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Optimum Design of a Pultruded FRP  
Bridge Deck.

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## OPTIMUM DESIGN OF A PULTRUDED FRP BRIDGE DECK

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### ABSTRACT

In this paper, an optimum design of GFRP bridge deck having a pultruded cellular cross-section is presented. The optimization process utilizes a modified genetic algorithm with the index technique. Based on the optimum design, viable cross-sectional dimension, volumes of fibers and matrix, fiber orientation, and stacking sequence for GFRP decks suitable for the pultrusion process are proposed.

### INTRODUCTION

The apparent advantages of FRP (fiber reinforced plastics) composites over the conventional structural materials may be attributed to their high specific strength and stiffness. Other affordable properties of FRPs including an excellent durability make them particularly attractive for the structures in severe service conditions. Therefore, the material and sectional properties of a FRP structural component should be designed to meet its specific requirements and service conditions.

Nowadays, many different types of structural shapes and materials are available for the FRP systems to achieve the optimal performance of the structures. Since the FRPs are relatively expensive materials than the conventional ones, both structural and material optimizations may be necessary for the design of FRP structural systems to reduce the fabrication cost.

Over two decades, a numerous studies have been performed for the structural optimizations, and most of them are related to the practical applications for the tractable engineering problems. Although many efficient structural optimization algorithms are currently available, the great number of the design variables and constraints are still required in the solution process of structural optimization. If the number of design variables and constraints is increased, a great deal of structural analysis is required in the optimization process. The highlights of recent efforts on structural optimization for the FRP decks are briefly discussed in the following.

Theoretical and practical applications of structural optimization are well summarized by Cohn and Dinovitzer (1). Burnside *et al.* (2) have proposed an optimization process for the FRP bridge decks having the cellular- and stiffened-box geometries. An optimal structural shape of FRP beams having a wide-flange section is presented by Qiao (3). In his study, the stacking sequence, volume ratios, number of ply, and ply angle of FRPs are considered as the major design variables. Based on the classical lamination theory, Mantell and Heiness (4) have also proposed an optimization procedure for a GFRP composite box beam. More recently, an optimum design of a precast FRP system is presented by Salem (5).

This paper deals with an optimum design of GFRP bridge deck having a pultruded cellular cross-section. The primary objectives of this study are to develop an efficient optimization procedure for the design of FRP systems and to design an optimum cross-sectional profile of GFRP bridge deck. The optimization process utilizes a modified genetic algorithm (GA) with the index technique. Based on the optimum design for both material and structure, a viable cross-sectional profile with the optimum material architecture for a GFRP deck was proposed.

## FORMULATION OF OPTIMIZATION

An optimum design problem generally involves both the material and structural design. Since the formulation of this problem usually employs a large number of design variables and constraints, the problem is solved by the time-consuming iteration process.

In order to perform an optimum design more efficiently, a GA-based optimization procedure was employed in this study. The design variables considered herein were the elastic modulus, Poisson's ratio, strength, fiber volume ratio, and fiber orientation of FRPs. On the other hand, cross-sectional dimensions and shape were selected for the structural design variables, as shown in Figure 1.

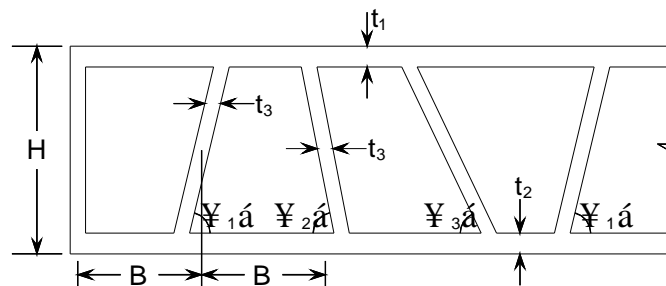


Figure 1. Thin-walled Cellular Section

In this study, an objective function was chosen to minimize the volume of a FRP structural system, as

$$\text{minimize } \sum_i (w_i \times t_i) \times L_i \quad (1)$$

where  $w$ ,  $t$ , and  $L$  are the width, thickness, and length of components within the section, i.e. flanges and webs. The subscript  $i$  represents the index for each component.

Three types of constraints were employed in this study: the constraints for design requirements; constraints for serviceability; and constraints for fabrication limits. The design and serviceability constraints are provided in Table 2. These constraints were established based on the design provisions specified in the Bridge Specifications [MOCT (6)], design criteria suggested in the FHWA's Advisory [FHWA (7)], and design codes in the Handbook [EUROCOM (8)]. The constraints provided in Table 3 were established to achieve the quality control of fabrication by considering the capability of fabricator in our country.

Table 2. Constraints for Design Requirements and Serviceability

Descriptions	Conditions
Maximum Flexural Moment	$M_n / M_u - 1 \leq 0$
Maximum Stress	$f / f_a - 1 \leq 0$
Local Buckling Load	$N / N_{cr} - 1 \leq 0$
Minimum Height of Deck	$h_{min} / h - 1 \leq 0$
Deflection Due to Live Load	$\delta / \delta_a - 1 \leq 0$

Table 3. Constraints for Fabrication Limits

Descriptions	Conditions
Thickness of Components	$t_{min} / t - 1 \leq 0, \quad 1 - t_{max} / t \leq 0$
Number of Ply	$n / 9 - 1 \leq 0$
Fiber Volume Ratio (Min., Max.)	$V_{f\_min} / V_f - 1 \leq 0, \quad V_f / V_{f\_max} - 1 \leq 0$
First Ply Failure	$P / P_{FPF} - 1 \leq 0$
Ply Angle	$\theta / 90 - 1 \leq 0, \quad 1 - 90 / \theta \leq 0$

## OPTIMIZATION ALGORITHM

Practical optimum design problems may be characterized by the mixed continuous discrete variables, and discontinuous and non-convex design spaces. If standard nonlinear programming techniques are used for these types of problem, they will be inefficient and computationally expensive. In most cases, these techniques may give a relative optimum that is close to the starting point [Vanderplaats (9)]. On the other hand, the optimization procedure based on the GAs may efficiently exploit the historical information to speculate on new search points with the improved performance. Therefore, a GA-based optimization procedure was employed and coded as a computer program. A concise flow of the overall optimization procedure is illustrated by a flowchart shown in Figure 2. As shown in the figure, three-dimensional structural computations were carried out by a commercial finite element (FE) analysis package, ABAQUS (10).

To treat optimization problem more efficiently, the index technique was employed in this study. The modified GA-based solution algorithm with the index technique is shown in Figure 3. Generally, the GA can be applied to an optimization problem with the non-constraints that are considered as the penalty parameters and maximum of fitness function.

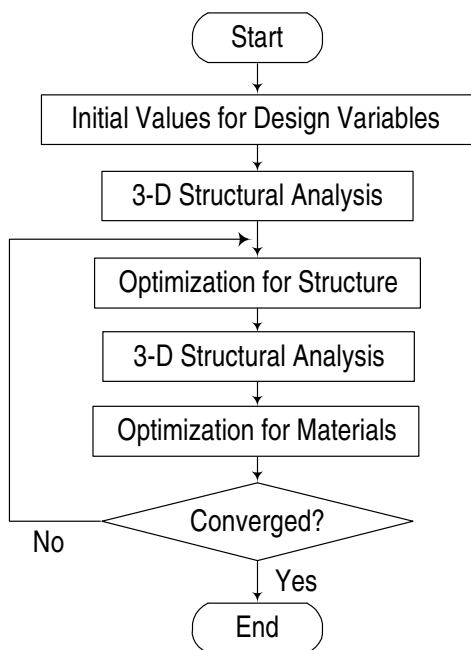


Figure 2. Flowchart for Optimization Procedure

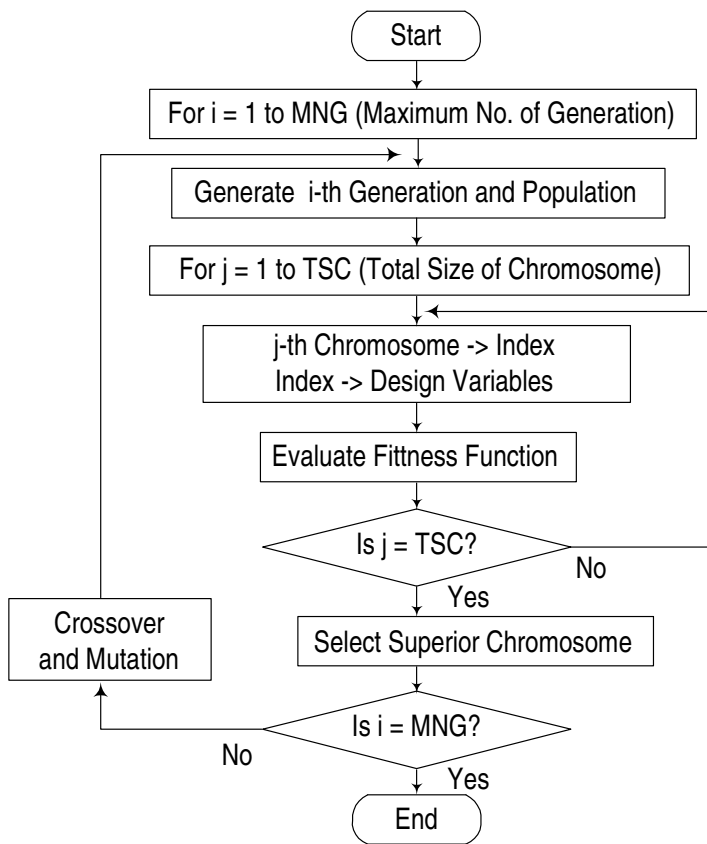


Figure 3. Modified GA-based Solution Algorithm

Accordingly, a penalty function used in this study is given as

$$f'(X) = f(X) + \begin{cases} R \sum_{j=1}^m \Phi(g_j(X))^\eta & \text{for } g(X) > 0 \\ 0 & \text{for } g(X) < 0 \end{cases} \quad (2)$$

in which  $\Phi$  is the penalty function with constraints  $g(X)$ , and  $R$  and  $\eta$  the penalty parameters, respectively. The fitness function,  $f(X)$ , can be written as

$$f(X) = \frac{C}{f'(X)} \quad (3)$$

where  $C$  is the parameter for fitness function.

The GA maximizes the fitness function in Eqn. (3). In the analysis, GA parameters are determined by the trial and error method. The values used for parameters of generation  $R$ , population  $N_p$ , and penalty  $\eta$  were 10,000, 30, and 6, respectively.

### DESIGN EXAMPLE

The optimization procedure briefly discussed above is implemented into a computer code. Using this code along with a commercial FE analysis engine, a GFRP deck for a prototype steel I-girder bridge was designed. As shown in Figure 4, the single-span bridge consists of a deck width of 12 m and is supported by five 40 m long steel girders spaced at 2.5 m, as provided in the Design Manuals [MOCT (11)]. The widths of the top and bottom flanges of the girder are 480 mm and 650 mm, respectively. The thicknesses of the top and bottom flanges, and web of the girder are 32 mm, 36 mm, and 12 mm, respectively. Standard DB-24 truck load shown in Figure 5 [MOCT (6)] was assumed for a design live load.

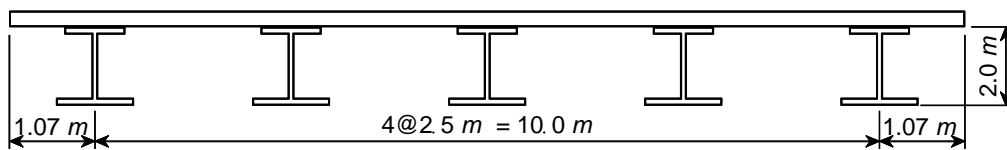


Figure 4. Front View of an Example Bridge

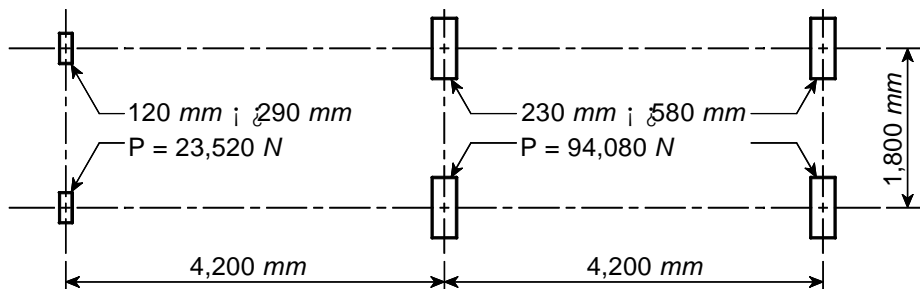


Figure 5. DB-24 Truck Load and Tire Contact Area

A DB-24 truck load is approximately 1.3 times heavier than AASHTO's HS20 truck load [AASHTO (12)]. For a steel girder, the elastic modulus of 205.8 GPa and Poisson's ratio of 0.3 were assumed. Deflection limit of  $L/800$  and ultimate safety factor of five that provided in the FHWA's Advisory [FHWA (7)] were applied. In the material design, the unidirectional E-glass roving, continuous strand mat (CSM), +45°/-45° woven fabric (WF), and vinylester were assumed for the constituents.

Figure 6 shows the results of structural optimization. A trapezoidal shape is identified as an optimum cross-sectional shape for a GFRP deck considered herein. On the other hand, Figure 7 shows the material compositions and stacking sequences for the flanges and web of a GFRP profile. Table 4 provides the results of material optimization along with the designed values of material properties. In the table, the materials properties used in the FE analysis are denoted as design.

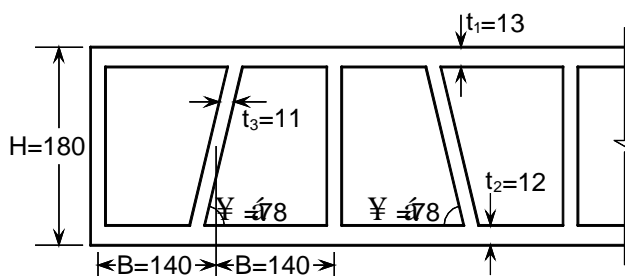


Figure 6. Optimum Structural Shape for a GFRP Deck (Units: mm)

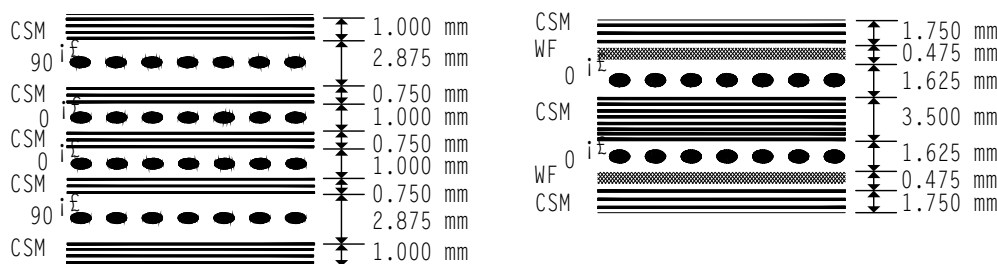


Figure 7. Material Architecture for Flange and Web

Table 5 provides the results of structural analysis for the example bridge along with the design limits. The results show that the induced stresses within the deck are smaller than those of the strength limits multiplied by the safety factor. The compressive stresses induced in the flanges and webs are also smaller than the local buckling strengths in flexure and compression modes that were computed by using the analytical equations provided in the Handbook [EUROCOM (8)].

In addition to an optimum design discussed above, a deck profile with a rectangular shape has also been optimized. Figure 8 shows the cross-section of the deck profile with a rectangular shape obtained by the structural optimization. The results of structural analysis and material properties of this profile are summarized in the reference by Kim *et al.* (13).

Table 4. Material Properties of FRP Deck with Trapezoidal Shape

Description		Flanges		Web	
		Design	Optimum	Design	Optimum
Elastic Moduli (GPa)	$E_{11}$	25.29	22.59	20.37	20.37
	$E_{22}$	16.00	16.00	12.79	11.51
Poisson's Ratio	$\nu_{12}$	0.30	0.26	0.30	0.26
Shear Moduli (GPa)	$G_{12}, G_{13}$	3.96	3.91	5.04	4.89
	$G_{23}$	2.32	2.24	1.41	1.35
Volume Ratio	$V_f$	0.49	-	0.45	-

Table 5. Results of Structural Analysis for a GFRP Deck System

Description	Design Limits	Results of Analysis	Factor of Safety
Stress in Fiber Direction ( $f_{11}$ , MPa)	392.4	76.64	5.1
Stress in Transverse Direction ( $f_{22}$ , MPa)	392.4	77.55	5.1
In-plane Shear Stress ( $f_{12}$ , MPa)	78.5	14.73	5.3
Tsai-Hill Failure Criterion	< 1.0	0.194	5.2
Buckling Strength of Web in Shear (kN/m)	2,572.6	1,254.9	2.0
Buckling Strength of Web in Flexure (MPa)	2,210.0	117.07	18.9
Buckling Strength of Flange in Flexure (MPa)	103.9	51.90	2.0
Maximum Deflection of Deck (L/800, mm)	50.00	23.58	2.1

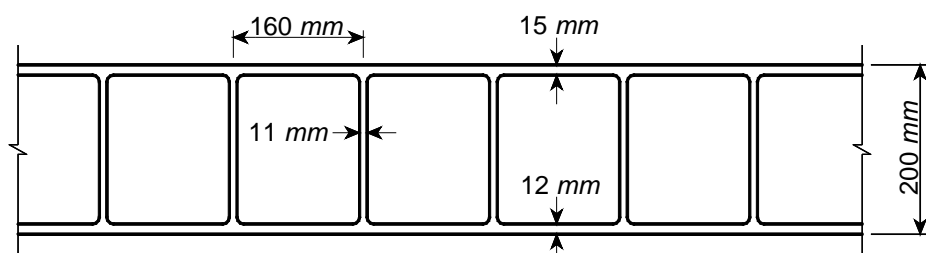


Figure 8. GFRP Deck Profile with Rectangular Shape

In this study, a sensitivity analysis has been performed to examine the impact of the design variables on the optimization. Figures 9, 10, and 11 respectively show the results of sensitivity analysis for the deflection, stress, and local buckling load, in accordance with the variations of design values. The results indicate that, as expected, the deflection of profile is sensitive for the thickness of flanges, whereas the local buckling load is sensitive for the thickness and height of the web. Overall, the geometrical design variables are more sensitive than those of material properties.



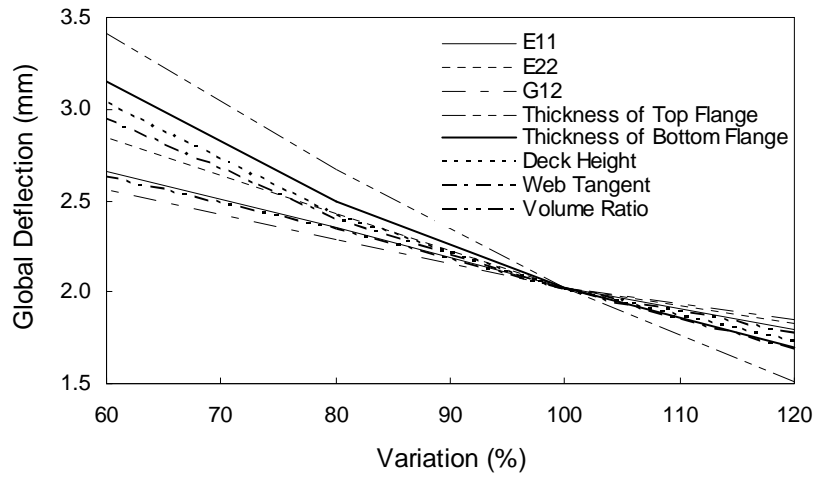


Figure 9. Sensitivity for Global Deflection

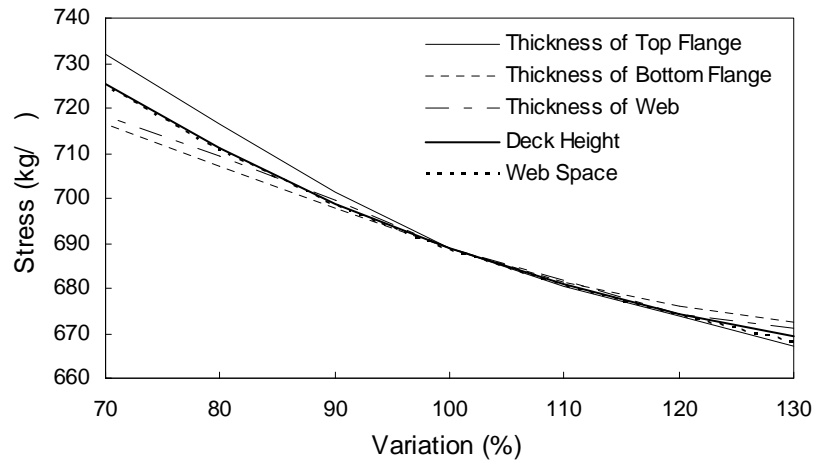


Figure 10. Sensitivity for Stress in Fiber Direction

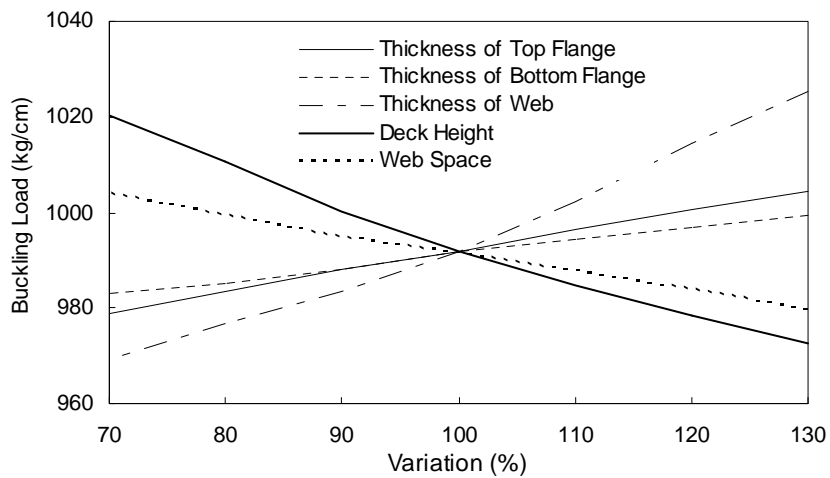


Figure 11. Sensitivity for Local Buckling Load

In Figure 12, the number iterations with different initial values for design variable, which is required to obtain the same accuracy of solutions, is compared. Regardless of initial values, the solutions obtained using the proposed optimization algorithm are converged in less than eight iterations.

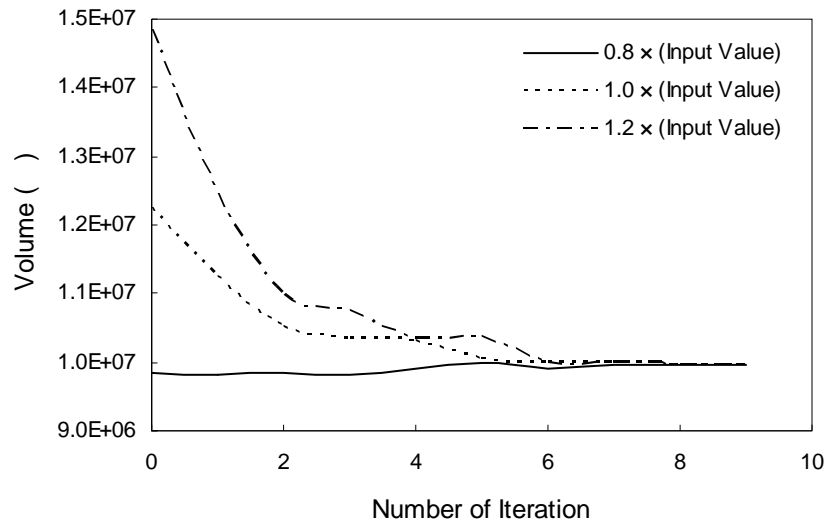


Figure 12. Convergence of Solutions

## CONCLUDING REMARKS

In this study, a GA-based optimization design algorithm is developed to design a GFRP bridge decks. Based on the results of this study, several conclusions can be drawn and are summarized in the following.

The developed algorithm is capable of optimizing the structure and material for GFRP deck system simultaneously. The convergence of solutions obtained using the developed computer code is quite fast. Since the overall optimization procedure involves three-dimensional structural analysis using a commercial FE analysis engine, this renders inefficient for practical application, especially for multi-cellular sections. Therefore, a structural analysis module should be implemented into the computer code to improve its efficiency.

The results of structural optimization for a prototype bridge indicate that a trapezoidal cross-section is an optimum shape for GFRP deck. However, the structural performance of a profile with trapezoidal section is slightly better than that of a rectangular section. As expected, the stiffness of deck is identified as a critical parameter for the design.

The results of sensitivity analysis indicate that the geometrical design variables are more sensitive than those of materials. The deflection of profile is greatly influenced by the thickness of flanges, while the local buckling load is sensitive for the dimension of the web.

## NOTATION

$E_{11}, f_{11}$  = elastic modulus and stress in the fiber direction, respectively;

- $E_{22}, f_{22}$  = elastic modulus and stress in the direction transverse to the fibers, respectively;
- $f, f_a$  = induced stress and ultimate strength of material multiplied by the factor of safety, respectively;
- $G_{12}, G_{13}$  = in-plane shear moduli;
- $G_{23}$  = out-of-plane shear modulus;
- $h, h_{\min}$  = height and minimum height of deck, respectively;
- $M_n, M_u$  = nominal moment and ultimate moment, respectively;
- $N, N_{cr}$  = in-plane load and local buckling load, respectively;
- $\alpha$  = angle between web tangent and bottom flange;
- $\delta, \delta_a$  = deflection due to live load and allowable deflection, respectively;
- $\nu_{12}$  = in-plane Poisson's ratio;
- $\phi$  = coefficient of strength reduction; and
- $\theta$  = ply angle in degree.

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