# EXPLORING CHILDREN'S IDEAS OF DISCRETE VARIABLES USING GRAPHS, TABLES AND ISOLATED CASES

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**Abstract.** A survey was conducted to analyse the effect of different representation of information on 6th graders mathematical reasoning (N=120), when discrete variables are involved. It addresses two questions: 1) Are there differences on children's performance when the information is represented by bar graphs, tables and isolated cases? 2) Are there differences on children's reasoning in each of these conditions? And 3) What difficulties do children present when solving problems with information presented using bar graphs, tables and isolates cases? The children were randomly assigned to work in one of the three groups: Graphs, Tables and Isolated Cases. The same problems were presented to all children using the representation of information of the group condition. The problems comprised simple and double proportions. Results show that the type of representation used to present discrete variables does not have an effect on students' performance.

## REPRESENTATION TOOLS AND MATHEMATICAL REASONING

This paper focuses on the effect of different representation of information on Portuguese children's mathematical reasoning, when discrete variables are involved.

An important issue on students' actions when solving problems with variables refers to the number of variables they have to deal with. Concerning the mathematical problems, Vergnaud (1983) distinguishes the additive and multiplicative structure problems: the additive structure problems are problems involving one variable and can be solved by combination; the multiplicative structure problems involve two or more variables. According to his theory, the problems of two variables – isomorphism of measures – refer to a simple direct proportion between variables. These problems can be represented by linear functions and by simple correspondence tables. The problems involving three or more variables – product of measures – refer to the Cartesian composition of two measures, $M_1$ and $M_2$, into a third one, $M_3$ (Vergnaud, 1983, 1997). The problems of the product of measures can be represented by tables or graphs. In the case of graphs, the two variables (number of girls and number of boys) would be represented by two distinct axes whereas the number of couples would be represented by dots in the plan.

The use of symbolic representations may have an important role in the understanding of additive and multiplicative structure problems. According to Nunes (1997) and Vergnaud (1997), the symbolic representations can be considered very helpful in conceptual building, as each symbolic model allows different approaches of conceptual properties. But the type of representation interferes with the understanding of mathematical relations.

Nunes (2004) distinguishes two categories of system of signs relevant for the understanding of mathematical relations: the analogous representations and the symbolic representations. The *analogous representations* can be figurative, numerical or by words. In this type of representation, the parts of the problem are explicitly given; the sign and the meaning of the sign are directly connected. The *symbolic representations* involve situations demanding the knowledge of basic mathematical procedures for the global understanding of symbolic aspects of the information. For example, when a child is asked to make additive compositions based on expressions such as "pay twenty-three pence" and "pay seven pence" different opportunities to think about the composition are involved. In the former, a verbal clue is given to consider '*20p + 1p + 1p + 1p*' as an additive group because for each written representation there is a correspondent numeric value that can be visually associated – the sign and the significance are directly linked; in the latest, there is an absence of this clue and the child needs to establish the additive group and consider, for instance, '*5p + 1p + 1p'*, demanding a previous understanding of relations to establish the additive compositions of numbers – the sign and the significance are only symbolically related, and the child needs to understand the additive composition of numbers to find the solution. According to Nunes (2004), in spite of being required an additive composition in both situations, the way the problem is presented to children has an effect on the way children reasoning about it.

**Representing discrete variables**

Concerning the representation of discrete variables, according to Nunes (2004), the use of isolated cases emphasizes the direct connections between the signs and the units they refer to, making them similar representations of the information. For the author, tables and graphs belong to the class of symbolic representations as they refer to relations between variables.

Watson and Moritz (2001) studied 90 students (8- to 18-years-old) to examine the effect of pictographs (cards with pictures) on the process of representation, interpretation and prediction of information. The authors argued that pictographs can be seen as a starting point for the development of more complex reasoning such as the analyses of relations between variables. Also Selva, Falcão and Nunes (2005) analysed the combination of isolated cases and bar graphs on 39 children (6- to 8-years-old) about the understanding of additive concepts. Their findings suggest that isolated cases using cards with pictures can have an important role on children's understanding of bar graphs. More recently Carvalho (2008) investigated whether bar graphs, tables and isolated cases influenced Brazilian students' reasoning when solving discrete variables tasks with 8$^{th}$ graders (n=99). The Brazilian students performed better when graphs were involved than when isolated cases or tables were used to present the information. In graphs, the relationships between variables are presented visually whereas in tables these aspects are condensed in a numeric way.

In Portugal, the children are introduced to graphs, tables and isolated cases since the primary school. Sixth graders should be able to represent data using absolute and relative frequency tables to represent discrete variables, bar graphs, and also pictograms. Based on the study of Carvalho (2008), this study aims to understand the effect of different representation of information on Portuguese children's mathematical reasoning, when discrete variables are involved. For that, it addresses three questions: 1) Are there differences on children's performance when the information is represented by bar graphs, tables and isolated cases? 2) Are there differences on children's reasoning in each of these conditions? And 3) What difficulties do children present when solving problems with information presented using bar graphs, tables and isolates cases?

## METHODS

A survey was conducted with 6$^{th}$ graders (n=120) from public Elementary Schools, from Braga, Portugal. The children were randomly assigned to work in one of the three groups: *Graphs*, the group of children who solved the problems presented only using bar graphs; *Tables*, the group who solved the problems with the information presented only by tables; and, *Isolated Cases*, the group who solved the problems with the information presented by isolated cases, using cards. The same problems were presented to all the participants using the representation of the information according to the group condition. There were 4 problems involving simple proportions and 2 involving double proportions. Table 1 gives examples of simple and double proportion problems.

The tasks presented to the children are an adaptation of those previously used by Carvalho (2008). Figure 1 gives examples of problems presented in each group. Each child of each group was asked to solve 6 problems, individually. Each child has a booklet with a printed problem in each page, in which they could solve the problems as they wish, and write their justifications. All the problems were presented to the class orally by the researcher in the children's classroom, with no interference of the teacher. In each group children took, approximately, 50 minutes to solve all the tasks. Data collected provided from students booklets.

Table 1 – Examples of problems involving simple and double proportions.

| *Simple proportion Problem* | *Variable values* | *Situation* |
|---|---|---|
| Do you think there is a greater chance of finding students with dark hair among students with blue eyes or among students with dark eyes? | Blue eyes - 7 cases (4 dark hair; 3 blond hair);<br>Dark eyes – 7 cases (5 dark hair; 2 blond hair). | Simple proportion controlling for the total amount. |
| *Double Proportion Problem* | *Variable values* | *Situation* |
| Do you think that there is a greater chance of finding students happy with CD1 or CD2? | CD1 = 12 cases (6 happy; 6 unhappy);<br>CD2 = 6 cases (3 happy; 3 unhappy) | Double variation in the whole and in the parts, using numbers with reference to half. |

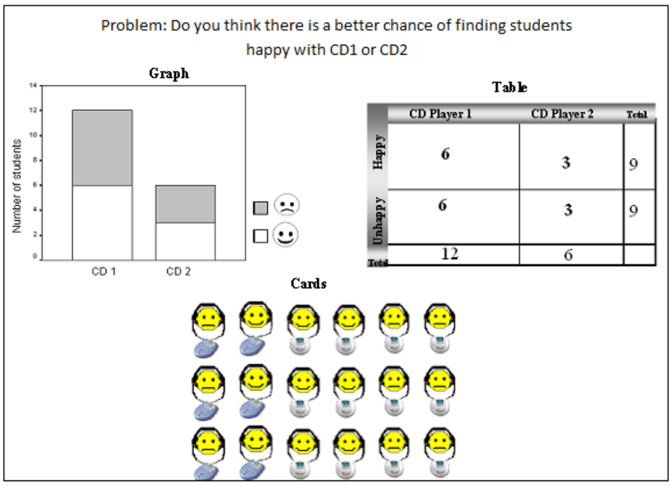



Figure 1 – Examples of problems presented to the children.

## RESULTS

Children's performance was analysed to identify possible difference related to the distinct types of representation. For each problem, 1 point was awarded for each children's correct answer and 0 for the others.  Table 2 presents the mean (standard deviation) of correct responses according to the type of representation.

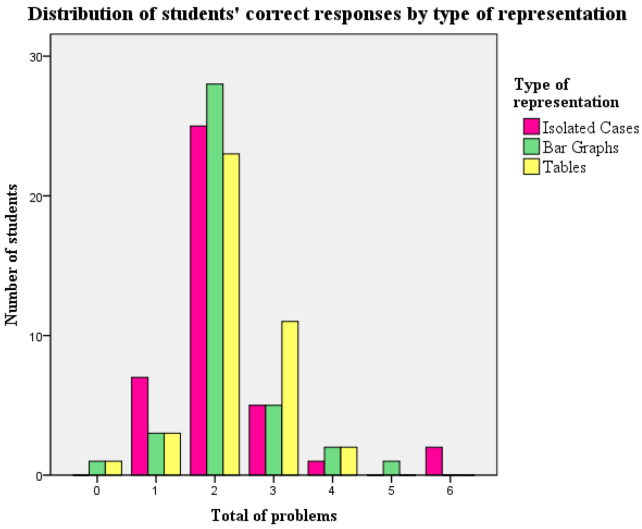
Table 2 - Mean (standard deviation) of correct responses by type of representation.

| | ***Type of representation*** | | |
|---|---|---|---|
| | *Isolated Cases* | *Bar Graphs* | *Tables* |
| Mean (s.d.) | 2,20 (1,09) | 2,18 (0.84) | 2,25 (0,78) |

The values of Table 2 suggest that children's performed similarly across the types of representation, when solving the problems. Graph 1 gives the distribution of the number of children's correct responses according to the type of representation used in the problem.

Graph 1 indicates that most of the children of each group could succeed in 2 of the 6 problems presented to them; 20,8% of the children of the Isolated Cases group, 23,3% of the children of the Bar graph group, and 19,2% of the Table group. The graph also indicates that 2 children of the Isolated Cases group solved correctly all the problems, and 1 child of the Bar Graph group succeeded in 5 of the 6 problems.

An ANOVA was conducted to analyse the effect of the type of representation (Isolated Cases, Bar Graph, Tables) on students' performance when solving the tasks (problems of simple proportions, problems of double proportions). Results indicate that differences on students' levels of performance are not dependent on the type of representation used in the problems, $F_{(117,2)}=0.84$, (n.s.). Thus, it seems that for the children of this level ($6^{th}$ graders) these problems are difficult.




Graph 1

In order to have an insight on children's reasoning when solving the problems presented in each condition, their explanations were analysed. Five categories were distinguished: 1) *quantification*, comprising arguments using numbers, relations or operations (e.g., "because 4 have blue eyes and 4 have dark eyes); 2) *non-quantification*, comprising justifications that do not include quantification (e.g., "because there are more students happy with CD2"); 3) *reference to total*, without any other relation to parts of variables (e.g. "Because there are more students with dark eyes, there are 9"); 4) *no explanation*, comprising the absence of justification; and 5) *inconclusive*, which includes expressions such "I don't know!", "Just because!", or any other explanation not related to the problem. Table 3 presents the number of types of arguments identified when solving the problems in each condition, for problems of simple and double proportion.

Table 3 suggests that, across the groups, the children used some type of quantification to explain their ideas and justify their answers, either in simple and double proportion problems. More research is needed to explore more children's ideas when solving these types of problems.

Table 3 - Number of types of arguments for simple proportion problems (máx. = 480 responses) and double proportion problems (máx. = 240 responses).

| | *Type of representation* | | |
|---|---|---|---|
| *Simple proportion problems* | *Isolated Case* | *Bar Graph* | *Table* |
| Quantification | 108 (22,5%) | 98 (20,4%) | 103 (21,5%) |
| Non-quantification | 22 (4,6%) | 26 (5,4%) | 10 (2,1%) |
| Reference to total | 5 (1%) | 8 (1,7%) | 12 (2,5%) |
| No explanation | 25 (5,2%) | 28 (5,8%) | 20 (4,2%) |
| Inconclusive | 0 | 0 | 15 (3,1%) |
| *Double proportion problems* | | | |
| Quantification | 61 (25,4%) | 63 (26,3%) | 61 (25,4%) |
| Non-quantification | 7 (2,9%) | 6 (2,5%) | 2 (0.8%) |
| Reference to total | 0 | 3 (1,3%) | 3 (1,3%) |
| No explanation | 0 | 0 | 3 (1,3%) |
| Inconclusive | 12 (5%) | 8 (3.3%) | 11 (4,6%) |

## DISCUSSION AND CONCLUSIONS

The findings of this study suggest that, the type of representation used to present discrete variables does not seem to have an effect on students' performance. These findings diverge from the previous results reported by Watson and Moritz (2001) who reported different levels of response to the representation of information with 8-18-years-old students. Carvalho (2008) investigated Brazilian 8$^{th}$ graders' reasoning when solving discrete variables tasks, argues that students performed better when graphs were involved than when isolated cases or tables were used to present the information. The findings of our study diverge from this idea, as differences between conditions were not identified. Children present low levels of performance despite of the type of representation of discrete variable. This suggest that the representation and interpretation of variables using graphs, tables and isolated cases should be a focus of more attention in the Portuguese classroom practices.

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