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EFFECT OF TEMPERATURE DEPENDENT VISCOSITY ON NATURAL CONVECTIVE BOUNDARY LAYER FLOW OVER A HORIZONTAL PLATE EMBEDDED IN A NANO-FLUID SATURATED POROUS MEDIUM

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ABSTRACT
The effect of temperature dependent viscosity on natural convective boundary-layer flow of a nano-fluid over an isothermal horizontal plate is investigated numerically. The viscosity of the fluid is assumed to have an exponentially decaying dependence on temperature $\nu = \nu_0 e^{-b(T-T_\infty)}$. A similarity analysis is presented for field equations embodying conservation of total mass, momentum, thermal energy and nano-particles. The analysis shows that velocity, temperature and nano-particle volume fraction profiles in the respective boundary layers depend on the viscosity parameter γ besides the pertinent parameters such as buoyancy ratio Nr, Brownian motion Nb, thermophoresis Nt and Lewis number Le. From physical considerations one expects that an increase in the viscosity must result in lower heat transfer rates. Results in accordance with the physical considerations are obtained in this study. The study also investigates the effect of the presence of an internal heat source in the porous medium by considering the internal heat generation term in the energy equation. In the presence of the heat source the heat transfer rate is lower whereas the nanoparticle volume fraction rate is higher than those obtained in the absence of the source term.

INTRODUCTION
Materials of nanometer size have unique chemical and physical properties. As a result nano-fluids finds its applications in industries such as electronics and automotive and also has applications in storage of nuclear waste material. Nanotechnology represents the most relevant technology that is being currently explored to enhance heat transfer. The coining of the term "nano-fluid" is credited to Choi [3]. The characteristic feature of nano-fluids is thermal conductivity enhancement, a phenomenon observed by Masuda et al [10]. Buongiorno [2] gave a detailed explanation for the abnormal increase of thermal conductivity and viscosity. He also focused on heat transfer enhancement in convective situation in nano-fluids. Buongiorno proposed a model incorporating the effects of Brownian diffusion and the thermophoresis. Detail explanations on Benard problem (the onset of convection in a horizontal layer uniform heated from below) for a nano-fluid flow in a porous medium (the Horton-Rogers-Lapwood problem) are given by Kuznetsov and Nield [6] – [8], Nield and Kuznetsov [13] – [16] and Tzou [17] – [18]. Recently Kuznetsov and Nield [9] gave a revised model on Cheng–Minkowycz problem of natural convection over a vertical plate in a porous medium saturated by nano-fluid. Abu-Nada et al [1] presented heat transfer enhancement in a differentially heated enclosure using variable thermal conductivity and variable viscosity of nano-fluid (Al_2O_3 - water and CuO - water). Experimental and theoretical study on the effective thermal conductivity and viscosity of nano-fluids was given by Murshed et al. [11]. They reported that the thermal conductivity and viscosity of nano-fluids increases with the nano-particle volume fraction.

Gorla and Chamkha [4] gave the similarity solution for the natural convection past an isothermal horizontal plate in porous medium saturated with nano-fluids. They gave the numerical results for friction factor, surface heat transfer rate and mass transfer rate was presented for parametric variations of the Brownian motion, buoyancy ratio, thermophoresis and Lewis number on nano-fluid. Uddin et al. [19] extended this work for thermal convective boundary condition on the boundary layer flow of a nano-fluid over a upward facing permeable horizontal plate. All the above mentioned research work was done without internal heat generation which inspire us to work on this dimension.

In the present article we study the effect of relevant parameters such as thermophoresis, Brownian motion, buoyancy ratio, Lewis number and variable temperature dependent viscosity on free convective heat transfer rates (Nusselt number), mass transfer rate (Sherwood number) and non-dimensional velocity in the presence of internal heat generation for an isothermal horizontal plate embedded in porous medium saturated with nano-fluid. Similarity solutions are obtained for exponentially decaying viscosity of the saturating fluid.

NOMENCLATURE

c	=	specific heat at constant pressure
D_B	=	Brownian diffusion coefficient
D_m	=	mass diffusivity of porous medium
D_T	=	thermophoretic diffusion coefficient

f	=	dimensionless stream function
g	=	acceleration due to gravity
K	=	permeability of porous medium
k_m	=	thermal conductivity of porous media
Le	=	Lewis number
n	=	power law fluid index
Nb	=	Brownian motion parameter
Nr	=	buoyancy ratio parameter
Nt	=	thermophoresis parameter
Nu_x	=	Nusselt number
q'''	=	internal heat generation per unit volume
q	=	heat flux
Ra_x	=	local Rayleigh number
Sh	=	Sherwood number
T	=	temperature
u, v	=	velocity component in x- and y-direction
x, y	=	Cartesian coordinates along the plate and normal to it respectively

Greek Symbols

α_m	=	effective thermal diffusivity
β	=	coefficient of thermal expansion
γ	=	viscosity parameter
ϕ	=	nanoparticle volume fraction
η	=	similarity variable
μ	=	dynamic viscosity
ν	=	kinematic viscosity
θ	=	dimensionless temperature
ρ	=	density
ω	=	rescale nanoparticle volume fraction
ψ	=	stream function

Subscripts

f	=	physical property related to fluid
p	=	physical property related to porous medium
w	=	wall condition
∞	=	ambient conditions

1 Formulation of the problem

We consider the problem of free convection boundary layer flow past a horizontal plate placed in a nano-fluid saturated porous medium in the presence of internal heat generation. We select a coordinate frame in which the x – axis is in the horizontal direction (along the plate) and y – axis is normal to the plate. At the surface (y = 0) the temperature T and the nano-particle fraction ϕ take constant values T_w and ϕ_w respectively. At the ambient ($y \to \infty$) values of temperature and nano-particle volume fraction denoted by T_∞ and ϕ_∞ respectively, with $T_w > T_\infty$ and $\phi_w > \phi_\infty$.

The Oberbeck-Boussinesq approximation is employed and the homogeneity and local thermal equilibrium in the porous medium are assumed. We consider a porous medium whose porosity is denoted by ε and permeability by K. The Darcy velocity is denoted by $\vec{v}$. The following four field equations embody the conservation of total mass, momentum, thermal energy and nani-particles respectively. The field variables are the Darcy velocity $\vec{v}$, the temperature T and the nano-particle volume fraction ϕ (Khan and Pop [5], Nield and Bejan [12]).

$$\nabla.\vec{v} = 0 \tag{1}$$

$$\frac{\rho_f}{\varepsilon}\frac{\partial \vec{v}}{\partial t} = -\nabla P - \frac{\mu}{K}\vec{v} + \left[\phi\rho_p + (1-\phi)\left\{\rho_f\left(1-\beta\left(T-T_\infty\right)\right)\right\}\right]g \tag{2}$$

$$\left(\rho c\right)_m \frac{\partial T}{\partial t} + \left(\rho c\right)_f \vec{v}.\nabla T = k_m \nabla^2 T + q''' + \varepsilon\left(\rho c\right)_p \left[D_B \nabla\phi.\nabla T + \frac{D_T}{T_\infty}\nabla T.\nabla T\right] \tag{3}$$

$$\frac{\partial \phi}{\partial t} + \frac{1}{\varepsilon}\vec{v}.\nabla\phi = D_B\nabla^2\phi + \frac{D_T}{T_\infty}\nabla^2 T \tag{4}$$

We consider a steady state flow and assume that the flow is slow so that the advective term and the Forchheimer quadratic term do not appear in the momentum equation.. In keeping with Oberbeck-Boussinesq approximation and an assumption that the nano-particle concentration is dilute, the momentum equation (2) can be written as (Kuznetsov, Nield [9] and Gorla, Chamkha [4])

$$0 = -\nabla P - \frac{\mu}{K}\vec{v} + \left[\left(\rho_p - \rho_{f_\infty}\right)\left(\phi - \phi_\infty\right) + \left(1-\phi_\infty\right)\rho_{f_\infty}\beta\left(T - T_\infty\right)\right]g \tag{5}$$

where dynamic viscosity μ is written as $\mu = \rho_{f_\infty}\nu$, ν is the kinematic viscosity. Making the standard boundary layer approximation based on scale analysis, we have the governing equations.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{6}$$

$$\frac{\partial P}{\partial x} = -\frac{\rho_{f_\infty}\nu}{K}u \tag{7}$$

$$\frac{\partial P}{\partial y} = \left(1-\phi_\infty\right)\rho_{f_\infty}\beta g\left(T - T_\infty\right) - \left(\rho_p - \rho_{f_\infty}\right)g\left(\phi - \phi_\infty\right) \tag{8}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_m\frac{\partial^2 T}{\partial y^2}+\frac{q'''}{(\rho c)_f}+ \tau\left[D_B\frac{\partial \phi}{\partial y}\frac{\partial T}{\partial y}+\left(\frac{D_T}{T_\infty}\right)\left(\frac{\partial T}{\partial y}\right)^2\right] \quad (9)$$

$$\frac{1}{\varepsilon}\left(u\frac{\partial \phi}{\partial x}+v\frac{\partial \phi}{\partial y}\right)=D_B\frac{\partial^2 \phi}{\partial y^2}+\frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2} \quad (10)$$

where $\alpha_m=\frac{k_m}{(\rho c)_f}$ is the thermal diffusivity of the fluid and $\tau=\frac{\varepsilon(\rho c)_p}{(\rho c)_f}$ is a parameter. Eliminating P from the equations [7], [8] we have

$$v(T)\frac{\partial u}{\partial y}+u\frac{\partial v(T)}{\partial y}=-(1-\phi_\infty)\beta gK\frac{\partial T}{\partial x}+ \frac{(\rho_p-\rho_{f_\infty})gK}{\rho_{f_\infty}}\frac{\partial \phi}{\partial x} \quad (11)$$

The appropriate boundary conditions for the problem are

$$v=0,\, T=T_w,\, \phi=\phi_w \quad \text{at} \quad y=0 \quad (12)$$

$$u\to 0,\, T\to T_\infty,\, \phi\to\phi_\infty \quad \text{as} \quad y\to\infty \quad (13)$$

The continuity equation (6) satisfied by introducing the stream function ψ defined as

$$u=\frac{\partial \psi}{\partial y},\, v=-\frac{\partial \psi}{\partial x} \quad (14)$$

We introduce the following similarity transformations

$$\psi=\alpha_m Ra_x^{\frac{1}{3}} f(\eta) \quad (15)$$

$$\eta=\frac{y}{x}Ra_x^{\frac{1}{3}} \quad (16)$$

$$T-T_\infty=\Delta T_w\theta(\eta) \quad (17)$$

$$\phi-\phi_\infty=\Delta\phi_w\omega(\eta) \quad (18)$$

$$v(T)=v_0 e^{-b(T-T_\infty)} \quad (19)$$

where $\Delta T_w=T_w-T_\infty$, $\Delta\phi_w=\phi_w-\phi_\infty$, b is a dimensional constant and $Ra_x=\frac{(1-\phi_\infty)\beta gK\Delta T_w x}{v_0\alpha_m}$ is the Rayleigh number. Using transformations (15) – (19) in equations (11), (9) and (10), we obtain the following system of non – linear ordinary differential equations

$$f''(\eta)=\gamma\theta'(\eta)f'(\eta)+\frac{2}{3}\eta e^{\gamma\theta(\eta)}\left[\theta'(\eta)-Nr\omega'(\eta)\right] \quad (20)$$

$$\theta''(\eta)+\frac{1}{3}f(\eta)\theta'(\eta)+Nb\omega'(\eta)\theta'(\eta)+Nt(\theta'(\eta))^2 +e^{-\eta}=0 \quad (21)$$

$$\omega''(\eta)+\frac{1}{3}Le\, f(\eta)\omega'(\eta)+\frac{Nt}{Nb}\theta''(\eta)=0 \quad (22)$$

where $q'''=\frac{k_m\Delta T_w}{x^2}Ra_x^{\frac{2}{3}}e^{-\eta}$. The transformed boundary conditions associated with the system (20) – (22) are

$$\text{At} \quad \eta=0,\ \theta=1,\ \omega=1,\ f=0 \quad (23)$$

$$\text{As} \quad \eta\to\infty,\ \theta\to 0,\ \omega\to 0,\ f'\to 0 \quad (24)$$

The primes denote differentiations with respect to η. The parameters Nr (buoyancy ratio), Nb (Brownian motion), Nt (thermophoresis), Le (Lewis number), γ (dimensionless viscosity) are defined as

$$Nr=\frac{(\rho_p-\rho_{f_\infty})\Delta\phi_w}{(1-\phi_\infty)\rho_{f_\infty}\beta\Delta T_w},\quad Nb=\frac{\tau D_B}{\alpha_m}\Delta\phi_w$$

$$Nt=\frac{\tau D_T}{\alpha_m T_\infty}\Delta T_w,\quad Le=\frac{\alpha_m}{\varepsilon D_B},\quad \gamma=b\Delta T_w$$

The heat transfer rate at the surface q_w and the local Nusselt number Nu_x are defined as.

$$q_w=-k_m\left.\frac{\partial T}{\partial y}\right|_{y=0} \quad (25)$$

$$Nu_x=\frac{q_w x}{\Delta T_w k_m}=-\theta'(0)Ra_x^{\frac{1}{3}} \quad (26)$$

The mass transfer rate at the surface N_w and Sherwood number Sh are defined as

$$N_w=-D_m\left.\frac{\partial \phi}{\partial y}\right|_{y=0} \quad (27)$$

$$Sh=\frac{N_w x}{\Delta\phi_w D_m}=-\omega'(0)Ra_x^{\frac{1}{3}} \quad (28)$$

where D_m is mass diffusivity of the porous medium.

2 Results and discussion

The system of non-linear ordinary differential equations (20)-(22) with boundary conditions (23)-(24) are solved numerically for different values of Nr , Nb, Nt, Le

and γ using shooting method. Table 1 gives the results for the heat transfer rate $Nu_x\left(Ra_x\right)^{-\frac{1}{3}}$, and the nano-particle volume friction rate $Sh\left(Ra_x\right)^{-\frac{1}{3}}$ at the leading edge x = 0 for $Nb = Nr = Nt = 0.5$ in the presence of internal heat generation (IHG) source and without internal heat generation (WIHG). It is observed that in both the cases the heat transfer rate and the nano-particle volume friction rate increases if the viscosity of the base fluid decreases that is as the viscosity parameter γ increases.

Table 1 Effect of γ and Le on $Nu_x\left(Ra_x\right)^{-\frac{1}{3}}$ and $Sh\left(Ra_x\right)^{-\frac{1}{3}}$ for $Nr = Nb = Nt = 0.5$ in the presence of internal heat generation (IHG) and without internal heat generation (WIHG).

γ	Le	$Nu_x\left(Ra_x\right)^{-\frac{1}{3}}$ IHG	$Nu_x\left(Ra_x\right)^{-\frac{1}{3}}$ WIHG	$Sh\left(Ra_x\right)^{-\frac{1}{3}}$ IHG	$Sh\left(Ra_x\right)^{-\frac{1}{3}}$ WIHG
0	5	- 0.2724	0.2371	1.5537	0.9959
	10	- 0.2496	0.2326	2.0334	1.4891
	50	- 0.2069	0.2234	4.1154	3.4503
	100	- 0.1938	0.2209	5.7139	4.8905
	500	- 0.1740	0.2172	12.5688	10.9303
0.1	5	- 0.2572	0.2434	1.5879	1.0285
	10	- 0.2348	0.2388	2.0893	1.5416
	50	- 0.1932	0.2293	4.2561	3.5820
	100	- 0.1805	0.2267	5.9160	5.0804
	500	- 0.1613	0.2229	13.0251	11.3640
0.2	5	- 0.2419	0.2500	1.6236	1.0623
	10	- 0.2199	0.2452	2.1473	1.5959
	50	- 0.1794	0.2354	4.4024	3.7186
	100	- 0.1671	0.2326	6.1261	5.2775
	500	- 0.1486	0.2288	13.4996	11.8144
0.5	5	- 0.1954	0.2710	1.7340	1.1709
	10	- 0.1746	0.2656	2.3356	1.7711
	50	-0.1375	0.2548	4.8763	4.1603
	100	- 0.1265	0.2517	6.8071	5.9149
	500	- 0.1099	0.2475	15.0399	13.2717
0.75	5	- 0.1556	0.2900	1.8475	1.2704
	10	-0.1360	0.2841	2.5102	1.9320
	50	- 0.1019	0.2723	5.3152	4.5675
	100	- 0.0918	0.2690	7.4380	6.5030
	500	- 0.0768	0.2645	16.4698	14.6175

The results of the heat transfer rate $Nu_x\left(Ra_x\right)^{-\frac{1}{3}}$ and the nano-particle volume fraction rate $Sh\left(Ra_x\right)^{-\frac{1}{3}}$ at the constant viscosity $\left(\gamma = 0\right)$ are compared with the results given by Uddin et al.[19]. Table 2 shows that the obtained results in this study are in good agreement with the known result for each values of Nr, Nb, Nt in the absence of internal heat generation. The effect of the viscosity parameter on dimensionless velocity $f'\left(\eta\right)$, temperature $\theta\left(\eta\right)$ and nano-particle volume fraction $\omega'\left(\eta\right)$ for a typical case for $Nr = Nb = Nt = 0.5$ in the presence or absence of internal heat generation, are illustrated in figures l – 6.

Table 2 Comparison of results with Uddin et al. [19] in the absence of internal heat generation term.

	PRESENT $Nu_x\left(Ra_x\right)^{-\frac{1}{3}}$	PRESENT $Sh\left(Ra_x\right)^{-\frac{1}{3}}$	UDDIN ET AL. (2012) $Nu_x\left(Ra_x\right)^{-\frac{1}{3}}$	UDDIN ET AL. (2012) $Sh\left(Ra_x\right)^{-\frac{1}{3}}$
Nr	$Nb = 0.3,\ Nt = 0.1,\ Le = 10,\ \gamma = 0$			
0.1	0.32586	1.48225	0.32578	1.48242
0.2	0.32393	1.46686	0.32385	1.46704
0.3	0.32195	1.45106	0.32188	1.45125
0.4	0.31993	1.43482	0.31985	1.43503
0.5	0.31784	1.41811	0.31777	1.41833
Nt	$Nb = 0.3,\ Nr = 0.5,\ Le = 10,\ \gamma = 0$			
0.1	0.31784	1.41811	0.31777	1.41833
0.2	0.30492	1.41471	0.30486	1.41491
0.3	0.29275	1.41542	0.2927	1.41561
0.4	0.28129	1.41975	0.28125	1.41991
0.5	0.27049	1.42723	0.27046	1.42737
Nb	$Nt = 0.1,\ Nr = 0.5,\ Le = 10,\ \gamma = 0$			
0.1	0.36729	1.32584	0.3672	1.32611
0.2	0.34279	1.39193	0.34271	1.39216
0.3	0.31784	1.41811	0.31777	1.41833
0.4	0.29405	1.43407	0.29399	1.43428
0.5	0.27165	1.44577	0.27161	1.44598

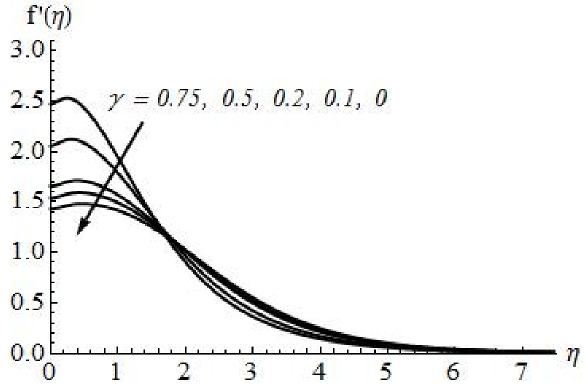

Figure 1: Effect of viscosity parameter γ on velocity profile for $Nr = Nb = Nt = 0.5$ and $Le = 50$ in the presence of internal heat generation

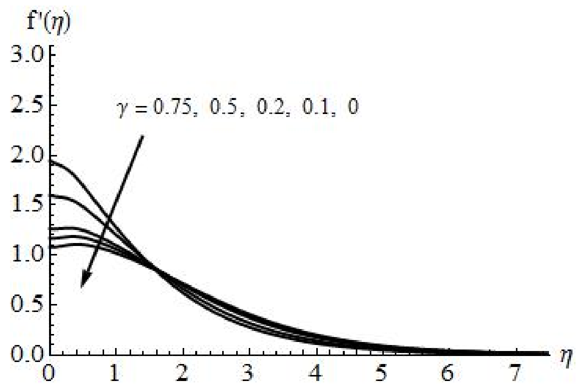

Figure 2: Effect of viscosity parameter γ on velocity profile for $Nr = Nb = Nt = 0.5$ and $Le = 50$ in the absence of internal heat generation

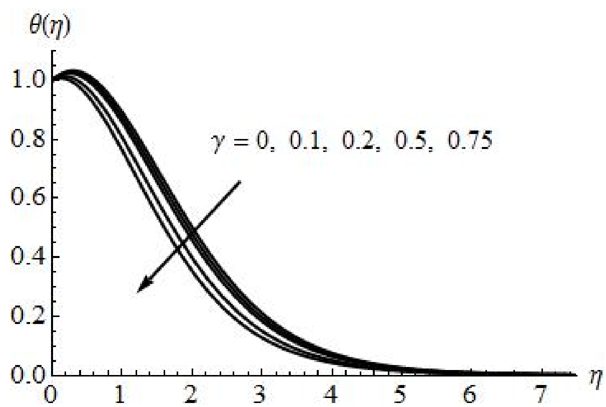

Figure 3: Effect of viscosity parameter γ on temperature profile for $Nr = Nb = Nt = 0.5$ and $Le = 50$ in the presence of internal heat generation

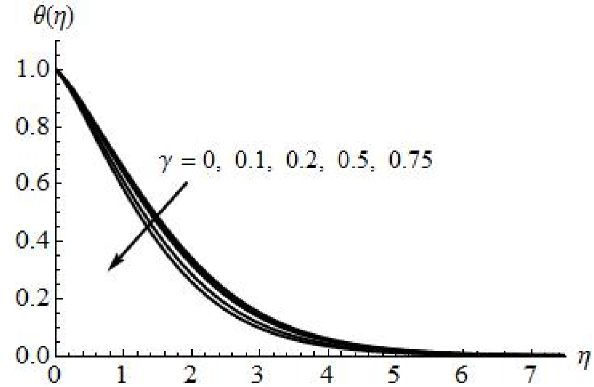

Figure 4: Effect of viscosity parameter γ on temperature profile for $Nr = Nb = Nt = 0.5$ and $Le = 50$ in the absence of internal heat generation

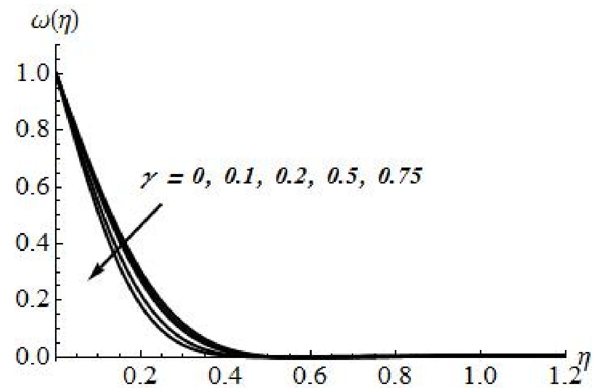

Figure 5: Effect of viscosity parameter γ on nano – particle volume fraction profile for $Nr = Nb = Nt = 0.5$ and $Le = 50$ in the presence of internal heat generation

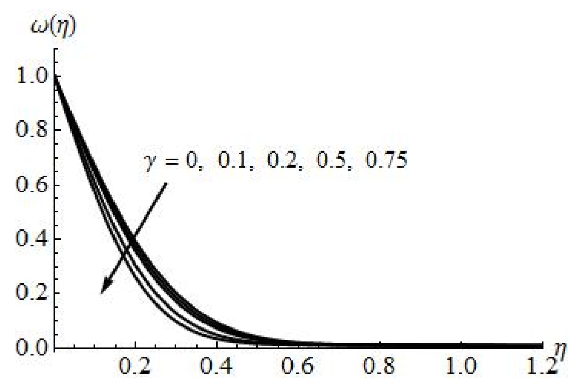

Figure 6: Effect of viscosity parameter γ on nano – particle volume fraction profile for $Nr = Nb = Nt = 0.5$ and $Le = 50$ in the absence of internal heat generation

CONCLUSIONS

We have examined the influence of viscosity on natural convection boundary layer flow over a horizontal plate in a porous medium saturated with nano-fluid. Numerical results for non-dimensional velocity, heat transfer rate and mass transfer rate have been presented for parametric variations of the non-dimensional viscosity parameter γ, buoyancy ratio Nr, Brownian motion parameter Nb, thermophoresis parameter Nt and Lewis number Le. We have assumed temperature and the nano-particle volume fraction are constant along the wall. The heat transfer rate (Nusselt number) and mass transfer rate (Sherwood number) decreases when Lewis number increases. It is also observed that the heat transfer rate and mass transfer rate is more as the viscosity parameter increases or in other words the viscosity of the fluid decreases.

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