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RESEARCH ARTICLE

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Key Points:

- The validity of Agarwal's method under ideal and non-ideal conditions is analyzed
- An alternative method is developed to overcome Agarwal's limitations, leading to a better interpretation, especially for early times data
- A new equivalent time is proposed to take into consideration the variable pumping rate during the recovery data interpretation process

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Generalizing Agarwal's Method for the Interpretation of Recovery Tests Under Non-Ideal Conditions

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Abstract Pumping tests are performed during aquifer characterization to gain conceptual understanding about the system through diagnostic plots and to estimate hydraulic properties. Recovery tests consist of measuring head response in observation and/or pumping wells after pumping termination. They are especially useful when the pumping rate cannot be accurately controlled. They have been traditionally interpreted using Theis' recovery method, which yields robust estimates of effective transmissivity but does not provide information about the conceptual model. Agarwal proposed a method that has become standard in the oil industry, to obtain both early and late time reservoir responses to pumping from recovery data. However, the validity of the method has only been tested to a limited extent. In this work, we analyze Agarwal's method in terms of both drawdowns and log derivatives for non-ideal conditions: leaky aquifer, presence of boundaries, and one-dimensional flow. Our results show that Agarwal's method provides excellent recovery plots (i.e., the drawdown curve that would be obtained during pumping) and parameter estimates for nearly all aquifer conditions, provided that a constant pumping rate is used and the log derivative at the end of pumping is constant, which is too limiting for groundwater hydrology practice, where observation wells are usually monitored. We generalize Agarwal's method by (1) deriving an improved equivalent time for time-dependent pumping rate and (2) proposing to recover drawdown curves by extrapolating the pumping phase drawdowns. These yield excellent diagnostic plots, thus facilitating the conceptual model analysis for a broad range of conditions.

1. Introduction

Hydraulic testing is the most widely used technique for aquifer systems characterization and the only one providing direct estimates of aquifer parameters through the interpretation of the aquifer response to pumping or other hydraulic perturbations. Well testing results are a function of the range and the quality of the drawdown and rate data available and of the approach used for their interpretation (Gringarten, 2008).

Pumping test interpretation emerged largely from the Theis (1935) analytical solution for the drawdown caused in an ideal homogeneous, infinite, and confined aquifer by pumping at a constant rate Q from a fully penetrating well. A feature of this solution is that, when sufficient time has elapsed since the beginning of pumping, the drawdown increases linearly with the logarithm of time. This feature prompted Cooper and Jacob (1946; CJ in the following) to develop the straight line method for applying Theis' method through a manageable logarithmic approximation of the analytical solution, which works very well also for a broad range of conditions, including heterogeneous formations (Halford et al., 2006; Meier et al., 1998).

Rereading the paper of Theis (1935) is joyful, because he himself identified the numerous limitations of his solution (we suspect that his USGS colleagues and reviewers must have helped). In reality, aquifers are rarely homogeneous or fully confined and they have boundaries. Storage release is not instantaneous but delayed with respect to head variations. Wells do not fully penetrate the aquifer and may have significant storage. Over the years, hydrologists and oil engineers developed numerous analytical solutions to overcome these limitations (see, e.g., Kruseman & de Ridder, 1990). While the main purpose of these methods was to estimate transmissivity T and storage coefficient S, they realized that drawdown curves contain a wealth of information about the well and the aquifer beyond the actual value of hydraulic parameters. Unfortunately, the large number of solutions and the subtle variations among them made it difficult to identify which one is best.

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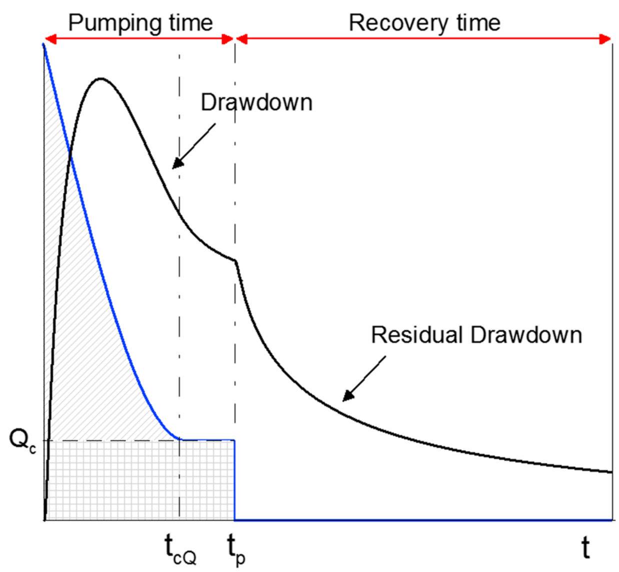

Figure 1. Drawdown and drawdown residual as a consequence of a pumping test carried out applying a variable pumping rate. The pumping rate decrease with the time, becoming constant for a while after pumping shutdown.

Prompted by the need to emphasize conceptual model identification, Bourdet et al. (1983, 1989) proposed using diagnostic plots (simultaneous plot of drawdowns and log derivatives) to highlight subtle variations in the aquifer response to pumping (Renard et al., 2009). Diagnostic plots are useful, because they complement other information (local geology, geophysics, well logs, etc.) in identifying non-ideal conditions during the pumping test. Given the qualitative nature of the conceptual model selection, it is nice to be able to examine diagnostic plots that can be compared to those of standard models or simply be used to assess how the resistance to flow evolves away from the pumping well.

But precisely because of their high sensitivity, log derivatives require exquisite test performance. While methods can be used to obtain smooth log derivatives (Ramos et al., 2017), they require a constant pumping rate, which is difficult to guarantee. Pumping rate can change for numerous reasons. For one thing, well owners do not care for transmissivity but for well production, which can be improved through development and ascertained through step pumping. Even when a pumping test is performed, maintaining a constant pumping rate during the whole test may be difficult. Pump efficiency may cause the pumping rate to decrease over time (Figure 1), variations of well efficiency often cause the pumping rate to increase over time, and electrical problems can cause short black outs (zero flow rate). A constant rate may prove impossible in low permeability formations, when the well often goes dry or when pumping by air lift. Things may get even worse when trying to interpret the hydraulic response to fracking. What can be done in all these cases is to monitor pumping rate and drawdowns recovery.

Recovery data (i.e., residual drawdowns measured after pumping has stopped) are much less noisy than pumping data because they are subject to less external perturbations during data acquisition (Figure 1). Specifically, pumping rate variability does not affect the aquifer response directly but only indirectly. However, residual drawdowns are affected by environmental fluctuations (e.g., seasonal changes in recharge, tidal effects, uncontrolled nearby pumping, and stage changes in close rivers), which usually dominate the late time recovery and must be filtered out prior to interpretation (see, e.g., Halford et al., 2012). The traditional method for recovery interpretation was also proposed by Theis (1935). The method is quite robust in estimating effective transmissivity values over a region that grows with the duration of pumping (Copty et al., 2011; Willmann et al., 2007) but lacks information about the storage coefficient. Numerous methods have been introduced to overcome this limitation (Agarwal, 1980; Ashjari, 2013; Ballukraya & Sharma, 1991; Banton & Bangoy, 1996; Chenaf & Chapuis, 2002; Çimen, 2015; Zheng et al., 2005), which suggests that recovery data contain information similar to that of the pumping phase of the test. Most of these methods are straight line methods based on CJ's approximation that allow evaluating hydraulic parameters but do not yield any information about the conceptual model. Yet Agarwal (1980) introduced a method, based on CJ's approximation, to reproduce the response of a pumping test using recovery test data. In other words, his method permits plotting recovery data as if they resulted from a constant pumping rate, facilitating not only the estimation of hydraulic parameters but also conceptual model assessment.

Given its simplicity, Agarwal's method should be considered as a very effective method for well test data interpretation. In fact, it has become the method of choice in the oil industry. Agarwal (1980) showed that his method reproduced the type curves of Earlougher and Kersch (1974) and Gringarten et al. (1979) that account for skin effect and wellbore storage. He also tested the validity for the case of a well intersected by a vertical fracture. However, the method lacks theoretical support for conditions other than those of CJ.

The objective of this work is to (1) analyze the conditions under which Agarwal's method is valid, to (2) generalize it for variable pumping rate, and to (3) propose alternative methods for recovery data interpretation.

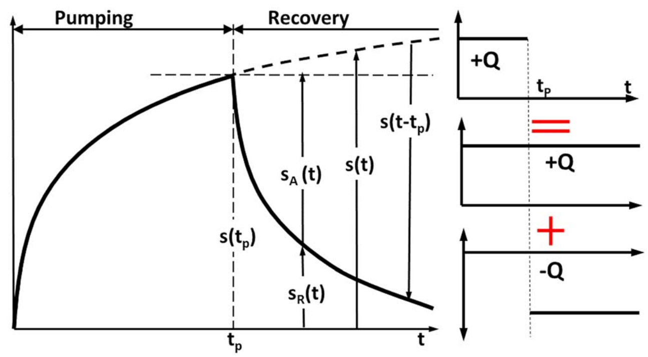

Figure 2. Pumping and recovery test. The left graph displays the residual drawdown, s_R, used in Theis recovery method and Agarwal's drawdown, s_A. Both can be computed by superposition (right) of a continuous pumping (+Q) and an injection (−Q) that start at the time t_p when pumping stopped in reality.

2. Methods

We first describe Agarwal's method. We then analyze the validity of the method, which naturally leads to a new approach that overcomes the limitations. Finally, we propose a method for variable pumping rate conditions.

2.1. Agarwal's Method

Agarwal initially developed his method for recovery test interpretation with the implicit assumption of an ideally large, homogeneous, and confined aquifer, subject to a constant pumping rate from a fully penetrating well for a sufficiently long time. Under these conditions, flow toward the well is radial, and the CJ equation yields a good approximation for late time drawdowns, that is, $s(t) = (Q/4\pi T)\ \ln\ (2.25Tt/r^2S)$, where Q is pumping rate and r is distance to the center of the pumping well or effective radius of the well when computing drawdowns at the pumping well itself. Based on the superposition principle, this author defined what we now call Agarwal drawdown as (Figure 2):

$$s_A(t) = s(t_p) - s_R(t) = s(t_p) - \left(s(t) - s(t-t_p)\right) \qquad t > t_p, \tag{1}$$

where t is the time elapsed since the beginning of pumping, t_p is the time at the end of pumping, and $s_R(t)$ is the (Theis) residual drawdown. The latter "will be the same as if discharge of the well had continued but a recharge well with the same flow had been introduced at the same point at the instant discharge stopped" (Theis, 1935).

That is, superposition implies that $s_R(t) = s(t) - s(t - t_p)$. Agarwal then used the CJ's approximation to the three drawdowns appearing in equation (1), which yields

$$s_A(t) = \frac{Q}{4\pi T}\left[\ln\left(\frac{2.25Tt_p}{r^2S}\right) - \ln\left(\frac{2.25Tt}{r^2S}\right) + \ln\left(\frac{2.25T(t-t_p)}{r^2S}\right)\right] = \frac{Q}{4\pi T}\ \ln\left(\frac{2.25T}{r^2S}t_A\right). \tag{2}$$

The resulting equation has exactly the same form as that of CJ for a pumping test but using t_A, implicitly defined as $t_A = t_p(t - t_p)/t$, instead of t. Therefore, Agarwal proposed, without further justification, treating s_A versus t_A as the drawdown curve caused by pumping Q. This is quite surprising because the CJ's approximation does not hold for small recovery times $(t - t_p)$, and yet the method works fine.

Four nice features of this approximation are worth pointing. First, t_A is comparable to $(t - t_p)$ when t_p and t are comparable (i.e., when t_p is large). Second, when t tends to infinity, t_A tends to t_p, that is, Agarwal's method will not yield a (pumping) drawdown curve longer than the pumping period. Third, s_A can be obtained directly from measured variables (drawdown at the end of pumping and residual drawdown). Fourth, as we shall see, the method works quite well. All of them explain the success of the method in the oil industry, where application is immediate, as the test simply consists of monitoring the pressure build-up after well shut-in. Still, general applications demand justification and, possibly, seeking alternative approximations.

2.2. Justification and Limitations of Agarwal's and Theis' Recovery Methods

This section analyzes the assumption of long time constant pumping rate needed by Agarwal's method to adopt the CJ's approximation. This limitation can be mathematically analyzed by approximating the Agarwal's drawdown solution with an infinite series. Let us recall first that, in reality, the CJ's approximation represents the leading term, for large times, of the full Theis solution, which we rewrite in dimensionless variables as

$$s_D(t_D) = -\gamma + \ln(4t_D) + \frac{1}{4t_D} - \frac{1}{2{\cdot}2!{\cdot}(4t_D)^2} + \frac{1}{3{\cdot}3!{\cdot}(4t_D)^3} - \ldots + \frac{(-1)^n}{n{\cdot}n!{\cdot}(4t_D)^n} + \ldots, \tag{3}$$

where γ is the Euler constant ($\gamma = 0.57721\ldots$), s_D is dimensionless drawdown ($s_D = 4\pi Ts/Q$) and t_D is dimensionless time ($t_D = t/t_c = Tt/Sr^2$). We have chosen to keep the term $4t_D$ without further simplifications in

10.1029/2018WR022684

equation (3) to facilitate comparison with the expression in terms of u ($1/4t_D$) typically used in well hydraulics. Substituting equation (3) in the dimensionless form of equation (1), that is, after multiplying (1) times $4\pi T/Q$, we obtain (after some elementary but tedious algebra):

$$s_{AD} = -\gamma + \ln(4t_{AD}) + \left(\frac{1}{4t_{AD}} - \frac{1}{4t_D}\right) - \frac{1}{2 \cdot 2!}\left(\frac{1}{(4t_{AD})^2} - \frac{t_{pD}^2 - t_D t_{pD} - 2t_D^2}{4^2 t_D^2 t_{pD}(t_D - t_{pD})}\right) + \ldots, \quad (4)$$

where $t_{pD} = t_p/t_c$ and $t_{AD} = t_A/t_c$. Comparing equations (3) and (4), it becomes obvious that they are identical, up to second order, except for the term $1/4t_D$ in (4), which becomes negligible when t_D is large. The first-order term is also the one controlling the error in the CJ's approximation of the Theis' solution (equation (3)), which suggests that Agarwal's method is valid under the same conditions that CJ's approximation would be valid at the end of the pumping phase. Note, however, that the second-order term can be large for small ($t_D - t_{pD}$) and declines only as $1/(t_D - t_{pD})$ when t_D increases (i.e., the error may persist for a sizeable time for small dimensionless times). These conditions, that is, large t_{pD} and ($t_D - t_{pD}$), are easy to meet at the pumping well where the characteristic time is small but not necessarily at the observation well (e.g., for an aquifer with $T = 100$ m^2/day, $S = 0.1$, and well radius $r = 0.1$ m, $t_c = 10^{-5}$day $\cong$ 1 s, but $t_c = 0.9$ day, at an observation located at $r = 30$ m). This explains the success of the method in the oil industry, where reservoir tests are routinely performed in production wells but suggests that it may fail at observation wells.

It is interesting to perform the same analysis for Theis (1935) recovery method, which consists of plotting residual drawdown (ideally, $s_R(t) = s(t) - s(t - t_p)$) versus $t_R = t/(t - t_p)$:

$$s_{RD} = \ln(t_R) - \frac{t_{pD}}{4t_D(t_D - t_{pD})} + \ldots, \quad (5)$$

where $s_{RD} = 4\pi T s_R/Q$. Note that the error is large for small ($t_D - t_{pD}$). Therefore, it is not, and was never meant to be, appropriate for early time recovery. But the error declines fast as both absolute time since the beginning of pumping, t_D, and recovery time, $t_D - t_{pD}$, increase. In fact, it becomes smaller than Agarwal method for very long recovery ($t_D \gg t_{pD}$). Moreover, it does not require t_{pD} to be large. In fact, it can be used to interpret slug tests. This fast decline explains the success of Theis recovery method but also highlights that it is important to keep measuring recovery for a long time. In practice, measuring recovery for a long time after the stop of pumping (and for a comparable time prior to the beginning) is of practical importance to detect head fluctuations caused by factors other than pumping, which is especially important in observation wells. A last comment, the error displayed in equation (5) can be eliminated (thus leaving only higher order terms) if CJ's approximation is extended to include the $1/4t_D$ term in equation (3), which leads to an extended and more accurate interpretation method (Çimen, 2015).

2.3. Proposed Method

The above analysis suggests an alternative method for interpreting recovery data that overcomes Agarwal's requirement of long pumping time and facilitates the use of early time data from which quantitative, but above all qualitative information can be gained. In fact, as many factors affect short-time data, a great deal of useful information can be obtained, helping in the selection of the most appropriate theoretical aquifer model (Gringarten et al., 1979; Ramey, 1970). The fact that the essential assumption behind the Agarwal's method is that the logarithmic approximation is valid (together with practical experience) suggests using CJ's approximation or any other that the modeler deems appropriate to transform the recovery test drawdown information into that of a pumping test:

$$s_M(t - t_p) = s_{ap}(t) - s_R(t) \qquad t > t_p, \quad (6)$$

where $s_{ap}(t)$ is an approximation of $s(t)$ that depends on the modeler's assumption about the behavior of the system. The two most immediate options are (1) $s_{ap}(t) = s(t_p) + m \cdot \ln(t/t_p)$, if the modeler assumes that flow is radial (dimension $n = 2$) and that a constant slope has been reached or (2) $s_{ap}(t) = s(t_p) + (2m/(2 - n))((t/t_p)^{1 - n/2} - 1)$, if the modeler assumes a power law behavior of the log derivative (i.e., that flow occurs with a dimension n other than 2). In either case, m is the log derivative at the end of the pumping phase, or the slope of the drawdown data, divided by 2.3, if they are plotted versus $\log_{10} t$ (traditional CJ semi-log plot).

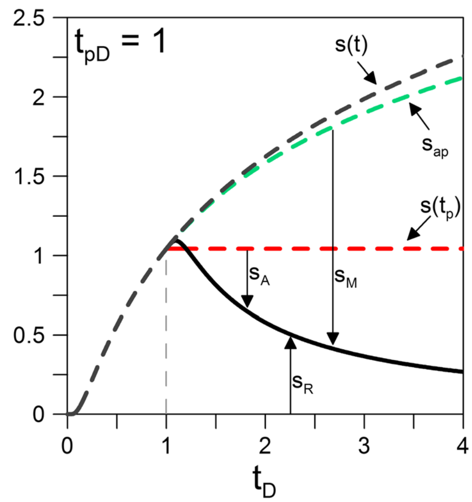

Figure 3. Graph showing the different drawdown terms used for the estimation of both Agarwal (s_A) and the proposed drawdown (s_M). It is important to underline the aquifer delay in responding to the pumping shut-in (residual drawdown s_R at early recovery times).

The new approximation (equation (6)) differs from that of Agarwal (equation (1)) as the delay in the aquifer response after pumping shut-down is taken into consideration. In fact, instead of a constant value $s(t_p)$, an extrapolated function $s_{ap}(t)$ has been considered to evaluate the pumping test drawdown that would have occurred at $t > t_p$ if a longer pumping test had been carried out (Figure 3). As the defined function is a straight line with slope m, the new approximation $s_{ap}(t)$ tends to CJ's one if quasi steady radial regime has been reached. Otherwise, in transient regime, a straight line with a lower slope will be generated. The latter condition is easy to meet in observation wells where the characteristic time is greater compared to that of the pumping well. Consequently, the aquifer response shows a delay (drawdown may increase for some time after pumping shut-in) that can be seen as if the pumping time period would last longer than that in the pumping well. As Agarwal's method was originally developed to analyze data recorded in the pumping well itself (as usually done in the oil and gas industry), the aquifer system behavior for transient time is not taken into account and insufficient pumping time periods lead to negative Agarwal's drawdown.

Applying the proposed method, it is important to underline that (1) we use recovery time ($t - t_p$) to plot residual drawdown, which means plot data on the same time scale of that of producing time; (2) the evaluation of s_M can be easily performed using the last pumping time data and recovery data series; (3) as we shall see, the method works quite well, allowing to obtain early-time data curves from recovery data even when the dimensionless duration of pumping is short; but (4) late time values of $s_M(t)$ are virtually identical to $s_{ap}(t)$ because $s_R(t)$ tends to 0 (recall Figure 2). Because of this last remark, $s_M(t)$ should not be used for recovery times longer than that at which $s_R(t)$ tends to 0.

2.4. Variable Pumping Rate

We generalize here Agarwal, Theis and other methods to acknowledge time-dependent pumping rate during the interpretation of recovery data. Time variability of pumping rate has been addressed by numerous authors, including Agarwal (1980). The goal has been typically to interpret pumping test data and the method consists of using either superposition (Birsoy & Summers, 1980; Neville & van der Kamp, 2012; van der Kamp, 1989) or deconvolution (von Schroeter et al., 2002). Given the difficulties of the latter, and the specificities of recovery test analysis, we adopt an approach similar to that of Birsoy and Summers (1980) but taking advantage of the fact that the last portion of the pumping phase is often performed at a constant rate. Therefore, we assume that the pumping rate fluctuates only up to time t_{cQ} (Figure 1). Thereafter, the pumping rate is constant (Q_c). We decompose the pumping rate as the sum of Q_c, which lasts over the whole pumping interval and a variable pumping rate $Q'(t)$:

$$Q(t) = Q_c + Q'(t), \tag{7}$$

where we assume that Q' becomes 0 after time t_{cQ}. Using superposition again, we write the total drawdown as $s(t) = s_c(t) + s'(t)$, where $s_c(t)$ and $s'(t)$ are the drawdowns caused by Q_c and $Q'(t)$, respectively. Assuming that $t \gg t_c$, so that steady radial regime has been established near the pumping well, $s'(t)$ can be calculated as the sum of the residual drawdowns produced by infinitesimal (or discrete, in the case of step tests) pumping rate steps, dQ', from the beginning of the step (at time τ) until the end (at time t_{cQ}):

$$s'(t) = \int_0^{t_{cQ}} \frac{1}{4\pi T} \frac{dQ'}{d\tau} \ln\left(\frac{t-\tau}{t-t_c}\right) d\tau = \frac{Q_c}{4\pi T} \ln(E_I), \tag{8}$$

$$s(t) = \frac{Q_c}{4\pi T} \ln\left(\frac{2.25Tt\, E_I(t)}{r^2 S}\right), \tag{9}$$

where E_I is implicitly defined in (8) as

$$E_I(t) = \exp\left[\int_0^{t_{cQ}} \frac{1}{Q_c} \frac{dQ'}{d\tau} \ln\left(\frac{t-\tau}{t-t_{cQ}}\right) d\tau\right] \tag{10}$$

We illustrate the use of equation (10) by application to a step drawdown test with $Q(t) = Q_i$, $t_i \le t < t_{i+1}$, $i = 1$, N, where N is the number of steps, and $t_N = t_{cQ}$ (pumping rate is Q_c during the last step), then $dQ'/d\tau = \sum_{i=1}^{N} (Q_i - Q_{i-1})\delta(t_i - \tau)$, with $Q_0 = Q_N = Q_c$ (Figure 2). With these definitions,

$$E_I(t) = \prod_{i=1}^{N-1} \left(\frac{t-t_i}{t-t_{cQ}}\right)^{\Delta Q_i/Q_c}, \tag{11}$$

where $\Delta Q_i = Q_i - Q_{i-1}$. This equation is similar to those of Birsoy and Summers (1980) and Agarwal (1980), except that they add each step independently. In the case of Birsoy and Summers (1980), their choice made sense because they were seeking an approximation of drawdown during pumping, but it is somewhat less accurate if Q' is indeed small and zero after t_{cQ}.

Mishra et al. (2013) argued that step approximations may not be appropriate for smoothly varying flow rates and proposed a Laplace transform solution for timewise linear flow rate. Equation (10) can also be easily integrated in this case. Assume that the pumping rate varies linearly between point measurements, $Q(t_i) = Q_i$, $i = 1, N$, with possible jumps at t_1 [$\Delta Q_1 = Q_1 - Q_c$] and $t_N = t_{cQ}$ [$\Delta Q_{N+1} = Q_c - Q_N$], when E_I becomes (again easy but tedious integration)

$$E_I(t) = \left(\frac{t-t_1}{t-t_{cQ}}\right)^{\Delta Q_1/Q_c} \left(\frac{t-t_N}{t-t_{cQ}}\right)^{\Delta Q_{N+1}/Q_c} \prod_{i=2}^{N} \left(\frac{t-t_{i-1}}{e(t-t_{cQ})}\right)^{\Delta Q_i/Q_c} \left(\frac{t-t_{i-1}}{t-t_i}\right)^{m_i(t-t_i)/Q_c}, \tag{12}$$

where $e = 2.718$ is the Euler's number. Using either equation (11) or (12) in (9) leads to an *equivalent time*, $tE_I(t)$, for all approximations of drawdowns caused by pumping (but not by recovery). Therefore, using (8) for $s(t)$ and $s(t_p)$ in equation (1) yields

$$s_{Ac}(t) = s(t_p) - s_R(t) = \frac{Q_c}{4\pi T} \cdot \ln\left[\frac{2.25T}{r^2 S} \frac{t_p(t-t_p)E_I(t_p)}{tE_I(t)}\right], \tag{13}$$

which suggests a modified Agarwal time:

$$\hat{t}_A = \frac{t_p(t-t_p)E_I(t_p)}{tE_I(t)}. \tag{14}$$

With the new corrected time $\hat{t}_A$, it becomes feasible to interpret recovery test data, as the mark left by the past pumping history over the recovery signal has been taken into consideration. Obviously, this equivalent time should be used also to correct Theis recovery time (t_R) or, for the proposed alternative (section 2.3), the recovery time ($t_r = t - t_p$).The modified times would result in

$$\hat{t}_R = \frac{tE_I(t_p)}{t-t_p} \quad \text{and} \quad \hat{t}_r = \frac{(t-t_p)E_I(t_p)}{E_I(t)}. \tag{15}$$

It is important to underline that the application of the proposed method (s_M) jointly with an equivalent time $(\hat{t}_r)$ would allow interpreting hydraulic tests characterized by a short pumping period and variable pumping rate.

This algorithm has been implemented in a spreadsheet available from the GHS-UPC software web page at https://h2ogeo.upc.edu/es/investigacion-hidrologia-subterrania/software/599-recovery-test-interpretation.

3. Performance of Recovery Test Interpretation Methods

3.1. Ideal Conditions: Importance of the Duration of Pumping

As discussed in section 2.2, Agarwal's method requires that the pumping period has been long enough to reach linear behavior in semilogarithmic scale. Thus, we test here how the pumping time t_{pD} can affect the

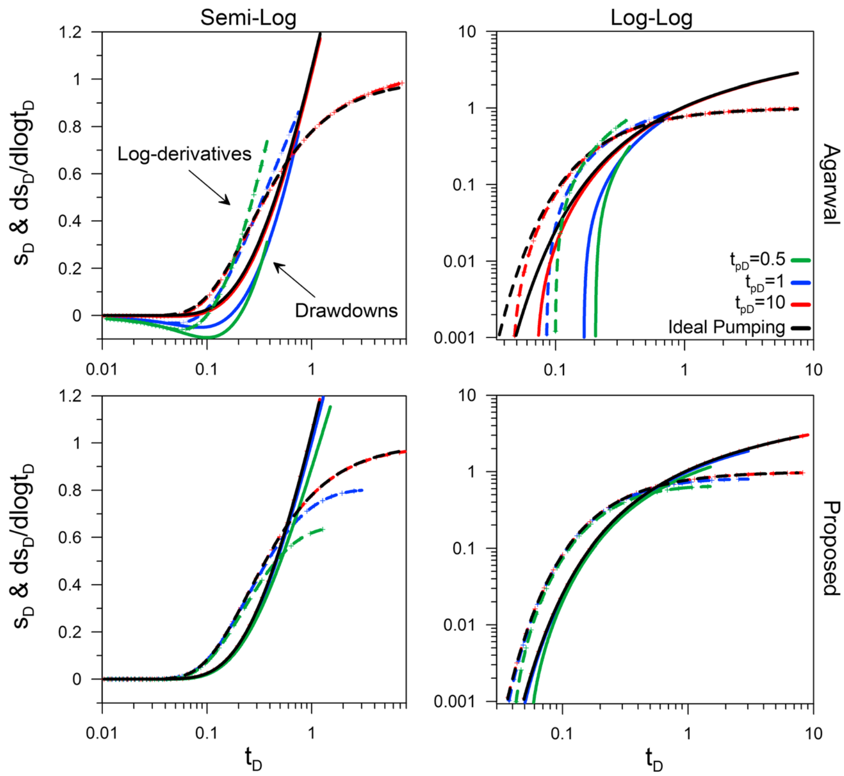

Figure 4. Diagnostic plots representing recovery test interpretation in an ideal Theis aquifer for three different pumping durations using Agarwal's recovery method (above) and the proposed method (below). The ideal pumping test solution (Theis) is shown as reference.

interpretation of recovery data for both Agarwal's and the proposed method (Figure 4). To this end, we compare the ideal pumping solution (Theis) with the corresponding diagnostic plot determined by (1) the Agarwal's drawdown curve, that is, the plot of s_A versus t_A obtained from the recovery test data and (2) the method proposed in section 2.3. The aim is to evaluate the effect of pumping and recovery duration on the recovery test interpretation, performed with both Agarwal's and the new proposed method, thus analyzing the validity of both.

The pumping durations t_p are chosen in terms of the characteristic time t_c of the observation well because pumping tests are usually designed bearing in mind that a sizable response should start at $0.1t_c$ (i.e., pumping time $t_{pD} = 0.1$, recall that $t_c = r^2S/T$) and CJ's slope starts developing at t_c ($t_{pD} = 1$). In well hydraulics, $u = 1/4t_D$ is often used instead of t_D, and the validity of CJ's approximation is usually restricted to $u < 0.03$ (i.e., $t_{pD} > 8$). Therefore, we have chosen three different pumping times ($t_{pD} = 0.5$, 1, 10). The recovery time $t_{rD} = t_D - t_{pD}$ has been fixed to be four times the pumping time period ($t_{rD} = 4t_{pD}$).

Figure 4 displays two different patterns in the application of Agarwal's method. First, the longer the producing period, the better the whole curves fit Theis' solution, approaching a perfect match when $t_{pD} = 10$. In fact, late time drawdowns do not reach a quasi steady state behavior (constant log derivative), unless a long producing period is applied. Second, the match of Agarwal curves to the ideal pumping solution improves with recovery time. Agarwal solution is poor for early times. In fact, when pumping is short, Agarwal yields negative drawdowns and negative log derivatives at early times. Both observations are consistent with the discussion of section 2.2.

The proposed method also improves with the producing period duration. However, the types of errors are complementary to those of Agarwal. On the one hand, the fit to the ideal Theis solution worsens for long recovery times, reaching a fictitious quasi steady state behavior, which reflects the log derivative at the end of the pumping period, which is different from the real one of the aquifer system. On the other hand, early times data perfectly match the ideal solution.

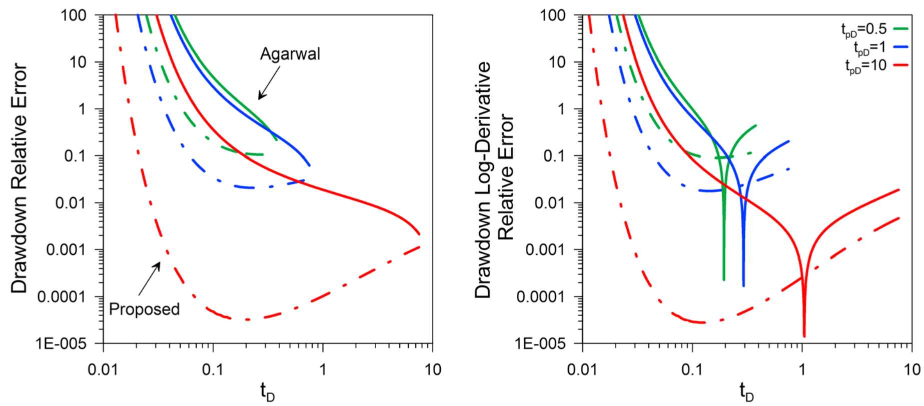

Figure 5. Relative errors of drawdowns and log derivatives resulting from both Agarwal's (line) and the proposed method (dash dot) under ideal conditions for three different pumping durations. Note that the errors of the proposed method are below those of Agarwal for any given pumping duration and that both methods improve with increasing pumping duration.

The accuracy of these methods can be further analyzed by estimating the absolute relative error of both the drawdown and its log derivative, defined as

$$\epsilon_s = \left| \frac{s_{\text{pump}}(t) - s_{\text{rec}}(t_{\text{corr}})}{s_{\text{pump}}(t)} \right| \quad \epsilon_m = \left| \frac{m_{\text{pump}}(t) - m_{\text{rec}}(t_{\text{corr}})}{m_{\text{pump}}(t)} \right| \qquad 0 < t = t_{\text{corr}} < t_p, \tag{16}$$

where s_{pump} and m_{pump} are respectively the theoretical drawdown curve and its log derivative associated with the Theis' solution, s_{rec} and m_{rec} are the corresponding corrected recovery values used in the interpretation, that is, $s_{\text{rec}} = s_A$ for the Agarwal's method and $s_{\text{rec}} = s_M$ for the proposed interpretation method, and t_{corr} is the corrected time determined as t_A and t_r depending on the interpretation method. It is important to underline that, assuming proportionality between estimated transmissivity and drawdown log derivative values (long pumping times), the relative error ϵ_m expresses that the estimated transmissivity value T_{est} differs from the exact value T by a factor given by $T_{\text{est}} = T\,(1 - \epsilon_m)^{-1}$.

The absolute value of the relative errors is shown in Figure 5, which displays in a log-log plot their recovery time evolution for three pumping durations. Figure 5 makes it clear that (1) the longer the producing time, the smaller the errors associated with both methods; (2) the errors of Agarwal's method for the drawdown and its log derivative decay with recovery time (which is consistent with equation (4)), although errors in log derivative tend to increase at late time; (3) the errors of the proposed method are smaller than those of Agarwal's method at any given time t_D. This effect is dramatically important when t_r is small, at early recovery times. Here more than 1 order of magnitude difference is noted; and (4) the errors of the proposed method rapidly decay initially (up to $t_D \approx 0.1 - 0.2$) but tend to increase slightly thereafter.

We finally acknowledge that the advantages given by the proposed method must be taken with care as the accuracy of the proposed method relies on how $s_{ap}(t)$ is selected in equation (6). In this context, we content that this conceptual decision will be only important at late times but not at early times where the performance of the method is best.

At this stage, it is worth mentioning here that the errors presented in Figure 5 are always positive, that is, Agarwal's method underestimates the drawdown and its log derivative, except for late recovery times (after the singularity where error values approach 0). In this region, the relative error of the drawdown log derivative given by Agarwal's method becomes negative. Importantly, the interpretation of the late-time recovery data with Agarwal's method will underestimate the transmissivity of an aquifer.

3.2. Non-Ideal Conditions: Effects of Boundaries and Flow Dimension

Aside from long pumping times, others assumptions were required for the development of Agarwal's method, as the aquifer is assumed to be large, homogeneous, confined, and characterized by radial flow. Thus, we analyze here the effect of departures from these assumptions: (1) presence of a no-flow boundary given by an impermeable barrier defined by $r'^2/r^2 = 100$, where r' is the radial distance from the image well (image well theory) to the observation well, (2) a constant head boundary (constant head defined by the same characteristic time as for the no-flow boundary condition), (3) a leaky aquifer characterized by the dimensionless number $B_D = B/r$, where B is the leakage factor and r is the distance from the pumping well to the observation well, and (4) a drainage line (flow dimension of one to test the impact of varying the flow dimension).

We compare the analytical solutions to these problems with the corresponding diagnostic plots determined by the Agarwal's drawdown curve (Figure 6, top rows) and those determined by the application of the proposed method (Figure 6, bottom rows). Still, the aim is to investigate the validity of the methods under non-ideal conditions.

Three different simulations have been carried out for each scenario with pumping times $t_{pD} = 1, 10, 100$. The recovery time has been fixed to $t_{rD} = 4t_{pD}$. It is important to underline that, compared to the previous analysis, we have now adopted longer pumping times t_{pD} to ensure that boundary effects can be observed.

As can be seen in the graphs, the performance of Agarwal's method for the one-dimensional flow is different than for other boundary effects. In fact, in the latter case it is possible to find the same pattern as that found for ideal conditions: The longer the producing period, the better the whole curves fit the analytical solution. In particular, early times present important mismatches for short producing periods and late times (late recovery times) do not clearly reproduce the ideal pumping curve. However, in the one-dimensional flow system solution, the curves fail to match the analytical solution for either short or long producing periods.

The proposed method works very well in matching the analytical solution for early times, even considering short producing periods. However, its performance for late recovery time data depends on the duration of pumping. Finally, the recovery test interpretation carried out for one-dimensional flow perfectly resembles the analytical solution.

3.3. Non-Ideal Conditions: Variable Pumping Rate

Until now, the validity of recovery test methods has been analyzed for constant pumping rate. However, as discussed in the introduction, this condition is often difficult to meet (in fact, it is this difficulty that motivates our emphasis in recovery analysis in the first place). We discuss here the effect of time-dependent pumping rate (see Figure 1) for ideal (Theis) aquifer conditions.

We have chosen a cubic law variable flow rate term Q' (recall equation (7)), written as

$$Q'(t) = Q'_{\max} - Q'_{\max}\left[\frac{3}{2}\left(\frac{t}{t_{cQ}}\right) - \frac{1}{2}\left(\frac{t}{t_{cQ}}\right)^3\right] \qquad 0 < t \leq t_{cQ}, \tag{17}$$

$$Q'(t) = 0 \qquad t_{cQ} < t \leq t_p, \tag{18}$$

where $Q'_{\max}$ is the maximum value of $Q'(t)$, which occurs at $t = 0$, t_{cQ} is the time at which $Q'(t)$ becomes equal to 0 and depends on the parameter R_t as $t_{cQ} = R_t t_p$. In addition, the constant pumping rate Q_c is proportional to $Q'_{\max}$ by a constant $R_q = Q_c/Q'_{\max}$. Fixing the pumping time $t_p = 10t_c$ and the recovery time $t_r = 4t_p$, multiple simulations have been run, considering different couples of R_t and R_q values. In this case, the aim is that of testing Agarwal's method, using both Agarwal's time (section 2.1) and the modified one (section 2.4). In order to compare different solutions, the drawdown produced by an equivalent pumping rate (constant rate, pumping the same water volume pumped during the hydraulic test in the same time period) has been calculated. Hereafter, it is referred to as the ideal pumping solution.

As in the previous two sections, diagnostic plots are compared in terms of dimensionless variables. For the comparison, we keep constant the dimensionless duration, t_{pD}, and the magnitude of the pumping rate variability, R_q, while changing the period over which pumping rate is variable, as measured by R_t. As shown in Figure 7, the longer the duration of pumping rate variation (high R_t), the worse Agarwal's solution reproduces that of an ideal pumping drawdown. The interpretation of the same hydraulic test through the adoption of the equivalent time (equation (14)) leads to a correct reproduction of the ideal solution. In short, if the pumping rate is variable, the variability must be acknowledged for the interpretation of recovery data.

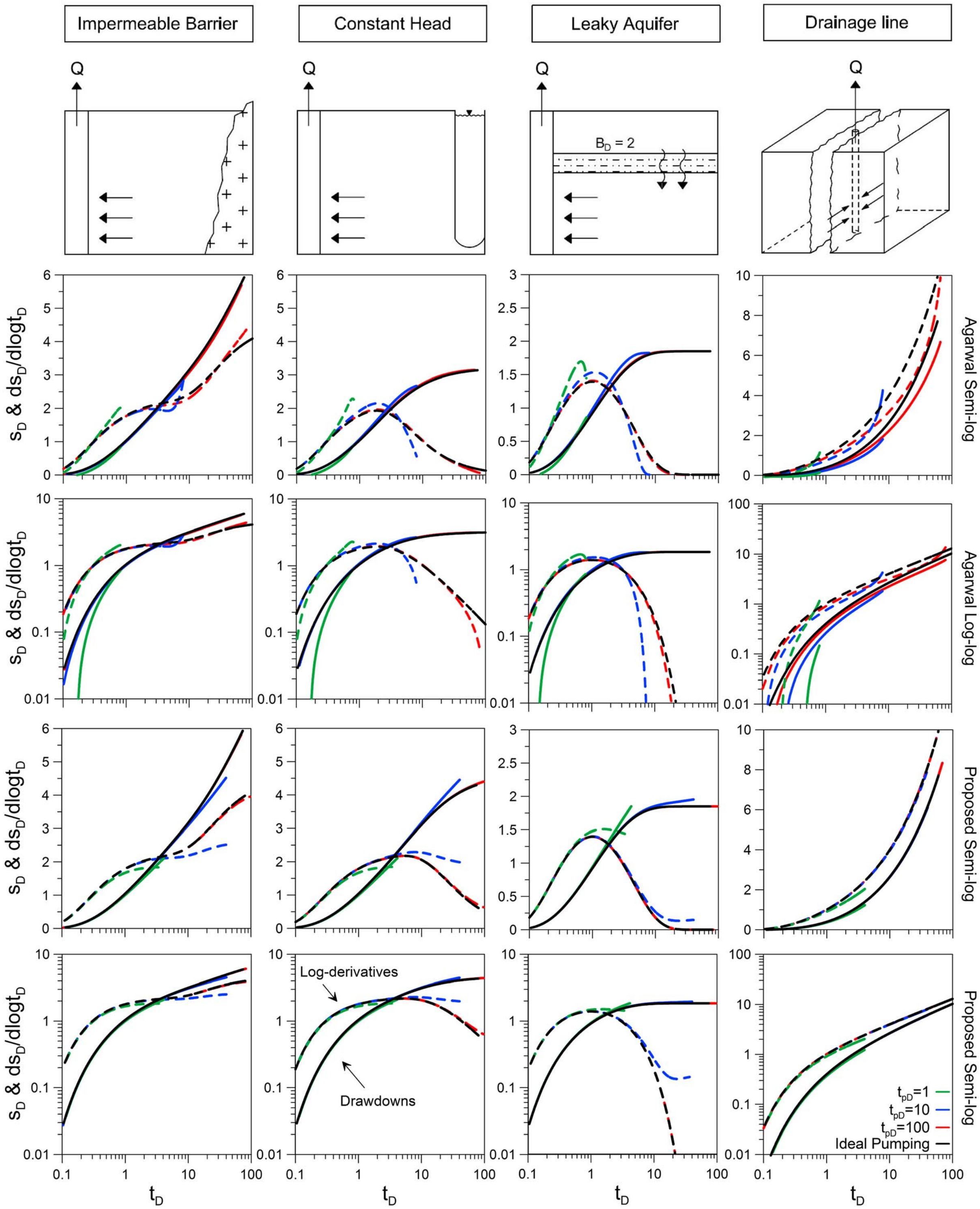

Figure 6. Diagnostic plots representing recovery test interpretation under non-ideal conditions. Three different pumping times have been analyzed applying both Agarwal (top two rows) and the proposed method (bottom two rows). The analytical solution for pumping is shown as a reference.

4. Application to Real-World Data Sets

The proposed method is used to interpret two cross-hole hydraulic tests performed at the Grimsel underground laboratory in Switzerland (FEBEX tunnel). The site description as well as the hydraulic test

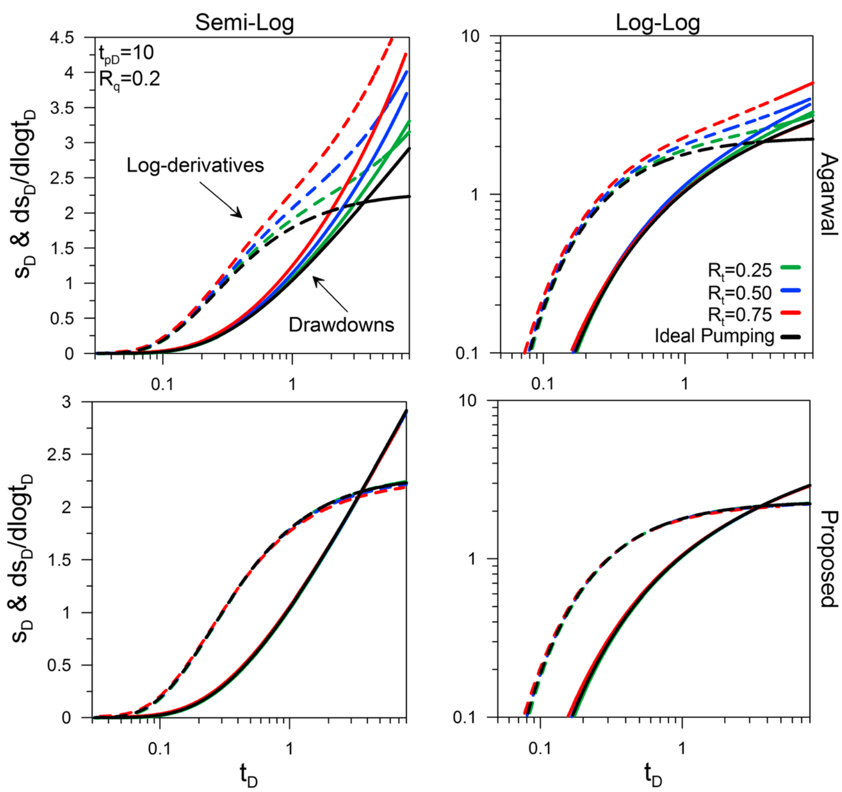

Figure 7. Diagnostic plots obtained from recovery data after pumping an ideal Theis aquifer at a variable rate (total pumping duration, t_{pD} = 10, and ratio between constant and maximum pumping rate deviation, R_q = 0.2). Three different values of R_t (ratio of variable rate to total pumping time) have been analyzed. The plots have been obtained both assuming constant pumping rate (above) and acknowledging its variability by means of equation (8)) (below).

performance and interpretation have been presented by Martinez-Landa and Carrera (2005, 2006). We have chosen these tests to illustrate the performance, strengths, and weaknesses of the methods.

4.1. Example A: Long Pumping Test and Fast Aquifer Response

This test corresponds to the water injection at point I2-1 and head measurements obtained at point F22-3, described by Martinez-Landa and Carrera (2006). The observation point is located 2.86 m away from the injection point and recorded drawdowns are produced by a constant pumping rate of 418.2 m^3/day for a pumping period t_p = 4.63 days and a recovery period t_r = 0.6t_p. The quasi steady state has been reached during pumping, which can be seen qualitatively in Figure 8 and confirmed quantitatively by the characteristic time of the observation point (t_c = 0.22 days, obtained from the transmissivity and storativity resulting from interpretation of the drawdown curve or a dimensionless pumping time t_{pD} = 21.47). Under these conditions, both Agarwal and the proposed method (section 2.3) should work given our previous discussion.

Figure 8a shows the resulting diagnostic plots, while the estimated hydraulic properties obtained during the recovery and the pumping period are reported in Table 1 (example A). The diagnostic plot shows three data sets: the drawdown produced during the actual pumping and the recovery data plotted as drawdown estimated by both Agarwal and the proposed method. As expected, both methods lead to good results: The curves present an almost perfect match with the actually measured drawdown data. Consequently, the same conceptual model can be inferred. Moreover, similar results have been achieved in terms of estimated hydraulic properties, as both transmissivity and storage coefficient values present quite insignificant differences compared to those obtained with the drawdown data.

4.2. Example B: Short Pumping Test and High Delay in Aquifer Response

In this second example, we interpret the cross-hole test resulting from injecting water at Fbx2-04 and observing head levels at point F13-2 (Martinez-Landa & Carrera, 2005). The observation point is located 10.72 m

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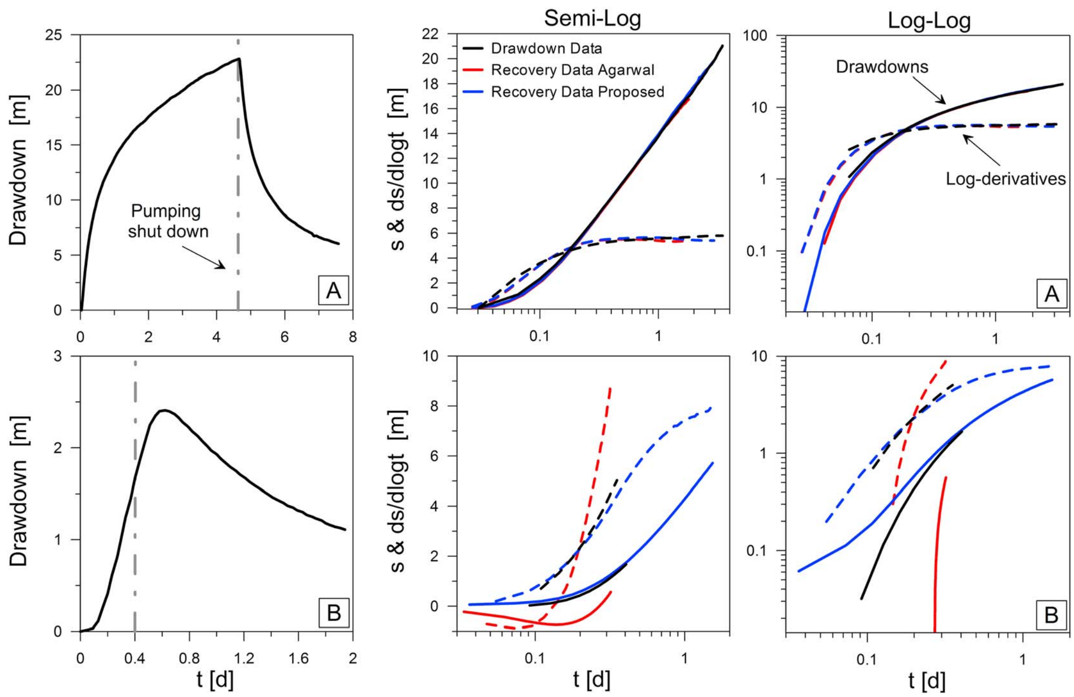

Figure 8. On the left side, the drawdown data related to two different hydraulic test performances (example A, above, and example B, below) have been plotted. On the right side, the diagnostic plots of the drawdown curves (black line), as well as of data resulting from the application of both Agarwal (red line) and the proposed method (blue line) have been presented.

away from the injection point and recorded drawdowns are produced by a constant flow rate of 496.8 m^3/day. As can be seen in Figure 8, the quasi steady state has not been reached, due to a unexpected shutdown of the pump that resulted in a pumping time t_p = 0.4 days and a long recovery time $t_r = 3.7t_p$ (data presented by Martinez-Landa and Carrera correspond to the full test). Remarkably, the residual drawdown clearly shows a high delay of the aquifer response as drawdown keeps increasing for a while after pumping shutdown. Both behaviors should be expected because of the long characteristic time of the observation point t_c = 3.03 days (value obtained from the interpretation of the drawdown curve proceeding from a long pumping period). It results in a dimensionless pumping time t_{pD} = 0.13, far from t_{pD} = 1 needed to get the whole transient response and clearly far from reaching quasi state conditions.

Given the circumstances, Agarwal and the proposed method (section 2.3) lead to different results. As expected, Agarwal performs very poorly. In fact, the recovery is so slow that the residual drawdown was larger than the drawdown at the end of pumping for a long time, so that Agarwal drawdowns remained

Table 1
Hydraulic Properties Estimated Applying the Method to Real-World Data Sets

Test	Drawdown log derivative (m)	Transmissivity (m^2/day)	Jacob time (day)	Storage coeff. (–)
Example A				
Drawdown data	5.808	$6.632 \cdot 10^{-5}$	0.096	$1.745 \cdot 10^{-6}$
Agarwal	5.410	$7.120 \cdot 10^{-5}$	0.079	$1.540 \cdot 10^{-6}$
Proposed	5.277	$7.299 \cdot 10^{-5}$	0.067	$1.356 \cdot 10^{-6}$
Example B				
Drawdown data	5.033	$9.091 \cdot 10^{-5}$	1.348	$2.401 \cdot 10^{-6}$
Agarwal	3.780	$1.210 \cdot 10^{-4}$	0.267	$6.332 \cdot 10^{-7}$
Proposed	3.214	$1.650 \cdot 10^{-4}$	0.265	$7.381 \cdot 10^{-7}$

Note. The estimated hydraulic parameters applying both Agarwal and the proposed method are presented and compared with those estimated using drawdown data obtained from long pumping tests. Example A refers to the hydraulic test with long pumping period, while example B refers to the one with short pumping period.

negative for a good portion of the recovery. Application of the proposed method led to a constant late time slope that was larger than the one at the end of pumping. While this is not surprising, given the short duration of pumping, it shows that recovery data contains information representative of a period that may be longer than that of pumping. Therefore, we followed an iterative process: The slope m, used to estimate $s_{ap}(t)$, has been iteratively replaced by the slope calculated using the last values of s_M. Throughout the iterations, the m value tends to a constant that has been used for the last estimation of s_M.

Figure 8b shows the resulting diagnostic plots. The corresponding estimated hydraulic properties are reported in Table 1 (example B), together with those estimated from the drawdown data that refers to a long pumping test performed after the unintentional switching off. It is clear that Agarwal's data do not reproduce the actual drawdown data. Instead, the proposed method results in a positive drawdown that matches quite well drawdown data. The log derivative is positive and fits well the actual pumping test log derivative at early times but diverge at late times, until reaching a fictitious quasi steady state regime, which turns out to be close to the one obtained from the long test described by Martinez-Landa and Carrera (2005). That means that we can adequately reproduce the aquifer response and therefore infer a proper conceptual model. In terms of hydraulic properties estimation, the difference is not as relevant as the information that one can obtained from the diagnostic plot.

5. Discussion and Conclusions

We have analyzed Agarwal's method validity under ideal and non-ideal conditions, generalizing it for time-dependent pumping rate and proposing an alternative method for recovery test interpretation to be applied especially under non-ideal conditions. Our results can be summarized as it follows:

1. Residual drawdowns contain as much information as pumping test data, which is qualitatively and quantitatively valuable. Moreover, they are much less influenced by pumping perturbations, resulting in a much gentler and complete time series to be interpreted. Therefore, the analysis of recovery data must be considered as a great choice for the characterization of aquifer systems through hydraulic testing. The only drawback of recovery data is their sensitivity to environmental head fluctuations (especially at observation points, where the head signal induced by pumping may be small, and at late times, when environmental fluctuations may be larger than the residual drawdown). Therefore, emphasis must be placed during test design, which should consider a long observation period both prior and, especially, after the pumping period to facilitate filtering of environmental head fluctuations to obtain residual drawdowns.
2. The Agarwal's method should be considered as a very effective method for recovery data interpretation because it is simple and may yield drawdown curves virtually identical to those obtained during pumping with constant rate, thus facilitating the use of diagnostic plots for quantitative and qualitative analyses. This is true under ideal conditions of radial flow (including boundary effects), provided that the dimensionless pumping time is long enough to develop the fully radial regime as ascertained by a constant CJ (semi-log plot) slope (log derivative).
3. The Agarwal method fails to reproduce the pumping drawdown curves in an appropriate way when the duration of pumping is so short that the constant slope has not started to develop (i.e., when the dimensionless time Tt/Sr^2 is less than 1). In practice, this restricts the applicability of the method to the pumping well, for which it was originally developed, unless the duration of pumping is very long or the aquifer is confined.
4. The Agarwal method may also fail for nonradial flow (i.e., when the late time slope is nonconstant). We tested the method for a flow dimension of one, and the method did not work even for long producing periods.
5. We proposed an alternative method (equation (6)) to overcome Agarwal's limitations that works very well both under ideal and non-ideal conditions, especially during early time recovery. The main limitation of the proposed method, when compared to Agarwal's, is that its duration is artificially unlimited (Agarwal time will never be longer than the pumping time). This implies that our late time drawdowns may suggest a quasi steady state regime that is fictitious, as it reflects the assumption made in approximating drawdowns during recovery. Surprisingly, the method performed quite well during application to a real case where the dimensionless pumping duration had been very short, which suggests that the proposed method may be valid for recovery times beyond the pumping duration.

6. The previous discussion suggests that a possibility would be to blend both methods (i.e., use the proposed method for early time recovery and Agarwal's for late time). This possibility has not been fully explored for two reasons. First, the proposed method tends to work better than Agarwal for the short dimensionless pumping durations typical of observation wells, which are frequent in hydrology (as opposed to petroleum engineering) and where we discourage the use of Agarwal method. Second, the primary use of the methods we are discussing here is for drawing diagnostic plots, which help in defining the conceptual model. Once this has been identified, quantitative interpretation can be best achieved through numerical modeling of the full test (pumping and recovery), while acknowledging the variability of pumping rates, so that whether radial flow regime has been reached becomes irrelevant.
7. We proposed an alternative equivalent time to treat recovery data from variable pumping rate (both stepwise and piecewise linear) that allows accounting for the influence of pumping rate variability during recovery data interpretation using our proposed method or Agarwal's. The test example (Figure 7) makes it clear that acknowledging for the time variability of pumping rate is critical. Therefore, pumping rate must be monitored carefully. This is also important for Theis' recovery method that remains the method of choice for the estimation of transmissivity from late time data.

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