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A Budget Feasible Mechanism for Hiring Doctors in E-Healthcare

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Abstract-Throughout the past decade there have been an extensive research on scheduling the hospital resources such as the operation theatre(s) (OTs) and the experts (such as nurses, doctors etc.) inside the hospitals. With the technological growth, mainly advancement in communication media (such as smart phones, video conferencing, smart watches etc.) one may think of taking the expertise by the doctors (distributed around the globe) from outside the in-house hospitals. Earlier this interesting situation of hiring doctors from outside the hospitals has been studied from monetary (with patient having infinite budget) and non-monetary perspectives in strategic setting. In this paper, the more realistic situation is studied in terms of hiring the doctors from outside the hospital when a patient is constrained by budget. Our proposed mechanisms follow the two pass mechanism design framework each consisting of allocation rule and payment rule. Through simulations, we evaluate the performance and validate our proposed mechanisms.

Keywords-e-healthcare; hiring doctors; budget; truthful

I. INTRODUCTION

Substantial literatures are available to schedule resources inside the hospitals in healthcare system [1][2][3][4]. However, how to hire (to schedule) resources (expert consultants (ECs) etc.) along with their pricing schemes from outside the hospitals are mostly unaddressed [5][6][7][8]. It is observed that, with the prodigious growth of the communication media (say video conferencing, Internet, smart phones etc.), it may be an usual phenomena to have the consultancies by the experts (especially doctors) from outside the hospital(s). It is to be noted that the doctors can provide consultancies by being present at the consultancy spot (where the patient is admitted) particularly virtually (using video conferencing, Internet, smart phones etc.) etc. making them pervasive. In our future references ECs and doctors will be used interchangeably. In this paper, an attempt is made to hire expert consultants from outside the hospital when a patient is budget limited. The detailing of the hiring concept in this paper is shown in Figure 1. In Sajal Mukhopadhyay Dept. of Computer Science and Engineering National Institute of Technology Durgapur 713209, India sajal@cse.nitdgp.ac.in

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our model, the hiring concept is shown as a two fold process. In the first fold, the accumulated hospital's budget can be utilized to detect the leaders in the social graph (representing ECs professional connections) to inform about the hiring concept to the substantial number of doctors.



Figure 1: System model

In the second fold, the subset of doctors will be selected from the set consisting of doctors as leaders and the informed doctors for the consultancy process, such that the total payment made to the doctors is within patient's budget.

The remainder of the paper is structured as follows. Section II elucidates the related works. Section III describes our proposed model. The proposed mechanisms are illustrated in section IV. The analysis of the proposed mechanisms are illustrated in section V. In section VI experiments and results are shown. Finally, conclusions are drawn in Section VII.

II. Related prior work

In past the handful of works have been done, focusing on scheduling inside the hospitals (or *internal scheduling*) in terms of operation theatres (OTs) scheduling [9][1][2][10] and internal staffs (such as *nurses* [3][11], *physicians* [12][4] etc.) scheduling. In [13][10] the works have been done for allocating OTs on time to increase OTs efficiency. In our future references, hospital(s), medical unit(s), organization(s) will be used interchangeably. As with the enhancement in the technologies, mainly communication media (say video conferencing, Internet, smart phones etc.), it may be an usual phenomena to think of the scheduling of medical staffs (mainly doctors) outside the in-house hospital [5][6][14]. In [5] a doctor is providing the expertise through video conferencing to a patient admitted to other hospital with prior contact. In [15] the context of the patient (such as age, sex, medical report etc.) is utilized to take the expertise of the doctors from outside the admitted hospitals in nonstrategic setting. In [6] the strategic case is considered and is solved using mechanism design with money and in [7][8] mechanism design without money is utilized. Despite some progress in the scenario of hiring ECs from outside the hospital(s), the patients with budget constraints case has been largely overlooked. In this paper, the problem of hiring doctors from outside the hospital is studied in this setting.

III. SYSTEM MODEL

A. Notation and Preliminaries

In this section, we formalize the doctors hiring problem where the multiple doctors are hired from outside of the hospital, for a patient having budget \mathcal{B}' . The patient's budget \mathcal{B}' is a *public* information. The hospital to which a patient is admitted is having an accumulated, publicly known budget \mathcal{B} , which will be utilized to inform about the hiring concept to the substantial number of ECs. The set of ECs is given as $S = \{s_1, s_2, \dots, s_m\}$; where each EC $s_i \in S$ is assumed to be professionally connected with some $\chi_i \subseteq S \setminus \{s_i\}$. The professional connections are given by a social graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$, where \mathcal{V} is the set of nodes representing ECs and \mathcal{E} is the set of edges representing their professional connections in the social graph. Each s_i is associated with a hospital $h_i \in \mathcal{H}$. Our model consists of two fold process. In the first fold, there is a social graph that is represented as $\mathcal{G}(\mathcal{V}, \mathcal{E})$ and publicly known *expert consultant activation function* given as $I : 2^{S} \to \mathcal{R}_{\geq 0}$. Given the subset $\Gamma \subseteq S$ the value $\mathcal{I}(\Gamma)$ represents the expected number of doctors that are made aware about the hiring concept *i.e.* $I(\Gamma) = |\bigcup \chi_i|$. Each node in the graph represents a doctor s_i that has a private cost (aka bid) c_i of being an initial adapter or the cost for spreading awareness about the hiring concept to other doctors. The cost vector of all the m doctors is given as: $C = \{c_1, c_2, \dots, c_m\}$. It is to be noted that, the ECs are rational and strategic in nature. It means that, the ECs can gain by misreporting their private cost. As the ECs are strategic; so each $s_i \in S$ may report their cost for being an initial adapter as c'_i instead of c_i in order to gain; where $c'_i \neq c_i$. The payment vector for the set Γ is given as $\mathcal{P}_{\Gamma} = \{\mathcal{P}_{\Gamma_1}, \mathcal{P}_{\Gamma_2}, \dots, \mathcal{P}_{\Gamma_k}\}$; where \mathcal{P}_{Γ_i}

is the payment of $s_i \in \Gamma$. The objective of the first fold is to maximize the expert consultant activation function while the total payment is at most hospital's budget \mathcal{B} . In the second fold, we have a set of doctors consisting of doctors acted as leaders in the first fold and the aware doctors given as $\hat{S} = \{s_1, s_2, \dots, s_{i-1}, s_i, \dots, s_n\}$ such that $n \leq m$. The quality vector of all the m ECs is given as $Q = \{Q_1, Q_2, \dots, Q_m\}$, where $Q_i \in Q$ is the quality of i^{th} doctor. In general, the quality Q_i of a doctor s_i can be estimated using various parameters calculated later in the section. The publicly known quality function is given as $\mathcal{D}: 2^{\hat{S}} \to \mathcal{R}_{\geq 0}$. Given a subset $\Upsilon \subseteq \hat{S}$, the value $\mathcal{D}(\Upsilon)$ denotes the sum of the qualities of all the doctors in Υ *i.e.* $\mathcal{D}(\mathcal{Y}) = \sum_{i \in \mathcal{Y}} Q_i$. For this fold, each doctor $s_i \in \hat{S}$ will bid afresh their cost (private) for providing consultancy to the patient and is given as \bar{c}_i . The cost vector of all the *n* doctors is given as: $C = \{\bar{c}_1, \bar{c}_2, \dots, \bar{c}_n\}$. The strategic behaviour of the doctors is continued in this fold also; so each $s_i \in \hat{S}$ may report their cost of consultancy as \bar{c}'_i instead of \bar{c}_i in order to gain; where $\bar{c}'_i \neq \bar{c}_i$. Our objective is to determine the subset $\hat{S}' \in \{\xi | \sum_{i \in \xi} \bar{c}'_i \leq \mathcal{B}'\}$ for which $\mathcal{D}(\hat{S}')$ is maximized and the total payment should not exceed the patient's budget \mathcal{B}' . The payment vector of the set $\hat{\mathcal{S}}'$ is given as $\hat{\mathcal{P}} = \{\hat{\mathcal{P}}_1, \hat{\mathcal{S}}'\}$ $\hat{\mathcal{P}}_2, \ldots, \hat{\mathcal{P}}_r$.

B. Quality determination

The parameters that determine the quality of each doctor s_i are: (1) qualification of s_i given as q_i (2) success rate of s_i given as sr_i (3) experience of s_i given as e_i (4) hospital to which s_i belong given as \hbar_i . So, the quality of doctor s_i is given as: $Q_i = (w_1 \cdot q_i + w_2 \cdot sr_i + w_3 \cdot e_i + w_4 \cdot \hbar_i)$; where, $w_i \in [0, 1]$ such that $\sum_i w_i = 1$. The weighted sum of the some of the parameters considered in our case will result in the quality of the doctors.

C. Budget distribution and utilization

In our scenario, each fold is utilizing the budget from two independent sources. Firstly, talking about the hospital's budget it can be thought of as 1) the accumulated fund from the previously admitted patients say adding 5-6% of the total fees of each patients to the hospital fund. 2) Donation to the hospital by high profile persons or communities. Next, the source of the budget utilized in the second phase is the patient itself.

Definition 1 (Marginal Contribution [16]). The marginal contribution of an EC $s_i \in S$ is the number of ECs informed about the hiring concept by the EC s_i given the set of i-1 ECs i.e. Γ_{i-1} already selected as the leaders. Mathematically, the marginal contribution of i^{th} EC given Γ_{i-1} is defined as: $\mathcal{M}_{C_i}(\Gamma_{i-1}) = \mathbf{I}(\Gamma_{i-1} \cup \{s_i\}) - \mathbf{I}(\Gamma_{i-1})$

Definition 2 (*Quality Contribution* [17]). *The quality contribution of an EC* $s_i \in S$ given a subset Υ_{i-1} of ECs already

been selected is given as: $\mathcal{D}_i(\Upsilon_{i-1}) = \mathcal{D}(\Upsilon_{i-1} \cup \{s_i\}) - \mathcal{D}(\Upsilon_{i-1})$ where $\mathcal{D}(\Upsilon_i)$ denotes the sum of the qualities of all the doctors in Υ_i i.e. $\mathcal{D}(\Upsilon_i) = \sum_i Q_i$ given $\Upsilon_i = \{1, \ldots, i\}$ and $\mathcal{D}(\Upsilon_0) = 0$ as $\Upsilon_0 = \phi$.

IV. PROPOSED MECHANISMS

In this section, we present proposed mechanisms: Nontruthful budget constraint (NoTBC) mechanism motivated by [18] and Truthful budget constraint (TBC) mechanism motivated by [16][17].

A. NoTBC mechanism

It is a two pass mechanism consisting of *Non-truthful budget constraint leader identification* (*NoTBC-LI*) *mechanism* and *Non-truthful budget constraint doctor selection* (*NoTBC-DS*) *mechanism*.

1) NoTBC-LI mechanism: In each iteration of while loop, a doctor with maximum marginal contribution per cost among the available doctors is considered and is selected only if its cost for being an initial adapter is less than the hospital's budget. The payment of each doctors as a leader is their revealed cost.

Algorithm 1 NoTBC-LI mechanism ($\mathcal{G}, \mathcal{S}, \mathcal{B}, C$)

Output: $\hat{S} \leftarrow \phi, \mathcal{P}_{\Gamma} \leftarrow \phi$ 1: $\bar{S} \leftarrow \phi$ ▶ Set containing all the informed doctors. 2: while $S \neq \phi$ do $s_i \leftarrow argmax_{j \in \mathcal{S}} \left[\frac{\mathcal{M}_{C_j}(\Gamma_{j-1})}{c_j} \right]$ 3: if $c_i \leq \mathcal{B}$ then 4: $\Gamma \leftarrow \Gamma \cup \{s_i\}; \bar{S} \leftarrow \bar{S} \cup \{\gamma_i\}; \mathcal{B} \leftarrow \mathcal{B} - c_i$ 5: 6: end if 7: $\mathcal{S} \leftarrow \mathcal{S} \setminus \{s_i\}$ 8: end while 9: $\hat{S} = \Gamma \cup \bar{S}$ for each $s_i \in \Gamma$ do 10: $\mathcal{P}_{\Gamma_i} \leftarrow c_i; \mathcal{P}_{\Gamma} \leftarrow \mathcal{P}_{\Gamma} \cup \{\mathcal{P}_{\Gamma_i}\}$ 11: 12: end for 13: return \hat{S} , \mathcal{P}_{Γ}

2) NoTBC-DS mechanism: In each iteration of while loop, a doctor with maximum quality contribution per cost among the selected doctors by NoTBC-LI mechanism is considered and is hired only if its cost for the consultancy is less than the patient's budget. The payment of each hired doctors is their revealed cost of consultancy.

Lemma 1. The NoTBC mechanism is computationally efficient.

Proof: In NoTBC-LI, for *m* iteration of *while* loop we have $O(m^2)$. Thus, the running time of NoTBC-LI is $O(m^2)$. In NoTBC-DS, for *m* iteration (in worst case) of *while* loop we have $O(m^2)$. Thus, the running time of NoTBC-DS is $O(m^2)$. In both the cases, the payment determination will

be linear in *m*. The computational complexity of NoTBC is given as $O(m^2)$.

Algorithm	2	NoTBC-DS	mechanism	$(\hat{S},$	$\mathcal{B}',$	Ē)
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Output: $\hat{S}' \leftarrow \phi, \hat{\mathcal{P}} \leftarrow \phi$ 1: while $\hat{S} \neq \phi$ do 2: 3: 4: 5: end if $\hat{S} \leftarrow \hat{S} \setminus \{s_i\}$ 6: 7: end while for each $s_i \in \hat{S}'$ do 8. $\hat{\mathcal{P}}_i \leftarrow \bar{c}_i; \hat{\mathcal{P}} \leftarrow \hat{\mathcal{P}} \cup \{\hat{\mathcal{P}}_i\}$ 9: 10: end for 11: return \hat{S}', \hat{P}



Proof: From line 11 of Algorithm 1, we can see $\mathcal{P}_{\Gamma_i} = c_i$ for each $s_i \in \Gamma$. Line 9 in Algorithm 2 shows that $\hat{\mathcal{P}}_i = \bar{c}_i$. Therefore, we have payment for any winner is its cost. Hence, NoTBC mechanism is individually rational.

Lemma 3. The NoTBC mechanism is budget feasible.

Proof: As it is clear that a doctor is included in the winning set only when the given condition in line 4 of Algorithm 1 and line 3 of Algorithm 2 is satisfied. As the payment in case of NoTBC is equal to the cost; the total payment will be at most the budget. Hence, NoTBC mechanism is budget feasible.

B. TBC mechanism

It is a two pass mechanism consisting of *Truthful budget* constraint leader identification (*TBC-LI*) and *Truthful* budget constraint doctor selection (*TBC-DS*) mechanisms.

1) TBC-LI mechanism: For first fold of hiring problem, we propose a TBC-LI mechanism motivated by [16][17].

Allocation rule: In this, a doctor with maximum marginal contribution per cost among the available doctors

is considered. But the doctor is selected as the leader only when the ratio between their cost as the initial adapter and budget is less than or equal to half of the ratio between their marginal contribution and the value of the subset already selected.

Example 1(a): Figure 2a show the initial configuration of the social graph along with cost distribution, and marginal contribution (m.c.). The quality vector of the nodes is given as: $Q = \{5, 1, 3, 5, 4, 5\}$. Higher the value, higher will be the quality. For understanding purpose we are taking the quality of the doctors as an integer value but in general it may not be the case. It is to be noted that the unit of cost and budget is taken as \$. We have considered hospital's budget to be 10. Using line 1 of the Algorithm 3 the node 4 is considered.



Figure 2: Detailed functioning of Algorithm 3

The condition $2 \le 5 \cdot \left(\frac{3}{3+0}\right)$ for node 4 is satisfied. So, $\Gamma = \{4\}$ and $\overline{S} = \{3, 5, 6\}$. Next, node 3 will be considered and $2.5 \le 5 \cdot \left(\frac{3}{3+3}\right)$ for node 3 is satisfied. So, $\Gamma = \{4, 3\}$ and $\overline{S} = \{3, 5, 6, 1, 2, 4\}$. So, we have $\hat{S} = \{3, 5, 6, 1, 2, 4\}$. *Payment rule:* The payment rule is motivated by [17].

Algorithm 4 TBC-LI pricing mechanism (Γ , \mathcal{B} , C) **Output:** $\mathcal{P}_{\Gamma} \leftarrow \phi$. 1: $\Gamma' \leftarrow \phi$ 2: for each $s_i \in \Gamma$ do $S' \leftarrow S \setminus \{s_i\}$ $s_j \leftarrow \operatorname{argmax}_{k \in S'} \left[\frac{M_{C_k}(\Gamma'_{k-1})}{c_k} \right]$ while $c_j \leq \mathcal{B}\left(\frac{M_{C_j}(\Gamma'_{j-1})}{M_{C_j}(\Gamma'_{j-1}) + I(\Gamma'_{j-1})} \right)$ do $\Gamma' \leftarrow \Gamma' \cup \{s_j\} \triangleright \Gamma' \text{ is the set of leaders when}$ 3: 4: 5: 6: s_i is not in the market. $\mathcal{S}' \leftarrow \mathcal{S}' \setminus \{s_j\}$ 7: $s_j \leftarrow argmax_{k \in \mathcal{S}'} \left[\frac{\mathcal{M}_{C_k}(\Gamma'_{k-1})}{c_k} \right]$ 8: 9: end while $\Gamma' \leftarrow \Gamma' \cup \{s_i\}$ 10: for each $s_i \in \Gamma'$ do 11: Calculate $C_i^j = \frac{\mathcal{M}_{C_i}^j(\Gamma'_{j-1}) \cdot c_j}{\mathcal{M}_{C_i}(\Gamma'_{i-1})}$ and $\Pi_i^j = \frac{\mathcal{B} \cdot \mathcal{M}_{C_i}^j(\Gamma'_{j-1})}{I(\Gamma'_{i-1} \cup \{i_i\})}$ 12: end for 13: $\mathcal{P}_{\Gamma_i} \leftarrow max_{j \in [1..\ell+1]} \{ min\{C_i^j, \Pi_i^j\} \}; \mathcal{P}_{\Gamma} \leftarrow \mathcal{P}_{\Gamma} \cup \{\mathcal{P}_{\Gamma_i}\} \}$ 14: 15: end for 16: return \mathcal{P}_{Γ}

In this, for each doctor $s_i \in \Gamma$ consider running line 3-9. Next, determine the smallest index ℓ in the sorting of $|\Gamma'|$ doctors (determined without s_i) such that the ratio between their cost as the initial adapter and budget is less than or equal to the ratio between their marginal contribution and the value of the subset already selected. Now for each point $j \in [1..\ell+1]$ find the maximal cost $C_i^j = \mathcal{M}_{C_i}^j(\Gamma'_{j-1}) \cdot \left(\frac{c_j}{\mathcal{M}_{C_j}(\Gamma'_{j-1})}\right)$ that doctor s_i can declare in order to be allocated instead of the doctor in the j^{th} place in the sorting; where $\mathcal{M}_{C_i}^j(\Gamma'_{j-1})$ is the marginal contribution of doctor s_i when considered on j^{th} place is given as: $\mathcal{M}_{C_i}^j(\Gamma'_{j-1}) = I(\Gamma'_{j-1} \cup \{s_i\}) - I(\Gamma'_{j-1})$. Now, if this cost does not exceed the threshold payment $\Pi_i^j = \mathcal{B} \cdot \frac{\mathcal{M}_{C_i}^{j}(\Gamma'_{j-1})}{I(\Gamma'_{j-1} \cup \{s_i\})}$ then the mechanism would have declared s_i as the leader. Considering the maximum of the values at $j \in [1..\ell+1]$ results in the payment of s_i .

Example 1(b): Figure 3 shows the payment calculation of node 4. So, placing node 4 outside the market and utilizing line 3 - 9 of Algorithm 4 on the configurations shown in Figure 3a, Figure 3b, and Figure 3c we find the critical point as $\ell = 2$ (index of node 3).



Figure 3: Payment calculation of node 4

Following Figure 3d at point 1 (index of node 2) the value $C_4^1 = 3 \cdot (\frac{1}{1}) = 3$, and $\Pi_4^1 = 10 \cdot (\frac{3}{3}) = 10$. So, min{3, 10} = 3. Similarly, at point 2 (index of node 3) the value $C_4^2 = 2 \cdot (\frac{2.5}{2}) = 2.5$, and $\Pi_4^2 = 10 \cdot (\frac{2}{3}) = 6.66$. So, min{2.5, 6.66} = 2.5. Considering point 3 (index of the first loser node *i.e* node 6) we get $C_4^3 = 2 \cdot (\frac{4}{1}) = 8$, and $\Pi_4^3 = 10 \cdot (\frac{2}{5}) = 4$. So, min{8, 4} = 4. The payment of node 4 is max{3, 2.5, 4} = 4.

2) *TBC-DS mechanism:* For the second fold of the doctors hiring problem, we propose a TBC-DS mechanism motivated by [16][17].

Allocation rule: In this, firstly the available doctors are sorted in decreasing order based on quality contribution by cost. Now, the doctors are greedily selected but will be hired only when the ratio of the selected doctor's cost of

consultation and the patient's budget is less than or equal to the ratio between the quality contribution by the selected doctor and the value of the quality of the selected subset.

 Algorithm 5 TBC-DS allocation mechanism $(\hat{S}, \mathcal{B}', \bar{C})$

 Output: $\hat{S}' \leftarrow \phi$.

 1: $Sort(\hat{S})$ > Sorting based on $\frac{\mathcal{D}_i(T_{i-1})}{\bar{c}_i}$ for all $s_i \in \hat{S}$

 2: for each $s_i \in \hat{S}$ do

 3: if $\frac{\bar{c}_i}{\mathcal{B}'} \leq \left(\frac{\mathcal{D}_i(T_{i-1})}{\mathcal{D}_i(T_{i-1}) + \mathcal{D}(T_{i-1})}\right)$ then

 4: $\hat{S}' \leftarrow \hat{S}' \cup \{s_i\}$

 5: end if

 6: end for

 7: return \hat{S}'

Example 1(c): Considering the set-up shown in Figure 2a. We have utilized the same cost vector as given in Figure 2. The patient's budget is given as 8. The quality vector is given as $Q = \{5, 1, 3, 5, 4, 5\}$. The set of nodes informed by the leaders $\{4, 3\}$ is given as $\{3, 5, 6, 4, 1, 2\}$. So, the nodes 3, 5, 6, 4, 1, and 2 are sorted based on quality contribution per cost and is given as: $\{1, 4, 6, 3, 2, 5\}$. First node 1 is considered and the condition $2 \le 8 \cdot (\frac{5}{5})$ for node 1 is satisfied. So, $\hat{S}' = \{1\}$. Next, node 4 will be considered and the condition $2 \le 8 \cdot (\frac{5}{10})$ for node 4 is satisfied. So, $\hat{S}' = \{1, 4\}$.

Payment rule: The Payment rule is motivated by [17]. For each s_i , it is defined as the minimum of the doctor's proportional share and the threshold payment.

Algorithm 6 TBC-DS Pricing Mechanism $(\hat{S}', \mathcal{B}', \bar{C})$ Output: $\hat{\mathcal{P}} \leftarrow \phi$ 1: for each $s_i \in \hat{S}'$ do 2: $\hat{\mathcal{P}}_i \leftarrow \min\left\{\frac{\mathcal{D}_i(T_{i-1})\cdot\mathcal{B}'}{\sum_{i\in\hat{S}'}\mathcal{D}(T_i)}, \frac{\mathcal{D}_i(T_{i-1})\cdot\bar{c}_{k+1}}{\mathcal{D}_{\ell+1}(T_\ell)}\right\}; \hat{\mathcal{P}} \leftarrow \hat{\mathcal{P}} \cup \hat{\mathcal{P}}_i$ 3: end for 4: return $\hat{\mathcal{P}}$

Example 1(d): The payment of doctors in $\hat{S}' = \{1, 4\}$ is: $\hat{\mathcal{P}}_1 = \min\{\frac{5\times 8}{10}, \frac{5\times 4}{5}\} = 4$ and $\hat{\mathcal{P}}_2 = \min\{\frac{5\times 8}{10}, \frac{5\times 4}{5}\} = 4$.

Lemma 4. The TBC mechanism is computationally efficient.

Proof: The time complexity is given as $O(m^2)$. Please see the full version of this paper [19] for details.

V. ANALYSIS OF TBC-LI AND TBC-DS

Lemma 5. In TBC-LI, the total payment made to the doctors are within hospital's budget \mathcal{B} .

Proof: The proof is motivated by [16]. As the maximum payment that any winning EC *i* can be paid is $\Pi_i^k = \frac{\mathcal{B} \cdot \mathcal{M}_{C_k}(T_{i-1})}{I(T_{i-1} \cup \{i\})}$. The total payment of the ECs as leaders *i.e.* $\mathcal{T}_I^{\mathcal{P}}$

is given as:

$$\mathcal{T}_{I}^{\mathcal{P}} = \sum_{i \in \hat{S}} \mathcal{P}_{i} = \sum_{i=1}^{k} \mathcal{B} \cdot \frac{\mathcal{M}_{C_{k}}(\Gamma_{i-1})}{I(\Gamma_{i-1} \cup \{s_{i}\})} = \sum_{i=1}^{k} \mathcal{B} \cdot \frac{\mathcal{M}_{C_{k}}(\Gamma_{i-1})}{I(\Gamma_{i})}$$

$$\leq \frac{\mathcal{B}}{I(\Gamma_{k})} \cdot \sum_{i=1}^{k} \mathcal{M}_{C_{k}}(\Gamma_{i-1}) = \frac{\mathcal{B}}{I(\Gamma_{k})} \cdot \sum_{i=1}^{k} \frac{I(\Gamma_{i-1} \cup \{s_{i}\})}{\prod_{\text{formed ECs}} - \prod_{\text{by set } \Gamma_{i-1}}}$$

$$= \frac{\mathcal{B}}{I(\Gamma_{k})} \cdot \sum_{i=1}^{k} \frac{I(\Gamma_{i})}{\prod_{\text{formed ECs}}} - \underbrace{I(\Gamma_{i-1})}{\prod_{\text{formed ECs}}} \Rightarrow \mathcal{T}_{I}^{\mathcal{P}} \leq \mathcal{B}$$

Hence, it is proved that the incentive compatible total payment do not exceed the budget.

Lemma 6. In TBC-LI mechanism, if any doctor s_i comes ahead of its current position say i' < i by declaring a cost $c_{i'} < c_i$ then,

$$\frac{\mathcal{B}}{2}\left(\frac{\mathcal{M}_{\mathcal{C}_{i'}}(\Gamma_{i'-1})}{\mathcal{M}_{\mathcal{C}_{i'}}(\Gamma_{i'-1})+\mathcal{I}(\Gamma_{i'-1})}\right) > \frac{\mathcal{B}}{2}\left(\frac{\mathcal{M}_{\mathcal{C}_{i}}(\Gamma_{i-1})}{\mathcal{M}_{\mathcal{C}_{i}}(\Gamma_{i-1})+\mathcal{I}(\Gamma_{i-1})}\right)$$

Proof: If the EC *i* by reporting $c_{i'}$ moves at position *i'* such that i' < i as depicted in *Figure* 4 below:

Figure 4: Pictorial representation

From the definition of $I(\cdot)$ we can say: $I(\Gamma_{i'-1}) < I(\Gamma_{i-1})$. As the set $\Gamma_{i'-1}$ is smaller as compared to the set Γ_{i-1} , so from the definition of the monotone sub-modular marginal contribution property, it can be said:

$$\underbrace{\mathcal{M}_{C_{i'}}(\Gamma_{i'-1})}_{\text{Marginal contribution}} > \underbrace{\mathcal{M}_{C_i}(\Gamma_{i-1})}_{\text{Marginal contribution}}$$
(1)

The number of ECs leaders by the set $\Gamma_{i'}$ will be less than the number of ECs leaders by Γ_i . Mathematically,

$$\underbrace{\mathcal{M}_{C_{i'}}(\Gamma_{i'-1})}_{\text{Marginal contribution}} + \mathcal{I}(\Gamma_{i'-1}) < \underbrace{\mathcal{M}_{C_{i}}(\Gamma_{i-1})}_{\text{Marginal contribution}} + \mathcal{I}(\Gamma_{i-1}) \\
\underbrace{\mathcal{M}_{C_{i'}}(\Gamma_{i'-1})}_{\mathbf{M}_{C_{i'}}(\Gamma_{i'-1}) + \mathcal{I}(\Gamma_{i'-1})} > \frac{1}{\mathcal{M}_{C_{i}}(\Gamma_{i-1}) + \mathcal{I}(\Gamma_{i-1})}$$
(2)

Combining equation 1, equation 2 and multiplying both side by $\frac{\mathcal{B}}{2}$, we get

$$\frac{\mathcal{B}}{2} \left(\frac{\mathcal{M}_{C_{i'}}(\Gamma_{i'-1})}{\mathcal{M}_{C_{i'}}(\Gamma_{i'-1}) + \mathcal{I}(\Gamma_{i'-1})} \right) > \frac{\mathcal{B}}{2} \left(\frac{\mathcal{M}_{C_i}(\Gamma_{i-1})}{\mathcal{M}_{C_i}(\Gamma_{i-1}) + \mathcal{I}(\Gamma_{i-1})} \right)$$

Hence, it is proved.

Theorem 1. TBC-LI mechanism is monotone.

Proof: Fix *i*, c_{-i} , c_i , and $c_{i'}$. For mechanism TBC-LI

mechanism to be monotone, we need to show that, any winning EC *i* with private cost c_i will still be considered in the winning set of ECs when declaring $c_{i'}$ such that $c_{i'} < c_i$ or any losing EC *i* with private cost c_i will still be considered in the losing set of ECs when declaring $c_{i'}$ such that $c_{i'} > c_i$. The proof is divided into two cases.

Case 1. In this case, the *i*th winning EC deviates and reveals a cost of consultation $c_{i'} < c_i$. Again two cases can happen. If the EC i shows a small deviation in his/her (henceforth his) cost c_i *i.e.* $c_{i'}$ such that $c_{i'} < c_i$ and the current position of the EC *i* remains unchanged. In this situation, it can still be considered in the winning set. It is to be noted that, if the EC *i* reports a large deviation in his cost c_i *i.e.* $c_{i'}$ such that $c_{i'} < c_i$, then in this case by definition:

$$\frac{\mathcal{M}_{\mathcal{C}_1}(\Gamma_0)}{c_1} \geq \frac{\mathcal{M}_{\mathcal{C}_2}(\Gamma_1)}{c_2} \geq \frac{\mathcal{M}_{\mathcal{C}_3}(\Gamma_2)}{c_3} \geq \ldots \geq \frac{\mathcal{M}_{\mathcal{C}_n}(\Gamma_{n-1})}{c_n}$$

EC *i* will be placed some position ahead (say i') of its current position say *i i.e.* i' < i. This scenario is depicted in *Figure* 5 below.

$$\bigcirc \begin{array}{c} c_{i'-1} & c_{i'} & c_{i-1} & c_{i} \\ \hline i'-1 & i' & i-1 & i \\ \end{array} \bigcirc \begin{array}{c} c_{i'} & c_{i'} \\ \hline c_{i'} & c_{i'} \\ c_{i'} & c_{i'} \\$$

Figure 5: Pictorial representation

From Lemma 6 it can be said that if EC i is placed some position ahead by revealing a cost $c_{i'} < c_i$ then it must satisfy

$$\frac{\mathcal{M}_{\mathcal{C}_{i'}}(\Gamma_{i'-1})}{\mathcal{M}_{\mathcal{C}_{i'}}(\Gamma_{i'-1}) + \mathcal{I}(\Gamma_{i'-1})} > \frac{\mathcal{M}_{\mathcal{C}_i}(\Gamma_{i-1})}{\mathcal{M}_{\mathcal{C}_i}(\Gamma_{i-1}) + \mathcal{I}(\Gamma_{i-1})}$$
(3)

Let us suppose for the sake of contradiction that, when the EC $i \in S$ comes ahead in ordering say at some position i'such that $c'_i < c_i$, then it is not considered in the winning set of the EC because it is not satisfying the given budget. If this is the case, then it means that:

$$c_{i'} > \frac{\mathcal{B}}{2} \left(\frac{\mathcal{M}_{\mathcal{C}_{i'}}(\Gamma_{i'-1})}{\mathcal{M}_{\mathcal{C}_{i'}}(\Gamma_{i'-1}) + \mathcal{I}(\Gamma_{i'-1})} \right)$$
(4)

Combining our assumption $c_{i'} < c_i$ and equation 4 it can be concluded that:

$$\frac{\mathcal{B}}{2} \left(\frac{\mathcal{M}_{C_{i'}}(\Gamma_{i'-1})}{\mathcal{M}_{C_{i'}}(\Gamma_{i'-1}) + \mathcal{I}(\Gamma_{i'-1})} \right) < c_{i'} < c_i$$
(5)

Using condition in line 2 of *Algorithm 3* and *equation 5*, we can say that:

$$\frac{\mathcal{B}}{2}\left(\frac{\mathcal{M}_{C_{i'}}(\Gamma_{i'-1})}{\mathcal{M}_{C_{i'}}(\Gamma_{i'-1}) + \mathcal{I}(\Gamma_{i'-1})}\right) < c_{i'} < c_i < \frac{\mathcal{B}}{2}\left(\frac{\mathcal{M}_{C_i}(\Gamma_{i-1})}{\mathcal{M}_{C_i}(\Gamma_{i-1}) + \mathcal{I}(\Gamma_{i-1})}\right)$$

$$\Rightarrow \frac{\mathcal{B}}{2} \left(\frac{\mathcal{M}_{C_{i'}}(\Gamma_{i'-1})}{\mathcal{M}_{C_{i'}}(\Gamma_{i'-1}) + I(\Gamma_{i'-1})} \right) < \frac{\mathcal{B}}{2} \left(\frac{\mathcal{M}_{C_i}(\Gamma_{i-1})}{\mathcal{M}_{C_i}(\Gamma_{i-1}) + I(\Gamma_{i-1})} \right)$$
$$\Rightarrow \frac{\mathcal{M}_{C_{i'}}(\Gamma_{i'-1})}{\mathcal{M}_{C_{i'}}(\Gamma_{i'-1}) + I(\Gamma_{i'-1})} < \frac{\mathcal{M}_{C_i}(\Gamma_{i-1})}{\mathcal{M}_{C_i}(\Gamma_{i-1}) + I(\Gamma_{i-1})}$$

So, it is a contradiction.

Case 2. Please see the full version of this paper [19] for details.

Hence, the theorem is proved.

Theorem 2. In TBC-DS, the function $\mathcal{D}: 2^{\mathcal{S}} \Rightarrow \mathcal{R}_{>0}$ is:

- (a) **monotone:** If $S \subseteq \mathcal{F}$ then $\mathcal{D}(S) \leq \mathcal{D}(\mathcal{F})$, and
- (b) submodular: If $\mathcal{D}(S \cup \{i\}) \mathcal{D}(S) \ge \mathcal{D}(\mathcal{F} \cup \{i\}) \mathcal{D}(\mathcal{F})$ $\forall S \subseteq \mathcal{F}.$

Proof: Please see the full version of this paper [19] for details.

Theorem 3. TBC-LI is truthful.

Proof: Fix EC j, c_{-i} , c_i , and $c_{i'}$. For TBC-LI mechanism to be truthful, we need to show that, it is not beneficial for any EC *j* to *underbid* or *overbid* say $c_{i'}$ such that $c_{i'} < c_i$ or $c_{i'} > c_i$ respectively. For each of the above possible scenarios, the proof is divided into two cases. Before going into the different cases let's consider the case where the EC j is reporting his true cost c_j . The pictorial representation of the possible set-up with n ECs are shown in Figure 6.



Figure 6: Pictorial representation

The values from 1 to *n* represents the position (or index). Currently, our analysis lies around the index k and k + 1; where k denote the index of the last EC ℓ that respects the allocation condition given in line 2 of Algorithm 3. As the ECs are sorted based on the marginal contribution per cost, so we can write

$$\frac{\mathcal{M}_{C_i}(\Gamma_{i-1})}{c_i} \ge \frac{\mathcal{M}_{C_j}(\Gamma_{i-1})}{c_j} \Rightarrow c_j \ge \frac{\mathcal{M}_{C_j}(\Gamma_{i-1}) \cdot c_i}{\mathcal{M}_{C_i}(\Gamma_{i-1})} \quad (6)$$

By using line 12 of *algorithm 4* and equation 6 it can be easily seen that, $c_j \ge C_j^i$. If this is the case then we can say $c_j \ge \Pi_i^i$. In order to be allocated EC j must satisfy $C_i^i \le \Pi_i^i$ otherwise $C_j^i > \Pi_j^i$ means not allocated. As we are taking the payment as:

- If $C_j^i = \Pi_j^i \Rightarrow \mathcal{P}_{\Gamma_j} = \min\{C_j^i, \Pi_j^i\} = \Pi_j^i = C_j^i \le c_j$ If $C_j^i > \Pi_j^i \Rightarrow \mathcal{P}_{\Gamma_j} = \min\{C_j^i, \Pi_j^i\} = \Pi_j^i = \Pi_j^i \le c_j$ If $C_j^i < \Pi_j^i \Rightarrow \mathcal{P}_{\Gamma_j} = \min\{C_j^i, \Pi_j^i\} = C_j^i \le c_j$

If this is the case, then it can be concluded that $C_i^i \leq c_i$ or $\Pi_{i}^{i} \leq c_{i}$. As the payment is less than the actual cost. Hence not allocated. Coming back to our underbid and overbid cases.

Scenario 1: Underbidding $(c_{j'} < c_j)$: In this case, the j^{th} EC deviates and reveals a cost of consultation $c_{j'} < c_j$. This scenario give rise to two cases.

Case 1:When EC j is in losing set.: If the EC j shows a small deviation in his/her (henceforth his) cost *i.e.* $c_{i'}$ such



Figure 7: Pictorial representation

that $c_{j'} < c_j$ and the current position of the EC *j* remains unchanged. In this situation, it can still be considered in the losing set. It is to be noted that, if the EC *j* reports a large deviation in his cost c_j *i.e.* $c_{j'}$ in this case it will belong to winning set and will appear before EC *i* as shown in Figure 7. As the ECs are sorted based on the marginal contribution per cost, so we can write:

$$\frac{\mathcal{M}_{C_j}(\Gamma_{i-1})}{c_{j'}} \ge \frac{\mathcal{M}_{C_i}(\Gamma_{i-1})}{c_i} \Rightarrow c_{j'} \le \frac{\mathcal{M}_{C_j}(\Gamma_{i-1}) \cdot c_i}{\mathcal{M}_{C_i}(\Gamma_{i-1})}$$

and $c_{j'} \leq \frac{\mathcal{M}_{C_j}(\Gamma_{i-1}) \cdot c_i}{\mathcal{M}_{C_j}(\Gamma_{i-1})} = C_j^i$ because from above we have got the relation $C_j^i \leq c_j$. This will lead to $c_{j'} \leq C_j^i \leq c_j$. The EC *j* is paid less than the actual cost.

Case 2: When EC j *is in winning set.*: If the EC j shows a deviation in his cost such that $c_{j'} < c_j$ it will still belong to winning set and will appear before EC i as shown in Figure 8. As the ECs are sorted based on the marginal contribution per cost, so we can write:

$$\frac{\mathcal{M}_{C_j}(\Gamma_{l-1})}{c_j} \ge \frac{\mathcal{M}_{C_i}(\Gamma_{i-1})}{c_i} \Rightarrow c_j \le \frac{\mathcal{M}_{C_j}(\Gamma_{l-1}) \cdot c_i}{\mathcal{M}_{C_i}(\Gamma_{i-1})} = C_j^l$$

and for the case when the EC j deviates, then

$$\frac{\mathcal{M}_{C_j}(\Gamma_{t-1})}{c_{j'}} \geq \frac{\mathcal{M}_{C_i}(\Gamma_{i-1})}{c_i} \Rightarrow c_{j'} \leq \frac{\mathcal{M}_{C_j}(\Gamma_{t-1}) \cdot c_i}{\mathcal{M}_{C_i}(\Gamma_{i-1})} = C_j^{1/2}$$

From above two equations it is clear that no matter what cost EC *j* is bidding, he will still be winning and be paid an amount C_j^l . Hence, considering *Case 1* and *Case 2* it can be concluded that EC *j* does not gain by underbidding there true cost. In similar fashion, we can write the above mentioned equation for any position *i* before *k* and in the same way we can prove that $C_i^i \le c_j$.



Figure 8: Pictorial representation

Scenario 2: Overbidding $(c_{j'} > c_j)$: Please see the full version of this paper [19] for details. Hence, the theorem is proved.

VI. PERFORMANCE EVALUATION

We compared our proposed mechanisms against the benchmark mechanism (random mechanism). In this, the doctors are selected randomly and are paid their declared cost. We have utilized the *coverage model* for the first fold of our hiring problem. The performance metric includes the Interested doctors set size, and Number of doctors hired. The unit of cost and budget is \$.

A. Simulation set-up

For our simulation purpose, a social graph is generated randomly using Networkx package of python. It consists of 1000 nodes (doctors) and approximately 28,250 edges. The maximum and minimum degree a node can have is 10% and 1% of the total available nodes respectively. The cost of each node as initial adapter is uniformly distributed over [30, 50], the cost of consultancy is uniformly distributed over [35, 50], and quality is uniformly distributed over [20, 50]. The budget is considered in range [100, 1000].

B. Result analysis

The simulation results shown in Figure 9 shows the comparison of the interested doctors set size *i.e.* the number of doctors acting as leaders and the number of doctors informed by the leaders about the hiring concept.



Figure 9: Interested doctors set *Figure 10:* Hired doctors with budsize with budget $\mathcal{B} \in [100, 1000]$ get $\mathcal{B}' \in [100, 1000]$

It is seen in Figure 9 that the interested doctors set size in case of NoTBC mechanism is higher than TBC mechanism and random mechanism. This nature of NoTBC mechanism is obvious due to the fact that the mechanisms (NoTBC-LI and NoTBC-DS) are utilizing almost the complete quota of the available budgets whereas TBC mechanism is utilizing only a part of total budget. With the increase in budget, one can easily see the increasing gap between NoTBC mechanism and TBC mechanism. It can be seen evidently in Figure 10 that the number of doctors hired in case of NoTBC mechanism is higher than TBC mechanism and random mechanism. Similar reasoning can be given as above.

VII. CONCLUSIONS AND FUTURE WORKS

In this paper, we have addressed the problem of hiring the doctors from outside of the hospital when a patient is constrained by budget. Designing a more general mechanism in this environment for the set-up consisting of multiple patients say n (each patient is associated to different hospitals) and m doctors can be thought of as the future work.

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