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Supplemental tables and R codes for the paper "Asymptotic cumulants of the minimum phi-divergence estimator for categorical data under possible model misspecification"

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This article supplements Ogasawara (2019a, b).

Part 1 gives numerical results under correct model specification in Tables S2.1 to S2.13. In the tables, ".00" indicates a rounded value of zero up to the second place while "0" indicates an exactly zero value.

Part 2 gives R codes and examples of running the programs.

References

Ogasawara, H. (2019a). Asymptotic cumulants of the minimum phi-divergence estimator for categorical data under possible model misspecification. To appear in *Communications in Statistics – Theory and Methods*.

Ogasawara, H. (2019b). Supplement to the paper "Asymptotic cumulants of the minimum phi-divergence estimator for categorical data under possible model misspecification". To appear in *Economic Review (Otaru University of Commerce), 70* (2 & 3). http://www.res.otaru-uc.ac.jp/~emt-hogasa/, https://barrel.repo.nii.ac.jp/.

Part 1. Numerical results under correct model specification

Table S2.1. True probabilities under correct model misspecification

Case	$\boldsymbol{\tau}'$			
A:	The genetics of plants (Fisher, 1970)			
	.6000	.1500	.1500	.1000
B:	3-category truncated Poisson variate (Bishop et al., 1975)			
	.3679	.3679	.2642	
C:	4-category truncated Poisson variate			
	.2231	.3347	.2510	.1912
D:	4-category redundant truncated Poisson variate			
	.2231	.3347	.2510	.1912

Table S2.2. Simulated and theoretical standard errors multiplied by $n^{1/2}$ for the MϕEs when models are true: $\beta_2^{1/2}$

Case	Parameter	Sim.(n) (50)	Sim.(n) (200)	Sim.(n) (800)	Sim.(n) (50)	Sim.(n) (200)	Sim.(n) (800)	Th.
		$\lambda = 0$ (G^2, ML)			$\lambda = -1$ (GM^2)			
A	θ	.801	.803	.801	.838	.812	.803	.800
B	θ	1.078	1.068	1.068	1.100	1.073	1.069	1.066
C	θ	1.286	1.271	1.278	1.339	1.283	1.280	1.272
D	θ_1	1.399	1.399	1.397	1.439	1.408	1.399	1.395
	θ_2	.418	.424	.422	.424	.425	.423	.423
		$\lambda = -2$ (Neyman)			$\lambda = 2/3$ (C-R)			
A	θ	.890	.826	.807	.792	.801	.801	.800
B	θ	1.452	1.078	1.070	1.066	1.064	1.067	1.066
C	θ	2.614	1.298	1.284	1.263	1.265	1.276	1.272
D	θ_1	1.499	1.419	1.402	1.381	1.394	1.396	1.395
	θ_2	.432	.427	.423	.416	.423	.422	.423
		$\lambda = 1$ (X^2, Pearson)			$\lambda = 2$			
A	θ	.790	.800	.801	.791	.801	.801	.800
B	θ	1.059	1.063	1.067	1.041	1.058	1.066	1.066
C	θ	1.254	1.262	1.275	1.237	1.257	1.274	1.272
D	θ_1	1.374	1.392	1.396	1.360	1.388	1.395	1.395
	θ_2	.414	.423	.422	.412	.422	.422	.423

Note. n = the number of observations, Sim. = simulated value, Th. = theoretical value = $\beta_2^{1/2}$, G^2 = the log-likelihood ratio statistic, GM^2 = the modified log-likelihood ratio statistic, Neyman = Neyman's statistic, C-R = the Cressie-Read statistic, X^2 = Pearson's statistic.

Table S2.2. (continued)

Case	n: 50 Z	50 NC	200 Z	200 NC	800 Z	800 NC
A	579	95	0	0	0	0
B	0	10	0	0	0	0
C	3	9	0	0	0	0
D	3	3251	0	2	0	0

Note. n = the number of observations, Z = the number of deleted cases with zero frequenc(ies), NC = the number of deleted cases due to non-convergence.

Table S2.3. Simulated and theoretical ratios of the higher- and lower-order asymptotic standard errors for the MϕEs when models are true

Case	n	50		200		800		50		200		800	
Parameter		Sim.	Th.	Sim.	Th.	Sim.	Th.	Sim.	Th.	Sim.	Th.	Sim.	Th.
		$\lambda = 0$ (G^2, ML)						$\lambda = -1$ (GM^2)					
A	θ	1.001	1.015	1.004	1.004	1.002	1.001	1.047	1.053	1.015	1.014	1.004	1.003
B	θ	1.012	.981	1.002	.995	1.002	.999	1.032	.999	1.006	1.000	1.003	1.000
C	θ	1.011	.975	.999	.994	1.004	.998	1.052	1.010	1.008	1.003	1.006	1.001
D	θ_1	1.002	.986	1.003	.996	1.001	.999	1.031	1.010	1.009	1.003	1.003	1.001
	θ_2	.990	.990	1.003	.998	.999	.999	1.003	1.006	1.006	1.002	1.000	1.000
		$\lambda = -2$ (Neyman)						$\lambda = 2/3$ (C-R)					
A	θ	1.113	1.113	1.032	1.029	1.008	1.007	.990	1.004	1.001	1.001	1.001	1.000
B	θ	1.362	1.017	1.011	1.004	1.004	1.001	1.000	.969	.999	.992	1.001	.998
C	θ	2.055	1.052	1.020	1.013	1.009	1.003	.992	.955	.994	.989	1.003	.997
D	θ_1	1.075	1.039	1.017	1.010	1.005	1.002	.990	.972	.999	.993	1.001	.998
	θ_2	1.022	1.024	1.010	1.006	1.001	1.002	.983	.981	1.001	.995	.999	.999
		$\lambda = 1$ (X^2, Pearson)						$\lambda = 2$					
A	θ	.988	1.002	1.000	1.001	1.001	1.000	.989	1.014	1.002	1.004	1.002	1.001
B	θ	.994	.962	.997	.991	1.001	.998	.977	.943	.993	.986	1.000	.997
C	θ	.985	.947	.992	.987	1.003	.997	.973	.925	.988	.982	1.001	.996
D	θ_1	.985	.965	.998	.991	1.000	.998	.975	.950	.995	.988	.999	.997
	θ_2	.980	.976	1.000	.994	.998	.999	.973	.963	.998	.991	.998	.998

Note. n = the number of observations, Sim. = simulated value = SD/ASE, SD = the standard deviation from simulation, ASE = $n^{-1/2}\bar{\beta}_2^{1/2}$, Th. = theoretical value = HASE/ASE, HASE=$(n^{-1}\beta_2 + n^{-2}\beta_{\Delta 2})^{1/2}$, G^2 = the log-likelihood ratio statistic, GM^2 = the modified log-likelihood ratio statistic, Neyman = Neyman's statistic, C-R = the Cressie-Read statistic, X^2 = Pearson's statistic.

Table S2.4. Simulated and theoretical biases multiplied by n for the MϕEs when models are true: β_1

Case	Parameter	Sim.(n) (50)	Sim.(n) (200)	Sim.(n) (800)	Th.	Sim.(n) (50)	Sim.(n) (200)	Sim.(n) (800)	Th.
		$\lambda = 0$ (G^2, ML)				$\lambda = -1$ (GM^2)			
A	θ	.09	-.03	.12	0	.30	.16	.30	.18
B	θ	.22	.24	.09	0	.23	.25	.09	0
C	θ	.17	.24	.06	0	.19	.26	.08	.02
D	θ_1	.89	.59	.35	0	1.45	1.08	.83	.48
	θ_2	-.01	.06	-.01	-.06	.09	.15	.08	.03
		$\lambda = -2$ (Neyman)				$\lambda = 2/3$ (C-R)			
A	θ	.52	.35	.48	.36	-.01	-.14	-.00	-.12
B	θ	.27	.25	.09	0	.22	.24	.09	0
C	θ	.36	.26	.09	.03	.16	.23	.05	-.01
D	θ_1	2.08	1.59	1.32	.96	.56	.27	.03	-.32
	θ_2	.20	.24	.17	.12	-.07	-.00	-.01	-.12
		$\lambda = 1$ (X^2, Pearson)				$\lambda = 2$			
A	θ	-.04	-.19	-.06	-.18	-.11	-.32	-.22	-.36
B	θ	.22	.24	.09	0	.21	.24	.09	0
C	θ	.15	.23	.05	-.02	.16	.22	.04	-.03
D	θ_1	.40	.11	-.12	-.48	-.03	-.36	-.60	-.96
	θ_2	-.10	-.03	-.10	-.15	-.18	-.12	-.19	-.24

Note. n = the number of observations, Sim. = simulated value, Th. = theoretical value = β_1, G^2 = the log-likelihood ratio statistic, GM^2 = the modified log-likelihood ratio statistic, Neyman = Neyman's statistic, C-R = the Cressie-Read statistic, X^2 = Pearson's statistic. The "0" indicates an exactly zero value.

Table S2.5. Simulated and theoretical skewnesses multiplied by $n^{1/2}$ for the MϕEs when models are true: $\beta_3 / \beta_2^{3/2}$

Case	Parameter	Sim.(n) (50)	(200)	(800)	Th.	Sim.(n) (50)	(200)	(800)	Th.
		$\lambda = 0$ (G^2, ML)				$\lambda = -1$ (GM^2)			
A	θ	.37	.05	-.30	.11	.60	.11	-.30	.11
B	θ	1.78	1.68	1.95	.42	1.81	1.69	1.95	.42
C	θ	1.30	1.33	1.29	.43	1.33	1.33	1.31	.43
D	θ_1	2.53	2.12	2.10	-.12	2.74	2.16	2.11	-.12
	θ_2	1.47	1.06	1.24	.98	1.55	1.07	1.24	.98
		$\lambda = -2$ (Neyman)				$\lambda = 2/3$ (C-R)			
A	θ	1.18	.30	-.26	.11	.42	.07	-.29	.11
B	θ	317	1.69	1.95	.42	1.78	1.68	1.95	.42
C	θ	343	1.31	1.32	.43	1.28	1.33	1.29	.43
D	θ_1	3.38	2.28	2.14	-.12	2.53	2.11	2.09	-.12
	θ_2	1.71	1.09	1.25	.98	1.46	1.07	1.24	.98
		$\lambda = 1$ (X^2, Pearson)				$\lambda = 2$			
A	θ	.45	.08	-.29	.11	.52	.13	-.27	.11
B	θ	1.78	1.68	1.95	.42	1.78	1.68	1.95	.42
C	θ	1.27	1.33	1.28	.43	1.19	1.32	1.27	.43
D	θ_1	2.55	2.12	2.09	-.12	2.63	2.14	2.09	-.12
	θ_2	1.46	1.07	1.24	.98	1.48	1.08	1.24	.98

Note. n = the number of observations, Sim. = simulated value, Th. = theoretical value = $\beta_3 / \beta_2^{3/2}$, G^2 = the log-likelihood ratio statistic, GM^2 = the modified log-likelihood ratio statistic, Neyman = Neyman's statistic, C-R = the Cressie-Read statistic, X^2 = Pearson's statistic.

Table S2.6. Simulated and theoretical kurtoses multiplied by n for the MϕEs when models are true: β_4 / β_2^2

Case	Parameter	Sim.(n)				Sim.(n)			
		(50)	(200)	(800)	Th.	(50)	(200)	(800)	Th.
		$\lambda = 0$ (G^2, ML)				$\lambda = -1$ (GM^2)			
A	θ	-8.1	-4.1	13.9	-.4	-8.3	-3.3	13.5	-.4
B	θ	7.2	5.9	18.6	-8.5	7.8	6.2	18.6	-8.5
C	θ	3.9	7.0	6.2	-10.4	5.8	7.0	6.9	-10.4
D	θ_1	11.7	12.2	6.5	-7.7	15.3	12.9	7.8	-10.3
	θ_2	-2.9	3.9	14.4	-.2	-2.1	3.7	14.7	-2.4
		$\lambda = -2$ (Neyman)				$\lambda = 2/3$ (C-R)			
A	θ	-2.8	.0	13.3	-.4	-7.1	-4.2	14.2	-.4
B	θ	2.5e5	6.9	18.6	-8.5	7.4	5.8	18.6	-8.5
C	θ	1.7e5	8.1	7.7	-10.4	4.0	7.2	5.8	-10.4
D	θ_1	32.4	15.2	9.4	-12.8	11.7	12.0	5.9	-6.0
	θ_2	.5	3.7	15.0	-4.6	-3.0	4.0	14.2	1.3
		$\lambda = 1$ (X^2, Pearson)				$\lambda = 2$			
A	θ	-6.8	-4.2	14.3	-.4	-6.4	-4.1	14.5	-.4
B	θ	7.7	5.9	18.6	-8.5	9.1	6.1	18.7	-8.5
C	θ	4.0	7.3	5.7	-10.4	3.1	7.4	5.1	-10.4
D	θ_1	11.9	12.0	5.5	-5.1	12.2	12.0	4.7	-2.5
	θ_2	-3.0	4.1	14.1	2.0	-2.8	4.3	13.8	4.2

Note. n = the number of observations, Sim. = simulated value, Th. = theoretical value = β_4 / β_2^2, G^2 = the log-likelihood ratio statistic, GM^2 = the modified log-likelihood ratio statistic, Neyman = Neyman's statistic, C-R = the Cressie-Read statistic, X^2 = Pearson's statistic, $x\,\mathrm{e}\,y = x10^y$.

Table S2.7. Simulated and squared biases and added higher-order asymptotic biases multiplied by n^2 for the MϕEs when models are true: β_1^2 and $\beta_{\Delta 2}$

Case	Parameter	β_1^2 Sim.(n) (50)	(200)	(800)	Th.	$\beta_{\Delta 2}$ Sim.(n) (50)	(200)	(800)	Th.= $\beta_{\Delta 2}$
		λ = -2 (Neyman)							
A	θ	.27	.12	.23	.13	7.6	8.4	8.4	7.6
B	θ	.07	.06	.01	0	48.6	5.2	7.7	1.9
C	θ	.13	.07	.01	.00	261	13.2	23.7	8.6
D	θ_1	4.31	2.54	1.73	.91	15.0	13.2	14.3	7.7
	θ_2	.04	.06	.03	.01	.4	.7	.2	.4
		λ = 2/3 (C-R)							
A	θ	.00	.02	.00	.01	-.7	.2	1.1	.2
B	θ	.05	.06	.01	0	-.0	-.6	2.1	-3.5
C	θ	.02	.05	.00	.00	-1.2	-3.7	7.9	-7.1
D	θ_1	.31	.07	.00	.10	-2.0	-.6	1.9	-5.4
	θ_2	.00	.00	.00	.01	-.3	.1	-.4	-.3
		λ = 2							
A	θ	.01	.10	.05	.13	-.7	.4	1.7	.9
B	θ	.05	.06	.01	0	-2.6	-3.4	-.7	-6.2
C	θ	.02	.05	.00	.00	-4.4	-7.8	3.6	-11.6
D	θ_1	.00	.13	.36	.91	-4.8	-4.1	-1.7	-9.6
	θ_2	.03	.02	.04	.06	-.5	-.1	-.6	-.6

Note. n = the number of observations, Sim. = simulated value, Th. = theoretical value = β_1^2 or $\beta_{\Delta 2}$, Neyman = Neyman's statistic, C-R = the Cressie-Read statistic. The "0" indicates an exactly zero value.

Table S2.8. Simulated and theoretical standard errors of the studentized MϕEs when models are true: $\beta_2^{1/2}$'

Case	Parameter	Sim.(n) (50)	(200)	(800)	Th.	Sim.(n) (50)	(200)	(800)	Th.
		$\lambda = 0$ (G^2, ML)				$\lambda = -1$ (GM^2)			
A	θ	1.079	1.021	1.005	1	1.093	1.025	1.006	1
B	θ	1.033	1.006	1.003	1	1.033	1.006	1.003	1
C	θ	1.035	1.004	1.006	1	1.038	1.005	1.006	1
D	θ_1	.991	1.002	1.002	1	.987	1.002	1.002	1
	θ_2	1.036	1.015	1.002	1	1.034	1.015	1.002	1
		$\lambda = -2$ (Neyman)				$\lambda = 2/3$ (C-R)			
A	θ	1.074	1.021	1.005	1	1.058	1.015	1.004	1
B	θ	1.859	1.006	1.003	1	1.030	1.006	1.003	1
C	θ	1.4e6	1.003	1.006	1	1.024	1.002	1.005	1
D	θ_1	.974	1.000	1.001	1	.989	1.002	1.002	1
	θ_2	1.030	1.014	1.002	1	1.035	1.015	1.002	1
		$\lambda = 1$ (X^2, Pearson)				$\lambda = 2$			
A	θ	1.049	1.011	1.003	1	1.028	.999	1.000	1
B	θ	1.027	1.006	1.003	1	1.014	1.005	1.003	1
C	θ	1.018	1.001	1.005	1	1.005	.995	1.004	1
D	θ_1	.989	1.002	1.002	1	.990	1.000	1.001	1
	θ_2	1.034	1.015	1.002	1	1.033	1.014	1.002	1

Note. n = the number of observations, Sim. = simulated value, Th. = theoretical value = $\beta_2^{1/2}$' =1, G^2 = the log-likelihood ratio statistic, GM^2 = the modified log-likelihood ratio statistic, Neyman = Neyman's statistic, C-R = the Cressie-Read statistic, X^2 = Pearson's statistic, $x\,\mathrm{e}\,y = x10^y$.

Table S2.9. Simulated and theoretical biases multiplied by $n^{1/2}$ for the studentized MϕEs when models are true: $\beta_1{}'$

Case	Parameter	Sim.(n) (50)	(200)	(800)	Th.	Sim.(n) (50)	(200)	(800)	Th.
		$\lambda = 0$ (G^2, ML)				$\lambda = -1$ (GM^2)			
A	θ	.03	-.10	.09	-.06	.26	.13	.32	.17
B	θ	-.48	-.43	-.57	-.21	-.48	-.43	-.57	-.21
C	θ	-.40	-.32	-.46	-.21	-.39	-.31	-.45	-.20
D	θ_1	-.03	-.26	-.43	.06	.31	.09	-.08	.40
	θ_2	-.60	-.44	-.58	-.64	-.38	-.22	-.36	-.42
		$\lambda = -2$ (Neyman)				$\lambda = 2/3$ (C-R)			
A	θ	.35	.31	.53	.39	-.13	-.25	-.06	-.20
B	θ	-.42	-.43	-.57	-.21	-.48	-.43	-.57	-.21
C	θ	3.2e4	-.30	-.44	-.19	-.39	-.32	-.47	-.22
D	θ_1	.59	.41	.25	.74	-.29	-.49	-.66	-.17
	θ_2	-.17	-.01	-.15	-.21	-.75	-.58	-.72	-.78
		$\lambda = 1$ (X^2, Pearson)				$\lambda = 2$			
A	θ	-.18	-.32	-.13	-.28	-.30	-.49	-.33	-.50
B	θ	-.47	-.43	-.57	-.21	-.46	-.43	-.57	-.21
C	θ	-.38	-.32	-.47	-.23	-.33	-.31	-.48	-.24
D	θ_1	-.42	-.61	-.77	-.28	-.79	-.97	-1.12	-.63
	θ_2	-.83	-.66	-.79	-.85	-1.04	-.87	-1.01	-1.07

Note. n = the number of observations, Sim. = simulated value, Th. = theoretical value = $\beta_1{}'$, G^2 = the log-likelihood ratio statistic, GM^2 = the modified log-likelihood ratio statistic, Neyman = Neyman's statistic, C-R = the Cressie-Read statistic, X^2 = Pearson's statistic, $x\,\mathrm{e}\,y = x10^y$.

Table S2.10. Simulated and theoretical skewnesses multiplied by $n^{1/2}$ for the studentized MϕEs when models are true: $\beta_3{}'$

Case	Parameter	Sim.(n) (50)	(200)	(800)	Th.	Sim.(n) (50)	(200)	(800)	Th.
		$\lambda = 0$ (G^2, ML)				$\lambda = -1$ (GM^2)			
A	θ	-.37	-.42	-.63	-.22	-.36	-.39	-.62	-.22
B	θ	-2.42	-2.31	-2.06	-.84	-2.43	-2.31	-2.06	-.84
C	θ	-1.97	-1.77	-1.79	-.85	-1.97	-1.77	-1.78	-.85
D	θ_1	-1.57	-2.01	-1.98	.24	-1.59	-2.01	-1.98	.24
	θ_2	-1.98	-2.46	-2.15	-1.96	-1.98	-2.47	-2.15	-1.96
		$\lambda = -2$ (Neyman)				$\lambda = 2/3$ (C-R)			
A	θ	-.71	-.53	-.66	-.22	-.62	-.48	-.64	-.22
B	θ	747	-2.31	-2.06	-.84	-2.36	-2.31	-2.05	-.84
C	θ	2236	-1.73	-1.77	-.85	-1.86	-1.75	-1.80	-.85
D	θ_1	-1.98	-2.10	-2.00	.24	-1.74	-2.06	-2.00	.24
	θ_2	-2.25	-2.51	-2.16	-1.96	-2.03	-2.47	-2.15	-1.96
		$\lambda = 1$ (X^2, Pearson)				$\lambda = 2$			
A	θ	-.72	-.51	-.64	-.22	-1.00	-.59	-.65	-.22
B	θ	-2.32	-2.31	-2.05	-.84	-2.18	-2.29	-2.05	-.84
C	θ	-1.82	-1.74	-1.80	-.85	-1.74	-1.68	-1.79	-.85
D	θ_1	-1.85	-2.09	-2.01	.24	-2.27	-2.20	-2.04	.24
	θ_2	-2.06	-2.47	-2.15	-1.96	-2.16	-2.50	-2.16	-1.96

Note. n = the number of observations, Sim. = simulated value, Th. = theoretical value = $\beta_3{}'$, G^2 = the log-likelihood ratio statistic, GM^2 = the modified log-likelihood ratio statistic, Neyman = Neyman's statistic, C-R = the Cressie-Read statistic, X^2 = Pearson's statistic.

Table S2.11. Proportions of a population value below the one-sided confidence intervals when models are true: n = 50 and $\lambda = -2$ (Neyman's statistic)

Case	Parameter	Method	.0050	.0250	.1000	.5000	.9000	.9750	.9950
			Nominal values						
A	θ		Z = 48, NC = 10						
		NF	.0240	.0590	.1546	.5299	.8794	.9507	.9815
		NR	.0054	.0207	.0858	.5299	.9142	.9660	.9840
		C-F	.0085	.0073	.0494	.4984	.9413	.9830	.9934
		Hall	.0003	.0064	.0487	.4986	.9439	.9844	.9949
B	θ		Z = 0, NC = 0						
		NF	.0026	.0198	.1003	.4992	.8655	.9455	.9828
		NR	.0039	.0233	.0936	.4992	.8753	.9508	.9821
		C-F	.0035	.0175	.0827	.5011	.9001	.9670	.9846
		Hall	.0031	.0171	.0827	.5011	.9001	.9670	.9846
C	θ		Z = 0, NC = 0						
		NF	.0050	.0300	.1106	.5039	.8596	.9415	.9794
		NR	.0035	.0189	.0825	.5039	.8940	.9621	.9867
		C-F	.0023	.0163	.0777	.5056	.9179	.9744	.9898
		Hall	.0019	.0155	.0776	.5061	.9183	.9752	.9900
D	θ_1		Z = 0, NC = 367						
		NF	.0062	.0325	.1218	.5458	.9097	.9711	.9937
		NR	.0014	.0169	.1043	.5458	.9116	.9707	.9909
		C-F	.0023	.0104	.0684	.5115	.9079	.9722	.9918
		Hall	.0011	.0096	.0684	.5115	.9082	.9722	.9919
	θ_2								
		NF	.0041	.0215	.0992	.5116	.8844	.9609	.9859
		NR	.0019	.0131	.0931	.5116	.8893	.9637	.9857
		C-F	.0021	.0131	.0896	.5020	.9098	.9798	.9954
		Hall	.0020	.0128	.0896	.5020	.9108	.9831	.9970

Note. NF = the normal approximation by the Fisher information matrix, NR = the normal approximation by the robust ASE estimate, C-F = the Cornish-Fisher expansion, Hall = Hall's (1992) monotonic cubic transformation, Z = the number of deleted cases with zero frequenc(ies), NC = the number of deleted case(s) due to non-convergence.

Table S2.12. Proportions of a population value below the one-sided confidence intervals when models are true: n = 50 and $\lambda = 2/3$ (the Cressie-Read statistic)

Case	Parameter	Method	.0050	.0250	.1000	.5000	.9000	.9750	.9950
A	θ		Z = 48, NC = 10						
		NF	.0105	.0317	.1125	.5048	.8833	.9606	.9873
		NR	.0127	.0380	.1096	.5048	.8827	.9553	.9831
		C-F	.0048	.0274	.1074	.5161	.9053	.9701	.9871
		Hall	.0042	.0240	.1074	.5161	.9054	.9847	.9893
B	θ		Z = 0, NC = 0						
		NF	.0016	.0147	.0798	.4992	.8872	.9604	.9875
		NR	.0016	.0169	.0831	.4992	.8831	.9584	.9852
		C-F	.0021	.0189	.0841	.4992	.8949	.9628	.9902
		Hall	.0021	.0189	.0841	.4992	.8949	.9629	.9904
C	θ		Z = 0, NC = 0						
		NF	.0016	.0173	.0879	.5053	.8894	.9642	.9899
		NR	.0025	.0192	.0895	.5053	.8863	.9592	.9874
		C-F	.0034	.0214	.1004	.5058	.8899	.9646	.9899
		Hall	.0034	.0213	.1004	.5058	.8904	.9653	.9906
D	θ_1		Z = 0, NC = 367						
		NF	.0022	.0156	.0832	.5030	.9000	.9689	.9932
		NR	.0026	.0168	.0856	.5030	.8944	.9660	.9930
		C-F	.0006	.0138	.0855	.5196	.8947	.9710	.9949
		Hall	.0006	.0122	.0855	.5196	.8947	.9710	.9949
	θ_2								
		NF	.0020	.0154	.0766	.4715	.8728	.9554	.9843
		NR	.0025	.0162	.0790	.4715	.8746	.9544	.9836
		C-F	.0038	.0217	.0987	.5001	.9045	.9800	.9955
		Hall	.0037	.0217	.0987	.5001	.9049	.9833	.9988

Note. NF = the normal approximation by the Fisher information matrix, NR = the normal approximation by the robust ASE estimate, C-F = the Cornish-Fisher expansion, Hall = Hall's (1992) monotonic cubic transformation, Z = the number of deleted cases with zero frequenc(ies), NC = the number of deleted cases due to non-convergence.

Table S2.13. Proportions of a population value below the one-sided confidence intervals when models are true: n = 50 and $\lambda = 2$

Case / Parameter / Method	.0050	.0250	.1000	.5000	.9000	.9750	.9950
A θ	Z = 48, NC = 10						
NF	.0091	.0347	.1067	.4945	.8771	.9589	.9862
NR	.0132	.0318	.0973	.4945	.8893	.9514	.9827
C-F	.0084	.0290	.0906	.5167	.8966	.9566	.9885
Hall	.0052	.0238	.0892	.5168	.9004	.9606	.9901
B θ	Z = 0, NC = 0						
NF	.0015	.0145	.0778	.4992	.8895	.9619	.9883
NR	.0024	.0191	.0832	.4992	.8823	.9584	.9827
C-F	.0041	.0234	.0932	.4992	.8754	.9468	.9730
Hall	.0041	.0234	.0932	.4992	.8754	.9472	.9744
C θ	Z = 0, NC = 0						
NF	.0010	.0149	.0832	.5041	.8939	.9660	.9908
NR	.0048	.0224	.0871	.5041	.8947	.9601	.9932
C-F	.0058	.0272	.1005	.5056	.8903	.9642	.9845
Hall	.0051	.0271	.1005	.5065	.8904	.9674	.9859
D θ_1	Z = 0, NC = 367						
NF	.0022	.0118	.0699	.4779	.8908	.9643	.9928
NR	.0014	.0133	.0757	.4779	.8900	.9538	.9895
C-F	.0013	.0156	.0832	.5164	.8996	.9681	.9899
Hall	.0012	.0154	.0832	.5164	.8996	.9681	.9899
θ_2							
NF	.0018	.0135	.0706	.4510	.8690	.9531	.9829
NR	.0021	.0134	.0675	.4510	.8533	.9564	.9834
C-F	.0046	.0236	.0993	.5109	.9132	.9683	.9932
Hall	.0046	.0232	.0993	.5109	.9137	.9725	.9947

Note. NF = the normal approximation by the Fisher information matrix, NR = the normal approximation by the robust ASE estimate, C-F = the Cornish-Fisher expansion, Hall = Hall's (1992) monotonic cubic transformation, Z = the number of deleted cases with zero frequenc(ies), NC = the number of deleted cases due to non-convergence.

Part 2. R codes and examples of running the programs

In Part 2, two examples are shown. In Example 1, log-linear models in a $2\times2\times2$ contingency table are analyzed using the program "mpe3a.R". In Example 2, log-linear models including the linear-by-linear model in a two-way contingency table are dealt with by the program "mpe2a.R". In these programs the function "mpese" is used.

Example 1.
The program "mpe3a.R"

```
#
#  Robust asymptotic standard errors of the minimizing phi-divergence
#  estimators (MPEs) for log-linear models in a 2x2x2 table
#  under possible model misspecification
#
#     <mpe3a.R>                                   April 16, 2018
#
#  Use as:   R prompt >source('file_name').
#  The results will be given in 'r.u'.
#

starttime=proc.time()
sink ('r.u')
print(date(),quote=F)

###### function mpese
mpese=function(n,pr,al,w,qo,iq,ik,im,imax){
if(im==1){
print(' ',quote=F)
print('methods (1:ML,2:GM^2,3-8:Lambda=(-2,-0.5,0.5,2/3,1,2))',quote=F)
print('.............................................................',quote=F)
}
delta=1e-8
q=qo
inv=0
se=rep(0,iq)
ser=rep(0,iq)

for (it in 1:(imax+1)){
```

```
if(it>imax){
print(paste('!!!! no convergence, the number of iterations=',
           it,sep=''),quote=F)
return(list(qhat=q,it=it,inv=inv,se=se,ser=ser))
}

pi=exp(w%*%q)
pi=pi[,1,drop=T]   # vectorized for later use
                  # as 'diag(pi,nrow=length(pi))'
pi=pi/sum(pi)
wp=t(w)%*%pi
wp=wp[,1,drop=T]   # vectorized for later use
                  # as 'wp%o%wp+...'
pq=matrix(0,ik,iq)
for (i in 1:ik){
for (j in 1:iq){
pq[i,j]=(w[i,j]-wp[j])*pi[i]
};}

pq2=array(0,c(ik,iq,iq))
for (i in 1:ik){
for (j1 in 1:iq){
for (j2 in 1:iq){
pq2[i,j1,j2]=(w[i,j1]-wp[j1])*pq[i,j2]
for (i1 in 1:ik){
pq2[i,j1,j2]=pq2[i,j1,j2]-w[i1,j1]*pq[i1,j2]*pi[i]
};};};}

g=rep(0,iq)
h=matrix(0,iq,iq)

for (i in 1:iq){
if(im==1)g[i]=-sum(pr/pi*pq[,i])    # note the minus sign
if(im==2)g[i]=sum((log(pi/pr)+1)*pq[,i])
if(im>=3 && im<=8)g[i]=sum(-al[im-2]*(pr/pi)^(al[im-2]+1)*pq[,i])

for (j in 1:i){
if(im==1)h[i,j]=-sum(-pr/pi^2*pq[,i]*pq[,j]+pr/pi*pq2[,i,j]) # note the
                                                     # minus sign
```

```
if(im==2)h[i,j]=sum(1/pi*pq[,i]*pq[,j]+(log(pi/pr)+1)*pq2[,i,j])
if(im>=3 && im<=8)h[i,j]=sum(al[im-2]*(al[im-2]+1)*pr^(al[im-2]+1)
/pi^(al[im-2]+2)*pq[,i]*pq[,j]-al[im-2]*(pr/pi)^(al[im-2]+1)*pq2[,i,j])

h[j,i]=h[i,j]
};}

aaa=eigen(h,symmetric=T)[['values']]   # not -h
if(min(abs(aaa))<delta){
print(paste('!!!! The following Hessian is singular!, iteration=',
      it,sep=''),quote=F)
print(h)
print('eigenvalues',quote=F)
print(aaa)
inv=1
return(list(qhat=q,it=it,inv=inv,se=se,ser=ser))
}

h1=h
h=solve(h)

if(max(abs(g))<delta){

fi=-wp%o%wp+t(w)%*%diag(pi,nrow=length(pi))%*%w
fi=solve(fi)
se=sqrt(fi[row(fi)==col(fi)]/n)

gp=matrix(0,iq,ik)

for (i in 1:iq){
for (k in 1:ik){
if(im==1)gp[i,k]=-1/pi[k]*pq[k,i] # note the minus sign
if(im==2)gp[i,k]=-1/pr[k]*pq[k,i]
if(im>=3 && im<=8)gp[i,k]=-al[im-2]*(al[im-2]+1)*
          pr[k]^al[im-2]/pi[k]^(al[im-2]+1)*pq[k,i]
};}

qp=-h%*%gp
qpi=qp%*%pr
```

```
qpi=qpi[,1,drop=T]  # vectorized

fir=-qpi%o%qpi+qp%*%diag(pr,nrow=length(pr))%*%t(qp)
ser=sqrt(fir[row(fir)==col(fir)]/n)

q=as.vector(q) # vectorized for ease of presentation

return(list(qhat=q,it=it,inv=inv,se=se,ser=ser))

} # end of if(max(abs(g))<delta)

iout=0
if(iout==1){
print(paste('###### iteration=',it,sep=''),quote=F)
print('q(parameters),g(gradients)',quote=F)
print(q)
print(g)
print('Hessian',quote=F)
print(h1)
}

q=q-h%*%g
q=as.vector(q) # vectorized for ease of presentation

} # end of it-loop
} # end of function mpese
######

al=c(-2,-0.5,0.5,2/3,1,2)  # powers in power divergences
                         # except those for G^2(ML) and GM^2

cat('¥n
Robust asymptotic standard errors of the minimizing phi-divergence ¥n
estimators (MPEs) for log-linear models in a 2x2x2 table ¥n
under possible model misspecification ¥n
for a survey on alcohol(A), cigarette(C) and marijuana(M) use ¥n
for US high school seniors using the data ¥n
by Agresti (2013, p.346) ¥n')
```

```
ik=8 # the number of categories
fn=c(911,538,44,456,3,43,2,279)  # a vectorized contingency table
                               # of Agresti (2013, p.346)
n=sum(fn)
pr=fn/n
ft=array(0,c(2,2,2),dimnames=list(c('A-Yes','A-No'),
              c('C-Yes','C-No'),c('M-Yes','M-No')))

ijk=0
for (i in 1:2){;for (j in 1:2){;for (k in 1:2){
ijk=ijk+1
ft[i,j,k]=fn[ijk] };};}

print(' ',quote=F)
print(paste('total frequency=',n,sep=''),quote=F)
print('frequencies',quote=F)
print(ft)
print('percentages',quote=F)
print(round(ft/n*100,1))

ws1=c(1,1,1,1,1,1,1, # The vectorized design matrix
      1,1,0,1,0,0,0, # for the saturated model
      1,0,1,0,1,0,0,
      1,0,0,0,0,0,0,
      0,1,1,0,0,1,0,
      0,1,0,0,0,0,0,
      0,0,1,0,0,0,0,
      0,0,0,0,0,0,0)

ws=matrix(data=ws1,8,7,byrow=T)
print('The K(the number  of catogories) by q(the number of parameters)',quote=F)
print('design matrix for the saturated model',quote=F)
print(ws)

an=c('Model 1 (A,C,M)','Model 2 (A,CM)','Model 3 (C,AM)','Model 4 (M,AC)',
     'Model 5 (AC,AM)','Model 6 (AC,CM)','Model 7 (AM,CM)',
     'Model 8 (AC,AM,CM)','Model 9 (ACM)(saturated model)')

wd1=c(1,1,1,0,0,0,0,  # Model 1
```

```
      1,1,1,0,0,1,0,  # Model 2
      1,1,1,0,1,0,0,  # Model 3
      1,1,1,1,0,0,0,  # Model 4
      1,1,1,1,1,0,0,  # Model 5
      1,1,1,1,0,1,0,  # Model 6
      1,1,1,0,1,1,0,  # Model 7
      1,1,1,1,1,1,0,  # Model 8
      1,1,1,1,1,1,1)  # Model 9
wd=matrix(wd1,9,7,byrow=T,dimnames=list(paste('Model ',1:9,sep=''),
         paste('p-use ',1:7,sep='')))
print(' ',quote=F)
print('p-use=the pattern of prameter use (1:use,0:not use)',quote=F)
print(wd)

imax=200

for (mdl in 1:9){
print(' ',quote=F)
print('===========================================================',quote=F)
print(an[mdl],quote=F)
iw=wd[mdl,]
iq=sum(iw)
print(paste('number of parameters=',iq,sep=''),qoute=F)
print('the pattern of parameter use (1;use,0:not use)',quote=F)
print(wd[mdl,])

iw=iw*1:7
iw=iw[iw>0]
w=ws[,iw]
print('the (reduced) design matrix for the model',quote=F)
print(w)

qo=rep(0,iq)  # arbitrary initial values for MLEs

for (im in 1:8){
print('............................................................',quote=F)

abc=mpese(n,pr,al,w,qo,iq,ik,im,imax)
```

```
print(paste('estimation method=',im,sep=''),quote=F)

itr=abc[['it']]
inv=abc[['inv']]
q=abc[['qhat']]
se=abc[['se']]
ser=abc[['ser']]

print('parameter estimates',quote=F)
print(q)
print('information-based ASEs',quote=F)
print(se)
print('robust ASEs(RASEs)',quote=F)
print(ser)
print('RASE/ASE',quote=F)
print(round(ser/se,3))

if(im==1)qo=q
}  # end of im-loop
}  # end of mdl-loop

print(proc.time()-starttime,quote=F)
print(date(),quote=F)
sink()
```

Results using the program “mpe3a.R”

```
[1] Thu Dec 06 10:42:19 2018


Robust asymptotic standard errors of the minimizing phi-divergence

estimators (MPEs) for log-linear models in a 2x2x2 table

under possible model misspecification

for a survey on alcohol(A), cigarette(C) and marijuana(M) use

for US high school seniors using the data

by Agresti (2013, p.346)
[1]
[1] total frequency=2276
[1] frequencies
, , M-Yes

      C-Yes C-No
A-Yes   911   44
A-No      3    2

, , M-No

      C-Yes C-No
A-Yes   538  456
A-No     43  279

[1] percentages
, , M-Yes

      C-Yes C-No
A-Yes  40.0  1.9
A-No    0.1  0.1

, , M-No

      C-Yes C-No
A-Yes  23.6 20.0
A-No    1.9 12.3

[1] The K(the number  of catogories) by q(the number of parameters)
[1] design matrix for the saturated model
     [,1] [,2] [,3] [,4] [,5] [,6] [,7]
[1,]    1    1    1    1    1    1    1
```

```
[2,]    1    1    0    1    0    0    0
[3,]    1    0    1    0    1    0    0
[4,]    1    0    0    0    0    0    0
[5,]    0    1    1    0    0    1    0
[6,]    0    1    0    0    0    0    0
[7,]    0    0    1    0    0    0    0
[8,]    0    0    0    0    0    0    0
[1] 
[1] p-use=the pattern of prameter use (1:use,0:not use)
        p-use 1 p-use 2 p-use 3 p-use 4 p-use 5 p-use 6 p-use 7
Model 1       1       1       1       0       0       0       0
Model 2       1       1       1       0       0       1       0
Model 3       1       1       1       0       1       0       0
Model 4       1       1       1       1       0       0       0
Model 5       1       1       1       1       1       0       0
Model 6       1       1       1       1       0       1       0
Model 7       1       1       1       0       1       1       0
Model 8       1       1       1       1       1       1       0
Model 9       1       1       1       1       1       1       1
[1] 
[1] ============================================================
[1] Model 1 (A,C,M)
[1] "number of parameters=3"
[1] the pattern of parameter use (1;use,0:not use)
p-use 1 p-use 2 p-use 3 p-use 4 p-use 5 p-use 6 p-use 7 
      1       1       1       0       0       0       0 
[1] the (reduced) design matrix for the model
     [,1] [,2] [,3]
[1,]    1    1    1
[2,]    1    1    0
[3,]    1    0    1
[4,]    1    0    0
[5,]    0    1    1
[6,]    0    1    0
[7,]    0    0    1
[8,]    0    0    0
[1] ............................................................
[1] 
[1] methods (1:ML,2:GM^2,3-8:Lambda=(-2,-0.5,0.5,2/3,1,2))
[1] ............................................................
[1] estimation method=1
[1] parameter estimates
[1]  1.7851115  0.6493063 -0.3154188
[1] information-based ASEs
[1] 0.05975941 0.04415095 0.04244461
[1] robust ASEs(RASEs)
[1] 0.05975941 0.04415095 0.04244461
[1] RASE/ASE
[1] 1 1 1
[1] ............................................................
```

```
[1] estimation method=2
[1] parameter estimates
[1]  3.62070082  1.47526026 -0.08688913
[1] information-based ASEs
[1] 0.13155570 0.05385366 0.04196175
[1] robust ASEs(RASEs)
[1] 0.28268430 0.11488380 0.08171046
[1] RASE/ASE
[1] 2.149 2.133 1.947
[1] ..........................................................
[1] !!!! The following Hessian is singular!, iteration=6
             [,1]          [,2]          [,3]
[1,] 9.491213e-34  2.794404e-18  2.794404e-18
[2,] 2.794404e-18 -5.547459e-16  1.432653e-16
[3,] 2.794404e-18  1.432653e-16 2.096498e-201
[1] eigenvalues
[1]  3.513827e-17 -3.171054e-19 -5.895671e-16
[1] estimation method=3
[1] parameter estimates
[1]  42.02769  36.50708 232.45397
[1] information-based ASEs
[1] 0 0 0
[1] robust ASEs(RASEs)
[1] 0 0 0
[1] RASE/ASE
[1] NaN NaN NaN
[1] ..........................................................
[1] estimation method=4
[1] parameter estimates
[1]  2.4459052  0.9183479 -0.3430608
[1] information-based ASEs
[1] 0.07737919 0.04641983 0.04254043
[1] robust ASEs(RASEs)
[1] 0.09906469 0.06414934 0.05833772
[1] RASE/ASE
[1] 1.280 1.382 1.371
[1] ..........................................................
[1] estimation method=5
[1] parameter estimates
[1]  1.4513710  0.5302778 -0.2114558
[1] information-based ASEs
[1] 0.05345371 0.04340437 0.04215671
[1] robust ASEs(RASEs)
[1] 0.05608016 0.03719939 0.03748794
[1] RASE/ASE
[1] 1.049 0.857 0.889
[1] ..........................................................
[1] estimation method=6
[1] parameter estimates
[1]  1.3764022  0.5053881 -0.1793792
```

```
[1] information-based ASEs
[1] 0.05224785 0.04326777 0.04209091
[1] robust ASEs(RASEs)
[1] 0.05582415 0.03591637 0.03668945
[1] RASE/ASE
[1] 1.068 0.830 0.872
[1] ..........................................................
[1] estimation method=7
[1] parameter estimates
[1]  1.2585712  0.4680354 -0.1230820
[1] information-based ASEs
[1] 0.05050034 0.04307535 0.04200159
[1] robust ASEs(RASEs)
[1] 0.05564573 0.03415946 0.03560498
[1] RASE/ASE
[1] 1.102 0.793 0.848
[1] ..........................................................
[1] estimation method=8
[1] parameter estimates
[1]  1.046233324  0.407716979 -0.005356147
[1] information-based ASEs
[1] 0.04779021 0.04279631 0.04192233
[1] robust ASEs(RASEs)
[1] 0.05589151 0.03184063 0.03395304
[1] RASE/ASE
[1] 1.170 0.744 0.810
[1]
[1] ===========================================================
[1] Model 2 (A,CM)
[1] "number of parameters=4"
[1] the pattern of parameter use (1;use,0:not use)
p-use 1 p-use 2 p-use 3 p-use 4 p-use 5 p-use 6 p-use 7
      1       1       1       0       0       1       0
[1] the (reduced) design matrix for the model
     [,1] [,2] [,3] [,4]
[1,]    1    1    1    1
[2,]    1    1    0    0
[3,]    1    0    1    0
[4,]    1    0    0    0
[5,]    0    1    1    1
[6,]    0    1    0    0
[7,]    0    0    1    0
[8,]    0    0    0    0
[1] ..........................................................
[1]
[1] methods (1:ML,2:GM^2,3-8:Lambda=(-2,-0.5,0.5,2/3,1,2))
[1] ..........................................................
[1] estimation method=1
[1] parameter estimates
[1]  1.7851115 -0.2351197 -2.7712291  3.2243089
```

```
[1] information-based ASEs
[1] 0.05975941 0.05551319 0.15198577 0.16098117
[1] robust ASEs(RASEs)
[1] 0.05975941 0.05551319 0.15198577 0.16098117
[1] RASE/ASE
[1] 1 1 1 1
[1] ...........................................................
[1] estimation method=2
[1] parameter estimates
[1]  3.40224851  0.09977517 -2.42208118  2.84599037
[1] information-based ASEs
[1] 0.11869392 0.05700172 0.14465901 0.15323224
[1] robust ASEs(RASEs)
[1] 0.28544261 0.06544265 0.15681141 0.16392242
[1] RASE/ASE
[1] 2.405 1.148 1.084 1.070
[1] ...........................................................
[1] !!!! The following Hessian is singular!, iteration=3
             [,1]          [,2]          [,3]          [,4]
[1,] -0.0696625479  0.0001115558 -0.0019400632 -0.0019400632
[2,]  0.0001115558 -0.0046929124 -0.0001389803 -0.0001389803
[3,] -0.0019400632 -0.0001389803 -0.0764020430 -0.0764020430
[4,] -0.0019400632 -0.0001389803 -0.0764020430 -0.0764020430
[1] eigenvalues
[1]  3.252607e-19 -4.692447e-03 -6.957232e-02 -1.528948e-01
[1] estimation method=3
[1] parameter estimates
[1]   4.211925   7.435819 -62.668405  58.062392
[1] information-based ASEs
[1] 0 0 0 0
[1] robust ASEs(RASEs)
[1] 0 0 0 0
[1] RASE/ASE
[1] NaN NaN NaN NaN
[1] ...........................................................
[1] estimation method=4
[1] parameter estimates
[1]  2.3010654 -0.1056044 -2.6500693  3.0413680
[1] information-based ASEs
[1] 0.07286796 0.05533851 0.14825942 0.15710751
[1] robust ASEs(RASEs)
[1] 0.08363237 0.05943445 0.15441332 0.16271760
[1] RASE/ASE
[1] 1.148 1.074 1.042 1.036
[1] ...........................................................
[1] estimation method=5
[1] parameter estimates
[1]  1.5332992 -0.2750535 -2.7966206  3.2919142
[1] information-based ASEs
[1] 0.05485746 0.05587158 0.15307516 0.16213350
```

```
[1] robust ASEs(RASEs)
[1] 0.05592170 0.05299905 0.15184018 0.16106101
[1] RASE/ASE
[1] 1.019 0.949 0.992 0.993
[1] ...........................................................
[1] estimation method=6
[1] parameter estimates
[1]  1.4757195 -0.2787266 -2.7965153  3.2999774
[1] information-based ASEs
[1] 0.05386142 0.05595584 0.15313835 0.16220649
[1] robust ASEs(RASEs)
[1] 0.05524171 0.05238165 0.15205009 0.16127635
[1] RASE/ASE
[1] 1.026 0.936 0.993 0.994
[1] ...........................................................
[1] estimation method=7
[1] parameter estimates
[1]  1.3832318 -0.2787648 -2.7909086  3.3050369
[1] information-based ASEs
[1] 0.05235464 0.05608109 0.15307457 0.16215053
[1] robust ASEs(RASEs)
[1] 0.05422613 0.05143172 0.15244168 0.16167401
[1] RASE/ASE
[1] 1.036 0.917 0.996 0.997
[1] ...........................................................
[1] estimation method=8
[1] parameter estimates
[1]  1.2075357 -0.2562814 -2.7614993  3.2862846
[1] information-based ASEs
[1] 0.04979820 0.05627964 0.15247260 0.16152366
[1] robust ASEs(RASEs)
[1] 0.05213885 0.04987298 0.15274596 0.16206301
[1] RASE/ASE
[1] 1.047 0.886 1.002 1.003
[1]
[1] ===========================================================
[1] Model 3 (C,AM)
[1] "number of parameters=4"
[1] the pattern of parameter use (1;use,0:not use)
p-use 1 p-use 2 p-use 3 p-use 4 p-use 5 p-use 6 p-use 7
      1       1       1       0       1       0       0
[1] the (reduced) design matrix for the model
     [,1] [,2] [,3] [,4]
[1,]    1    1    1    1
[2,]    1    1    0    0
[3,]    1    0    1    1
[4,]    1    0    0    0
[5,]    0    1    1    0
[6,]    0    1    0    0
[7,]    0    0    1    0
```

```
[8,]    0    0    0    0
[1] ..........................................................
[1]
[1] methods (1:ML,2:GM^2,3-8:Lambda=(-2,-0.5,0.5,2/3,1,2))
[1] ..........................................................
[1] estimation method=1
[1] parameter estimates
[1]  1.1271857  0.6493063 -4.1651136  4.1250877
[1] information-based ASEs
[1] 0.06412196 0.04415095 0.45067236 0.45294452
[1] robust ASEs(RASEs)
[1] 0.06412196 0.04415095 0.45067235 0.45294450
[1] RASE/ASE
[1] 1 1 1 1
[1] ..........................................................
[1] estimation method=2
[1] parameter estimates
[1]  2.106684  1.347147 -3.132103  3.067634
[1] information-based ASEs
[1] 0.09145557 0.05179729 0.42236054 0.42457389
[1] robust ASEs(RASEs)
[1] 0.1387445 0.1129191 0.4999101 0.4984764
[1] RASE/ASE
[1] 1.517 2.180 1.184 1.174
[1] ..........................................................
[1] !!!! The following Hessian is singular!, iteration=8
              [,1]          [,2]          [,3]          [,4]
[1,] -3.981748e-07 -9.525347e-20  3.140589e-07 -8.411584e-08
[2,] -9.525347e-20  2.184960e+02 -1.263444e-14 -1.572626e-19
[3,]  3.140589e-07 -1.263444e-14 -3.468130e-07 -9.613071e-17
[4,] -8.411584e-08 -1.572626e-19 -9.613071e-17 -8.411584e-08
[1] eigenvalues
[1]  2.184960e+02 -9.958067e-09 -1.252174e-07 -6.939282e-07
[1] estimation method=3
[1] parameter estimates
[1]   2.2605520   0.4054651  24.0481213 -25.3655070
[1] information-based ASEs
[1] 0 0 0 0
[1] robust ASEs(RASEs)
[1] 0 0 0 0
[1] RASE/ASE
[1] NaN NaN NaN NaN
[1] ..........................................................
[1] estimation method=4
[1] parameter estimates
[1]  1.5103048  0.8576832 -3.7655976  3.6203349
[1] information-based ASEs
[1] 0.07126124 0.04583647 0.42872928 0.43103321
[1] robust ASEs(RASEs)
[1] 0.07919432 0.05908070 0.45573468 0.45743781
```

```
[1] RASE/ASE
[1] 1.111 1.289 1.063 1.061
[1] .............................................................
[1] estimation method=5
[1] parameter estimates
[1]  0.9148656  0.5475605 -4.3852091  4.4504034
[1] information-based ASEs
[1] 0.06163574 0.04350317 0.46946715 0.47170316
[1] robust ASEs(RASEs)
[1] 0.06432889 0.03929165 0.45086202 0.45348532
[1] RASE/ASE
[1] 1.044 0.903 0.960 0.961
[1] .............................................................
[1] estimation method=6
[1] parameter estimates
[1]  0.8668759  0.5246981 -4.4348982  4.5284530
[1] information-based ASEs
[1] 0.06121019 0.04337316 0.47448745 0.47671443
[1] robust ASEs(RASEs)
[1] 0.06477607 0.03857981 0.45095993 0.45364686
[1] RASE/ASE
[1] 1.058 0.889 0.950 0.952
[1] .............................................................
[1] estimation method=7
[1] parameter estimates
[1]  0.7926992  0.4886477 -4.5116900  4.6526624
[1] information-based ASEs
[1] 0.06065383 0.04317967 0.48289679 0.48510947
[1] robust ASEs(RASEs)
[1] 0.06562382 0.03784033 0.45105026 0.45380997
[1] RASE/ASE
[1] 1.082 0.876 0.934 0.935
[1] .............................................................
[1] estimation method=8
[1] parameter estimates
[1]  0.6686971  0.4225198 -4.6399334  4.8724512
[1] information-based ASEs
[1] 0.06003456 0.04286117 0.49912282 0.50131064
[1] robust ASEs(RASEs)
[1] 0.06745441 0.03792329 0.45084606 0.45365032
[1] RASE/ASE
[1] 1.124 0.885 0.903 0.905
[1]
[1] ============================================================
[1] Model 4 (M,AC)
[1] "number of parameters=4"
[1] the pattern of parameter use (1;use,0:not use)
p-use 1 p-use 2 p-use 3 p-use 4 p-use 5 p-use 6 p-use 7
      1       1       1       1       0       0       0
[1] the (reduced) design matrix for the model
```

```
     [,1] [,2] [,3] [,4]
[1,]    1    1    1    1
[2,]    1    1    0    1
[3,]    1    0    1    0
[4,]    1    0    0    0
[5,]    0    1    1    0
[6,]    0    1    0    0
[7,]    0    0    1    0
[8,]    0    0    0    0
[1] ............................................................
[1]
[1] methods (1:ML,2:GM^2,3-8:Lambda=(-2,-0.5,0.5,2/3,1,2))
[1] ............................................................
[1] estimation method=1
[1] parameter estimates
[1]  0.5762534 -1.8097133 -0.3154188  2.8737341
[1] information-based ASEs
[1] 0.07455682 0.15905298 0.04244461 0.16729609
[1] robust ASEs(RASEs)
[1] 0.07455682 0.15905298 0.04244461 0.16729609
[1] RASE/ASE
[1] 1 1 1 1
[1] ............................................................
[1] estimation method=2
[1] parameter estimates
[1]  1.6314458 -0.8720668 -0.2469873  2.2939163
[1] information-based ASEs
[1] 0.12066127 0.20322858 0.04224226 0.21037819
[1] robust ASEs(RASEs)
[1] 0.3602700 0.4377359 0.1096806 0.4124869
[1] RASE/ASE
[1] 2.986 2.154 2.596 1.961
[1] ............................................................
[1] !!!! The following Hessian is singular!, iteration=13
              [,1]          [,2]          [,3]          [,4]
[1,] -2.193640e-06  1.493926e-08 -4.482037e-16 -2.178701e-06
[2,]  1.493926e-08 -4.308265e-07  1.077543e-17 -2.845620e-14
[3,] -4.482037e-16  1.077543e-17  6.032793e+00 -4.481966e-16
[4,] -2.178701e-06 -2.845620e-14 -4.481966e-16 -2.178701e-06
[1] eigenvalues
[1]  6.032793e+00 -7.194313e-09 -4.310606e-07 -4.364913e-06
[1] estimation method=3
[1] parameter estimates
[1]  -3.326420  19.287532  -2.662588 -14.305043
[1] information-based ASEs
[1] 0 0 0 0
[1] robust ASEs(RASEs)
[1] 0 0 0 0
[1] RASE/ASE
[1] NaN NaN NaN NaN
```

```
[1] ............................................................
[1] estimation method=4
[1] parameter estimates
[1]  0.8099698 -1.6128774 -0.4033236  2.7746509
[1] information-based ASEs
[1] 0.08214896 0.16764309 0.04277751 0.17559115
[1] robust ASEs(RASEs)
[1] 0.08775606 0.17766999 0.05441635 0.18662614
[1] RASE/ASE
[1] 1.068 1.060 1.272 1.063
[1] ............................................................
[1] estimation method=5
[1] parameter estimates
[1]  0.5128898 -1.8568149 -0.2313024  2.8302470
[1] information-based ASEs
[1] 0.07134895 0.15353539 0.04220285 0.16186388
[1] robust ASEs(RASEs)
[1] 0.07520578 0.16136632 0.03804372 0.16861910
[1] RASE/ASE
[1] 1.054 1.051 0.901 1.042
[1] ............................................................
[1] estimation method=6
[1] parameter estimates
[1]  0.5049230 -1.8621036 -0.2074611  2.8086844
[1] information-based ASEs
[1] 0.07063586 0.15212139 0.04214793 0.16046066
[1] robust ASEs(RASEs)
[1] 0.07541919 0.16210747 0.03716219 0.16915711
[1] RASE/ASE
[1] 1.068 1.066 0.882 1.054
[1] ............................................................
[1] estimation method=7
[1] parameter estimates
[1]  0.4967147 -1.8671780 -0.1656515  2.7660457
[1] information-based ASEs
[1] 0.06950300 0.14977871 0.04206606 0.15812974
[1] robust ASEs(RASEs)
[1] 0.07571069 0.16305092 0.03581640 0.16981202
[1] RASE/ASE
[1] 1.089 1.089 0.851 1.074
[1] ............................................................
[1] estimation method=8
[1] parameter estimates
[1]  0.49162992 -1.86987983 -0.07685628  2.66703317
[1] information-based ASEs
[1] 0.06741072 0.14529992 0.04195314 0.15366765
[1] robust ASEs(RASEs)
[1] 0.07597795 0.16377327 0.03323355 0.17001995
[1] RASE/ASE
[1] 1.127 1.127 0.792 1.106
```

```
[1]
[1] ============================================================
[1] Model 5 (AC,AM)
[1] "number of parameters=5"
[1] the pattern of parameter use (1;use,0:not use)
p-use 1 p-use 2 p-use 3 p-use 4 p-use 5 p-use 6 p-use 7
      1       1       1       1       1       0       0
[1] the (reduced) design matrix for the model
     [,1] [,2] [,3] [,4] [,5]
[1,]    1    1    1    1    1
[2,]    1    1    0    1    0
[3,]    1    0    1    0    1
[4,]    1    0    0    0    0
[5,]    0    1    1    0    0
[6,]    0    1    0    0    0
[7,]    0    0    1    0    0
[8,]    0    0    0    0    0
[1] ............................................................
[1]
[1] methods (1:ML,2:GM^2,3-8:Lambda=(-2,-0.5,0.5,2/3,1,2))
[1] ............................................................
[1] estimation method=1
[1] parameter estimates
[1] -0.08167244 -1.80971327 -4.16511363  2.87373412  4.12508776
[1] information-based ASEs
[1] 0.0780971 0.1590530 0.4506724 0.1672961 0.4529445
[1] robust ASEs(RASEs)
[1] 0.08341921 0.15905298 0.45067237 0.16729609 0.45294453
[1] RASE/ASE
[1] 1.068 1.000 1.000 1.000 1.000
[1] ............................................................
[1] estimation method=2
[1] parameter estimates
[1] -0.7432232 -1.8479935 -4.6283525  3.4903467  4.6904931
[1] information-based ASEs
[1] 0.0834269 0.1519561 0.5322889 0.1641620 0.5342567
[1] robust ASEs(RASEs)
[1] 0.1347046 0.1634865 0.6219219 0.1946395 0.6254193
[1] RASE/ASE
[1] 1.615 1.076 1.168 1.186 1.171
[1] ............................................................
[1] !!!! The following Hessian is singular!, iteration=7
              [,1]          [,2]          [,3]          [,4]          [,5]
[1,] -1.290155e-05  9.618684e-07  9.799231e-09 -7.536576e-11 -2.284777e-12
[2,]  9.618684e-07 -2.039938e-05 -7.282805e-10 -1.943744e-05  2.465179e-12
[3,]  9.799231e-09 -7.282805e-10 -5.990683e-07  2.352082e-12 -5.892667e-07
[4,] -7.536576e-11 -1.943744e-05  2.352082e-12 -1.943744e-05  2.294826e-12
[5,] -2.284777e-12  2.465179e-12 -5.892667e-07  2.294826e-12 -5.892667e-07
[1] eigenvalues
[1] -4.876704e-09 -4.387827e-07 -1.183450e-06 -1.291990e-05 -3.937970e-05
```

```
[1] estimation method=3
[1] parameter estimates
[1]  13.637221  -2.518663  -7.181809 -10.630457  -9.463397
[1] information-based ASEs
[1] 0 0 0 0 0
[1] robust ASEs(RASEs)
[1] 0 0 0 0 0
[1] RASE/ASE
[1] NaN NaN NaN NaN NaN
[1] ............................................................
[1] estimation method=4
[1] parameter estimates
[1] -0.3138298 -1.8316042 -4.4321792  3.1076961  4.3858503
[1] information-based ASEs
[1] 0.07909004 0.15581548 0.49880917 0.16527515 0.50088362
[1] robust ASEs(RASEs)
[1] 0.09265513 0.16148501 0.51695891 0.17351200 0.51961269
[1] RASE/ASE
[1] 1.172 1.036 1.036 1.050 1.037
[1] ............................................................
[1] estimation method=5
[1] parameter estimates
[1]  0.05913331 -1.79188405 -3.89367230  2.71348876  3.89470083
[1] information-based ASEs
[1] 0.07803361 0.16146867 0.40392857 0.16903949 0.40644343
[1] robust ASEs(RASEs)
[1] 0.08080089 0.15687250 0.48345300 0.16417623 0.48525264
[1] RASE/ASE
[1] 1.035 0.972 1.197 0.971 1.194
[1] ............................................................
[1] estimation method=6
[1] parameter estimates
[1]  0.09352406 -1.78807130 -3.81292069  2.67219709  3.82910849
[1] information-based ASEs
[1] 0.07808069 0.16216416 0.39080111 0.16957493 0.39339542
[1] robust ASEs(RASEs)
[1] 0.08031267 0.15609411 0.49947647 0.16326557 0.50116052
[1] RASE/ASE
[1] 1.029 0.963 1.278 0.963 1.274
[1] ............................................................
[1] estimation method=7
[1] parameter estimates
[1]  0.1491953 -1.7835243 -3.6709297  2.6039786  3.7164104
[1] information-based ASEs
[1] 0.07820985 0.16344721 0.36870460 0.17059956 0.37144640
[1] robust ASEs(RASEs)
[1] 0.07957022 0.15461894 0.52480521 0.16162130 0.52633070
[1] RASE/ASE
[1] 1.017 0.946 1.423 0.947 1.417
[1] ............................................................
```

```
[1] estimation method=8
[1] parameter estimates
[1]  0.2515631 -1.7843301 -3.3809681  2.4756117  3.4972851
[1] information-based ASEs
[1] 0.07863296 0.16658667 0.32769003 0.17325901 0.33076344
[1] robust ASEs(RASEs)
[1] 0.07827543 0.15198863 0.55261916 0.15870356 0.55395581
[1] RASE/ASE
[1] 0.995 0.912 1.686 0.916 1.675
[1]
[1] ============================================================
[1] Model 6 (AC,CM)
[1] "number of parameters=5"
[1] the pattern of parameter use (1;use,0:not use)
p-use 1 p-use 2 p-use 3 p-use 4 p-use 5 p-use 6 p-use 7
      1       1       1       1       0       1       0
[1] the (reduced) design matrix for the model
     [,1] [,2] [,3] [,4] [,5]
[1,]    1    1    1    1    1
[2,]    1    1    0    1    0
[3,]    1    0    1    0    0
[4,]    1    0    0    0    0
[5,]    0    1    1    0    1
[6,]    0    1    0    0    0
[7,]    0    0    1    0    0
[8,]    0    0    0    0    0
[1] ............................................................
[1]
[1] methods (1:ML,2:GM^2,3-8:Lambda=(-2,-0.5,0.5,2/3,1,2))
[1] ............................................................
[1] estimation method=1
[1] parameter estimates
[1]  0.5762534 -2.6941394 -2.7712291  2.8737341  3.2243089
[1] information-based ASEs
[1] 0.07455682 0.16257385 0.15198577 0.16729609 0.16098117
[1] robust ASEs(RASEs)
[1] 0.07455682 0.16867486 0.15198577 0.16729609 0.16098117
[1] RASE/ASE
[1] 1.000 1.038 1.000 1.000 1.000
[1] ............................................................
[1] estimation method=2
[1] parameter estimates
[1]  0.5867064 -3.7661408 -3.2674343  3.9190234  3.7592763
[1] information-based ASEs
[1] 0.07476418 0.25666594 0.19054343 0.25963278 0.19784751
[1] robust ASEs(RASEs)
[1] 0.07893518 0.38500977 0.27880730 0.37952182 0.28431162
[1] RASE/ASE
[1] 1.056 1.500 1.463 1.462 1.437
[1] ............................................................
```

```
[1] estimation method=3
[1] parameter estimates
[1]  0.5300749 -4.5844949 -4.0438607  4.7404595  4.5628822
[1] information-based ASEs
[1] 0.07452326 0.36693100 0.27659477 0.36908580 0.28168682
[1] robust ASEs(RASEs)
[1] 0.0827107 0.5770305 0.6525872 0.5741721 0.6548953
[1] RASE/ASE
[1] 1.110 1.573 2.359 1.556 2.325
[1] ............................................................
[1] estimation method=4
[1] parameter estimates
[1]  0.589673 -3.135789 -2.961575  3.304283  3.425468
[1] information-based ASEs
[1] 0.0746865 0.1955481 0.1654277 0.1994526 0.1737497
[1] robust ASEs(RASEs)
[1] 0.07634255 0.20669261 0.16909582 0.20400869 0.17761494
[1] RASE/ASE
[1] 1.022 1.057 1.022 1.023 1.022
[1] ............................................................
[1] estimation method=5
[1] parameter estimates
[1]  0.5612236 -2.4482495 -2.6570020  2.6309742  3.1138446
[1] information-based ASEs
[1] 0.07444836 0.14701924 0.14466211 0.15224486 0.15408738
[1] robust ASEs(RASEs)
[1] 0.07405539 0.16765607 0.15252408 0.16815772 0.16125501
[1] RASE/ASE
[1] 0.995 1.140 1.054 1.105 1.047
[1] ............................................................
[1] estimation method=6
[1] parameter estimates
[1]  0.556848 -2.393284 -2.629807  2.576190  3.088984
[1] information-based ASEs
[1] 0.07441907 0.14378091 0.14299525 0.14912270 0.15252645
[1] robust ASEs(RASEs)
[1] 0.07402519 0.16750570 0.15293945 0.16860471 0.16161049
[1] RASE/ASE
[1] 0.995 1.165 1.070 1.131 1.060
[1] ............................................................
[1] estimation method=7
[1] parameter estimates
[1]  0.5492365 -2.3084704 -2.5864044  2.4911810  3.0505026
[1] information-based ASEs
[1] 0.07436986 0.13894437 0.14039411 0.14446742 0.15009749
[1] robust ASEs(RASEs)
[1] 0.07404363 0.16702877 0.15358176 0.16917652 0.16217897
[1] RASE/ASE
[1] 0.996 1.202 1.094 1.171 1.080
[1] ............................................................
```

```
[1] estimation method=8
[1] parameter estimates
[1]  0.5336424 -2.1638004 -2.5083417  2.3445205  2.9851442
[1] information-based ASEs
[1] 0.07427613 0.13112717 0.13589607 0.13696303 0.14591926
[1] robust ASEs(RASEs)
[1] 0.07427614 0.16568140 0.15455259 0.17003058 0.16310429
[1] RASE/ASE
[1] 1.000 1.264 1.137 1.241 1.118
[1]
[1] ============================================================
[1] Model 7 (AM,CM)
[1] "number of parameters=5"
[1] the pattern of parameter use (1;use,0:not use)
p-use 1 p-use 2 p-use 3 p-use 4 p-use 5 p-use 6 p-use 7
     1       1       1       0       1       1       0
[1] the (reduced) design matrix for the model
     [,1] [,2] [,3] [,4] [,5]
[1,]    1    1    1    1    1
[2,]    1    1    0    0    0
[3,]    1    0    1    1    0
[4,]    1    0    0    0    0
[5,]    0    1    1    0    1
[6,]    0    1    0    0    0
[7,]    0    0    1    0    0
[8,]    0    0    0    0    0
[1] ............................................................
[1]
[1] methods (1:ML,2:GM^2,3-8:Lambda=(-2,-0.5,0.5,2/3,1,2))
[1] ............................................................
[1] estimation method=1
[1] parameter estimates
[1]  1.1271857 -0.2351197 -6.6209236  4.1250875  3.2243089
[1] information-based ASEs
[1] 0.06412196 0.05551319 0.47371264 0.45294446 0.16098117
[1] robust ASEs(RASEs)
[1] 0.06412196 0.05551319 0.49045848 0.45294440 0.16098117
[1] RASE/ASE
[1] 1.000 1.000 1.035 1.000 1.000
[1] ............................................................
[1] estimation method=2
[1] parameter estimates
[1]  1.3866267 -0.2416015 -6.7152864  4.2072392  3.2622248
[1] information-based ASEs
[1] 0.07006960 0.05645955 0.54223632 0.52434585 0.16024436
[1] robust ASEs(RASEs)
[1] 0.08956285 0.06642767 0.58682340 0.56031784 0.16773061
[1] RASE/ASE
[1] 1.278 1.177 1.082 1.069 1.047
[1] ............................................................
```

```
[1] estimation method=3
[1] parameter estimates
[1]  1.8303925 -0.0723899 -6.2842636  3.8415783  3.0994144
[1] information-based ASEs
[1] 0.08244917 0.05694385 0.55372571 0.53659661 0.15823225
[1] robust ASEs(RASEs)
[1] 0.1670253 0.0841692 0.6275273 0.6002464 0.1758406
[1] RASE/ASE
[1] 2.026 1.478 1.133 1.119 1.111
[1] ............................................................
[1] estimation method=4
[1] parameter estimates
[1]  1.2398450 -0.2539314 -6.7515457  4.2382149  3.2643174
[1] information-based ASEs
[1] 0.06658249 0.05600861 0.51959139 0.50069074 0.16103558
[1] robust ASEs(RASEs)
[1] 0.07130090 0.05997769 0.53645793 0.50753728 0.16423027
[1] RASE/ASE
[1] 1.071 1.071 1.032 1.014 1.020
[1] ............................................................
[1] estimation method=5
[1] parameter estimates
[1]  1.0368174 -0.2030364 -6.3573661  3.8954172  3.1628244
[1] information-based ASEs
[1] 0.06232453 0.05507772 0.41464904 0.39110344 0.16015877
[1] robust ASEs(RASEs)
[1] 0.06193008 0.05301371 0.57731932 0.52125495 0.16031239
[1] RASE/ASE
[1] 0.994 0.963 1.392 1.333 1.001
[1] ............................................................
[1] estimation method=6
[1] parameter estimates
[1]  1.0107805 -0.1914056 -6.2615709  3.8115026  3.1420482
[1] information-based ASEs
[1] 0.06183944 0.05495403 0.39637964 0.37180503 0.15987018
[1] robust ASEs(RASEs)
[1] 0.06172081 0.05254417 0.61758515 0.55671323 0.16034489
[1] RASE/ASE
[1] 0.998 0.956 1.558 1.497 1.003
[1] ............................................................
[1] estimation method=7
[1] parameter estimates
[1]  0.9641650 -0.1683224 -6.0830305  3.6543965  3.1041640
[1] information-based ASEs
[1] 0.06100846 0.05473966 0.36524822 0.33862770 0.15943861
[1] robust ASEs(RASEs)
[1] 0.06164986 0.05197494 0.67821597 0.61152081 0.16005457
[1] RASE/ASE
[1] 1.011 0.949 1.857 1.806 1.004
[1] ............................................................
```

```
[1] estimation method=8
[1] parameter estimates
[1]  0.8603861 -0.1084225 -5.7126489  3.3258146  3.0251793
[1] information-based ASEs
[1] 0.05932593 0.05430971 0.31019036 0.27844951 0.15928362
[1] robust ASEs(RASEs)
[1] 0.06219327 0.05166710 0.72958879 0.65836744 0.15845056
[1] RASE/ASE
[1] 1.048 0.951 2.352 2.364 0.995
[1]
[1] ============================================================
[1] Model 8 (AC,AM,CM)
[1] "number of parameters=6"
[1] the pattern of parameter use (1;use,0:not use)
p-use 1 p-use 2 p-use 3 p-use 4 p-use 5 p-use 6 p-use 7
     1       1       1       1       1       1       0
[1] the (reduced) design matrix for the model
     [,1] [,2] [,3] [,4] [,5] [,6]
[1,]    1    1    1    1    1    1
[2,]    1    1    0    1    0    0
[3,]    1    0    1    0    1    0
[4,]    1    0    0    0    0    0
[5,]    0    1    1    0    0    1
[6,]    0    1    0    0    0    0
[7,]    0    0    1    0    0    0
[8,]    0    0    0    0    0    0
[1] ............................................................
[1]
[1] methods (1:ML,2:GM^2,3-8:Lambda=(-2,-0.5,0.5,2/3,1,2))
[1] ............................................................
[1] estimation method=1
[1] parameter estimates
[1]  0.487719 -1.886669 -5.309042  2.054534  2.986014  2.847889
[1] information-based ASEs
[1] 0.0757672 0.1626970 0.4751970 0.1740643 0.4646780 0.1638394
[1] robust ASEs(RASEs)
[1] 0.07578959 0.16422965 0.48432895 0.17565960 0.46773287 0.16352188
[1] RASE/ASE
[1] 1.000 1.009 1.019 1.009 1.007 0.998
[1] ............................................................
[1] estimation method=2
[1] parameter estimates
[1]  0.488160 -1.884680 -5.345490  2.052237  3.020513  2.850064
[1] information-based ASEs
[1] 0.07577525 0.16264703 0.48244025 0.17403877 0.47198534 0.16402133
[1] robust ASEs(RASEs)
[1] 0.07582508 0.16430701 0.49894061 0.17581954 0.48237421 0.16403440
[1] RASE/ASE
[1] 1.001 1.010 1.034 1.010 1.022 1.000
[1] ............................................................
```

```
[1] estimation method=3
[1] parameter estimates
[1]  0.4886384 -1.8824705 -5.3779379  2.0496918  3.0508872  2.8523821
[1] information-based ASEs
[1] 0.07578315 0.16257455 0.48889114 0.17398931 0.47848148 0.16420149
[1] robust ASEs(RASEs)
[1] 0.07588175 0.16454615 0.53138496 0.17619115 0.51303139 0.16478538
[1] RASE/ASE
[1] 1.001 1.012 1.087 1.013 1.072 1.004
[1] ............................................................
[1] estimation method=4
[1] parameter estimates
[1]  0.4879297 -1.8857265 -5.3276442  2.0534444  3.0036786  2.8489344
[1] information-based ASEs
[1] 0.07577119 0.16267610 0.47889498 0.17405568 0.46841072 0.16392916
[1] robust ASEs(RASEs)
[1] 0.07580485 0.16425456 0.48832981 0.17572004 0.47200461 0.16374754
[1] RASE/ASE
[1] 1.000 1.010 1.020 1.010 1.008 0.999
[1] ............................................................
[1] estimation method=5
[1] parameter estimates
[1]  0.487538 -1.887461 -5.289985  2.055452  2.967778  2.846976
[1] information-based ASEs
[1] 0.07576342 0.16270743 0.47140271 0.17406279 0.46084303 0.16375502
[1] robust ASEs(RASEs)
[1] 0.07577665 0.16419849 0.48805353 0.17559809 0.47084925 0.16333279
[1] RASE/ASE
[1] 1.000 1.009 1.035 1.009 1.022 0.997
[1] ............................................................
[1] estimation method=6
[1] parameter estimates
[1]  0.4874857 -1.8876847 -5.2835860  2.0557127  2.9616197  2.8467083
[1] information-based ASEs
[1] 0.07576222 0.16270830 0.47012675 0.17405977 0.45955217 0.16372856
[1] robust ASEs(RASEs)
[1] 0.07577229 0.16418059 0.49101098 0.17556991 0.47358366 0.16327215
[1] RASE/ASE
[1] 1.000 1.009 1.044 1.009 1.031 0.997
[1] ............................................................
[1] estimation method=7
[1] parameter estimates
[1]  0.4873947 -1.8880663 -5.2707852  2.0561576  2.9492452  2.8462355
[1] information-based ASEs
[1] 0.07575993 0.16270589 0.46757118 0.17404979 0.45696473 0.16367868
[1] robust ASEs(RASEs)
[1] 0.07576292 0.16412775 0.49927527 0.17549491 0.48145360 0.16314772
[1] RASE/ASE
[1] 1.000 1.009 1.068 1.008 1.054 0.997
[1] ............................................................
```

```
[1] estimation method=8
[1] parameter estimates
[1]  0.4872396 -1.8886492 -5.2331209  2.0568476  2.9123670  2.8453705
[1] information-based ASEs
[1] 0.07575415 0.16266522 0.46002223 0.17398853 0.44930482 0.16355740
[1] robust ASEs(RASEs)
[1] 0.0757276 0.1638260 0.5378484 0.1751104 0.5200499 0.1627182
[1] RASE/ASE
[1] 1.000 1.007 1.169 1.006 1.157 0.995
[1]
[1] =============================================================
[1] Model 9 (ACM)(saturated model)
[1] "number of parameters=7"
[1] the pattern of parameter use (1;use,0:not use)
p-use 1 p-use 2 p-use 3 p-use 4 p-use 5 p-use 6 p-use 7
      1       1       1       1       1       1       1
[1] the (reduced) design matrix for the model
     [,1] [,2] [,3] [,4] [,5] [,6] [,7]
[1,]    1    1    1    1    1    1    1
[2,]    1    1    0    1    0    0    0
[3,]    1    0    1    0    1    0    0
[4,]    1    0    0    0    0    0    0
[5,]    0    1    1    0    0    1    0
[6,]    0    1    0    0    0    0    0
[7,]    0    0    1    0    0    0    0
[8,]    0    0    0    0    0    0    0
[1] ............................................................
[1]
[1] methods (1:ML,2:GM^2,3-8:Lambda=(-2,-0.5,0.5,2/3,1,2))
[1] ............................................................
[1] estimation method=1
[1] parameter estimates
[1]  0.4912810 -1.8700117 -4.9380646  2.0353774  2.5997614  2.2754768
[7]  0.5895107
[1] information-based ASEs
[1] 0.07600797 0.16382931 0.70963669 0.17576052 0.72698314 0.92745532
[7] 0.94236408
[1] robust ASEs(RASEs)
[1] 0.07600797 0.16382931 0.70963669 0.17576052 0.72698314 0.92745532
[7] 0.94236408
[1] RASE/ASE
[1] 1 1 1 1 1 1 1
[1] ............................................................
[1] estimation method=2
[1] parameter estimates
[1]  0.4912810 -1.8700117 -4.9380646  2.0353774  2.5997614  2.2754768
[7]  0.5895107
[1] information-based ASEs
[1] 0.07600797 0.16382931 0.70963669 0.17576052 0.72698314 0.92745532
[7] 0.94236408
```

```
[1] robust ASEs(RASEs)
[1] 0.07600797 0.16382931 0.70963669 0.17576052 0.72698314 0.92745532
[7] 0.94236408
[1] RASE/ASE
[1] 1 1 1 1 1 1 1
[1] ..........................................................
[1] estimation method=3
[1] parameter estimates
[1]  0.4912810 -1.8700117 -4.9380646  2.0353774  2.5997614  2.2754768
[7]  0.5895107
[1] information-based ASEs
[1] 0.07600797 0.16382931 0.70963669 0.17576052 0.72698314 0.92745532
[7] 0.94236408
[1] robust ASEs(RASEs)
[1] 0.07600797 0.16382931 0.70963669 0.17576052 0.72698314 0.92745532
[7] 0.94236408
[1] RASE/ASE
[1] 1 1 1 1 1 1 1
[1] ..........................................................
[1] estimation method=4
[1] parameter estimates
[1]  0.4912810 -1.8700117 -4.9380646  2.0353774  2.5997614  2.2754768
[7]  0.5895107
[1] information-based ASEs
[1] 0.07600797 0.16382931 0.70963669 0.17576052 0.72698314 0.92745532
[7] 0.94236408
[1] robust ASEs(RASEs)
[1] 0.07600797 0.16382931 0.70963669 0.17576052 0.72698314 0.92745532
[7] 0.94236408
[1] RASE/ASE
[1] 1 1 1 1 1 1 1
[1] ..........................................................
[1] estimation method=5
[1] parameter estimates
[1]  0.4912810 -1.8700117 -4.9380646  2.0353774  2.5997614  2.2754768
[7]  0.5895107
[1] information-based ASEs
[1] 0.07600797 0.16382931 0.70963669 0.17576052 0.72698314 0.92745532
[7] 0.94236408
[1] robust ASEs(RASEs)
[1] 0.07600797 0.16382931 0.70963669 0.17576052 0.72698314 0.92745532
[7] 0.94236408
[1] RASE/ASE
[1] 1 1 1 1 1 1 1
[1] ..........................................................
[1] estimation method=6
[1] parameter estimates
[1]  0.4912810 -1.8700117 -4.9380646  2.0353774  2.5997614  2.2754768
[7]  0.5895107
[1] information-based ASEs
```

```
[1] 0.07600797 0.16382931 0.70963669 0.17576052 0.72698314 0.92745532
[7] 0.94236408
[1] robust ASEs(RASEs)
[1] 0.07600797 0.16382931 0.70963669 0.17576052 0.72698314 0.92745532
[7] 0.94236408
[1] RASE/ASE
[1] 1 1 1 1 1 1 1
[1] ...........................................................
[1] estimation method=7
[1] parameter estimates
[1]  0.4912810 -1.8700117 -4.9380646  2.0353774  2.5997614  2.2754768
[7]  0.5895107
[1] information-based ASEs
[1] 0.07600797 0.16382931 0.70963669 0.17576052 0.72698314 0.92745532
[7] 0.94236408
[1] robust ASEs(RASEs)
[1] 0.07600797 0.16382931 0.70963669 0.17576052 0.72698314 0.92745532
[7] 0.94236408
[1] RASE/ASE
[1] 1 1 1 1 1 1 1
[1] ...........................................................
[1] estimation method=8
[1] parameter estimates
[1]  0.4912810 -1.8700117 -4.9380646  2.0353774  2.5997614  2.2754768
[7]  0.5895107
[1] information-based ASEs
[1] 0.07600797 0.16382931 0.70963669 0.17576052 0.72698314 0.92745532
[7] 0.94236408
[1] robust ASEs(RASEs)
[1] 0.07600797 0.16382931 0.70963669 0.17576052 0.72698314 0.92745532
[7] 0.94236408
[1] RASE/ASE
[1] 1 1 1 1 1 1 1
   user  system elapsed
   2.31    0.04    2.35
[1] Thu Dec 06 10:42:21 2018
```

The program “mpe2a.R”

```
#
#  Robust asymptotic standard errors of the minimizing phi-divergence
#  estimators (MPEs) for log-linear models including the linear-by-
#  linear model in a two-way contingency table with two sets
#  of ordinal categories under possible model misspecification
#
#     <mpe2a.R>                                April 16, 2018
#
#  Use as:   R prompt >source('file_name').
#  The results will be given in 'r.u'.
#

starttime=proc.time()
sink ('r.u')
print(date(),quote=F)

###### function mpese
mpese=function(n,pr,al,w,qo,iq,ik,im,imax){
if(im==1){
print('methods (1:ML,2:GM^2,3-8:Lambda=(-2,-0.5,0.5,2/3,1,2))',quote=F)
print('............................................................',quote=F)
}
delta=1e-8
q=qo
inv=0
se=rep(0,iq)
ser=rep(0,iq)

for (it in 1:(imax+1)){
if(it>imax){
print(paste('!!!! no convergence, the number of iterations=',
           it,sep=''),quote=F)
return(list(qhat=q,it=it,inv=inv,se=se,ser=ser))
}

pi=exp(w%*%q)
```

```
pi=pi[,1,drop=T]   # vectorized for later use
                  # as 'diag(pi,nrow=length(pi))'
pi=pi/sum(pi)
wp=t(w)%*%pi
wp=wp[,1,drop=T]   # vectorized for later use
                  # as 'wp%o%wp+...'
pq=matrix(0,ik,iq)
for (i in 1:ik){
for (j in 1:iq){
pq[i,j]=(w[i,j]-wp[j])*pi[i]
};}

pq2=array(0,c(ik,iq,iq))
for (i in 1:ik){
for (j1 in 1:iq){
for (j2 in 1:iq){
pq2[i,j1,j2]=(w[i,j1]-wp[j1])*pq[i,j2]
for (i1 in 1:ik){
pq2[i,j1,j2]=pq2[i,j1,j2]-w[i1,j1]*pq[i1,j2]*pi[i]
};};};}

g=rep(0,iq)
h=matrix(0,iq,iq)

for (i in 1:iq){
if(im==1)g[i]=-sum(pr/pi*pq[,i])    # note the minus sign
if(im==2)g[i]=sum((log(pi/pr)+1)*pq[,i])
if(im>=3 && im<=8)g[i]=sum(-al[im-2]*(pr/pi)^(al[im-2]+1)*pq[,i])

for (j in 1:i){
if(im==1)h[i,j]=-sum(-pr/pi^2*pq[,i]*pq[,j]+pr/pi*pq2[,i,j]) # note the
                                                           # minus sign
if(im==2)h[i,j]=sum(1/pi*pq[,i]*pq[,j]+(log(pi/pr)+1)*pq2[,i,j])
if(im>=3 && im<=8)h[i,j]=sum(al[im-2]*(al[im-2]+1)*pr^(al[im-2]+1)
/pi^(al[im-2]+2)*pq[,i]*pq[,j]-al[im-2]*(pr/pi)^(al[im-2]+1)*pq2[,i,j])

h[j,i]=h[i,j]
};}
```

```
aaa=eigen(h,symmetric=T)[['values']]   # not -h
if(min(abs(aaa))<delta){
print(paste('!!!! The following Hessian is singular!, iteration=',
      it,sep=''),quote=F)
print(h)
print('eigenvalues',quote=F)
print(aaa)
inv=1
return(list(qhat=q,it=it,inv=inv,se=se,ser=ser))
}

h1=h
h=solve(h)

if(max(abs(g))<delta){

fi=-wp%o%wp+t(w)%*%diag(pi,nrow=length(pi))%*%w
fi=solve(fi)
se=sqrt(fi[row(fi)==col(fi)]/n)

gp=matrix(0,iq,ik)

for (i in 1:iq){
for (k in 1:ik){
if(im==1)gp[i,k]=-1/pi[k]*pq[k,i] # note the minus sign
if(im==2)gp[i,k]=-1/pr[k]*pq[k,i]
if(im>=3 && im<=8)gp[i,k]=-al[im-2]*(al[im-2]+1)*
          pr[k]^al[im-2]/pi[k]^(al[im-2]+1)*pq[k,i]
};}

qp=-h%*%gp
qpi=qp%*%pr
qpi=qpi[,1,drop=T]  # vectorized

fir=-qpi%o%qpi+qp%*%diag(pr,nrow=length(pr))%*%t(qp)
ser=sqrt(fir[row(fir)==col(fir)]/n)

q=as.vector(q) # vectorized for ease of presentation
```

```
return(list(qhat=q,it=it,inv=inv,se=se,ser=ser))

} # end of if(max(abs(g))<delta)

iout=0
if(iout==1){
print(paste('###### iteration=',it,sep=''),quote=F)
print('q(parameters),g(gradients)',quote=F)
print(q)
print(g)
print('Hessian',quote=F)
print(h1)
}

q=q-h%*%g
q=as.vector(q) # vectorized for ease of presentation

} # end of it-loop
} # end of function mpese
######

al=c(-2,-0.5,0.5,2/3,1,2)  # powers in power divergences
                           # except those for G^2(ML) and GM^2

cat('¥n
Robust asymptotic standard errors of the minimizing phi-divergence ¥n
estimators (MPEs) for log-linear models including the linear-by- ¥n
linear model in a two-way contingency table with two sets ¥n
of ordinal categories under possible model misspecification ¥n
on opinions about premarital sex (X) and teenage birth control (Y) ¥n
using the data by Agresti (2013, p.386) ¥n')

i0=4 # the number of row categories
j0=4 # the number of column categories
ik=i0*j0 # the number of categories

fn=c(81, 68, 60, 38,  # a vectorized contingency table
     24, 26, 29, 14,  # of Agresti (2013, p.386)
     18, 41, 74, 42,
```

```
     36, 57,161,157)
n=sum(fn)
pr=fn/n
ft=matrix(fn,4,4,byrow=T,dimnames=list(paste('X',1:4,sep=''),
                                      paste('Y',1:4,sep='')))

print(' ',quote=F)
print(paste('total frequency=',n,sep=''),quote=F)
print('frequencies',quote=F)
print(ft)
print('percentages',quote=F)
print(round(ft/n*100,1))

print('row marginal(%)',quote=F)
print(round(apply(ft/n*100,1,sum),1))
print('column marginal(%)',quote=F)
print(round(apply(ft/n*100,2,sum),1))

ws1=c(1,0,0,1,0,0,0,1,0,0,0,0,0,0,0,0, # The vectorized design
      1,0,0,0,1,0,0,0,1,0,0,0,0,0,0,0, # matrix for the linear-by-
      1,0,0,0,0,1,0,0,0,1,0,0,0,0,0,0, # linear and saturated models
      1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0, # (the 7th column is null,
      0,1,0,1,0,0,0,0,0,0,1,0,0,0,0,0, # which will be filled by
      0,1,0,0,1,0,0,0,0,0,0,1,0,0,0,0, # the linear-by-linear
      0,1,0,0,0,1,0,0,0,0,0,0,1,0,0,0, # model)
      0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
      0,0,1,1,0,0,0,0,0,0,0,0,0,1,0,0,
      0,0,1,0,1,0,0,0,0,0,0,0,0,0,1,0,
      0,0,1,0,0,1,0,0,0,0,0,0,0,0,0,1,
      0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,
      0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,
      0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,
      0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,
      0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0)

ws=matrix(data=ws1,16,16,byrow=T)

print(' ',quote=F)
print('The K(the number of catogories) by K',quote=F)
```

```
print('design matrix for the linear-by-linear and saturated models',
      quote=F)
print('(the 7th column is null, which will be filled by the',quote=F)
print('linear-by-linear model)',quote=F)
print(ws)

an=c('Model 1 (X,Y)','Model 2A (X,Y,linA)','Model 2B (X,Y,linB)',
     'Model 3 (XY)(saturated)')

wd1=c(1,1,1,1,1,1,0,0,0,0,0,0,0,0,0,0,  # Model 1
         1,1,1,1,1,1,1,0,0,0,0,0,0,0,0,0,  # Model 2A
         1,1,1,1,1,1,1,0,0,0,0,0,0,0,0,0,  # Model 2B
         1,1,1,1,1,1,0,1,1,1,1,1,1,1,1,1)  # Model 3(saturated)
wd=matrix(wd1,4,16,byrow=T,
        dimnames=list(an,paste('p-use ',1:16,sep='')))
print(' ',quote=F)
print('p-use=the pattern of prameter use (1:use,0:not use)',quote=F)
print(wd)
print(' ',quote=F)

ua=c(1,2,3,4)
va=ua
print('unstandadized u for Model (X,Y,linA)',quote=F)
print(ua)
print('unstandardized v for Model (X,Y,linA)',quote=F)
print(va)
print('unstandardized u*v¥' for Model (X,Y,linA)',quote=F)
print(round(ua%o%va,3))

pr1=apply(ft/n,1,sum)
pr2=apply(ft/n,2,sum)
ua=(ua-sum(ua*pr1))/sqrt(sum(ua^2*pr1)-sum(ua*pr1)^2)
va=(va-sum(va*pr2))/sqrt(sum(va^2*pr2)-sum(va*pr2)^2)
print('standardized u for Model (X,Y,linA)',quote=F)
print(round(ua,3))
print('standardized v for Model (X,Y,linA)',quote=F)
print(round(va,3))
print('standardized u*v¥' for Model (X,Y,linA)',quote=F)
print(round(ua%o%va,3))
```

```
ub=c(1,2,4,5)
vb=ub
print(' ',quote=F)
print('unstandadized u for Model (X,Y,linB)',quote=F)
print(ub)
print('unstandardized v for Model (X,Y,linB)',quote=F)
print(vb)
print('unstandardized u*v¥' for Model (X,Y,linB)',quote=F)
print(round(ub%o%vb,3))

ub=(ub-sum(ub*pr1))/sqrt(sum(ub^2*pr1)-sum(ub*pr1)^2)
vb=(vb-sum(vb*pr2))/sqrt(sum(vb^2*pr2)-sum(vb*pr2)^2)
print('standardized u for Model (X,Y,linB)',quote=F)
print(round(ub,3))
print('standardized v for Model (X,Y,linB)',quote=F)
print(round(vb,3))
print('standardized u*v¥' for Model (X,Y,linB)',quote=F)
print(round(ub%o%vb,3))

imax=200

for (mdl in 1:4){
print(' ',quote=F)
print('============================================================',quote=F)
print(an[mdl],quote=F)
iw=wd[mdl,]
iq=sum(iw)
print(paste('number of parameters=',iq,sep=''),qoute=F)
print('the pattern of parameter use (1;use,0:not use)',quote=F)
print(wd[mdl,])

if(mdl==2)ws[,i0+j0-1]=as.vector(va%o%ua)  # not ua%o%va
if(mdl==3)ws[,i0+j0-1]=as.vector(vb%o%ub)  # not ub%o%vb

iw=iw*1:ik
iw=iw[iw>0]
w=ws[,iw]
print('the (reduced) design matrix for the model',quote=F)
```

```
print(round(w,3))

qo=rep(0,iq)  # arbitrary initial values for MLEs

for (im in 1:8){
print('...........................................................',quote=F)

abc=mpese(n,pr,al,w,qo,iq,ik,im,imax)

print(paste('estimation method=',im,sep=''),quote=F)

itr=abc[['it']]
inv=abc[['inv']]
q=abc[['qhat']]
se=abc[['se']]
ser=abc[['ser']]

print('parameter estimates',quote=F)
print(q)
print('information-based ASEs',quote=F)
print(se)
print('robust ASEs(RASEs)',quote=F)
print(ser)
print('RASE/ASE',quote=F)
print(round(ser/se,3))

if(im==1)qo=q
}  # end of im-loop
}  # end of mdl-loop

print(proc.time()-starttime,quote=F)
print(date(),quote=F)
sink()
```

Results using the program "mpe2a.R"

```
[1] Thu Dec 06 10:38:49 2018


Robust asymptotic standard errors of the minimizing phi-divergence

estimators (MPEs) for log-linear models including the linear-by-

linear model in a two-way contingency table with two sets

of ordinal categories under possible model misspecification

on opinions about premarital sex (X) and teenage birth control (Y)

using the data by Agresti (2013, p.386)
[1]
[1] total frequency=926
[1] frequencies
   Y1 Y2  Y3  Y4
X1 81 68  60  38
X2 24 26  29  14
X3 18 41  74  42
X4 36 57 161 157
[1] percentages
    Y1  Y2   Y3   Y4
X1 8.7 7.3  6.5  4.1
X2 2.6 2.8  3.1  1.5
X3 1.9 4.4  8.0  4.5
X4 3.9 6.2 17.4 17.0
[1] row marginal(%)
  X1   X2   X3   X4
26.7 10.0 18.9 44.4
[1] column marginal(%)
  Y1   Y2   Y3   Y4
17.2 20.7 35.0 27.1
[1]
[1] The K(the number of catogories) by K
[1] design matrix for the linear-by-linear and saturated models
[1] (the 7th column is null, which will be filled by the
[1] linear-by-linear model)
     [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10] [,11] [,12] [,13]
 [1,]    1    0    0    1    0    0    0    1    0     0     0     0     0
 [2,]    1    0    0    0    1    0    0    0    1     0     0     0     0
 [3,]    1    0    0    0    0    1    0    0    0     1     0     0     0
 [4,]    1    0    0    0    0    0    0    0    0     0     0     0     0
 [5,]    0    1    0    1    0    0    0    0    0     0     1     0     0
 [6,]    0    1    0    0    1    0    0    0    0     0     0     1     0
```

```
 [7,]    0    1    0    0    0    1    0    0    0     0     0     0     1
 [8,]    0    1    0    0    0    0    0    0    0     0     0     0     0
 [9,]    0    0    1    1    0    0    0    0    0     0     0     0     0
[10,]    0    0    1    0    1    0    0    0    0     0     0     0     0
[11,]    0    0    1    0    0    1    0    0    0     0     0     0     0
[12,]    0    0    1    0    0    0    0    0    0     0     0     0     0
[13,]    0    0    0    1    0    0    0    0    0     0     0     0     0
[14,]    0    0    0    0    1    0    0    0    0     0     0     0     0
[15,]    0    0    0    0    0    1    0    0    0     0     0     0     0
[16,]    0    0    0    0    0    0    0    0    0     0     0     0     0
     [,14] [,15] [,16]
 [1,]     0     0     0
 [2,]     0     0     0
 [3,]     0     0     0
 [4,]     0     0     0
 [5,]     0     0     0
 [6,]     0     0     0
 [7,]     0     0     0
 [8,]     0     0     0
 [9,]     1     0     0
[10,]     0     1     0
[11,]     0     0     1
[12,]     0     0     0
[13,]     0     0     0
[14,]     0     0     0
[15,]     0     0     0
[16,]     0     0     0
[1]
[1] p-use=the pattern of prameter use (1:use,0:not use)
                      p-use 1 p-use 2 p-use 3 p-use 4 p-use 5 p-use 6
Model 1 (X,Y)                1       1       1       1       1       1
Model 2A (X,Y,linA)          1       1       1       1       1       1
Model 2B (X,Y,linB)          1       1       1       1       1       1
Model 3 (XY)(saturated)      1       1       1       1       1       1
                      p-use 7 p-use 8 p-use 9 p-use 10 p-use 11 p-use 12
Model 1 (X,Y)                0       0       0        0        0        0
Model 2A (X,Y,linA)          1       0       0        0        0        0
Model 2B (X,Y,linB)          1       0       0        0        0        0
Model 3 (XY)(saturated)      0       1       1        1        1        1
                      p-use 13 p-use 14 p-use 15 p-use 16
Model 1 (X,Y)                 0        0        0        0
Model 2A (X,Y,linA)           0        0        0        0
Model 2B (X,Y,linB)           0        0        0        0
Model 3 (XY)(saturated)       1        1        1        1
[1]
[1] unstandadized u for Model (X,Y,linA)
[1] 1 2 3 4
[1] unstandardized v for Model (X,Y,linA)
[1] 1 2 3 4
[1] unstandardized u*v' for Model (X,Y,linA)
```

```
    [,1] [,2] [,3] [,4]
[1,]    1    2    3    4
[2,]    2    4    6    8
[3,]    3    6    9   12
[4,]    4    8   12   16
[1] standardized u for Model (X,Y,linA)
[1] -1.442 -0.645  0.151  0.948
[1] standardized v for Model (X,Y,linA)
[1] -1.650 -0.691  0.268  1.227
[1] standardized u*v' for Model (X,Y,linA)
       [,1]   [,2]   [,3]   [,4]
[1,]  2.380  0.996 -0.387 -1.770
[2,]  1.065  0.446 -0.173 -0.792
[3,] -0.250 -0.105  0.041  0.186
[4,] -1.565 -0.655  0.254  1.164
[1]
[1] unstandadized u for Model (X,Y,linB)
[1] 1 2 4 5
[1] unstandardized v for Model (X,Y,linB)
[1] 1 2 4 5
[1] unstandardized u*v' for Model (X,Y,linB)
    [,1] [,2] [,3] [,4]
[1,]    1    2    4    5
[2,]    2    4    8   10
[3,]    4    8   16   20
[4,]    5   10   20   25
[1] standardized u for Model (X,Y,linB)
[1] -1.426 -0.842  0.325  0.909
[1] standardized v for Model (X,Y,linB)
[1] -1.574 -0.902  0.443  1.115
[1] standardized u*v' for Model (X,Y,linB)
       [,1]   [,2]   [,3]   [,4]
[1,]  2.244  1.286 -0.631 -1.590
[2,]  1.326  0.759 -0.373 -0.939
[3,] -0.512 -0.293  0.144  0.363
[4,] -1.431 -0.820  0.403  1.014
[1]
[1] ============================================================
[1] Model 1 (X,Y)
[1] "number of parameters=6"
[1] the pattern of parameter use (1;use,0:not use)
 p-use 1  p-use 2  p-use 3  p-use 4  p-use 5  p-use 6  p-use 7  p-use 8
       1        1        1        1        1        1        0        0
 p-use 9 p-use 10 p-use 11 p-use 12 p-use 13 p-use 14 p-use 15 p-use 16
       0        0        0        0        0        0        0        0
[1] the (reduced) design matrix for the model
     [,1] [,2] [,3] [,4] [,5] [,6]
 [1,]    1    0    0    1    0    0
 [2,]    1    0    0    0    1    0
 [3,]    1    0    0    0    0    1
```

```
 [4,]    1    0    0    0    0    0
 [5,]    0    1    0    1    0    0
 [6,]    0    1    0    0    1    0
 [7,]    0    1    0    0    0    1
 [8,]    0    1    0    0    0    0
 [9,]    0    0    1    1    0    0
[10,]    0    0    1    0    1    0
[11,]    0    0    1    0    0    1
[12,]    0    0    1    0    0    0
[13,]    0    0    0    1    0    0
[14,]    0    0    0    0    1    0
[15,]    0    0    0    0    0    1
[16,]    0    0    0    0    0    0
[1] ...........................................................
[1] methods (1:ML,2:GM^2,3-8:Lambda=(-2,-0.5,0.5,2/3,1,2))
[1] ...........................................................
[1] estimation method=1
[1] parameter estimates
[1] -0.5092049 -1.4859937 -0.8538072 -0.4565487 -0.2679576  0.2552906
[1] information-based ASEs
[1] 0.08050884 0.11482934 0.09026281 0.10135764 0.09587699 0.08408617
[1] robust ASEs(RASEs)
[1] 0.08050884 0.11482934 0.09026281 0.10135764 0.09587699 0.08408617
[1] RASE/ASE
[1] 1 1 1 1 1 1
[1] ...........................................................
[1] estimation method=2
[1] parameter estimates
[1] -0.5900943 -1.5132182 -0.8172789 -0.5905505 -0.2549424  0.3108265
[1] information-based ASEs
[1] 0.08192023 0.11515865 0.08838669 0.10577443 0.09558068 0.08313713
[1] robust ASEs(RASEs)
[1] 0.09514767 0.12698725 0.09383876 0.12440861 0.10599820 0.08884008
[1] RASE/ASE
[1] 1.161 1.103 1.062 1.176 1.109 1.069
[1] ...........................................................
[1] estimation method=3
[1] parameter estimates
[1] -0.7159232 -1.5999395 -0.7950218 -0.7619474 -0.3011114  0.3526301
[1] information-based ASEs
[1] 0.08394540 0.11735047 0.08623481 0.11098265 0.09600212 0.08169733
[1] robust ASEs(RASEs)
[1] 0.1206917 0.1567767 0.0986794 0.1645183 0.1291253 0.0974380
[1] RASE/ASE
[1] 1.438 1.336 1.144 1.482 1.345 1.193
[1] ...........................................................
[1] estimation method=4
[1] parameter estimates
[1] -0.5425720 -1.4928240 -0.8356766 -0.5161433 -0.2564579  0.2832485
[1] information-based ASEs
```

```
[1] 0.08111389 0.11480501 0.08942373 0.10336824 0.09567980 0.08367823
[1] robust ASEs(RASEs)
[1] 0.08625639 0.11879942 0.09158328 0.10933041 0.09964655 0.08552722
[1] RASE/ASE
[1] 1.063 1.035 1.024 1.058 1.041 1.022
[1] ............................................................
[1] estimation method=5
[1] parameter estimates
[1] -0.4875521 -1.4873785 -0.8688151 -0.4101978 -0.2826759  0.2297618
[1] information-based ASEs
[1] 0.08008370 0.11505980 0.09087097 0.09975734 0.09610047 0.08440270
[1] robust ASEs(RASEs)
[1] 0.07682780 0.11382946 0.09007916 0.09843038 0.09411921 0.08439530
[1] RASE/ASE
[1] 0.959 0.989 0.991 0.987 0.979 1.000
[1] ............................................................
[1] estimation method=6
[1] parameter estimates
[1] -0.4823368 -1.4888431 -0.8729089 -0.3972004 -0.2874998  0.2220574
[1] information-based ASEs
[1] 0.07997520 0.11515828 0.09102568 0.09930713 0.09617257 0.08449414
[1] robust ASEs(RASEs)
[1] 0.07594985 0.11393049 0.09023049 0.09817771 0.09386881 0.08476143
[1] RASE/ASE
[1] 0.950 0.989 0.991 0.989 0.976 1.003
[1] ............................................................
[1] estimation method=7
[1] parameter estimates
[1] -0.4741725 -1.4925053 -0.8797197 -0.3743045 -0.2964883  0.2079566
[1] information-based ASEs
[1] 0.07979862 0.11536623 0.09127306 0.09851673 0.09630696 0.08466085
[1] robust ASEs(RASEs)
[1] 0.07465291 0.11454079 0.09077221 0.09836340 0.09374491 0.08571429
[1] RASE/ASE
[1] 0.936 0.993 0.995 0.998 0.973 1.012
[1] ............................................................
[1] estimation method=8
[1] parameter estimates
[1] -0.4609923 -1.5040889 -0.8909048 -0.3252497 -0.3167800  0.1751952
[1] information-based ASEs
[1] 0.07949446 0.11594063 0.09165647 0.09684356 0.09660616 0.08505676
[1] robust ASEs(RASEs)
[1] 0.07390511 0.11795395 0.09360268 0.10153685 0.09544380 0.08904868
[1] RASE/ASE
[1] 0.930 1.017 1.021 1.048 0.988 1.047
[1]
[1] ============================================================
[1] Model 2A (X,Y,linA)
[1] "number of parameters=7"
[1] the pattern of parameter use (1;use,0:not use)
```

```
 p-use 1  p-use 2  p-use 3  p-use 4  p-use 5  p-use 6  p-use 7  p-use 8
      1        1        1        1        1        1        1        0
 p-use 9 p-use 10 p-use 11 p-use 12 p-use 13 p-use 14 p-use 15 p-use 16
      0        0        0        0        0        0        0        0
[1] the (reduced) design matrix for the model
      [,1] [,2] [,3] [,4] [,5] [,6]   [,7]
 [1,]    1    0    0    1    0    0  2.380
 [2,]    1    0    0    0    1    0  0.996
 [3,]    1    0    0    0    0    1 -0.387
 [4,]    1    0    0    0    0    0 -1.770
 [5,]    0    1    0    1    0    0  1.065
 [6,]    0    1    0    0    1    0  0.446
 [7,]    0    1    0    0    0    1 -0.173
 [8,]    0    1    0    0    0    0 -0.792
 [9,]    0    0    1    1    0    0 -0.250
[10,]    0    0    1    0    1    0 -0.105
[11,]    0    0    1    0    0    1  0.041
[12,]    0    0    1    0    0    0  0.186
[13,]    0    0    0    1    0    0 -1.565
[14,]    0    0    0    0    1    0 -0.655
[15,]    0    0    0    0    0    1  0.254
[16,]    0    0    0    0    0    0  1.164
[1] ...........................................................
[1] methods (1:ML,2:GM^2,3-8:Lambda=(-2,-0.5,0.5,2/3,1,2))
[1] ...........................................................
[1] estimation method=1
[1] parameter estimates
[1] -0.5789915 -1.4473969 -0.7938966 -0.5298733 -0.1907992  0.3519626
[7]  0.3740116
[1] information-based ASEs
[1] 0.08657981 0.11664395 0.09141289 0.10810097 0.09961080 0.08660537
[7] 0.03694896
[1] robust ASEs(RASEs)
[1] 0.08737509 0.11595021 0.09150934 0.10813442 0.10030711 0.08794389
[7] 0.03852044
[1] RASE/ASE
[1] 1.009 0.994 1.001 1.000 1.007 1.015 1.043
[1] ...........................................................
[1] estimation method=2
[1] parameter estimates
[1] -0.5887809 -1.4539706 -0.8026313 -0.5296210 -0.1821819  0.3617084
[7]  0.3819869
[1] information-based ASEs
[1] 0.08681603 0.11680542 0.09159032 0.10844797 0.09982603 0.08674519
[7] 0.03715562
[1] robust ASEs(RASEs)
[1] 0.08872597 0.11717539 0.09275217 0.10968989 0.10194202 0.08939896
[7] 0.03955495
[1] RASE/ASE
[1] 1.022 1.003 1.013 1.011 1.021 1.031 1.065
```

```
[1] ............................................................
[1] estimation method=3
[1] parameter estimates
[1] -0.5994112 -1.4586835 -0.8098657 -0.5299413 -0.1725591  0.3729617
[7]  0.3912852
[1] information-based ASEs
[1] 0.08711539 0.11692333 0.09175562 0.10886197 0.10007788 0.08690411
[7] 0.03741070
[1] robust ASEs(RASEs)
[1] 0.09208819 0.11969709 0.09512518 0.11389433 0.10544635 0.09237493
[7] 0.04195529
[1] RASE/ASE
[1] 1.057 1.024 1.037 1.046 1.054 1.063 1.121
[1] ............................................................
[1] estimation method=4
[1] parameter estimates
[1] -0.5837653 -1.4508834 -0.7983998 -0.5296878 -0.1865967  0.3566578
[7]  0.3778397
[1] information-based ASEs
[1] 0.08669044 0.11672972 0.09150237 0.10826685 0.09971376 0.08667312
[7] 0.03704671
[1] robust ASEs(RASEs)
[1] 0.08782081 0.11636855 0.09197630 0.10861013 0.10091251 0.08851810
[7] 0.03890969
[1] RASE/ASE
[1] 1.013 0.997 1.005 1.003 1.012 1.021 1.050
[1] ............................................................
[1] estimation method=5
[1] parameter estimates
[1] -0.5744749 -1.4435798 -0.7892146 -0.5301674 -0.1948251  0.3475958
[7]  0.3704864
[1] information-based ASEs
[1] 0.08648300 0.11654937 0.09132323 0.10794907 0.09951697 0.08654143
[7] 0.03686142
[1] robust ASEs(RASEs)
[1] 0.08731812 0.11595989 0.09136150 0.10816401 0.10007626 0.08761312
[7] 0.03831483
[1] RASE/ASE
[1] 1.010 0.995 1.000 1.002 1.006 1.012 1.039
[1] ............................................................
[1] estimation method=6
[1] parameter estimates
[1] -0.5730277 -1.4422458 -0.7876287 -0.5302888 -0.1961331  0.3462087
[7]  0.3693756
[1] information-based ASEs
[1] 0.08645358 0.11651613 0.09129352 0.10790135 0.09948765 0.08652088
[7] 0.03683434
[1] robust ASEs(RASEs)
[1] 0.08737255 0.11606124 0.09138258 0.10826709 0.10007422 0.08754811
[7] 0.03827667
```

```
[1] RASE/ASE
[1] 1.011 0.996 1.001 1.003 1.006 1.012 1.039
[1] ............................................................
[1] estimation method=7
[1] parameter estimates
[1] -0.5702210 -1.4394996 -0.7844361 -0.5305663 -0.1987036  0.3435314
[7]  0.3672464
[1] information-based ASEs
[1] 0.08639873 0.11644739 0.09123456 0.10781007 0.09943186 0.08648083
[7] 0.03678314
[1] robust ASEs(RASEs)
[1] 0.08757530 0.11641047 0.09152693 0.10858823 0.10017242 0.08747703
[7] 0.03823487
[1] RASE/ASE
[1] 1.014 1.000 1.003 1.007 1.007 1.012 1.039
[1] ............................................................
[1] estimation method=8
[1] parameter estimates
[1] -0.5624890 -1.4308006 -0.7748655 -0.5316748 -0.2060950  0.3362209
[7]  0.3615483
[1] information-based ASEs
[1] 0.08626263 0.11622710 0.09106401 0.10756731 0.09928578 0.08636802
[7] 0.03665094
[1] robust ASEs(RASEs)
[1] 0.08876222 0.11855830 0.09270210 0.11019863 0.10117436 0.08765867
[7] 0.03828761
[1] RASE/ASE
[1] 1.029 1.020 1.018 1.024 1.019 1.015 1.045
[1]
[1] ============================================================
[1] Model 2B (X,Y,linB)
[1] "number of parameters=7"
[1] the pattern of parameter use (1;use,0:not use)
 p-use 1  p-use 2  p-use 3  p-use 4  p-use 5  p-use 6  p-use 7  p-use 8
      1        1        1        1        1        1        1        0
 p-use 9 p-use 10 p-use 11 p-use 12 p-use 13 p-use 14 p-use 15 p-use 16
      0        0        0        0        0        0        0        0
[1] the (reduced) design matrix for the model
      [,1] [,2] [,3] [,4] [,5] [,6]   [,7]
 [1,]    1    0    0    1    0    0  2.244
 [2,]    1    0    0    0    1    0  1.286
 [3,]    1    0    0    0    0    1 -0.631
 [4,]    1    0    0    0    0    0 -1.590
 [5,]    0    1    0    1    0    0  1.326
 [6,]    0    1    0    0    1    0  0.759
 [7,]    0    1    0    0    0    1 -0.373
 [8,]    0    1    0    0    0    0 -0.939
 [9,]    0    0    1    1    0    0 -0.512
[10,]    0    0    1    0    1    0 -0.293
[11,]    0    0    1    0    0    1  0.144
```

```
[12,]    0    0    1    0    0    0  0.363
[13,]    0    0    0    1    0    0 -1.431
[14,]    0    0    0    0    1    0 -0.820
[15,]    0    0    0    0    0    1  0.403
[16,]    0    0    0    0    0    0  1.014
[1] ............................................................
[1] methods (1:ML,2:GM^2,3-8:Lambda=(-2,-0.5,0.5,2/3,1,2))
[1] ............................................................
[1] estimation method=1
[1] parameter estimates
[1] -0.5795756 -1.4688187 -0.8042863 -0.5286763 -0.2282483  0.3255431
[7]  0.3721345
[1] information-based ASEs
[1] 0.08650037 0.11687094 0.09095776 0.10743179 0.09923428 0.08534599
[7] 0.03587000
[1] robust ASEs(RASEs)
[1] 0.08703396 0.11638053 0.09070466 0.10673898 0.09977986 0.08565320
[7] 0.03643739
[1] RASE/ASE
[1] 1.006 0.996 0.997 0.994 1.005 1.004 1.016
[1] ............................................................
[1] estimation method=2
[1] parameter estimates
[1] -0.5838589 -1.4710607 -0.8195749 -0.5241862 -0.2262246  0.3330449
[7]  0.3745184
[1] information-based ASEs
[1] 0.08649825 0.11676808 0.09128244 0.10752307 0.09941259 0.08538768
[7] 0.03589420
[1] robust ASEs(RASEs)
[1] 0.08755863 0.11679762 0.09210622 0.10755621 0.10100091 0.08642853
[7] 0.03673744
[1] RASE/ASE
[1] 1.012 1.000 1.009 1.000 1.016 1.012 1.023
[1] ............................................................
[1] estimation method=3
[1] parameter estimates
[1] -0.5881048 -1.4729835 -0.8345630 -0.5198470 -0.2252611  0.3393236
[7]  0.3769678
[1] information-based ASEs
[1] 0.08650388 0.11666130 0.09160863 0.10758826 0.09958929 0.08542453
[7] 0.03591541
[1] robust ASEs(RASEs)
[1] 0.08877127 0.11775366 0.09504082 0.10952173 0.10330792 0.08781803
[7] 0.03736947
[1] RASE/ASE
[1] 1.026 1.009 1.037 1.018 1.037 1.028 1.040
[1] ............................................................
[1] estimation method=4
[1] parameter estimates
[1] -0.5817118 -1.4699843 -0.8119540 -0.5264203 -0.2271009  0.3294318
```

```
[7]  0.3733124
[1] information-based ASEs
[1] 0.08649814 0.11682038 0.09111960 0.10748054 0.09932315 0.08536741
[7] 0.03588236
[1] robust ASEs(RASEs)
[1] 0.08721281 0.11651657 0.09121055 0.10700095 0.10025558 0.08596129
[7] 0.03654874
[1] RASE/ASE
[1] 1.008 0.997 1.001 0.996 1.009 1.007 1.019
[1] ...........................................................
[1] estimation method=5
[1] parameter estimates
[1] -0.5774702 -1.4675597 -0.7966044 -0.5309379 -0.2296645  0.3214168
[7]  0.3709943
[1] information-based ASEs
[1] 0.08650519 0.11691909 0.09079755 0.10737731 0.09914691 0.08532352
[7] 0.03585734
[1] robust ASEs(RASEs)
[1] 0.08700783 0.11639425 0.09058094 0.10675852 0.09956373 0.08551005
[7] 0.03639430
[1] RASE/ASE
[1] 1.006 0.996 0.998 0.994 1.004 1.002 1.015
[1] ...........................................................
[1] estimation method=6
[1] parameter estimates
[1] -0.5767784 -1.4671190 -0.7940468 -0.5316902 -0.2301947  0.3199959
[7]  0.3706241
[1] information-based ASEs
[1] 0.08650740 0.11693450 0.09074462 0.10735798 0.09911826 0.08531581
[7] 0.03585309
[1] robust ASEs(RASEs)
[1] 0.08702981 0.11643214 0.09062139 0.10682358 0.09954691 0.08549960
[7] 0.03639331
[1] RASE/ASE
[1] 1.006 0.996 0.999 0.995 1.004 1.002 1.015
[1] ...........................................................
[1] estimation method=7
[1] parameter estimates
[1] -0.5754138 -1.4662068 -0.7889445 -0.5331889 -0.2313394  0.3170961
[7]  0.3698998
[1] information-based ASEs
[1] 0.08651274 0.11696425 0.09063964 0.10731771 0.09906187 0.08530010
[7] 0.03584459
[1] robust ASEs(RASEs)
[1] 0.08711505 0.11655697 0.09081833 0.10703437 0.09959224 0.08553449
[7] 0.03640892
[1] RASE/ASE
[1] 1.007 0.997 1.002 0.997 1.005 1.003 1.016
[1] ...........................................................
[1] estimation method=8
```

```
[1] parameter estimates
[1] -0.5715111 -1.4632411 -0.7738462 -0.5375967 -0.2353878  0.3080624
[7]  0.3678682
[1] information-based ASEs
[1] 0.08653597 0.11704401 0.09033343 0.10718616 0.09890161 0.08525108
[7] 0.03581951
[1] robust ASEs(RASEs)
[1] 0.08763916 0.11729764 0.09222437 0.10820276 0.10030422 0.08606547
[7] 0.03656742
[1] RASE/ASE
[1] 1.013 1.002 1.021 1.009 1.014 1.010 1.021
[1]
[1] ==============================================================
[1] Model 3 (XY)(saturated)
[1] "number of parameters=15"
[1] the pattern of parameter use (1;use,0:not use)
 p-use 1  p-use 2  p-use 3  p-use 4  p-use 5  p-use 6  p-use 7  p-use 8
       1        1        1        1        1        1        0        1
 p-use 9 p-use 10 p-use 11 p-use 12 p-use 13 p-use 14 p-use 15 p-use 16
       1        1        1        1        1        1        1        1
[1] the (reduced) design matrix for the model
      [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10] [,11] [,12] [,13]
 [1,]    1    0    0    1    0    0    1    0    0     0     0     0     0
 [2,]    1    0    0    0    1    0    0    1    0     0     0     0     0
 [3,]    1    0    0    0    0    1    0    0    1     0     0     0     0
 [4,]    1    0    0    0    0    0    0    0    0     0     0     0     0
 [5,]    0    1    0    1    0    0    0    0    0     1     0     0     0
 [6,]    0    1    0    0    1    0    0    0    0     0     1     0     0
 [7,]    0    1    0    0    0    1    0    0    0     0     0     1     0
 [8,]    0    1    0    0    0    0    0    0    0     0     0     0     0
 [9,]    0    0    1    1    0    0    0    0    0     0     0     0     1
[10,]    0    0    1    0    1    0    0    0    0     0     0     0     0
[11,]    0    0    1    0    0    1    0    0    0     0     0     0     0
[12,]    0    0    1    0    0    0    0    0    0     0     0     0     0
[13,]    0    0    0    1    0    0    0    0    0     0     0     0     0
[14,]    0    0    0    0    1    0    0    0    0     0     0     0     0
[15,]    0    0    0    0    0    1    0    0    0     0     0     0     0
[16,]    0    0    0    0    0    0    0    0    0     0     0     0     0
      [,14] [,15]
 [1,]     0     0
 [2,]     0     0
 [3,]     0     0
 [4,]     0     0
 [5,]     0     0
 [6,]     0     0
 [7,]     0     0
 [8,]     0     0
 [9,]     0     0
[10,]     1     0
[11,]     0     1
```

```
[12,]    0    0
[13,]    0    0
[14,]    0    0
[15,]    0    0
[16,]    0    0
[1] ..........................................................
[1] methods (1:ML,2:GM^2,3-8:Lambda=(-2,-0.5,0.5,2/3,1,2))
[1] ..........................................................
[1] estimation method=1
[1] parameter estimates
 [1] -1.41865965 -2.41718848 -1.31857619 -1.47272687 -1.01319454  0.02515856
 [7]  2.22958986  1.59511608  0.43159984  2.01172337  1.63223375  0.70307994
[13]  0.62542901  0.98909699  0.54123692
[1] information-based ASEs
 [1] 0.1807905 0.2789229 0.1737209 0.1847896 0.1546392 0.1121633 0.2698308
 [8] 0.2548234 0.2357182 0.3837218 0.3657915 0.3442266 0.3369158 0.2685387
[15] 0.2233912
[1] robust ASEs(RASEs)
 [1] 0.1807905 0.2789229 0.1737209 0.1847896 0.1546392 0.1121633 0.2698308
 [8] 0.2548234 0.2357182 0.3837218 0.3657915 0.3442266 0.3369158 0.2685387
[15] 0.2233912
[1] RASE/ASE
 [1] 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
[1] ..........................................................
[1] estimation method=2
[1] parameter estimates
 [1] -1.41865965 -2.41718848 -1.31857619 -1.47272687 -1.01319454  0.02515856
 [7]  2.22958986  1.59511608  0.43159984  2.01172337  1.63223375  0.70307994
[13]  0.62542901  0.98909699  0.54123692
[1] information-based ASEs
 [1] 0.1807905 0.2789229 0.1737209 0.1847896 0.1546392 0.1121633 0.2698308
 [8] 0.2548234 0.2357182 0.3837218 0.3657915 0.3442266 0.3369158 0.2685387
[15] 0.2233912
[1] robust ASEs(RASEs)
 [1] 0.1807905 0.2789229 0.1737209 0.1847896 0.1546392 0.1121633 0.2698308
 [8] 0.2548234 0.2357182 0.3837218 0.3657915 0.3442266 0.3369158 0.2685387
[15] 0.2233912
[1] RASE/ASE
 [1] 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
[1] ..........................................................
[1] estimation method=3
[1] parameter estimates
 [1] -1.41865965 -2.41718848 -1.31857619 -1.47272687 -1.01319454  0.02515856
 [7]  2.22958986  1.59511608  0.43159984  2.01172337  1.63223375  0.70307994
[13]  0.62542901  0.98909699  0.54123692
[1] information-based ASEs
 [1] 0.1807905 0.2789229 0.1737209 0.1847896 0.1546392 0.1121633 0.2698308
 [8] 0.2548234 0.2357182 0.3837218 0.3657915 0.3442266 0.3369158 0.2685387
[15] 0.2233912
[1] robust ASEs(RASEs)
```

```
 [1] 0.1807905 0.2789229 0.1737209 0.1847896 0.1546392 0.1121633 0.2698308
 [8] 0.2548234 0.2357182 0.3837218 0.3657915 0.3442266 0.3369158 0.2685387
[15] 0.2233912
[1] RASE/ASE
 [1] 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
[1] ............................................................
[1] estimation method=4
[1] parameter estimates
 [1] -1.41865965 -2.41718848 -1.31857619 -1.47272687 -1.01319454  0.02515856
 [7]  2.22958986  1.59511608  0.43159984  2.01172337  1.63223375  0.70307994
[13]  0.62542901  0.98909699  0.54123692
[1] information-based ASEs
 [1] 0.1807905 0.2789229 0.1737209 0.1847896 0.1546392 0.1121633 0.2698308
 [8] 0.2548234 0.2357182 0.3837218 0.3657915 0.3442266 0.3369158 0.2685387
[15] 0.2233912
[1] robust ASEs(RASEs)
 [1] 0.1807905 0.2789229 0.1737209 0.1847896 0.1546392 0.1121633 0.2698308
 [8] 0.2548234 0.2357182 0.3837218 0.3657915 0.3442266 0.3369158 0.2685387
[15] 0.2233912
[1] RASE/ASE
 [1] 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
[1] ............................................................
[1] estimation method=5
[1] parameter estimates
 [1] -1.41865965 -2.41718848 -1.31857619 -1.47272687 -1.01319454  0.02515856
 [7]  2.22958986  1.59511608  0.43159984  2.01172337  1.63223375  0.70307994
[13]  0.62542901  0.98909699  0.54123692
[1] information-based ASEs
 [1] 0.1807905 0.2789229 0.1737209 0.1847896 0.1546392 0.1121633 0.2698308
 [8] 0.2548234 0.2357182 0.3837218 0.3657915 0.3442266 0.3369158 0.2685387
[15] 0.2233912
[1] robust ASEs(RASEs)
 [1] 0.1807905 0.2789229 0.1737209 0.1847896 0.1546392 0.1121633 0.2698308
 [8] 0.2548234 0.2357182 0.3837218 0.3657915 0.3442266 0.3369158 0.2685387
[15] 0.2233912
[1] RASE/ASE
 [1] 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
[1] ............................................................
[1] estimation method=6
[1] parameter estimates
 [1] -1.41865965 -2.41718848 -1.31857619 -1.47272687 -1.01319454  0.02515856
 [7]  2.22958986  1.59511608  0.43159984  2.01172337  1.63223375  0.70307994
[13]  0.62542901  0.98909699  0.54123692
[1] information-based ASEs
 [1] 0.1807905 0.2789229 0.1737209 0.1847896 0.1546392 0.1121633 0.2698308
 [8] 0.2548234 0.2357182 0.3837218 0.3657915 0.3442266 0.3369158 0.2685387
[15] 0.2233912
[1] robust ASEs(RASEs)
 [1] 0.1807905 0.2789229 0.1737209 0.1847896 0.1546392 0.1121633 0.2698308
 [8] 0.2548234 0.2357182 0.3837218 0.3657915 0.3442266 0.3369158 0.2685387
```

```
[15] 0.2233912
[1] RASE/ASE
 [1] 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
[1] ...........................................................
[1] estimation method=7
[1] parameter estimates
 [1] -1.41865965 -2.41718848 -1.31857619 -1.47272687 -1.01319454  0.02515856
 [7]  2.22958986  1.59511608  0.43159984  2.01172337  1.63223375  0.70307994
[13]  0.62542901  0.98909699  0.54123692
[1] information-based ASEs
 [1] 0.1807905 0.2789229 0.1737209 0.1847896 0.1546392 0.1121633 0.2698308
 [8] 0.2548234 0.2357182 0.3837218 0.3657915 0.3442266 0.3369158 0.2685387
[15] 0.2233912
[1] robust ASEs(RASEs)
 [1] 0.1807905 0.2789229 0.1737209 0.1847896 0.1546392 0.1121633 0.2698308
 [8] 0.2548234 0.2357182 0.3837218 0.3657915 0.3442266 0.3369158 0.2685387
[15] 0.2233912
[1] RASE/ASE
 [1] 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
[1] ...........................................................
[1] estimation method=8
[1] parameter estimates
 [1] -1.41865965 -2.41718848 -1.31857619 -1.47272687 -1.01319454  0.02515856
 [7]  2.22958986  1.59511608  0.43159984  2.01172337  1.63223375  0.70307994
[13]  0.62542901  0.98909699  0.54123692
[1] information-based ASEs
 [1] 0.1807905 0.2789229 0.1737209 0.1847896 0.1546392 0.1121633 0.2698308
 [8] 0.2548234 0.2357182 0.3837218 0.3657915 0.3442266 0.3369158 0.2685387
[15] 0.2233912
[1] robust ASEs(RASEs)
 [1] 0.1807905 0.2789229 0.1737209 0.1847896 0.1546392 0.1121633 0.2698308
 [8] 0.2548234 0.2357182 0.3837218 0.3657915 0.3442266 0.3369158 0.2685387
[15] 0.2233912
[1] RASE/ASE
 [1] 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
   user  system elapsed
   7.14    0.03    7.17
[1] Thu Dec 06 10:38:56 2018
```