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Modified Rhie-Chow/PISO Algorithm for Collocated Variable Finite Volume Porous Media Flow Solvers

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Abstract

A modified Rhie-Chow / PISO (Pressure-Implicit with Splitting of Operators) segregated algorithm is proposed, which by construction avoids the development of spurious oscillations in the solution fields for low Mach number flow in heterogeneous, isotropic porous media. The collocated variable finite volume based algorithm modifies the commonly used Rhie-Chow interpolation to maintain a strong pressure-velocity coupling when large discontinuous implicit momentum sources are present. This Redistributed Resistivity PISO (rdrPISO) algorithm is based on a redistribution of the flow resistivity over the neighboring grid cells to the discontinuity. The proposed algorithm is successfully verified against published data for the velocity and pressure for two cases. The robustness of the proposed algorithm is also demonstrated for high Reynolds number flows and low Darcy numbers, for which oscillatory free solutions are achieved.

Keywords: Porous, PISO, Rhie-Chow, spurious oscillations, collocated, segregated, finite volume method, low Mach

1 Introduction

Fluid flow through porous media is fundamental to many natural and industrial processes, such as groundwater flows, filtration, and chemical and biomass processing. In order to efficiently simulate these processes and predict their performance, robust mathematical and numerical models are of high importance. While the equations governing the flow and heat transfer in porous media are readily specified using the method of volume averaging [1] in conjunction with suitable closure models, it often remains challenging to obtain physically acceptable solutions in the vicinity of fluid-porous interfaces, where the porosity is discontinuous. Without special care in the algorithm development, such discontinuity may yield spurious oscillations in the solution variables. This is especially true for high Reynolds (Re) number flows and for low Darcy (Da) number heterogeneous porous media, for which the jump in flow resistivity and/or porosity is high. The occurrence of spurious oscillations is partic-

ularly pronounced when segregated algorithms are applied, in which the velocity and pressure equations are decoupled and an iterative solution process is required. Numerical schemes that avoid spurious oscillations at sharp interfaces were proposed in [2] and [3] for both structured and unstructured grids using a collocated variable, finite volume block-coupled solver, which solves pressure and velocity simultaneously instead of in a segregated way as in the PISO algorithm [4]. Their rather complex schemes, which treat fluid-porous interfaces consistently, work well for high Re number flows, but require special, local treatment of the fluid-porous interfaces.

Furthermore, a correction to the Rhie-Chow interpolation for when the fluid is subject to large body forces was suggested in [5] and a force field discretization rule for the volume-of-fluid method was suggested in [6] as remedies for the pressure-velocity decoupling and their resulting spurious velocities. These correction schemes may be applied for explicit treatments of the flow resistance source terms occurring in porous media flow. However, explicit flow resistance source terms impose severe time-step limitations and implicit implementation of the flow resistance is therefore preferable.

In contrast to [2] and [3], this work focuses on segregated algorithms for collocated variable finite volume methods for general porous media flow. A modified Rhie-Chow / PISO algorithm is proposed, which by construction avoids the development of spurious oscillations in the solution fields when large discontinuous implicit momentum sources are present. The algorithm is based on a redistribution of the discontinuous flow resistivity over nearby grid cells, in a similar way as was proposed in [7] for discontinuous body force fields. In order to not create time step restrictions for high Re number or low (Da) number flows, for which the flow-induced resistance is high, the flow resistance term is treated implicitly. The proposed algorithm is successfully verified against published data for the velocity and pressure fields for the incompressible, isothermal flow through a porous plug and for flow parallel to a porous medium. The robustness of the proposed algorithm is also demonstrated for high Re number flows and for low Da numbers.

The organization of this paper is as follows. In Sec-

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tion 2, the governing equations for low Mach number flow and heat transfer in isotropic, heterogeneous porous media are introduced, followed by their finite volume discretization and an analysis of the discretized equations for large, discontinuous implicit source terms in Section 3. Thereafter, a Redistributed Resistivity PISO (rdr-PISO) algorithm is put forward in Section 4 followed by its validation in Section 5 and conclusions in Section 6.

2 Governing equations

Fluid flow and heat transfer in conjugate fluid/porous domains is governed by the volume-averaged mass, momentum and energy conservation equations [1]. Following the derivations in [1] and the closure modeling in [8] and [9], the equations governing the flow and heat transfer in isotropic, heterogeneous porous media, constituted by a fluid phase α and a solid phase β, can in its closed general conservation form, assuming local thermal non-equilibrium between the phases, be written as:

$$\partial_t(\phi\langle\rho_\alpha\rangle^\alpha) + \partial_i(\langle\rho_\alpha\rangle^\alpha\langle u_i\rangle) = 0 \tag{1}$$

$$\partial_t(\langle\rho_\alpha\rangle^\alpha\langle u_i\rangle) + \partial_j(\phi^{-1}\langle\rho_\alpha\rangle^\alpha\langle u_j\rangle\langle u_i\rangle) = \\ -\phi\partial_i\langle p_\alpha\rangle^\alpha + \partial_j\langle\tau_{ij}\rangle + \phi\langle f_i\rangle^\alpha - \phi D\langle u_i\rangle \tag{2}$$

$$\langle c_{p,\alpha}\rangle^\alpha \left(\partial_t(\phi\langle\rho_\alpha\rangle^\alpha\langle T_\alpha\rangle^\alpha) + \partial_i(\langle\rho_\alpha\rangle^\alpha\langle u_i\rangle\langle T_\alpha\rangle^\alpha)\right) = \\ \partial_i(\lambda_\alpha^e\partial_i\langle T_\alpha\rangle^\alpha) + h_{\alpha\beta}a_{\alpha\beta}(\langle T_\beta\rangle^\beta - \langle T_\alpha\rangle^\alpha) \\ + \partial_t(\phi\langle p_\alpha\rangle^\alpha) + \langle u_i\rangle\partial_i\langle p_\alpha\rangle^\alpha \tag{3}$$

$$\langle c_{p,\beta}\rangle^\beta\langle\rho_\beta\rangle^\beta\partial_t((1-\phi)\langle T_\beta\rangle^\beta) = \partial_i(\lambda_\beta^e\partial_i\langle T_\beta\rangle^\beta) \\ - h_{\alpha\beta}a_{\alpha\beta}(\langle T_\beta\rangle^\beta - \langle T_\alpha\rangle^\alpha) \tag{4}$$

where t is the time, ρ the density, u_i the velocity, p the pressure, and f_i a momentum body source. The volume-averaged rate of strain tensor is $\langle\tau_{ij}\rangle = \langle\mu_\alpha\rangle^\alpha(\partial_j\langle u_i\rangle + \partial_i\langle u_j\rangle) - (\frac{2}{3}\langle\mu_\alpha\rangle^\alpha - \langle\kappa_\alpha\rangle^\alpha)\delta_{ij}\partial_k\langle u_k\rangle$ with μ the dynamic viscosity, δ_{ij} the Kronecker delta tensor and κ the dilatational viscosity. The operator ∂_t is the temporal partial derivative and ∂_i is the partial derivative with respect to the spatial coordinate i. $D = \langle\mu_\alpha\rangle^\alpha(K^{-1} + K^{-1}F)$ is the isotropic resistivity, ϕ is the porosity, K is the isotropic permeability and $F = \frac{\langle\rho_\alpha\rangle^\alpha}{\langle\mu_\alpha\rangle^\alpha}K^{1/2}|\langle u_j\rangle|c_E$ is the non-Darcy resistivity, where c_E is the form drag coefficient. In the energy equations, $c_{p,m}$ is the specific heat, T_m the temperature, λ_m^e is the effective thermal conductivity for the phase $m \in \{\alpha, \beta\}$. The interfacial heat transfer coefficient is denoted with $h_{\alpha\beta}$ and $a_{\alpha\beta}$ is the specific surface area.

The extrinsic, or superficial, average and the intrinsic volume average of a property φ are according to [1] defined as: $\langle\varphi_\alpha\rangle = \frac{1}{V}\int_{V_\alpha}\varphi_\alpha dV$ and $\langle\varphi_\alpha\rangle^\alpha = \frac{1}{V_\alpha}\int_{V_\alpha}\varphi_\alpha dV$, respectively, where V is the representative elementary volume containing both fluid and solid constituents and

V_α is the volume occupied by constituent α.

Note that the volume-averaged equations can be used in both the pure fluid and in the porous region by setting $\phi = 1$, $D = \lambda_\beta^e = h_{\alpha\beta} = a_{\alpha\beta} = 0$ and $\lambda_\alpha^e = \lambda_\alpha$, where λ_α is the fluid thermal conductivity, in the pure fluid region. In the next section the discretization of the volume-averaged governing equations for the fluid flow in conjugate fluid/porous domains is presented.

3 Finite volume discretization and derivation of pressure equation

In order to discretize the governing volume-averaged equations using a collocated finite-volume method, the equations must be integrated over a typical control volume V_P centered about a node P and bounded by N faces with surface area vector $S_{i,f}$ and face centers f. Applying Gauss divergence theorem, the semi-discretized form of the volume-averaged mass, momentum and energy equations for an implicit dominated collocated variable discretization yields:

$$V_P\partial_t(\phi_P\rho_P) + \sum_f(\rho u_i)_f S_{i,f} = 0 \tag{5}$$

$$V_P\partial_t(\rho_P u_{i,P}) + \sum_f(\phi^{-1}\rho u_j)_f u_{i,f}S_{j,f} = \\ -\phi_P\sum_f p_f S_{i,f} + \sum_f \tau_{ij}S_{j,f} + \phi_P f_{i,P}V_P \\ -\phi_P D_P u_{i,P}V_P \tag{6}$$

$$V_P c_{p,P}[\partial_t(\phi_P\rho_P T_P) + \sum_f(\rho u_i)_f T_f S_{i,f}] = \\ \sum_f \lambda_f^e(\partial_i T)_f S_{i,f} + V_P h_{\alpha,\beta,P}a_{\alpha\beta,P}(T_{\beta,P} - T_P) \\ + V_P\partial_t(\phi_P p_P) + u_{i,P}\sum_f p_f S_{i,f} \tag{7}$$

$$V_P(c_{p,\beta}\rho_\beta)_P\partial_t((1-\phi_P)T_{\beta,P}) = \sum_f \lambda_{\beta,f}^e(\partial_i T_\beta)_f S_{i,f} \\ - V_P h_{\alpha\beta,P}a_{\alpha\beta,P}(T_{\beta,P} - T_P) \tag{8}$$

Here and below the spatial averaging operators $\langle\rangle^\alpha$ and $\langle\rangle$ and the subscript α for the fluid phase are dropped for brevity. Note that u_i hereinafter refers to the superficial velocity and the other variables are intrinsic.

In the discretized equations above, some of the properties and variables are required at the cell centers and some at the face centers. In a collocated variable representation, the properties and variable are all stored in the cell-centers and need to be interpolated by suitable interpolation schemes to the face centers. The mass flow rate in the convective term is found in line with the collocated variable method of Rhie and Chow [10] as described later. Applying suitable discretization and interpolation

schemes for the terms and variables in (6), dividing them by V_P and replacing the discretized pressure gradient with its non-discretized form, the semi-discretized momentum equation can for the P node be written as:

$$A_P u_{i,P} = H_{i,P} - \phi_P (\partial_i p)_P \qquad (9)$$

where $A_P = a_P / V_P$, $H_{i,P} = (1/V_P) \sum_{nb} a_{nb} u_{i,nb}$ and a_P and a_{nb} are the finite volume discretization coefficients for the node P and its neighboring nodes nb, respectively. Dividing by A_P results in the following equation for the cell-centered velocity:

$$u_{i,P} = A_P^{-1} H_{i,P} - A_P^{-1} \phi_P (\partial_i p)_P \qquad (10)$$

3.1 Discretized pressure equation and Rhie-Chow interpolation

It can be noted that there is no explicit equation for the pressure in the conservation equations. Thus, a pressure equation must be derived from the mass and momentum conservation equations. This can be realized by assuming that $u_{i,f}$ can be expressed in a similar semi-discretiized form as (10). Multiplying the expression by ρ, inserting it into the mass conservation equation (5) and replacing ρ in the time derivative with the equation of state:

$$\rho = \psi p \qquad (11)$$

where ψ is the compressibility, the following pressure equation emerges:

$$V_P \partial_t (\phi_P \psi_P p_P) + \sum_f (\rho A^{-1} H_i)_f S_{i,f} =$$
$$\sum_f (\rho A^{-1} \phi)_f (\partial_i p)_f S_{i,f} \qquad (12)$$

after having made the assumption that $(\rho A^{-1} \phi \partial_i p)_f = (\rho A^{-1} \phi)_f (\partial_i p)_f$ in accordance to the method of Rhie and Chow [10].

As seen in the derived pressure equation, interpolations from cell-centered values to the face centers are required for all terms inside the surface summations. The terms $(\rho A^{-1} H_i)_f$ and $(\rho A^{-1} \phi)_f$ can be interpolated to the face centers from the nodes P and nb by linear interpolation according to:

$$\Gamma_f = (1 - r)\Gamma_P + r\Gamma_{nb} \qquad (13)$$

where $r = |d_{i,Pf}| / |d_{j,Pnb}|$, $d_{i,Pf}$ is the distance vector between the node P and the face center f, $d_{j,Pnb}$ is the distance vector between node P and node nb and Γ is the property to interpolate. To avoid pressure-velocity decoupling for smooth flows, Rhie and Chow [10] suggested to use the directly calculated pressure gradient at the face, using the neighboring nodes according to:

$$(\partial_i p)_f = \hat{n}_{i,f} \frac{p_{nb} - p_P}{|d_{j,Pnb}|} \qquad (14)$$

where $\hat{n}_{i,f}$ is the face normal vector, instead of interpolating the cell-centered pressure gradients to the face. The mass flux at the faces can then be expressed as:

$$(\rho u_i)_f S_{i,f} =$$
$$(\rho A^{-1} H_i)_f S_{i,f} - (\rho A^{-1} \phi)_f (\partial_i p)_f S_{i,f} \qquad (15)$$

where the pressure gradient at the face is directly computed using (14).

3.2 Pressure-velocity decoupling for large discontinuities

The Rhie-Chow interpolation and the PISO algorithm were proposed for pure fluid flows, for which the properties and field variables are smooth and only slowly varying due to temperature and density changes. The Rhie-Chow interpolation as described previously has been shown to result in pressure-velocity decoupling and spurious pressure and velocity oscillations in the vicinity of discontinuities in properties, field variables or large source terms in the governing equations [2, 3, 5, 7].

The main causes of the pressure-velocity decoupling when solving the porous media flow equations using a segregated PISO algorithm are:

i The interpolation of $(\rho A^{-1} H_i)_f$ and $(\rho A^{-1} \phi)_f$ to match to the directly calculated $(\partial_i p)_f$ in the Rhie-Chow interpolation in (12).

ii The interpolation of face pressure to balance the discontinuous cell-centered flow resistivity in the discretized momentum equation, (6), in the predictor step and in (10) in the velocity corrector step of the PISO algorithm.

4 Modified Rhie-Chow interpolation and the Redistributed Resistivity PISO algorithm

In this section, a novel modified Rhie-Chow interpolation and Redistributed Resistivity PISO algorithm preserving the pressure-velocity coupling at the interfaces by construction is proposed. This algorithm addresses the main causes of the pressure-velocity decoupling mentioned previously.

4.1 Modified Rhie-Chow interpolation

To avoid interpolation of the discontinuous ϕ from the cell-centers to the face centers in the Laplacian term of the pressure equation (12), the discretized momentum equation (6), is divided by ϕ_P. Further division by V_P and replacement of the discretized pressure gradient with its non-discretized form, yields:

3

$$\frac{1}{\phi_P}\partial_t(\rho_P u_{i,P}) + \frac{1}{\phi_P V_P}\sum_f (\phi^{-1}\rho u_j)_f u_{i,f} S_{j,f} =$$
$$-(\partial_i p)_P + \frac{1}{\phi_P V_P}\sum_f \tau_{ij,f} S_{j,f} + f_{i,P} - D_P u_{i,P} \quad (16)$$

Instead of discretizing $D_P u_{i,P}$ into A_P and $H_{i,P}$, as was done in (9), $D_P u_{i,P}$ is left in its non-discretized form together with the pressure gradient. The rest of the terms are discretized according to:

$$A_P u_{i,P} - H_{i,P} = \frac{1}{\phi_P}\partial_t(\rho_P u_{i,P})$$
$$+ \frac{1}{\phi_P^2 V_P}\sum_f (\rho u_j)_f u_{i,f} S_{j,f} - \frac{1}{\phi_P V_P}\sum_f \tau_{ij,f} S_{j,f} - f_{i,P}.$$
$$(17)$$

Note that the reciprocal porosity in the convective term of (17) has been moved outside of the sum to ensure consistent treatment of the porosity discontinuity. This approximation restricts the discontinuity to the interface only and prevents it to spread to the neighboring cells when discretizing. The resulting semi-discretized momentum equation takes the form:

$$A_P u_{i,P} + D_P u_{i,P} = H_{i,P} - (\partial_i p)_P. \quad (18)$$

Dividing it first by A_P and then collecting the velocity terms, the cell-centered velocity becomes:

$$u_{i,P} = B_P(A_P^{-1}H_{i,P} - A_P^{-1}(\partial_i p)_P) \quad (19)$$

where $B_P = (1 + D_P A_P^{-1})^{-1}$. Assuming that the face velocity can be expressed similarly to (19) and multiplying it with ρ, the mass flux through a face f yields:

$$(\rho u_i)_f S_{i,f} = (\rho B A^{-1}H_i)_f S_{i,f} -$$
$$(\rho B A^{-1}\partial_i p)_f S_{i,f}. \quad (20)$$

To maintain a strong pressure-velocity coupling at the discontinuity and to interpolate the discontinuous variables consistently to the interface in the face mass flux expression, B is interpolated separately to the faces resulting in the following modified Rhie-Chow interpolation:

$$(\rho u_i)_f S_{i,f} = B_f[(\rho A^{-1}H_i)_f S_{i,f}$$
$$- (\rho A^{-1})_f (\partial_i p)_f S_{i,f}] \quad (21)$$

where $B_f = (1 + D_f(A^{-1})_f)^{-1}$. The terms $(\rho A^{-1}H_i)_f$, $(\rho A^{-1})_f$, D_f and $(A^{-1})_f$ are found using the linear interpolation in (13), and $(\partial_i p)_f$ is discretized using (14). Note that $(\rho A^{-1}\partial_i p)_f = (\rho A^{-1})_f(\partial_i p)_f$ in accordance with the approximation by Rhie and Chow [10]. In this way, the original Rhie-Chow interpolation is used for the continuous or slowly varying variables, and the main contribution from the large discontinuity is multiplied to it consistently for both terms in the face flux expression.

Inserting the modified Rhie-Chow interpolation into the discretized mass conservation equation (5) and replacing ρ_P in the time derivative with (11) the following modified pressure equation is found:

$$V_P \partial_t(\phi_P \psi_P p_P) + \sum_f B_f(\rho A^{-1}H_i)_f S_{i,f} =$$
$$\sum_f B_f(\rho A^{-1})_f(\partial_i p)_f S_{i,f} \quad (22)$$

4.2 Flow resistivity redistribution

To avoid the generation of spurious velocity oscillations in the vicinity of the discontinuity, when solving the discretized momentum equation, (18) or correcting the velocity with (19), a balance between the discontinuous flow resistance field and the cell-centered pressure gradient is required. In [2] and [6], special face pressure interpolation schemes were proposed to estimate the face pressure at porous interfaces consistently. These schemes require either explicit corrections with the mass flux or extrapolations of the cell-centered pressure from each side of the interface. Here, another method is proposed, which conserves the mass and momentum on the discrete level. In this method, p_f is determined by linear interpolation, (13). Since the pressure gradients in the two cells neighboring the discontinuity are not balancing the cell-centered flow resistivity in these cells, the cell-centered resistances must be modified to balance the cell-centered pressure gradients calculated from the linearly interpolated face pressures.

In order to determine the cells requiring a redistributed resistivity, a cell indicator function is defined as:

$$\Omega_P = \frac{\sum_f |(\Delta\phi)_f|}{max(\sum_f |(\Delta\phi)_f|, \varepsilon)} \quad (23)$$

where $(\Delta\phi)_f = \phi_{nb} - \phi_P$ and ε is a small number in the order of 10^{-15} to avoid division by zero for when $\sum_f |(\Delta\phi)_f| = 0$. Ω_P has the value 1 for cells requiring a redistributed resistivity and 0 otherwise. Another indicator function θ_f is defined as:

$$\theta_f = \frac{|(\Delta(\Omega\phi))_f|}{max(|(\Delta(\Omega\phi))_f|, \varepsilon)} \quad (24)$$

where $\Delta(\Omega\phi)_f = \Omega_{nb}\phi_{nb} - \Omega_P\phi_P$, in order to determine the faces required for the calculation of the redistributed flow resistivity in the cells where $\Omega_P = 1$. The required faces for the redistribution of the resistivity are stipulated by the linear interpolation stencil used to compute $(\partial_i p)_P$. θ_f has the value 1 for required faces and 0 otherwise. The redistributed resistivity D_P^{rd} is then computed using inverse distance weighting for the required faces according to:

$$D_P^{rd} = (1 - \Omega_P)D_P + \frac{\Omega_P}{\sum_f \omega_f \theta_f}\sum_f \omega_f \theta_f D_f \quad (25)$$

4

where $\omega_f = 1/|d_{i,Pf}|$ and D_f is interpolated to the face centers by the linear interpolation in (13).

The modified discretized momentum equation with the redistributed resistivity yields:

$$\frac{1}{\phi_P}\partial_t(\rho_P u_{i,P}) + \frac{1}{\phi_P^2 V_P}\sum_f (\rho u_j)_f u_{i,f} S_{j,f} =$$

$$-\frac{1}{V_P}\sum_f p_f S_{i,f} + \frac{1}{\phi_P V_P}\sum_f \tau_{ij,f} S_{j,f} + f_{i,P} - D_P^{rd} u_{i,P}$$

(26)

and the expression for the velocity correction reads:

$$u_{i,P} = B_P^{rd} A_P^{-1}(H_{i,P} - (\partial_i p)_P)$$

(27)

where $B_P^{rd} = (1 + D_P^{rd} A_P^{-1})^{-1}$. The face velocity $u_{i,f}$ in the convective term can be discretized by any suitable scheme and the mass flux $(\rho u_i)_f S_{i,f}$ is calculated from (21).

4.3 Redistributed Resistivity PISO algorithm

The original PISO algorithm [4] is a non-iterative technique for the solution of the implicitly discretized time-dependent general flow equations. The non-iterative solution is accomplished at each time-step through a sequential predictor-corrector process by which the different dependent variables are updated individually. The two-stage PISO scheme in [4] takes, while following the notation in [4], the following form:

1. *Momentum predictor step:* $u_{i,P}^n$, ρ_P^n, μ_P^n, p_P^n, D_P^n and $(\rho u_i)_f^n S_{i,f}$ from the last time-step are used to calculate A_P^n, $H_{i,P}^n(u_{i,P}^*)$ and $D_P^{rd,n}$ in, (26), which is then solved using $(\partial_i p)_P = 1/V_P \sum_f p_f^n S_{i,f}$ to give $u_{i,P}^*$.

2. *Momentum corrector step 1:* $u_{i,P}^*$ generally does not satisfy the mass conservation equation [4]. The pressure equation (22) is therefore solved to find an intermediate pressure p_P^*. The mass flux is then updated to $(\rho u_i)_f^* S_{i,f}$ using (21). Thereafter, the density is updated from $\rho_P^* = \psi_P^n p_P^*$ and the velocity is corrected to $u_{i,P}^{**}$ using (27). The fields ρ_P^*, $u_{i,P}^{**}$ satisfies the mass conservation equation.

3. *Energy predictor step:* The equations (7) and (8) are solved with $\rho_{\beta,P}$, $\lambda_P^{e,n}$, $\lambda_{\beta,P}^{e,n}$, $h_{\alpha\beta,P}^n$, $a_{\alpha\beta,P}^n$, $c_{p,P}^n$, $c_{p,\beta,P}^n$, ρ_P^*, p_P^*, $(\rho u_i)_f^* S_{i,f}$, $u_{i,P}^{**}$ to get T_P^* and $T_{\beta,P}^*$

4. *Momentum corrector step 2:* The compressibility is updated to ψ_P^* and the pressure equation, (22), is thereafter solved again using ρ_P^*, A_P^n, D_P^n, $H_{i,P}^n(u_{i,P}^{**})$, to get p_P^{**}. The mass flux is updated to $(\rho u_i)_f^{**} S_{i,f}$ using (21) followed by an update of the density using $\rho_P^{**} = \psi_P^* p_P^{**}$. The velocity is thereafter corrected using (27) to get $u_{i,P}^{***}$. The corrected

field variables are then taken over to the next time-step.

5 Validation

The proposed rdrPISO and the original PISO algorithms described in Sections 4 and 3, respectively, have been implemented using the OpenFOAM® open source computational fluid dynamics C++ library (version 2.2.0). The porous plug case for flow perpendicular to a porous region and the Beaver-Joseph problem for flow parallel to a porous region are considered, in order to demonstrate the accuracy and robustness of the proposed algorithm. Their respective geometries can be seen in Fig. 1. The flow is assumed to be incompressible and isothermal and the porous media isotropic. The fluid and porous properties, initial and boundary conditions and computational grids are specified according to the setups in [2]. U is the average velocity in the pure fluid portion of the channel at fully developed conditions, $Da = K/h^2$ with h as the channel height and $Re = \frac{\rho U h}{\mu}$. The time derivative is discretized by a second-order implicit backward differencing scheme and the convective term by the second-order linear upwind differencing (LUD) scheme [11]. The discretized momentum equations for each coordinate direction are solved using a smooth solver with a Gauss-Seidel smoother down to the tolerance 10^{-11} and the pressure equations are solved with a Preconditioned Conjugate Gradient (PCG) solver with Faster Diagonal Incomplete-Cholesky (FDIC) preconditioner down to the tolerance 10^{-12}.
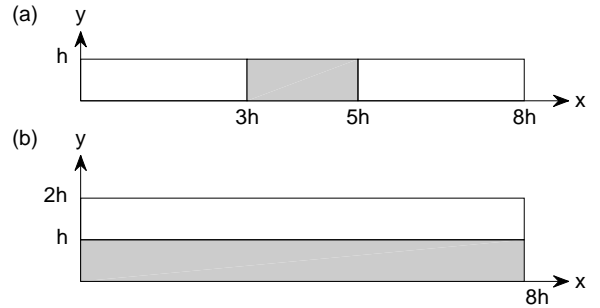

Figure 1: Geometries for (a) porous plug and (b) Beaver-Joseph cases.

It can be seen in Fig. 2 that an oscillatory free solution, in agreement with the reference data [2], is obtained with the proposed rdrPISO algorithm with modified Rhie-Chow interpolation, whereas if a original Rhie-Chow / PISO scheme is used, spurious oscillations of the velocity in the vicinity of the porous interface occur, already at a low Da and Re number flow.

The rdrPISO and the original PISO algorithms perform equally well (nearly overlapping), compared to the reference data [2], for the Beaver-Joseph case, see Fig. 3,
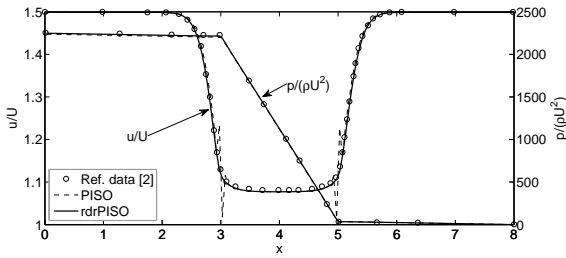
Figure 2: Centerline velocity for porous plug problem for $Re = 1$ and $Da = 10^{-3}$.

for both Da numbers tested. This is due to the fact that the pressure gradient over the porous interface is close to zero, leading to a negligible flow over the discontinuous interface, and a trivial pressure-velocity coupling as a result.
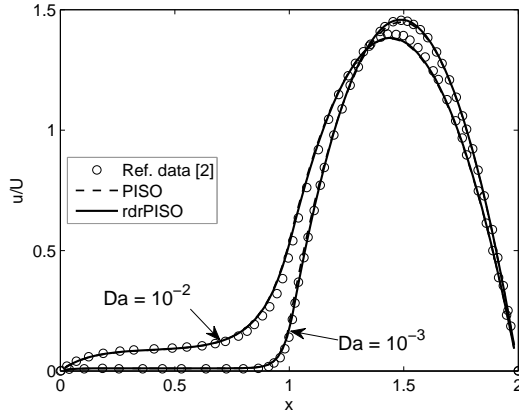

Figure 3: Fully developed velocity profiles for the Beaver-Joseph problem for $Re = 1$ and $Da = 10^{-2}$ and $Da = 10^{-3}$.

In order to demonstrate the robustness of the proposed rdrPISO algorithm, simulations of the flow at both high Re numbers and low Da numbers were carried out. It can be seen in Fig. 4 that the rdrPISO algorithm generates oscillatory free solutions for all Re and Da numbers tested. This demonstrates that that the rdrPISO algorithm is robust for a wide range of flow conditions and porous media applications. Ongoing work focuses on further validation of the proposed algorithm for density varying flow.

6   Conclusions

A Redistributed Resistivity PISO (rdrPISO) algorithm was proposed for low Mach number flow and heat transfer in heterogeneous, isotropic porous media. The algorithm is based on a modified Rhie-Chow interpolation and a redistribution of the flow resistivity over the neighboring cells of the discontinuity. The algorithm was validated to literature data for the incompressible and isothermal
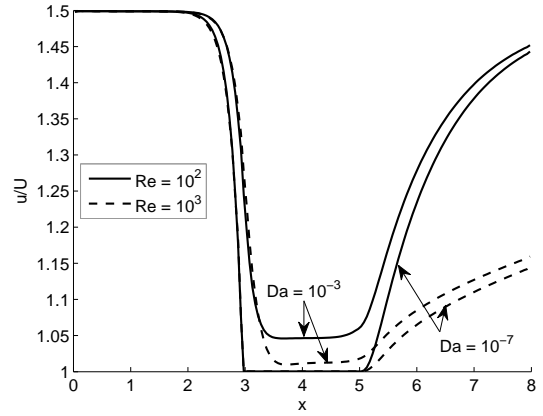

Figure 4: Centerline velocity for porous plug problem for $Re \in \left\{ 10^2, 10^3 \right\}$, and $Da \in \left\{ 10^{-3}, 10^{-7} \right\}$.

flow parallel and perpendicular to a porous region and was found to agree well with the reference velocity and pressure data for both cases. It was also compared to an original PISO algorithm, which was shown to generate undesired pressure-velocity decoupling in the vicinity of the discontinuity, whereas the rdrPISO algorithm generated smooth, non-oscillatory results. Moreover, the robustness of the algorithm was demonstrated for high Re flows up to $Re = 10^3$ and for Da numbers as low as $Da = 10^{-7}$.

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