Doppler Cooling a Microsphere

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Doppler cooling the center-of-mass motion of an optically levitated microsphere via the velocity-dependent scattering force from narrow whispering gallery mode resonances is described. Light that is red detuned from the whispering gallery mode resonance can be used to damp the center-of-mass motion in a process analogous to the Doppler cooling of atoms. The scattering force is not limited by saturation but can be controlled by the incident power. Cooling times on the order of seconds are calculated for a 20 μm diameter silica microsphere trapped within optical tweezers.

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Doppler cooling has been an extremely successful technique for cooling atomic species to temperatures in the μK regime, opening up new areas in atomic [1], molecular [2], condensed matter [3], and many-body physics [4]. It has allowed the creation of atomic gases in the quantum regime, including the creation of Bose-Einstein condensates of atomic gases [5] and Fermi gases [6]. More recently, there has been considerable interest in the cavity cooling of atoms and molecules because a wider range of particles can, in principle, be cooled as no internal resonance is required [7]. A resonance is, however, required in the form of an external optical cavity, and a single atom [8] and ion [9] and atomic ensembles [10] have been cooled. For molecular and atomic species that cannot be laser cooled, cavity cooling of a trapped species appears attractive because it does not rely on the detailed internal level structure. Over the past ten years micro- and nano-optomechanical systems have been cooled down to temperatures where the quantum mechanical nature of their motion will soon be apparent [11–13]. Like cavity cooling of atoms and molecules, blueshifted photons are scattered from the cavity with respect to the incident photons, thus extracting energy. In this process, at least one degree of freedom, such as a cavity mirror or intracavity membrane, is damped or cooled by interaction with the cavity field [13–17]. An important system of this type is the cooling of the internal mechanical modes of a high-Q, whispering gallery mode (WGM) resonator formed by a toroidal or spherical structure [18]. Very recently, there have been proposals to cool optically levitated particles by using cavity cooling [19–21]. Levitation isolates the particle from the environment, increasing the prospects of cooling the center-of-mass motion to its quantum ground state. While this latter scheme is attractive for cooling nano-particles, it does not appear to be practical for larger particles, which would significantly perturb the cavity field, reducing and potentially inhibiting cooling.

In this Letter, we describe a scheme that links laser Doppler cooling and cavity cooling. It differs from cavity cooling, where the particle is cooled within the cavity, or optomechanics, where part of the cavity is cooled. Instead, we cool the whole cavity in a process analogous to Doppler cooling where the required frequency-dependent scattering force is provided by the high-Q WGM of the microsphere.

Whispering gallery modes occur in cylindrical and spherical dielectric particles which act as high-Q ($>10^8$) optical cavities for light that propagates by total internal reflection around its annulus. Figure 1 illustrates, from a geometrical optics perspective, the propagation of trapped rays of light from an incident plane wave within one plane through the microsphere. In spherical particles, the excitation of these WGM resonances increases the scattering cross section and therefore the radiation pressure on them. The spectral widths of these resonances can vary considerably depending on wavelength and the size of the sphere, but, like atomic resonances, they can have spectral widths of a few megahertz. For these modes the sphere can be seen as a high-Q spherical ringlike cavity. The effect of these resonances on the radiation pressure forces was first observed in the early work on levitating spheres by Ashkin and Dziedzic [22]. Although the coupling of light into these modes is not efficient for a free space propagating optical field, it was observed that the scattering cross section, as well as the optical force from levitation experiments, is enhanced when the incident light is resonant with

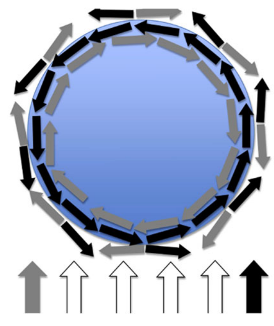

FIG. 1 (color online). The propagation of counterpropagating light rays (black and gray) from an incident plane wave within a microsphere on a whispering gallery mode resonance. The white rays do not couple into the WGM. The isotropic leakage of photons from the sphere is analogous to the spontaneous emission of photons from an atom following excitation.

these modes. Although not discussed by them, the very narrow resonances could be used to damp the motion of the spheres via the Doppler effect, which transforms a frequency-dependent force to a velocity-dependent force. It is stressed that it is the damping of the center-of-mass motion of the whole microsphere resonator structure that we consider and not the internal degrees of freedom in spherical and toroidal resonators which has previously been explored.

To illustrate the forces on a resonator we first consider light incident on an idealized Fabry-Perot resonator, as shown in Fig. 2(a), which is moving towards the beam. We consider the simple case where there are no losses in the mirrors or the medium between them and the reflectivity of each mirror is close to unity. The force on the cavity in a vacuum in the direction of propagation of an incident plane wave is calculated from the incident (P_i), reflected (P_r), and transmitted power (P_t) and is given by $F = 1/c(P_i + P_r - P_t)$, where c is the speed of light. When the incident light is not near the cavity resonance, almost all the incident light is reflected from the cavity and the force is at its maximum $F \approx 2P_i/c$. On resonance, all light is transmitted and $F \approx 0$. The force is frequency-dependent near the cavity resonance as shown in Fig. 2(a). The resonance is inverted when compared to an atomic resonance, such that through the Doppler effect more force would be felt by the resonator when it is moving towards an incident field that is blue detuned with respect to the resonance. The WGMs in a microsphere

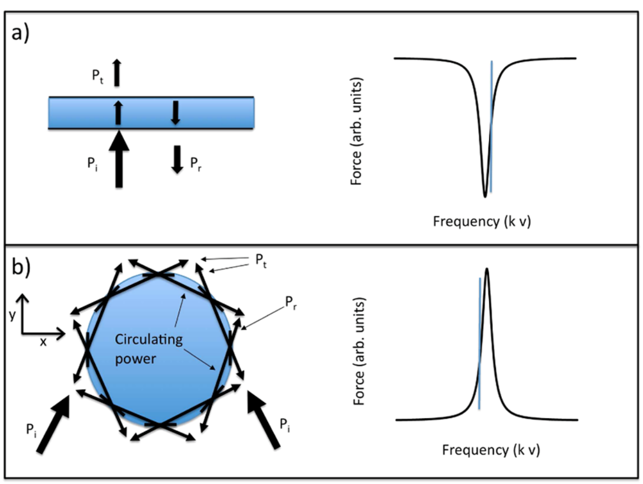

FIG. 2 (color online). (a) Incident (P_i), reflected (P_r), and transmitted (P_t) power from a Fabry-Perot cavity in vacuum. Also shown is the variation in optical force as a function of frequency near the transmission peak. The vertical line is the detuning with respect to resonance which would damp motion of the cavity towards the incident field. (b) An eight-mirrored ring cavity used to illustrate the radiation pressure force on a WGM-like ring cavity which approximates the WGM of a microsphere. Also shown is a diagram illustrating the variation in force in the y direction as a function of frequency near resonance. The vertical line represents the red detuning required to damp the motion of the resonator moving in the negative y direction.

are, however, more like the ring cavity as shown in Fig. 1. Two rays near the annulus of the sphere are coupled and counterpropagate around the sphere. To understand how this resonator is affected by radiation pressure, we consider a more simplistic ring cavity of eight mirrors shown in Fig. 2(b). Two rays of equal power from an incident field are coupled into the side mirrors. Any other rays from an optical field that are not resonant because of their angle of incidence or position on the cavity will be reflected and/or refracted, producing a force that is not strongly frequency-dependent. We again consider the incident, reflected, and transmitted light at each mirror and the resulting forces due to the change in momentum, assuming that all mirrors have the same reflectivity and transmissivity with no losses. Unlike the Fabry-Perot resonator, the transmitted rays are distributed evenly in a plane. By symmetry, the resulting forces around the ring due to the transmitted rays cancel each other out on average, just as in the case of isotropic spontaneous emission from an excited atom. The force in the y direction is then only due to the reflected and the incident field. Off resonance, all the incident light is reflected and there is no net force in the y direction. The force only acts to maintain the sphere position along the x axis. On resonance, all light is transmitted and the net forces act to push the sphere in the y direction. There is no net force along the x axis, and the sphere maintains its position along this axis. Figure 2(b) also shows a plot of the force in the y direction due to these two rays derived from the reflection of an n mirrored cavity as a function of frequency, where $F = 2/c(\vec{P}_i - \vec{P}_r) \cdot \hat{y}$ and $P_r = \frac{P_i}{c}|(r - \frac{(r-r^3)e^{-ikL}}{1-r^n e^{-ikL}})|^2$, where r is the amplitude reflection coefficient and L is the cavity length [23]. In contrast to the Fabry-Perot cavity, the force due to radiation pressure is maximized on resonance, just as in the atomic case, and the sphere acts in this respect like a large two-level atom.

The discussion above served to illustrate the basic physics behind the radiation pressure at resonance. We now calculate a more accurate force due to radiation pressure from an incident plane wave and the scattered light fields via Lorenz-Mie theory. The radiation pressure on a nonabsorbing sphere in vacuum is given by $F = \frac{P}{c} Q_{\rm rad}$, where P is the power incident on the area of the sphere. The size parameter $x = \frac{2\pi}{\lambda} a$, where a is the sphere radius and λ is the wavelength of light. The radiation pressure cross section normalized by the sphere cross section is given by $Q_{\rm rad} = Q_{\rm ext} - 4/x^2 \sum_{n=0}^{\infty} \{\frac{n(n+2)}{n+1} {\rm Re}(a_n a_{n+1}^* + b_n b_{n+1}^*) + \frac{2n+1}{n(n+1)} {\rm Re}(a_n b_n^*)\}$, where $Q_{\rm ext} = 2/x^2 \sum_{n=0}^{\infty} (2n+1){\rm Re}(a_n + b_n)$. The values for the Mie coefficients a_n and b_n can be found from standard texts [24], where n represents the nth partial wave for the a_n and b_n modes, respectively. The WGM resonances can be found by solving for ${\rm Im}(a_n) = 0$ and ${\rm Im}(b_n) = 0$. The mode order l is the root of the partial wave of mode number n, with the lowest order $l = 1$ producing the narrowest resonance for each mode number. Figure 3 is a plot of the radiation pressure force calculated for an incident power of

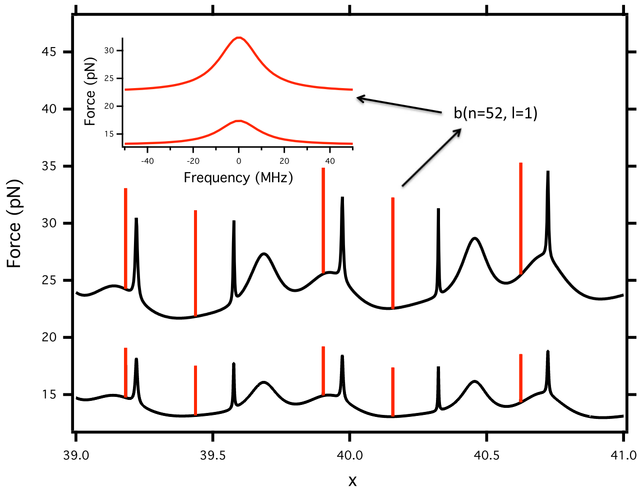

FIG. 3 (color online). Force on microsphere for size parameters $x = 39$–41. The inset is a higher resolution plot of the force as a function of frequency for the b_n ($n = 52$ and $l = 1$) resonance corresponding to $x = 40.157\,183$. In both the main and inset graphs, the lower curve is for plane wave illumination and the upper curve for Gaussian beam illumination.

10 mW on the cross-sectional area of the sphere for $x = 39$–41. A similar plot, also shown in the figure, is obtained by using generalized Lorenz-Mie theory for a nearly collimated Gaussian beam with a beam waist of 64 μm of which 10 mW power is incident on the cross section of the sphere [25]. A range of resonances in the scattering force can be observed with very different widths. The narrowest are not well resolved and are indicated by the solid vertical lines. The inset graph is a higher resolution plot on an expanded scale for the b_n ($n = 52$ and $l = 1$) resonance for $x = 40.157\,183$. This corresponds to a 10 μm radius sphere with $\lambda = 773$ nm. The scale has been converted to frequency, and the resonance has a Lorentzian profile with a half width half maximum of $\delta = 2\pi \times 11$ MHz. On resonance, the force for the plane wave or Gaussian beam case is $17/33$ pN ($Q_{\rm rad} = 0.52/0.99$), offset by a constant force of $13/24$ pN due to light that is not coupled into the WGM. The force near any narrow Mie resonance can be approximated by $F_{\rm rad} = \frac{P_0}{c} + \frac{P_p}{c} \times \frac{\delta^2}{(\omega-\omega_0)^2+\delta^2}$, where $\frac{P_0}{c} = F_0$ is constant over the frequency range considered and $\frac{P_p}{c} = F_p$ is the peak resonant force. Here ω is the frequency of the light that is detuned from the resonant frequency ω_0. The force is dependent on the velocity v of the microsphere via the Doppler effect and is given by $F_{\rm rad} = \frac{P_0}{c} + \frac{P_p}{c} \frac{\delta^2}{(\Delta - kv)^2+\delta^2}$, where Δ is the detuning from resonance of the stationary sphere. For the small velocities expected of a trapped microsphere, the force can be expanded about $v = 0$ to give $F_{\rm rad} = \frac{P_0}{c} + \frac{P_p}{c}\frac{\delta^2}{\Delta^2+\delta^2} + \frac{P_0}{c}\frac{2k\Delta\delta^2}{(\Delta^2+\delta^2)^2} v$. Unlike laser Doppler cooling of atoms, there is no saturation of the cooling force.

The dissipative force on a single sphere cooled by a 1D optical molasses is $F = \beta v$, where $\beta = \frac{4kP_p\Delta\delta^2}{c(\Delta^2+\delta^2)^2}$. The e^{-1} velocity damping time or cooling time is $\tau \approx (\beta/m)^{-1}$. Using 100 mW incident on a sphere of mass $m = 4 \times 10^{-12}$ kg and $a = 10$ μm SiO_2 sphere gives a characteristic cooling time of $\tau = 2$ s for $\delta = 2\pi \times 11$ MHz HWHM resonance or 178 ms for a $2\pi \times 1$ MHz resonance width. Like laser Doppler cooling of atoms, this will lead to a 1D cooling limit based on a balance between the average energy damping rate or cooling power expressed as $\langle P \rangle = \beta \langle v^2 \rangle$ and heating by diffusion through recoil from both the resonant (WGM) and nonresonant scattered light. In 1D where $1/2k_bT = 1/2m\langle v^2\rangle$ and $\Delta = \delta$, the Doppler cooling limit is given by $k_bT = \hbar\delta\frac{\Gamma_{\rm total}}{\Gamma_{\rm wgm}} = \hbar\delta(1 + 2\frac{F_0}{F_p}) \approx 10\hbar\delta$ [26], where $\Gamma_{\rm wgm} = \frac{F_p}{\hbar\omega}\frac{\delta^2}{(\Delta+kv)^2+\delta^2}$ is the scattering rate for the light that is coupled into the WGM and $\Gamma_{\rm total} = \frac{F_0}{\hbar\omega} + \frac{F_p}{\hbar\omega}\frac{\delta^2}{(\Delta+kv)^2+\delta^2}$ is the total scattering rate. For the 11 MHz resonance this corresponds to a temperature of 2.6 mK. The conventional Doppler limit is recovered if the light is coupled into the WGM only and $T = 261$ μK.

The Doppler limit establishes a lower temperature limit for cooling microspheres. However, WGM structures are subject to thermal, Kerr, and radiation pressure induced mechanical mode fluctuations which can affect the ability to cool the center-of-mass motion of a sphere. The relative importance of each has been shown to be strongly dependent on laser detuning with respect to the WGM and the power coupled into it [27]. Despite thermal instabilities in silica, a laser can be locked to the red side of a WGM, as required for the center-of-mass cooling described here [18,28,29]. We estimate that the predominant effects on the cooling of the center-of-mass motion are fluctuations in the force due to thermally driven mechanical motion. For a 20 μm diameter sphere, spheroidal mechanical mode oscillations occur at frequencies on the order of 100 MHz [30]. The magnitude of these fluctuations can be estimated from the temperature of the levitated sphere in vacuum. For an incident power of $P_i = 100$ mW, a temperature of $T = 1200$ K [31] is determined for an absorption coefficient of 1 dB/km and emissivity $\epsilon = 0.1$ [32]. A characteristic mean square displacement of the sphere radius can be calculated from a typical value of a mechanical mode which affects the WGM resonance frequency. This is given by $\langle d^2 \rangle = \frac{k_bT}{m_{\rm eff}\omega_m^2}$, where $m_{\rm eff}$ is the effective mass and ω_m the mode frequency. The characteristic thermal fluctuation in the WGM frequency is then determined by $\Delta_{\rm th} = \omega_{\rm wgm}\frac{\langle d^2\rangle^{1/2}}{a}$. For $\omega_m = 100$ MHz and $m_{\rm eff} = 50$ ng [29], the displacement at 1200 K is $\langle d^2\rangle^{1/2} = 1 \times 10^{-15}$ m, resulting in a WGM frequency fluctuation of $\Delta_{\rm th} = 0.17$ MHz. This displacement noise has been shown to be Lorentzian, which will act to broaden the WGM resonance, reduce the cooling rate, and increase the ultimate achievable temperature by the ratio $\Delta_{\rm th}/2\delta = 1.5 \times 10^{-2}$ for the

11 MHz resonance and approximately $\Delta_{\mathrm{th}}/2\delta = 1.5 \times 10^{-1}$ for 10 uncorrelated modes with the same characteristics [15,29]. While this would reduce the cooling rate by only 15%, it will have a greater effect on narrower WGM resonances. Mechanical oscillations will also induce sidebands onto the WGM, reducing the cooling rate and thus the ultimate temperature. This effect can be estimated by determining the fraction of power transferred to the first sideband from modulation index $\alpha = \frac{\omega_{\mathrm{wgm}}\langle d^2\rangle^{1/2}}{a\omega_m}$. The fraction of power in the first sideband for the mechanical mode considered above is $|J_1(\alpha)|^2 = -74$ dB [15]. Even for 10 similar modes, this would reduce the main WGM resonance only by -64 dB, and thus this mechanism is not considered to play an important role in determining the temperature limit. A single, red detuned, collimated beam could be used to cool a microsphere while it is optically tweezed, electrostatically trapped, or attached to a cantilever. As microspheres are often optically trapped in air, the damping due to gas viscosity and Doppler cooling is compared. The equation of motion along one axis of the trap is given by $mx''(t) = -\omega_0^2 x(t) + (\beta - \Gamma_0)x'(t) + F_0 + F_f(t)$, where $\omega_0 = \sqrt{\kappa/m}$ is the trap frequency and κ is the spring constant of the trap. The drag coefficient for damping of a spherical particle by the gas viscosity η is given by $\Gamma_0 = 6\pi\eta a$. The microsphere will also be subjected to a time-varying Langevin force $F_f(t)$. At 288 K, $\Gamma_0 = 3.4 \times 10^{-9}$ kg s^{-1}, while the maximum value of the optical damping coefficient for a 100 mW incident beam is 2 orders of magnitude less at $\beta = 1 \times 10^{-11}$ kg s^{-1}. These values are equal at a pressure of 22 mTorr, based on the drag coefficient of a sphere in the free molecular flow regime where $\Gamma_0 = (4/3 + 3\pi/16)\pi\rho\langle v\rangle a^2$, $\langle v\rangle$ is the mean velocity of the gas particles, and ρ is the gas density [33]. The optical damping is 6 orders of magnitude greater than that due to the background gas at 1×10^{-6} torr.

A method for cooling the center-of-mass motion of a microsphere by using the velocity-dependent force inherent in the whispering gallery modes was presented. This type of Doppler cooling has much in common with laser cooling of trapped atoms and ions. A 3D optical molasses could be used to cool a microsphere in all three dimensions, or a single beam could be used when the sphere is trapped by using optical or electrostatic fields or is attached to a cantilever. Such a scheme may also be used to sympathetically cool optically bound, cotrapped particles which do not possess whispering gallery mode resonances. The lowest temperatures will be achieved when light is more efficiently coupled into the WGM, for example, by evanescent fields from a fiber or prism. The effects of mechanical fluctuations of the sphere on the cooling process have been estimated to be minimal for the case we consider here. Like laser cooling, ultimate temperatures in the μK to mK range appear feasible, with cooling times on the order of seconds.

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[1] S. Chu, Science **253**, 861 (1991).
[2] K. K. Ni *et al.*, Science **322**, 231 (2008).
[3] R. Jördens *et al.*, Nature (London) **455**, 204 (2008).
[4] I. Bloch, J. Dalibard, and W. Zwerger, Rev. Mod. Phys. **80**, 885 (2008).
[5] M. H. Anderson *et al.*, Science **269**, 198 (1995).
[6] A. K. Truscott *et al.*, Science **291**, 2570 (2001).
[7] P. Horak *et al.*, Phys. Rev. Lett. **79**, 4974 (1997).
[8] P. Maunz *et al.*, Nature (London) **428**, 50 (2004).
[9] D. R. Leibrandt *et al.*, Phys. Rev. Lett. **103**, 103001 (2009).
[10] H. W. Chan, A. T. Black, and V. Vuletic, Phys. Rev. Lett. **90**, 063003 (2003).
[11] J. D. Teufel *et al.*, Phys. Rev. Lett. **101**, 197203 (2008).
[12] R. G. Knobel and A. N. Cleland, Nature (London) **424**, 291 (2003).
[13] C. H. Metzger and K. Karrai, Nature (London) **432**, 1002 (2004).
[14] T. J. Kippenberg and K. J. Vahala, Science **321**, 1172 (2008).
[15] A. Schliesser *et al.*, Nature Phys. **5**, 509 (2009).
[16] J. D. Thompson *et al.*, Nature (London) **452**, 72 (2008).
[17] D. K. Armani, T. J. Kippenberg, S. M. Spillane, and K. J. Vahala, Nature (London) **421**, 925 (2003).
[18] A. Schliesser *et al.*, Phys. Rev. Lett. **97**, 243905 (2006).
[19] P. F. Barker and M. N. Shneider, Phys. Rev. A **81**, 023826 (2010).
[20] D. E. Chang *et al.*, Proc. Natl. Acad. Sci. U.S.A. **107**, 1005 (2010).
[21] O. Romero-Isart *et al.*, New J. Phys. **12**, 033015 (2010).
[22] A. Ashkin and J. M. Dziedzic, Phys. Rev. Lett. **38**, 1351 (1977).
[23] A. E. Seigman, *Lasers* (University Science Books, Sausalito, 1986).
[24] C. F. Bohren and D. R. Huffman, *Scattering of Light by Small Particles* (Wiley, New York, 1983).
[25] T. A. Nieminen *et al.*, J. Opt. A **9**, S196 (2007).
[26] W. M. Itano and D. J. Wineland, Phys. Rev. A **25**, 35 (1982).
[27] T. J. Kippenberg and K. J. Vahala, Opt. Express **15**, 17 172 (2007).
[28] A. Schliesser *et al.*, Nature Phys. **4**, 415 (2008).
[29] Y. S. Park and H. Wang, Nature Phys. **5**, 489 (2009).
[30] R. Ma *et al.*, Opt. Lett. **32**, 2200 (2007).
[31] This is determined by equating the power absorbed by the sphere with the power emitted by blackbody radiation where $Q_{\mathrm{abs}}P_i = 4\pi a^2\sigma\epsilon T^4$ and $Q_{\mathrm{abs}} = Q_{\mathrm{ext}} - Q_{\mathrm{scat}}$, $Q_{\mathrm{scat}} = 2/x^2 \sum_{n=0}^{\infty}(2n+1)[\mathrm{Re}(a_n)^2 + \mathrm{Re}(b_n)^2]$, where σ is the Stefan-Boltzmann constant.
[32] A. Ashkin and J. M. Dziedzic, Appl. Phys. Lett. **28**, 333 (1976).
[33] P. S. Epstein, Phys. Rev. **23**, 710 (1924).