The use of interactivity in the controller design: Loop shaping versus closed-loop shaping

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Abstract: Frequency domain is one of the most popular and powerful framework to design control system. Usually this procedure is done using the open-loop transfer function. In this work, it is explored the possibility to perform the controller design by closed-loop shaping. It is analyzed how graphical representation and interactivity can be used in this framework to help automatic control students during the learning process.

Keywords: Graphical representation, Interactivity, Control education, loop-shaping.

1. INTRODUCTION

Nowadays, computers offer excellent graphical and interactivity functionalities, even small cellphones and tablets can be used to play computer games. All these capabilities can also be used in interactive teaching and design.

Control engineering is a multidisciplinary subject that is part of the curriculum of aerospace, industrial, mechanical, electrical and chemical engineering students. Typically, an introductory undergraduate course on fundamentals of control systems includes the following contents (Dorf, 1967; Astrom and Murray, 2012; Longchamp, 2006): mathematical models of dynamic systems, analysis of the time-response of dynamic systems, analysis of the root locus, analysis of the frequency response of dynamic systems, design of basic feedback control systems using lead compensators, lag compensators, lag-lead compensators, and PID controllers.

Most of these contents have a nice an intuitive graphical representation: time series plot, poles-zeros map, root locus, and frequency domain plot (Bode plot, Nyquist plot, or Nichols plot). These diagrams are related to each other. For example, a change in the position of a pole of a linear system in the poles-zeros map produces a modification of its time response, root locus and frequency response. Likewise, a change in the corner frequency of a pole in a frequency plot produces a change in the time response, poles-zeros map and root locus. These characteristics have been extensively used to design graphical and interactive tools to design control systems and learn automatic control (Longchamp, 2006; Astrom and Murray, 2012; Dormido et al., 2002; Piguet and Longchamp, 2006; Guzmán Sánchez et al., 2012; Costa-Castelló et al., 2016; Guzmán et al., 2016).

Between others the frequency domain is one of the most powerful tools that is currently used in controller design (Lewis, 2004; Sánchez-Peña and Sznaier, 1998). Using this methodology, it is possible to analyze closed-loop steady-state performance by analyzing the closed-loop frequency response, to study the closed-loop stability by examining the open-loop frequency response, and to determine robustness properties of the closed loop system through this framework (using both the open-loop or the closed-loop frequency response). At the same time, some time domain characteristic like settling time and overshoot can be indirectly obtained from the frequency domain analysis. Finally, the well known limitations (Seron, 2010; Sánchez-Peña and Sznaier, 1998) in the design are better described in the frequency design.

Due to this nice properties in this work it is analyzed how the frequency domain can be used to develop graphical and interactive tools to design, learn and teach control systems.

The paper is organized as follows, section 2 describes the proposed controller architecture and the most important control objectives. Section 3 presents fundamental concepts used in open-loop shaping. Section 4 describes fundamental concepts behind the closed-loop shaping methodology based on controller parametrization. Finally section 5 contains some conclusions and future works.

2. PROBLEM FORMULATION

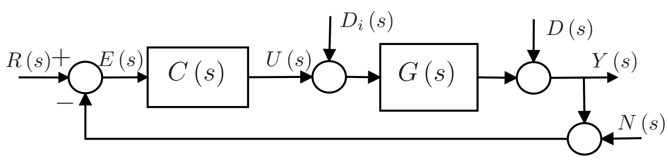

Figure 1. Closed-loop Control System architecture

Figure 1, shows the most popular control structure (Dorf, 1967; Astrom and Murray, 2012). In this scheme $G(s)$ stands for the plant (system to be controlled), $C(s)$ is the control system to be designed, $Y(s)$ is the output signal to be regulated, $R(s)$ is the reference, $E(s)$ is the error signal, $U(s)$ is the control action, $N(s)$ is the measurement noise, and $D(s)$ and $D_i(s)$ are the output and input disturbances respectively.

Most relevant control objectives to be achieved are that the closed-loop system is stable and robust, the output should track the reference, the controller system should try to reject the disturbance effect and to be as insensitive as possible to the measurement noise.

The first step to analyze the closed-loop system behavior is obtaining the most relevant transfer functions. These transfer function are:

- Open-loop transfer function: $L(s) = C(s)G(s)$
- Sensitivity transfer function: $S(s) = \frac{1}{1+L(s)}$
- Complementary sensitivity transfer function: $T(s) = \frac{L(s)}{1+L(s)}$

From these transfer functions it is possible to build the relationship between the system inputs and most relevant points in the control system (Figure 1):

$$\begin{bmatrix} Y(s) \\ E(s) \\ U(s) \end{bmatrix} = \begin{bmatrix} T(s) & G(s)S(s) & S(s) & -T(s) \\ S(s) & G(s)S(s) & -S(s) & -S(s) \\ C(s)S(s) & T(s) & -C(s)S(s) & -C(s)S(s) \end{bmatrix} \begin{bmatrix} R(s) \\ D_i(s) \\ D(s) \\ N(s) \end{bmatrix}$$

Each transfer function has it own desired values. As an example, it can easily analyzed that to force the output to track the reference the complementary sensitivity function must be equal to 1, similarly to guarantee that the disturbances are rejected the sensitivity function must be equal to 0. These two values are compatible because it is well known that: $T(s) + S(s) = 1$. Additionally in order to make the system insensitive to noise it is necessary that the complementary sensitivity function must be equal to 0, which clearly it is not compatible with previous statements.

In order to design controllers which can make the system work as expected designers take profit from the fact that the different signals in Figure 1 have different frequency components. By default it is assumed that the reference and the disturbances have frequency components below ω_{band} while the noise has frequency components over ω_{noise}. This assumption induces a partition of the frequency range which automatically defines the desired value of most relevant transfer functions:

- *Low-frequency range*, this region corresponds to frequencies in the range $[0, \omega_{band})$. In this frequency range the goal is to track references and reject disturbances, consequently $T(j\omega) \approx 1$ and $S(j\omega) \approx 0$ for $\omega \in [0, \omega_{band})$.
- *High-frequency range*, this region corresponds to frequencies in the range $[\omega_{noise}, \infty)$. In this frequency range the goal is not to amplify noise, consequently $T(j\omega) \approx 0$ and $S(j\omega) \approx 1$ for $\omega \in [\omega_{noise}, \infty)$.
- *Mid-frequency range*, this region corresponds to frequencies in the range $[\omega_{band}, \omega_{noise})$. In this frequency range no specific specifications are made, it corresponds to a frequency range where the frequency response changes from one value to another.

Previous specifications must be combined with stability and robustness. In most systems robustness margins are defined in the mid-frequency range.

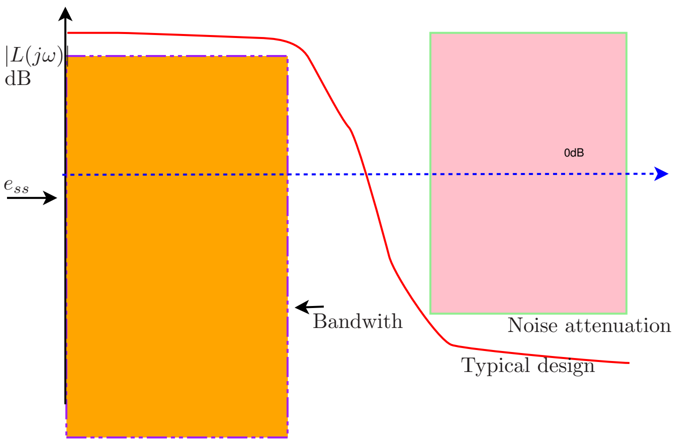

Figure 2. Open-loop transfer function, $L(s)$, specifications in the magnitude bode diagram.

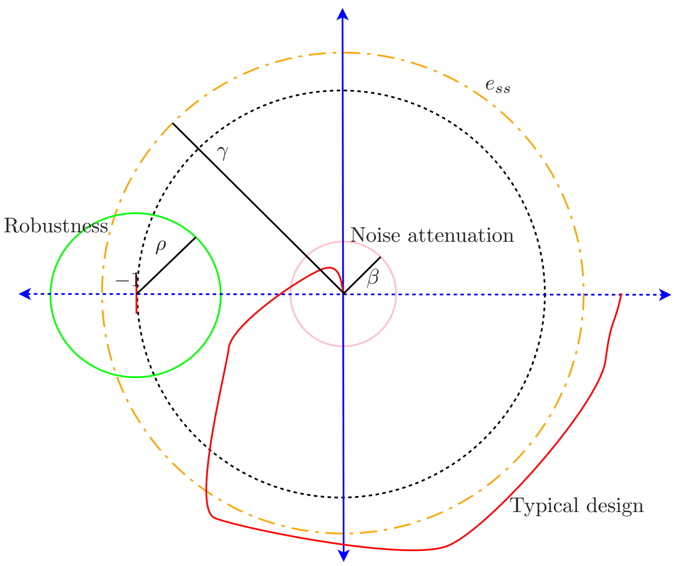

Figure 3. Open-loop transfer function, $L(s)$, specifications in the Nyquist diagram.

3. LOOP SHAPING

Most popular shaping methods are based on modifying the controllers, $C(s) = \frac{N_C(s)}{D_C(s)}$, so that the open-loop transfer function:

$$L(s) = C(s)G(s) = \frac{N_C(s)}{D_C(s)} \frac{N_G(s)}{D_G(s)}.$$

takes the appropriate shape. Note that the poles and zeros of $C(s)$ are directly poles and zeros of $L(s)$ so it is simple to shape $L(s)$ modifying/adding/removing poles of zeros in $C(s)$. This straightforward idea is the foundation of several different design methods called *loop shaping* (Astrom and Murray, 2012; Barratt and Boyd, 1992).

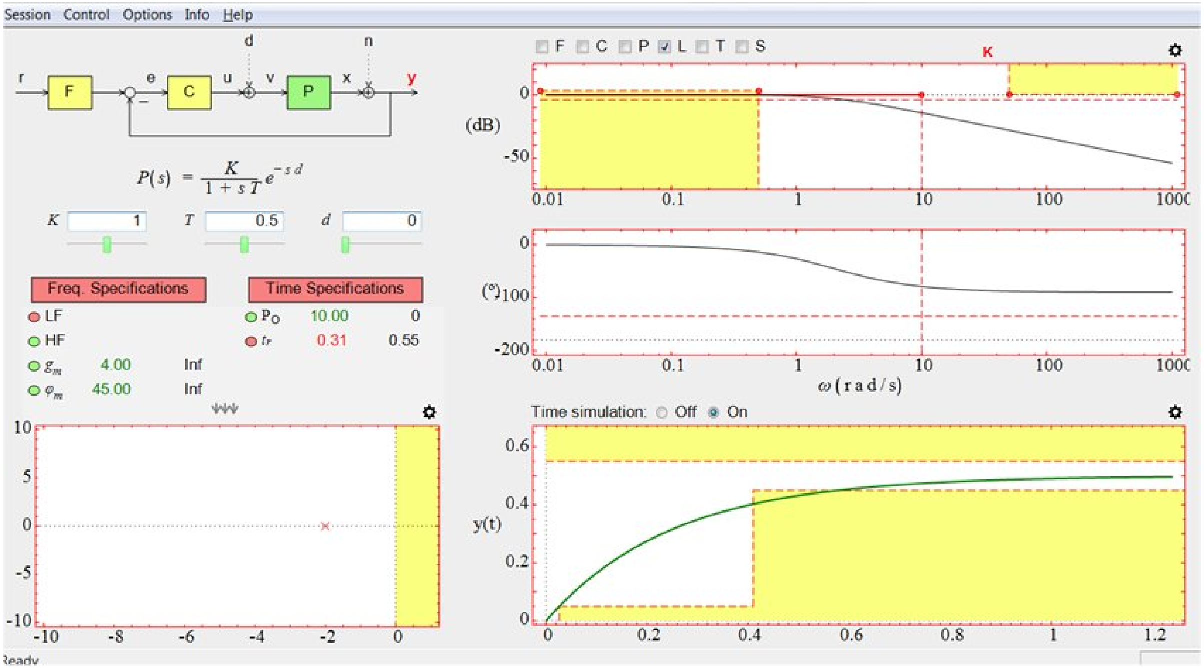

Figure 4. The Linear Control System Design (LCSD) (Diaz et al., 2017) main screen.

For a stable $L(s)$, stability implies that the magnitude, $|L(j\omega)|$, must be small (less than 1) when the argument, $\angle L(j\omega)$, is more negative than -180º (gain margin) and that must be greater than -180º when $|L(j\omega)|$ is close to 1 (phase margin). In summary, a typical set of loop shaping specifications are the following:

- $|L(j\omega)| > L_{e_{ss}}$ for $\omega \in [0, \omega_{band})$
- $|L(j\omega)| < L_{noise}$ for $\omega \in [\omega_{noise}, \infty)$
- $-150^o < \angle L(j\omega) < -30^o$ for $\omega \in [\omega_{band}, \omega_{noise})$

The values of $L_{e_{ss}}$ and L_{noise} are defined according to the desired steady-state error in the low-frequency range and the desired noise attenuation in the high-frequency range respectively.

The loop shaping design can be performed in a Bode, Nyquist, or Nichols diagram. In each representation, the specifications can be associated to a different geometric shape:

- *Bode diagram*: Figure 2 shows the structure of $L(j\omega)$ fulfilling the desired specifications diagram used for loop-shaping over the Bode plot.
- *Nyquist diagram*: Figure 3 shows the structure of $L(j\omega)$ fulfilling the desired specifications diagram used for loop-shaping over the polar plot.

  In the polar plot those points that have the same module are over the same circle centered in the origin. Accordingly, to meet the specifications in $[0, \omega_{band})$ the open-loop frequency response must be out of a big circle of radius defined by the allowed steady-state error, while in $[\omega_{band}, \omega_{noise})$ the frequency response must be inside a small circle with a radius defined by the required noise attenuation.

  Regarding to robustness, open-loop frequency response must be out of a circle centered in $(-1, 0)$ of radius defined according to the robustness specification.

  As the polar plot combines phase and gain in one diagram it is possible to simultaneously consider closed-loop stability and robustness. The open-loop frequency response must combine high gain with a reduced phase to guarantee performance in a robust manner.

  One drawback about using the Nyquist plot for loop shaping is that there is no straightforward connection between the movements of the poles and zeros of the controller and the changes in the curve at a given point. Additionally, it needs to display values with big module and others with small module. This is, in general, difficult to be done because of the linear nature of the Nyquist plot.
- *Nichols diagram* is another very popular diagram where loop-shaping is performed. Quantitative Feedback Theory (QFT) (Houpis et al., 2005) is a technique based on this diagram.

Interactive software tools have proven to be useful techniques with high impact on control education. This kind of interactive tools have demonstrated in the past that students learn in a much more active way. A very nice example is The Linear Control System Design (LCSD) (Diaz et al., 2017) Tool. It is an interactive tool for analysis and design of linear control systems with special emphasis on the open-loop shaping design. The software tool is implemented in Sysquake, a MATLAB-like language with fast execution and excellent facilities for interactive graphics and is delivered as a stand-alone executable that is readily accessible to students and instructors. Figure 4 shows its main screen, as it can be seen in just one screen previous concepts are shown and the user can interactively shape the open-loop transfer function.

Interactiviy has also been exploited in the QFT framework (Diaz et al., 2005).

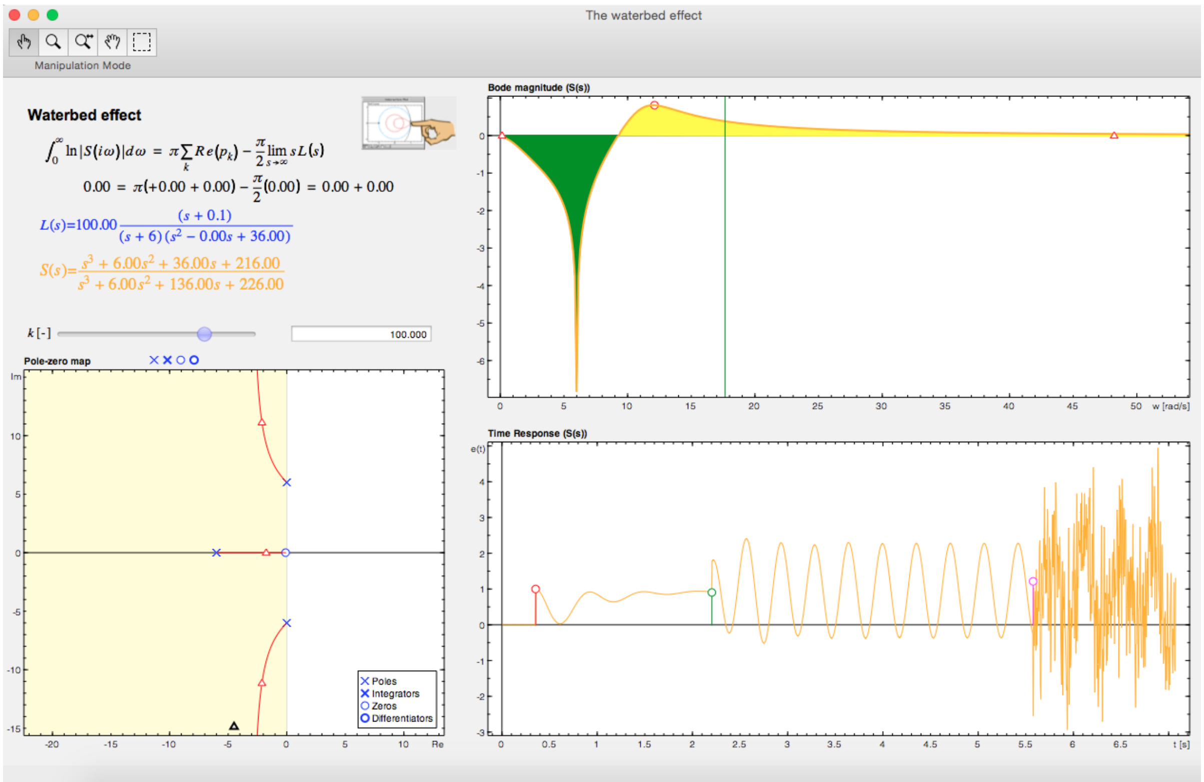

Figure 5. Waterbed analysis Interactive Tool : main screen (Costa-Castelló and Dormido, 2015).

4. CLOSED-LOOP SHAPING

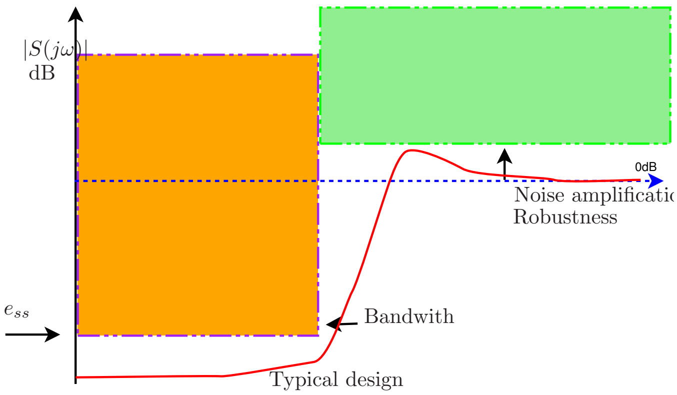

Figure 6. Sensitivity transfer function, $S(s)$, specifications in the magnitude bode diagram.

As it is shown in section 2, a closed-loop system has different transfer functions, which are tightly coupled between them, so it is necessary to select one of them to simplify the design procedure. From the authors point of view, an excellent candidate is the sensitivity function. This function can be developed as:

$$S(s) = \frac{1}{1+L(s)} = \frac{1}{1+C(s)G(s)} \quad (1)$$

$$= \frac{D_c(s)D_G(s)}{D_c(s)D_G(s)+N_c(s)N_G(s)}. \quad (2)$$

The sensitivity function has very interesting properties

- Usually, it is relative degree 0 and $\lim_{s\to j\infty} S(s) = 1$.

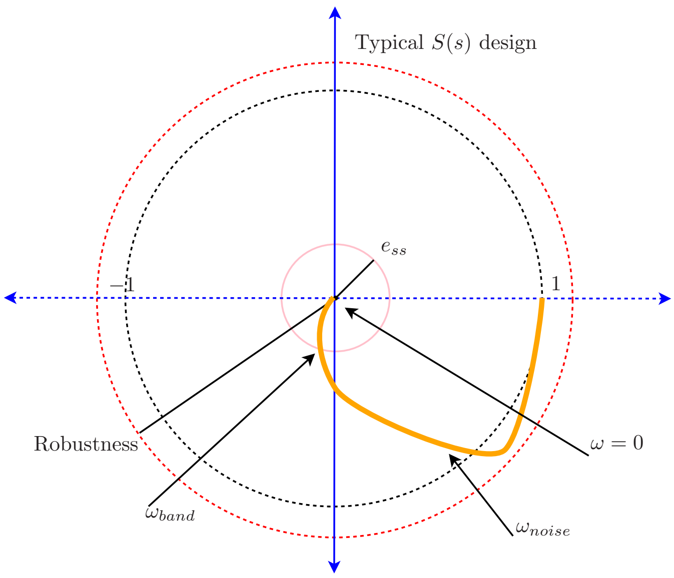

Figure 7. Sensitivity transfer function, $S(s)$, specifications in the Nyquist diagram.

The closed-loop poles are directly the poles of the closed-loop system while the zeros are the poles of the plant and the poles of the controller.

- $\|S(s)\|_\infty$ provides a quantitative robustness measure. The distance between the open-loop frequency response, $L(j\omega)$, and the -1 point in the Nyquist plot can be computed as:

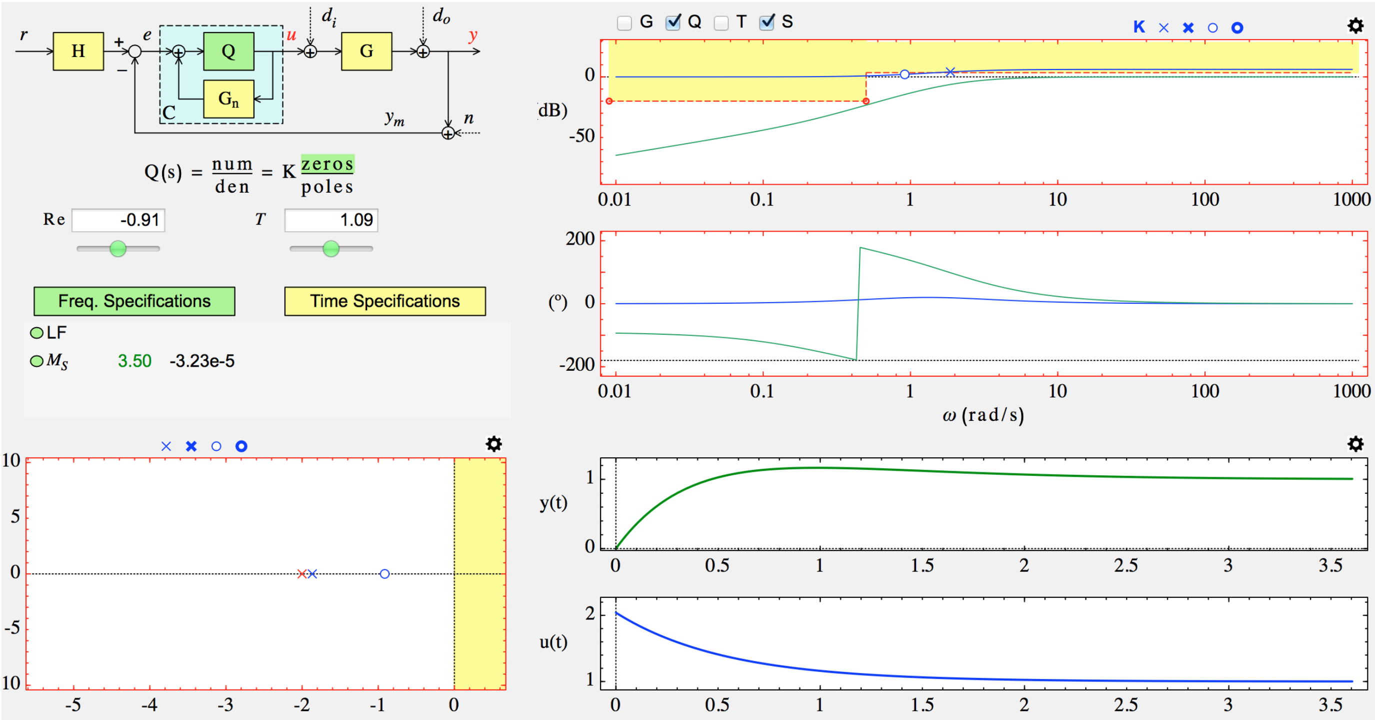

Figure 8. Closed-loop Interactive loop-shaping tool : main screen.

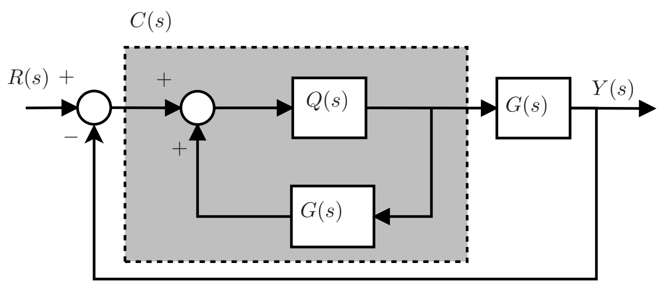

Figure 9. Closed-loop control system using affine parametrization.

$$d(-1, L(j\omega)) \\ = \inf_{\omega} |-1 - L(j\omega)| = \inf_{\omega} |1 + L(j\omega)| \\ = \left[\sup_{\omega} \frac{1}{|1 + L(j\omega)|}\right]^{-1} = \|S(s)\|_{\infty}^{-1}.$$

- Waterbed effect. The sensitivity function fulfills the following (Costa-Castelló and Dormido, 2015):

$$\int_0^{\infty} \ln |S(j\omega)| d\omega = -\kappa\frac{\pi}{2} + \pi \sum_{k}^{n_{nmp}} p_k$$

where $p_k \in \mathbb{C}^+$ are the unstable poles of $L(s)$, n_{nmp} are the number of unstable poles of $L(s)$ and $\kappa = \lim_{s\to\infty} sL(s)$.

The waterbed effect is a complex phenomena that induces very important consequences over the sensitivity function frequency response. Interactivity has been used to introduce this concept to students with excellent results (Costa-Castelló and Dormido, 2015). Figure 5 shows the main screen of an interactive tool used to introduce students to this concept.

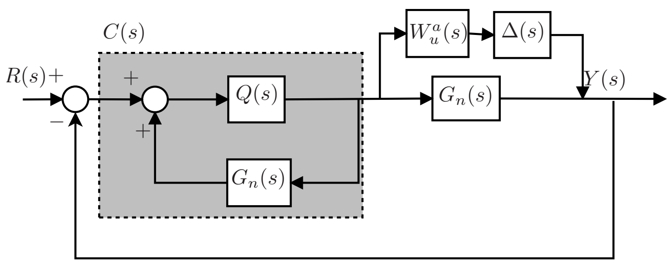

Figure 10. Closed-loop control system using affine parametrization with additive uncertainty.

To shape the sensitivity function it is necessary to identify its appropriate values in the different frequency ranges.

- *Bode plot*. In the low-frequency range the magnitude of $S(j\omega)$ must be below a certain value which directly defines the desired steady-state error, while in the high frequency range it must be below a given maximum value which corresponds to the noise amplification and the robustness. Figure 6 shows a scheme with the usual design.

  In this simple magnitude Bode plot, performance, robustness and bandwidth are simultaneously shown.
- *Nyquist plot*. In the low-frequency range $S(j\omega)$ must be inside a small circle centered at the origin with a radius defined by the maximum allowed steady-state error while in the high-frequency range should be inside a circle corresponding to allowed resonances (robustness) while converging to 1. Figure 6 shows a scheme with the usual design.

An important drawback of shaping $S(s)$ is that controller parameters (gain, poles and zeros) are not easily connected with closed-loop transfer function characteristics. A simple way to address this issue is using the affine parametrization (Sánchez-Peña and Sznaier, 1998) (Figure 9):

$$C(s) = \frac{Q(s)}{1 - G(s)Q(s)}. \quad (3)$$

This controller generates the following closed-loop transfer functions:

$$T(s) = Q(s)G(s)$$
$$S(s) = 1 - Q(s)G(s)$$
$$C(s)S(s) = Q(s)$$

with $Q(s)$ is a filter to be designed. If both $G(s)$ and $Q(s)$ are stable the closed-loop stability is guaranteed by construction. As it can be seen the closed-loop poles are the poles of $G(s)$ and $Q(s)$. Also the zeros of $T(s)$ and $C(s)S(s)$ can be easily connected with the design filter $Q(s)$.

Another important characteristic is that this framework allows to design controller for plants with additive uncertainty, i.e:

$$G(s) = G_n(s) + W_u^a(s)\Delta(s),$$

with $G_n(s)$ being the nominal plant, $W_u^a(s)$ the uncertainty weighting function and $\|\Delta(s)\|_\infty < 1$. For this type of plant the robust stability condition is defined by:

$$\|C(s)S(s)W_u^a(s)\|_\infty = \|Q(s)W_u^a(s)\|_\infty < 1. \quad (4)$$

Which can be transformed in the following bounds over the closed-loop frequency response:

$$|Q(j\omega)| < \frac{1}{|W_u^a(j\omega)|} \ \forall\omega. \quad (5)$$

Currently, the authors are developing an interactive tool, shown in Figure 8, which will allow the interactively design controller using the framework described in this section.

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5. CONCLUSIONS

In this work the use of loop shaping and closed-loop shaping techniques using interactive tools has been analyzed and compared. Although the most people use open-loop loop shaping, the authors suggest a new paradigm where the shaping problem is performed over the closed-loop transfer function.

The authors are currently developing a new interactive tool which will allow to design control system using closed-loop shaping. The tool will allow to deal with nominal performance, nominal stability, robust stability and robust performance in a homogeneous framework.

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