Universitat Politècnica de Catalunya (UPC)
Universitat Autònoma de Barcelona (UAB)
Universitat de Barcelona (UB)
Institut de Ciències Fotòniques (ICFO)

http://www.photonicsbcn.eu

Master in Photonics

MASTER THESIS WORK

Long-range polarimetric propagation in turbid media

Maria Ballesta Garcia

Supervised by Dr./Prof. Santiago Royo, (UPC, CD6)

Presented on date 6th September 2018

Registered at

Escola Tècnica Superior d'Enginyeria de Telecomunicació de Barcelona

Long-range polarimetric propagation in turbid media

Maria Ballesta Garcia

Beamagine S.L., Carrer Bellesguard 16, 08755, Castellbisbal, Spain

E-mail: maria.ballesta.garcia@alu-etsetb.edu

6th September 2018

Abstract. Imaging through nebulous media encountered in nature, like fog, rain, and light haze, is on the most up-to-date research problems in the field of navigation as a result of the recent increasing interest in autonomous vehicles. However, it is still an unresolved problem. One of the proposed approaches to solve it is based on polarization, as in a turbid medium, polarized light is known to preserve its polarization properties deeper into the medium. Thus, the main aim of this thesis is to create from scratch a model of the interaction between polarized light and the scattering particles present in this kind of medium. As a result, this work presents the development and validation of a model based on a combination of Mie theory and the Monte Carlo method, for its later use as tool to study the feasibility and characteristics of a polarimetric imaging technique for long-range imaging in nebulous media.

Keywords: Polarimetric imaging, Propagation through turbid media, Scattering, Monte Carlo, Mie theory

1. Introduction

Imaging of light sources and objects hidden behind a turbid medium could have a wide profit in areas such as medical diagnostics, remote sensing, transport and navigation. Specifically, imaging through nebulous media encountered in nature, like fog, is still an issue that attracts a lot of attention, and, in fact, is the main topic of this Thesis. Enhanced vision in such weather conditions has indeed tremendous applications, for example, in all kind of autonomous vehicles [1]. The problem of imaging through such kind of media has been addressed using various techniques that essentially rely on discriminating forward scattered photons from multiple scattered (diffuse) photons traversing through the turbid medium. One of these techniques is based on polarization. In a turbid medium, polarized light is known to preserve its polarization properties deeper into the medium [2]. Thus, a device capable of recording the polarization information of received light may provide a simple solution for enhancing visibility through a turbid medium. Therefore, the aim of this Master Thesis is to develop a model of the interaction between polarized light and the scattering particles present in the medium, in order to check the feasibility and characteristics of a polarimetric imaging technique for long-range imaging in this kind of medium.

The radiative transport equation (RTE) provides a mathematical description for the physical phenomenon of energy transfer, in the form of electromagnetic (EM) radiation, through turbid media. However, this has a limited applicability in real systems as their solutions are considered analytically unfeasible. Hence, since there are not analytical solutions, it must be either approximated or solved numerically. Nowadays, several modelling techniques are used, being Monte Carlo (MC) method on of the prevalent. MC methods are robust and provide accurate solutions: they are considered to be the gold standard methodology in light propagation modelling throughout turbid media [3]. Hence, this work is devoted to the development of an own model to properly describe the propagation of polarized beams in turbid media. We will first review the State-of-the-art in Sec. 2. In Sec. 3, a brief theoretical background of the EM theory used in our models is presented. Sec. 4 presents the MC simulation method for radiometric and polarimetric approaches. Results are shown in Sec. 5, which essentially include the validation of the presented models; and, finally, in Sec. 6, conclusions and future work are disclosed. An Annex with more detailed information on the contents of Sec. 3 and Sec. 4 has been included due to the limits in used space of this thesis.

2. State of the art

Various approaches have been studied to solve the problem of imaging through turbid medium in the field of navigation, such as time-gated imaging, spatial filtering techniques or intensity modulation schemes [1]. An alternative approach involves light polarization, based on the preservation of the state of polarization during propagation through thick scattering media. This so-called polarization memory effect has already been analyzed in numerous references [2]. This can be exploited using a polarization-sensitive imaging device to enhance the visibility of a source that emits polarized light. This approach has already proved to be efficient and been reported in a number of laboratory experiments. Recently, Fade and Panigrahi [1, 4] have been the ones with deeper acknowledgment of the topic with an already developed experimental implementation of long-range polarimetric imaging through fog over kilometric distances in real atmospheric conditions.

On the other hand, the interest in research on the propagation of polarized light in randomly scattering media has also increased due to the potential existing applications. Kattawar and Plass [5] were the first to calculate the status of polarization of the light after multiple scattering using a MC method. Since then, several research groups have developed numerical methods based on MC models that describe the propagation of polarized light in scattering media under the goal of a better understanding of this process [3,6]. All of them used MC algorithms based on the Stokes-Mueller formulation, the formulation that we are going to use.

3. Theoretical background

To attain imaging through turbid media, it is needed to understand the behaviour of light as it traverses through a scattering medium. Since we are mainly interested in the propagation of light through nebulous media, like fog, and those media are characterized for having spherical particles (water droplets), we are going to center our attention in developing specifically the theory of diffusion by spherically shaped particles of arbitrary radius and known refractive index, based on the Mie theory [7].

Firstly, it is desirable to present polarization in a systematic way. The state of polarization and intensity of a beam is specified by a 4 x 1 vector, known as Stokes vector (**S**), in the following form:

$$S = \begin{bmatrix} E_l E_l^* + E_r E_r^* \\ E_l E_l^* - E_r E_r^* \\ E_l E_r^* + E_r E_l^* \\ i(E_l E_r^* - E_r E_l^*) \end{bmatrix} = \begin{bmatrix} I \\ Q \\ U \\ V \end{bmatrix} \tag{1}$$

where, E_r and E_l are two orthogonal electrical field components in a plane perpendicular to the propagation direction, the elements Q, U, V define the state of polarization of the light beam and I denotes the intensity. The Stokes parameters are time averages, therefore no coherence effects are considered. These parameters are real numbers and obey the inequality: $I^2 \geq Q^2 + U^2 + V^2$.

When Stokes vectors are used to describe the propagation of light through optical components such as lenses, each of these components can be represented uniquely by a 4 x 4 Mueller matrix **M**. Consequently, when the Stokes-Mueller formalism is employed to describe scattering events, the scattering properties of a medium are defined by the Mueller scattering matrix. By multiplying the Stokes vector of the incident polarization state with this Mueller matrix, one obtains the resulting Stokes vector of the scattered light. For scattering in a medium composed of spherical scatterers, this matrix can be derived rigorously by use of the Mie theory.

Once the polarization nomenclature has been presented, a brief comment about Mie theory (in Stokes-Mueller formulation) is going to be disclosed. For a more detailed and deeper exposure of the development of this theory and, also, for the explicit calculations of the functions mentioned next, see the 1st section of the Annex attached to this work.

The scattering in any direction is described by four amplitude functions: S_1, S_2, S_3 and S_4; that depend, in general, on the scattering angle θ and the azimuthal angle ϕ; which form a matrix $\mathbf{S}(\theta, \phi)$ of four elements (do not confuse this **S** with the Stokes vector). The formulae for spherical particles results in a null value for two of these amplitude functions, so only two of them are of interest: $S_1(\theta)$ and $S_2(\theta)$, which are independent of ϕ. These results hold without restriction for light of arbitrary polarization, provided that the scattering particles are homogeneous and spherical. The amplitude of the field scattered by an arbitrary particle is a linear function of the amplitude of the incident field. This relationship between incident and scattered fields is conveniently written in matrix form as:

$$\begin{bmatrix} E'_\perp \\ E'_\parallel \end{bmatrix} = \frac{e^{-ik(r-z)}}{-ikr} \begin{bmatrix} S_2 & 0 \\ 0 & S_1 \end{bmatrix} \begin{bmatrix} E_\perp \\ E_\parallel \end{bmatrix} \tag{2}$$

where elements $E_\perp$ and $E_\parallel$ corresponds to the perpendicular and parallel components of the electric field relative to the the plane of scattering, which is the plane defined by the direction of propagation of the incident plane wave and the direction of scattering.

Thus, the relationship between the incident and scattered Stokes parameters is:

$$\begin{bmatrix} I_s \\ Q_s \\ U_s \\ V_s \end{bmatrix} = \frac{1}{k^2r^2} \begin{bmatrix} m_{11} & m_{12} & 0 & 0 \\ m_{12} & m_{11} & 0 & 0 \\ 0 & 0 & m_{33} & -m_{34} \\ 0 & 0 & m_{34} & m_{33} \end{bmatrix} \begin{bmatrix} I_i \\ Q_i \\ U_i \\ V_i \end{bmatrix} \tag{3}$$

where:

$$\begin{aligned} m_{11} &= \frac{1}{2}(\mid S_2 \mid^2 + \mid S_1 \mid^2), \quad m_{12} = \frac{1}{2}(\mid S_2 \mid^2 - \mid S_1 \mid^2), \\ m_{33} &= \frac{1}{2}(S_2^* S_1 + S_2 S_1^*), \qquad m_{34} = \frac{i}{2}(S_2^* S_1 - S_2 S_1^*) \end{aligned} \tag{4}$$

In order to obtain quantitative results from the Mie theory and calculate each of the coefficients of the scattering Mueller matrix, it is necessary a lengthy, although straightforward, procedure. This approach includes the calculation of what are known as the Mie angular functions (π_n and τ_n) and the scattering coefficients (a_n and b_n). The EM field has to be written as an expansion of a plane wave in vector spherical harmonics: the electromagnetic normal modes of the spherical particle. At this point, it is convenient to define π_n and τ_n which determine the θ dependence of the fields. Then, in general, the scattered field is a superposition of those normal modes, each weighted by the appropriate coefficient a_n or b_n. Moreover, armed with this approach, it is also possible to calculate other important parameters for this problem, such as the phase function, and the scattering (μ_s) and absorption (μ_a) coefficients which are used to characterize the optical properties of the medium.

4. Methods and materials: simulations

Monte Carlo method refers to a technique to simulate physical processes using a stochastic model. In the radiative transfer problem (RTP), the MC method consists of recording photons histories as they are scattered and absorbed. Thus, MC simulations show the expected movement of individual photons, which are treated as particles of light that behave according to certain probability density functions for their movement.

4.1. Radiometric MC model

First of all we wish to develop a Monte Carlo model with the conceptual simplicity of radiation transfer [8] –a photon is emitted, it travels a distance, and then something happens to it– upon which, later, we will add the polarization treatment. As long as the Monte Carlo method for RTP is a well-established and well-known technique, we are not going to to detail it. In order to obtain a deeper knowledge of each of the building blocks of the code, please look at the 2nd section of the attached Annex.

Our code is based on a variance reduction technique called implicit capture: many photons (a packet) are propagated along each pathway. The size of this packet is called its weight (w) and after each propagation step it is reduced by the probability of absorption.

The MC method begins by launching a packet (from now on called photon) into the medium with $w = 1$. Once launched, the photon is moved a straight path $\Delta s = -ln\xi/\mu_t$, being $\mu_t = \mu_a + \mu_s$ and ξ a random number uniformly distributed between 0 and 1. This is a variable-step size approach and the step is chosen in such a way that it is the distance at which the photon interacts with an obstacle. Scheme shown in Fig. 1 summarizes the whole code structure, once the first packet is initialized.

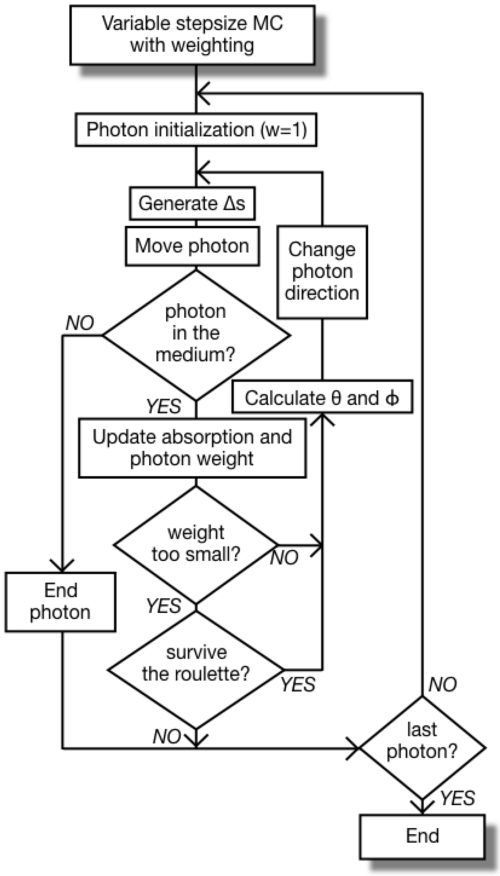


Figure 1. Flux diagram of the MC method used

After each propagation step, the weight of the photon is reduced by the probability of absorption. The absorbed fraction is placed in a bin **W** that encloses the current photon position: $W(x', y', z') = (1-a)w$, and the new scattered photon's weight is actualized: $w' = aw$, where $a = \mu_s/\mu_t$. As the weight of the photon falls below a certain threshold, the so-called Russian Roulette technique is applied: it can be stochastically decided to either terminate the photon or to let it travel.

The angle at which the light is bent when it strikes an obstacle is described by two angles of scatter: θ and ϕ, the deflection and azimuthal scattering angles, respectively. A normalized phase function describes the probability density function for the angle at which a photon is scattered. If the phase function is not isotropic, then a parameter called the average cosine of the phase function (g) is used to describe the degree of anisotropy of the phase function. The most commonly used phase function is the Henyey-Greenstein (HG) function.

Once all the photons have been run, the data has been stored in the bin **W**, in units of weight/bin. In order to have understandable values some changes are needed from which it is possible to obtain the fluence rate ϕ $[Watts/cm^2]$:

$$\phi(x,y,z) = \frac{W(x,y,z) \cdot \#\ \textit{of photons per packet} \cdot \textsc{Incident Power}}{\mu_a \cdot V \cdot \textsc{Total number of photons}} \tag{5}$$

4.2. Polarimetric MC model

MC techniques to simulate the propagation of light of polarized light in turbid media are based on the Stokes-Mueller formulation [3] presented before. Except for some differences, the general scheme of the polarized approach is similar to those of the scalar approach, treated previously.

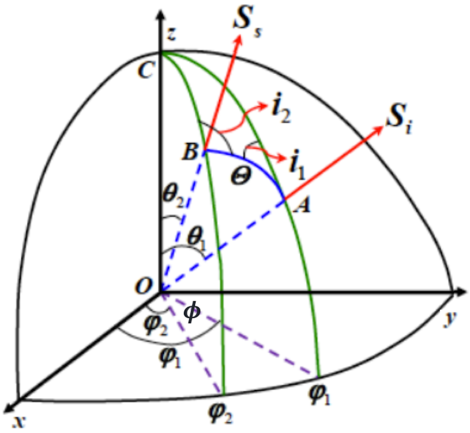


Figure 2. Geometry of a scattering event.

4.2.1. Scattering of the Stokes vector. The status of polarization of a field is defined with respect to a reference plane. Seeing Fig. 2, the incident Stokes vector (S_i) is defined with respect to its meridian plane (AOC) that is created by the direction of propagation and the Z-axis. In order to compute the change in polarization, S_i needs to be expressed in the scattering plane (AOB) before to be multiplied by the particle scattering matrix. Thus, once the scattering angles (Θ and Φ) are determined, a first rotation to express S_i in the AOB plane is done, $\mathbf{R}(\Phi)$. Then, the rotated Stokes vector is multiplied by the Mueller scattering matrix $\mathbf{M}(\Theta)$ to obtain the scattered Stokes vector (S_s) in the scattering plane. Next, a second rotation, $\mathbf{R}(i_2)$, is needed to express S_s in its meridian plane (BOC). Hence, the status of polarization of the field after a scattering event in the new local coordinate system is given by:

$$S_s = \mathbf{R}(i_2)\mathbf{M}(\Theta)S_i\mathbf{R}(\Phi) \tag{6}$$

where $\mathbf{M}(\Theta)$ is the Mueller scattering matrix deduced from Mie theory presented in previous section (matrix in Eq. 3); $\mathbf{R}$ is a rotation matrix that connects two Stokes vectors that express the same polarization state in two different reference planes:

$$R(\beta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(2\beta) & \sin(2\beta) & 0 \\ 0 & -\sin(2\beta) & \cos(2\beta) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{7}$$

being β the angle of rotation between the planes; and i_2 the angle that relates the scattering plane and the new meridian plane, and can be derived from geometry.

4.2.2. Computation of the angles. A fundamental problem in this MC model with polarization information is the choice of the scattering angles, Θ and Φ. Generally, an incident plane wave is scattered over the whole solid angle, resulting in an intensity distribution $I(\Theta, \Phi)$. As long as we are dealing with discrete packets and, consequently, discrete deflection angles, it is necessary to properly sample Θ and Φ according to a logical probability function. The preferred approach is to directly sample the angles so that the probability density ρ of a packet being scattered under these angles is proportional to the expected intensity. Thus, ρ is obtained in terms of the Stokes components of an incident state by applying $\rho(\Theta, \Phi) \propto I'(\Theta, \Phi)$, with:

$$I'(\Theta, \Phi) = m_{11}(\Theta)I + m_{12}(\Theta)[Q\cos(2\Phi) + U\sin(2\Phi)] \tag{8}$$

Then, the so-called rejection method is applied. It is a technique that may be used to generate random variables with a particular distribution. For a bivariate distribution, as our case, three random numbers are generated: (i) P_{rand} is a uniform distributed random number between 0 and 1; (ii) Θ_{rand} is generated uniformly between 0 and π; and (iii) Φ_{rand} which is uniformly distributed between 0 and 2π. If $P_{rand} \leq \rho(\Theta_{rand}, \Phi_{rand})$, the angles are accepted as the new scattering angles. If not, P_{rand}, Θ_{rand} and Φ_{rand} are regenerated and the test is repeated.

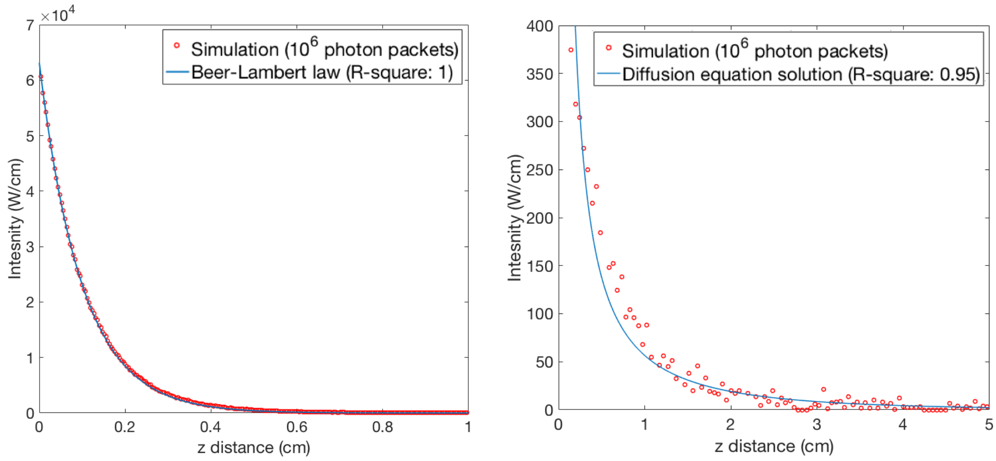
4.2.3. Computed values. In order to ensure the conservation of energy, a normalization must be applied to S_s. That is to say the I-Stokes element of the photon must be always kept as one as it propagates through. Thus, the normalized Stokes vector is written as: $S_s = (1, Q/I, U/I, V/I)'$. In other words, the polarization information is always normalized. The photon is removed from the system using the same approach as in the scalar model, the Russian Roulette, and the fluence rate is obtained throughout absorption $\mathbf{W}(x, y, z)$, Eq. 5. If the photon exits the media with a certain weight w the corresponding Stokes vector is multiplied by w to keep into account the photon attenuation: $S_{out} = (w, w \cdot Q, w \cdot U, w \cdot V)'$.

5. Results

5.1. Validation of the Radiometric MC model

First of all, we are interested in verifying the Radiometric MC model that we have built as starting point. To do so, we have chosen two limit cases that already have established analytic models which we can compare with our simulations.
To begin with, it is well known that when absorption prevails over scattering ($\mu_a >> \mu_s$), Beer-Lambert Law [9] prevails. Next, we have developed a case in the diffusion regime [9] ($\mu_a << \mu_s$, $\mu_s \approx 10\mu_a$), where an analytical solution, based on the theory of photon density waves in steady state conditions, source-free and homogeneous media conditions, exists.

In Fig. 3, we can clearly see that our simulations follow the expected theoretical models. Thus, we have checked that our radiometric model, including the implementation of the MC method, is valid and works properly, enabling us to move further towards the polarimetric model.



(a)Absorption prevailing over scatt. ($\mu_a = 10$, $\mu_s = 0.05$, $g = 0.99$)

(b) Diffusion regime. ($\mu_a = 0.05$, $\mu_s = 2$, $g = 0.5$)

Figure 3. Validation of the radioemtric model for two known limits

5.2. Validation of the Polarimetric MC model

Next, we needed to validate the Polarimetric MC model. Unfortunately, a full validation of the code has not been possible due to the limited time extension of this work. However, some intermediate validations could be implemented. To do so, we have used the consulted literature to compare our intermediate results. This includes the verification of the first five Mie angular functions (Fig. 4), the scattering coefficients for an specific case (Table 1), the matrix elements $\mid S_2 \mid^2$ and $\mid S_1 \mid^2$ for the previous case (Fig. 5) and the four elements of the Mueller scattering matrix (Fig. 6); all of them numerically computed based on the Mie theory.

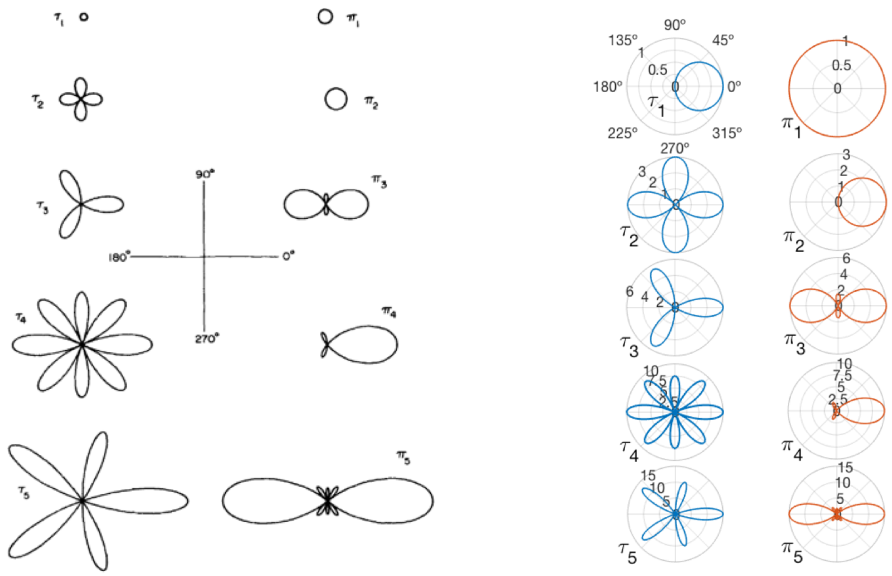
The results and data values obtained using our code are exactly the same as the ones from the literature. The next step is to use them for properly implementing the full MC-polarimetric model, which has been coded but not validated.

n	a_n	b_n
1	$5.1631 \times 10^{-1} - 4.9973i \times 10^{-1}$	$7.3767 \times 10^{-1} - 4.3990i \times 10^{-1}$
2	$3.4192 \times 10^{-1} - 4.7435i \times 10^{-1}$	$4.0079 \times 10^{-1} - 4.9006i \times 10^{-1}$
3	$4.8467 \times 10^{-2} - 2.1475i \times 10^{-1}$	$9.3552 \times 10^{-3} - 9.6269i \times 10^{-2}$
4	$1.0346 \times 10^{-3} - 3.2148i \times 10^{-2}$	$6.8811 \times 10^{-5} - 8.2949i \times 10^{-3}$

Table 1. Scattering coefficients for a water droplet in air with size parameter $x = 3$ and complex refractive index $m = 1.33 + i10^{-8}$. Values computed with our code, exactly equal to the ones in [7]

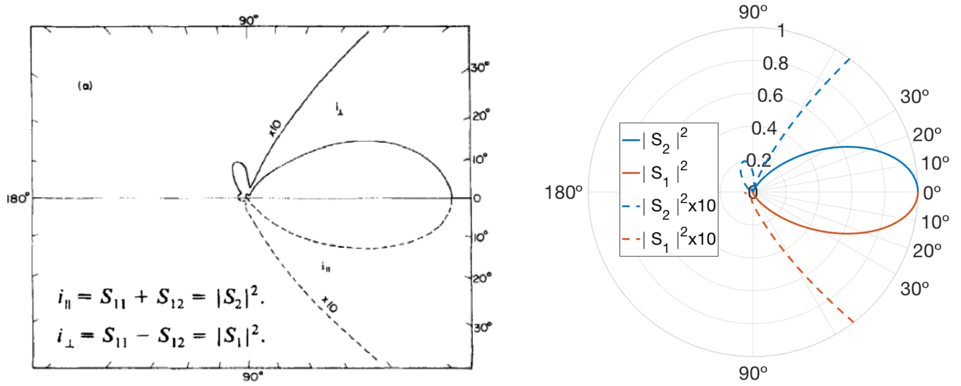
6. Conclusions and future work

A polarimetric tool based on a radiometric MC method was developed in a Matlab framework for its later use for assisting in the design of polarimetric imaging techniques for long-range imaging. The structure of the code has been verified with its



(a)Extracted from [7] (b)Our results

Figure 4. Polar plots of the first five angle dependent functions π_n and τ_n, exhibiting complete coincidence

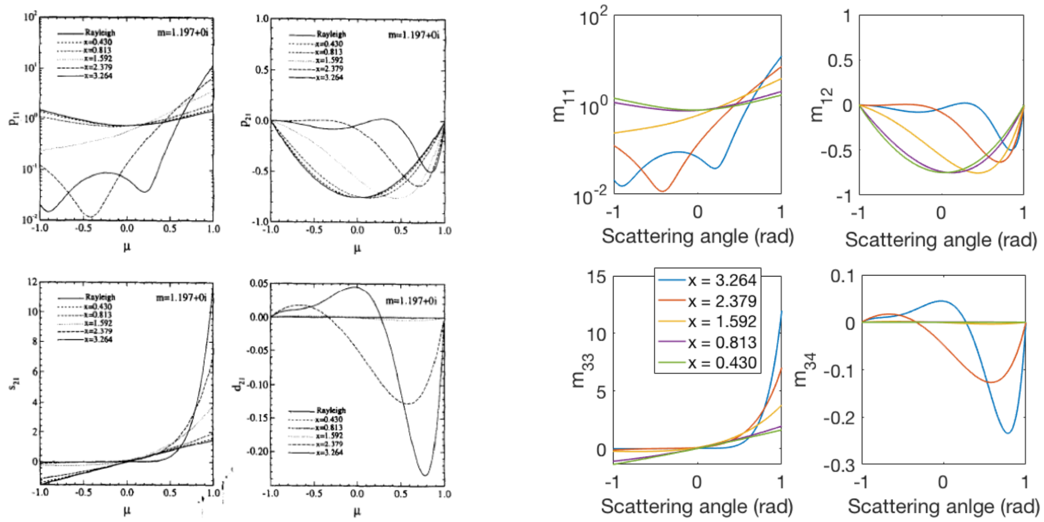


(a)Extracted from [7] (b)Our results

Figure 5. Absolute normalized matrix elements $| S_2 |^2$ and $| S_1 |^2$ for a water droplet in air with size parameter $x = 3$ and complex refractive index $m = 1.33 + i10^{-8}$

corresponding theoretical models (when available) and its performance has been checked from data in the literature. Due to the limited time existing for the development of this MSc Thesis, it was not possible to consolidate a full radiometric model for polarized beams. However, the code has been prepared and preliminary results for the polarized MC propagation have been obtained but they are pending from validation.

Taking into account the above mentioned, the immediate future work is to validate a full polarimetric radiative simulation based on MC method, to follow with a suitable experiment in a foggy space (or turbid media of spherical particles) in order to compare our simulated results with experimental measurements in well-established and well-controlled conditions. Once its performance will have been validated, an extra effort will be necessary to adapt this model to outdoor imaging conditions, as long as the combined



(a)Extracted from [6] (b)Our results

Figure 6. Elements of the Mueller scattering function for $m = 1.197 + i0$ and different size parameters, showing full accordance

effects of solar background, surrounding artificial illumination, visibility evolution, change of density and size of the scatterers, and varying atmospheric conditions are extremely difficult to mimic or anticipate in a laboratory. In fact, reproducing all these effects in a model, that would be our gold standard, is expected to be developed in a future PhD Thesis at CD6.

As mid-term goals, some simpler initial improvements will lead to a better performance of the simulation including: adding a distribution of the size of the scatterers (water droplets) because in this first model we have only considered a single size; and a C++ implementation to enable faster computation times.

References

[1] J. Fade and et al., "Long-range polarimetric imaging through fog," *Applied optics*, vol. 53, no. 18, pp. 3854–3865, 2014.

[2] F. MacKintosh, J. Zhu, D. Pine, and D. Weitz, "Polarization memory of multiply scattered light," *Physical Review B*, vol. 40, no. 13, p. 9342, 1989.

[3] J. C. Ramella-Roman, S. A. Prahl, and S. L. Jacques, "Three monte carlo programs of polarized light transport into scattering media: part i," *Optics Express*, vol. 13, no. 12, pp. 4420–4438, 2005.

[4] J. Fade, *Physics of light depolarization*. PhD thesis, Institut dOptique, 2017.

[5] G. W. Kattawar and G. N. Plass, "Radiance and polarization of multiple scattered light from haze and clouds," *Applied optics*, vol. 7, no. 8, pp. 1519–1527, 1968.

[6] A. Ambirajan and D. C. Look, "A backward monte carlo study of the multiple scattering of a polarized laser beam," *JQSRT*, vol. 58, no. 2, pp. 171–192, 1997.

[7] C. F. Bohren and D. R. Huffman, *Absorption and scattering of light by small particles*. John Wiley & Sons, 2008.

[8] S. A. Prahl, "A monte carlo model of light propagation in tissue," in *Dosimetry of laser radiation in medicine and biology*, vol. 10305, p. 1030509, SPIE, 1989.

[9] D. A. Boas, C. Pitris, and N. Ramanujam, *Handbook of biomedical optics*. CRC press, 2016.