez y Gonzalez (2012) in their study on how mathematics teachers perceive the differences between position-time graphs and path of object's movement while modeling motion problems. This research revealed that math teachers are willing to apply math modeling in their math classes, but they face obstacles in finding sound scientific methodology that could help with organizing the modeling activities processes that would nurture students' learning. Another unanswered question centers on the linkage of the modeling process with the contents of other academic disciplines. Some researchers claim that the goal of math modeling should be limited to formulating a mathematical representation and that no further conceptual discussions of the formulated patterns are needed. Although valid from a mathematical point of view, this stance does not mediate with a commonly accepted contemporary approach to teaching mathematics and science (e.g., Niess, 2005) and it is at odds with STEM education. Instead, shouldn't mathematical modeling be perceived as a bridge linking mathematics with other academia? The core phase of modeling constituted by a discussion about proving or disproving the validity of a derived mathematical model for fitting into scientific principles that the given system displays may also have a profound impact on eliminating many science-math misconceptions. Li (2007) recommended that "the differences between school science and mathematics concepts should be noted when we try to develop a model to explain students' misconceptions in school mathematics" (p. 6). Since college students' preparation to link mathematical world and real world is fragile (e.g., Carrejo & Marshall, 2007; Klymchuk & Zverkova, 2001), placing more emphasis on integrating these areas in high school emerges as a recommendation that would strengthen the links.

In sum, the major meta-analyses and qualitative syntheses reported positive learning effects when simulations were used to enhance math learning objectives. Yet, the information associated with the type of instructional support that appears to be of high significance is limited. This study attempted to fill in the gap and enrich the analyses by placing an emphasis on this construct. It is evident that mathematical modeling has an established voice in mathematics education research. Its cognitive and affective effects on students' math knowledge and aptitude are well exploited and researched. However, as this synthesis has revealed, there are unanswered questions and unresolved issues regarding, for example, instrumental implementation of this learning method in school mathematics. Successful implementation of computer technology must rely on sound instructional strategies (Coley, Cradler, & Engel, 2000). Thus, in order to have students capitalize on this learning method and maximize their learning potential, further research is needed.

Research Methods

Although a literature review is usually undertaken with one research method—either narrative, quantitative, or meta-analytic — guiding the study, this study intertwined two methods: a systematic approach proposed by Glass (1976), called *meta-analysis,* and a systematic summary of qualitative and mixed-methods research. The general search criteria were set to be similar for both lines of research, and differentiation was made on research findings' evaluation. The meta-analytic part was concluded with calculating main effect size statistics along with moderator effects, and the qualitative part concluded with a formulation of common themes and emergence of a grounded theory that reflected on subjects' perceptions of various constructs, referred to modeling activities. Considering each study as individual informatory, the analysis of the qualitative pool of studies was guided by methods described by Lincoln and Guba (1985). Content wise, this study attempted to synthesize research on applying mathematical modeling to support the process of mathematical knowledge acquisition and improving students' problem-solving techniques at the high school and college levels. The modern theory on research design on mathematical modeling (Zawojewski, 2010) identified two types of research objectives: (a) development and evaluation of the models formulated by learners, and (b) instructional tools and learning media applied during the modeling activities. While in positivistic paradigms, inferences are made due to quantifiable data, in naturalistic paradigms each informant is a source of valuable district data (Lincoln & Guba, 1985) providing multiple yet holistic information. Undertaken with such scope, this research intended to zoom in and reflect on findings of both; quantitative empirical studies that provide measurable outcomes and qualitative studies that reflect on learners thought processes. It was hoped that by embracing the research in this strategy, a more comprehensive picture of current research on mathematical modeling would be generated and the inferences would be broader.

Research Questions

The formulation of the research questions was supported by (a) the research conceptual framework, (b) suggestions found in the prior literature, (c) development of modern views on the role of mathematical modeling in school practice, and (d) the type of research methods employed. Intertwining these four pillars, the following research questions emerged:

1. What are the magnitude and direction of the learning effect size when students are instructed in mathematical modeling as compared to conventional methods of learning?
2. What are the possible moderators that affect students' achievement during modeling activities?
3. What are recommendations for a design of modeling activities as based on students' perspectives?

While question (1) and (2) will be answered by applying meta-analytic techniques and synthesizing experimental-pretest posttest studies, question (3) will be answered by synthesizing qualitative research findings.

The Main Key Term Descriptions

Several key description terms were formulated to guide the qualitative and quantitative research literature search. As the listing below summarizes all the keys, some, for instance, *effect size*, were used to scrutinize only experimental pre-posttest quantitative research.

Mathematical modeling. The virtue of mathematical modeling is supported by Crouch and Haines's (2004) description that defines it as a process of moving from a real-word situation to a model, working with that model, and using it to further understand and develop or solve real-word problems. Thus, this definition was taken with a broader scope, and the research encompassed not only research involving activities concluding with a model but also those that used the model to find a unique solution.

Model-eliciting activities. MEAs provide a theoretical framework for mathematical modeling activities used as a treatment design in the accumulated research. Due to this framework being relatively new in the mathematical research community, studies that satisfied major phases of MEAs but that did not explicitly highlight following all phases of MEAs were also included in the pool.

Student achievement in mathematics. Student achievement is defined as a percent score or their equivalent decimal form on solving various mathematical structures adequate to high school and college math curricula where MEAs or some of their phases were used as a treatment. The basis for calculating student achievement was their performances on standardized or locally developed tests.

Effect size statistic. Effect size is a statistical parameter used to quantify student achievement scores in the meta-analytic part of the study. It is computed using mean posttest scores of treatment and control groups along with the coupled standard deviation of both groups. If these quantifications are not available, other statistic parameters such as F-ratio, t-statistics, or p-values were used according to formulas formulated by Lipsey and Wilson (2001).

Grounded theory. Grounded theory is a result of synthesis of qualitative research (Lincoln & Guba, 1985). It formulates general inferences and recommendations based on that research.

Themes. Themes are common inferences from debriefing qualitative research that were used to formulate grounded theory.

Data Collection Criteria and Descriptions of Coding Study Features

Due to the dual focal point of this synthesis—qualitative and quantitative—the initial search criteria included the following: (a) time span: this study intended to synthesize research published between January 1, 2000, and February 31, 2013, on applying mathematical modeling to support student math learning at the high school and college levels; (b) type of research design: qualitative and quantitative; and (c) sample size: the minimum sample size established in this meta-analysis was 10 participants for experimental pretest-posttest research, and no limit was established qualitative research. In the meta-analysis part, we allowed only experimental research that provided means of calculated effect size statistics. The following defines features that were extracted from the accumulated research.

Descriptive parameters. Descriptive parameters encompassed the following: the grade level of the group under investigation, the locale where the studies were conducted, the sample size representing the number of subjects in experimental and control groups, the date of the study publication, the duration of the study, and the total time interval that the subjects were under treatment. The total treatment time was introduced due to a high diversity of treatment frequency; thus, for instance, if the study lasted 2 months and the treatment was applied twice a week for 3 hours each session, the reporting on this study would be shown as 2 months/48 h.

Inferential parameters. In order to compute study effect sizes, posttest mean scores of experimental and control groups and their corresponding standard deviations were extracted. If these were not provided, F-ratios or t-statistics were recorded. Although most of the studies reported more than one effect size, for example, Schoen and Hirsch (2003) and Wang, Vaughn, and Liu (2011), who also reported on students' attitudes toward mathematical modeling activities, the current study focused on reporting effects of student achievement only and mediated by highlighted moderators.

The research authors. A complete list of research-leading authors and co-authors involved in each study completion will be provided in the general tabularization summary. As the analysis progressed, each primary study was denoted by its leading author and the year of research conduct.

Publication bias. We focused on extracting the studies that were peer-reviewed and published as journal articles; thus, no additional category in the summaries was created. By embracing the study publication selection in these criteria, a publication bias was reduced.

Group assignment. Shadish, Cook, and Campbell's (2002) definitions supported identification of group assignments. During the coding process, two main categories emerged: (a) randomized, where the participants were randomly selected and assigned to the treatment and control group; and (b) quasi-experimental, where the participants were assigned by the researchers.

Type of research design. Experimental studies that provided pretest-posttest means or other statistic parameters allowing the means calculations along with experimental mixed-method research were scrutinized.

Medium used for model construction. Medium for modeling is defined as information presented as data tables, a written text problem, computerized scenario, or real experiment. Any of these type of media will be categorized.

Descriptions of Moderators

A total of 12 moderators were formulated to be extracted from each study that met the general criteria. The moderator group classifications along with their levels are presented in Table 11.

Table 11

Summary of Group of Variables and Their Classes

Group	Variables
Study general characteristics	Research authors
	School level (high school or college)
	Subject area (calculus, statistics, or algebra)
	Locale of the research (country where the study was conducted)
	Year of publication (year when research was published)
	Type of publication (peer-reviewed journal articles, conference proceeding)
Study methodological characteristics	Instrumentation (computer-supported activity or pen and paper)
	Reliability of measure (researcher-developed instrument (local) or standardized tests)
	Type of research (qualitative, quantitative, or mixed methods)
	Group assignment (randomized or quasi-experimental)
	Sample size (number of participants in control and experimental groups)
Study design characteristics	Program used, research specifications (verbal descriptions)
	Duration of treatment (in semesters, weeks, or days)
	Frequency of treatment assignment (in hours per day or other metrics provided)
	Type of model eliciting activities
	Medium for model construction (computer or context provided on paper)

In the process of collecting the research literature, ERIC (Ebsco), Educational Full Text (Wilson), Professional Development Collection, and ProQuest educational journals, as well as Science Direct, Google Scholar, and other resources available through the university library, were used. In the process of locating the relevant literature, the following terms were utilized: *mathematical modeling*, *model eliciting activities, simulations*, *computers in mathematics, mathematics education, student achievement*, *high school, college*. These search criteria returned 387 articles. After a review, it was revealed that 19 of these research studies satisfied the criteria, including eight studies of a qualitative nature. Most of the rejected studies focused on examining formulated models in the professional fields of engineering or medicine. In order to increase the statistical significance of this review, a further search was undertaken with broader conceptual definitions. This search included auxiliary terms that were found in descriptions of mathematical modeling activities, such as *investigations in mathematics, techniques of problem solving*, *exploratory learning in mathematics, and computerized animations and learning*. These modifications allowing for adjusting the contexts and strengthening the relevance of the study returned 82 research papers. After an additional scrutiny, 13 studies were coded as satisfying the research conditions, resulting in 32 primary studies, out of which 13 were quantitative, 16 were quantitative, and 3 were conducted using mixed research design. The validity of the coding and the extracted data was supported by a double research rating at the initial and at the concluding stages of the study. Any potential discrepancies were resolved.

Descriptive Analysis of the Accumulated Research Pool

The accumulated pool of studies that constituted raw data for the current research was at first analyzed descriptively. The purpose of such an undertaking was an attempt to summarize accumulated research in a meaningful way so that some patterns could be formulated. Once descriptive computations were concluded, further analysis was diverted in two independent channels: a meta-analysis of learning effect sizes of the quantitative studies and a synthesis of themes that emerged from qualitative studies. With such an aim, the data for the current research was constituted by 32 primary studies and a total of 2,925 participants. A general descriptive analysis of the studies is displayed in Figure 13 and Figure 14 that follow. In both of these graphs, the vertical axes represent the number of studies.

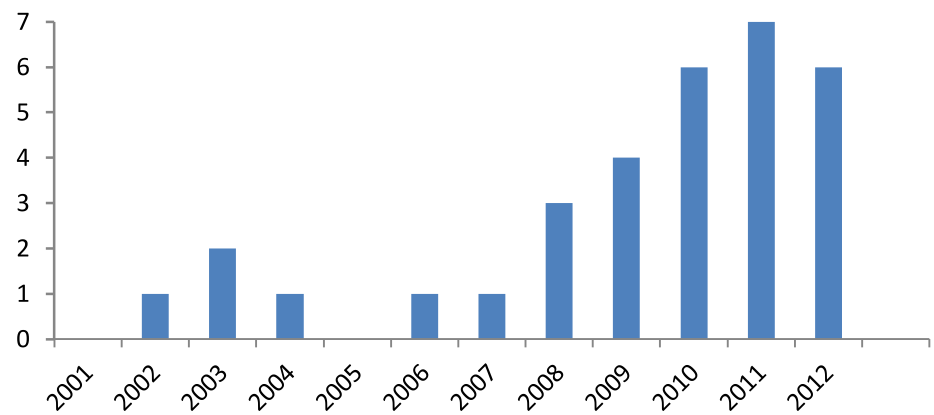

Figure 13. Distribution of studies per date of publication.

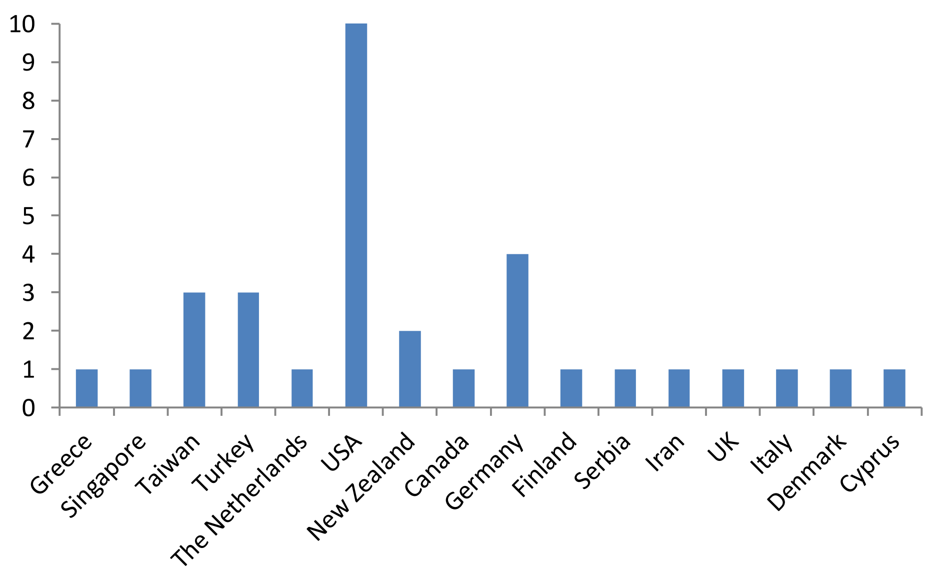

Figure 14. Distribution of studies per locale.

Most research (26, or 81%) was conducted within the past 5 years, which signalized a rapid increase of interest in mathematical modeling among the mathematical research community. Considering the locale and frequency of studies, the United States, with 9, or 28%, had the modal number of studies, followed by Germany, then Taiwan, and Turkey, each contributing three studies, or 9%. A high diversity of countries where the studies were conducted indicates a high global interest in conducting research on mathematical modeling. Table 15 shows categorization of the pool of studies as qualitative, mixed method, and qualitative and displays their relative frequencies.

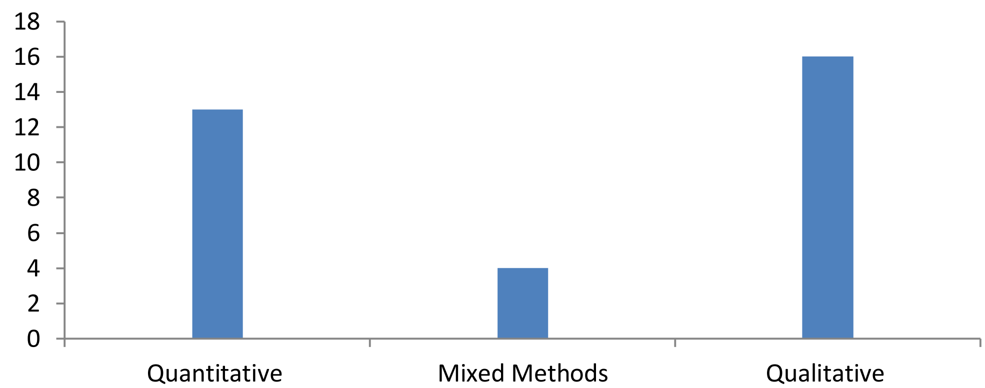

Figure 15. Distribution of studies per type of research.

As mixed-methods studies could provide findings for either or both of these general categorizations (meta-analysis or qualitative synthesis), one of them satisfied the condition to be meta-analyzed, and three were placed in qualitative synthesis. Further classification will reveal that as quantitative research was equally distributed among high school and college levels, qualitative substantially dominated the latter. The following section presents a detailed meta-analysis of the experimental research.

Meta-Analysis of Experimental Studies

Out of 32 studies, 13 studies (or 41%) satisfied these conditions. These studies underwent rigorous meta-analytic data quantifications. Thirteen experimental pre-post studies and 14 primary effect sizes were used for the meta-analytic part of the study. The total number of participants was 1,670. Table 12 provides a summary of the extracted features of these studies.

Table 12

General Characteristics of the Studies' Features

Authors	Date	Locale	RD	SS	School Level/ Subject	Research Duration/ Frequency	Learn. Setting	Medium of Learning
Young, Ramsey, Georgiopoulos, Hagen, Geiger, Dagley-Falls, Islas, Lancey, Straney, Forde, & Bradbury	2011	USA	QE	265	College/ Calculus	1 semester 1h/week	SC	Comp
Wang, Vaughn, & Liu	2011	Taiwan	QE	123	College/ Statistics	1 semester NP	SC	Comp
Voskoglou & Buckley	2012	Greece	QE	90	College/ Calculus	1 semester NP	SC	Comp
Laakso, Myller, & Korhonen	2009	Finland	R	75	College/ Statistics	2 weeks 2h/week	SC	Comp

Table 12 *Continued*

Authors	Date	Locale	RD	SS	School Level/ Subject	Research Duration/ Frequency	Learn. Setting	Medium of Learning
Milanovic, Takaci, & Milajic	2011	Serbia	QE	50	HS/ Calculus	1 week 4.5h	SC	Comp
Baki, Kosa, & Guven	2011	Turkey	R	96	College/ Geometry	1semester NP	SC	Comp
Bos	2009	USA	R	95	HS Algebra	8 days 55min/day	SC	Comp
Mousoulides, Christou, & Sriraman	2008	Cyprus	QE	90	HS Statistics and Geometry	3 months 3h	SC	Comp
Schoen & Hirsch	2003	USA	QE	341	HS Algebra	1 semester NP	SC	PP
Scheiter, Gerjets, & Schuh	2010	Germany	QE	32	HS Algebra	1 session 2h	SC	Comp
Eysink, de Jong, Berthold, Kolloffel, Opfermann, & Wouters	2009	The Nether-lands and Germany	QE	272	HS Probability	1 week	SC	Comp
Bahmaei	2012	Iran	R	60	College/ Calculus	1 semester 15 sessions	SC	PP
Baki & Guveli	2008	Turkey	QE/ MS	80	HS Algebra	1 semester NP	SC	Comp

Note. R = randomized, QE = quasi-experimental, DN = design, SC = student centered, MS = mixed methods, Comp = computer PP = pen and pencil, HS = high school, SS = sample size, NP = not provided.

The majority of the studies (9, or 70%) were designed as quasi-experimental, while five (30%) were randomized. The study durations ranged from 1 semester to 2 hours. Where it was provided, a frequency of treatment was also coded and reported. The average sample size for the study pool was 123 participants, with the highest being 272 in a study conducted by Eysink et al. (2009) and the lowest sample being 32 students in a study conducted by Milanovic et al. (2010). When categorized by school level, college and high school were uniformly represented, with six high school studies (or 46%) and seven college studies (or 54%). When categorized by learning setting, all of the studies were student centered, meaning that students worked on derived models of the given scenarios using the teachers' expertise only when needed. Model construction was supported by using computerized simulations in 11 (or 85%) of the studies; only two studies (or 15%) used the traditional pen-and-pencil approach.

Inferential Analysis

The inferential analysis of this study pool was initially performed using SPSS 21 (Statistical Package for the Social Sciences) software. We used the program and built in graphical capability to verify the homogeneity of the study pool, as suggested by Cooper (2010). For this stage, we calculated the effect size for each study present in the pool using posttest means on experimental and control groups, as suggested by Lipsey and Wilson (2001).

Such standardized individual effect sizes were then corrected for population bias, as suggested by Hedges (1992), and weighted. After weighted effect sizes were computed, the overall weighted mean effect size statistic for the study pool was calculated. The homogeneity statistics was also calculated ($Q_T = 329.74$, with $d_f = 16$, $p < 0.01$) and indicated that the pool of effect sizes variation was statistically significant;

thus, a random-effect model was adopted for the further data analysis. In order to visually justify the degree of heterogeneity of the study pool and depict the relations between the overall mean effect size, individual effect sizes, and the confidence interval of individual effect sizes, a funnel plot was generated.

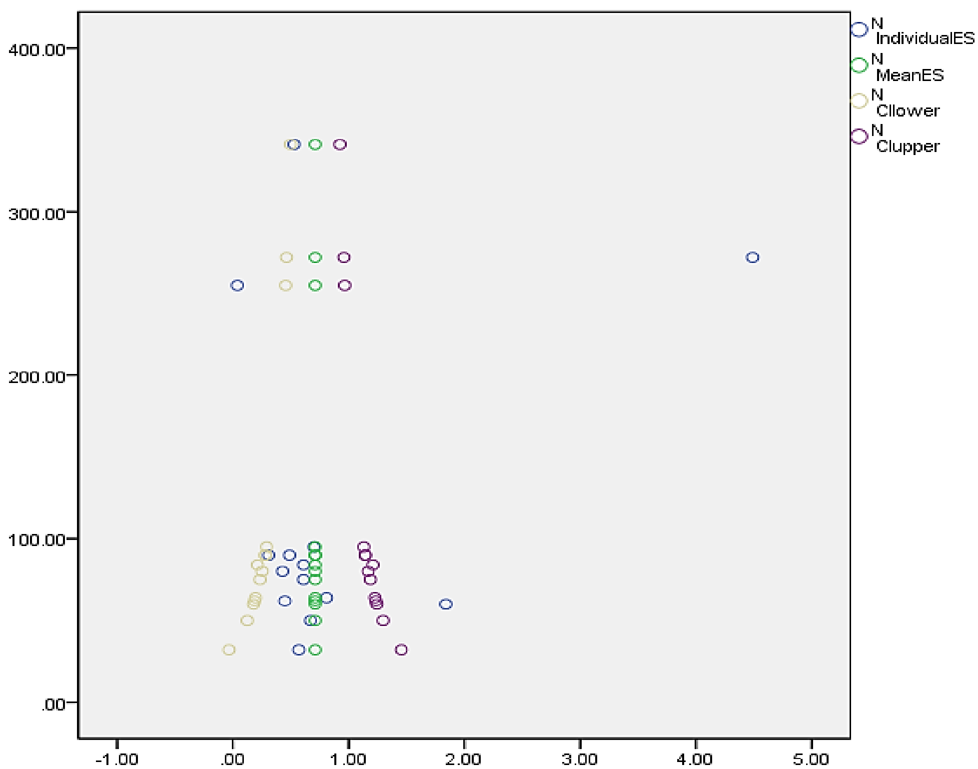

Figure 16. Funnel plot for the data.

The funnel plot (see Figure 16) shows that the effect sizes of three of the studies (or 23%) were outside of their confidence intervals, yet the majority—10 (or 77%)—were located within the confidence intervals. The mean effect size for the 13 primary studies (14 primary effect sizes) was reported to have a magnitude of 0.69 (SE = 0.05) and a positive direction. A 95% confidence interval around the overall mean—C_{lower} = 0.59 and C_{upper} = 0.79—supported its statistical significance and its precision (Hunter & Schmidt, 1990). The numerical magnitude of the effect size of 0.69 is classified by Lipsey and Wilson (2001) as having a moderate size. Its positive direction indicated that the score of an average student in the experimental groups, who used mathematical modeling and computer programs to enhance problem-solving techniques, was 0.69 of standard deviation above the score of an average student in the control groups, who was taught the process of problem-solving using traditional methods of instruction. Another examination of the computed effect size incorporating modeling processes scored higher on unit tests than 70% of students who learned the same concepts being taught by traditional methods. Hence, it can be concluded that using mathematical modeling to support the process of problem-solving has a profound impact on students' achievement compared to conventional methods of teaching.

Table 13

Effect Sizes of Applying Mathematical Modeling in High School and College

Study (First Author)	ES	SE	95% CI		Reliability of Measure	Program Used, Research Findings, Research Specifications
			Lower	Upper		
Bos (2009)	0.70	0.21	0.18	1.01	Used Standardized Texas state assessment. Kuder-Richardon formula 20 for reliability: r_{pret} = .80 and r_{postt} = .90.	Used TI Interactive Instructional environment.
Young (2011)	0.61	0.13	0.10	1.09	Used (UCF) university faculty Math Department tests. Inter-rater reliability: r_{pret} = .82 and r_{postt} = .92.	Research modeling activities (Excel) supported by computer.

Table 13 *continued*

Study (First Author)	ES	SE	95% CI		Reliability of Measure	Program Used, Research Findings, Research Specifications
			Lower	Upper		
Baki (2011)	0.81	0.26	0.09	1.11	Used PCVT test with KR-20 of r_{pret} = .82 and r_{pret} = .80 (Branoff, 1998).	Used interactive geometry software.
Young (2011)	0.04	0.13	0.34	0.85	Used University (UCF) faculty Math Department tests. Reliability: r_{pre}= .82 and r_{post} = .92.	Used research modeling activities (Excel) supported by computer.
Wang (2011)	0.45	0.26	0.08	1.11	Used researcher-developed 20-item test, Conbach's α = .91.	Developed dynamic computer program that modeled real situations to test hypothesis.
Voskoglou (2009)	0.49	0.22	0.17	1.03	Used researcher-developed test graded by two faculty members.	Contextualized differential equations using computer programs.
Laakso (2009)	0.61	0.24	0.12	1.07	Used researcher-developed test.	Used Trakla2 to have learners developed probability principles.
Milanovic (2010)	0.67	0.29	0.02	1.18	Used researcher-developed test, items the same on both pretest and posttest.	Developed simulated program to evaluate integrals. Used Macromedia Flash 10.
Mousoulides (2008)	0.31	0.22	0.17	1.03	Used researcher-developed test.	Used researcher-designed activities aimed at various math model formulations.
Schoen (2003)	0.53	0.11	0.38	0.81	Used standardized calculus readiness test items.	Developed new math curriculum that focused on modeling.
Scheiter (2009)	0.57	0.36	-0.14	1.33	Used researcher-developed test aligned with Reed (1999) categorization.	Used computer programs to enhance modeling through animated situations.
Eysink (2009)	4.49	0.12	0.35	0.84	Used researcher-developed 44-item test. Reliability was determined by Cranach's α = .64 and α = .82.	Used different multimedia settings to investigate the effect on students' math inquiry skills.
Bahmaei (2012)	1.84	0.26	0.07	1.13	Used researcher-developed test items.	Used researcher-developed activities.

Table 13 *continued*

Study (First Author)	ES	SE	95% CI		Reliability of Measure	Program Used, Research Findings, Research Specifications
			Lower	Upper		
Baki (2008)	0.43	0.23	0.14	1.05	Used researcher-developed test items with reliability of r_{postt} = .62.	Used web-based mathematics teaching material (WBMTM).

Note. ES = effect size, SE = standard error.

Table 13 provides a summary of the individual effect sizes of the meta-analyzed studies along with their confidence intervals, standard errors, and general descriptions of the treatment and computer programs used as a medium for model constructions. All meta-analyzed studies reported a positive effect size when an exploratory learning environment was applied. The highest effect size of ES = 4.49 was reported by Eysink et al. (2009), who investigated the effect of multimedia on students' modeling skills, and the lowest of ES = 0.04 was reported by Wang et al. (2011), who investigated the effect of using computerized programs on Calculus 2 students' skills of modeling differential equations. Several researchers (e.g., Wang et al., 2011) applied the Crnobach's α-coefficient or other reliability measures, such as Kruder – Richardon's formula 20, to support reliability of the assessment instrument. A reliability coefficient of the assessment instrument was induced in six (or 46%) of the studies. Table 2 also contains additional information provided by the primary researchers that distinguish the applied exploratory environment within the study pool. In the majority of the studies, the modeling activities were supported by researcher-developed contexts consistent with the curriculum. The studies were further aggregated into four subgroups to identify potential moderator effects.

Analysis of Moderator Effects

The process of computing subgroup effects allowed for uncovering moderators that optimized the magnitude of the effect size statistic and consequently helped with the design and implementation of mathematical modeling activities in school setting. We realized that to have the most accurate data and most accurate inferences, the activities used during these studies would have to be coded according to the MEA principles defined by Lesh and Kelly (2000). However, such extractions from the studies were not feasible at a high extent, due to perhaps MEA principles not being converted into providing quantitative constructs yet. The outcomes of designing activities following MEA principles more rigorously were found in several qualitative studies, which will be summarized in the following section of the current study.

A set of four moderators was identified: school level, instrumentation**,** treatment duration, and math content domain. This categorization resulted in 10 subgroups whose effects were individually computed and summarized in Table 14. The mathematical calculations associated with this part of the analysis were performed following Cooper (2010), who suggested giving more weight to effect sizes with larger sample populations according to the formula of w = inverse of the variance in the effect calculations. Calculation of corresponding confidence intervals and standard errors helped summarize the effect sizes according to the formulated moderators and their subgroup levels.

Table 14

Summary of Subgroups' Weighted Effect Sizes

Moderators and Their Classes	N	ES	SE	95 % CI	
				Upper	Lower
Grade Level					
High school	7	0.94	0.07	0.79	1.08
College	7	0.45	0.08	0.30	0.61
Medium Supporting MEA					
Computer simulations	12	0.72	0.06	0.60	0.85
Pen and paper activities	2	0.68	0.10	0.48	0.88
Treatment Duration					
Semester	8	0.46	0.06	0.34	0.59
Shorter than one semester	6	1.31	0.10	0.11	1.50
Content Domain					
Algebra	4	0.73	0.09	0.55	0.91
Calculus	5	0.38	0.09	0.19	0.56
Probability and Statistics	4	3.11	1.17	3.11	3.80
Geometry	1	0.81	0.26	0.09	1.11

Note, N = number of participants, ES = effect size, SE = standard error.

The grouping into levels provided a relatively helpful source of information about the effects of modeling activities on students' mathematics achievement at the high school and college levels that can be used as suggestions for the activities' designs. A more detailed discussion of each moderator effect follows.

Are the effect sizes of student achievement different across the school levels? This block was created to mediate the effect sizes of students' achievement between the high school and college levels. Although it was intended to differentiate not only among high school grade levels but also among college majors, due to the limited pool of studies, this idea was aborted and two general group levels—high school and college—

were formulated. The computed effect size showed differences across the school levels, with high school students achieving a larger effect size of ES = 0.94 (SD = 0.07), and college-level students achieving an effect size of moderate magnitude of ES = 0.45 (SD = 0.08). It is apparent that high school students benefit even more by being involved in modeling activities than college-level students. This result can be accounted for by other mediators (silent in these studies), such as the difference in difficulty level of high school and college math or better acquaintance of high school students in learning mainly from computerized modeling activities. As modeling is a relatively new math learning method, some college students might find it difficult to alter their views and habits of considering mathematics as a subject of *drill and practice* to a subject that provides a basis for explorations and opportunities for genuine applications. The element of previous experience might have an impact on the students' achievement, although this is just a hypothesis that would need further research. The data accumulated in the pool do not provide the basis for supporting such a hypothesis; however, if it proves to be true, calling for a broader implementation of modeling activities even at lower school levels such as primary or middle, as advocated by English and Watters (2005), has our full support. Developing modeling skills and techniques that require high-order skills of analyzing and synthesizing knowledge of multiple subject areas requires certain time. It seems that the sooner such skills are initiated and brought forth, the sooner the learner will become a proficient modeler and problem solver.

Does the medium used to elicit modeling activities affect students' achievement? Two media—computer simulations and written pen-and-pencil activities—were identified in the gathered pool of qualitative studies. Computers were used in 12 (or 86 %) of the studies to support modeling activities, and written pen-and-pencil methods were used in two of these studies (or 14%). The learning effect size produced by simulations was higher (ES = 0.72, SD = 0.06) when compared to traditional pen-and-pencil activities (ES = 0.68, SD = 0.10). An unquestionable advantage of computer simulations is their interactivity that allows for variable manipulations and easiness of principle identification in generalization and mathematization. Simulations also allow for a clear verification of the constructed mathematical model by manipulating system parameters and observing the changes. Thus, using them in school practice to support modeling activities is highly recommended. Yet, the medium itself, as noted by Noble, Nemirovsky, Wright, and Tierney (2001), will not generate learning because concepts, principles, and ideas do not reside in physical materials or classroom activities but in what students actually do and experience. It seems that careful inquiry planning coupled with learning mediums are the prerequisites for initiating students' engagements and their learning. Research (Young et al., 2011) shows that providing students with detailed descriptions of procedures to follow without letting them explore and discover relations on their own is not an effective inquiry planning. The virtue of mathematical inquiry design as seen from the subject perspective will be undertaken in detail in the qualitative research synthesis.

Does the length of treatment have an effect on student achievement? Two different classes were formulated to answer this question: one semester and shorter than one semester. At the college level, some of the research was designed in the form of a math modeling course lasting one semester with modeling activities embedded during the course.

The effect size computation for this subgroup showed that shorter treatments produced a higher effect on students' achievement (ES = 1.31, SD = 0.10) than longer treatments (ES = 0.46, SD = 0.06). Consequently, it seems that shorter treatments, considered additions to courses, support the learning objectives more effectively than longer ones. Depending on the content domain modeled, modeling activities might be difficult for students (e.g., modeling that involves differential equations; Milanovic et al., 2011), yet any improvement as compared with traditional teaching methods is worthy of implementing.

Does the effect size depend on the math domain being modeled? Four different domains were formulated for this subgroup: algebra, calculus, probability and statistics, and geometry. The frequency of studies in each level was highly dispersed, ranging from one that examined modeling the concepts of geometry to five modeling calculus concepts. According to the computations, probability and statistics produced the highest effect size (ES = 3.11, SD = 1.17). The magnitude of this effect size was strongly affected by an outlier of 4.49 (Eysink et al., 2009). If this study were rejected, the effect size would have been ES = 0.21, and SD = 0.17. The concepts of algebra and its sub-domain, function analysis, produced a moderately high effect size of ES = 0.73. It is to note that calculus concepts were the most frequently researched, although they did not produce the maximum effect size. Calculus, a study of change and accumulation, provides a wide range of sophisticated apparatus for inducing mathematical modeling activities, but it seems that how students apply calculus to learn about real-life phenomena and how to support their learning through modeling activities needs further research.

Summary of Quantitative Research Findings

Combining all of the inferences from this part of the study, it is apparent that modeling activities generate positive learning effects when compared to traditional teaching methods. Further, a subgroup moderator analysis revealed that the setting producing the maximum effect was a high school mathematics model, conducted in the form of short additional activities enhanced by computer simulations. A need for preparing instructional materials was voiced frequently in the research, despite MEAs' design being proposed. The activities developed by researchers used as an instrument were mostly locally developed, that their transition to be used by other schools is limited. A need for firming the type of inquiry and bridging it to other subjects involved during modeling processes also emerged. Strengthening these phases will not only help the learners develop a scientific view of knowledge acquisition but also help achieve the goals of STEM education.

One of the problem questions not being explicitly stated in this research but hoped to materialize during the literature review was the relation of word problem-solving and mathematical modeling. Should both be separately taught, or should one be a complement of the other? If so, which process is more general? Although modeling activities are to provide general inquiry methods for problem-solving not only in mathematics classes but also in other courses, especially science and engineering (e.g., Hestens, 1985), these two areas seem to be disjointed in the current research. Perhaps one of the reasons is that researchers are still debating the place of modeling in mathematics education. As many educators would see modeling as a separate activity, for example, Blum and Niss (1991), others, for example, English and Sriraman (2010), have proposed otherwise. It seems that to find more convincing support for one or the other approach, the voices of the learners and the teachers conducting such activities should be heard. How do they perceive these two activity strands? Does modeling help students solve word problems in mathematics? If so, what phases of modeling are especially helpful? The qualitative part of the research is posited to provide more insight into this domain.

Synthesis of Qualitative Research

While meta-analysis provided measurable effect sizes of using mathematical modeling activities along with quantification of formulated moderators, qualitative research was used to extract findings that provide information about how students learn by being immersed in such activities as well as what are their concerns and recommendations for improving their learning experiences. Thus, it is hoped that the qualitative part along with several mixed-methods research will enrich the findings and provide a better picture of the role of modeling activities in school practice.

Descriptive Analysis

This synthesis encompasses findings from 19 studies conducted with over 1,256 students at the high school and college levels. Table 15 provides general descriptive characteristics of these studies and short summaries of their key features.

Table 15

Summary of General Futures of Qualitative Research Pool

Authors	Date	Locale	Research Type and Assessment Instrument	SS	School Level/ Subject	RD/TD/
Yoon, Dreyfus, &Thomas	2010	New Zealand	QUAL Interview	18	College/ Calculus	1 hour/NP
Lim, Tso, & Lin	2009	Taiwan	QUAL Questionnaires Interview	26	College/ Applied Math	2 month/sNP
Liang, Parsons, Wu, & Sedig	2010	Canada	QUAL Interview	30	HS/ Geometry	3 days/7.5h
Leutner	2002	Germany	QUAL Repeated measure	228	College/ HS/ Algebra	1 day/70 min
Chinnappan	2010	USA	QUAL Observations	28	HS/ Algebra	1 day/45 min
Crouch & Haines	2004	UK	MM	23	College/ Calculus	1 day/45min
Yu & Chang	2011	Taiwan	QUAL Observation, Video Questionnaires	16	College/ Teachers	9 weeks/18h
Diefes-Dux, Zawojewski, Hialamarson, & Cardella	2012	USA	QUAL Questionnaires	200	College/ Engineering	1 day/4 hours
Faraco & Gabriele	2012	Italy	MM One group design	59	College/ Mathematica l Methods	NP/NP
Iversen & Larsen	2006	Denmark	MM One group design	35	HS/ Calculus	7 weeks/NP
Klymchuk, Zverkova, Gruenwald, & Sauerbier	2008	New Zealand/ Germany	QUAL Questionnaires	147 25	College/ Engineering	1 semester/NP
Schorr & Koellner-Clark	2003	USA	QUAL Questionnaires Observations	58	College Teachers	1 day/ 1 h

Table 15 *continued*

Authors	Date	Locale	Research Type/ Measuring Instrument	SS	School Level/ Subject	RD/TD
Turker, Saglam, & Umay	2010	Turkey	QUAL Survey and Interview	60	College/ Teachers	1 day/ 1 h
Soon, Lioe, & McInnes	2011	Singapore	QUAL Survey	50	College/ Engineering	1 semester/ NP
Yildirim, Shuman, & Besterfield-Sacre	2010	USA	QUAL Interview	5	College Calculus	1 semester/ NP
Cory & Garofalo	2011	USA	QUAL	3	College/ Calculus	1day/ 1 hour
Schukajlow, Leiss, Pekurun, Blum, Muller, & Messner	2012	Germany	QUAL Interview Survey	224	HS/ Calculus	NP/ 10 lessons
Sokolowski & Gonzalez y Gonzalez	2012	USA	QUAL Interview	6	Mathematics Teachers	6 days/ 3 h
Carrejo & Marshall	2007	USA	QUAL Observation Interview	15	College/ Teachers program	5 weeks/ 15h

Note. MM = mixed methods, SS = sample size, HS = high school, NP = not provided. RD/TD = research duration/treatment duration.

Several evaluation instruments were used in the qualitative research which included; interviews with participants, surveys, observations, and questionnaires. The sample sizes of this research pool ranged from three subjects (Cory & Garofalo, 2011) to 228 subjects (Leutner, 2002); the average sample size was 60 subjects. When categorized by school level, 14 of these studies (or 74%) were conducted on the college level involving mainly calculus students, and four (or 21%) were conducted on a high school level. The populations included practicing teachers that were found in two studies (e.g., Schorr & Koellner-Clark, 2003). Four studies (or 21%; e.g., Turker et al., 2010) were conducted at the college level and involved students from teacher preparatory programs (e.g., Carrejo et al., 2007). This trend indicates that preparing teachers to teach students modeling techniques has gained popularity in mathematics education. Considering the ratios of the populations, it is evident that the interest in examining the effects of applying mathematical modeling in mathematics gravitates toward college-level education (in the meta-analysis, the number of research studies at each school level was similar). There was a noticeable diversity in the studies' duration, ranging from 1 hour (e.g., Cory & Garofalo, 2011) to 1 semester (e.g., Klymchuk et al., 2008; Yildirim et al., 2010).

Inferential Analysis and Themes Formulation

While qualitative research unfolds as data are gathered, each study considered as an individual source of information was further scrutinized. With a goal of searching for key features that reflected the most promising features of modeling activities— as seen from the students' perspective— and also their shortfalls, general treatment descriptions along with the study findings were tabularized. Table 16 displays summaries of these findings and the analysis of these findings follows.

Table 16

Summary of Treatment Descriptions and Research Findings

Leading Authors	Treatment Description and General Findings	Medium Applied
Yoon (2010)	Used MEAs after an instructional unit to support the process of integration. Investigated change of students' perception and interpretation of calculus tools.	PP
Lim (2009)	Used MATLAB and computer to model real scientific happening (the process of volcanic ash fall). Investigated change in students' attitude toward mathematics. Mathematics appeared to generate a friendlier environment to students. Focused on having students interpret partial derivatives that emerged from given differential equations.	PP
Liang (2010)	Used interactive 3D objects to formulate patterns for volume and surface area computing. Investigated students' change of interpretations of some abstract geometry terms.	COMP
Leutner (2002)	Used dynamic simulation called SimCity to enhance problem solving through modeling skills. Measured participants' comprehension skills due to applied modeling processes.	COMP
Chinnappan (2010)	Observations were made during one lesson on students' discussion of modeling techniques and approaches. Students' descriptions of math terms were more detailed and focused.	PP
Crouch (2004)	Analyzed reflective questionnaire and used interview to distinguish between novice and expert modelers. Expert modelers used math tools with a greater flexibility.	PP
Yu (2011)	Teachers engaged in MEA. They solved modeling problems and designed some. Teachers perceived modeling as a bridge to problem solving.	PP
Diefes-Dux (2012)	Used web-based MEA resources to support modeling activities. Evaluated grading processes of modeling activities by instructors. General suggestions for students' modeling activities emerged. MEA was applied after certain math concepts were introduced.	PP
Faraco (2012)	Students used Lab VIEW to develop program that simulated physical phenomena. Conclusion: principle understanding is needed for successful modeling techniques.	PP
Iversen (2006)	Modeled real-life situations. Derived algebraic functions and evaluated the functions following MEAs. Focused on assessing strengths and weaknesses of the modeling processes.	PP

Table 16 continued

Leading Authors	Treatment Description and General Findings	Medium Applied
Klymchuk (2008)	Students took course on modeling being given non-traditional, interactive life contexts. Differential equations were provided. Students were asked to interpret variables. Students priced correlations of the tasks to reality and claimed that mathematical modeling improved their problem-solving skills.	PP
Schorr (2003)	Future teachers provided feedback about teaching the processes of modeling. Positive changes in students' attitudes and knowledge emerged as an impetus for changing teachers' instructional methods.	NP
Turker (2010)	Participants worked on modeling activities. They were surveyed and stated that mathematics concepts became more tangible to them.	PP
Soon (2011)	Students worked on modeling activities involving DE and linear algebra. Auxiliary steps were provided.	PP
Yildirim (2010)	Investigated students' process on MEAs. Students had difficulties with hypothesis stating.	PP
Cory (2011)	Used sketchpad to visualize the concept of limits.	COMP
Schukajlow (2012)	Students worked on diverse modeling problems in two different learning settings such as student and teacher centered. Their perception on problem solving was analyzed. Student-centered modeling benefited the students the most.	PP
Sokolowski (2012)	Interviews with teachers were conducted and aggregated into themes. Teachers expressed a need to have modeling activities available to put in practice.	NP
Carrejo (2007)	Teachers were involved in mathematical modeling activities. Need for implementing mathematical modeling in teacher preparatory programs has arisen.	Real Lab

Note. PP = pen and paper, COMP = computer, NA = not applicable.

When categorized by medium-supporting modeling activities, traditional pen-and-pencil activities dominated these studies (used in 13); computers were used in three of the studies, and a real lab as a medium was applied in one study. According to procedures of analyzing a qualitative research study, its inference concludes with formulations of categories of concepts that are used to formulate a research grounded theory (Lincoln & Guba, 1985). Constant comparisons of the accumulated research justified by inductive reasoning helped formulate inferences of this study pool. The following themes emerged from this analysis:

- Concerns about college-level modeling.
- Teachers' role during modeling activities.
- The degree of contextual support during modeling activities.
- Sequencing of modeling activities in math curriculum.
- Problem solving and modeling.

Discussion and synthesis of these themes led to proposing a grounded theory embodied as a mathematical modeling cycle whose design will be presented and discussed.

Concerns about college-level modeling. The pool of college undergraduate-level modeling with 75% of studies dominated the qualitative research investigations. This substantial contribution indicates a high importance of qualitative research methods in examining mathematical modeling. Thus, one can conclude that the math research community is not only interested in computed learning effects sizes but also in knowing; (a) *what* the obstacles that the learner still faces are and (b) *why* mathematical modeling benefits the learner, (c) *how* the learner moves through the modeling cycle, and (d) *what* is the teacher's role during modeling activities.

Phase of converting reality into mathematical symbolism. A major concern voiced frequently in the accumulated research was students' inability to transfer scenario text description into its mathematical embodiment (e.g., Soon et al., 2011; Yoon et al., 2010). This phase is essential in modeling, and a deeper analysis of this deficiency is necessary to formulate suggestions for assistance. While analyzing the accumulated research with an intention to find these answers, two questions seemed to be left not discussed by research: Is the deficiency due to a weak student understanding of mathematical structures (e.g., the properties of periodic functions, the differences between rate of change and a percent change, the techniques of solving differential equations and the like), or is the deficiency due to difficulties in identifying conceptual patters in given problems and mapping the patterns on corresponding mathematical embodiments? If students cannot identify algebraic functions that would reflect given behaviors, then the reason for the deficiency is their lack of their mathematical knowledge or skills. If the difficulty lies in recognizing the properties of system behavior, then this deficiency can be attributed to a lack of scientific inquiry skills or lack of the contextual knowledge embedded in the modeling activity. Thus, the interface of integrating of the two different worlds—real situations and their corresponding mathematical models, as defined by Blum and Leiss (2007)— needs further investigation, and its importance should be augmented by the research community. As the college-level modeling encompasses all types of system behaviors (e.g. multivariable rate of change, two —or three— dimensional motion), the high school level will focus more on singular structures and on developing general foundations of the modeling processes.

Concerns about hypothesis formulation. Several researchers (e.g., Crouch & Haines, 2004; Faraco et al., 2012) pointed out concerns with weak student skills in formulating hypotheses for given problems and following through the process of proving or disproving these hypotheses. Hypotheses reflect closely on problem statement. Thus the hypothesis context builds on the problem stated in the activity. Once formulated, a hypothesis focuses the investigator's attention on a narrower area of investigation. Hypothesis can be perceived as the investigator's proposed theory explaining why something happens based on the learner's prior knowledge (Felder & Brent, 2004).

The role of a hypothesis is to confirm or correct an investigator's understanding of what the content of the modeling activity presents. As hypotheses in mathematical modeling activities will most likely be verbalized aiming at testing mathematical concepts rather than scientific, yet the contextual balance between these two academic domains needs to be established. Reducing problem statement to gearing students to formulating only mathematical dependence will not nurture the connection between real world and mathematical world as defined by Blum and Leiss (2007). Yet, due to mathematical modeling concluded often with a mathematical structure, hypotheses in mathematics will focus more on testing students' knowledge in applying these structures. For instance, if students are to derive Newton's second law of motion, then in mathematics classes their hypothesis will try to answer a question about what type of algebraic function can be used to describe the type of mathematical dependence between an object's acceleration and the net force, while in science classes their hypothesis will answer the question of how an object's acceleration depends on the net force acting on it. These hypotheses are mutually inclusive, and one can be perceived as a complement of the other, yet it is suggested that their formulation is subject domain dependent. Thus, a hypothesis plays a central role in the process of modeling. Students' difficulties in its formulation call for amplifying the hypothesis role and broader the discussions in mathematics classes where the term hypothesis is rather rarely used.

A need for more elaboration is also noted in the differentiation between hypothesis and prediction. As a hypothesis proposes an explanation for some puzzling observation, a prediction is defined as an expected outcome of a test of some elements of the hypothesis (Lawson, Oehrtman, & Jensen, 2008). In modeling, a hypothesis will reflect on general mathematical structures, whereas a prediction will constitute an extension of the activity supporting its further validation. The usage of these essential terms of scientific inquiry during mathematical modeling activities is not visible in the current research, yet it seems that its importance is high. An extension of this idea, empowering hypothesis testing with its statistical interpretation, seems to be a task worthy of further investigation. In sum, the general purpose of hypothesis formulation during modeling activities along with the process its proving or disproving will be congruent to process of hypotheses testing that students encounter in other subjects.

Classifying variables. Students' difficulties with isolating variables and classifying the variables for the purpose of formulating a mathematical structure also frequently surfaced in the research (e.g., Stenberg, 1997; Carrejo & Marshall, 2007;

Diefes-Dux et al., 2012; Faraco et al., 2012). While the categorization of variables as dependent and independent appears when the variables are labeled in the Cartesian plane or in the function notation is straightforward, these distinguishing categories cause doubts when realistic contexts are presented and the variables are not explicitly pictured. This difficulty concerns also the science research community (e.g., Halloun & Hestenes, 1985) thus, amplifying this phase— not explicated in the Lesh and Kelly (2000) MEA design —seems to be necessary. The research shows that classifying quantities as *given* and *required* traditionally done during problem solving in science is not sufficient to succeed during the process of mathematical modeling (Lim, Tso, & Lin, 2009). It is then hypothesized, based on research findings, that students do not transfer this technique to math classes automatically.

Furthermore, while in science classes students often reduce the process of solving problems to mapping the given set of variables to available formulas (Redish, & Steinberg, 1999), in mathematics classes the identification of *given* and *required* takes another step—it is often used to perceive the *given* and *required* as function parameters (e.g., slope, coordinates of vertex, period, initial value, etc.). The required quantities are then being classified as independent and dependent reflecting the context of modeling. Thus, while constructing the algebraic function or structure (e.g., rate, ratio, proportion, and the like), the goal of extracting given and required quantities from the problem or scenario requires another step, apparently often omitted in the current research on modeling. It is hypothesized that extracting these differences may help students with variables' identification and their classification.

Teacher's role during modeling activities. Although modeling activities are classified as student centered, instructors, even at a college level, play a vital a role. Diefes-Dux et al. (2012) suggested that instructors or graduate teaching assistants should be partners of innovation during modeling processes. They should suggest certain solutions when needed and correct certain modeling processes if the processes will not lead to a correct model formulation. A phrase *corrective guidance* surfaces description of the instructor assistance during college level modeling. Mason, Stephens, and Watson (2009) stated that teachers need to possess strategies and tactics for extracting structural relationships and bring them to the fore for the students. They also pinpointed teachers' enthusiasm toward MEA implementation as a significant factor: "If the instructor appreciates the potential benefits that the students can receive from MEA, he/she should more readily make the extra effort to properly guide the students" (Yildrim et al., 2010, p. 838). The teacher's role as a subtle guider through activities with modeling discourse is also advocated in by Hestenes (2013) who further suggested that the teacher promotes framing of *all* classroom discourse in terms of models and modeling in the aim to synthesize students to the structure of scientific knowledge. It is suggested that this recommendation is extended to designing modeling activities at any mathematics school level.

The degree of contextual support during modeling activities. Calculus as a study of change is the subject where modeling is most frequently exercised. Accumulated research shows that at the college level, mathematical modeling is perceived as applications of differential equations (DEs) and it is supported by providing students with general structures of DEs (e.g., Cory & Garofalo, 2011; Klymchuk et al., 2008). Students are expected to interpret the models and use the models to infer about embedded science/business or other domain-specific principles. While students generally possess the skills to solve DEs algebraically by separating the variables and taking antiderivatives of both sides of DEs, the scientific interpretations of these results have lacked precision (e.g., see Chaachoua & Saglam, 2006). This conclusion draws on students' weak understanding of the scientific context and consequently on the reality principle of the MEA designs (Lesh & Kelly, 2000) not being verified prior the activity assign.

In sum, the modeling activities at the college level are perceived as opportunities to put learned mathematical tools in practice (Soon et al., 2011). They do not represent standalone learning experiences but are to be blended into the curriculum and their role is to show undergraduate students ways of inducing modeling techniques into the real world. Derived models are used to predict future system behavior or compute quantities not observable or measureable by using directly the experiment outcomes. Math analysis is presented as a subject providing tools to delve deeper into the system behavior and extend the inferences beyond given parameters in the experiment. Although there several pitfalls of these processes, the math research community strives to remove them. As a subject, modeling becomes more frequently considered as an independent courses taught (e.g. Klymchuk et al., 2008; Yildirim et al., 2010).

Placing modeling activities within high school math curriculum. One of the themes that emerged from the high school modeling research findings was the sequencing of modeling activities within a chapter domain. There are two distinct voices raised in this matter: one advocated by Blum et al. (2007) and Lesh et al. (2007) that suggests that modeling activities be implemented prior to new content being taught, and the opposite view presented, for example, by Leutner (2002) and Chinnappan (2010) suggesting that modeling activities be implemented after new content is delivered. Both strategies seem to benefit the learners, yet caution needs to be given for the inquiry design of the activities. Lesh et al. (2007) suggested to place modeling activity at the beginning of new chapter. They supported their claim by pointing out that if the MEA is implemented as a concluding activity of the instructional unit, it guides students along necessary trajectories and turns the activity into mathematics applications, which is not what modeling activities should be about. A legitimate question in this context arises: Is associating mathematical modeling with exercising applications of mathematics diminishing the virtue of modeling activities? Furthermore, if the sequencing has not been a concern in college-level modeling activities, the question is why it would be a concern at the lower— high school — levels? MEAs as defined by Lesh and Kelly (2000) are not sensitive to where they are inserted into the curriculum. Considering the content of the simplicity principle (see Lesh & Kelly, 2000) that students must possess necessary mathematical tools and knowledge before engaging in modeling activities, implementing such activities after the content is delivered is concluded. There are further supports for such sequencing; Leutner (2002) advocated that students' pre-domain knowledge correlates with their achievement in problem solving modeling activities. A similar conclusion was researched by Chinnappan (2010), who stated that if the goal of teaching mathematics is developing students' structural understanding of concepts and embedding the concepts in realistic contexts, students need to learn the structures before exercising their applications. In his study, the students succeeded on the modeling processes being provided with structures along with scaffolds of the processes. Even at the college level, to have students succeed on modeling activities, they need to possess knowledge of mathematical structures that the problems involve. The complexity of the processes of uncovering and identifying the structures will depend on students' background, yet mathematical apparatus needs to be learned prior to the activity. For instance, if a quadratic function is to be used in a given problem, one needs to recognize that the given situation must suggest that the relation takes an extreme value; if a linear dependence is to be used, one needs to recognize that the rate of change between involved variables remains constant and so forth. Without providing means for recognizing these parameters, activities of modeling might result in endless trials, leaving the students frustrated and unmotivated to be involved in further such tasks. Constructing different MEA designs, undertaken with different theoretical scopes, would place more diversity on the sequencing. One of the solutions to this debate would be proposing an implementation of math modeling course to high school curriculum, where sequencing could be exhibited with a higher flexibility. It is to note that in the accumulated research pool no information was found about such math class designed.

Problem solving and modeling. As a relatively new theoretical framework in mathematical education research, investigating how MEAs lead students to problem solving, what are their strongholds, and which elements appear to be still under discussion emerged as pivotal themes from the qualitative analyses. In their study, Yu and collegues (2011) concluded that “developing the modeling ability promotes students problem solving ability” (p. 152). However, they also noticed a lack of theoretical background on how to transition the process of mathematical modeling to problem solving. The formulation of MEA theoretical framework directed to designing the content of modeling activities does not appear to be sufficient to help the instructor bridge students’ thinking to problem solving. Niss (2010) claimed that knowing mathematical theories does not guarantee that this knowledge is transferred automatically to an ability of solving real-life problems. However, there is a strong research supporting the thesis that carefully designed modeling environments can foster and solidify students’ problem-solving skills. While scaffolding is found to produce a positive effect on students’ math learning (Anghileri, 2006), the type of inquiry to apply, inductive or deductive, is still unanswered. Investigating the impact of the scaffolds and/or its removal might not be sufficient, as was suggested by Chinnappan (2010). It seems that the extent to which modeling activities help the learner with problem solving skills is rooted in the design of modeling activities and their capacities to develop students’ analytic skills and abilities to mediate contexts with mathematical tools. The phase of transitioning between modeling and problem solving needs much more attention not only form the modeling but also form the problem solving perspective. This conclusion has developed as another recommendation for further research.

Emerged Modeling Cycle

The findings of both parts of the research led to formulation of a mathematical modeling cycle (see Figure 17) that encompasses the global research recommendations. More specifically, it integrates elements of scientific inquiry as well as recommendations from the problem-solving math research community that were raised in the accumulated research pool. The proposed cycle highlights phases that were silent or absent in the quantitative part but whose importance has emerged through student responses, summarized in the qualitative part of this study. The purpose of this section is bifocal: it is to elaborate on the general structure of the proposed modeling cycle by pinpointing particular research findings that led to its emergence and to discuss its applications.

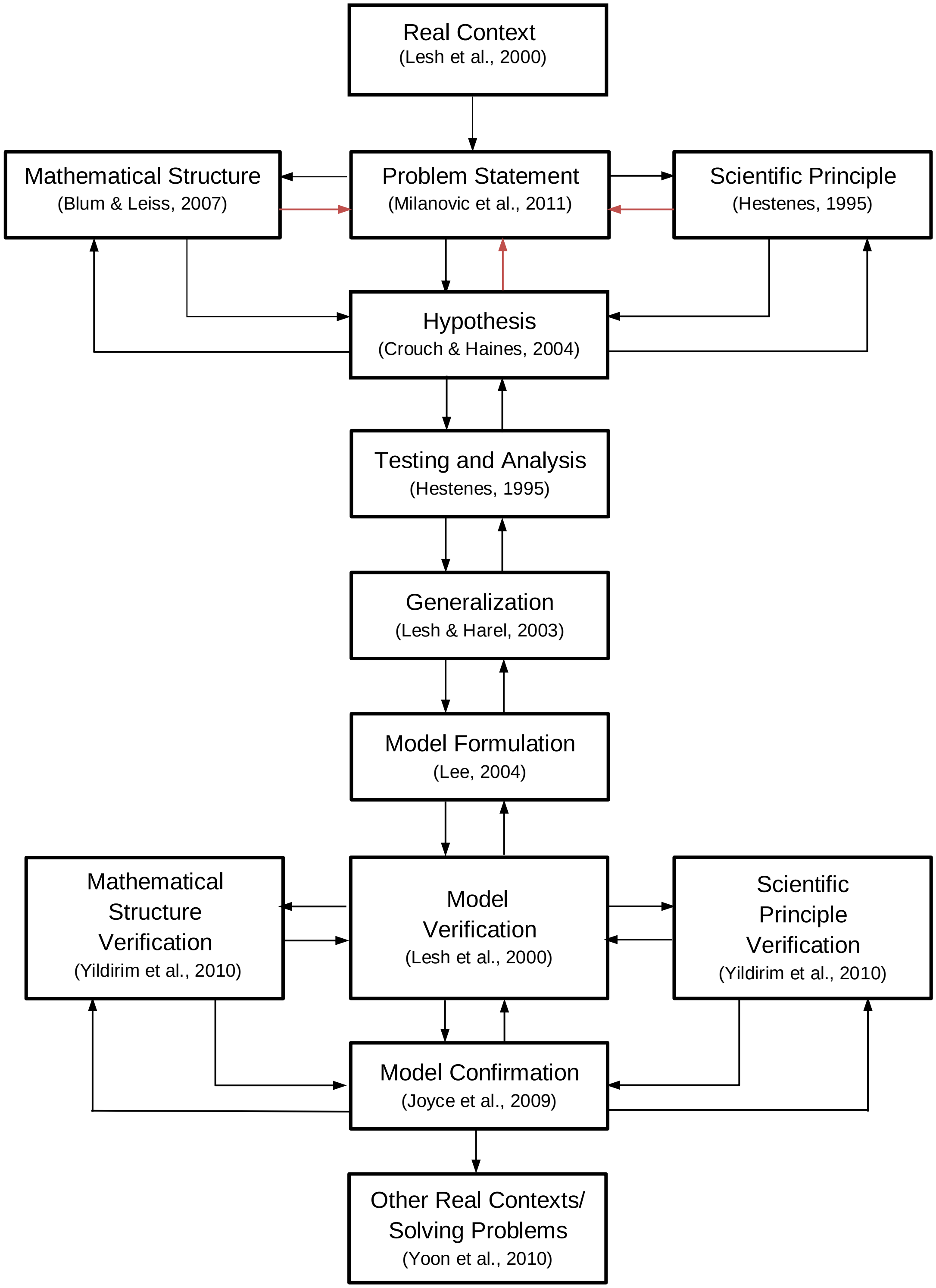

Figure 17. Proposed integrated math-science modeling cycle.

Deciding About the Type of Inquiry

One of the main questions seeking an answer was the selection of type of a suitable inquiry method for mathematical modeling processes. The inquiry method applied during modeling activities has not been discussed in detail in the accumulated research, and the literature has not provided a coherent view on what inquiry method should be used. Because the general method of reasoning affects the design and task formulation of the activity, a need for its establishment has emerged. Hestenes (2013) claimed that throughout their schooling, students must be engaged in scientific inquiry so that they learn how to form and justify rational opinions on their own. There are two main types of reasoning used in science, mathematics, and engineering: deductive and inductive (Prince & Felder, 2006). While deductive inquiry denotes the process of reasoning from a set of general premises to reaching a logically valid conclusion, inductive inquiry is a process of reasoning from specific observations to reaching a general conclusion (Christou & Papageorgiou, 2007). Viewed through these lenses, deductive thinking draws out conclusions, whereas inductive thinking adds information (Klauer, 1989). Because mathematical modeling processes are about pattern formulation and generalization (Lee, 2004; Lesh & Harel, 2003), inductive inquiry emerged as a leading form of reasoning for mathematical modeling. This selection is further supported by NCTM (2000) standards that recommend students' familiarity with this learning method and by research on using inductive thinking in general math knowledge-acquisition processes. For instance, Harverty, Koedinger, Klahr, and Alibali (2000) proved that inductive reasoning plays a significant role in problem solving, concept learning, and the development of math expertise. Inductive reasoning includes a range of instructional methods such as inquiry learning, problem-based learning, project-based learning, case-based teaching, discovery learning, and just-in-time teaching (Prince & Felder, 2006). Discovery-type instructional methods that accompany mathematical modeling (English & Sriraman, 2010) further supported inductive inquiry selection for mathematical modeling activities.

Type of Modeling Medium

The modeling processes (see Figure 11) that are set forth to mathematize processes happening in the real world (Blum & Leiss, 2007) are initiated by providing the learner with *Real Contexts*. The contexts must satisfy certain conditions: They must be exploratory (Flum & Kaplan, 2006) and must obey the five principles of MEAs formulated by Lesh and colleagues (2000). As the research pool shows (see Table 13 and Table 14), the contexts can be supplied by various means such as a written paper-and-pencil test, data provided by a table of values, or a real lab, or it can be presented by computerized simulations. The moderator effects of some of the means were summarized in Table 13. The complexity of the contexts will depend on (a) the math grade level taught, (b) activity objective that is to be achieved, and (c) time interval allocated for its completion

The Formulation of Problem Statement and Hypothesis

The catalyst for inquiry initiation is *Problem Formulation* or *Problem Statement* followed by *Hypothesis*. Research has shown (e.g., Crouch & Haines, 2004; Faraco et al., 2012; Milanovic et al., 2011) that students have difficulties with formulating the first two phases of the inquiry process. As students' confidence in formulating the two initial phases will have an impact on their engagement throughout the activity, this part of the modeling design demands more elaboration. If the learner falsely identifies the *problem* embedded in the given activity, he/she might not be able to extract quantifiable variables to prove or disprove the hypothesis. How can teachers help the learner with this stage? This question can be further reduced to whether the problem statement formulation should be a part of tasks assigned to the students or if the problem statement should instead be provided to students. The research pool did not elaborate on this issue; thus, we propose our position. It is important to note that the problem statement formulation does not necessarily have to have the form of a problem traditionally found in mathematics textbooks under the heading *Problem Solving*. Since a well-established precept in education is that a strong motivation to learn is generated by a strong desire to know (Albanese & Mitchell, 1993), we suggest that the problem statement be provided to the learner and the hypothesis formulation be assigned as a student task. This organization is further supported by the fact that the hypothesis is formulated based on the problem statement, and the problem statement is formulated through establishing certain control of the *Real Context* to reflect on the activity design (if prepared by the instructor).

The hypothesis formulation is very important because it focuses the students' attention and determines the analysis process. Furthermore, since hypothesis formulation depends on students' judgment and their prior knowledge (Felder & Brent, 2004), it will drive students' motivation to prove whether they were right or wrong. We recommend that in math modeling activities, the hypothesis formulation not only involve providing suggestions for extracting the mathematical structure but that it also require the students to incorporate the verification scientific principle rooted in the activity. For example, if the context of the activity refers to producing and selling goods, the mathematical structures used to model this context will most likely be polynomial functions along with an analysis of their intersecting points, while the principle of the context will be the law of supply and demand. If the context refers to modeling projectile motion, then mathematical structures will include parametric equations and the principle will refer to properties of gravitational field.

Discussion of Model Eliciting Phases

The contents and tasks of the next phases of the modeling process such as *Analysis*, *Generalization* and *Model Formulation* are similar to those proposed by Blum and Leiss (2007) and Hestenes (1995). Yet, as Blum and his colleagues suggest, separation of mathematical results and real results, we propose that both types of results mediate during the analysis and model-eliciting phases. While the analysis of the variables and its generalization is the stage at which the learner integrates the knowledge of math with the real world, the next phase, the *Model Verification*, is proposed to take two different paths, called in our modeling process *Scientific Principle Verification* and *Mathematical Structure Verification*. Having students verify mathematical structure along with the embedded scientific principle has not been emphasized in the prior modeling cycles. However, this modification reflects researchers' concerns (e.g., see Klymchuk et al., 2008; Yildirim et al., 2010) that students fail to validate formulated mathematical structures or have difficulty formulating the verification processes. This phase seems to be very important because it warrants reusability of the derived model (eg., see Lesh et al., 2000) and provides means of reflecting on the hypothesis and its math/other subject duality. In order to have the learner validate the derived model, special tasks targeting verification of the structure supported by contextual interpretation of tasks would require the students to solve the problems algebraically and also justify coherence of the model to the scientific principle under investigation. Once these two distinct verification processes are enacted, the formulated model is ready to be confirmed and deployed to other similar contexts outside of the activity (e.g., physics, economics, and statistics). This will constitute its final phase, called *Other Real Contexts* to *Solving Additional Problems.* Research has shown (Hestenes, 2013) that students are thrilled when they realize that a single model can be used to solve multiple problems. The modeling cycle also proposes revision processes, often omitted in the current modeling. The stage of revision depends on the particular model and the nature of its lack of fit. Thus, it can begin from revising *Testing and Analysis* or even from *Hypothesis*. The arrows in red, emerging from the *Problem Statement,* refer to a case when the *Problem Statement* is formulated by the students.

Zooming Deeper into the Modeling and Problem Solving Interface

The phase of verification of the model in new contexts resembles typical problem solving—the learner uses the derived mathematical representation to answer additional questions. Viewed as such, we support the position that problem solving is an extension of modeling activities and constitutes its integrated part that is nested in the verification process. Thus, problem solving, in our view, emerges as a subset of mathematical modeling. Both of these cognitive activities are very important in mathematical knowledge acquisition, but these processes complement rather than contradict each other. If the problem-solving process does not require the students to analyze the situation and raise its mathematical structure, when the process does not include modeling, it reduces to a repetition of procedures (e.g., see findings of Mousoulides et al., 2008, and Yoon et al., 2010), and that is not what mathematical methods have to offer. Concurrently, this conclusion presents our position on the relation between problem solving and mathematical modeling that is illustrated in Figure 18.

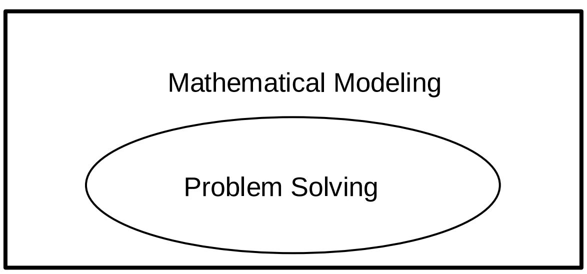

Figure 18. Proposed relation between math modeling and problem-solving.

Mathematical modeling appears in the relation as superior to problem solving in the sense that it provides students with methods and techniques that are broader and more comprehensive. As such, modeling can also be perceived as an activity of shifting the learner's focus from deductively finding a particular solution to inductively developing mathematical structure-based contexts and then using the developed structures to find the particular solution. Through modeling activities, students learn that the solution to a problem follows directly from a mathematical model of the problem. The modeling process applies also to solving artificial textbook problems and significant real-world problems of great complexity (Hestenes, 2013). A number of studies (e.g., Gravemeijer & Doorman, 1999; Malone, 2008) find that modeling instructions promotes expert problem-solving behavior in students.

Modeling activities cannot replace problem solving, but it is hypothesized that when students work on such activities, their problem-solving techniques take a different, more scientific approach that will benefit them throughout their high school, college, and professional endeavors.

Sequencing Modeling Activities

In answering the question of where mathematical activity should be placed in the curriculum as viewed through the proposed modeling process, the answer is that it depends on the goal of the activity: (1) if the goal of modeling is to have students learn how to apply mathematical tools, such activity should conclude the chapter; (2) if the goal of the activity is to introduce a concept and provide the learner with its introduction, then such activity should be introduced before new material is delivered. If the latter is used, the process as described in Figure 17 will be significantly reduced, retaining only its conceptual formulations. Further research designed to reflect on both paths of modeling is suggested to quantify an effect of each type of sequencing on student achievement.

Limitations and Suggestions for Future Research

This study, as any other research, carries certain limitations. Some of them can be attributed to the limited number of studies available to be meta-analyzed, and some can be attributed to diversity of math curricula across the countries where the research on modeling was conducted. Although sensitivity of smaller quantitative sample sizes was restored, the significance of the mean effect statistic would promote the replication of the findings more accurately by being computed over a larger study pool. The qualitative synthesis generated a wealth of themes, yet the diversity the curriculum systems could constitute some mediators that remained silent. Accounting for such diversity was not possible in the present study. Varied methods of student achievement evaluation used in the quantitative pool of the research also limited, to a certain degree, the study findings and its generalizability. Moreover, as in the line of this research mathematical modeling is to support problem solving, a moderator link testing the modeling impact on students' problem-solving techniques could not be established either. In some of these primary studies, the students' achievement due to mathematical modeling activities was evaluated as taken with a broader scope, seen through general students' math concept understanding. This conclusion prompts impulses for generating another more sophisticated research, focusing on investigating students perceptions of transitioning from mathematical modeling to problem solving.

As the initial intent of this undertaking was to examine the utilization of scientific simulations to support mathematical modeling activities, due to limited number of studies, we broadened the literature search and included a substantial amount of literature that examined pen-and-pencil activities, as well as these that utilized real experiments. We believe that this modification benefited this research. Situated mainly in naturalistic paradigms, these studies contributed substantially to formulating the general modeling process proposed in Figure 11. Analyzing these studies another conclusion was reached; if mathematical modeling activities are set to model reality, then the real sceneries need to be supplied to the learners. In the era of highly interactive multimedia availability, simulated scientific experiments can be easily brought to a mathematics classroom and serve as a rich basis for inducing mathematical modeling with all of its phases. This research revealed that this great opportunity is not fully exploited yet in mathematics classrooms.

Another conclusion that emerged is the role of modeling in the current math curriculum. Students' skills and techniques on modeling are not being tested on standardized tests yet. If modeling is to be wider implemented, students need to be tested on these skills as well. The current research shows a need for a stronger link established between mathematical modeling and problem solving in school practice. It has been proven, especially in the qualitative part of this research, that mathematical modeling even being taught with isolation to problem solving helps accomplish multiple math learning objectives. Yet, it seems that with a goal of being set as a governing method to problem solving techniques, its impact on students' mathematical knowledge acquisition will be much higher.

CHAPTER V

SUMMARY AND CONCLUSIONS

While several studies have been conducted about the effects of visualization on students' math achievement, this research sought to examine the effects of exploratory visual environments on students' understanding of math concepts and their skills on applying these concepts to solving real life problems. Undertaken with a large scope that included the entire range of math schooling levels, this study revealed that visualization embodied by either static diagrams or by computer simulated programs supported the learning of mathematics at any level. This study also examined effectiveness of one of the emerging instructional methods that extensively uses visualization; mathematical modeling. This study revealed that mathematical modeling when compared to traditional teaching produces a high positive effect size of ES = 0.69 (SE = 0.05, 95% CI: 0.59–0.79) signaling a need for its wider implementation to school practice.

Yet, as was discussed earlier, using computers or visualization purely as a method of instruction will not increase students' math understanding. This synthesis of contemporary literature allowed formulations of several valuable inferences about teaching and learning mathematics and also generated questions for further research. While the first article revealed that exploratory environments help students understand new concepts, the question remains how exploratory environments can help students with the transfer of math concepts to new situations. If the notion of teaching is to enact, in students minds, an integrated math-science-technology approach to problem solving that will continue and advance a rapid development in this area, how can teachers balance the inclusion of science content in math classes or math content in science classes? Should widely established schemata for problem solving in lower math grades be consistently introduced throughout more advanced math levels? Or should rather simplified modeling cycles proposed by math research (e.g., Blum, 1996; Blum & Leiss, 2007) be introduced to lower math grades? Cheng (1999) proposed four learning stages that are supposed to lead the learner to developing concept understanding through using schemata such as domain, external representation, concept, and internal network of concepts. These stages, however, show certain limitations, which are that (1) their very general forms make them difficult to apply in school, (2) they do not provide the teacher with a framework for lesson organization, and (3) they do not bridge the learning process with other processes that are applied in higher-level math courses. As in the process of moving from one stage to another, the learner is immersed in four processes: observation, modeling, acquisition, and integration. These processes reassemble formal modeling as defined by Dreyfus and Thomas (2010); thus, a potential for unifying them exists. The majority of the gathered pool of studies did not refer to these inquiry processes and focused instead on applying fixed models without conditioning them.

While schemata-based representations produced a moderate effect size ES = 0.49 (SE = 0.09, 95% CI: 0.31–0.67) a need to learn more about how students perceive these mathematical structures would benefit a further implementation of this supporting learning tool. Thus, not only does quantitative research contribute to inferences about learning effects, but also recommended is qualitative research that reflects the underlying students thought processes. Although the moderator of explorations could not be applied to this research, letting students explore and find meanings of the representations was frequently raised by researchers (e.g., Terwel et al., 2009; Perkins and Unger, 1994).

The idea of using exploratory environment, in the form of computerized simulations to support concept understanding and problem solving was further examined in the Chapter 3 (Manuscript #2). This strictly meta – analytic study revealed that using computerized programs exploratory environments produced high ES = 0.60 (SE = 0.03, 95% CI: 0.53–0.66) learning effect size across elementary and middle school mathematics levels. High dynamism of simulated medium and multiple opportunities for investigating relations between variables deem to be the primary factors for this inference. Despite this high positive effect, mathematics curriculum with its rather rigid structures is challenged to adapt to the new methods of its content learning. Simply having students explore some relations without purposeful objectives that relate to their prior experiences might distort applicability of explorations. Another research area that opens for further investigation is the transition from exploration to problem solving. Problem-solving skills are merged on two independent paths: (a) They are developed on the basis of understanding the context through identifying the principles of the system's behavior and (b) they require fluency and flexibility in applying computational skills. Research shows (Hestenes, 2013; Yildrim et al., 2010) that students succeed on problem solving if both paths are balanced and both are developed prior to having the learners work on such problems. The solver must be equipped with tools that he/she will use to solve given tasks. The process of accumulating these skills is highly intertwined and includes (a) verbal and syntactic processing, (b) special-representation storage and retrieval in short- and long-term memory, (c) algorithmic learning, and (d) its most complex element—conceptual understanding (Goldin, 1992). Computerized programs offering bases for investigations display great potential for improving problem conceptual understanding, yet this study shows that this area is not fully explored yet. Taking full advantage of such learning environments emerges as a next stage of this research. Extending the problem analysis, thorough explorations to focus the learners' attention on its underpinning principles and then formulate patterns and generalize the patterns using mathematical apparatus is worthy of further investigation. Of special attention in these investigations is a structure of theoretical framework that would provide directions for methodological design of activities. Manuscript #3 (Chapter 4) provides such a framework that is rooted in mathematical modeling processes. Although several such theoretical frameworks have been already created (e.g., Blum & Leiss, 2007; Pollak, 1978), the exploratory factor linking mathematics to other subjects, in particular to science, has not been explored in those designs. This missing link is believed to have a diminishing impact on students' achievement when modeling activities are applied. Departing from this premise, a new and enriched modeling cycle was proposed. It not only encompassed qualitative elements of experimentations but its entire structure was supported by inductive inquiry that dominates school reasoning nowadays (Prince & Felder, 2006). Moving through the reasoning cycle, students are constantly reminded to verify and revise their math applications techniques not only considering correctness of mathematical structures but also adherence to scientific principles embedded in the given context.

In most math assessment tests, students' skills and techniques in modeling are not being tested yet, so another concern that emerged from the research was the inclusion of an assessment that would evaluate students' math modeling skills. As the content taught is reflected on standardized assessment tests, a question arises of whether mathematical modeling skills should also be evaluated. If modeling is to be more widely implemented, students need to be tested on these skills as well. If so, how should we measure and evaluate these skills? It seems that requiring only numerical answers as solutions to text problems is not sufficient and that assessments should also require verbal justifications of these answers and reflections of thought processes that led the students to their conclusions.

The current research shows also a need for the establishment of a stronger link between mathematical modeling and problem solving in school practice at any level. More specifically, there is a need to research and explicate how these methods of math knowledge acquisition are interrelated. As the analysis of the contemporary research allowed formulating an integrated math modeling cycle, its experimental testing emerged as an intermediate task.

Even if mathematical modeling is taught in isolation from problem solving, it helps accomplish multiple math learning objectives (Yoon et al., 2010). Yet, if the goal of including modeling activities in school practice is to support problem-solving techniques, the link needs to be explicitly formulated in the math curriculum.

Each of the three studies has certain limitations attributed to either: (a) diversity of treatment designs, (b) diversity of math curriculum, or (c) inability to evaluate the degree of interactivity of the computer programs applied. Although through the process of weighting, sensitivity to smaller sample sizes was restored, the replication of the findings would be more significant if the primary studies had larger sample counts.

Widely varied methods used to assess student achievement, ranging from traditional multiple-choice exams mostly locally developed to new assessment techniques such as standardize-based assessments also decreased validity of the primary research findings and consequently decreased the validity of their corresponding effect sizes. Although some of these studies reported a Cronbach's alpha reliability coefficient, most did not, or used a different reliability measure thus not allowing for comparisons.

While initially the effect of computerized simulations on students' problem solving was to be examined, this idea was soon abandoned due to lack of available research. Thus, the effect of mathematical modeling on students' problem-solving techniques was not investigated as it was anticipated. The idea of examining the effect of modeling on students' problem solving techniques emerged as one of the themes for further research. It is hoped that this study enriched the knowledge about using exploratory environment to support the processes of mathematical learning. By providing answers to stated research problems, it also generated prompts and themes for other more detailed investigations in this domain.

REFERENCES

Albanese M.A., & Mitchell S. (1993). Problem-based learning: a review of literature on the outcomes and implementation issues. *Academic Medicine, 68*(1), 52-81.

*Alibali, M. W., Phillips, K. M., & Fischer, A. D. (2009). Learning new problem-solving strategies leads to changes in problem representation. *Cognitive Development*, *24*(2), 89-101.

Anderson, L., & Krathwohl, D. A. (2001). *Taxonomy for learning, teaching and assessing: A revision of Bloom's taxonomy of educational objectives.* New York, NY: Longman.

Anghileri, J. (2006). Scaffolding practices that enhance mathematics learning. *Journal of Mathematics Teacher Education*, *9*(1), 33-52.

Arthur, J. D., & Nance, R. E. (2007). Investigating the use of software requirements and engineering techniques in simulation modeling. *Journal of Simulation*, *1*(3), 159-174.

Artzt, A. F., & Armour-Thomas, E. (1992). Development of a cognitive-metacognitive framework for protocol analysis of mathematical problem solving in small groups. *Cognition and Instruction*, *9*(2), 137-175.

*Bahmaei, F. (2012). Mathematical modeling in university, advantages and challenges. *Journal of Mathematical Modeling and Application*, *1*(7), 34-49.

*Baki, A., & Güveli, E. (2008). Evaluation of a web based mathematics teaching material on the subject of functions. *Computers & Education*, *51*(2), 854-863.

*Baki, A., Kosa, T., & Guven, B. (2011). A comparative study of the effects of using dynamic geometry software and physical manipulatives on the spatial visualization skills of pre-service mathematics teachers. *British Journal of Educational Technology*, *42*(2), 291-310.

Berry, J., & Davies, A. (1996). Written reports. In C. R. Haines & S. Dunthorne (Eds.), *Mathematics learning and assessment*: *Sharing innovative practices* (pp. 301-311). London, England: Arnold.

Bleich, L., Ledford, S., Hawley, C., Polly, D., & Orrill, C. (2006). An analysis of the use of graphical representation in participants' solutions. *The Mathematics Educator, 16*(1), 22-34.

Blum, W. (1996). Anwendungsbezuge im mathematikumterricht – trends und perspectiven. In G. Kadunz, H. Kautschitsch, G. Ossimitz & E. Schneider (Ed.), *Trends und perspektiven* (pp.15-38). Wien, Austria: Holder-Pichler-Tempsky.

Blum, W., & Booker, G. (1998). *Mathematical modeling: Teaching and assessment in a technology-rich world*. West Sussex, UK: Horwood.

Blum, W., Galbraith, P. L., Henn, H. W., & Niss, M. (2007). *Modeling and applications in mathematics education: The 14th ICMI Study*. New York, NY: Springer.

Blum, W., & Leiss, D. (2007). How do students and teachers deal with modeling problems. In C. Haines, P. Galbraith, W. Blum & S. Khan (Eds.), *Mathematical Modeling* (pp. 222-231). Chichester, England: Horwood Publishing Limited.

Blum, W., & Niss, M. (1991). Applied mathematical problem solving, modeling, applications, and links to other subjects—State, trends and issues in mathematics instruction. *Educational Studies in Mathematics*, *22*(1), 37-68.

*Booth, J. L., & Siegler, R. S. (2008). Numerical magnitude representations influence arithmetic learning. *Child Development*, *79*(4), 1016-1031.

Bos, B. (2009). Virtual math objects with pedagogical, mathematical, and cognitive fidelity. *Computers in Human Behavior*, *25*(2), 521-528.

Bransford, J. D., & Stein, B. S. (1984). *The ideal problem solver. A guide for improving thinking, learning, and creativity*. New York: Freeman.

Bruner, J. (1961). *The process of education*. Cambridge, MA: Harvard University Press.

Carpenter, T. P., & Moser, J. M. (1984). The acquisition of addition and subtraction concepts in grades one through three. *Journal for research in Mathematics Education*, *15*, 179-202.

Carrejo, D. J., & Marshall, J. (2007). What is mathematical modeling? Exploring prospective teachers' use of experiments to connect mathematics to the study of motion. *Mathematics Education Research Journal*, *19*(1), 45-76.

*Casey, B., Erkut, S., Ceder, I., & Young, J. M. (2008). Use of a storytelling context to improve girls' and boys' geometry skills in kindergarten. *Journal of Applied Developmental Psychology*, *29*(1), 29-48.

*Castle, K., & Needham, J. (2007). First graders' understanding of measurement. *Early Childhood Education Journal*, *35*(3), 215-221.

Chaachoua, H., & Saglam, A. (2006). Modeling by differential equations. *Teaching Mathematics Applications*, *25*(1), 15-22.

*Chang, K. E., Sung, Y. T., & Lin, S. F. (2006). Computer-assisted learning for mathematical problem solving. *Computers & Education*, *46*(2), 140-151.

Chen, C. H. (2010). Promoting college students' knowledge acquisition and ill-structured problem solving: Web-based integration and procedure prompts. *Computers & Education, 55*(1), 292-303.

*Chen, C. J., & Liu, P. L. (2007). Personalized computer-assisted mathematics problem-solving program and its impact on Taiwanese students. *Journal of Computers in Mathematics and Science Teaching*, *26*(2), 105-121.

Cheng, P. C. H. (1999). Unlocking conceptual learning in mathematics and science with effective representational systems. *Computers & Education*, *33*(2), 109-130.

*Chinnappan, M. (2010). Cognitive load and modeling of an algebra problem. *Mathematics Education Research Journal*, *22*(2), 8-23.

Christmann, E., Badgett, J., & Lucking, R. (1997). Microcomputer-based computer-assisted instruction within differing subject areas: A statistical deduction. *Journal of Educational Computing Research*, *16*(3), 281-296

Christiansen, I. M. (2001). Modeling and mathematics education. In J. F. Matos, W. Blum, S. K. Houston, & S. P. Carreira (Eds.), *Applications in science and technology* (p. 311). Chichester, England: Harwood Publishing.

Christou, C., & Papageorgiou, E. (2007). A framework of mathematics inductive reasoning. *Learning and Instruction*, *17*(1), 55-66.

Clark, R. E. (1994). Media will never influence learning. *Educational Technology Research and Development*, *42*(2), 21-29.

Clark, R. C., & Mayer, R. E. (2011). *e-Learning and the science of instruction: Proven guidelines for consumers and designers of multimedia learning.* San Francisco, CA: Pfeiffer.

Coley, R. J., Cradler, J., & Engel, P. K. (2000). *Computers and the classroom: The status of technology in U.S. schools*. Retrieved from http://www.ets.org/research/policy_research_reports/pic-compclssl.

Cooper, H. (2010). *Research synthesis and meta-analysis* (4th ed.). Thousand Oaks, CA: Sage.

Corwin, R. B., Russell, S. J., & Tierney, C. C. (1990). *Seeing fractions: A unit for the upper elementary grades*. Sacramento, CA: California Dept. of Education.

*Cory, B. L., & Garofalo, J. (2011). Using dynamic sketches to enhance preservice secondary mathematics teachers' understanding of limits of sequences. *Journal for Research in Mathematics Education*, *42*(1), 65-97.

*Coquin-Viennot, D., & Moreau, S. (2003). Highlighting the role of the episodic situation model in the solving of arithmetical problems. *European Journal of Psychology of Education*, *18*(3), 267-279.

Coquin J., Viennot, D., & Moreau, S. (2007). Arithmetic problems at school: When there is an apparent contradiction between the situation model and the problem model. *British Journal of Educational Psychology*, *77*(1), 69-80.

*Crouch, R., & Haines, C. (2004). Mathematical modeling: Transitions between the real world and the mathematical model. *International Journal of Mathematical Education in Science and Technology*, *35*(2), 197-206.

*Csíkos, C., Szitányi, J., & Kelemen, R. (2012). The effects of using drawings in developing young children's mathematical word problem solving: A design experiment with third-grade Hungarian students. *Educational Studies in Mathematics*, *81*(1), 47-65.

Cuoco, A. (2001). Mathematics for teaching. *Notice of the AMS*, *48*(2), 168-174.

*David, M. M., & Tomaz, V. S. (2012). The role of visual representations for structuring classroom mathematical activity. *Educational Studies in Mathematics*, *80*, 413-431.

De Bock, D., Verschaffel, L., Janssens, D., Van Dooren, W., & Claes, K. (2003). Do realistic contexts and graphical representations always have a beneficial impact on students' performance? Negative evidence from a study on modeling non-linear geometry problems. *Learning and Instruction*, *13*(4), 441-463.

Dekkers, J., & Donatti, S. (1981). The integration of research studies on the use of simulation as an instructional strategy. *Journal of Educational Research, 74*, 424-427.

*Deliyianni, E., Monoyiou, A., Elia, I., Georgiou, C., & Zannettou, E. (2009). Pupils' visual representations in standard and problematic problem solving in mathematics: Their role in the breach of the didactical contract. *European Early Childhood Education Research Journal*, *17*(1), 95-110.

Devlin, K. (1996). *Mathematics: The science of patterns: The search for order in life,* *mind and the universe.* New York, NY: Henry Holt and Company.

Diefes-Dux, H. A., & Salim, W. W. A. W. (2012). Transforming the First-Year Engineering Experience through Authentic Problem-Solving: Taking a Models and Modeling Perspective. *Procedia-Social and Behavioral Sciences*, *56*, 314-332.

Diefes-Dux, H. A., Zawojewski, J. S., & Hjalmarson, M. A. (2010). Using educational research in the design of evaluation tools for open-ended problems. *International Journal of Engineering Education*, *26*(4), 807.

*Diefes-Dux, H. A., Zawojewski, J. S., Hjalmarson, M. A., & Cardella, M. E. (2012). A framework for analyzing feedback in a formative assessment system for mathematical modeling problems. *Journal of Engineering Education*, *101*(2), 375-406.

Dienes, Z. P. (1971). *Building up mathematics.* London, England: Hutchinson Educational Dreyfus, T. (1991). *Advanced mathematical thinking processes*. In D. Tall (Ed.), Advanced mathematical thinking (pp. 25-41). Dordrecht, The Netherlands: Kluver.

*Eid, G. K. (2005). An investigation into the effects and factors influencing computer-based online math problem-solving in primary schools. *Journal of Educational Technology Systems*, *33*(3), 223-240

Eisenberg, T., & Dreyfus T. (1991). On the reluctance to visualize in mathematics. In W. Zimmermann & S. Cunningham (Eds.), *Visualization in teaching and learning* *mathematics* (pp. 26–37).Washington, DC: Mathematical Association of America.

English, L. (2004). Mathematical modeling in the primary school. In *Mathematics education for the third millennium: Towards 2010 (Proceedings of the 27th Annual Conference of the Mathematics Education Research Group of Australasia*; Vol. 1, pp. 207-214). Sydney, Australia: MERGA.

English, L., & Sriraman, B. (2010). Problem solving for the 21st century. In G. Kaiser, R. Borromeo, W., Blum (Eds.), *Theories of mathematics education* (pp. 263-290). Berlin, Germany: Springer.

English, L. D., & Watters, J. J. (2005). Mathematical modeling in the early school years. *Mathematics Education Research Journal*, *16*(3), 58-79.

*Erbas, A. K., & Yenmez, A. A. (2011). The effect of inquiry-based explorations in a dynamic geometry environment on sixth grade students' achievements in polygons. *Computers & Education*, *57*(4), 2462-2475.

Evan, R., & Lappan, G. (1994). Constructing meaningful understanding of mathematics content. In D. Aichele & A. Coxford (Eds.), *Professional development for teachers of mathematics* (pp. 128-143). Reston, VA: NCTM

*Eysink, T. H., de Jong, T., Berthold, K., Kolloffel, B., Opfermann, M., & Wouters, P. (2009). Learner performance in multimedia learning arrangements: An analysis across instructional approaches. *American Educational Research Journal*, *46*(4), 1107-1149.

*Faraco, G., & Gabriele, L. (2007). Using LabVIEW for applying mathematical models in representing phenomena. *Computers & Education*, *49*(3), 856-872.

Felder, R. M., & Brent, R. (2004). The intellectual development of science and engineering students. *Journal of Engineering Education, 93*(4), 269-277.

Ferri, R. B. (2006). Theoretical and empirical differentiations of phases in the modeling process. *ZDM*, *38*(2), 86-95.

Ferrini-Mundy, J., & Güçler, B. (2009). Discipline-based efforts to enhance undergraduate STEM education. *New Directions for Teaching and Learning*, *117*, 55-67.

Fey, J. T. (1989). Technology and mathematics education: A survey of recent development and important problems. *Educational Studies in Mathematics*, *20*(3), 237-272.

Flum, H., & Kaplan, A. (2006). Exploratory orientation as an educational goal. *Educational Psychologist, 4*(2), 99-110.

Francisco, J. M., & Maher, C. A. (2005). Conditions for promoting reasoning in problem solving: Insights from a longitudinal study. *The Journal of Mathematical Behavior*, *24*(3), 361-372.

*Fuchs, L. S., Fuchs, D., Finelli, R., Courey, S. J., & Hamlett, C. L. (2004). Expanding schema-based transfer instruction to help third graders solve real-life mathematical problems. *American Educational Research Journal*, *41*(2), 419-445.

*Fuchs, L. S., Fuchs, D., Finelli, R., Courey, S. J., Hamlett, C. L., Sones, E. M., & Hope,

S. K. (2006). Teaching Third Graders about Real Life Mathematical Problem Solving: A Randomized Controlled Study. *The Elementary School Journal*, *106*(4), 293-311.

*Fujimura, N. (2001). Facilitating children's proportional reasoning: A model of reasoning processes and effects of intervention on strategy change. *Journal of Educational Psychology, 93*(3), 589-603.

*Gamo, S., Sander, E., & Richard, J.-F. (2010). Transfer of strategy by semantic recording in arithmetic problem solving. *Learning and Instruction*, *20*(5), 400-410.

Geiger, V. (2011). Factors affecting teachers' adoption of innovative practices with technology and mathematical modeling. In G. Kaiser, W. Blum, R. Borromeo, and F. G. Stillmsn (Eds.), *Trends in Teaching and Learning of Mathematical Modeling* (pp. 305-314). New York, NY: Springer.

Gerofsky, S. (2004). *A man left Albuquerque heading east: Word problem as genre in mathematics education* (Vol. 5). New York, NY: Peter Lang.

Gershman, J., & Sakamato, E. (1981). Computer-assisted remediation and evaluation: A CAI project for Ontario secondary schools. *Educational Technology*, *21*(3), 40-43.

Giancoli, D. C. (2005). *Physics: Principles with applications* (6th ed.). Upper Saddle River, NJ: Prentice Hall.

Gick, M. L., & Holyoak, K. J. (1983). Schema induction and analogical transfer. *Cognitive Psychology*, *15*(1), 1-38.

Gijbels, D., Dochy, F., Van den Bossche, P., & Segers, M. (2005). Effects of problem-based learning: A meta-analysis from the angle of assessment. *Review of Educational Research*, *75*(1), 27-61.

Glass, G. V. (1976). Primary, secondary, and meta-analysis of research. *Educational Researcher*, *5*(10), 3-8.

Goldin, G. A. (1992). Meta-analysis of problem-solving studies: A critical response. *Journal for Research in Mathematics education*, *23*(3), 274-283.

Goldin, G. A., & Shteingold, N. (2001). Systems of representation and the development of mathematical concepts. In A. A. Cuoco & F. R. Curcio (Eds.), *The roles of representation in school mathematics* (2nd ed.; pp. 1-23). Reston, VA: NCTM.

Gravemeijer, K. (1997). Solving word problems: A case of modeling? *Learning and Instruction*, *7*(4), 389-397.

Gravemeijer, K., & Doorman, M. (1999). Context problems in realistic mathematics education: A calculus course as an example. *Educational Studies in Mathematics, 39*(1), 111-129.

Grouws, D. A., & Cebulla, K. J. (2000). *Improving student achievement in mathematics*. Geneva, Switzerland: International Academy of Education.

*Guven, B. (2012). Using dynamic geometry software to improve eight grade students' understanding of transformation geometry. *Australasian Journal of Educational Technology*, *28*(2), 364-382.

Haas, M. (2005). Teaching methods for secondary algebra: A meta-analysis of findings. *NASSP Bulletin, 89*(642), 24-46.

Halloun, I. A., & Hestenes, D. (1985). Common sense concepts about motion. *American Journal of Physics*, *53*(11), 1056-1065.

Hart, J. M. (1996). The effect of personalized word problems. *Teaching Children Mathematics, 2*(8), 504-505.

Harter, C. A., & Ku, H. Y. (2008). The effects of spatial contiguity within computer-based instruction of group personalized two-step mathematics word problems. *Computers in Human Behavior*, *24*(4), 1668-1685.

Haverty, L. A., Koedinger, K. R., Klahr, D., & Alibali, M. W. (2000). Solving inductive reasoning problems in mathematics: Not-so-trivial pursuit. *Cognitive Science*, *24*(2), 249-298.

Hedges, L. V. (1992). Meta-analysis. *Journal of Educational and Behavioral Statistics*, *17*(4), 279-296.

Hestenes, D. (2013). Remodeling science education. *European Journal of Science and Mathematics Education, 1*(1), 13-22.

Hestenes, D. (1995). Modeling software for learning and doing physics. In C. Bernardini, C. Tarsitani, M. Vicentini (Eds.), *Thinking physics for teaching* (pp. 25-65). New York, NY: Plenum Press.

Hedges, L. V. (1992). Meta-analysis. *Journal of Educational and Behavioral Statistics*, *17*(4), 279-296.

Hiebert, J. (1988). A theory of developing competence with written mathematical symbols. *Educational Studies in Mathematics*, *19*(3), 333-355.

Hiebert, J., & Carpenter, T. (1992). Learning and teaching with understanding. In D. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 65-97). New York, NY: Simon & Schuster Macmillan.

Hoffler, T., & Leutner, D. (2007). Instructional animation versus static pictures: A meta-analysis. *Learning and Instruction*, *17*(6), 722-738.

Hoyt, J. E., & Sorensen, C. T. (2001). High school preparation, placement testing, and college remediation. *Journal of Developmental Education*, *25*(2), 26-34.

*Huang, T. H., Liu, Y. C., & Chang, H. C. (2012). Learning achievement in solving word-based mathematical questions through a computer-assisted learning system. *Educational Technology & Society, 15*(1), 248-259.

Hunter, J. E., & Schmidt, F. L. (1990). *Methods of meta-analysis: Correcting error and bias in research findings.* Newbury Park, CA: Sage.

*Hwang, W. Y., & Hu, S. S. (2013). Analysis of peer learning behaviors using multiple representations in virtual reality and their impacts on geometry problem solving. *Computers & Education*, *62*, 308-319.

*Iversen, S. M., & Larson, C. J. (2006). Simple thinking using complex math vs. complex thinking using simple math—A study using model eliciting activities to compare students' abilities in standardized tests to their modeling abilities. *ZDM*, *38*(3), 281-292.

*Jitendra, A. K., Griffin, C. C., Haria, P., Leh, J., Adams, A., & Kaduvettoor, A. (2007). A comparison of single and multiple strategy instruction on third-grade students' mathematical problem solving. *Journal of Educational Psychology*, *99*(1), 115-127.

Jitendra, A. K., Griffin, C. C., McGoey, K., Gardill, M. C., Bhat, P., & Riley, T. (1998). Effects of mathematical word problem solving by students at risk or with mild disabilities. *The Journal of Educational Research*, *91*(6), 345-355.

Jitendra, A. K., Star, J. R., Rodriguez, M., Lindell, M., & Someki, F. (2011). Improving students' proportional thinking using schema-based instruction. *Learning and Instruction*, *21*(6), 731-745.

Jonassen, D. H. (2003). *Learning to solve problems: An instructional design guide* (Vol. 3). San Francisco, CA. Pfeiffer.

Jonassen, D. H. (1997). Instructional design models for well-structured and ill-structured problem-solving learning outcomes. *Educational Technology Research and Development*, *45*(1), 65-94.

Joyce, B., Weil, M., Calhoun, E. (2009). *Models of teaching* (8th ed.). Boston, MA: Pearson.

Kalyuga, S. (2006). Assessment of learners' organized knowledge structures in adaptive learning environments. *Applied Cognitive Psychology*, *20*(3), 333-342.

*Kapa, E. (2007). Transfer from structured to open-ended problem solving in a computerized metacognitive environment. *Learning and Instruction*, *17*(6), 688-707.

Kaput, J. (1991). Notations and representations as mediators of constructive processes. *Radical Constructivism in Mathematics Education*, *7*, 53-74.

Kaput, J. (1989). *Linking representation in the symbol systems of algebra.* Hillsdale, NJ: Erlbaum.

Kaput, J., & Roschelle, J. (1997). Deepening the impact of technology beyond assistance with traditional formalism in order to democratize access to ideas underlying calculus. In E. Pehkonen (Ed.), *Proceedings of the 21st conference of the international group for the psychology of mathematics education* (pp. 105-112). Lahti, Finland: University of Helsinki.

Kaput, J. J., & Thomson, P. W. (1994). Technology in mathematics education research: The first 25 years in *JRME. Journal for Research in Mathematics Education*, *25*(6), 676-684.

Kaufman, A. S., & Kaufman, N. L. (1983). *K-ABC Kaufman assessment battery for children: Administration and scoring manual*. Circle Pines, MN: American Guidance Service.

Kieran, C. (1984). Cognitive mechanisms underlying the equation-solving errors of algebra novices. *Proceedings of PME-VIII, Sydney, Australia*, 70-77.

Kieran, C., & Hillel, J. (1990). It's tough when you have to make the triangles angle: Insights from a computer-based geometry environment. *The Journal of Mathematical Behavior*, *9*(2), 99-127.

Kim, M. C., & Hannafin, M. J. (2011). Scaffolding problem solving in technology-enhanced learning environments (TELEs): Bridging research and theory with practice. *Computers & Education, 56*(2), 403-417.

Kintch, W. (1998). *Comprehension: A paradigm for cognition*. Cambridge, England: Cambridge University Press.

Klauer, K. J. (1990). Paradigmatic teaching of inductive thinking. *Learning and instruction*, *2*, 23-45.

*Kong, S. C. (2007). The development of a cognitive tool for teaching and learning fractions in the mathematics classroom: A design-based study. *Computers & Education*, *51*(2), 886-899.

Klymchuk, S., & Zverkova, T. (2001). Modeling and mathematics education. In J. F. Matos, W. Blum, S. K. Houston, & S. P. Carreira (Eds.), *Role of mathematical modeling and applications in university service courses: An across countries study* (pp. 227-235). Chichester, England: Horwood Publishing.

*Klymchuk, S., Zverkova, T., Gruenwald, N., & Sauerbier, G. (2008). Increasing engineering students' awareness to environment through innovative teaching of mathematical modelling. *Teaching Mathematics and its Applications*, *27*(3), 123-130.

*Ku, H. Y., & Sullivan, H. J. (2002). Student performance and attitudes using personalized mathematics instruction. *Educational Technology Research and Development*, *50*(1), 21-34.

Kuhn, D. (2007). Is direct instruction the answer to the right question? *Educational*

Psychologist, 42(2), 109-114.

Kulik, J. (2003). Effects of using instructional technology in elementary and secondary schools: What controlled evaluation studies say. *SRI International*. Project # P10446001. Retrieved from http://bestevidence.org.

*Laakso, M. J., Myller, N., & Korhonen, A. (2009). Comparing learning performance of students using algorithm visualizations collaboratively on different engagement levels. *Journal of Educational Technology & Society*, *12*(2), 267-282.

*Lai, K., & White, T. (2012). Exploring quadrilaterals in a small group computing environment. *Computers & Education*, *59*(3), 963-973.

*Lan, Y. J., Sung, Y. T., Tan, N. C., Lin, C. P., & Chang, K. E. (2010). Mobile-device-supported problem-based computational estimation instruction for elementary school students. *Educational Technology & Society*, *13*(3), 55-69.

Lawson, A. E., Oehrtman, M., & Jensen, J. (2008). Connecting science and mathematics: The nature of scientific and statistical hypothesis testing. *International Journal of Science and Mathematics Education*, *6*(2), 405-416.

Lee, J. (1999). Effectiveness of computer-based instructional simulation: A meta-analysis. *International Journal of Instructional Media, 26*(1) 71-85.

Lee, V.S., (2004). *Teaching and learning through inquiry.* Sterling, Virginia: Stylus Publishing.

Lee, J. S., & Ginsburg, H. P. (2009). Early childhood teachers' misconceptions about mathematics education for young children in the United States. *Australasian Journal of Early Childhood*, *34*(4), 37.

Lesh, R., & Harel, G. (2003). Problem solving, modeling, and local conceptual development. Revealing activities for students and teachers. *Mathematical Thinking and Learning: An International Journal, 5*(2/3), 157-190.

Lesh, R., & Kelly, E. (2000). Multi-tiered teaching experiments. In E. Kelly & R. Lesh (Eds.), *Handbook of research design in mathematics and science education* (pp. 591-646). Mahwah, NJ: Lawrence Erlbaum.

Lesh, R., & Lehrer, R. (2003). Models and modeling perspectives on the development of students and teachers. *Mathematical Thinking and Learning*, *5*(2-3), 109-129.

Lesh, R., & Zawojewski, J. (2007). Problem solving and modeling. *Second handbook of research on mathematics teaching and learning*, *2*, 763-804.

Lester, F. K., Jr., Masingila, J. O., Mau, S. T., Lambdin, D. V., dos Santon, V. M., & Raymond, A. M. (1994). Learning how to teach via problem solving. In D. Aichele & A. Coxford (Eds.), *Professional development for teachers of mathematics* (pp. 152-166). Reston, VA: NCTM.

*Leutner, D. (2002). The fuzzy relationship of intelligence and problem solving in computer simulations. *Computers in Human Behavior*, *18*(6), 685-697.

Lewis, R., Stoney, S., & Wild, M. (1998). Motivation and interface design: Maximizing learning opportunities. *Journal of Computer Assisted Learning*, *14*(1), 40-50.

Li, Q., & Ma, X. (2010). A meta-analysis of the effects of computer technology on school students' mathematics learning. *Educational Psychology Review*, *22*, 215-243.

Li, Y. (2007). Curriculum research to improve teaching and learning. *School Science and Mathematics Journal*, *107*(5), 166-168.

Li, Y., & Li, J. (2009). Mathematics classroom instruction excellence through the platform of teaching contests. *ZDM Mathematics Education, 41*(3), 263-277.

*Liang, H. N., Parsons, P. C., Wu, H. C., & Sedig, K. (2010). An exploratory study of interactivity in visualization tools: “Flow” of interaction. *Journal of Interactive Learning Research*, *21*(1), 5-45.

*Lim, L. L., Tso, T. Y., & Lin, F. L. (2009). Assessing science students’ attitudes to mathematics: A case study on a modeling project with mathematical software. *International Journal of Mathematical Education in Science and Technology*, *40*(4), 441-453.

Lincoln, Y. S., & Guba, E. G. (1985). *Naturalistic inquiry*. Newbury Park, CA: Sage.

Lingefjärd, T. (2005). To assess students’ attitudes, skills and competencies in mathematical modeling. *Teaching Mathematics Applications*, *24*(3), 123-133.

Lipsey, M. W., & Wilson, D. B. (2001). *Practical meta-analysis.* Thousand Oaks, CA: Sage.

Malone, K. L. (2008). Correlations among knowledge structures, force concept inventory, and problem-solving behaviors. *Physical Review Special Topics - Physics Education Research, 4*(2), 020107.

Marshall, S. P. (1995). *Schemas in problem solving*. Cambridge University Press.

Mason, J., Stephens, M., & Watson, A. (2009). Appreciating mathematical structure for all. *Mathematics Education Research Journal*, *21*(2), 10-32.

Mayer, R. E., & Anderson, R. B. (1992). The instructive animation: Helping students build connections between words and pictures in multimedia learning. *Journal of Educational Psychology, 84*, 444-452.

McBride, J. W., & Silverman, F. L. (1991). Integrating elementary/middle school science and mathematics. *School Science and Mathematics*, *91*(7), 285-292.

*McNeil, N. M., & Alibali, M. W. (2004). You'll see what you mean: Students encode equations based on their knowledge of arithmetic. *Cognitive Science*, *28*(3), 451-466.

Merrill, M. D. (1991). Constructivism and instructional design. *Educational Technology, 31*(5), 45-53.

Merrill, M. D., & Gilbert, C. G. (2008). Effective peer interaction in problem-centered instructional strategy. *Distance Education*, *29*(2), 199-207.

Milovanović, M., Takači, Đ., & Milajić, A. (2011). Multimedia approach in teaching Mathematics: Example of lesson about the definite integral application for determining an area. *International Journal of Mathematical Education in Science and Technology*, *42*(2), 175-187.

Monk, S. (2003). Representation in school mathematics: Learning to graph and graphing to learn. In J. Kilpatrick (Eds.), *A research companion* to *principles and standards for school mathematics* (pp. 250-262). Reston, VA: NCTM

*Moseley, B., & Okamoto, Y. (2008). Identifying fourth graders' understanding of rational number representations: A mixed methods approach. *School Science and Mathematics*, *108*(6), 238-250.

*Mousoulides, N. G., Christou, C., & Sriraman, B. (2008). A modeling perspective on the teaching and learning of mathematical problem solving. *Mathematical Thinking and Learning*, *10*(3), 293-304.

National Council of Teachers of Mathematics (NCTM) (2000). *Principles and standards for school mathematics*. Reston, VA: NCTM.

National Research Council. (2000). *Inquiry and the national science education: A guide for teaching and learning*. Washington, DC: National Research Press.

Nesher, P., Hershkovitz, S., & Novotna, J. (2003). Situation model, text base and what else? Factors affecting problem solving. *Educational Studies in Mathematics*, *52*(2), 151-176.

Neves, R. G., Silva, J. C., & Teodoro, V. D. (2011). Improving learning in science and mathematics with exploratory and interactive computational modeling. *Trends in Teaching and Learning of Mathematical Modeling*, *1*(4), 331-339.

Niess, M. L. (2005). Preparing teachers to teach science and mathematics with technology: Developing a technology pedagogical content knowledge. *Teaching and Teacher Education*, *21*(5), 509-523.

Niss, M. (2010). Modeling a crucial aspect of students' mathematical modeling. In R. Lesh, C. R. Haines, P. L. Galbraith, & A. Hurford (Eds.), *Modeling students' mathematical modeling competencies* (pp. 43-59). New York, NY: Springer.

Noble, T., Nemirovsky, R., Wright, T., & Tierney, C. (2001). Experiencing change: The mathematics of change in multiple environments. *Journal for Research in Mathematics Education*, *31*(1), 85-108.

Nunokawa, K. (2005). Mathematical problem solving and learning mathematics: What we expect students to obtain. *The Journal of Mathematical Behavior*, *24*(3), 325-340.

Ozgun-Koca, S. A. (1998). *Students' use of representations in mathematics education.* Paper presented at the annual meeting of the North America Chapter of the International Group for the Psychology of Mathematics Education, Raleigh, NC.

Owen, E., & Sweller, J. (1985). What do students learn while solving mathematics problems?. *Journal of Educational Psychology*, *77*(3), 272.

Paas, F., Renkl, A., & Sweller, J. (2003). Cognitive load theory and instructional design: Recent developments. *Educational Psychologist*, *38*, 1-4.

Pape, S. J., & Tchoshanov, M. A. (2001). The role of representation(s) in developing mathematical understanding. *Theory into Practice, 40*(2), 118-125.

*Panaoura, A. (2012). Improving problem solving ability in mathematics by using a mathematical model: A computerized approach. *Computers in Human Behavior*, *28*(6), 2291-2297.

*Papadopoulos, I., & Dagdilelis, V. (2008). Students' use of technological tools for verification purposes in geometry problem solving. *The Journal of Mathematical Behavior*, *27*(4), 311-325

Perkins, D. N., & Unger, C. (1994). A new look in representations for mathematics and science learning. *Intructional Science*, *22*, 1-37.

Perrenet, J., & Zwaneveld, B. (2012). The many faces of the mathematical modeling cycle. *Journal of Mathematical Modeling and Application, 1*(6), 3-21.

*Pilli, O., & Aksu, M. (2013). The effects of computer-assisted instruction on the achievement, attitudes and retention of fourth grade mathematics students in North Cyprus. *Computers & Education, 62*, 62-71.

Podolefsky, N. S., Perkins, K. K., & Adams, W. K. (2010). Factors promoting engaged exploration with computer simulations. *Physics Review Special Topics - Physics Educational Research, 6*(2), 117-128.

*Poland, M., van Oers, B., & Terwel, J. (2009). Schematizing activities in early childhood education. *Educational Research and Evaluation*, *15*(3), 305-321.

Pollak, H. O. (1978). On mathematics applications and real problem solving. *School Science and Mathematics*, *78*(3), 232-239.

Polya, G. (1957). *How to solve it.* Princeton, NJ: Lawrence Erlbaum.

Porter, A., McMaken, J., Hwang, J., & Yang, R. (2011). Assessing the common core standards: Opportunities for improving measures of instruction. *Educational Researcher, 40*(4), 186-188.

Porzio, D. T. (1999). Effects of differing emphasis in the use of multiple representations and technology on students' understanding of calculus concepts. *Focus on Learning Problems in Mathematics, 21*(3), 1-29.

Prince, M. J., & Felder, R. M. (2006). Inductive teaching and learning methods: Definitions, comparisons, and research bases. *Journal of Engineering Education*, *95*(2), 123-138.

Rakes, C. R., Valentine, J. C., McGatha, M. B., & Ronau, R. N. (2010). Methods of instructional improvement in algebra: A systematic review and meta-analysis. *Review of Educational Research*, *80*(3), 372-400.

Redish, E. F., & Steinberg, R. N. (1999). Teaching physics: Figuring out what works. *Physics Today*, *52*, 24-31.

Reid, D. J., Zhang, J., & Chen, Q. (2003). Supporting scientific discovery learning in a simulation environment. *Journal of Computer Assisted Learning, 19,* 9-20.

Remillard, J. T., & Bryans, M. B. (2004). Teachers' orientations toward mathematics curriculum materials: Implications for teacher learning. *Journal for Research in Mathematics Education*, *35*(5), 352-388.

Reusser, K., & Stebler, R. (1997). Every word problem has a solution—The social rationality of mathematical modeling in schools. *Learning and Instruction*, *7*(4), 309-327.

Riley, M. S., Greeno, J. G., & Heller, J. 1.(1983). Development of children's problem-solving ability in arithmetic. In H.P. Ginsburg (Ed.), *The development of mathematical thinking*, (pp. 153-196). New York, NY: Academic.

*Rittle-Johnson, B., Siegler, R. S., & Alibali, M. W. (2001). Developing conceptual understanding and procedural skill in mathematics: An iterative process. *Journal of Educational Psychology*, *93*(2), 336-346.

*Roschelle, J., Shechtman, N., Tatar, D., Hegedus, S., Hopkins, B., Empson, S., & Gallagher, L. P. (2010). Integration of technology, curriculum, and professional development for advancing middle school mathematics: Three large-scale studies. *American Educational Research Journal*, *47*(4), 833-878.

Rosenshine, B., Meister, C., & Chapman, S. (1996). Teaching students to generate questions: A review of the intervention studies. *Review of Educational Research*, *66*(2), 181-221.

Ross, A., & Willson, V. (2012). The effects of representations, constructivist approaches, and engagement on middle school students' algebraic procedure and conceptual understanding. *School Science and Mathematics*, *112*(2), 117-128.

Santos-Trigo, M. (2007). Mathematical problem solving: An evolving research and practice domain. *Mathematics Education*, *39*(5/6), 523-536.

*Saxe, G. B., Taylor, E. V., McIntosh, C., & Gearhart, M. (2005). Representing fractions with standard notation: A developmental analysis. *Journal for Research in Mathematics Education*, *36*(2), 137-157.

*Scheiter, K., Gerjets, P., & Schuh, J. (2010). The acquisition of problem-solving skills in mathematics: How animations can aid understanding of structural problem features and solution procedures. *Instructional Science*, *38*(5), 487-502.

Schifter, D., & Fosnot, C. (1993). *Reconstructing mathematics education*. New York, NY: Teachers College Press.

Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 334-370). New York, NY: Macmillan.

*Schoen, H. L., & Hirsch, C. R. (2003). Responding to calls for change in high school mathematics: Implications for collegiate mathematics. *The American Mathematical Monthly*, *110*(2), 109-123.

Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 334-370). New York, NY: Macmillan.

*Schorr, R. Y., & Koellner-Clark, K. (2003). Using a modeling approach to analyze the ways in which teachers consider new ways to teach mathematics. *Mathematical Thinking and Learning*, *5*(2-3), 191-210.

*Schukajlow, S., Leiss, D., Pekrun, R., Blum, W., Müller, M., & Messner, R. (2012). Teaching methods for modelling problems and students' task-specific enjoyment, value, interest and self-efficacy expectations. *Educational Studies in Mathematics*, *79*(2), 215-237.

Schwarz, C. V., & White, Y. (2005). Metamodeling knowledge: Developing students' understanding of scientific modeling. *Cognition and Instruction, 23*(2), 165-205.

Shadish, W. R., Cook, T. D., & Campbell, D. T. (2002). *Experimental and quasi-experimental designs for generalized causal inference*. Belmont, CA: Wadsworth Cengage Learning.

Shepard, R. (1967). Recognition memory for words, sentences and pictures. *Journal of Verbal Learning and Verbal Behavior, 6*, 156-163.

Skemp, R. (1971). *The psychology of mathematics.* Baltimore, MD: Penguin Books.

Slough, S. W., & Rupley, W. H. (2010). Re-creating a recipe for science instructional programs: Adding learning progressions, scaffolding, and a dash of reading variety. *School Science and Mathematics*, *110*(7), 352-362.

Sokolowski, A., & Gonzalez y Gonzalez, E. (2012). Teachers' perspective on utilizing graphical representations to enhance the process of mathematical modeling. In M. Koehler & P. Mishra (Eds.), *Research highlights in technology and teacher education: Society for information technology & teacher education* (pp. 157-164). Chesapeake, VA: SITE.

*Soon, W., Lioe, L. T., & McInnes, B. (2011). Understanding the difficulties faced by engineering undergraduates in learning mathematical modeling. *International Journal of Mathematical Education in Science and Technology*, *42*(8), 1023-1039.

Stenberg, R. J. (1997). Cognitive Conceptions of Expertise. In P. J. Feltovich, K. M. Ford, & R. R. Hoffman (Eds.), *Expertise in context: Human and machine* (pp. 149-160). Menlo Park, CA: AAAI Press/The MIT Press.

Stern, L., Barnea, N., & Shauli, S. (2008). The effect of a computerized simulation on middle school students' understanding of the kinetic molecular theory. *Journal of Science Education and Technology*, *17*(4), 305-315.

Stillman, G. A., & Galbraith, P. L. (1998). Applying mathematics with real-world connections: Metacognitive characteristics of secondary students. *Educational Studies in Mathematics*, *36*, 157-195.

*Suh, J., & Moyer-Packenham, P. (2007). Developing students' representational fluency using virtual and physical algebra balances. *Journal of Computers in Mathematics and Science Teaching*, *26*(2), 155-173.

Swafford, J. O., & Langrall, C. W. (2000). Grade 6 students' preinstructional use of equations to describe and represent problem situations. *Journal for Research in Mathematics Education*, *31*(1), 89-112.

Swing, S. R., Stoiber, K. C., & Peterson, P. L. (1988). Thinking skills versus learning time: Effects of alternative classroom-based interventions on students' mathematics problem solving. *Cognition and Instruction*, *5*(2), 123-191.

Tall, D. (1986). Using the computer as an environment for building and testing mathematical concepts: A tribute to Richard Skemp. In *Papers in honor of Richard Skemp* (pp. 21-36). Retrieved from http://homepages.warwick.ac.uk /staff/David.Tall/pdfs/dot1986h-computer-skemp.pdf

*Terwel, J., van Oers, B., van Dijk, I., & van den Eeden, P. (2009). Are representations to be provided or generated in primary mathematics education? Effects on transfer. *Educational Research and Evaluation*, *15*(1), 25-44.

Thomas, J. M., & Young, R. M. (2011). Dynamic guidance for task-based exploratory learning. In G. Biswas, S. Bull, J. Kay, A. Mitrovic (Eds.), *Artificial intelligence in education* (pp. 369-376). Berlin, Germany: Springer.

Thomas, M. O. J., Yoon, C., & Dreyfus, T. (2009). Multimodal use of semiotic resources in the construction of antiderivative. In R. Hunter, B. Bicknell, & T. Burgess

(Eds.), *Proceedings of the 32nd conference of the mathematics education research group of Australasia* (Vol. 2; pp. 539-546). Wellington, NZ: MERGA.

Thomas, R., & Hooper, E. (1991). Simulations: An opportunity we are missing. *Journal of Research on Computing in Education*, *23*(4), 497-513.

*Türker, B., Sağlam, Y., & Umay, A. (2010). Preservice teachers' performances at mathematical modeling process and views on mathematical modeling. *Procedia-Social and Behavioral Sciences*, *2*(2), 4622-4628.

Tzur, R. (1999). An integrated study of children's construction of improper fractions and the teacher's role in promoting that learning. *Journal for Research in Mathematics Education*, 30*(4)*, 390-416.

Van Loon-Hillen, N., van Gog, T., & Brand-Gruwel, S. (2012). Effects of worked examples in a primary school mathematics curriculum. *Interactive Learning Environments, 20*(1), 89-99.

Van Oers, B. (1998). From context to contextualizing. *Learning and Instruction*, *8*(6), 473-488.

Von Glasersfeld, E. (1990). An exposition of constructivism: Why some like it radical? In *Constructivism views on the teaching and learning of mathematics* (pp. 19-29). Reston, VA: National Council of Teachers of Mathematics.

*Voskoglou, M. G., & Buckley, S. (2012). Problem solving and computational thinking in a learning environment. *Egyptian Computer Science Journal*, *36*(4), 28-45.

*Wang, P. Y., Vaughn, B. K., & Liu, M. (2011). The impact of animation interactivity on novices' learning of introductory statistics. *Computers & Education*, *56*(1), 300-311.

Weber-Russell, S., & LeBlanc, M. D. (2004). Learning by seeing by doing: Arithmetic word problems. *The Journal of the Learning Sciences*, *13*(2), 197-220.

Wells, M., Hestenes, D., & Swackhamer, G. (1995). A modeling method for high school physics instruction. *American Journal of Physics*, *63*(7), 606-619.

Willson, V. L. (1983). A meta-analysis of the relationship between science achievement and science attitude: Kindergarten through college. *Journal of Research in Science Teaching*, *20*(9), 839-850.

Winsberg, E. (2003). Simulated experiments: Methodology for a virtual world. *Philosophy of Science, 70*(1), 105-125.

Xin, Y. P., & Jitendra, A. K. (1999). The Effects of Instruction in Solving Mathematical Word Problems for Students with Learning Problems A Meta-Analysis. *The Journal of Special Education*, *32*(4), 207-225.

Xin, Y. P., Jitendra, A. K., & Deatline-Buchman, A. (2005). Effects of Mathematical Word Problem—Solving Instruction on Middle School Students with Learning Problems. *The Journal of Special Education*, *39*(3), 181-192.

*Xin, Y. P., Zhang, D., Park, J. Y., Tom, K., Whipple, A., & Si, L. (2011). A comparison of two mathematics problem-solving strategies: Facilitate algebra-readiness. *The Journal of Educational Research*, *104*(6), 381-395.

*Yildirim, T. P., Shuman, L., Besterfield-Sacre, M. (2010). Model eliciting activities: Assessing engineering student problem solving and skill integration processes. *International Journal of Engineering Education*, *26*(4), 831-845.

Yimer, A., & Ellerton N. F. (2009). A five-phase model for mathematical problem solving: Identifying synergies in pre-service-teachers' metacognitive and cognitive actions. *ZDM Mathematics Education, 42*, 245-261.

*Yoon, C., Dreyfus, T., & Thomas, M. O. (2010). How high is the tramping track? Mathematising and applying in a calculus model-eliciting activity. *Mathematics Education Research Journal*, *22*(2), 141-157.

*Young, C. Y., Georgiopoulos, M., Hagen, S. C., Geiger, C. L., Dagley-Falls, M. A., Islas, A. L., Bradbury, E. E. (2011). Improving student learning in calculus through applications. *International Journal of Mathematical Education in Science and Technology*, *42*(5), 591-604.

*Yu, S. Y., & Chang, C. K. (2011). What did Taiwan mathematics teachers think of model- eliciting activities and modeling teaching? In G. Kaiser, W. Blum, R. Borromeo, & F. G. Stillman (Eds.), *Trends in teaching and learning of mathematical modeling* (pp. 147- 156). New York, NY: Springer.

*Yuzawa, M., Bart, W. M., Yuzawa, M., & Junko, I. (2005). Young children's knowledge and strategies for comparing sizes. *Early Childhood Research Quarterly*, *20*(2), 239-253.

Zaakis R., Liljedahl, P. (2004). Understanding Primes: The Role of Representation. *Journal for Research in Mathematics Education, 35*(3), 164-186.

Zawojewski, J. (2010). Problem solving versus modeling. In R. Lesh, P. L. Galbraith, Ch. R. Haines, A. Hurford (Eds.), *Modeling students' mathematical* modeling *competencies* (pp. 237-243). New York, NY: Springer.

Zbiek, R. M., & Conner, A. (2006). Beyond motivation: Exploring mathematical modeling as a context for deepening students' understandings of curricular mathematics. *Educational Studies in Mathematics*, *63*(1), 89-112.

Zhang, J. (1997). The nature of external representations in problem solving. *Cognitive Science, 21*(2), 179-182.

Zheng, X., Swanson, H. L., & Marcoulides, G. A. (2011). Working memory components as predictors of children's mathematical word problem solving. *Journal of Experimental Child Psychology*, *110*(4), 481-498.