Nonlinear Resonances in δ-Kicked Bose-Einstein Condensates

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We investigate the effect of atomic interactions on δ-kicked cold atoms. We show that the clearest signature of the nonlinear dynamics is a surprisingly abrupt cutoff that appears on the leading resonances. We show that this is due to an excitation path combining both Beliaev and Landau processes, with some analogies to nonlinear self-trapping. Investigation of dynamical instability reveals further symptoms of nonlinearity such as a regime of exponential oscillations.

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Cold atoms subjected to time-periodic driving from standing waves of light provide a rich arena for the investigation of nonlinear dynamics and the quantum regime, including, for example, quantum chaos [1] and quantum ratchets or directed motion [2]. In both of these, one can highlight the role of the quantum resonance (QR) regime of cold atoms subjected to short pulses (δ-kicks) applied at regularly spaced kick period T: if T is a rational fraction of $T = 4\pi$ (the so-called Talbot time), absorption of energy by the atomic cloud peaks at a complex series of narrow resonances. These were analyzed theoretically in [3] in terms of a novel "image" classical dynamics. The interesting dynamics of the QR regime stimulated a large number of delicate experiments [4]. Further theory [5] includes proposed applications such as the realization of a quantum random walk algorithm [6].

Most recent QR experiments employed atomic Bose-Einstein condensates (BECs), albeit in a weakly interacting limit. But this suggests a new and quite different possibility: the largely unexplored regime where nonlinear dynamics, arising from the many-body nature of the BEC, combine with the δ-kicked quantum dynamics. To date, the deep understanding acquired from other areas of BEC physics on collective excitations has not been applied to the unique dynamical features of the δ-kicked atoms. In addition, the conditions for the onset of dynamical instability and exponential behavior remain poorly understood.

A few theoretical studies have considered the role of interactions at the Talbot time (or a rational multiple): in [7] dephasing and loss of resonant behavior as a function of nonlinearity parameter g was found; in [8] exponential growth of noncondensate atoms was predicted for certain parameters at half the Talbot time; in [9] significant differences were reported for resonant dynamics between attractive and repulsive interactions. Only recently, though, was it demonstrated [10] that an approach based on Bogoliubov phonon modes is essential and that certain instability borders found in [8] corresponded to parametric resonances. Parametric instabilities through periodic driving of collective modes have been investigated in several BEC studies [11]. Higher-order effects, resulting from phonon-phonon interactions, have been experimentally studied, e.g., in the context of excitation lifetimes [12] and in a nonlinear coupling between two phonon modes [13]. Beliaev and Landau (BL) couplings provide the dominant contribution to such nonlinear mode conversions; their importance was demonstrated in a series of recent experiments with BECs excited by optical Bragg pulses [14].

Here we have, for the first time, mapped the position and stability parameters of the resonances of a δ-kicked BEC with strong interactions. We show that the Talbot time no longer plays a significant role: the resonances shift away from multiples of $T = \pi$. Our key finding is that the leading resonances acquire an extraordinarily sharp cutoff at their maximum; this behavior is due to a nonlinear feedback process, originating from a combination of *both* Beliaev and Landau coupling. In other nonlinear resonances, we find novel features not previously seen, such as *exponential oscillations* and Fano-like profiles. We calculate the local Lyapunov exponents and model the Fano profiles quantitatively. The kicked BEC experiments to date correspond to effective values of the nonlinearity ($g \lesssim 0.5$) only slightly smaller than those ($g \sim 1$) needed for the effects we find.

For noninteracting single-particle dynamics, an attractive feature of the δ-kicked system, for quantum chaologists, is the extensively investigated simple quantum map which stroboscobically evolves the system from kick n to kick $n + 1$. For example, expressing our quantum state in a momentum basis, $\psi(x, t) = \sum_{l=0}^{\infty} a_l(t)|l\rangle$, we write

$$\mathbf{a}[(n+1)T] = \mathbf{U}_{g=0}(T)\mathbf{a}(t = nT), \tag{1}$$

where $\mathbf{a}(t = nT)$ is a vector with the amplitudes a_l. The atoms experience a kicking potential $V_{\text{kick}}(x, t) = K \cos x \sum_n \delta(t - nT)$. The corresponding unitary time evolution operator factors (exactly) into a free-evolution part U_{free} and a kick part U_{kick}, i.e.,

$$\mathbf{U}_{g=0}(T) = U_{\text{free}} U_{\text{kick}} = e^{-i(\hat{L}^2 T/2)} e^{-iK\cos x}. \tag{2}$$

The first exponential term represents the free evolution under the kinetic energy operator in some units where the atom mass $M = 1$ and $\hbar = 1$. Clearly, since the atomic momentum $l = 0, \pm 1, \pm 2, \ldots$, is quantized in units of the recoil momentum, $T = 4\pi$ implies $U_{\rm free} = 1$ for *all* momentum states, so consecutive kicks add in phase. The result is a phase-matched absorption of energy from the field yielding ballistic transport. In contrast, the nonresonantly kicked atoms experience diffusive growth in energy.

We consider a uniform, tightly confined, effectively 1D BEC with periodic boundary conditions. The corresponding field-free, many-body Hamiltonian, in a momentum representation, is

$$H = \sum_l \epsilon_l \hat{a}_l^\dagger \hat{a}_l + \frac{g_{\rm 1D}}{2L} \sum_{l,j,m} \hat{a}_l^\dagger \hat{a}_j^\dagger \hat{a}_m \hat{a}_{j+l-m}, \qquad (3)$$

where $\epsilon_k = \hbar^2 k^2/(2M)$ and L is the BEC size. The 1D interaction constant $g_{\rm 1D} \simeq 2\hbar a_s \omega_\perp$ depends on the atomic scattering length a_s and the transverse trap frequency $\omega_\perp$. In the presence of linearized perturbations around the macroscopically occupied $k = 0$ ground state, H may be diagonalized up to quadratic order by the Bogoliubov transformation $\hat{a}_k = u_k \hat{b}_k - v_k \hat{b}_{-k}^\dagger$, for $k \neq 0$, with $u_k - v_k = 1/(u_k + v_k) = \sqrt{\hbar\omega_k}$, where $\hbar\omega_k = [\epsilon_k(\epsilon_k + 2g_{\rm 1D}n)]^{1/2}$ and $n = N/L$ is the atom density. Then we can expand H in the orders of $\sqrt{N}$: $H = {\rm const} + H^{(2)} + H^{(3)} + H^{(4)}$, where $H^{(2)} = \sum_{k\neq 0} \hbar\omega_k \hat{b}_k^\dagger \hat{b}_k$ and the cubic part $H^{(3)}$ describes the leading order contribution to the interactions between phonons:

$$H^{(3)} = \kappa \sum_{q,p} (\Gamma_{qp} \hat{b}_q^\dagger \hat{b}_p^\dagger \hat{b}_{-q-p}^\dagger + \Delta_{qp} \hat{b}_p^\dagger \hat{b}_q^\dagger \hat{b}_{q+p} + {\rm H.c.}), \quad (4)$$

where q, p, $(q + p) \neq 0$ and $\kappa = \sqrt{N} g_{\rm 1D}/L$. The coefficient Γ_{qp} represents a process in which three phonons are created or annihilated and Δ_{qp} is a process in which one phonon with momentum $q + p$ decays into two phonons with momenta q and p (Beliaev term) or its inverse in which two merge to produce a third phonon (Landau term). Since the energy of the excitations around the ground state is positive, the processes described by Γ_{qp} are suppressed by energy conservation. In terms of the Bogoliubov amplitudes, we obtain

$$\begin{aligned} \Delta_{qp} &= u_p u_q u_{q+p} + 2u_p v_q (v_{q+p} - u_{q+p}) - v_p v_q v_{q+p}, \\ \Gamma_{qp} &= u_q v_p v_{q+p} - u_p u_q v_{q+p}. \end{aligned} \qquad (5)$$

We assume macroscopic occupancy for the low-lying modes of interest and that phase fluctuations of the 1D condensate may be neglected. Hence we will treat the Bogoliubov mode amplitudes classically. The δ-kicked map (2), in the presence of the quadratic Hamiltonian $H^{(2)}$ alone, requires only a straightforward modification to its free-evolution part:

$$\mathbf{U}_g(T) = \mathcal{B}^{-1} \mathbf{e}^{-\mathbf{i}\omega\mathbf{T}} \mathcal{I} \mathcal{B} U_{\rm kick}, \qquad (6)$$

where $\mathcal{B}$ denotes the Bogoliubov transform for each l. In terms of momentum amplitudes, the classical mode amplitudes are simply given by $b_l = u_l a_l + v_l a_{-l}^*$. The term $\exp(-\mathbf{i}\omega\mathbf{T})$ is a row vector with the mode frequencies, and $\mathcal{I}$ is the identity. The eigenvalues of the nonunitary matrix $\mathbf{U}_g(T)$ indicate dynamical stability. It is easy to prove that they generally come in quartets λ, $1/\lambda$, λ^*, $1/\lambda^*$. Then $|\lambda_{\max}| > 1$ (where $\lambda_{\max}$ is the largest eigenvalue and the local Lyapunov exponent) imply dynamical instability and exponential growth in the relevant modes (at least for short times).

We compare Eq. (6) to the numerical solutions of the 1D Gross-Pitaevski equation (GPE) (here rescaled to dimensionless units [15]) for an initially uniform BEC:

$$i\partial_t \psi(x,t) = (-\tfrac{1}{2}\partial_x^2 + g|\psi(x,t)|^2 + K\cos x F_t)\psi(x,t), \quad (7)$$

where $F_t = \sum_n \delta(t - nT)$. We study $g \simeq 0$–10, realistic with the current experiments [14,16], for $K < 1$, which allows only the creation of discrete low-lying phonon excitations.

Figure 1(a) shows GPE numerics; it maps the average BEC response, $\langle 1 - |a_0(t)|^2 \rangle_t = \frac{1}{t} \sum_{n=1}^t 1 - |a_0(nT)|^2$ averaged over the first t kicks, for $K = 0.5$. At $g \simeq 0$, the Talbot-time resonance at $T = 4\pi$ is perfectly symmetrical (as are the fractional resonances on either side). For $g \simeq 0$–1 an asymmetry develops, due here to the lifting of the degeneracy between the lowest modes: the main resonance splits into mode 1 resonance $\omega_1 T \simeq 2\pi$ and mode 2 resonance $\omega_2 T \simeq 8\pi$. Mode 2 resonance rapidly decays away as the gap $\omega_2 - \omega_1$ increases: direct coupling $\langle 0|U_{\rm kick}|2\rangle \sim J_2(K)$ with the condensate is small. A slight asymmetry was noted in the GPE numerics in Ref. [7] for the Talbot-time resonance at $g \simeq 0.1$, which we now attribute to this regime.

The most striking feature is the very sharp "cutoff" appearing at $g \gtrsim 1$ (and still exists even at $g = 20$). It is also evident in the second harmonic of the resonance (upper half of the graph). Figure 1(b) shows that even a grid of 100 points per unit of T (each star representing a GPE simulation for 30 kicks and $g = 5$) is too coarse to resolve the cutoff: there are no stars on the order-of-magnitude drop seen at $T \simeq 6.65$. In comparison, such a grid could resolve the famously narrow $g = 0$ Talbot-time resonance. The dotted line shows the Bogoliubov map (6) here incorrectly produces a symmetric resonance and fails to shift the resonance away from $\omega_1 T = 2\pi$. Hence we need to include the neglected phonon-phonon interaction terms between the kicks from $H^{(3)}$.

The free-ringing $\exp(-\mathbf{i}\omega\mathbf{T})$ part of the map [Eq. (6)] must be replaced by a set coupled equations following from the Heisenberg equations $d\hat{b}_k/dt = -i\omega_k \hat{b}_k - i[\hat{b}_k, H^{(3)}/\hbar]$, for $k \neq 0$, where we replace $\hat{b}_k$ by the rescaled classical amplitudes $b_k \rightarrow \langle \hat{b}_k \rangle/\sqrt{N}$. Figure 1(b) shows that by including only the lowest four excitations ($k = \pm 2, \pm 1$) one obtains excellent agreement with the GPE : even the cutoff is accurately reproduced (for regimes where depletion of the ground state is small ($\lesssim 10\%$).

We simplify further by transforming (by symmetry $b_k = b_{-k}$) to the basis $|l\rangle \rightarrow \frac{1}{\sqrt{2}}(|l\rangle + |-l\rangle)$, for $l \neq 0$, so that

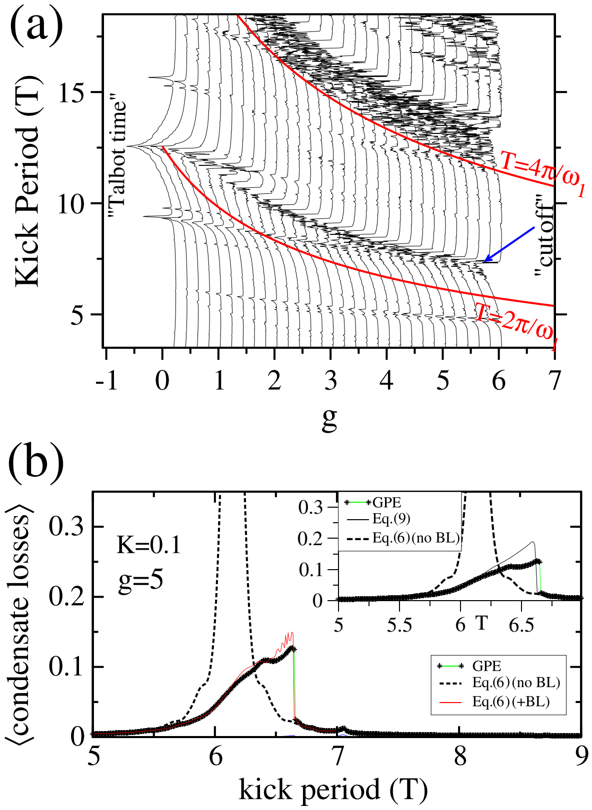

FIG. 1 (color online). (a) Losses from the unperturbed condensate $\langle 1 - |a_0(t)|^2 \rangle_t$ averaged over time, calculated from the GPE, for kick strength $K = 0.5$. The BEC resonance evolves from the well-known Talbot-time QR (at $T = 4\pi M$, $M = 1, 2, \ldots$, for $g = 0$) and develops an abrupt cutoff for any $g \gtrsim 1$. (b) Comparison between GPE numerics (points shown as black stars) and the model. The grid of black stars at intervals $\Delta T = 10^{-2}$ strikingly demonstrates the sharpness of the cutoff at $T \simeq 6.65$: the drop occurs over an interval $\Delta T \ll 10^{-2}$, so contains no stars at all. The quantum map *excluding* BL processes, incorrectly gives a symmetric, unshifted resonance (dashed line). The model *including* BL coupling reproduces perfectly not only the shift but also the sharp cutoff. The simple Eq. (9) also provides reasonable agreement (inset) with the GPE.

$\langle l|U_{\rm kick}|n\rangle = U_{nl} = i^{l-n}J_{n-l}(K) + i^{-(l+n)}J_{l+n}(K)$ if n, $l > 0$, but $U_{0l} = \sqrt{2}i^{-l}J_l(K)$ and $U_{00} = J_0(K)$. We then need only two coupled equations to accurately reproduce the cutoff. Moreover, neglecting the small Γ_{qp} terms, we obtain (from the free-ringing plus BL terms), for the dynamics *between the kicks*,

$$\begin{aligned} \dot{b}_1 &= -i[\omega_1 + 2C_1 \mathrm{Re}(b_2)]b_1 + 2C_2 b_1^* b_2, \\ \dot{b}_2 &= -i\omega_2 b_2 - i[C_1|b_1|^2 + C_2 b_1^2], \end{aligned} \tag{8}$$

where $C_1 = \bar{\kappa}(\Delta_{-1,2} + \Delta_{2,-1})$, $C_2 = \bar{\kappa}\Delta_{1,1}$, and $\bar{\kappa} = g/(2\sqrt{2}\pi)$ (note $\Delta_{-1,2} = \Delta_{1,-2}$, etc.). We can even obtain a reasonable cutoff if we set the smaller term $C_2 = 0$. Hence, the main effect of mode 2 is simply to provide a phase shift on ω_1. If we integrate b_2 for $b_2(0) = 0$ while keeping $|b_1|$ constant, $b_2(t) \simeq (e^{i\omega_2 t} - 1)C_1|b_1|^2/\omega_2$, we can approximate the *full map* by

$$\dot{b}_1 \simeq -i\omega_1 b_1 + iA\sin^2(\omega_2 t/2)|b_1|^2 b_1 - i\mathcal{R}(t), \tag{9}$$

where $A = 4C_1^2/\omega_2$, $\mathcal{R}(t) = \sqrt{2}J_1(K)(u_1 - v_1)F(t)$. Figure 1(b) (inset) shows that we still get reasonable agreement with the full model for weak $K = 0.1$. In this regime, $|b_0(t)|^2 \simeq 1 - |b_1(t)|^2$. Writing $b_1 = \rho e^{i\theta}$, a phase space analysis in the ρ, θ plane reveals a separatrix curve which appears at the cutoff parameters and bounds the value of ρ. We may thus also describe the mechanism: if the $0 \to 1$ transition is initially only slightly off resonant, the kicking starts populating mode 1 effectively; with population in mode 1, BL processes begin to populate the empty modes $l = \pm 2$. This mode 2 population provides a phase shift bringing mode 1 closer into resonance; the nonlinear feedback accelerates the growth in b_1 which in turns brings mode 1 further into resonance. However, if the $0 \to 1$ transition is initially too far off resonant (beyond the cutoff), the nonlinear feedback cycle cannot start. An analogous model of two-mode dynamics with continuous driving rather than δ kicks is reminiscent of a macroscopic self-trapping effect in a BEC in a double-well potential [17], but in the present work the cutoff is *considerably sharper.*

Figure 2 maps the average probability of mode 2 (averaged over 100 kicks) for $K = 0.5$. The right-hand side maps regions of dynamical instability $|\lambda_{\max}| > 1$. We analyze dynamical stability by mapping the eigenvalues of $\mathbf{U}_g(T)$ for all the resonances of the lowest three excited modes. We divide the resonances into (i) the "linear" family $L(n, l)$ (i.e., those which evolve from the linear case and converge at $g = 0$ to a rational fraction of the Talbot time. The resonance in Fig. 1(a) is the $L(1, 1)$ (first resonance of mode $l = 1$). (ii) The "nonlinear" resonances N_n and ν_n which vanish in the absence of interactions, at $g = 0$; the N_n correspond to $(\omega_1 + \omega_2)T \simeq 2\pi n$, while ν_n are somewhat analogous to "counterpropagating mode" resonances found in modulated traps [11] and imply $2\omega_n T \simeq 2\pi$. Contrary to the suggestion of [10] where no Liapunov exponents were calculated, we find that none of the $L(n, l)$ resonances have any $|\lambda| > 1$. They are all stable, including $L(1, 1)$, and are by far the strongest of all, but counterintuitively, they are associated with a much stronger BEC response, even after a very long time (100 kicks), than the nonlinear resonances N_n and ν_n that are unstable.

The reason for this is clear from Fig. 2(b). The mode 2 populations from the GPE (for both N_1 and N_3) grow exponentially for a finite time, then decay exponentially; the inset shows this behavior on a log scale. The map with BL corrections here is quantitative for only the first 10–20 kicks, so we cannot model this behavior from $H^{(3)}$ alone. But it is tempting to attribute it to regimes where either the λ, λ^* or the $1/\lambda$, $1/\lambda^*$ eigenvalues are predominant. The cluster of interacting resonances N_2, ν_1, and ν_2 lies in a region of very low ground state depletion (it lies in the

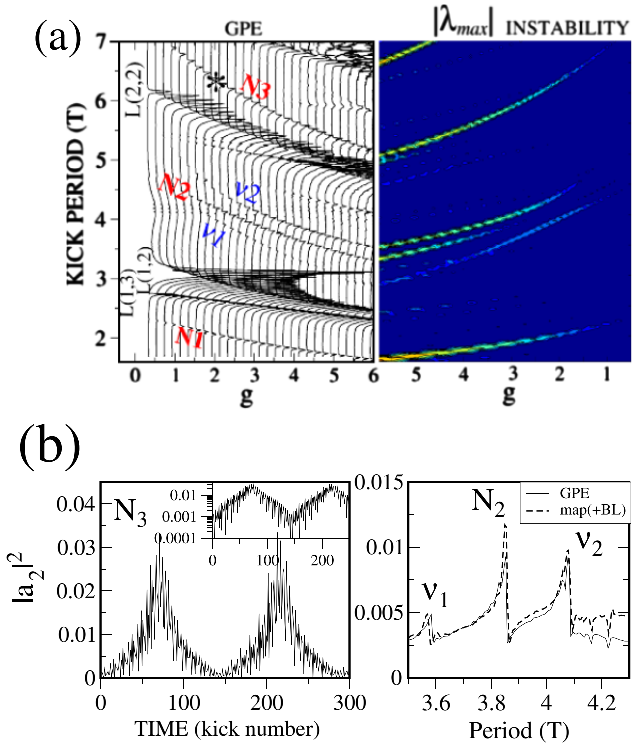

FIG. 2 (color online). Panel (a): (left) Probability for mode 2 averaged over 100 kicks for $K = 0.5$; (right) $|\lambda_{max}|$, largest eigenvalue of U_g. Bright regions denote $|\lambda| > 1$ and hence exponential behavior (dynamical instability). The unstable N_1, $N_2, \ldots, N_n$ series of nonlinear resonances (which only appear for $g \gtrsim 1$) correspond to $(\omega_1 + \omega_2)T \simeq 2n\pi$. The asterisk denotes the position of the "instability border" found by [8], which we thus show is due to N_3. The L series are resonances which evolved from partial or full resonances of the Talbot-time at $g = 0$. They are stable, yet in spite of this they are *much* stronger than the exponential resonances. Panel (b): (Left) Mode 2 probability of N_3, near the asterisk on the left-hand side of panel (a). $T = 6.12$, $g = 2.5$. The exponential growth persists for only a finite time; it is then replaced by exponential decay, leading to *exponential oscillations* (log scale shown in inset). (Right) Mode 2 near N_2. The cluster of three overlapping resonances have "Fano-like" profiles. These are well reproduced by Eq. (6) corrected with $H^{(3)}$ in Eq. (4), including only the seven lowest modes.

minimum of the dominant $L(1, 1)$ tail). The map (6), corrected by BL terms $H^{(3)}$ in Eq. (4) (with the lowest seven modes), reproduces very well the characteristic Fano-like profiles seen in all three peaks, while the uncorrected map produces only symmetric resonance profiles. To our knowledge neither the exponential oscillations nor the Fano profiles are seen in comparable nonequilibrium BEC dynamics. While not fully understood, they indicate that the δ-kicked systems offer new and experimentally accessible BEC dynamics.

While the Talbot-time $g = 0$ resonances have been proposed for metrological applications (e.g., for measurement of gravity), the similarly sharp BEC cutoff suggests analogous possibilities as it provides a sharp excitation threshold. A rotating BEC in a large ring at threshold kicking frequencies, e.g., could provide a sensitive probe of rotation for small changes in the resonance frequency between different angular momentum (vortex) states.

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