Application of the edge-based finite element method for fusion plasma simulations

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I. Extended Abstract

Fusion is a clean energy source which shows promise as a future nuclear energy resource. One of the ideas of the nuclear fusion on Earth is that the very high temperature ionized particles forming a plasma can be controlled by a magnetic field, called magnetically confined plasma. This is essential, because no material can be sustained against the high temperature reached in a fusion reactor. In the ideal case, the plasmas in such reactors would remain well confined within the magnetic field in order to allow their core to reach the temperature needed for thermonuclear fusion. Unfortunately such a quiescent confined state is, in general, not observed, and both turbulent small scale motion and collective bulk motion lead to complicated dynamics that cannot be computed analytically and require numerical approaches. The goal of the work is to develop a useful computational tool for fusion applications using the infrastructures given by the PETGEM code [1] which is based on edge elements, a preferable numerical scheme for electromagnetic physics including plasma physics rather than the nodal element approach. The goal of the work is to develop an user-friendly code based on Python which is one of the remarkable growing major programming languages so that the code can be applicable for industrial applications.

The abstract introduces PETGEM code in Section 2. Section 3 shows the achievement of the work such as the mesh generation and the implementation of the initial profiles for the input of PETGEM. The conclusion and the perspectives of the work are summarised in Section 4.

II. PETGEM: based on Edge Finite Element Method (EFEM)

The Parallel Edge-based Tool for Geophysical Electromagnetic Modelling (PETGEM) is a Python HPC scalable tool based on Nédélec Finite Element Method. This code has been developed as open-source (under GPLv3 license) at Computer Applications in Science & Engineering (CASE) of the Barcelona Supercomputing Center (BSC). PETGEM is aimed to solve the marine Controlled-Source Electromagnetic method (CSEM) which is an important technique for reducing ambiguities in data interpretation for hydrocarbon exploration. So as to solve CSEM, one must solve Maxwell's equations:

$$\nabla \times \mathbf{E} = iw\mu_0 \mathbf{H} \qquad \nabla \times \mathbf{H} = \mathbf{J}_s + \sigma \mathbf{E} \tag{1}$$

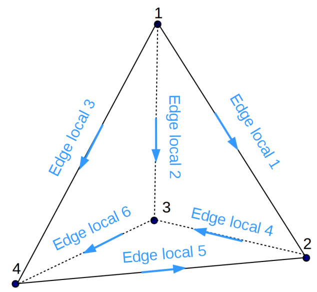

Fig. 1. Tetrahedron discretization.

where ω is the frequency and σ the electric conductivity. Using a perturbation approach ($\mathbf{E} = \mathbf{E}_s + \mathbf{E}_p$ and $\sigma = \sigma_s + \triangle\sigma$) and following the work of [2] the equations to solve become a single equation:

$$\nabla \times \nabla \times \mathbf{E}_s + iw\mu_0\sigma\mathbf{E}_s = -iw\mu_0 \triangle\sigma\ \mathbf{E}_p \tag{2}$$

where $\mathbf{p}$ refers to primary and $\mathbf{s}$ to secondary field. The primary part is the input and the secondary is the output of the PETGEM calculation. In order to obtain $\mathbf{E_s}$ for unstructured meshes, Nédélec Finite Element (a type of edge elements) offers a good balance between accuracy and number of degrees of freedom (DOFs). Nédélec formulation uses vector basis functions defined on the edges of the corresponding elements. As Fig. 1 shows, the discretization method selected is a tetrahedral mesh as these meshes are the easiest to scale-up to very large domains or arbitrary shape. PETGEM is an HPC code due to the fact that Nédélec elements offer a good scalability and it is exploited through the Python Package Petsc4py [3].

III. Results

A. Generation of the finite element mesh

The mesh generation has been performed with GMSH [4] which allows to generate most of kinds of meshes and refine them. The arbitrary function of $\mathbf{E_p}$ is implemented in the generated mesh. Figure 2 shows the generated mesh for finite element method for cylindrical geometry plasma. The initial profile of the electric field $\mathbf{E_p} = -\nabla\phi$ where the electric potential ϕ has been successfully implemented. In the absence of velocity field $\mathbf{u}$ and the simple assumption of the uniform magnetic resistivity η, the profile of the current density $\mathbf{J}$ can be proportional to the electric field through Ohm's law $\mathbf{E} = \eta\mathbf{J} - \mathbf{u} \times \mathbf{B}$ where $\mathbf{B}$ indicates the magnetic field. The cylindrical geometry refers to the mirror plasma confinement

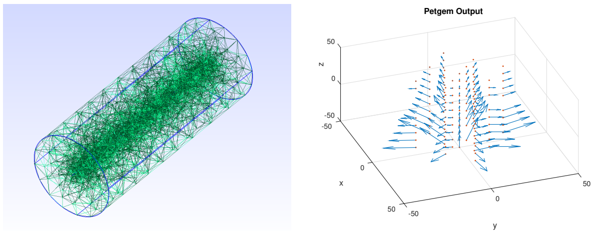

Fig. 2. (Left panel) The generated mesh of the inhomogeneous mesh density for cylindrical geometry. (Right panel) The implemented initial profile of the electric field $E_p = \nabla\phi$ where the electric potential $\phi = e^{-0.02z} \cdot e^{-(x/30)^2-(y/30)^2}$.

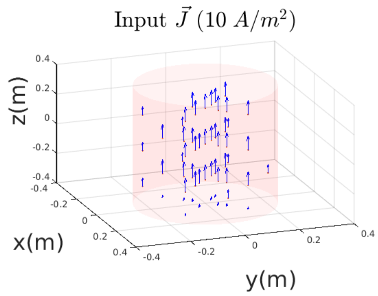

Fig. 3. Initial profile of the antenna current density to be solved by the full wave equation.

devices which are similar with SLPM [5], PANTA [6], etc. The plasma confined in the cylindrical geometry is a useful approach to study the plasma instabilities. The density of the simulation grid can be arbitrarily chosen in the simulation domain. The location of the mesh concentration can be chosen depending on the physics problem to be aimed to study.

B. Steady state plasma

The full wave equation in a magnetically confined plasma with an antenna current as a boundary condition is given by Eq. 3:

$$\begin{aligned} \nabla \times \nabla \times \mathbf{E} - \frac{w^2}{c^2}\mathbf{E} = \frac{4\pi wi}{c^2}\left(\mathbf{J^p} + \mathbf{J^a}\right) \\ \mathbf{J^p}(\mathbf{r}) = \int d\mathbf{r}'\sigma(\mathbf{r},\mathbf{r}')\mathbf{E}(\mathbf{r}') \end{aligned} \tag{3}$$

where $\mathbf{J^a}$ is the current of the antenna and σ is the conductivity tensor. This equation models the wave equation with a boundary condition corresponding to the current density introduced by the antenna while Eq. 2 is the perturbation solution of a wave propagating in the earth. Figure 3 shows the initial profile of the current density $\mathbf{J} = \mathbf{J^p} + \mathbf{J^a} = \hat{\mathbf{z}}/f(r)$ where $f(r) = (1 + (r/a)^{2\Lambda})^{1+1/\Lambda}$ and $a = 0.25$m is the radius of the cylinder and $\Lambda = 4$ [7]. The implemented current profile is reasonable approach to compute the initial profile of the current density and the electric field for fusion plasmas [8]. The time variation of the current density will be implemented in the future in order to investigate the interactions between the effect of the antenna and the plasma response.

IV. Conclusions and Perspectives

The work demonstrates the generation of the simulation mesh for finite element method for fusion plasma in the cylindrical geometry. The location and the density of the mesh concentration can be arbitrary adopted according to the physics problem which is aimed to investigate. The implementation of the initial field of any quantities such as electric field and current density, and the application of the reasonable current profile which is specifically aimed for the fusion plasma research have been carried out.

The future work is to solve the full wave equation in the cylindrical geometry which can be solved by current version of PETGEM in order to analyze the precise profile of the electric field and the current density in the steady state plasma. The long-term objective of the work is to go beyond the calculation of the steady state plasma, i.e. implementation of the time integration of the plasma dynamics, for example, magnetohydrodynamic (MHD) which is the combination of the electromagnetism system i.e. Maxwell's equations and the fluid system i.e. Navier-Stokes equation. The primitive approach is to develop the fluid modelling part considering an incompressible ($\nabla \cdot \mathbf{u} = 0$), diffusive model: $\frac{\partial \mathbf{u}}{\partial t} = -\nu \ \nabla \times \nabla \times \mathbf{u}$. is ongoing to be implemented in PETGEM.

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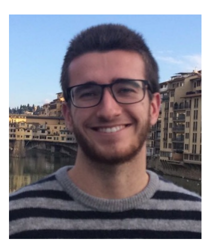

Marc Fuster Rullan is a Physics bachelor student at UAB. He is at his fourth year and is currently carrying out his bachelor thesis at the Fusion group at the Barcelona Supercomputing Center (BSC). He has won a gold medal at the international physics contest UPHYSICS. His actual work focuses on the computational modelling for fusion. More specifically, he is applying the Edge Finite Element Method code PETGEM to plasma physics.