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2013

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### Recommended Citation

Aditya Karnik, Ajay M. R, and Mohit Tandon, "Numerical Investigation of the Hydrodynamics of Cylindrical Fluidized Bed" in "The 14th International Conference on Fluidization – From Fundamentals to Products", J.A.M. Kuipers, Eindhoven University of Technology R.F. Mudde, Delft University of Technology J.R. van Ommen, Delft University of Technology N.G. Deen, Eindhoven University of Technology Eds, ECI Symposium Series, (2013). [http://dc.engconfintl.org/fluidization\\_xiv/67](http://dc.engconfintl.org/fluidization_xiv/67)

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# Numerical Investigation of the Hydrodynamics of Cylindrical Fluidized Bed

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## ABSTRACT

Numerical simulation for 4 different ratios of initial bed heights (H) to base diameter (D), were performed; viz. 0.5, 1, 2 and 3. Glass beads of density 2600kg/m<sup>3</sup> and with an average diameter of 550µm were used for all the simulations. Simulations were performed using the commercial CFD software, STAR-CCM+. The minimum fluidization velocity was identified by measuring pressure drop across the entire domain and found to remain same for all the above mentioned ratios. Comparison between experiment[1] and simulation is done.

## INTRODUCTION

Fluidized beds have a wide range of application in the chemical, pharmaceutical, mineral and oil-gas industries. Manufacturing of polyethylene and polypropylene, the synthesis of various fuels, roasting and heat exchangers are some of the industrial application of fluidized beds. The reason for their widespread usage is the better mixing properties and the high contact surface area it provides between the 2 phases. This high contact area improves the efficiency of catalysts.

Depending on the type of the bubbles, within the bed, the flow is classified into different regimes---packed bed, bubbly flow, slug flow, churn flow and annular flow [2]. The bubbling regime occurs at moderate superficial velocities and contains small particles with very less transverse movement. There is no coalescence or break up of bubbles and the size of the bubbles formed is determined by the properties of fluid, particle and the distribution of the gas.

Several complexities are involved in numerical modeling of fluidized beds, the presence of gas-solid intermixing media-with a continuously changing interface, the transient nature and the interaction between the phases. This compounded nature of fluidized beds has been a hindrance in completely understanding the physics involved. With the advent of CFD, considerable progress has been made in conducting investigative studies relating to bed hydrodynamics. Two main numerical techniques have emerged in solving multi-phase problems, Eulerian model [3] [4] [5] [6] [7] [8] [9] and the Lagrangian model [10] [11] [12] [13] [14] [15]. The 2 methods have been compared by Gera et al, 1998 [16]. In the present study, Eulerian model is used for all the simulations.

The Eulerian model, assumes the 2 phases are continuous and inter penetrating. The general Navier-Stokes equation is solved for both the media, but additional closure laws are required to model the particle phase as continuous media. The inter phase interaction is accounted by the drag model and hence, utmost care has to be taken in choosing them. Studies relating to heat transfer [7], horizontal jet penetration [8] and particle rotation for segregation [5] has been conducted using the Eulerian model.

A 2D Cartesian simulation is performed as opposed to a 3D cylindrical geometry, to save simulation time. 2D simulations must be used with caution and should be used only for

sensitivity analysis, they predict the bed height and pressure drop with good accuracy but, for predicting the spatial position of particles it is preferable to use 3D simulations. Xie et al [6] have done extensive work in comparing results from 2D Cartesian, 2D axisymmetric and 3D calculations for bubbling, slugging and turbulent flow regime.

Minimum fluidization velocity is one of the most important parameters to characterize a bed [17]. It is the velocity at which the weight of the bed is just balanced by the inertial force carried by the air coming into the bed. At velocities just equal to or above minimum fluidization velocity the bed attains a suspended state. This velocity is a characteristic property because it depends on the particle property/geometry, bed geometry and fluid properties [18]. Gunn and Hilal [19] and Cranfield and Geldart [20] both showed that  $U_{mf}$  is independent of bed height for a certain types of beds like spouting beds and pseudo 2D beds.

## CONDITIONS IN THE ACTUAL EXPERIMENT

The exact details about the experimental setup and procedure are explained in the paper by D.Escudero and T.J.Heindel, 2010 [1].

## COMPUTATIONAL MODEL

The transport equations for momentum and continuity are solved for both the gas and the solid phase. The equations for the 2 phases are linked together through the drag law. The solid phase has additional equations solved for the kinetic, collisional and frictional regime fundamentally based on the kinetic theory of granular flow.

### Continuity equation

The continuity equations solved are the conventional multi phase eulerian equations.

### Momentum equation

Separate momentum equations for gas and solid phase are solved. The conventional method is used for the pressure and stress terms in the equation.

### Kinetic theory of granular flow

Details of the model were first explained by Gidaspow [21] [22]. Assuming local dissipation of the granular energy, the granular temperature ( $\theta_s$ ) is evaluated using an algebraic equation which account for the collisions between particles. It is modelled as follows:

$$\theta_s = \left[ \frac{-K_1 \varepsilon_s tr(\bar{\tau}_s) + \sqrt{K_1^2 tr^2(\bar{\tau}_s) \varepsilon_s^2 + 4K_4 \varepsilon_s [K_2 tr^2(\bar{\tau}_s) + 2K_3 tr(\bar{\tau}_s^2)]}}{2\varepsilon_s K_4} \right]^2 \quad (1)$$

$tr(\bar{\tau}_s)$  is the stress tensor and the K's are defined as:

$$K_1 = 2(1 + e)\rho_s g_o \quad (2)$$

$$K_2 = \frac{4}{3\sqrt{\pi}} d_s \rho_s (1 + e) \varepsilon_s g_o - \frac{2}{3} K_3 \quad (3)$$

$$K_3 = \frac{d_s \rho_s}{2} \left( \frac{\sqrt{\pi}}{3(3-e)} \left[ 1 + \frac{2}{5} (1 + e)(3e - 1) \varepsilon_s g_o \right] + \frac{8\varepsilon_s}{5\sqrt{\pi}} g_o (1 + e) \right) \quad (4)$$

$$K_4 = \frac{12(1-e^2)\rho_s g_o}{d_s \sqrt{\pi}} \quad (5)$$

$$P_k = \rho_s \varepsilon_s \theta_s \quad (6)$$

$$P_c = 2g_o \rho_s \varepsilon_s^2 \theta_s (1 + e) \quad (7)$$

$$\mu_c = \frac{4}{5} \varepsilon_s \rho_s d_s (1 + e) \sqrt{\frac{\theta_s}{\pi}} \quad (8)$$

$$\mu_k = \frac{2\mu_{dil}}{g_o(1+e)} \left[ 1 + \frac{4}{5}(1+e)\varepsilon_s g_o \right]^2 \quad (9)$$

$$\mu_{dil} = \frac{5\sqrt{\pi}}{96} (\varepsilon_s \rho_s) \left( \frac{d_s}{\varepsilon_s} \right) \sqrt{\theta_s} \quad (10)$$

### Schaeffer model

In regions where the contact between the particles is not instantaneous but continuous the friction between particles has to be considered. The model equations were originally described by Schaeffer [23] which describe the plastic flow of a granular material and relate the shear stress to the normal stress. The Schaeffer model is only activated when the volume fraction of the particle exceeds a certain maximum packing limit (which is set as 0.65 in our case). The frictional pressure is modeled according to the following equation:

$$P_f = \begin{cases} 10^{25}(\varepsilon_s - \varepsilon_s^{max})^{10}, & \varepsilon_s > \varepsilon_s^{max} \\ 0 & \varepsilon_s \leq \varepsilon_s^{max} \end{cases} \quad (11)$$

$$\mu_f = \begin{cases} \min\left(\frac{P_f \sin(\Phi)}{\sqrt{4I_{2D}}}, \mu_m^{max}\right), & \varepsilon_s > \varepsilon_s^{max} \\ 0 & \varepsilon_s \leq \varepsilon_s^{max} \end{cases} \quad (12)$$

$$\text{and } \mu_m^{max} = 1000P \quad (13)$$

$$I_{2D} = \frac{1}{6} [(D_{s11} - D_{s22})^2 + (D_{s22} - D_{s33})^2 + (D_{s33} - D_{s11})^2] + D_{s12}^2 + D_{s23}^2 + D_{s31}^2 \quad (14)$$

and

$$D_{sij} = \frac{1}{2} \left( \frac{\partial u_{si}}{\partial x_j} + \frac{\partial u_{sj}}{\partial x_i} \right) \quad (15)$$

The over-all solid pressure is as solved as follows

$$P_s = P_f + P_k + P_c \quad (16)$$

The viscosity for the solid is modeled as

$$\mu_s = \mu_f + \mu_k + \mu_c \quad (17)$$

### Drag models

Drag force is the most important force in fluidized beds as it is the only source of inter-phase interaction in fluidized beds. Some drag laws are obtained by experimental pressure drop data of packed beds. Ergun equation is one such mathematical model obtained for a packed bed. The Gidaspow drag model has a complementary Wen and Yu [24] model for lower volume fraction of particles (i.e., fluidized bed). Some details of Ergun and Wen & Yu equations are given in the paper by Robert K Niven [25]. The Gidaspow [26] model was used in current study and is formulated as follows:

$$I_{gs} = \beta_{gs}(u_g - u_s) \quad (18)$$

where  $\beta_{gs}$  is the inter phase drag coefficient and for the Gidaspow model given as follows:

$$\beta_{gs} = \frac{3}{4} C_D \frac{\varepsilon_s \varepsilon_g \rho_g |u_g - u_s|}{d_s} \varepsilon_g^{-2.65} \text{ for } \varepsilon_g > 0.8 \quad (19)$$

$$C_D = \begin{cases} \frac{24}{\varepsilon_g Re_s} \left[ 1 + 0.15(\varepsilon_g Re_s)^{0.687} \right], & Re_s < 1000 \\ 0.44 & Re_s > 1000 \end{cases} \quad (20)$$

$$\beta_{gs} = 150 \frac{\varepsilon_s^2 \mu_g}{\varepsilon_g d_s^2} + 1.75 \frac{\varepsilon_s \rho_g |u_g - u_s|}{d_s} \text{ for } \varepsilon_g < 0.2 \quad (21)$$

## NUMERICAL SIMULATION

### Mesh

Square cells were used in the domain with a refinement near the inlet to better capture the bed and the bubbles that form in the bed (Figure 1). The grid size and the refinement size near the inlet are given in Table 2. Similar mesh sizes have been used in previous work by Hosseini et al [27]. A mesh independence test was done by simulating a few cases with a refined mesh (33344 cells); the cell size in the entire domain was reduced by half.

### Initial conditions

The maximum packing fraction was set as 0.65, as explained earlier, and the initial packing fraction of the bed was chosen based on the bulk density reported in the experiment. The density of the particle is the same in all cases, and the ratio of mean bulk density-to-particle density gives the average volume fraction of particle in the bed, before starting the air flow. The bulk density values are reported in Table 1.

An initial velocity is given to the air which is equal to the superficial velocity by the volume fraction of air in the domain. This is done to give a good guess to the initial condition and so as to achieve quasi steady state quicker. Initial gauge pressure was set as 0 Pa throughout the domain.

### Boundary conditions

Extrapolation condition was used for granular temperature on all the boundaries. This way, we are not explicitly specifying the granular temperature to the particles but extrapolating to the boundary from the first layer of cells. The walls, on the sides, were given a no-slip condition for the fluid phase but a slip boundary condition for the particle phase.

### Post processing

The pressure drop was measured by measuring the difference between the surface averaged pressure across the bottom (inlet) and the top boundary (outlet). The average of this is taken from 5s-15s with data acquisition at each time step. The averaging was started after 5 seconds of physical time as the bed achieves a quasi-steady state after approximately 5 seconds time.

Pressure plot Vs. time was plotted so as to see the trend as quasi-steady state is approached. The amplitude of oscillations was found to be increasing with increasing superficial velocities at superficial velocities greater than the minimum fluidization velocity; below minimum fluidization velocity the oscillations are negligible.

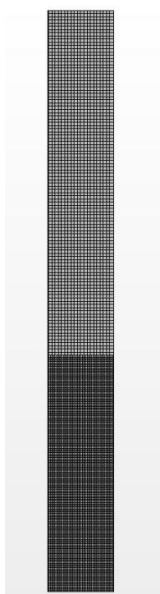


Figure 1. Mesh used in all simulations

	Glass beads		
H/D	Bed mass(g)	Bulk density(kg/m <sup>3</sup> )	Volume fraction
0.5	670	1610±70	0.62
1	1320	1590±70	0.61
2	2560	1540±70	0.59
3	3610	1440±70	0.55
Diameter(μm)	500-600		
Particle Density(kg/m <sup>3</sup> )	2600		

Table 1. Bed material characteristics

Description	Value
Particle density	2600kg/m <sup>3</sup>
Gas density	1.2kg/m <sup>3</sup>
Mean particle diameter(d)	550 μm
Coefficient of restitution(e)	0.9
Superficial gas velocity( <i>U</i> )	0.1m/s-0.3m/s
Bed width(D)	0.102m
Free board height	0.91m
Static bed height(H)	0.051m-0.306m
Grid spacing	0.005m
Grid refinement	0.0025m
Time step	0.0001-0.0005s
Maximum physical time	16s

**Table 2. Simulation parameters**

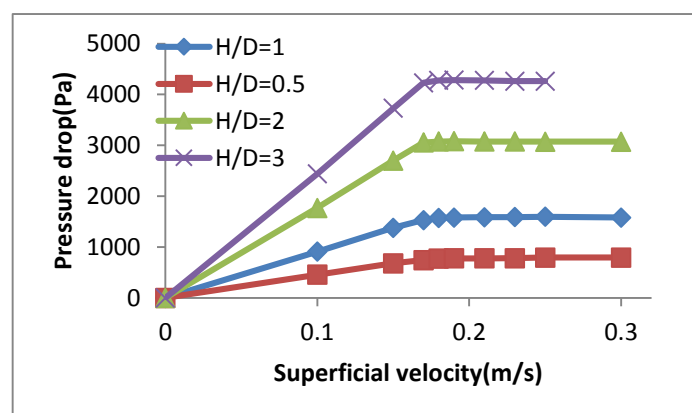
## RESULTS AND DISCUSSION

The pressure drop across the bed increases with increase in H/D ratio; this is related to the increase in mass of the bed. On the other hand the minimum fluidization velocity (the velocity where the knee of the graph is obtained in Figure 2) is approximately same for the different H/D ratios. Hence, it can be concluded that there is no correlation between minimum fluidization velocity and bed height for cylindrical fluidized beds. The value of minimum fluidization velocity is approximately obtained to be at around 0.18m/s as shows in Figure 2. The exact of value of minimum fluidization can only be obtained by performing more simulations near this value.

A force balance between the pressure drop and the weight of the bed is plotted as shown in Figure 3. The value of the knee along the y-axis is approximately 1 showing that beyond minimum fluidization the inertial force of the incoming air balances the weight of the bed.

The time history of pressure drop across the bed is shows in Figure 4. The pressure drop oscillates for velocities above minimum fluidization. Similar behavior was reported by Goldschmidt et al [9].

The plot of experimental and simulation results of pressure drop are shown in Figure 5. The plots do not exactly coincide below the minimum fluidization velocity. A possible reason for this is the absence of wall friction and also, as reported by previous works, the Johnson and Jackson friction model works better than Schaeffer friction model.



**Figure 2. Pressure dropt as a function of superficial velocity**

## CONCLUSION

Simulations were performed and the minimum fluidization velocity was determined to be independent of bed height for cylindrical beds. As discussed in literature, bed height affects minimum fluidization only in certain beds. The data obtained in this research corroborate with the data presented in the literature

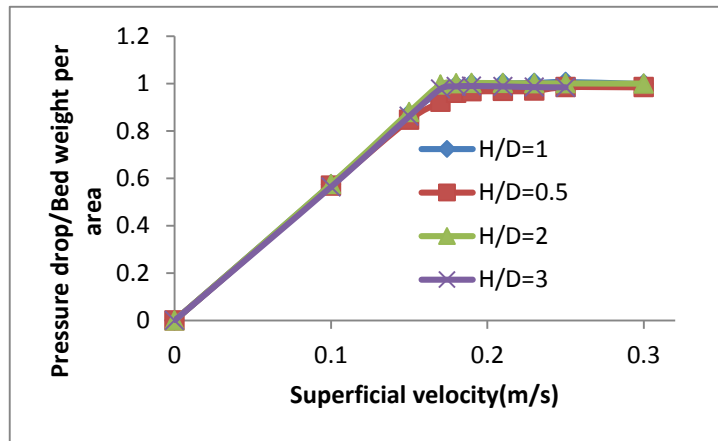


Figure 3. Bed pressure force/ Bed weight as a function of superficial velocity

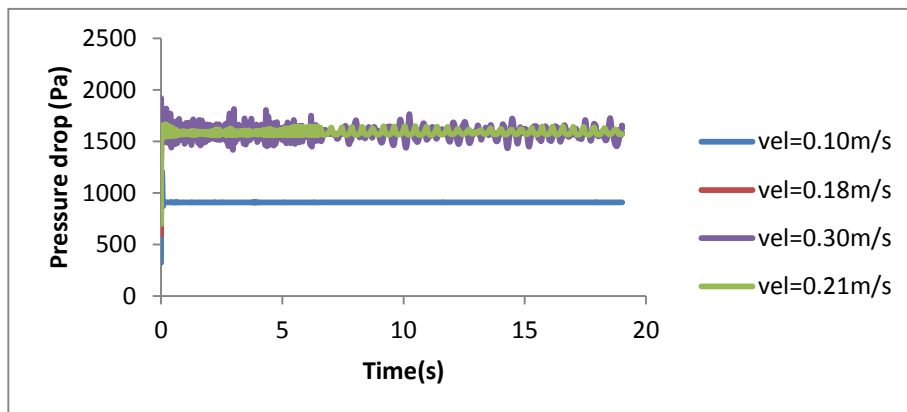


Figure 4. Time history of pressure drop across the bed

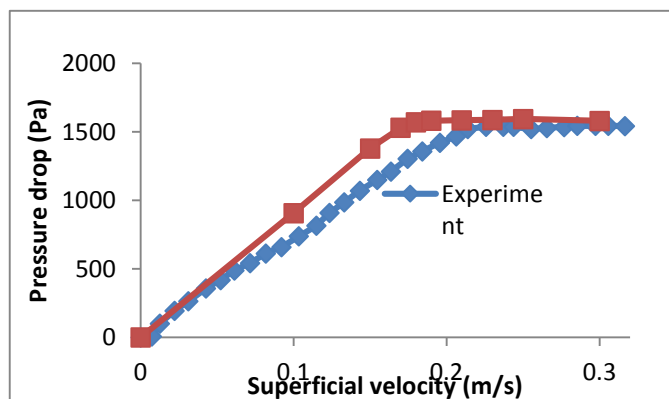


Figure 5. Simulation and experimental pressure drop across the bed for H/D=1

## NOTATION

$\varepsilon_s$	Volume fraction of particle/solid	$d_s$	Diameter of particle
$\varepsilon_g$	Volume fraction of air	$\varepsilon_s^{max}$	Maximum volume fraction of particle
$\rho_s$	Density of particle	$\mu_m^{max}$	Maximum viscosity of particle
$\rho_g$	Density of air	$C_D$	Standard drag coefficient
$\vec{V}_s$	Velocity of solid (vector)	$Re_s$	Particle Reynolds number
$\vec{V}_g$	Velocity of gas (vector)	$d$	Mean particle diameter
$\bar{\tau}_s$	Stress tensor for solid	$U_{mf}$	Minimum fluidization velocity
$\bar{\tau}_g$	Stress tensor for gas	$u_s^{adv}$	Advection velocity of solid
$\lambda_g$	Bulk viscosity of gas	$u_g^{adv}$	Advection velocity of gas
$P_s$	Solid pressure	$I_{gs}$	Drag force
$P_f$	Frictional pressure	$\mu_s$	Solid phase viscosity
$P_k$	Kinetic pressure	$\mu_f$	Frictional viscosity
$P_c$	Collisional pressure	$\mu_k$	Kinetic viscosity
$\beta_{gs}$	Drag coefficient	$\mu_c$	Collisional viscosity
$g_o$	Radial distribution function	$\mu_{dil}$	Dilute viscosity
$e$	Coefficient of restitution		

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