

## Spatial variability of ocean waves, from in-situ measurements

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I.G.C. Ashton\*, J-B. Saulnier, G.H. Smith

Renewable energy group, C.E.M.P.S., University of Exeter, Cornwall Campus,

**E-mail:** [I.G.C.Ashton@exeter.ac.uk](mailto:I.G.C.Ashton@exeter.ac.uk);

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### Abstract

This paper describes the analysis of the spatial properties of ocean waves using measurements from an array of four directional wave buoys installed in a square formation, with side 500m, in the Celtic Sea, UK. Wave measurements in this area have been installed to support resource assessment and design for wave energy devices at the Wave Hub Site off the North Cornwall coast. This unique deployment of multiple directional sensors provides high quality direct measurements of the spatial properties of the wave field. Spectral parameters measured simultaneously by all four buoys within the array are compared and it is demonstrated that wave conditions cannot be considered stationary across the measurement area. Differences in the measured wave fields were observed primarily in the low frequencies and are observed to be of a level sufficient to impact the assessment of site characteristics. The results demonstrate the potential effect of measurement variability on the monitoring of wave energy sites.

**Keywords:** Wave measurement, marine renewable energy, resource assessment, wave stationarity/homogeneity, spatial variability of wave fields.

## **1. Introduction**

The development of devices to convert energy in ocean waves into electrical energy is underway throughout the world, with the aim of providing a meaningful contribution to global electricity supply. Leading technologies in this field are being deployed at sea at full-scale, either in designated test sites, or as commercial ventures. These early deployments are a critical opportunity for research and development of technology in real sea conditions. In order to maximise the benefits of a test deployment, accurate measurements of the physical conditions are essential to support analysis, and as such, test sites are often heavily instrumented. In addition to progression of the technology itself, test sites also offer an opportunity to develop procedures and strategies for the management of wave energy sites, including how wave conditions are monitored and assessed.

Published standards recommend that an active campaign of direct wave measurements at a wave energy site are combined with numerical modelling, to provide an accurate estimate of the long-term wave climate (Smith and Taylor, 2007, Pitt, 2009a). These data can be applied to estimate extreme conditions, and to assess the wave energy available at the site (Mollison, 1994, Pontes, 1996). Once a site becomes operational, direct measurements will also be used for the short-term analysis of device performance, and may be used to inform methods for monitoring the environmental impact of a wave farm (e.g. Millar et al. 2007).

However, spatial variability in wave conditions across the site has the potential to affect the accuracy of in-situ measurements. The practicalities of marine operations will commonly prohibit the deployment of instrumentation where a device is to be situated, particularly once devices have been deployed. For floating instruments, a safety margin will be required to prevent collision between wave energy converters (WEC) and wave sensors. A significant difference in the wave climate between a point of measurement and the location at which a device is to be situated will affect the assessment of the resource, and the performance of said device.

Best practice for the collection and analysis of wave measurements at a wave energy site has been the subject of significant research, and the results are published in the form of standards, or protocols. These have been developed in the UK to cover resource assessment, (Smith and Taylor, 2007, Pitt, 2009a) and performance assessment (Pitt, 2009b), for wave energy. Furthermore, the EQUIMAR project reviewed

standards for all aspects of data capture and analysis, and was developed from a European basis (Ingram et al., 2011) that will feed into the ongoing formation of international standards, through collaboration with the International Electrotechnical Commission (IEC). Each of these documents acknowledge the potential impact that spatial variability in the wave field will have on analysis for wave energy, and recommend that measurement locations are chosen to avoid systematic differences between measurement site and devices. Pitt (2009b) makes the specific recommendation for shallow water that where a difference in excess of 5% is predicted in the incident wave power, between measurement location and device location, a 'correction factor' should be derived for the power statistics applied to the device. A

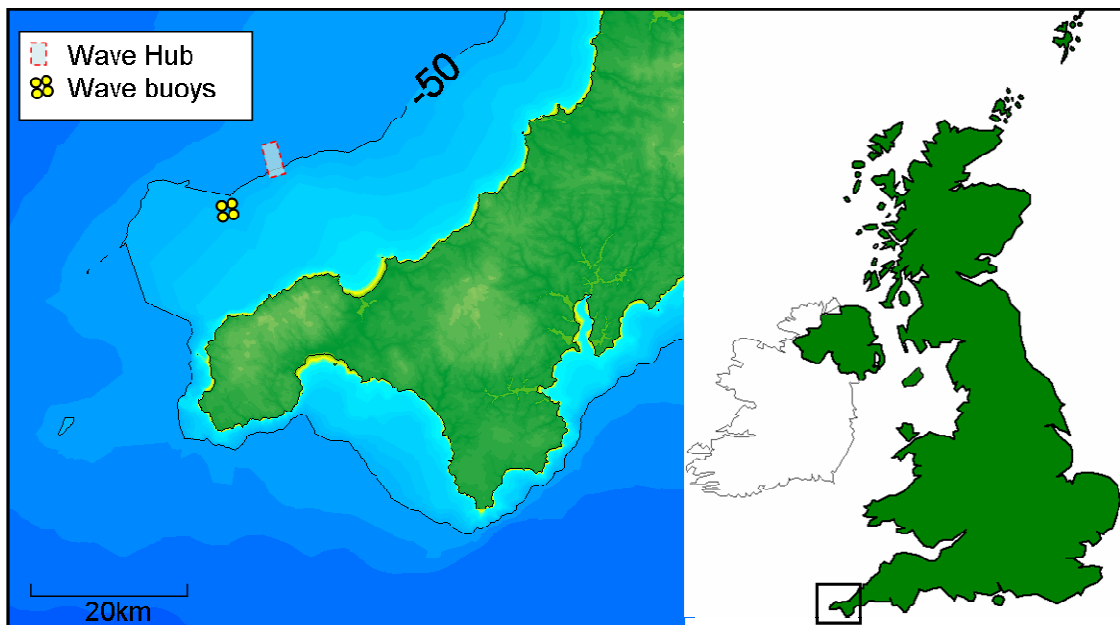


Figure 1. The location of the wave buoy array.

standard methodology for obtaining or applying this correction factor is not provided.

Despite the importance of spatial variability in wave conditions to the assessment of wave energy conversion, little direct data exists with which to examine the potential effect. Remote sensing technologies can provide spatial data sets for a wave energy site. However, the benefits of deploying multiple point measurements to describe spatial changes in the wave field are that the data collected are of excellent quality, with direct measurement of surface motions. As such, they are suitable for detailed analysis of wave processes across the area covered. Temporal resolution is also excellent, with

continuous monitoring possible. When using floating wave buoys, spatial resolution is limited by the excursion radius of the device, which is commonly in the region of 50m. Perhaps the greatest difficulty of such a system is that costs for deployment and servicing increase significantly when deploying multiple instruments offshore, relative to a single sensor.

In the past, multiple point measurements have been used for the intercomparison of wave sensors, and a review of this process is given in Krogstad et al. (1999). Such analysis commonly assumes that the wave processes are the same at all measurement points, or differences are negligible relative to the accuracy of the measurement process. An interesting example of such a project is the WADIC experiment (Allender 1989) which compared the results from a variety of floating wave buoys, corroborated with other technologies and visual recordings. Sova and Wyatt, (2005) used data from the WADIC experiment to compare simultaneous measurements in terms of the spatial variability of the wave field. Significant differences between data sets were primarily related to variability in wind fields affecting high-frequency waves. However, a key limitation of this study was that the measurements were derived from various sensors, and differences in their operation and performance could not be ruled out. More recently, two directional wave buoys have been installed at the European Marine Energy Centre

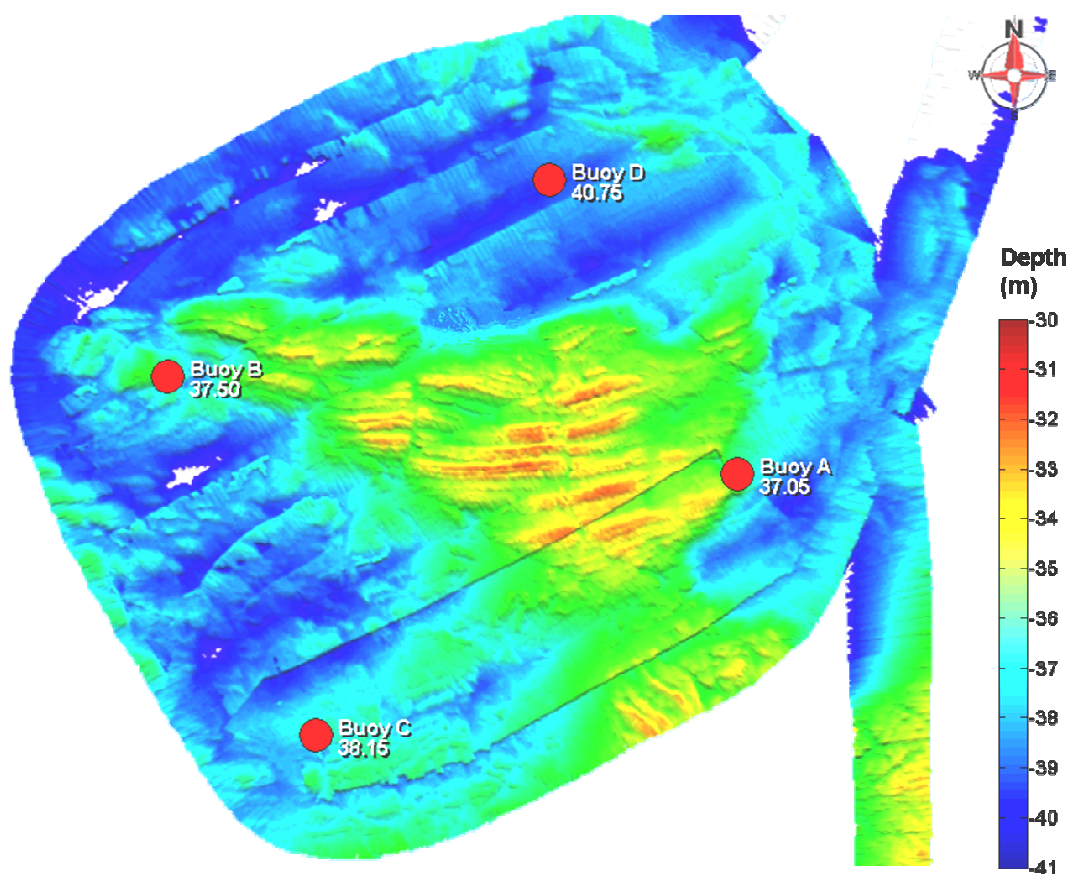


Figure 2. The bathymetry at the wave buoy site, showing depths at LAT.

(EMEC) wave energy test site, separated on similar scales of 500m – 2km. Results from these buoys have been analysed for spatial variability and results indicate a significant difference in wave conditions across the EMEC test site (Smith et al., 2008, Barrett et al., 2009, Mackay, 2009).

This paper describes results obtained from a deployment of four directional wave buoys separated by 500m, which is representative of the expected size of a wave energy test site. This deployment was conducted in an area close to the Wave Hub test site, Cornwall, UK, chosen such that the wave climate was broadly representative of that at a full-scale deployment site. Initial descriptions of the wave measurement site close to the Wave Hub wave energy test site in Cornwall, UK, and the instrumentation installed are provided in section 1.1. Section 2 provides a review of the statistical analysis of wave data, and sampling variability associated with spectral estimates. Sections 2.2 and 2.3 describe the methods used for comparison of data between the data sets that were captured, before section 3 provides results of the application of this analysis to spectral parameters and to spectral energy density in individual frequency bands.

### **1.1 The Wave Buoy Array**

During 2009 and 2010, four Fugro Oceanor SeaWatch mini II directional wave buoys were deployed by the University of Exeter, at a site close to the Wave Hub test site, Cornwall UK (Fig 1.). The buoys were arranged in a square, with sides approximately 500m.

A key upgrade made by Fugro Oceanor to the instruments that were supplied, was that the buoys are phase-locked so that measurements at each buoy co-incide. The clocks are synchronised with Co-ordinated Universal Time (UTC) via a GPS satellite receiver on each buoy, to an accuracy of +/- 0.01s.

The separation of 500m between buoys was chosen as it is representative of the size of a single berth in the Wave Hub test site. It is also equivalent to the size of the safety zone imposed around the Wave Hub itself, within which marine operations, including the deployment of scientific instrumentation, are prohibited. As such, it corresponds to the likely minimum displacement of wave sensors from operating devices.

The deployment site is situated on the continental shelf of the UK, open to the Atlantic Ocean. As such, it has up to 6000km of fetch and receives both swell and local wind waves from low pressure systems in the North Atlantic, which are more energetic during winter. Fetch from the northerly sector is limited to approximately 300km by

Ireland, which blocks swell propagation from the seas surrounding Iceland and Greenland.

The seabed at the deployment site is a mix of exposed bedrock and sandy gravel sediment. The bedrock protrudes 5m above the sea bed, with depths in the site between 36m and 41m above chart datum (fig. 2). This represents deep water for waves of wavelength below 72m and 82m ( $f \sim 0.147$  and  $0.138\text{Hz}$ ) respectively, and intermediate waters for all lower frequencies in the gravity wave spectrum. It follows that interactions between waves and the seabed can be expected to alter the properties of the wave field across the array.

Data for the tidal conditions at the site were extracted from the national tidal model, Polpred (POL, 2011). The model data provides estimates of tidal level, flow speed (integrated for depth) and direction, on a grid of resolution 500m every 30 seconds. A grid output situated close to the centre of the buoy array, was taken to be representative of tidal conditions at all buoys. The tidal range is 5.8m, inducing maximum currents of 1.2m/s. Tidal flows are predominantly from the south west ( $230^\circ$ ) to the north east ( $50^\circ$ ) during a flood tide, and from the north east to the south west during an ebb tide.

Process	Data returns
Measure data at 4 buoys	<b>Over 13 months, servicing and failures cause gaps in the data. 89-93%</b>
↓	
Quality control	<b>All data are quality controlled and errored records are removed 88 - 91%</b>
↓	
Limit data to simultaneous records	<b>Data limited to records where 4 buoys simultaneously return records that are error-free 80%</b>

Figure 3. The data availability associated with each buoy, and how this is limited when data are restricted to records where 4 buoys were measuring simultaneously.

Raw measurements of the water surface elevation were captured at a sampling frequency of 2Hz for 1028 secs (17mins 4secs) every 30 minutes, during a 13 month period between October 2009 and October 2010. Raw data were quality controlled in

accordance with international guidelines for the analysis of oceanographic data (NOAA, 2005). Data processing was designed to follow the guidelines set out in (Tucker, 1993) and (Tucker and Pitt, 2001), in accordance with recommendations in (Sanmuganathan, 2009). Figure 3. demonstrates the steps in data processing and the associated data returns at each stage.

For this research, comparing measurements between the wave buoys, it is critical that errors in the data set are minimised. Failure to do so will introduce inaccuracies on the same scale as the characteristics that are under investigation. Further confidence is gained through the limitation of all data sets to records where all buoys were measuring simultaneously and were error free. This process was seen to decrease the data available to less than the amount captured by the worst performing buoy. 16320 records were available from the worst performing buoy position, although only 15834 records made up the combined data set, which reduced to 14673 after quality control.

An extended gap in the data during November and December 2009 occurred where the buoys were retrieved after a mooring failure (fig. 4). This gap is during the more active winter months and although data are captured during 12 months, annual statistics such as the average power cannot be considered representative of the annual wave climate. A second gap appears during April, which was caused by battery failure in one buoy.

## 2. Statistical analysis of wave measurements

The buoys convert raw measurements of acceleration, heading and tilt into displacement time-series in three dimensions (vertical, east, north). Sampling frequency is 2Hz and takes place during 1024 seconds (17 minutes 4 secs), every 30 minutes, giving time-series with 2048 data points.

Spectral analysis was applied to decompose the time-series of surface elevation,  $\eta(t)$ , into  $N$  component regular waves of different frequencies, represented by a Fourier series as,

$$\eta(t) = \sum_{i=1}^N [a_i \cos(2\pi f_i t) + b_i \sin(2\pi f_i t)] \quad (1)$$

The amplitudes  $a_i$  and  $b_i$  were calculated from the time-series for each Fourier harmonic using a periodogram method, and the spectral variance density was calculated as,

$$\hat{S}_j(f)\Delta f_j = \frac{1}{2} \sum_{i=1}^k (\hat{a}_i^2 + \hat{b}_i^2) \quad (2)$$

for  $j = \{1, 2, \dots, N\}$ , where hats denote estimated quantities. Data were averaged over 8 frequencies ( $k = 8$ ), giving a variance density spectrum for 128 individual frequency bands, between 0 and the Nyquist frequency (1Hz), with resolution,  $\Delta f = 0.0078$  Hz.

Spectral moments were calculated from the averaged, spectrum as,

$$\hat{m}_n = \int_{f_1}^{f_2} f^n \hat{S}(f) df \quad (3)$$

where  $n$  represents the order of the spectral moment. Spectral filters removed data above  $f_2$  and below  $f_1$  were applied to eliminate errors in the very high and very low frequencies. The limit  $f_2 = 0.5$  Hz limit was set by the instrument manufacturer (Sanmuganathan, 2009). The low frequency filter,  $f_1$ , was 0.04 Hz, although an automated inspection of the variance spectrum was used to eliminate errors observed in the low frequencies ( $< 0.05$  Hz). This process was used in response to specific low frequency errors, and full details can be found in (Ashton 2011). Spectral moments were then used to calculate the significant wave height,

$$H_{m0} = 4\sqrt{m_0} \quad (4)$$

and mean wave period,

$$T_{m02} = \sqrt{\frac{m_0}{m_2}} \quad (5)$$

The dispersion equation (equation 6) dictates that speed of propagation is dependent on the depth of the water,  $h$ , and the wavenumber,  $k$ . Hence, in intermediate or shallow waters, it is necessary to consider the speed of propagation of component waves when calculating wave power.

$$\omega^2 = gk \tanh kh \quad (6)$$

The phase velocity of the waves is given by,

$$c_p = \left( \frac{g}{k} \tanh kh \right)^{1/2} \quad (7)$$

The group velocity is given by,

$$c_g = \frac{1}{2} c_p \left( 1 + \frac{2kh}{\sinh 2kh} \right) \quad (8)$$



In deep waters, wavenumber (and equivalently wavelength,  $\lambda = 2\pi/k$ ) are solely dependent on wave frequency,

$$\lambda_0 = \frac{gT^2}{2\pi} \quad (9)$$

As water depth decreases, wave frequency remains constant. At the point  $\frac{h}{\lambda_0} \leq 0.5$ , the dispersion relation (7) dictates that the wavelength will change with depth, however it cannot be solved for the wavelength directly. To approximate the true value the wavenumber,  $k$ , was calculated using an iterative procedure. The incident wave power was then calculated as,

$$P_i = \rho g \int_{f=0.04}^{f=0.5} c_g(f) \hat{S}(f) df \quad (10)$$

$P_i$  represents the total power per m wave crest, incident on the buoy from all directions.

## 2.1 Sampling variability in spectral estimates

Wave spectra, and spectral parameters are statistical summaries of a finite measurement of a continuous process (wave surface elevation). As a result, each parameter contains a random error component, and represents a value drawn at random from a distribution whose mean is the true value of that parameter in the sea state. In practice, the sea state may change during the sampling record, but for the purpose of analysis is assumed to be constant (temporal stationarity). Random variability can be described by the statistical distribution of parameter estimates about the mean (or true) value,

$$\hat{X} = X + \varepsilon \quad (11)$$

where  $X$  denotes the true value of parameter,  $X$ , and  $\varepsilon$  is an error parameter drawn at random from a particular distribution. Statistical comparison between parameters requires a definition of this error distribution. The variance (error variance), or standard deviation (standard error) of this distribution are also used to describe the magnitude of random statistical variability in a given parameter estimate.

Equation (2) expresses spectral estimates,  $\hat{S}(f)$ , as the sum of the squares of two Gaussian distributed random variables, which are known to follow a scaled Chi-Squared distribution. The degrees of freedom (which govern the magnitude of the error

variance) are defined by the averaging used in spectral processing, as (Tucker and Pitt, 2001)

$$\nu = 2p \quad (12)$$

where  $p$  is the number of averages used to derive  $\hat{S}(f)$ , in this case,  $p = 8$ ,  $\nu = 16$ .

Krogstad *et al.* (1999) provides a definition of the covariance in spectral moments,

$$cv(m_r, m_s) = \frac{1}{D} \int_{f_1}^{f_2} f^{r+s} S^2(f) df \quad (13)$$

where  $m_r$  is a spectral moment of the order,  $r$ , and  $D$  is the duration of the record, in seconds. Setting  $r = s$ , this formula can be used to calculate the error variance in a spectral moment of the order  $r$ . Krogstad *et al.* (1999) go on to derive the following equations for the error variance in estimates of key wave parameters including  $H_{m0}$  and  $T_{m02}$  (equations 4 & 5),

$$\begin{aligned} \sigma^2(\hat{H}_{m0}) &= 4cv(m_0 m_0) / m_0 \\ \sigma^2(\hat{T}_{m02}) &= \frac{1}{4} \left( \frac{cv(m_0 m_0)}{m_0 m_2} - 2 \frac{cv(m_0 m_2)}{m_2^2} + \frac{m_0 cv(m_2 m_2)}{m_2^3} \right) \end{aligned} \quad (14)$$

where  $\sigma^2(\hat{H}_{m0})$  is the error variance of an individual estimate of  $H_{m0}$ . Spectral parameters are assumed to follow a Gaussian distribution about their mean (or true value for the sea state) and equations (13) and (14) demonstrates that the variance will be dependent on the shape of the measured spectrum.

The classification of variability in spectral parameters requires quantification of the degrees of freedom in that parameter. Young (1986) shows that the effective degrees of freedom for a spectral parameter is not equivalent to the sum of the degrees of freedom for each spectral estimate, rather it is calculated as a weighted sum that depends on spectral shape,

$$d.o.f. = \frac{\nu \left( \sum_1^N S_j(f) \right)^2}{\sum_1^N S_j^2(f)} \quad (15)$$

Elgar et al. (1987) demonstrate that when using discrete frequency bands, equation (15) will introduce a bias, which reduces with increasing averages. An equation to calculate this bias is given as,

$$d.o.f._u = \frac{1}{1 + \frac{2}{\nu}} d.o.f. \quad (16)$$

which shows that the maximum bias is a factor of 2, when  $\nu = 2$ , and decreases as the number of averages is increased. By using both equations (15) and (16), the number of effective degrees of freedom for spectral moments can be calculated.

## 2.2 Quantifying differences between summary parameters

Differences between wave conditions can be quantified in the first instance through the comparison of long-term mean values of summary parameters.

However, where simultaneous data are available, a more sensitive analysis can be achieved through comparison of data on a record by record basis using the proportional differences between measurements of a given parameter, from a pair of buoys, for a record,  $i$  as follows,

$$dX_i = \frac{X_{1i} - X_{2i}}{\frac{1}{2}(X_{1i} + X_{2i})} \quad (17)$$

where  $X_{1i}$  and  $X_{2i}$  are simultaneous measurements of a given parameter,  $X$ . This methodology was applied to  $\hat{H}_{m0}$ ,  $\hat{T}_{m02}$ ,  $\hat{P}_t$  and  $\hat{S}(f)$  data to calculate the proportional differences between simultaneous measurements.

Random variability in wave fields means that separate measurements of the same sea state will exhibit differences in the instantaneous or short term. However, the proportional differences,  $dX$ , would be expected to have a long-term mean of 0. The statistical properties of  $dX$  will be dependent on the sampling statistics of the parameter in question,  $X$ , which will vary on a record by record basis, and is dependent on the spectral shape of the underlying sea state (equation 13).

Goda (1977) and Mackay (2009) express variability in measured data using the coefficient of variation (*c.o.v.*), which can be defined as follows,

$$c.o.v. = \frac{\sigma}{\mu} \quad (18)$$

This can be related to the proportional differences between two simultaneous measurements as,

$$c.o.v. \approx \sqrt{2}\sigma(dX) \quad (19)$$

where  $\sigma(dX_i)$  represents the standard deviation of the proportional differences of a generic parameter,  $X$ . This formulation is used below to allow direct comparison with the literature (Goda, 1977, Mackay, 2009). Bias, or residual differences between the pairs of measurements, sampling variability, and any further sources of errors, such as instrument noise will affect this value.

The inclusion of theoretical predictions of sampling variability for each datum, allows calculation of the measured differences that are not explained by theoretical sampling variability. This important standardisation allows direct comparison of the measurement differences, or errors, from data sets measured in different wave climates.

Assuming the spectral parameter,  $X_i$ , to have a Gaussian error distribution, the difference between two independent values of  $X_i$ , will also be Gaussian distributed, with a variance equal to the sum of the error variances in the two contributing parameters. It follows that the standard error of the difference between two measurements of the same parameter,  $X_{1i}$  and  $X_{2i}$ , can be calculated for a record,  $i$ ,

$$\sigma(\Delta X_i) = \sqrt{\sigma^2 X_{1i} + \sigma^2 X_{2i}} \quad (20)$$

where  $\Delta X_i$  is the difference between  $X_{1i}$  and  $X_{2i}$ .

Sova and Wyatt (2005) use this formulation to derive a statistic,  $Z$ , that represents the ratio of the instantaneous differences between summary parameters and the theoretical standard error of that difference on a record by record basis,

$$Z_i = \frac{X_{1i} - X_{2i}}{\sqrt{\sigma^2(X_{1i}) + \sigma^2(X_{2i})}} \quad (21)$$

where  $\sigma^2(X_i)$  is the theoretical variance for a parameter calculated using equation (14). This formulation assumes independence between the two simultaneous estimates of  $X_i$ . Where this is not the case, a co-variance term must be included with the denominator in equation (21). For the purpose of this study, simultaneous measurements were assumed to be independent, although it is acknowledged that this may not be valid for all data.

The  $Z_i$  statistic was calculated using summary parameters for each pair of records in the data set. Where there is no difference between the underlying wave fields from which  $X_1$  and  $X_2$  were measured, the values of  $Z_i$  would be expected to follow a standard Gaussian distribution, with mean,  $\mu Z = 0$ , and standard deviation  $sZ = 1$ . If aggregated  $Z$  values do not have a mean of 0, the wave fields measured at the two points are not homogeneous. This was tested statistically using a two-tailed t-test with the null hypothesis,  $H_0$ , that there is no difference in the sea states at two measurement locations.

$$\begin{aligned} H_0 &\rightarrow \mu Z = 0 \\ H_1 &\rightarrow \mu Z \neq 0 \end{aligned} \quad (22)$$

A statistic,  $t$ , was calculated as,

$$t = \frac{\mu Z - \mu}{sZ/\sqrt{n}} \quad (23)$$

where  $\mu$  is the theoretical mean, and was set to 0, and  $n$  is the number of paired records. If  $\mu Z = \mu = 0$ ,  $t$  will be a value from a student's  $T$  distribution with  $n - 1$  degrees of freedom. The probability of a value drawn at random from the cumulative  $T$  distribution, with  $n - 1$  degrees of freedom, exceeding the calculated value for  $t$ ,  $p(t < T)$ , was identified using look up tables. When  $p(t < T)$  was below a critical threshold of 0.025, or above 0.975,  $H_0$  was rejected and it was concluded that the differences in the wave field measured between two locations were significant to a 95% probability.

This methodology was applied to  $\hat{H}_{m0}$ ,  $\hat{T}_{m02}$  and  $\hat{P}_t$  estimates to assess the significance of observed differences between simultaneous data.

### 2.3 Quantifying differences between spectral estimates

In order to analyse the significance of observed differences between spectral energy densities at each frequency,  $\hat{S}(f)$ , it is more convenient to use the ratio of simultaneous measurements,  $\hat{r}(f)$ , defined as follows (Krogstad et al. 1999),

$$\hat{r}(f) = \frac{\hat{S}(f)_2}{\hat{S}(f)_1} \quad (24)$$

where  $\hat{S}(f)_1$  represents the estimated spectral energy density, measured at location 1, for a given frequency band, centred on the frequency,  $f$ . Krogstad et al. (1999) also provide an unbiased estimator for  $\hat{r}(f)$ , with expected value of 1, as

$$\hat{r}_u(f) = \frac{\hat{S}(f)_2}{\hat{S}(f)_1} \frac{\nu_1 - 2}{\nu_1} \quad (25)$$

with variance,

$$\sigma^2(\hat{r}_u(f)) = r^2(f) \frac{2(\nu_1 + \nu_2 - 2)}{\nu_2(\nu_1 - 4)} \quad (26)$$

Both  $\hat{r}_u(f)$ , and  $\hat{r}(f)$  were calculated for each record,  $i = \{1, 2, 3 \dots n\}$ , at each frequency,  $f$ . A long-term mean was calculated then calculated using,

$$\bar{r}(f) = \frac{\sum_{i=1}^n \hat{r}_u(f)}{n} \quad (27)$$

where  $N$  is the number of records in the data set. Equation (27) gives a mean spectral ratio at each frequency,  $\bar{r}(f)$ .

As described above, spectral estimates are chi-square distributed and the ratio  $\hat{r}(f)$  will follow a scaled F distribution, with degrees of freedom defined by those of the spectral estimates (equation 15). For these data, all spectral estimates have 16 degrees of freedom. Therefore, any value of  $\hat{r}(f)$ , would be expected to be drawn from the distribution,  $F_{(16,16)}$ , and, where the underlying wave states measured are the same, any combination of  $\hat{r}(f)$  values would be expected to follow the distribution  $F_{(16,16)}$  (were the unbiased ratio used,  $\hat{r}_u(f)$ , the F distribution would be scaled according to the second term on the RHS of equation (25)).

This assumption can form the basis for a hypothesis test, with the null hypothesis,  $H_0$  that both samples are drawn from the same distribution, and therefore, that  $\hat{r}(f)$  values follow an F distribution,

$$\begin{aligned} H_0 &\rightarrow \hat{r}(f) = F_{(16,16)} \\ H_1 &\rightarrow \hat{r}(f) \neq F_{(16,16)} \end{aligned} \quad (28)$$

For an individual value, the probability that it was drawn from the appropriate distribution was calculated from the cumulative density function for the F distribution as

$p(\hat{r}(f) < F)$ . Where  $p(\hat{r}(f) < F)$  was below a critical threshold of 0.05, or above 0.95, the null hypothesis was rejected with a confidence limit of 90%. For all 6 available pairs of buoy locations, this procedure was repeated for every record in the data set, giving a percentage of records for which the null hypothesis was retained. Based on the confidence limit set, this percentage has an expected value of 90%. Values below 90% indicate that for a given frequency, the differences between measurements from a pair of buoy locations cannot be explained by sampling variability alone.

## 2. Physical processes affecting wave propagation at the measurement site

The depth of the water at the measurement site is such that waves propagating across the site will be interacting with the seabed. In linear wave theory, waves for which,  $\frac{h}{\lambda} \geq 0.5$  are considered deep water, as it is at this ratio that the sea bed begins to influence wave processes. It follows that the interaction of waves and the seafloor at the site will only affect waves for which the deep water wavelength,  $\lambda_0 \geq 76$  m. This relates to frequency components for which,  $f \leq 0.143$  Hz. The interaction with the seafloor will increase with lower frequencies resulting in a complex pattern of energy loss and re-distribution throughout the measurement site. The key physical processes acting are, energy dissipation, which through bed friction will serve to remove energy from the waves, whilst refraction will alter the direction of wave crests and may focus or de-focus energy.

### 2.1 Bottom friction

Energy dissipation occurs through friction between the water column and the seabed in the boundary layer. Flows in the boundary layer are considered turbulent, and energy dissipation can be predicted as the time-averaged product of tangential shear stress and bottom velocity, giving, (see Holthuijsen (2007) for derivation)

$$D_{bfr} = -\rho C_{bfr} u_b^3 \quad (29)$$

where shear stress,  $\tau_b = \rho C_{bfr} u_b^2$ ,  $\rho = 1025 \text{ kg/m}^3$  is the density of water,  $C_{bfr}$  is the bottom friction parameter, discussed below, and  $u_b$  is water velocity at the seabed. In the absence of ambient currents, the velocity at the seabed can be calculated using linear wave theory for a uniform periodic wave. For a regular periodic wave with angular frequency,  $\omega$ , wavenumber,  $k$  and height,  $H$ , average energy dissipation per unit area per second becomes,

$$\bar{D}_f = \rho C_{bfr} \left( \frac{\omega}{\sinh kh} \right) \frac{H^3}{6\pi} \quad (30)$$

where the phase term in the velocity calculation is averaged over a wave cycle,  $\sin^3(\omega t) = 8/6\pi$ . The metric used in this analysis was the proportional reduction in energy as it propagates across the site. This can be expressed as the ratio of energy removed to the energy contained in the original wave,  $E = \rho g a^2$ , giving

$$\Delta E_{bfr} = \frac{\bar{D}_f}{E} = \frac{2}{3\pi g} H C_{bfr} \left( \frac{\omega}{\sinh kh} \right)^3 \quad (31)$$

Notably, this formulation varies with  $H$ , showing that the energy lost through dissipation will rely on the frequency and the magnitude of the incoming wave train.

The constant,  $C_{bfr}$ , varies with the physical properties of the seabed.  $C_{bfr}$ , can be formulated in terms of the grain-size (or roughness) of the sediment, and Mirfenderesk 2003 gives a comprehensive list of different formulations of which that of Madsen et al. 1988 is perhaps the most widely used. Wave induced bed formations will also affect  $C_{bfr}$ . Tolman (1994) describes a combined grain size and moveable bed model, the application of which requires detailed sediment data and consideration of the sediment regime beyond the scope of this paper. Bottom friction in the widely used spectral wave model, SWAN, can be formulated in terms of grain size (Booij 1996). However, in the absence of detailed information about the seabed properties, a co-efficient that remains constant with the root mean square bottom velocity for all spectral components,  $\chi$ , is commonly used,  $C_{bfr} = \chi / g u_{rms}$  (Hasselmann *et al.* 1973).  $\chi$  was originally defined as  $0.038 \text{ m}^2/\text{s}^{-3}$ , although this is often considered applicable to swell conditions, and different values have been proposed for fully developed seas (Holthuisen 2007).

For this research, equation (31) was applied using a constant bottom friction parameter across the array, per frequency. This was applied to provide an estimate of the potential influence of bottom friction on wave energy propagating through the measurement site, to inform conclusions drawn from wave measurements. It was beyond the scope of this paper to attempt a detailed examination of sediment conditions in order to calculate an accurate estimation of wave conditions across the measurement array for incident sea states.

## 2.2 Refraction



Refraction of waves occurs when waves are slowed due to interactions with the sea-floor. This process can be predicted based on the linear model for propagation in finite water depth. The classic method is to plot a refraction diagram, which shows the propagation of wave rays, that begin as parallel lines, and bend according to the profile of wave crests. It is assumed that the energy flow between two rays remains constant. Therefore, changing their direction of propagation will concentrate or dissipate energy along the wave crest. The change in energy as a result of refraction can be quantified by the change in distance between wave rays. A shortening of the distance indicates a concentration of energy, whilst a widening of the gap indicates energy dissipation. Goda (2000) derives a coefficient of refraction,  $k_r$ , in terms of the change in wave heights for a regular wave,

$$\frac{H}{H_0} = \sqrt{\frac{l_0}{l}} = k_r \quad (32)$$

For this research, the statistic of interest is the relative change in energy, which is equivalent to  $\frac{l_0}{l}$ , and here is denoted as  $\Delta E_{rfr}$ .

Differences in the measured data will result from refraction of complex wave fields propagating from a range of different directions. For this research, a refraction diagram function was applied to calculate the effect of refraction on simple harmonic waves with frequencies centred on the frequency bands in the measured spectra, which was then repeated for waves propagating from a range of directions. The maximum changes in energy on the lee side of the measurement array were noted for each run, and represent the largest potential changes in wave energy affected by refraction on waves of different frequencies at this site.

Goda (2000) discusses refraction diagrams, and shows that the classic regular wave diagram will give inaccurate results for the case of a random, irregular sea. He demonstrates that when considering the combined effect of refraction in the whole spectrum, changes in wave height parameters (e.g.  $H_{m0}$ ), are smoothed by the varying effect across the active frequency range. The quantification of refraction at individual frequencies described in this paper, was used to demonstrate the potential impact of refraction on waves of varying frequencies at this site, and results will not be of equal magnitude when examining summary parameters.

### 2.3 Surface winds and Tides

In addition to interactions with the seabed, tidal flows, and surface winds will interact with the surface wave fields. The influence of surface winds has been derived empirically for a standardised spectrum in the Jonswap experiment (Hasselmann *et al.* 1973) and will predominantly transfer energy into the higher frequencies in the surface gravity wave spectrum. Wave-current interactions with tidal flows will alter the absolute propagation speed of waves of all frequencies. Similar to interactions of waves with varying seabed topography, tidal flow effects will cause energy bunching as waves get steeper, and refraction, which will alter the spatial distribution of energy across the measurement site. However, conversely to seabed effects, the effect of tidal flows will be proportionally greater on higher frequency waves.

For this experiment, ADCP flow meters were deployed to measure the current at the site. However, this provided a point measurement rather than a complete spatial analysis of current conditions, without which it is not possible to draw conclusions about the effects of ambient currents on the spatial distribution of wave conditions across the measurement site. It is acknowledged that the complex bathymetry may cause some spatial variability in the tidal flows. Without more data, it is not possible to predict whether this will have a significant effect on the short-term variability of the wave field, or long-term systematic differences in measurements across the array.

Wind conditions were not measured at the site, although in the absence of any nearby landmass, there seems to be justification in assuming that there is no systematic spatial differences in the wind field at the site. This would suggest that random variability in the wind field may increase variability in the high frequency waves, but not produce any difference in the long-term averages.

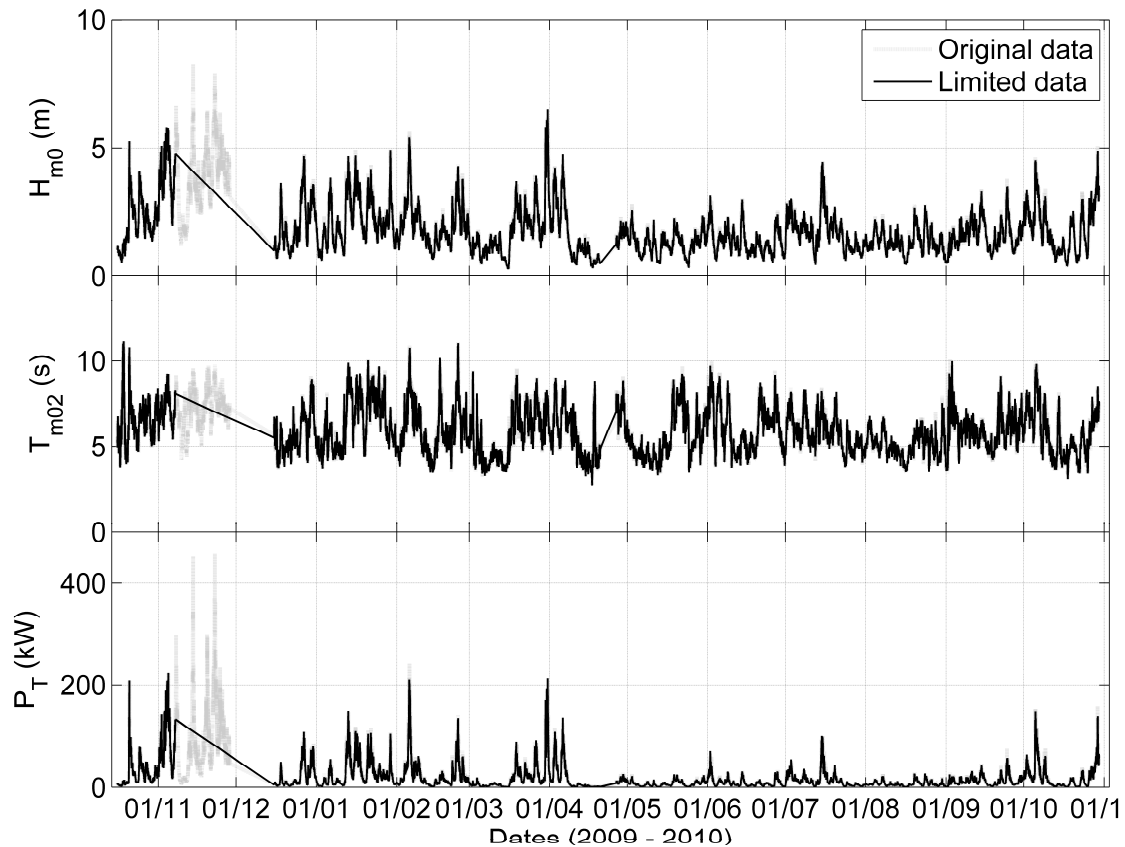


Figure 4. Time-series of key parameters,  $H_{m0}$ ,  $T_{m02}$  and  $P_t$  for buoy location B. The grey line shows the data collected, and the black line shows the limitations imposed by ensuring simultaneous data with the other buoys in the array.

### 3. Results

Figure 4 shows the time series of key spectral parameters,  $H_{m0}$ ,  $T_{m02}$  and  $P_t$ , and demonstrates the wave conditions measured during the deployment. Measurements exhibited a close agreement between data from the four locations, although periods when they are not operational vary significantly between buoy locations. This reinforces the requirement for limiting the data to only records where 4 buoys were measuring simultaneously.

Table 1. The mean values of three key parameters at each location within the array.

Parameter	Mean value at buoy			
	A	B	C	D
$H_{m0}$ (m)	1.78	1.74	1.78	1.76
$T_{m02}$ (s)	5.94	5.84	5.94	5.86
$P_t$ (kW)	17.38	15.98	17.49	16.41

Table 2. The mean proportional differences in key parameters between pairs of buoys, expressed as a percentage.

Parameter	Mean proportional differences between buoys (%)					
	A:B	A:C	A:D	B:C	B:D	C:D
$\mu(dH_{m0})$	2.14	-0.06	1.09	-2.20	-1.06	1.15
$\mu(dT_{m02})$	1.46	0.03	1.25	-1.42	-2.1	1.22
$\mu(dP_t)$	5.76	-0.04	3.47	-5.80	-2.30	3.53

### 3.1 Consideration of average differences between summary parameters

The mean value of each of the key parameters are presented in table 1. These values demonstrate differences between the data from different positions within the array with a maximum difference of 1.5kW/m (9%) in the mean incident power measured between locations C and B. The differences between the other parameters were 0.05m (2.6%) in  $H_{m0}$ , and 0.09s (1.6%) in  $T_{m02}$ .

### 3.2 Record by record differences between summary parameters

The proportional differences in the key parameters between locations were calculated on a record by record basis using equation (17), giving three data sets of differences for each pair,  $dH_{m0}$ ,  $dT_{m02}$  and  $dP_t$  (tab. 2).

$P_t$  again exhibits the largest mean difference between pairs, which is greatest at -5.8% between locations B and C, and smallest at -0.04% between A and C. However, the maximum difference of -2.1% occurs for the parameter  $T_{m02}$ , between the pair, B and D (tab. 2).

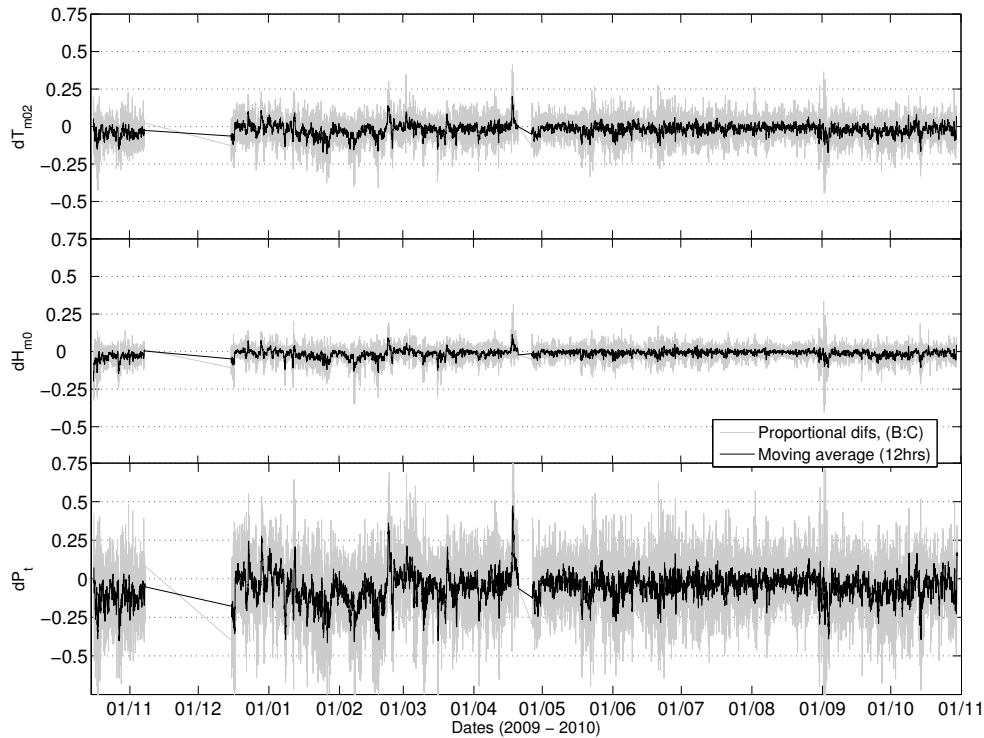


Figure 5. Proportional differences between locations B and C for the three key parameters,  $dH_{m0}$ ,  $dT_{m02}$  and  $dP_t$  calculated using equation (12). Also shown is a moving average of each parameter, taken over 12 consecutive records.

The *c.o.v.* values, calculated using equation (19) were seen to be consistent across all pairs, and had mean values of 5.2%, 3.2%, and 12.42% for of  $H_{m0}$ ,  $T_{m02}$  and  $P_t$  respectively. These values agree closely with Mackay (2009), who calculated the *c.o.v.* of  $H_{m0}$  and  $T_{m02}$  estimates, from a single pair of buoys, separated by 1500m at the EMEC site to be 6.2% and 4.5% respectively.

The time series of  $dH_{m0}$ ,  $dT_{m02}$  and  $dP_t$  for a single pair of buoy locations are presented in figure 5. These graphs demonstrate periods over which the parameters persistently deviate from a zero mean, which is not consistent with the assumption that sampling variability the sole reason for observed differences (fig. 5). Key examples include the first period of deployment, October and November 2009, is a sustained period during which the proportional differences between measurements at locations B and C were persistently less than 0, for each parameter. These differences are again greatest in  $P_t$ , and during 4 weeks, the mean difference was -12.41%. A second period

where measurements at location B are consistently less than at location C can be identified during January and February 2010, contributing to an average difference of -14.0% between the 14<sup>th</sup> January and the 20<sup>th</sup> February 2010.

The differences presented in figure 5 are for the pair B:C, and are of the largest magnitude in terms of the parameter  $dP_t$ . However, other pairs demonstrate the same temporal properties, of a different scale, which suggests the same underlying causes affecting spatial differences between all measurements.

In order to establish the statistical significance of observed differences between summary parameters, a set of  $Z$  statistics were calculated for each pair of buoys for the three key parameters using equation (21). The mean of the  $Z$  statistics ranged from 0.00 to 0.38 (absolute values). Notably, the largest values were consistently attributed to the  $T_{m02}$  parameter, and values for the pairs B:C and A:B were the largest for all parameters. The associated  $t$  values were also calculated using equation (23) and for most records, the mean of the  $Z$  statistic was found to be significantly different from 0. This indicates that differences in measurements of these parameters across the site are significant to a 95% probability. The two exceptions to this result occur for the pair of measurements at locations A and C where the mean  $Z$  statistic for  $H_{m0}$  was small,  $\mu Z(H_{m0}) = 0.001$ , (for which,  $t = 0.11$  and  $p = 0.9$ ). Therefore, the null hypothesis was retained, and the differences were not deemed significant. This was also the case for the wave power measurements,  $\mu Z(P_t) = 0.014$  with  $t = 1.7$  and  $p = 0.09$ . In contrast, for wave period measurements at locations A and C,  $\mu Z(T_{m02}) = 0.050$ , which was deemed to be significant.

In all but one cases, the standard deviation of the  $Z$  statistic is greater than 1, with the highest value,  $\sigma Z(T_{m02}) = 1.15$  occurring for the pair B:C. This indicates that the variability in measured data is greater than that predicted by sampling theory alone. The exception is the result for incident wave power,  $\sigma Z(P_t) = 0.98$ , for the pair B:D, which indicates that the variability between these measurements is less than predicted by sampling theory. In contrast to the mean values,  $\mu Z$ , the standard deviations,  $\sigma Z$ , were consistently smallest for  $P_t$ , and greatest for  $T_{m02}$ . Variability between estimates of the parameter  $P_t$  are within, or close to the levels predicted by sampling theory, whereas the estimates for other parameters are not. Furthermore, whilst results in tables 1 and 2

indicate that bias has the greatest effect on  $P_i$ , analysis of the  $Z$  statistics shows that measurements differences are greatest in the parameter  $T_{m02}$ , relative to sampling variability.

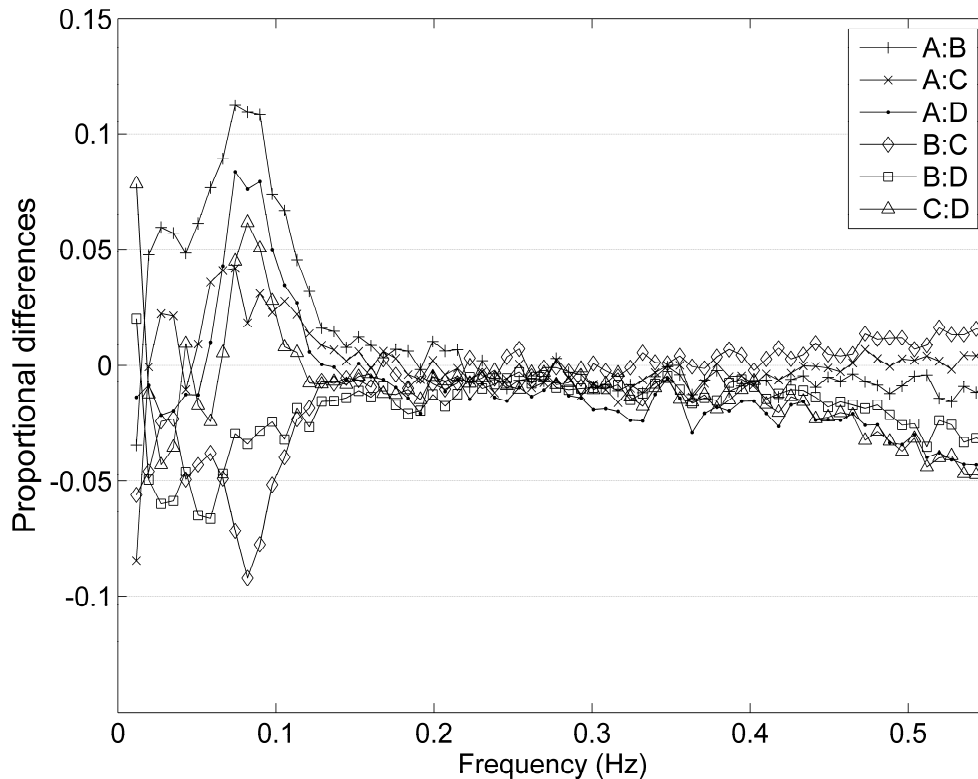


Figure 6. The mean proportional difference between spectral energy density at each frequency band, for pairs of buoys.

### 3.3 Record by record differences in spectral estimates

Spectral analysis of the raw time-series provided  $n$  variance density spectra, with 128 discrete frequency bands, for the 6 pairs of buoy location data sets within the array (where  $n$  is the number of records in the data set).

The proportional differences between spectral estimates,  $\hat{S}(f)$ , at each of the 128 discrete frequency bands in variance spectra, were calculated on a record by record basis using equation (17). The results provide  $n$  values of the parameter,  $d\hat{S}(f)$ , for each pair of buoy locations in the array. The mean values,  $d\bar{S}(f)$ , plotted in figure 6,

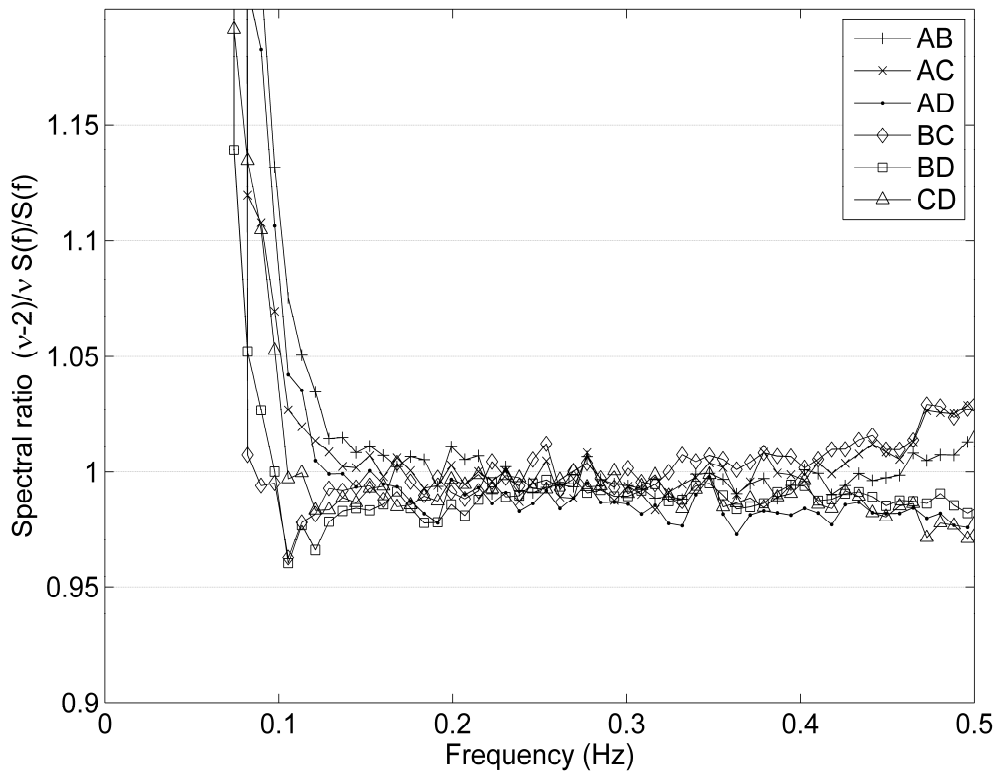


Figure 7. The mean spectral ratio,  $\mu r$ , for each pair of buoys

reveal that the largest mean proportional differences between locations occur in the low frequency measurements for all pairs.

The greatest proportional differences were between estimates for locations A and B, which reached 11.25% at 0.07Hz ( $T = 13.5s$ ). Reflecting the results for parameters, some variability in the magnitude of the observed differences was observed for different pairs. Pairs A:C and B:D showed notably smaller differences in the low frequencies than the other pairs. However, for all pairs, differences were seen to increase for estimates of frequencies below 0.13Hz. An increase in mean differences was also observed for high frequencies, and at the 0.5Hz frequency cut-off used for parameter estimation, the maximum difference was 4% between locations C and D.

Notably, the pair A:B exhibited the greatest difference in the low frequencies and the least difference in the high frequencies. This explains results in table 3 where the pairs exhibiting the greatest differences between parameters was not the same for  $P_t$ ,



which is strongly dependent on low frequencies, and  $T_{m02}$ , which is more dependent on high frequencies through its inclusion of  $m_2$  (equation 5).

For each pair of locations, the spectral ratio,  $\hat{r}_u(f)$ , was calculated using equation (25), and the mean value was found using equation (27), to provide an unbiased estimator of the ratio between variance density spectra. Figure 7 displays the calculated values of  $\bar{r}(f)$  for each pair of locations in the array. Values at frequencies below 0.1Hz increase rapidly with lower frequencies and below 0.07Hz, are orders of magnitude different than 1, reflecting the large differences indicated in figure 6. For clarity, very large values at low frequencies are not shown on figure 7. Across the range of frequencies 0.15Hz – 0.4Hz,  $\bar{r}(f)$  is consistently close to 1. There is some variation between the pairs, and within this frequency range, the pair B:C exhibits values closest to 1, despite demonstrating the largest differences between spectral parameters (tab. 2).

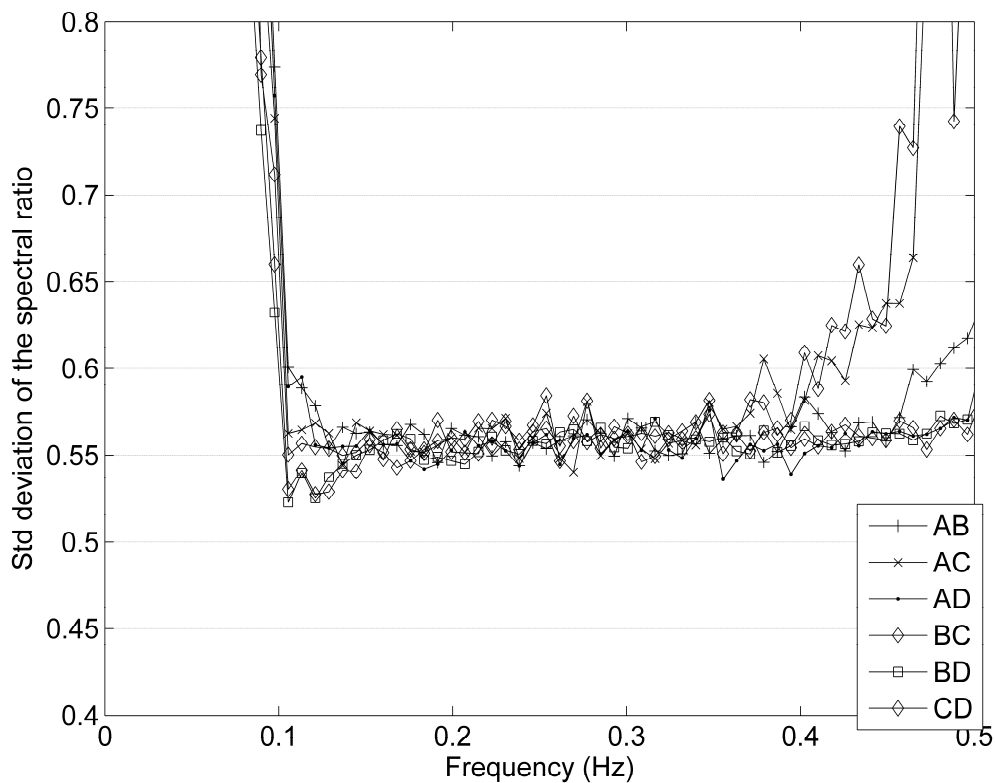


Figure 8. The standard deviation of the spectral ratio,  $\hat{r}(f)$ , for each pair of buoys

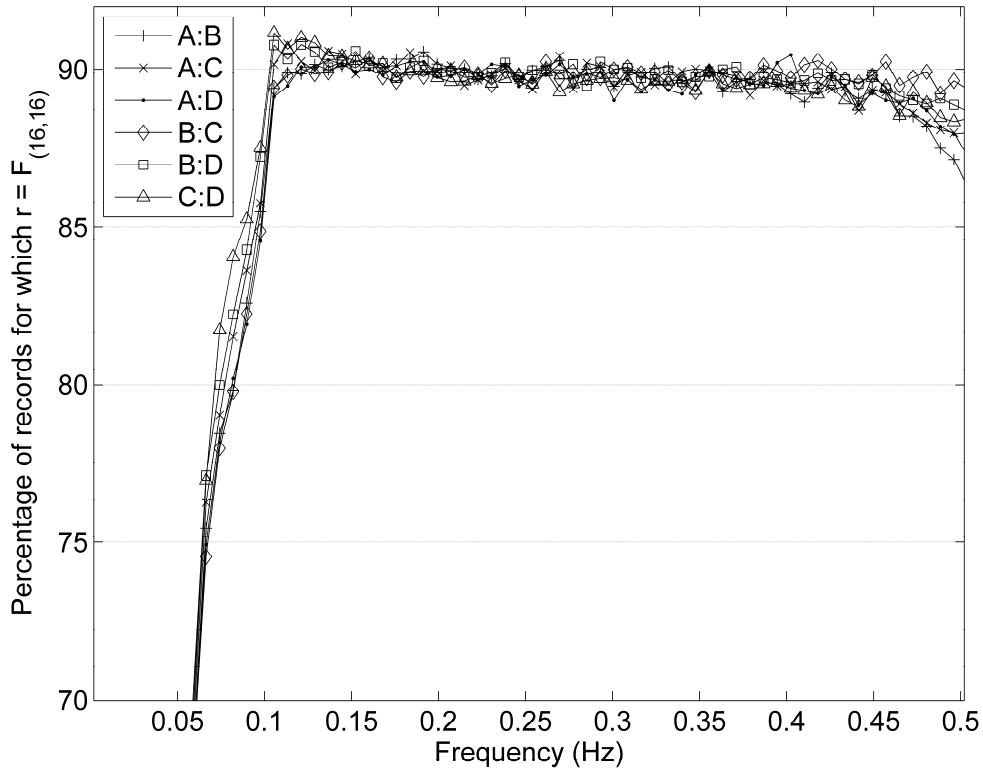


Figure 9. The percentage of records for which the spectral ratio,  $\hat{r}_i(f)$ , was deemed to be drawn from the distribution  $F_{(16,16)}$ , by a hypothesis test with 90% significance level, for each frequency band in the wave spectrum.

The standard deviation of the spectral ratio,  $\sigma^2(\hat{r}_u(f))$ , was calculated using equation (26), to have an expected value,  $E[\sigma r] = 0.56$  for all frequencies. Variability in low and high frequencies causes large values of  $\sigma^2(\hat{r}_u(f))$  below 0.1Hz and above 0.35Hz. However, over the central frequency range, the standard deviations of the measured data are close to this value (fig. 8). This demonstrates that theoretical sampling variability is suitable to explain differences in the measured data. Within this range, the variability can be seen to decrease at lower frequencies, potentially due to increasing correlation between the wave records.

The hypothesis test, on the ratio,  $\hat{r}_i(f)$ , calculated using equation (24) for the  $n$  records in the data set was applied with a 90% significance level and the results are shown in figure 9. In the frequency range,  $0.13 < f < 0.4$  Hz, the null hypothesis,  $H_0$

was retained for approximately 90% of values. This result demonstrates that for these frequencies, the ratio between simultaneous estimates follows an  $F$  distribution with appropriate degrees of freedom, and supports previous results that measurements in this region are consistent across the four data sets. For higher frequencies,  $f > 0.4$  Hz, a decrease in the proportion of records retaining the null hypothesis was observed, which can be attributed to the influence of the bias (figs. 6 and 7) and increased variability at higher frequencies (fig. 8). For three pairs, A:C, B:D and C:D, more than 90% of records retained the null hypothesis for frequencies,  $0.1 < f < 0.13$  Hz. This indicates that measured differences between spectral estimates at these frequencies are less than predicted due to theoretical sampling variability. This is potentially due to correlation in the underlying time-series, which would be expected to primarily affect low frequencies.

Results displayed in figure 9 for frequencies below 0.1 Hz, reflect those in figures 6 and 7, with a dramatic decrease in the proportion of records retaining the null hypothesis with frequency.

## **5. The influence of physical processes**

To aid conclusions relating to the observed differences in measured data across the array, this section estimates the potential contribution of physical processes on the wave field. The cumulative effect of two key source terms, bottom friction (equation 31) , and refraction (equation 32), were calculated for the same frequency bands that were estimated in the measured wave spectra. This was based on the bathymetry data plotted in figure 2. The aim of this study was not to predict the measured differences described in section 4. Rather, it was to predict the potential impact of these source terms, and their frequency distribution, in order to establish whether they are of a similar magnitude to the observed differences.

### **5.1 Bottom Dissipation**

In order to predict the potential influence of energy dissipation through bottom friction, the drag-flow model described by equation (29) above, was applied to calculate the cumulative energy loss due to bottom friction across the measurement array. This was achieved by calculating the cumulative energy loss for each metre in the direction of travel of the waves for the range of frequencies measured by the buoys,  $0.04 \leq f \leq 0.5$  Hz, using equation (30). A constant bottom friction co-efficient with value 0.01 was used, which represents an approximation, albeit one that is commonly used as

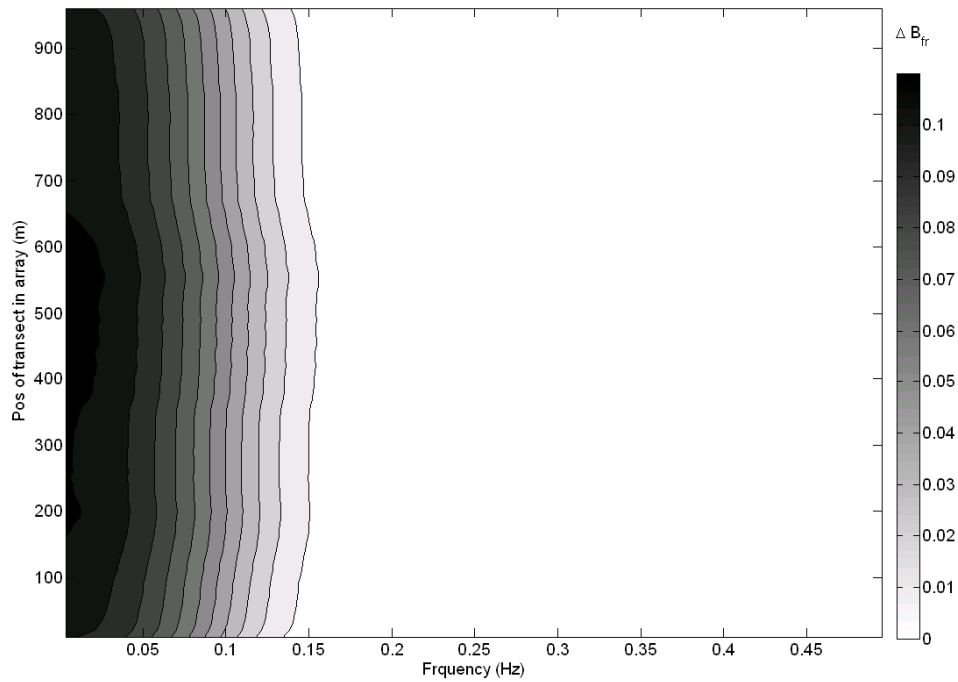


figure 10. The proportional difference in spectral energy due to bottom friction as waves propagate through the array. The energy associated with each frequency band is constant

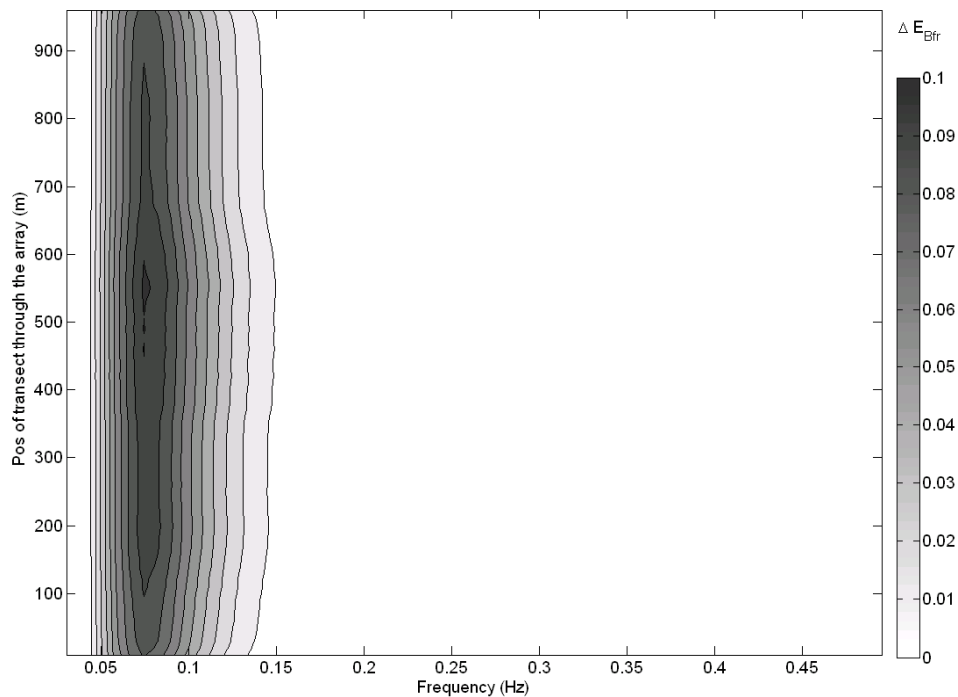


figure 11. The proportional difference in energy due to bottom friction as waves propagate through the array in a westerly direction. The initial energy within each frequency band was set as the mean spectral energy measured at the Northerly position in the array, and the co-efficient,  $C_{bfr}$ , was set to 0.01.

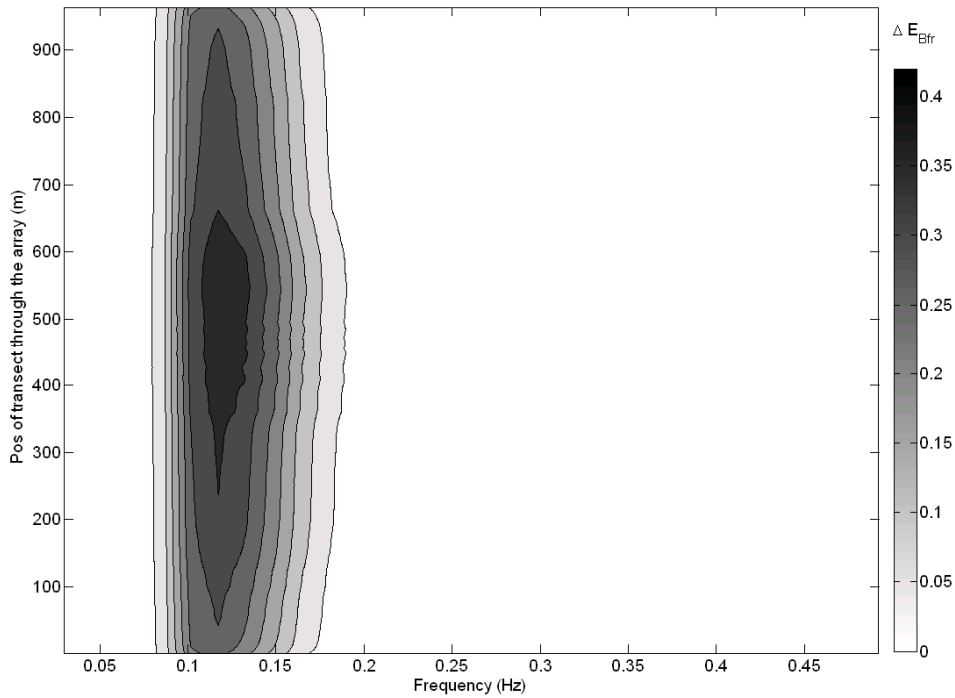


figure 12. The proportional difference in energy due to bottom friction as waves propagate through the array in a westerly direction. The initial energy within each frequency band was set as the maximum spectral energy measured at the Northerly position in the array, and the co-efficient  $C_{bfr}$  was set to 0.01.

as the proportional difference between an eastern boundary, a north-south line passing through the location of buoy B, and a western boundary, a similar line passing through the location of buoy A on figure 2. Locations with no bathymetry data were represented with a constant value of -44m. This will render the most northerly and most southerly values inaccurate, due to the limited bathymetry for these transects (fig. 2).

Figure 10 shows the proportional energy dissipated, per frequency, for waves of varying frequency, but constant energy ( $H = 1$ ), propagating through the array from west to east. An increase of the effect of bottom friction with decreasing frequency can be observed, demonstrating the increased potential for energy dissipation in waves of lower frequency and longer wavelength. However, energy dissipation is also dependent on the initial energy in the wave train. Therefore, for comparison with the measured data, it is important to take into account the levels of energy that can be experienced at this site, for waves of different frequencies.

The proportional energy dissipation was also calculated using  $H$  values defined for each frequency band to represent a regular wave with the same energy as the measured mean (figure 11) and max (figure 12) spectral values in the data measured at the westerly position (buoy B). The magnitude of the observed proportional differences reached a maximum of 11% for the mean case, increasing to 42% for the maximum energy per frequency. The dependency on frequency observed in figure x is tempered by the dependency on the initial energy for individual components.

Through examination of the maximum difference per frequency for all transects through the array (fig. a), it is observed that the difference in energy exceeds 1% for frequencies,  $0.04 \leq f \leq 0.15$  Hz, both for mean (fig. 13(a)), and maximum (fig. a(b)) spectral values. This can be compared with results presented in figures 6 and 7, where significant differences in the spectral estimates were observed at these frequencies.

## 5.2 Refraction

A refraction diagram was plotted for the centre of each frequency band in the measured spectra. Figure b gives an example refraction diagram, for a simple harmonic wave of frequency 0.066Hz, with an initial direction from west to east. The maximum changes in energy on the lee side of the measurement array, along a line orientated north-south level with buoy A, were noted for each frequency,  $\max(\Delta E_{rfr}(f))$ , and represent the largest potential changes in wave energy affected by refraction on waves of different frequencies for waves propagating in this direction (fig. c). Differences are observed for frequencies below 0.15, although they do not increase consistently at lower frequencies. Rather, the highest value of  $\max(\Delta E_{rfr}(f))$ , is observed for,  $f =$  Hz.

This parameter was seen to be dependent on the direction of propagation, which was assessed through calculating  $\max(\Delta E_{rfr}(f))$  for 10 separate directions of propagation, spread evenly between 0 and  $360^\circ$ . The average maximum change in energy due to refraction,  $\max(\Delta E_{rfr}(f))$ , exceeds 1% only for frequencies below 0.15Hz. It is also seen to increase consistently with a decrease in wave frequency (fig. d). The maximum values provided here are indicative of the potential for re-distribution of wave energy however, their definition as the maximum change across the array, means that their magnitude cannot be compared directly to the measured values.

## 4. Discussion

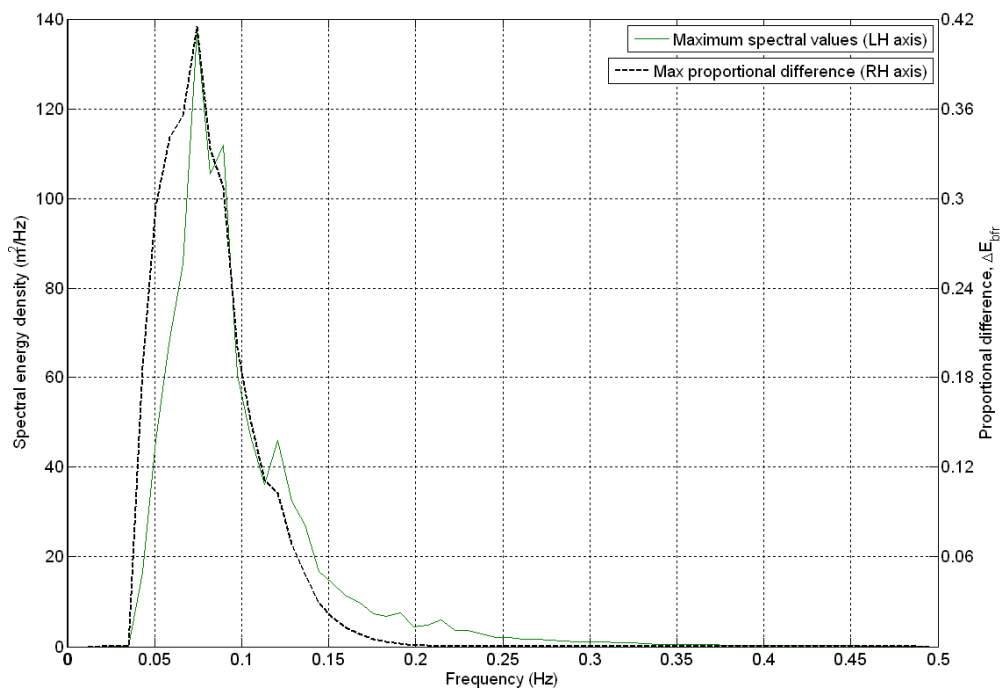
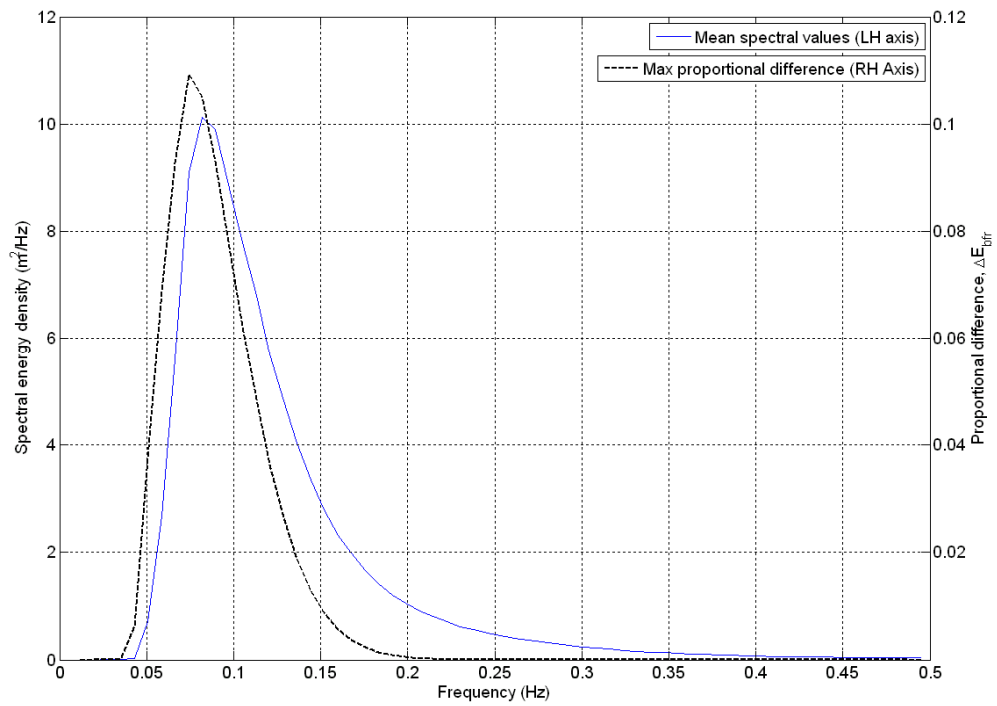


figure 13. The maximum proportional difference in energy due to bottom friction, of all transects through the array in a westerly direction. The co-efficient  $C_{bfr}$  was set to 0.01, and the initial energy in each frequency band was set by a) the maximum and b) the mean spectral energy density measured at the northerly position within the array.

used to analyse the spatial properties of ocean wave fields on the of proposed wave energy arrays, or wave farms. A detailed statistical investigation demonstrates that key parameters vary significantly between the data sets. Quantifying the observed differences in terms of the theoretical variability for each datum provides a robust assessment of the measured data. The methodology is recommended for future analysis of similar data sets as it ensures that results directly comparable between data sets.

Of the parameters compared, the measured differences were greatest for the mean incident wave power, where an 8.6% difference was observed for 12 months of data. This parameter is critical for resource assessment of a proposed wave energy site, and indicates that the placement of wave sensors can play a critical role in the accuracy of assessment of long-term wave statistics.

When examined for each frequency band in the spectrum, the greatest differences between buoys were observed in the low frequencies,  $f < 0.1\text{Hz}$ . It is this frequency dependence that increased the bias in the low-order summary parameter,  $P_t$ , relative to other parameters.

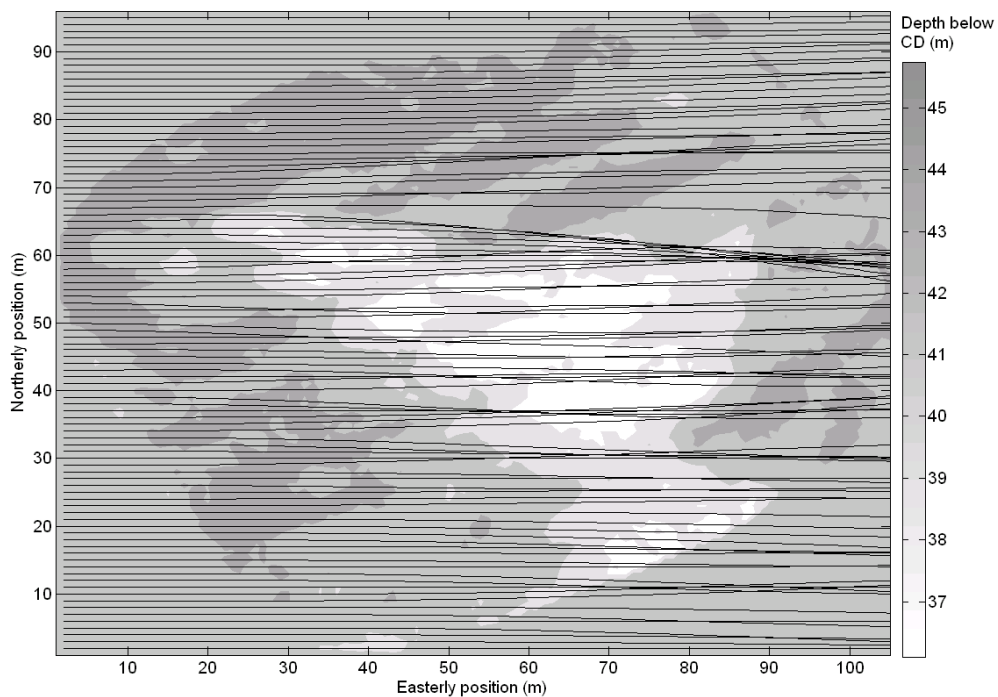
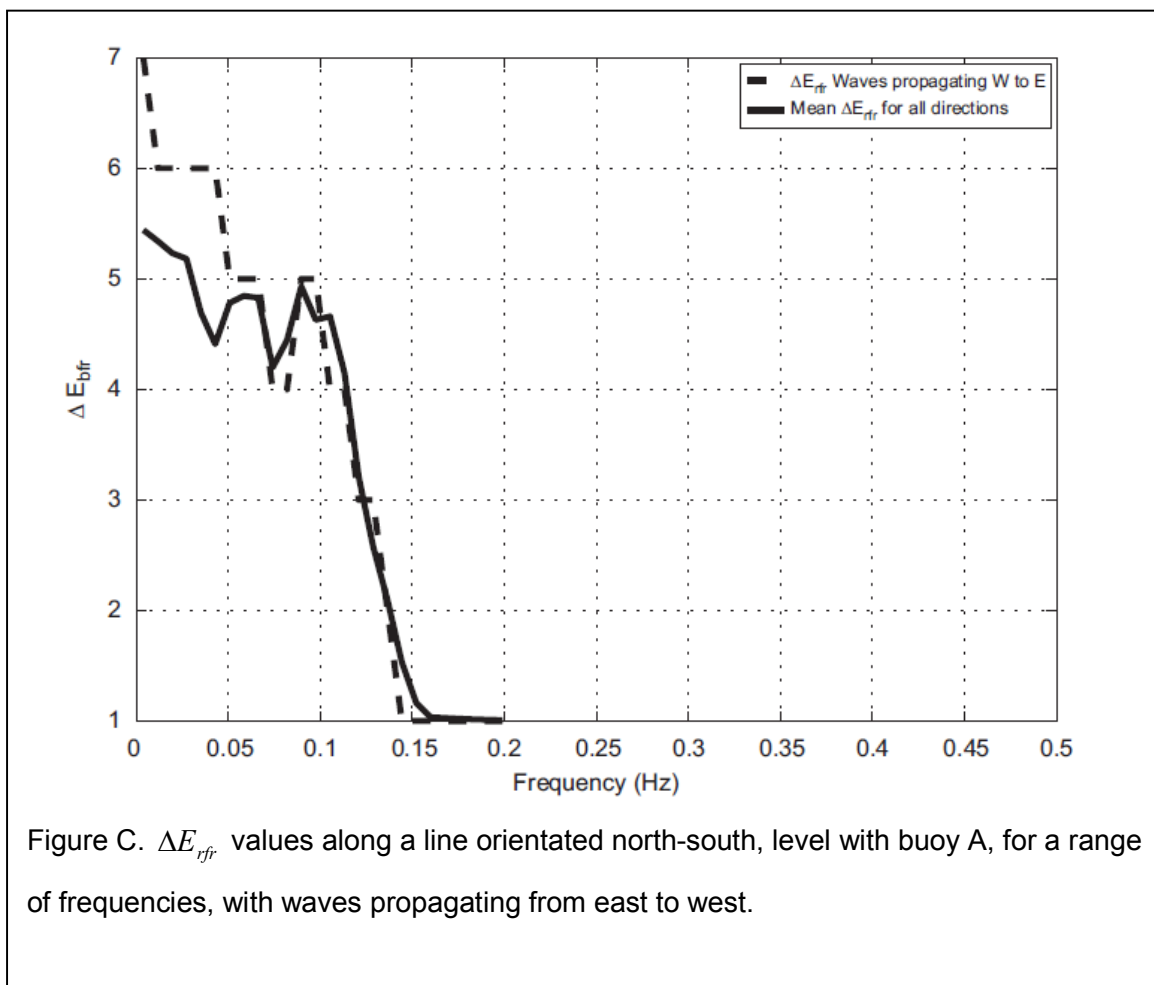


Figure 14. An example refraction diagram, for a simple harmonic wave of frequency 0.066Hz, with an initial wave direction from west to east.



A small bias was also observed for high frequencies,  $f > 0.35\text{Hz}$ , although random variability was relatively large. This agrees with observations that the variability in the parameter,  $T_{m02}$ , are large relative to the theoretical sampling variability, indicated by variability in the statistic,  $Z(T_{m02})$ . This shows that a spatial separation introduces an increased random error term, which may be driven by spatial variability in the winds fields, as suggested by Sova and Wyatt, (2005).

Low frequency waves would be expected to interact with the local bathymetry in this area and the observed differences may be caused by deterministic spatial differences in wave conditions. Theoretical calculations of the potential effect of energy dissipation due to bottom friction, and the spatial re-distribution of energy through refraction have been presented. The frequency profile of these two effects are comparable to that of the differences observed in the measured data. This indicates that these processes could be expected to cause differences in the data measured across



the array and are likely to contribute to the observations.

The outcome suggests that the rocky protrusion in the sea bed situated within the array affects the flow of wave energy between the measurement locations. The differences observed in the measured data are sufficient to have a significant effect on the wave statistics for the site. It is therefore essential that physical processes are considered, for an accurate assessment of wave conditions when using separated wave sensors in intermediate waters. Published standards for the assessment of wave energy sites suggest the use of wave models to provide wave data across the site (e.g. Ingram 2011). The results presented in this paper demonstrate that the accuracy of operational wave models at resolving bottom friction and refraction must be considered carefully. Inaccurate, or incomplete formulation of these processes, will lead to significant errors in subsequent assessments.

Interactions with the seabed can be related to observed differences in the low frequencies. However, tidal effects will also be affecting the wave field across the measurement site. Spatial variability in the tidal flows will cause spatial variability in the high frequency measurements and this may be responsible for differences observed in figures 6 and 7. Furthermore, bottom dissipation is calculated using the water particle velocity at the seabed. It can therefore be expected that the presence of an ambient current, will affect this process. A detailed assessment of the current field would benefit from an intensive measurement effort to measure the variability of tidal flows, in order to assess their influence on the wave measurements across the array.

When considering causes for the observed differences, one must also consider the operation of the wave sensor. A clear benefit of this facility over previous multi point deployments (e.g. Allender 1989, Barrett *et al.* 2009) is that all the sensors are the same, have the same mooring setup, and are time-synchronised. Nevertheless, inaccuracies due to variability in the operation of individual components cannot be ruled out, and a more accurate quantification of the effect of deterministic processes on the wave field at this location would allow a more accurate assessment of the accuracy of the wave sensor. A further deployment of multiple sensors in close proximity in deep water would also achieve the same goal.

The work presented here offers a detailed and robust statistical analysis of wave data from a unique deployment. the site is typical of that proposed for full scale wave energy test sites, and the distances considered are relevant to separations seen at such sites. The results demonstrate that for WEC deployments, physical processes must be

taken into account for an accurate assessment of the wave properties from spatially removed point wave sensors. The methodology used is suitable for a comparative assessment of differences where multiple point measurements are available.

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Figure D. The maximum  $\Delta E_{rfj}$  per frequency, along a line orientated north-south, level with buoy A, with waves propagating from east to west.