# Robust Point-Location Method for Linear and High Order Meshes. Application to Particle Transport 

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#### Abstract

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## I. Extended Abstract

## II. Introduction

In computational geometry the point-location problem is a fundamental topic. In the Finite Elements Method (FEM) context, it is used to find which is the host element of a given point in the computational domain. This process is required in many application such as the measurement of flow properties on specific points (probes) in computational fluid dynamics (CFD), the projection from one mesh to an- other in adaptivity or for Lagrangian particle transport simulations.

## III. Point-location algorithm

In this work, an efficient solution to the point-location problem applied to FEM is presented. The robustness of the proposed approach is evaluated in the context of particle transport simulations in the respiratory system airways. Respiratory system simulations involve CFD and millions of transported particles along with tens or hundreds of thousands of time-steps [1]. As a consequence, the location process is one of the critical parts of the simulation. In other words, an efficient and robust inclusion test becomes essential not only for the accuracy of the results, but also to achieve a good computational efficiency.

Our algorithm is composed of four main steps: three consecutive filters are applied, followed by the evaluation of the iso-parametric coordinates of the point within the hosting element.

- Filter 1: Bin/Oct tree. A list of host element candidates is created using a bin/oct tree strategy [2] in the initial injection. After this, the list of candidates is only formed by direct neighbours of the previous host element.
- Filter 2: Bounding box of the element. The list of candidates is looped. The candidate elements which do not contain the target point within its bounding box are discarded.
- Filter 3: The inclusion test. An inclusion test method based on ray casting [3] is used to check if the candidate element is the host element between the remaining candidates. This method is based on counting ray intersections with edges or nodes and apply the
odd/even parity rule (if the number of intersections is odd, the point belongs to the element). Care must be taken to avoid a ray intersections an edge of the element because invalidates the parity rule.
- Calculation: Iso-parametric coordinates. Once the host element is known, the iso-parametric coordinates of the point inside the element can be calculated. To solve the shape function equation which allow to transform global coordinates into local ones, a Newton-Raphson iterator is used.

In figure 1, a flowchart is shown outlining the aforesaid steps.


Fig. 1. Flowchart of the point-location process.

## IV. Hihg order meshes

In the particular case of high order meshes, the ray casting strategy has been modified because of the peculiarites of intersection calculus for high order element, which analytic solution may become too complicated or not exist. For this reason, a root finding algorithm comes as the most obvious solution. Thus, a second Newton-Rapshon iterative approach is included.

## V. Conclusions and perspectives

In this work, special emphasis is put on two aspects. First, on the numerical and implementation details that ensure the robustness and efficiency. Second, on the peculiarities of intersection calculus for high order elements, showing a new method based on a modified ray casting strategy which has been proved to properly work with first order meshes.

Both points together allow us to introduce a general solution to the point-location problem, suitable to large and high order meshes.

## References

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Edgar Olivares was born in Barcelona in 1985 He received a B.Sc. degree in Physics in UB and a M.Sc. degree in Computational Physics in UPC. He is about to finish his PhD about Lagrangian particle transport simulations in BSC and has recently moved to Fusion group, also in BSC. He has experience in particle transport algorithms, using them in different applications such as Computational Fluid Dynamics (CFD) or plasma simulations. His work has been always focused on a High Performance Computing (HPC) environment, making him developing applied mathematics and computational skills.

