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Review Article

A Review on Fatigue Life Prediction Methods for Metals

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Metallic materials are extensively used in engineering structures and fatigue failure is one of the most common failure modes of metal structures. Fatigue phenomena occur when a material is subjected to fluctuating stresses and strains, which lead to failure due to damage accumulation. Different methods, including the Palmgren-Miner linear damage rule- (LDR-) based, multiaxial and variable amplitude loading, stochastic-based, energy-based, and continuum damage mechanics methods, forecast fatigue life. This paper reviews fatigue life prediction techniques for metallic materials. An ideal fatigue life prediction model should include the main features of those already established methods, and its implementation in simulation systems could help engineers and scientists in different applications. In conclusion, LDR-based, multiaxial and variable amplitude loading, stochastic-based, continuum damage mechanics, and energy-based methods are easy, realistic, microstructure dependent, well timed, and damage connected, respectively, for the ideal prediction model.

1. Introduction

Avoiding or rather delaying the failure of any component subjected to cyclic loadings is a crucial issue that must be addressed during preliminary design. In order to have a full picture of the situation, further attention must be given also to processing parameters, given the strong influence that they have on the microstructure of the cast materials and, therefore, on their properties.

Fatigue damage is among the major issues in engineering, because it increases with the number of applied loading cycles in a cumulative manner, and can lead to fracture and failure of the considered part. Therefore, the prediction of fatigue life has an outstanding importance that must be considered during the design step of a mechanical component [1].

The fatigue life prediction methods can be divided into two main groups, according to the particular approach used. The first group is made up of models based on the prediction of crack nucleation, using a combination of damage evolution rule and criteria based on stress/strain of components. The key point of this approach is the lack of dependence from loading and specimen geometry, being the fatigue life determined only by a stress/strain criterion [2].

The approach of the second group is based instead on continuum damage mechanics (CDM), in which fatigue life is predicted computing a damage parameter cycle by cycle [3].

Generally, the life prediction of elements subjected to fatigue is based on the "safe-life" approach [4], coupled with the rules of linear cumulative damage (Palmgren [5] and Miner [2]). Indeed, the so-called Palmgren-Miner linear damage rule (LDR) is widely applied owing to its intrinsic simplicity, but it also has some major drawbacks that need to be considered [6]. Moreover, some metallic materials exhibit highly nonlinear fatigue damage evolution, which is load dependent and is totally neglected by the linear damage rule [7]. The major assumption of the Miner rule is to consider the fatigue limit as a material constant, while a number of studies showed its load amplitude-sequence dependence [8–10].

Various other theories and models have been developed in order to predict the fatigue life of loaded structures [11–24].

Among all the available techniques, periodic *in situ* measurements have been proposed, in order to calculate the macrocrack initiation probability [25].

The limitations of fracture mechanics motivated the development of local approaches based on continuum damage mechanics (CDM) for micromechanics models [26]. The advantages of CDM lie in the effects that the presence of microstructural defects (voids, discontinuities, and inhomogeneities) has on key quantities that can be observed and measured at the macroscopic level (i.e., Poisson's ratio and stiffness). From a life prediction point of view, CDM is particularly useful in order to model the accumulation of damage in a material prior to the formation of a detectable defect (e.g., a crack) [27]. The CDM approach has been further developed by Lemaitre [28, 29]. Later on, the thermodynamics of irreversible process provided the necessary scientific basis to justify CDM as a theory [30] and, in the framework of internal variable theory of thermodynamics, Chandrakanth and Pandey [31] developed an isotropic ductile plastic damage model. De Jesus et al. [32] formulated a fatigue model involving a CDM approach based on an explicit definition of fatigue damage, while Xiao et al. [33] predicted high-cycle fatigue life implementing a thermodynamics-based CDM model.

Bhattacharya and Ellingwood [26] predicted the crack initiation life for strain-controlled fatigue loading, using a thermodynamics-based CDM model where the equations of damage growth were expressed in terms of the Helmholtz free energy.

Based on the characteristics of the fatigue damage, some nonlinear damage cumulative theories, continuum damage mechanics approaches, and energy-based damage methods have been proposed and developed [11–14, 21–23, 34, 35].

Given the strict connection between the hysteresis energy and the fatigue behavior of materials, expressed firstly by Inglis [36], energy methods were developed for fatigue life prediction using strain energy (plastic energy, elastic energy, or the summation of both) as the key damage parameter, accounting for load sequence and cumulative damage [18, 37–45].

Lately, using statistical methods, Makkonen [19] proposed a new way to build design curves, in order to study the crack initiation and to get a fatigue life estimation for any material.

A very interesting fatigue life prediction approach based on fracture mechanics methods has been proposed by Ghidini and Dalle Donne [46]. In this work they demonstrated that, using widespread aerospace fracture mechanics-based packages, it is possible to get a good prediction on the fatigue life of pristine, precorroded base, and friction stir welded specimens, even under variable amplitude loads and residual stresses conditions [46].

In the present review paper, various prediction methods developed so far are discussed. Particular emphasis will be given to the prediction of the crack initiation and growth stages, having a key role in the overall fatigue life prediction. The theories of damage accumulation and continuum damage mechanics are explained and the prediction methods based on these two approaches are discussed in detail.

2. Prediction Methods

According to Makkonen [19], the total fatigue life of a component can be divided into three phases: (i) crack initiation, (ii) stable crack growth, and (iii) unstable crack growth. Crack initiation accounts for approximately 40–90% of the total fatigue life, being the phase with the longest time duration [19]. Crack initiation may stop at barriers (e.g., grain boundaries) for a long time; sometimes the cracks stop completely at this level and they never reach the critical size leading to the stable growth.

A power law formulated by Paris and Erdogan [47] is commonly used to model the stable fatigue crack growth:

$$\frac{da}{dN} = C \cdot \Delta K^{m}, \tag{1}$$

and the fatigue life N is obtained from the following integration:

$$N = \int_{a_i}^{a_f} \frac{da}{C \cdot (\Delta K)^{m}}, \tag{2}$$

where ΔK is the stress intensity factor range, while C and m are material-related constants. The integration limits a_i and a_f correspond to the initial and final fatigue crack lengths.

According to the elastic-plastic fracture mechanics (EPFM) [48–50], the crack propagation theory can be expressed as

$$\frac{da}{dN} = C' \Delta J^{m'}, \tag{3}$$

where ΔJ is the J integral range corresponding to (1), while C' and m' are constants.

A generalization of the Paris law has been recently proposed by Pugno et al. [51], where an instantaneous crack propagation rate is obtained by an interpolating procedure, which works on the integrated form of the crack propagation law (in terms of S-N curve), in the imposed condition of consistency with Wöhler's law [52] for uncracked material [51, 53].

2.1. Prediction of Fatigue Crack Initiation. Fatigue cracks have been a matter of research for a long time [54]. Hachim et al. [55] addressed the maintenance planning issue for a steel S355 structure, predicting the number of priming cycles of a fatigue crack. The probabilistic analysis of failure showed that the priming stage, or rather the crack initiation stage, accounts for more than 90% of piece life. Moreover, the results showed that the propagation phase could be neglected when a large number of testing cycles are performed [55].

Tanaka and Mura [56] were pioneers concerning the study of fatigue crack initiation in ductile materials, using the concept of slip plastic flow. The crack starts to form when the surface energy and the stored energy (given by the dislocations accumulations) become equal, thus turning the dislocation dipoles layers into a free surface [56, 57].

In an additional paper [58], the same authors modeled the fatigue crack initiation by classifying cracks at first as

(i) crack initiation from inclusions (type A), (ii) inclusion cracking by impinging slip bands (type B), and (iii) slip band crack emanating from an uncracked inclusion (type C). The type A crack initiation from a completely debonded inclusion was treated like crack initiation from a void (notch). The initiation of type B cracks at the matrix-particle interface is due to the impingement of slip bands on the particles, but only on those having a smaller size compared to the slip band width. This effect inhibits the dislocations movements. The fatigue crack is generated when the dislocation dipoles get to a level of self-strain energy corresponding to a critical value. For crack initiation along a slip band, the dislocation dipole accumulation can be described as follows:

$$(\Delta\tau - 2k)\, N_1^{1/2} = \left[\frac{8\mu W_s}{\pi d}\right]^{1/2}, \tag{4}$$

where W_s is the specific fracture energy per unit area along the slip band, k is the friction stress of dislocation, μ is the shear modulus, $\Delta\tau$ is the shear stress range, and d is the grain size [58]. Type C was approximated by the problem of dislocation pile-up under the stress distribution in a homogeneous, infinite plane. Type A mechanism was reported in high strength steels, while the other two were observed in high strength aluminum alloys. The quantitative relations derived by Tanaka and Mura [58] correlated the properties of matrices and inclusions, as well as the size of the latter, with the fatigue strength decrease at a given crack initiation life and with the reduction of the crack initiation life at a given constant range of the applied stress [58].

Dang-Van [59] also considered the local plastic flow as essential for the crack initiation, and he attempted to give a new approach in order to quantify the fatigue crack initiation [59].

Mura and Nakasone [60] expanded Dang-Van's work to calculate the Gibbs free energy change for fatigue crack nucleation from piled up dislocation dipoles.

Assuming that only a fraction of all the dislocations in the slip band contributes to the crack initiation, Chan [61] proposed a further evolution of this theory.

Considering the criterion of minimum strain energy accumulation within slip bands, Venkataraman et al. [62–64] generalized the dislocation dipole model and developed a stress-initiation life relation predicting a grain-size dependence, which was in contrast with the Tanaka and Mura theory [56, 58]:

$$(\Delta\tau - 2k)_i^{\alpha} = 0.37\left(\frac{\mu d}{eh}\right)\left(\frac{\gamma_s}{\mu d}\right)^{1/2}, \tag{5}$$

where γ_s is the surface-energy term and e is the slip-irreversibility factor ($0 < e < 1$). This highlighted the need to incorporate key parameters like crack and microstructural sizes, to get more accurate microstructure-based fatigue crack initiation models [61].

Other microstructure-based fatigue crack growth models were developed and verified by Chan and coworkers [61, 65–67].

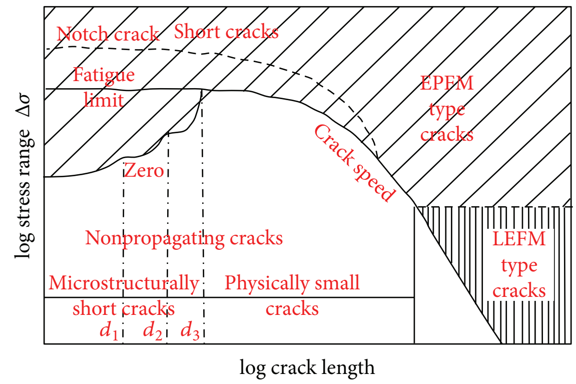


Figure 1: A modified Kitagawa-Takahashi $\Delta\sigma$-a diagram, showing boundaries between MSCs and PSCs and between EPFM cracks and LEFM cracks [69, 71].

Concerning the metal fatigue, after the investigation of very-short cracks behavior, Miller and coworkers proposed the immediate crack initiation model [68–71]. The early two phases of the crack follow the elastoplastic fracture mechanics (EPFM) and were renamed as (i) microstructurally short crack (MSC) growth and (ii) physically small crack (PSC) growth. Figure 1 shows the modified Kitagawa-Takahashi diagram, highlighting the phase boundaries between MSC and PSC [69, 71].

The crack dimension has been identified as a crucial factor by a number of authors, because short fatigue cracks (having a small length compared to the scale of local plasticity, or to the key microstructural dimension, or simply smaller than 1-2 mm) in metals grow at faster rate and lower nominal stress compared to large cracks [72, 73].

2.1.1. Acoustic Second Harmonic Generation. Kulkarni et al. [25] proposed a probabilistic method to predict the macrocrack initiation due to fatigue damage. Using acoustic nonlinearity, the damage prior to macrocrack initiation was quantified, and the data collected were then used to perform a probabilistic analysis. The probabilistic fatigue damage analysis results from the combination of a suitable damage evolution equation and a procedure to calculate the probability of a macrocrack initiation, the Monte Carlo method in this particular case. Indeed, when transmitting a single frequency wave through the specimen, the distortion given by the material nonlinearity generates second higher level harmonics, having amplitudes increasing with the material nonlinearity. As a result, both the accumulated damage and material nonlinearity can be characterized by the ratio A_2/A_1, where A_2 is the amplitude of the second harmonic and A_1 is the amplitude of the fundamental one. The ratio is expected to increase with the progress of the damage accumulation. It is important to point out that this A_2/A_1 acoustic nonlinearity characterization [74] differs from the approach given by Morris et al. [75].

In the work of Ogi et al. [74] two different signals were transmitted separately into a specimen, one at resonance frequency f_r and the other at half of this frequency ($f_r/2$).

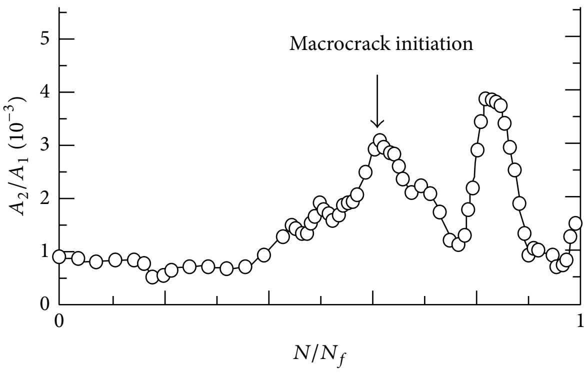


Figure 2: Typical evolution of the ratio $A_2(f_r/2)/A_1(f_r)$ for 0.25 mass% C steel, $N_f = 56,000$ [74].

The transmission of a signal at f_r frequency generates a measured amplitude $A_1(f_r)$, and while the signal is transmitted at frequency $f_r/2$, the amplitude $A_2(f_r/2)$ was received. The measurement of both signals ensures the higher accuracy of this method [74]. Figure 2 shows that the $A_2(f_r/2)/A_1(f_r)$ ratio increases nearly monotonically, and at the point of the macrocrack initiation a distinct peak can be observed. This result suggests that the state of damage in a specimen during fatigue tests can be tracked by measuring the ratio A_2/A_1.

According to the model of Ogi et al. [74], Kulkarni et al. [25] showed that the scalar damage function can be written as $D(N)$, designating the damage state in a sample at a particular fatigue cycle. The value $D = 0$ corresponds to the no-damage situation, while $D = 1$ denotes the appearance of the first macrocrack. The damage evolution with the number of cycles is given by the following equation:

$$\frac{dD}{dN} = \frac{1}{N_c}\left(\frac{\Delta\sigma/2 - r_c(\overline{\sigma})}{\Delta\sigma/2}\right)^m \frac{1}{(1-D)^n}. \qquad (6)$$

When $\Delta\sigma/2$ is higher than the endurance limit ($r_c(\overline{\sigma})$), otherwise the rate dD/dN equals zero.

2.1.2. Probability of Crack Initiation on Defects. Melander and Larsson [76] used the Poisson statistics to calculate the probability P_x of a fatigue life smaller than x cycles. Excluding the probability of fatigue crack nucleation at inclusions, P_x can be written as

$$P_x = 1 - \exp(-\lambda_x), \qquad (7)$$

where λ_x is the number of inclusions per unit volume. Therefore, (7) shows the probability to find at least one inclusion in a unity volume that would lead to fatigue life not higher than x cycles.

In order to calculate the probability of fatigue failure P, de Bussac and Lautridou [77] used a similar approach. In their model, given a defect of size D located in a volume adjacent to the surface, the probability of fatigue crack initiation was assumed to be equal to that of encountering a discontinuity with the same dimension:

$$P = 1 - \exp(-N_v D), \qquad (8)$$

with N_v as the number of defects per unit volume having diameter D. As in the model developed by Melander and Larsson [76], also in this case an equal crack initiation power for different defects having the same size is assumed [77, 78].

In order to account for the fact that the fatigue crack initiation can occur at the surface and inside a material, de Bussac [79] defined the probability to find discontinuities of a given size at the surface or at the subsurface. Given a number of load cycles N_0, the survival probability P is determined as the product of the survival probabilities of surface and subsurface failures:

$$P = [1 - p_s(D_s)][1 - p_v(D_v)], \qquad (9)$$

where D_s and D_v are the diameters of the discontinuities in surface and subsurface leading to N_0 loads life. $p_s(D_s)$ and $p_v(D_v)$ are the probabilities of finding a defect larger than D_s and D_v at surface and subsurface, respectively. It must be stressed that this model does not rely on the type of discontinuity but only on its size [79].

Manonukul and Dunne [80] studied the fatigue crack initiation in polycrystalline metals addressing the peculiarities of high-cycle fatigue (HCF) and low-cycle fatigue (LCF). The proposed approach for the prediction of fatigue cracks initiation is based on the critical accumulated slip property of a material; the key idea is that when the critical slip is achieved within the microstructure, crack initiation should have occurred. The authors developed a finite-element model for the nickel-based alloy C263 where, using crystal plasticity, a representative region of the material (containing about 60 grains) was modeled, taking into account only two materials properties: (i) grain morphology and (ii) crystallographic orientation.

The influence on the fatigue life due to the initial conditions of the specimens was studied deeply by Makkonen [81, 82], who addressed in particular the size of the specimens and the notch size effects, both in the case of steel as testing material.

The probability of crack initiation and propagation from an inclusion depends on its size and shape as well as on the specimen size, because it is easier to find a large inclusion in a big component rather than in a small specimen [77, 79, 83–85].

2.2. Fatigue Crack Growth Modeling. The growth of a crack is the major manifestation of damage and is a complex phenomenon involving several processes such as (i) dislocation agglomeration, (ii) subcell formation, and (iii) multiple microcracks formation (independently growing up to their connection) and subsequent dominant crack formation [39].

The dimensions of cracks are crucial for modeling their growth. An engineering analysis is possible considering the relationships between the crack growth rates associated with the stress intensity factor, accounting for the stress conditions at the crack tip. Of particular interest is the behavior and modeling of small and short cracks [4, 86], in order to determine the conditions leading to cracks growth up to a level at which the linear elastic fracture mechanics (LEFM) theory becomes relevant. Fatigue cracks can be classified as short if one of their dimensions is large compared to the microstructure, while the small cracks have all dimensions similar or smaller than those of the largest microstructural feature [87].

Tanaka and Matsuoka studied the crack growth in a number of steels and determined a proper best-fit relation for room temperature growth conditions [88, 89].

2.2.1. Deterministic Crack Growth Models. While the Paris-Erdogan [48] model is valid only in the macrocrack range, a deterministic fatigue crack model can be obtained starting from the short crack growth model presented by Newman Jr. [93]. Considering N cycles and a medium crack length μ, the crack growth rate can be calculated as follows:

$$\frac{d\mu}{dN} = \exp\left[m \ln\left(\Delta K_{\mathrm{eff}}\right) + b\right]; \quad \mu\left(N_0\right) = \mu_0 > 0, \tag{10}$$

where ΔK_{eff} is the linear elastic effective stress intensity factor range and m and b are the slope and the intercept of the linear interpolation of the (log scale) $\Delta K_{\mathrm{eff}} - d\mu/dN$, respectively. In order to determine the crack growth rate, Spencer Jr. et al. [94, 95] used the cubic polynomial fit in $\ln\left(\Delta K_{\mathrm{eff}}\right)$. Therefore, the crack growth rate equation can be written in the continuous-time setting as follows [96, 97]:

$$\frac{d\mu}{dt} = \frac{(\partial\Phi/\partial S)\cdot(dS/dt)}{(1-\partial\Phi)/\partial\mu}; \quad \mu_0\left(t_0\right) = \mu_0 > 0. \tag{11}$$

Manson and Halford [98] introduced an effective crack growth model accounting for the processes taking place meanwhile, using

$$a = a_0 + \left(a_f - a_0\right) r^q, \tag{12}$$

where a_0, a, and a_f are initial ($r = 0$), instantaneous, and final ($r = 1$) cracks lengths, respectively, while q is a function of N in the form $q = BN^{\beta}$ (B and β are material's constants).

A fracture mechanics-based analysis addressing bridges and other steel structures details has been made by Righiniotis and Chryssanthopoulos [99], accounting for the acceptability of flaws in fusion welded structures [100].

2.2.2. Stochastic Crack Growth Models. The growth of a fatigue crack can be also modeled by nonlinear stochastic equations satisfying the Itô conditions [94–101]. Given that $E[c(\omega, t_0)] = \mu_0$ and $\mathrm{cov}[c(\omega, t_0)] = P_0$, the stochastic differential equation for the crack growth process $c(\omega, t)$ can be written according to the deterministic damage dynamics as

$$\frac{dc(\omega,t)}{dt} = \exp\left[z(\omega,t) - \frac{\sigma_z^2(t)}{2}\right]\cdot\frac{d\mu}{dt} \quad \forall t \geq t_0. \tag{13}$$

If ω and t represent the point and time of the sample in the stochastic process, the auxiliary process $z(\omega, t)$ is assumed to be a stationary Gauss-Markov one having variance $\sigma_z^2(t)$, which implies the rational condition of a lognormal-distributed crack growth [96].

In order to clarify the influence of the fracture peculiarities on the failure probability of a fatigue loaded structure, Maljaars et al. [102] used the linear elastic fracture mechanics (LEFM) theory to develop a probabilistic model.

Ishikawa et al. [103] proposed the Tsurui-Ishikawa model, while Yazdani and Albrecht [104] investigated the application of probabilistic LEFM to the prediction of the inspection interval of cover plates in highway bridges. As for the welded structures, a comprehensive overview of probabilistic fatigue assessment models can be found in the paper of Lukić and Cremona [105]. In this study, the effect of almost all relevant random variables on the failure probability is treated [105].

The key feature of the work of Maljaars et al. [102] with respect to other LEFM-based fatigue assessment studies [87, 103–105] is that it accounts for the fact that, at any moment in time, a large stress cycle causing fracture can occur. Therefore, the probability of failure in case of fatigue loaded structures can be calculated combining all the failure probabilities for all time intervals.

Considering the stress ranges as randomly distributed, the expectation of da/dN can be written as a function of the expectation of the stress range $\Delta\sigma$, as follows (see (14)):

$$\begin{aligned} E\left(\frac{da}{dN}\right) &= A_1 E\left[\Delta\sigma^{m_1}\right]_{\Delta\sigma_{\mathrm{th}}}^{\Delta\sigma_{\mathrm{tr}}} \cdot \left(\left[\frac{B_{\mathrm{nom}}}{B}\right]^P C_{\mathrm{load}} C_{\mathrm{glob}} C_{\mathrm{scf}} C_{\mathrm{sif}} Y_a \sqrt{\pi a}\right)^{m_1} \cdots \\ &\quad + A_2 E\left[\Delta\sigma^{m_2}\right]_{\Delta\sigma_{\mathrm{tr}}}^{\infty} \cdot \left(\left[\frac{B_{\mathrm{nom}}}{B}\right]^P C_{\mathrm{load}} C_{\mathrm{glob}} C_{\mathrm{scf}} C_{\mathrm{sif}} Y_a \sqrt{\pi a}\right)^{m_2}, \end{aligned} \tag{14}$$

$$E\left[\Delta\sigma^m\right]_{s_1}^{s_2} = \int_{s_1}^{s_2} s^m f_{\Delta\sigma}(s)\, ds, \tag{15}$$

where $f_{\Delta\sigma}(s)$ is the probability density function of the stress ranges $\Delta\sigma$ and s represents the stress range steps. The C-factors are the uncertainties of the fluctuating load mode. In (14) B_{nom} represents the plate thickness (considering a welded plate) used in the calculation of the stress, while P is the thickness exponent. The probabilistic LEFM model applied on fatigue loaded structures typical of civil enegineering showed that modeling the uncertainity factors is the key during the assessment of the failure probability, which is quite independent of the particular failure criterion. The partial factors introduced to meet the reliability requirements of civil engineering structures and derived for various values of the reliability index (β) appeared to be insensitive to other parameters such as load spectrum and geometry [102].

2.3. Stochastic Methods for Fatigue Life Prediction. Ting and Lawrence [106] showed that, for an Al-Si alloy, there was a remarkable difference between the size distributions of the casting pores and of those that initiated dominant fatigue cracks.

In order to consider also the size of the defects, Todinov [78] classified them into categories according to their size and then divided them into groups, according to the probability of fatigue crack initiation. Other than type and size, an additional separation was done, when needed, based on the shape of the defects. Considering $M-1$ groups (where M is the index reserved for the matrix), having a p_i average probability of fatigue crack initiation each, it is possible to calculate the probability P_i that at least one fatigue crack initiated in the ith group as

$$P_i = 1 - (1 - p_i)^{N_i}, \quad (16)$$

where $(1-p_i)^{N_i}$ is the probability that none of the group's defects initiated a fatigue crack, while N_i represents the number of discontinuities in the group. The fatigue crack is supposed to be generated on the jth defect of the ith group ($j = 1, N_i$). The probability P_i is not affected by the presence of other groups of defects, because they do not affect the fatigue stress range.

Figure 3 shows that, given a circle having unit area and corresponding to the matrix, the area of the overlapping domains located in this circle is numerically equal to the total probabilities P_i. The i index domain overlaps with greater index domains, thus resulting in the following relations between the average fatigue lives of the groups: $L_i \leq L_{i+1} \leq \cdots \leq L_M$ [78]. f_i is the frequency of failure or rather the probability that, in the ith group of defects, a dominant fatigue crack initiates. The shortest fatigue life is given by the cracks generated in the first group (all dominant) and therefore $f_1 = P_1$. Recurrent equations can be used to express the dependence between the probabilities and the fatigue failure frequencies:

$$\begin{aligned} f_1 &= P_1, \\ f_2 &= P_2 (1 - P_1), \\ f_3 &= P_3 (1 - P_1)(1 - P_2), \\ f_M &= P_M (1 - P_1)(1 - P_2) \cdots (1 - P_{M-1}). \end{aligned} \quad (17)$$

This can be reduced to

$$P_i = \frac{f_i}{1 - \sum_{j-1}^{i-1} f_j} = \frac{f_i}{\sum_{j=i}^{M} f_j}, \quad i = 1, M. \quad (18)$$

Considering a new distribution N_i' of defects in the groups, the probabilities P_i' are calculated according to (16), while failure frequencies are calculated as follows:

$$f_i' = P_i' \left(1 - \sum_{j=1}^{i-1} f_j'\right), \quad i = 2, M. \quad (19)$$

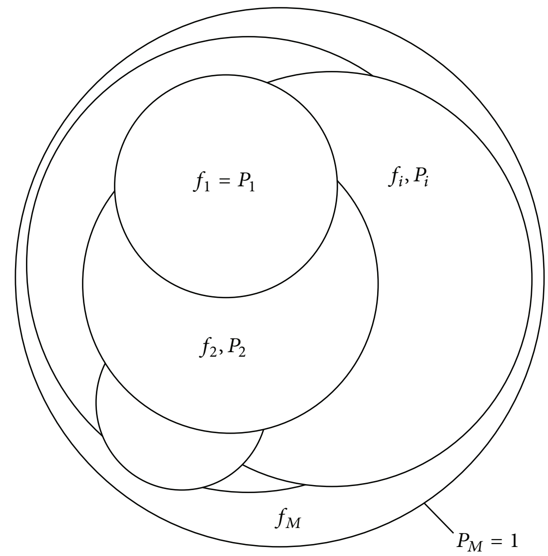


Figure 3: Probabilities that at least one fatigue crack had been nucleated in the ith group of defects (P_M is the probability for nucleation in the matrix). Fatigue failure frequencies f_i (the probability for initiation of a dominant fatigue crack in the ith group) are numerically equal to the area of the nonoverlapped regions (adapted from [78]).

With the new failure frequencies being f_i' ($i = 1, M$) and the average fatigue lives being L_i, $i = 1, M$, the expected fatigue life $\overline{L}$ can be determined as follows [78]:

$$\begin{aligned} \overline{L} &= \sum_{i=1}^{M} f_i' L_i \\ &= P_1' L_1 + P_2' (1 - P_1') L_2 + \cdots \\ &\quad + P_M' (1 - P_1')(1 - P_2') \cdots (1 - P_{M-1}') L_M. \end{aligned} \quad (20)$$

The Monte-Carlo simulation showed that, in the case of cast aluminum alloys, the fatigue life depends more on the type and size of the defects where the fatigue crack arises than on other parameters concerning the fatigue crack initiation and propagation. In general, it can be stated that the probability of fatigue crack initiation from discontinuities and the variation of their sizes produce a large scatter in the fatigue life [78].

A nonlinear stochastic model for the real-time computations of the fatigue crack dynamics has been developed by Ray and Tangirala [96].

Ray [107] presented a stochastic approach to model the fatigue crack damage of metallic materials. The probability distribution function was generated in a close form without solving the differential equations; this allowed building algorithms for real-time fatigue life predictions [107].

Many static failure criteria such as Shokrieh and Lessard, Tsay-Hill, Tsai-Wu, and Hashin can be converted into fatigue failure criterion, by replacing the static strength with fatigue strength in the failure criterion [108–111].

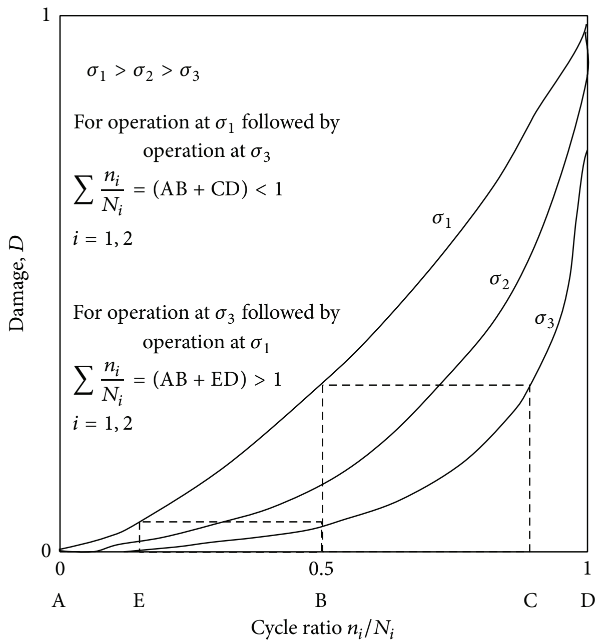


Figure 4: A schematic representation for the Marco-Starkey theory [90].

2.4. Cumulative Damage Models for Fatigue Lifetime Calculation. The most popular cumulative damage model for fatigue life prediction is based on the Palmer intuition [5] of a linear accumulation. Miner [2] was the first researcher who wrote the mathematical form of this theory. The Palmgren-Miner rule, also known as linear damage accumulation rule (LDR), stated that at the failure the value of the fatigue damage D reaches the unity [2]:

$$D = \sum \frac{n_i}{N_i} = 1. \quad (21)$$

The LDR theory is widely used owing to its intrinsic simplicity, but it leans on some basic assumptions that strongly affect its accuracy such as (i) the characteristic amount of work absorbed at the failure and (ii) the constant work absorbed per cycle [39]. From the load sequence point of view, the lack of consideration leads to experimental results that are lower than those obtained by applying the Miner rule under the same loading conditions for high-to-low load sequence and higher results for the opposite sequence.

In order to overcome the LDR shortcomings, firstly Richart and Newmark [112] proposed the damage curve, correlating the damage and the cycle ratio ($D - n_i/N_i$ diagram). Based on this curve and with the purpose of further improving the LDR theory accuracy, the first nonlinear load dependent damage accumulation theory was suggested by Marco and Starkey [90]:

$$D = \sum \left(\frac{n_i}{N_i}\right)^{C_i}. \quad (22)$$

The Palmgren-Miner rule is a particular case of the Marco-Starkey theory where $C_i = 1$, as reported in Figure 4.

The effect of the load sequence is highlighted in Figure 4, since for low-to-high loads $\sum(n_i/N_i) > 1$, while for the high-to-low sequence the summation is lower than the unity [90].

Other interesting theories accounting for load interaction effects can be found in literature [113–122].

The two-stage linear damage theory [123, 124] was formulated in order to account for two types of damage due to (i) crack initiation ($N_i = \alpha N_f$, where α is the life fraction factor for the initiation stage) and (ii) crack propagation ($N_i = (1 - \alpha)N_f$) under constant amplitude stressing [123, 124]. This led to the double linear damage theory (DLDR) developed by Manson [125]. Manson et al. further developed the DLDR providing its refined form and moving to the damage curve approach (DCA) and the double damage curve approach (DDCA) [98, 126, 127]. These theories lean on the fundamental basis that the crack growth is the major manifestation of the damage [39] and were successfully applied on steels and space shuttles components (turbo pump blades and engines) [128, 129].

In order to account for the sequence effects, theories involving stress-controlled and strain-controlled [130–135] cumulative fatigue damage were combined under the so-called hybrid theories of Bui-Quoc and coworkers [136, 137]. Further improvements have been made by the same authors, in order to account for the sequence effect, when the cyclic loading includes different stress levels [138–141], temperature [142, 143], and creep [144–150].

Starting from the Palmgren-Miner rule, Zhu et al. [92] developed a new accumulation damage model, in order to account for the load sequence and to investigate how the stresses below the fatigue limit affect the damage induction. The specimens were subjected to two-stress as well as multi-level tests, and the authors established a fuzzy set method to predict the life and to analyze the evolution of the damage.

Hashin and Rotem [151], Ben-Amoz [152], Subramanyan [153], and Leipholz [154] proposed in the past a variety of life curve modification theories; also various nonlinear cumulative damage fatigue life prediction methods can be found in literature [155–158].

Recently Sun et al. [159] developed a cumulative damage model for fatigue regimes such as high-cycle and very-high-cycle regimes, including in the calculation some key parameters, such as (i) tensile strength of materials, (ii) sizes of fine grain area (FGA), and (iii) sizes of inclusions. Fatigue tests on GCr15 bearing steel showed a good agreement with this model [159].

2.4.1. Fatigue Life Prediction at Variable Amplitude Loading. Usually, the fatigue life prediction is carried out combining the material properties obtained by constant amplitude laboratory tests and the damage accumulation hypothesis by Miner and Palmgren [2, 5]. Popular drawbacks of this method are the lack of validity of the Palmgren-Miner rule accounting for sequential effects, residual stresses, and threshold effects, but the biggest one is that, for loading cycles having amplitude below the fatigue limit, the resulting fatigue life according to the LDR is endless ($N = \infty$) [160]. This is not acceptable, especially when variable amplitude loadings are applied [6, 161].

To overcome these issues, a variety of approaches has been proposed and can be found in literature [6, 162–168]. Among these, a stress-based approach was introduced by the Basquin relation [162, 163]:

$$\sigma_a = \frac{(E \cdot \Delta\varepsilon)}{2} = \sigma'_f \left(2N_f\right)^b, \tag{23}$$

where σ'_f, E, and b are the fatigue strength coefficient, Young's modulus, and the fatigue strength exponent, respectively. The values of these three terms should be determined experimentally. A modification of the Basquin relation has been proposed by Gassner [164] in order to predict the failures of materials and components in service.

A strain-based approach was developed by means of the Coffin-Manson relation:

$$\varepsilon_a^p = \frac{(\Delta\varepsilon^p)}{2} = \varepsilon'_f \left(2N_f\right)^c, \tag{24}$$

with ε_a^p, c, and ε'_f correspond to plastic strain amplitude, fatigue ductility exponent, and fatigue ductility, respectively [51, 169]. The combination of (24) and (25) leads to a widely used total strain life expression:

$$\varepsilon_a = \varepsilon_a^p + \varepsilon_a^e = \varepsilon'_f \left(2N_f\right)^c + \left(\frac{\sigma'_f}{E}\right)\left(2N_f\right)^b, \tag{25}$$

which has been implemented in fatigue life calculation software.

A different approach was proposed by Zhu et al. [92], who extended the Miner rule to different load sequences with the aid of fuzzy sets.

A detailed description of the effects of variable amplitude loading on fatigue crack growth is reported in the papers of Skorupa [170, 171].

Schütz and Heuler [167] presented a relative Miner rule, which is built using constant amplitude tests to estimate the parameter β; afterwards, spectrum reference tests are used to estimate the parameter α. According to Miner's equation [2] (see (21)), for every reference spectrum test the failure occurs at

$$D^* = \frac{N_f}{\widehat{\alpha}} \sum_i \nu_i S_i^{\widehat{\beta}} = \frac{N_f}{N_{\text{pred}}}, \tag{26}$$

where N_f is the number of cycles to failure and N_{pred} is the predicted life according to Palmgren-Miner. When more than one reference test is conducted, the geometric mean value is used. The failure is predicted at a damage sum of D^* (not 1 as in the Palmgren-Miner equation), and the number of cycles to failure becomes

$$N_i^* = D^* \cdot N_i. \tag{27}$$

A stochastic life prediction based on the Palmgren-Miner rule has been developed by Liu and Mahadevan [172]. Their model involves a nonliner fatigue damage accumulation rule and accounts for the fatigue limit dependence on loading, keeping the calculations as simple as possible. Considering an applied random multiblock loading, the fatigue limit is given by

$$\sum \frac{n_i}{N_i} \sum \frac{1}{A_i/\omega_i + 1 - A_i}, \tag{28}$$

where ω_i is the loading cycle distribution and A_i is a material parameter related to the level of stress.

Jarfall [173] and Olsson [165] proposed methods where the parameter α needs to be estimated from laboratory tests, while the exponent β is assumed as known.

In the model suggested by Johannesson et al. [174], load spectra considered during laboratory reference tests were scaled to different levels. The authors defined S_{eq} as the equivalent load amplitude for each individual spectrum, as

$$S_{\text{eq}} = \sqrt[\beta]{\sum_k \nu_k S_k^{\beta}}, \tag{29}$$

where β is the Basquin equation ($N = \alpha S^{-\beta}$) exponent. Considering the load amplitude S_k, its relative frequency of occurrence in the spectrum is expressed by ν_k. The features of the load spectra, such as shape and scaling, are chosen in order to give different equivalent load amplitudes. The modified Basquin equation is then used to estimate the material parameters α and β:

$$N = \alpha S_{\text{eq}}^{-\beta}. \tag{30}$$

This estimation can be done combining the Maximum-Likelihood method [175] with the numerical optimization. Considering a new load spectrum $\{\widehat{\nu}_k, \widehat{S}_k;\ k = 1, 2, \ldots\}$, the fatigue life prediction results from the following calculations:

$$\widehat{N} = \widehat{\alpha}\widehat{S}_{\text{eq}}^{-\widehat{\beta}} \quad \text{where } \widehat{S}_{\text{eq}} = \sqrt[\widehat{\beta}]{\sum_k \widehat{\nu}_k \widehat{S}_k^{\widehat{\beta}}}. \tag{31}$$

Using continuum damage mechanics, Cheng and Plumtree [13] developed a nonlinear damage accumulation model based on ductility exhaustion. Considering that in general the fatigue failure occurs when the damage D equals or exceeds a critical damage value D_C, the damage criterion can be written as

$$D \geq D_C \text{ fatigue failure.} \tag{32}$$

In the case of multilevel tests, the cumulative damage is calculated considering that n_i is the cycle having a level stress amplitude of $\Delta\sigma_i^a$ ($i = 1, 2, 3, \ldots$), while N_i is the number of cycles to failure. Therefore, the fatigue lives will be N_1, N_2, and N_3 at the respective stress amplitudes $\Delta\sigma_1^a$, $\Delta\sigma_2^a$, and $\Delta\sigma_3^a$. The damage for a single level test can be written as [13]

$$D_1 = D_C \left\{1 - \left[1 - \left(\frac{n_1}{N_1}\right)^{1/(1+\psi)}\right]^{1/(1+\beta_1)}\right\}, \tag{33}$$

where ψ is the ductility and β is a material constant. The cumulative damage for a two-stage loading process can be therefore written as

$$\frac{n_1}{N_1} + \left(\frac{n_2}{N_2}\right)^{\{(1-\varphi)[(1+\beta_2)/(1+\beta_1)]\}} = 1. \quad (34)$$

In order to account for variable amplitude loading under the nominal fatigue limit, Svensson [15] proposed an extension of the Palmgren-Miner rule considering that the fatigue limit of a material decreases as the damage increases, owing to damage accumulation due to crack growth. The effects on fatigue behavior and cyclic deformation due to the load sequence were investigated for stainless steel 304L and aluminum alloy 7075-T6 by Colin and Fatemi [176], applying strain- and load-controlled tests under variable amplitude loading. The investigation under different load sequences showed that, for both materials, the L-H sequence led to a bigger sum (longer life) compared to the H-L sequence.

A few models explaining the macrocrack growth retardation effect under variable amplitude loading can be found in literature [177–183], also combined with crack closure effects [179].

Recently, starting from the model proposed by Kwofie and Rahbar [184, 185] using the fatigue driving stress parameter (function of applied cyclic stress, number of loading cycles, and number of cycles to failure), Zuo et al. [186] suggested a new nonlinear model for fatigue life prediction under variable ampitude loading conditions, particularly suitable for multilevel load spectra. This model is based on a proper modification of the Palmgren-Miner's linear damage accumulation rule, and the complete failure (damage) of a component takes place when

$$D = \sum_{i=1}^{n} \beta_i \frac{\ln N_{fi}}{\ln N_{f1}} = 1, \quad (35)$$

where

$$\beta_i = \frac{n_i}{N_{fi}}. \quad (36)$$

β_i is the expended life fraction at the loading stress S_i, and N_{fi} is the failure life of S_i [184]. Compared with the Marco-Starkey [90] LDR modification, the present model is less computationally expensive and is easier to use compared to other nonlinear models. In order to account for random loading conditions, Aïd et al. [160] developed an algorithm based on the S-N curve, further modifed by Benkabouche et al. [187].

2.4.2. Fatigue Life Prediction under Multiaxial Loading Conditions. Generally speaking, a load on an engineering component in service can be applied on different axis contemporary (multiaxial), instead of only one (uniaxial). Moreover, the loads applied on different planes can be proportional (in phase), or nonproportional (out of phase). Other typical conditions that can vary are changes in the principal axes, or a rotation of these with respect to time, a deflection in the crack path, an overloading induced retardation effect, a nonproportional straining effect, a multiaxial stress/strain state, and many more [188–190]. These factors make the multiaxial fatigue life prediction very complicated. A number of theories and life predictions addressing this issue have been proposed in the past [15, 191–215].

For the multiaxial fatigue life prediction, critical plane approaches linked to the fatigue damage of the material can be found in literature; these approaches are based either on the maximum shear failure plane or on the maximum principal stress (or strain) failure plane [190]. The critical plane is defined as the plane with maximum fatigue damage and can be used to predict proportional or nonproportional loading conditions [216]. The prediction methods can be based on the maximum principal plane, or on the maximum shear plane failure mode, and also on energy approaches. The models can be classified into three big groups as (i) stress-based models, involving the Findley et al. [217] and McDiarmid [218] parameters, (ii) strain-based models (i.e., Brown and Miller [195]), and (iii) stress- and strain-based models, involving the Goudarzi et al. [194] parameter for shear failure and the Smith-Watson-Topper (SWT) parameter for tensile failure [216, 219]. Stress-based damage models are useful under high-cycle fatigue regimes since plastic deformation is almost negligible, while strain-based criteria are applied under low-cycle fatigue regimes but can be also considered for the high-cycle fatigue.

The application of these methodologies on the fatigue life prediction of Inconel 718 has been recently made by Filippini et al. [220].

Among all the possible multiaxial fatigue criteria, those belonging to Sines and Waisman [221] and Crossland [222] resulted in being very easy to apply and have been extensively used for engineering design [223].

In the case of steels, by combining the Roessle-Fatemi method with the Fatemi-Socie parameter it is possible to estimate the fatigue limit for loadings being in or out of phase, and the hardness is used as the only material's parameter [216, 224].

In particular, Carpinteri and Spagnoli [206] proposed a prediction method for hard metals based on the critical plane determination; a nonlinear combination of the shear stress amplitude and the maximum normal stress acting on the critical plane led to the fatigue life prediction. According to (37), the multiaxial stress state can be transformed into an equivalent uniaxial stress:

$$\sigma_{\text{eq}} = \sqrt{N_{\max}^2 + \left(\frac{\sigma_{af}}{\tau_{af}}\right)^2 C_a^2}, \quad (37)$$

where C_a is the shear stress amplitude, $N_{\max}$ is the maximum normal stress, and σ_{af}/τ_{af} is the endurance limit ratio [206]. Further modifications proposed for this model are available in literature [225–227].

Papadopoulos [228] also proposed a critical plane model in order to predict multiaxial high-cycle fatigue life using a stress approach [218, 229, 230].

For high-cycle fatigue, Liu and Mahadevan [208] developed a criterion based on the critical plane approach, but this model can be even used for the prediction of fatigue life under different loading conditions, such as (i) in phase, (ii) out of phase, and (iii) constant amplitude. The same authors [231] proposed also a unified characteristic plane approach for isotropic and anisotropic materials where the cracking information is not required.

Bannantine and Socie [191] stated that there is a particular plane which undergoes the maximum damage and this is where the fatigue damage events occur. All the strains are projected on the considered plane, and the normal strain (shear strain) is then cycle-counted using the rainflow-counting algorithm [232]. The strain to be counted is selected according to the predominant cracking mode for the considered material. The fatigue damage connected with every cycle can be calculated with the tensile mode (see (38)) or the shear mode (see (39)):

$$\frac{\Delta\varepsilon}{2}\sigma_{\max} = \frac{\left(\sigma_f'\right)^2}{E}\left(2N_f\right)^{2b} + \sigma_f'\varepsilon_f'\left(2N_f\right)^{b+c}, \tag{38}$$

$$\begin{aligned}\frac{\Delta\widehat{\gamma}}{2} &\equiv \frac{\Delta\gamma}{2}\left(1+\frac{\sigma_{n,\max}}{\sigma_y}\right)\\ &= \frac{\left(\tau_f^*\right)^2}{G}\left(2N_f\right)^{b} + \gamma_f^*\left(2N_f\right)^{c},\end{aligned} \tag{39}$$

where for each cycle $\Delta\varepsilon$ is the strain range and $\sigma_{\max}$ is the maximum normal stress. The maximum shear strain range is given by $\Delta\gamma$, while the maximum normal stress on the maximum shear plane is expressed by $\sigma_{n,\max}$. Other constants and symbols are explained deeply in the paper of Goudarzi et al. [194], where a modification of Brown and Miller's critical plane approach [195] was suggested. It is worth mentioning that, among the conclusions reached by Fatemi and Socie [195], it is stated that since by varying the combinations of loading and materials different cracking modes are obtained, a theory based on fixed parameters would not be able to predict all the multiaxial fatigue situations. Therefore, this model can be applied only to combinations of loading and materials resulting in a shear failure. The Bannantine and Socie method [191] is unable to account for the cracks branching or for the consequent involvement of the multistage cracks growth process. Therefore, this model should be considered only when the loading history is composed of repeated blocks of applied loads characterized by short length, because the cracks would grow essentially in one direction only [189].

Given that cycling plastic deformation leads to fatigue failure, Wang and Brown [189, 192, 233, 234] identified that a multiaxial loading sequence can be assimilated into cycles, and therefore the fatigue endurance prediction for a general multiaxial random loading depends on three independent variables: (i) damage accumulation, (ii) cycle counting, and (iii) damage evaluation for each cycle. Owing to the key assumption of the fatigue crack growth being controlled by the maximum shear strain, this method has been developed only for medium cycle fatigue (MCF) and low cycle fatigue (LCF) [189, 233]. Application of the Wang and Brown criteria showed that, allowing changes of the critical plane at every reversal, a loading history composed of long blocks is suitable for fatigue life prediction [192].

It is worth mentioning that a load path alteration (torsion before tension or tension before torsion, etc.) in multiaxial fatigue strongly affects the fatigue life [235–237].

A situation of constant amplitude multiaxial loading (proportional and nonproportional) was used to develop a new fatigue life prediction method by Papadopoulos [228].

Aid et al. [238] applied an already developed Damage Stress Model [160, 239–241] for uniaxial loadings to study a multiaxial situation, combining it with the material's strength curve (S-N curve) and the equivalent uniaxial stress.

More recently, Ince and Glinka [242] used the generalized strain energy (GSE) and the generalized strain amplitude (GSA) as fatigue damage parameters, for multiaxial fatigue life predictions. Considering the GSA, the multiaxial fatigue damage parameter can be expressed as follows:

$$\begin{aligned}\frac{\Delta\varepsilon^*_{\text{gen}}}{2} &= \left(\frac{\tau_{\max}}{\tau_f'}\frac{\Delta\gamma^e}{2} + \frac{\Delta\gamma^p}{2} + \frac{\sigma_{n,\max}}{\sigma_f'}\frac{\Delta\varepsilon_n^e}{2} + \frac{\Delta\varepsilon_n^p}{2}\right)_{\max}\\ &= f\left(N_f\right),\end{aligned} \tag{40}$$

where the components of the shear (τ, γ) and normal (σ, ε) strain energies can be spotted. Application of this model to Incoloy 901 super alloy, 7075-T561 aluminum alloy, 1045 HRC 55 steel, and ASTM A723 steel gave good agreement with experimental results [242].

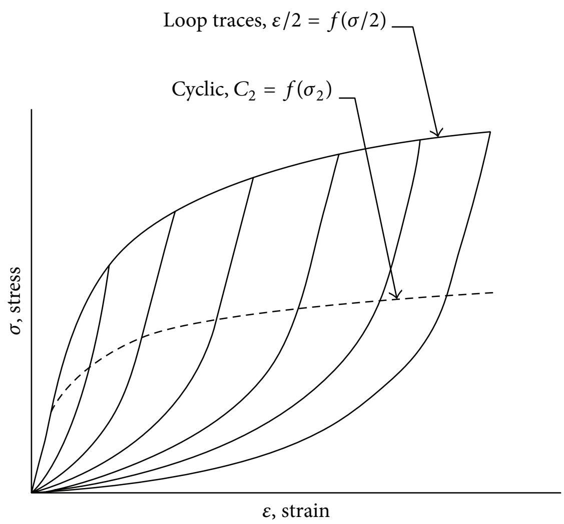
A comparison of various prediction methods for multiaxial variable amplitude loading conditions under high-cycle fatigue has been recently performed by Wang et al. [243]. Results showed that, for aluminum alloy 7075-T651, the maxium shear stess together with the main auxiliary channels counting is suitable for multiaxial fatigue life predictions.

2.5. Energy-Based Theories for Fatigue Life Prediction. As mentioned before in this paragraph, linear damage accumulation methods are widely used owing to their clarity. These methods are characterized by three fundamental assumptions: (i) at the beginning of each loading cycle, the material acts as it is in the virgin state; (ii) the damage accumulation rate is constant over each loading cycle; and (iii) the cycles are in ascending order of magnitude, despite the real order of occurrence [18]. These assumptions allow predicting the fatigue failure for high cycles (HCF) in a sufficiently appropriate manner, but the same cannot be said for the low-cycle fatigue (LCF), where the dominant failure mechanism is identified as the macroscopic strain.

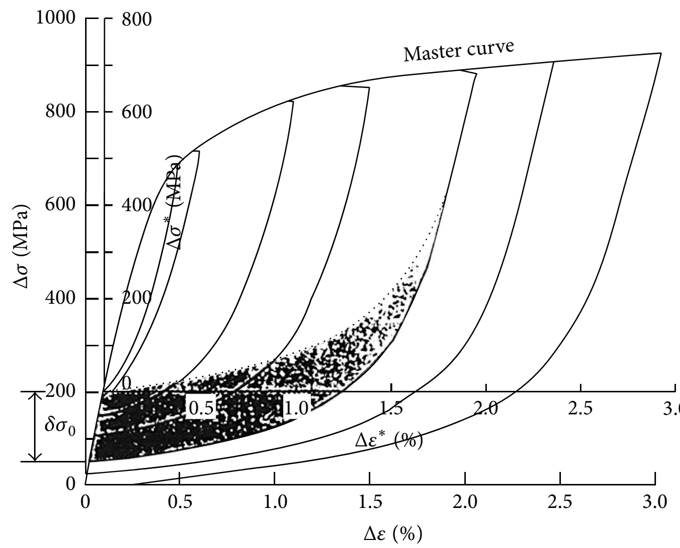
In order to describe and predict the damage process and, therefore, the fatigue life under these two regimes, a unified theory based on the total strain energy density was presented by Ellyin and coworkers [12, 244, 245].

The total strain energy per cycle can be calculated as the sum of the plastic (ΔW^p) and elastic (ΔW^e) strain energies:

$$\Delta W^t = \Delta W^p + \Delta W^e. \tag{41}$$



(a) Masing-type deformation



(b) Non-Masing type deformation

FIGURE 5: Materials exhibiting hysteresis loops with (a) Masing-type deformation and (b) non-Masing type deformation (Adapted from [39]).

The plastic portion of strain is the one causing the damage, while the elastic portion associated with the tensile stress facilitates the crack growth [12]. This theory can be applied to Masing and non-Masing materials [246] (Figure 5) and, in both cases, a master curve can be drawn (translating loop along its linear response portion, for the non-Masing materials).

Therefore, the cyclic plastic strain can be written as follows:

Non-Masing behavior is

$$\Delta W^p = \frac{1-n^*}{1+n^*}\left(\Delta\sigma - \delta\sigma_0\right)\Delta\varepsilon^p + \delta\sigma_0\Delta\varepsilon^p. \quad (42)$$

Ideal Masing behavior is

$$\Delta W^p = \frac{1-n'}{1+n'}\Delta\sigma\Delta\varepsilon^p, \quad (43)$$

where n^* and n' are the cyclic strain hardening exponents of the mater curve and of the idealized Masing material, respectively. Further improvements to this theory have been made, in order to account for the crack initiation and propagation stages [245–247].

Concerning the low-cycle fatigue (LCF), Paris and Erdogan [47] proposed an energy failure criterion based on the energy expenditure during fatigue crack growth.

Interesting energy-based theories were proposed also by Xiaode et al. [248], after performing fatigue tests under constant strain amplitude and finding out that a new cyclic stress-strain relation could have been suggested, given that the cyclic strain hardening coefficient varies during the tests as introduced also by Leis [249].

The theory of Radhakrishnan [250, 251] postulating the proportionality between plastic strain energy density and crack growth rate is worth mentioning. The prediction of the life that remains at the m-load variation can be expressed as

$$r_m = 1 - \sum_{i=1}^{m-1}\frac{W_{fi}}{W_{fm}}r_i, \quad (44)$$

where W_{fi} and W_{fm} are the total plastic strain energy at failure for the ith and the mth levels, respectively, under cycles at constant amplitude. In the case of harmonic loading cycles, Kliman [252] proposed a similar concept for fatigue life prediction.

Based on the energy principle and on a cumulative damage parameter, Kreiser et al. [18] developed a nonlinear damage accumulation model (NLDA), which is particularly suitable for materials and structures subjected to loadings of high amplitude applied for low cycles (substantial plastic strain). This model, starting from the Ellyin-Golos approach [12], accounts for the load history sequence which reflects in the progressive damage accumulation. Considering the LCF Coffin-Manson relation [253], the plastic strain range was considered as an appropriate damage parameter for metals showing stable hysteresis, while in the case of unstable hysteresis also the stress range must be included [18, 254, 255]. The cumulative damage function (φ) depends on a material parameter (p_m) and on the cumulative damage parameter (ψ), and it can be defined using the ratio between the plastic energy density (ΔW_p) and the positive (tensile) elastic strain energy density (ΔW_e^+):

$$\varphi = f\left(\psi, p_m\right) = \frac{1}{\log 10\left(\Delta W_p/\Delta W_e^+\right)}. \quad (45)$$

With the universal slopes method, Manson [256, 257] derived the following equation for the fatigue life prediction:

$$\Delta\varepsilon = \Delta\varepsilon_e + \Delta\varepsilon_p = 3.5\frac{\sigma_B}{E}N_f^{-0.12} + \varepsilon_f^{0.6}N_f^{-0.6}, \quad (46)$$

where ε_f is the fracture ductility and σ_B is the tensile strength. As can be seen in (39), the values of the slopes have been universalized by Manson and are equal to −0.6 for the elastic part and −0.12 for the plastic part.

Considering a wide range of materials steels, aluminum and titanium alloys, Muralidharan and Manson [258] derived a modified universal slopes method, in order to estimate the fatigue features only from the tensile tests data obtained at temperatures in the subcreep range. This method gives a higher accuracy than the original one and is described by the following equation:

$$\begin{aligned} \Delta\varepsilon = {} & 1.17\left(\frac{\sigma_B}{E}\right)^{0.832} N_f^{-0.09} \\ & + 0.0266\varepsilon_f^{0.155}\left(\frac{\sigma_B}{E}\right)^{-0.53} N_f^{-0.56}. \end{aligned} \quad (47)$$

Obtaining the elastic slope from tensile strength, Mitchell [259] proposed a modified elastic strain life curve. A further modification of this method was proposed by Lee et al. [260], in order to estimate the life of some Ni-Based superalloys exposed to high temperatures.

Bäumel and Seeger [261] established a uniform materials law that uses only the tensile strength as input data. In the case of some steels (low-alloyed or unalloyed), the equation can be expressed as follows:

$$\begin{aligned} \frac{\Delta\varepsilon}{2} &= \frac{\Delta\varepsilon_e}{2} + \frac{\Delta\varepsilon_p}{2} \\ &= 1.50\frac{\sigma_B}{E}\left(2N_f\right)^{-0.087} + 0.59\psi\left(N_f\right)^{-0.58}, \end{aligned} \quad (48)$$

where

$$\begin{aligned} &\psi = 1 \quad \text{when } \frac{\sigma_B}{E} \le 0.003, \\ &\psi = 1.375 - 125.0\frac{\sigma_B}{E} \quad \text{when } \frac{\sigma_B}{E} > 0.003. \end{aligned} \quad (49)$$

For titanium and aluminum alloys, the equation is modified as follows:

$$\begin{aligned} \frac{\Delta\varepsilon}{2} &= \frac{\Delta\varepsilon_e}{2} + \frac{\Delta\varepsilon_p}{2} \\ &= 1.67\frac{\sigma_B}{E}\left(2N_f\right)^{-0.095} + 0.35\left(2N_f\right)^{-0.69}. \end{aligned} \quad (50)$$

The application of these techniques to the prediction of the fatigue life of gray cast iron under conditions of high temperatures has been recently conducted by K.-O. Lee and S. B. Lee [262].

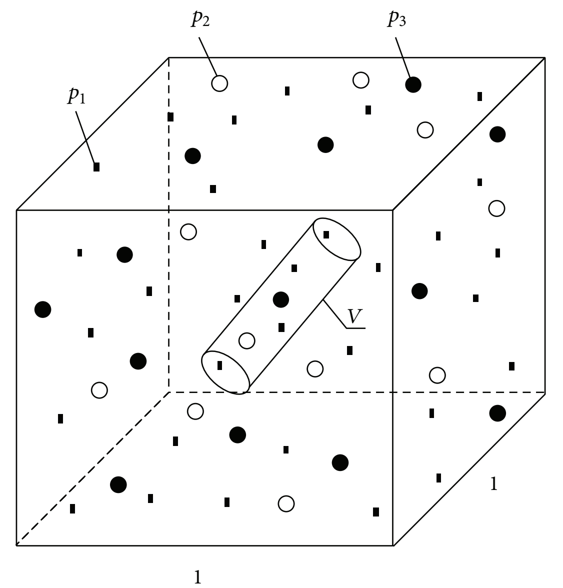


Figure 6: Three groups of defects characterized by individual probabilities $p_1 = 0.2$, $p_2 = 0.3$, and $p_3 = 0.4$ of fatigue crack initiation used in the Monte Carlo simulations. The region with volume V is characterized by a fatigue stress range $\Delta\sigma$ [91].

Under multiaxial loading conditions, the fatigue life can be also estimated using energy criteria based on elastic energy [40, 263, 264], plastic energy [40, 265–267], or a combination of these two [219, 268–276].

2.6. Probability Distribution of Fatigue Life Controlled by Defects. Todinov [91] proposed a method to determine the most deleterious group of defects affecting the fatigue life. For a given material, the effect of discontinuities on the cumulative fatigue distribution can be calculated [91]. As already proposed in a previous work by the same author [78], the defects have been divided into groups according to their type and size, each group being characterized by a different average fatigue life. The individual probability p_i depends on the fatigue stress range $\Delta\sigma$ but not on other groups of defects.

Considering a volume region V (Figure 6) subjected to fatigue loading and having a fatigue stress range $\Delta\sigma$, the probability that at least one fatigue crack will start in this volume can be obtained by subtracting from the unity the no-fatigue crack initiation (in V) probability. Therefore, the probability equation $P^0_{(r)}$ can be written as follows:

$$P^0_{(r)} = \frac{(\mu V)^r e^{-\mu V}}{r!}\left[1 - p(\Delta\sigma)\right]^r, \quad (51)$$

where μ represents the defects' density reducing the fatigue life to a number of cycles lower than x. The absence of fatigue crack initiation at $\Delta\sigma$ can be defined by

$$P^0 = \sum_{r=0}^{\infty} P^0_{(r)} = \sum_{r=0}^{\infty} \frac{(\mu V)^r e^{-\mu V}}{r!}\left[1 - p(\Delta\sigma)\right]^r, \quad (52)$$

which can be simplified as follows:

$$P^0 = e^{-\mu V} \sum_{r=0}^{\infty} \frac{\left[\mu V \left(1 - p\left(\Delta\sigma\right)\right)\right]^r}{r!} = e^{-\mu V} e^{\mu V[1-p(\Delta\sigma)]} = e^{-\mu V \cdot p(\Delta\sigma)}. \tag{53}$$

Equation (53) shows how to obtain the fatigue crack initiation probability (P^0) at a fatigue stress range $\Delta\sigma$. This probability P^0 equals the probability of at least a single fatigue crack initiation in a space volume V and is given by

$$G\left(\Delta\sigma\right) = 1 - e^{-V \sum_{i=1}^{M} \mu_i p_i(\Delta\sigma)}. \tag{54}$$

The equivalence in (54) leads to a fatigue life smaller than or equal to x cycles as reported in

$$F\left(X \leq x\right) = 1 - e^{-V \sum_x \mu_x p_x(\Delta\sigma)}. \tag{55}$$

This calculation is extended only to the defects' groups inducing a fatigue life equal to or smaller than x cycles. The same groups are considered for the expected fatigue life $\overline{L}$ determination, obtained by calculating the weighted average of the fatigue lives L_i:

$$\overline{L} = f_0 L_0 + \sum_{i=1}^{M} f_i L_i, \tag{56}$$

where f_i $(i = 0; M)$ are the fatigue failure frequencies.

The most influential group of defects for the fatigue life is determined by calculating the expected fatigue life rise, when the ith group of defects is removed:

$$\Delta\overline{L}_i = -f_i L_i + \frac{G_i}{1 - G_i} \left(f_0 L_0 + \sum_{j=i+1}^{M} f_j L_j \right). \tag{57}$$

Therefore, the largest value of $\Delta\overline{L}_m$ corresponding to the removal of the m-indexed group of defects identifies the most deleterious ones.

Su [277] studied the connection between the microstructure and the fatigue properties using a probabilistic approach. The key assumption in this case is that the fatigue life decreases monotonically with the size of the local microstructural features which are responsible for the generation of the fatigue crack. Addressing the cast aluminum topic, Su [277] obtained that micropores are the crucial feature affecting the fatigue life. It must be stressed out that the local microstructural feature having the highest impact on the fatigue properties depends on the particular material (e.g., for cast aluminum alloy 319 [278], fatigue cracks are used to initiate from micropores [278–281]). Caton et al. [281] found a correlation between the critical pore size and the fatigue life, the latter being monotonically decreasing with the porosity size. Therefore, the growth of a fatigue crack can be calculated as follows:

$$\frac{da}{dN} = C \left[\left(\varepsilon_{\max} \frac{\sigma_a}{\sigma_y} \right)^s a \right]^t, \tag{58}$$

where a is the crack length, whose initial value is equal to the pore diameter of the discontinuity that generates the crack. Integrating (58), the relationship between the initial pore size and the fatigue life can be written as follows:

$$N = K \left(a_f^{-t+1} - D_0^{-t+1} \right) \quad \text{where } K = \frac{1}{1-t} C^{-1} \left(\varepsilon_{\max} \frac{\sigma_a}{\sigma_y} \right)^{-st}, \tag{59}$$

where D_0 is the crack length, while C, s, and t are material parameters. σ_a, σ_y, and $\varepsilon_{\max}$ are the alternating stress magnitude, the yield stress, and the maximum strain, respectively. It can be useful to invert (58) in order to express the critical pore dimension as a function of the fatigue life:

$$D_0 = \left[a_f^{-t+1} - (1-t) C \left(\varepsilon_{\max} \frac{\sigma_a}{\sigma_y} \right)^{st} N \right]^{1/(1-t)}. \tag{60}$$

2.7. Continuum Damage Mechanics (CDM) Models for Fatigue Life Prediction. Starting from the concept that the accumulation of damage due to environmental conditions and/or service loading is a random phenomenon, Bhattacharaya and Ellingwood [27, 282] developed a continuum damage mechanics (CDM) based model for the fatigue life prediction. In particular, starting from fundamental thermodynamic conditions they proposed a stochastic ductile damage growth model [27]. The damage accumulation equations resulting from other CDM-based approaches suffer usually from a lack of continuity with the first principles of mechanics and thermodynamics [283], owing to their start from either a dissipation potential function or a kinetic equation of damage growth. Battacharaya and Ellingwood [27] considered instead the dissipative nature of damage accumulation and accounted for the thermodynamics laws that rule it [284]. If a system in diathermal contact with a heat reservoir is considered, the rate of energy dissipation can be calculated from the first two laws of thermodynamics and can be written as follows according to the deterministic formulation:

$$\Gamma \equiv -\dot{K}_E + \dot{W} - \frac{\partial \Psi}{\partial \underline{\varepsilon}} \cdot \dot{\underline{\varepsilon}} - \frac{\partial \Psi}{\partial D} \dot{D} \geq 0, \tag{61}$$

where K_E is the kinetic energy and W is the work done on the system. The Helmholtz free energy $\Psi(\theta, \underline{\varepsilon}, D)$ is a function of the damage variable D, the symmetric strain tensor ($\underline{\varepsilon}$), and the temperature (θ).

The system is at the near equilibrium state and is subjected to continuous and rapid transitions of its microstates, causing random fluctuations of state variables around their mean values. Therefore, the stochastic approach [27] is the most suitable to describe the free energy variation and as per (61), the first one can be written as follows:

$$\delta\Psi(t) = \delta\int_{t_0}^{t}\left(\dot{W} - \dot{K}_E\right)dt - \delta\int_{t_0}^{t}\Gamma\, dt + \delta B(t) \approx 0, \quad (62)$$

where $B(t)$ represents the free energy random fluctuation, t_0 is the initial equilibrium state, and t is an arbitrary instant of time ($t > t_0$). For a body that undergoes an isotropic damage due to uniaxial loading, the following can be assumed:

$$\sigma_\infty + \psi_D\frac{dD}{d\varepsilon} + s_b = 0, \quad (63)$$

where σ_∞ is the far-field stress acting normal to the surface and ψ_D is the partial derivative of the free energy per unit volume (ψ) with respect to D. The quantity s_b, which is equal to $(\partial^2 B)/(\partial\varepsilon\partial V)$, represents the random fluctuation imposed on the stress field existing within the deformable body. Assuming that s_b is described by the Langevin equation, it is possible to write what follows:

$$\frac{ds_b}{d\varepsilon} = -c_1 s_b + \sqrt{c_2\xi(\varepsilon)}, \quad (64)$$

where c_1 and c_2 are positive constants, while $\xi(\varepsilon)$ corresponds to the Gaussian white noise indexed with the strain. In this particular case, three key assumptions are necessary: (i) s_b should be a zero-mean process characterized by equiprobable positive and negative values, (ii) compared to the macroscopic rate of damage change, the fluctuation rate is extremely rapid, and (iii) the s_b mean-square fluctuation should be time or strain independent [27]. Considering the scale of time/strain typical of structural mechanics, the fluctuations are extremely rapid and, therefore, the damage growth stochastic differential equation can be written as follows [285]:

$$dD(\varepsilon) = -\frac{\sigma_\infty}{\psi_D}d\varepsilon - \frac{\sqrt{c_2/c_1}}{\psi_D}dW(\varepsilon), \quad (65)$$

where $W(\varepsilon)$ is the standard Wiener process. This formulation allows considering the presence of negative damage increments over a limited time interval at the microscale, even though the calculated increment of damage should be nonnegative in absence of repair.

According to Battacharaya and Ellingwood [27], in presence of a uniaxial monotonic loading, the free energy per unit volume can be calculated as follows:

$$\psi = \int\sigma d\varepsilon - \gamma. \quad (66)$$

In (66), γ is the energy associated with the defects formation (per unit volume) due to damage evolution. With the aid of the Ramberg-Osgood monotonic stress-strain relations, the total strain can be estimated as follows:

$$\varepsilon = \frac{\tilde{\sigma}}{E} + \left(\frac{\tilde{\sigma}}{K}\right)^M, \quad (67)$$

where the first term is the elastic strain (ε_e) and the second is the plastic strain (ε_p), while M and K are the hardening exponent and modulus, respectively. The second term of (66) can be instead estimated as

$$\gamma = \frac{3}{4}\sigma_f D, \quad (68)$$

where σ_f is the failure strain, assuming that (i) discontinuities are microspheres not interacting with each other and having different sizes, (ii) stress amplifications can be neglected, and (iii) there is a linear relation between force and displacement at the microscale. Equation (65) can be rewritten as follows:

$$\begin{aligned} dD(\varepsilon_p) &= A(\varepsilon_p)\left(1 - D(\varepsilon_p)\right)d\varepsilon_p \\ &\quad + B(\varepsilon_p)\,dW(\varepsilon_p), \end{aligned} \quad (69)$$

where A and B are coefficients depending on ε_p, σ_f, M, K, and ε_0. If the materials properties $\underline{\Omega} = \{\varepsilon_0, \sigma_f, K, M\}$ are considered deterministic and $D_0 = D(\varepsilon_0)$ is the initial damage that can be either deterministic or Gaussian, the result is that damage is a Gaussian process as defined by the damage variable [27].

Concerning nonlinear models, Dattoma et al. [286] proposed theory applied on a uniaxial model based on continuum damage mechanics [287]. In the formulation of this nonlinear model [286] the authors started from the nonlinear load dependent damage rule, firstly formulated by Marco and Starkey [90] as follows:

$$D = \sum_{i=1}^{n} r_i^{x_i}, \quad (70)$$

where x_i is a coefficient depending on the ith load. The experimental results showed a good agreement with the data calculated with this method, although for each load it is necessary to recalculate the x_i coefficients.

The mechanical deterioration connected to fatigue and creep through the continuum damage theory was introduced by Kachanov [288] and Rabotnov [289]. Later, Chaboche and Lemaitre [290, 291] formulated a nonlinear damage evolution equation, so that the load parameters and the damage variable D result in being nondissociable:

$$\delta D = f(D, \sigma)\,\delta n, \quad (71)$$

where n is the number of cycles at a given stress amplitude and σ is the stress amplitude [291, 292].

This fatigue damage can be better defined as follows:

$$\begin{aligned} \delta D &= \left[1 - (1 - D)^{\beta+1}\right]^{\alpha(\sigma_{\max}, \sigma_{\mathrm{med}})} \\ &\quad \cdot \left[\frac{\sigma_a}{M_0\left(1 - b\sigma_{\mathrm{med}}\right)(1 - D)}\right]^{\beta}\delta n, \end{aligned} \quad (72)$$

where β, M_0, and b depend on the material, while α depends on the loading. $\sigma_{\max}$ and σ_{med} are the maximum and the mean stress of the cycle, respectively. The stress amplitude σ_a is calculated as $\sigma_a = \sigma_{\max} - \sigma_{\mathrm{med}}$. This approach has been adopted by various authors [13, 14, 33, 292] with integrations and modifications. All these CDM models were developed for the uniaxial case, but also thermomechanical models based on mechanics of the continuous medium can be found in literature [11, 293]. According to Dattoma et al. [286] the number of cycles to failure (N_f) for a given load can be written as follows:

$$N_f = \frac{1}{1-\alpha}\frac{1}{1+\beta}\left[\frac{\sigma_a}{M_0}\right]^{-\beta}, \tag{73}$$

which is a good approximation of the linear relation between $\log S$ and $\log N$ [33]. The main result highlighted by this formulation is that damage is an irreversible degradation process which increases monotonically with the applied cycles (see (74)). Moreover, the higher the load, the larger the fatigue damage (see (74)):

$$\frac{\delta D}{\delta n} > 0, \tag{74}$$

$$\frac{\delta^2 D}{\delta n \delta \sigma} > 0. \tag{75}$$

This model was employed for the fatigue life calculation of a railway axle built with 30NiCrMoV12, running onto a European line for about 3000 km, taking care of its load history. Final experimental tests were conducted with high-low, low-high, and random sequence using cylindrical specimens. Results showed that this model has a good capacity to predict the final rupture when complex load histories are considered [286, 287].

A lot of fatigue life prediction theories based on nonlinear continuum damage mechanics can be found in literature [13, 14, 39, 294–298], addressing various situations such as fatigue combined to creep, uniaxial fatigue, and ductile failure.

2.8. Other Approaches. Addressing the particular case of aluminum alloys, Chaussumier and coworkers developed a multicrack model for fatigue life prediction, using the coalescence and long and short crack growth laws [299]. Based on this work, further studies on the prediction of fatigue life of aluminum alloys have been conducted by the same authors [300, 301]. The approach used in the paper of Suraratchai et al. [300] is worth mentioning, where the effect of the machined surface roughness on aluminum alloys fatigue life was addressed. Considering an industrial frame, the particular purpose of this work was the fatigue life prediction of components when changing machining parameters and processes, in order to avoid tests that could be expensive and time-consuming. Accounting for other methods present in literature [302–309] that usually consider the surface roughness as a notch effect in terms of stress, the theory developed by Suraratchai et al. [299] modeled how the geometric surface condition affects the fatigue properties of structures. In this theory [299] the surface roughness is considered responsible for the generation of a local stress concentration, controlling the possible surface crack propagation or nonpropagation.

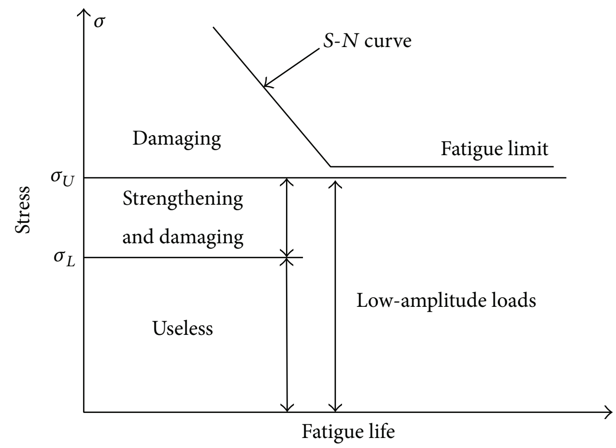


Figure 7: Load region and its strengthening and damaging effect (adapted from [92]).

Thermomechanical fatigue (TMF), mainly related to single crystal superalloys operating at high temperatures, was studied by Staroselsky and Cassenti [310], and the combination of creep, fatigue, and plasticity was also addressed [311]. The low-cycle fatigue-creep (LCF-C) prediction has been studied by several authors [70, 312–317] for steels and superalloys applications. Strain range partitioning- (SRP-) based fatigue life predictions [318–320] or frequency modified fatigue life (FMFL) [320, 321] can be also found in this field.

Zhu et al. [92] used the fuzzy set method to predict the fatigue life of specimen under loadings slightly lower than the fatigue limit, accounting for strengthening and damaging effects, as well as for the load sequence and load interaction. A schematic representation of this theory is reported in Figure 7.

Other particular fatigue life prediction models can be found in literature, based on significant variations of physical and microstructural properties [322–327], on thermodynamic entropy [328, 329], and on entropy index of stress interaction and crack severity index of effective stress [330].

Even though a look at the frequency domain for the fatigue life estimation has been given in the past [331–334], recently new criteria have been developed by several authors, highlighting the rise of a brand new category of fatigue life estimation criteria that will surely get into the engineering spotlights [335–342].

3. Summary

Starting from the introduction of the linear damage rule, a huge number of fatigue life prediction models have been proposed, yet none of these can be universally accepted. Authors all over the world put efforts into modifying and extending the already existing theories, in order to account for all the variables playing key roles during cyclic applications of loadings. The complexity of the fatigue problem makes this topic actual and interesting theories using new approaches arise continuously. The range of application of each model varies from case to case and, depending on the particular application and to the reliability factors that must be considered,

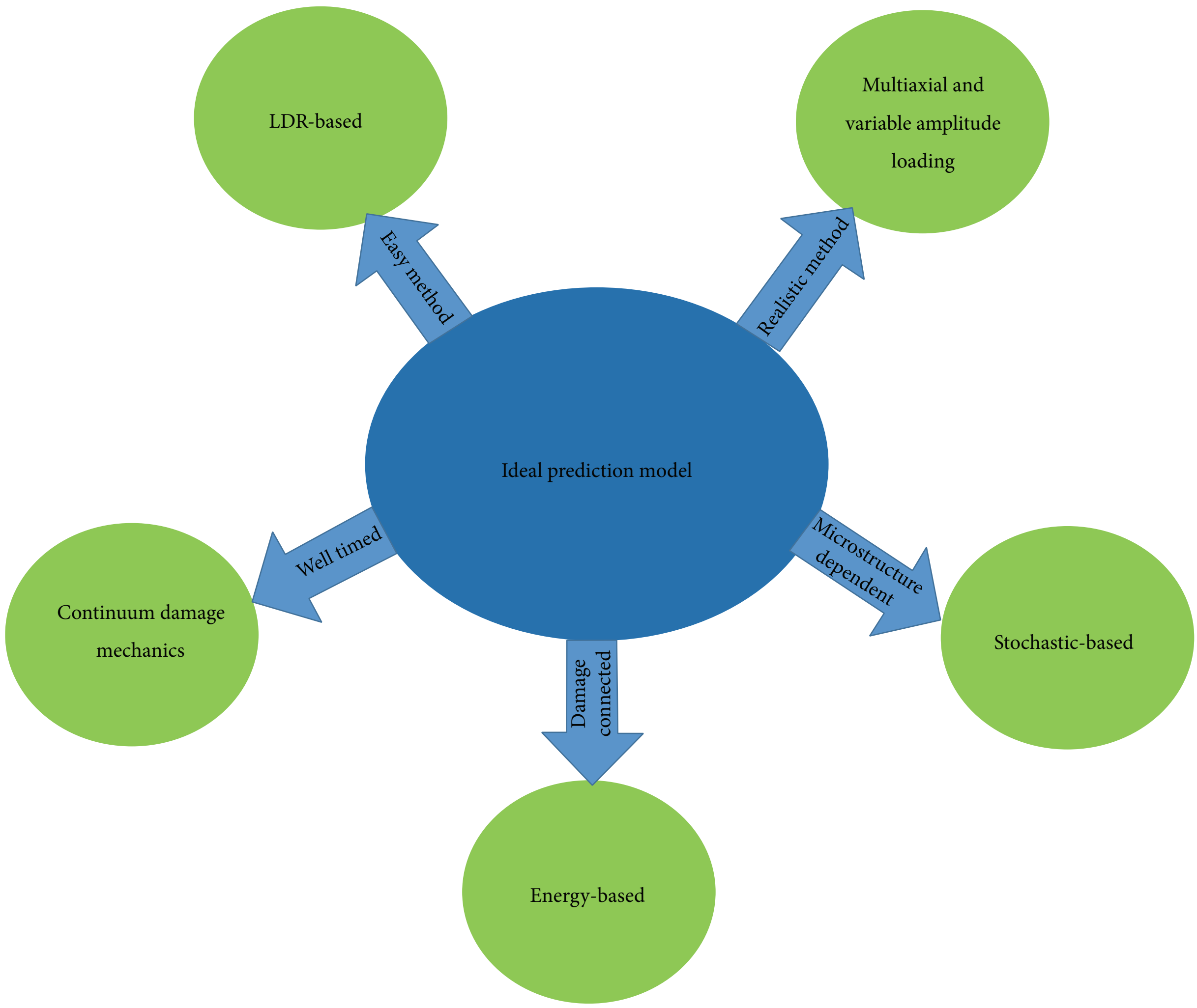


Figure 8: Features of an ideal fatigue life prediction model.

researchers can opt for multivariable models to be computed, or approaches easier to handle leading to a "safe-life" model. As reported in Figure 8, an ideal fatigue life prediction model should include the main features of those already established, and its implementation in simulation systems could help engineers and scientists in a number of applications.

Competing Interests

The authors declare that they have no competing interests.

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