UNIVERSITAT POLITÈCNICA
DE CATALUNYA
BARCELONATECH

Influence of coordinate and dynamic formulations on solving biomechanical optimal control problems

Gil Serrancolí, Rosa Pàmies-Vilà
gil.serrancoli@upc.edu, rosa.pamies@upc.edu

Introduction

Optimal control problems have become popular in recent years in biomechanical movement predictions mainly due to an increase of computational capacities and development of new optimization software [1]. The **convergence** of optimal control problems can be influenced by the type of coordinates used to describe the model, as well as by the dynamic formulation used to introduce the equations of motion.

Methods

In this study we present a **comparison** of optimal control problems using 9 models with different **complexity** (from 2 to 10 degrees of freedom – DoF -), modeled using different types of **coordinates** (absolute, relative and natural) and solved by means of two dynamic **formulations** (explicit and implicit) in CasADi [1]. Note that natural coordinates lead to a constant mass matrix [2].

Dynamic formulation

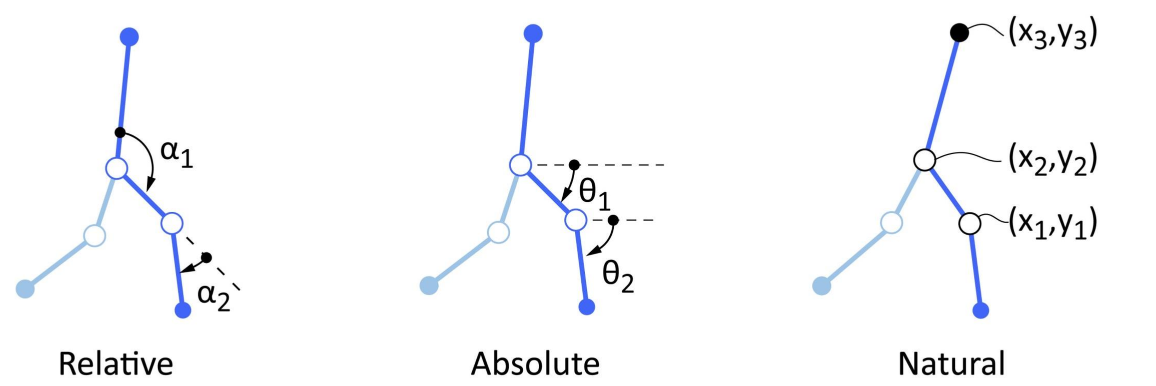
Implicit

$$M\ddot{q}+C\left(q,\dot{q}\right)+G\left(q\right)-\tau=0$$

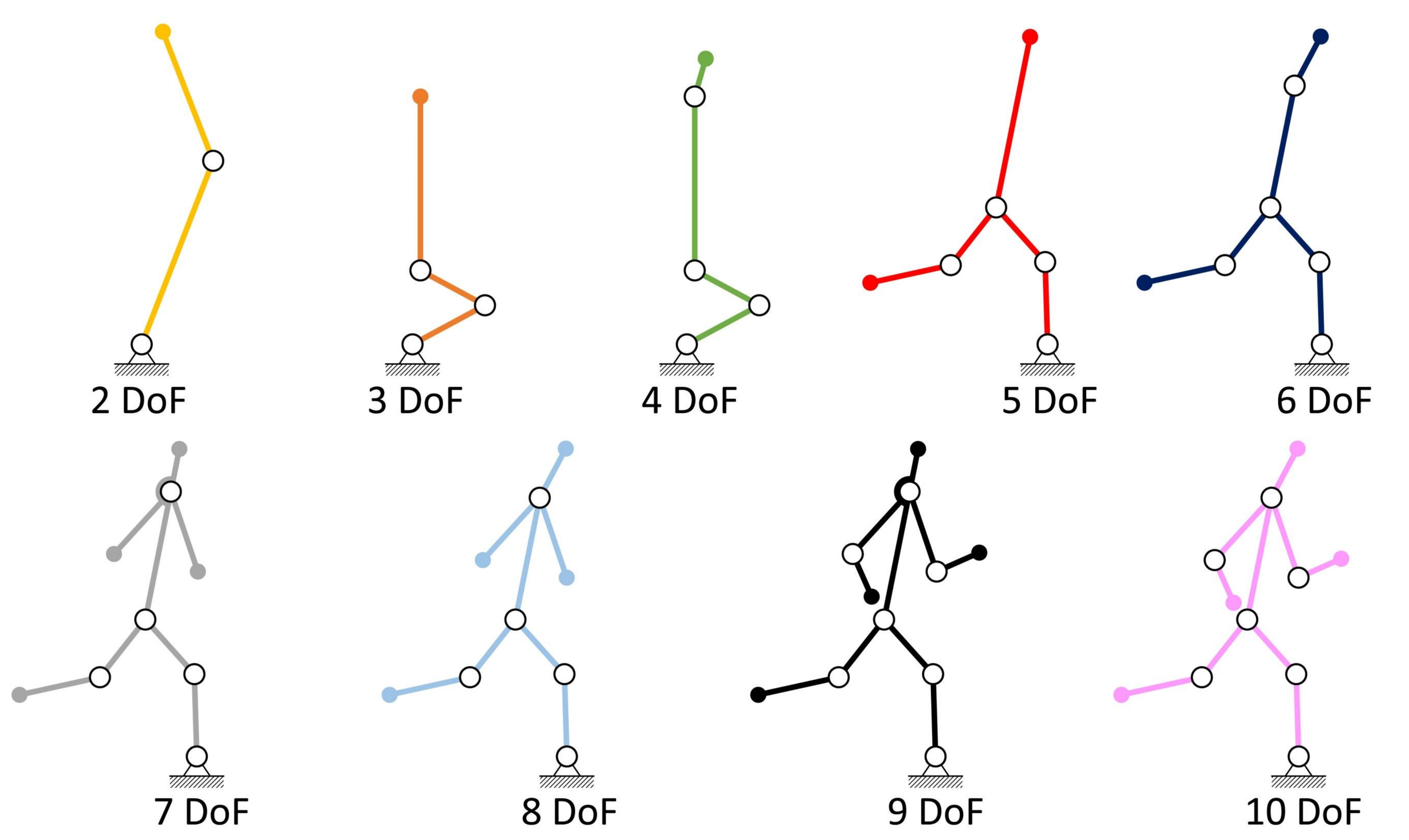
Explicit

$$\ddot{q}=M^{-1}\left(-C\left(q,\dot{q}\right)-G\left(q\right)+\tau\right)$$

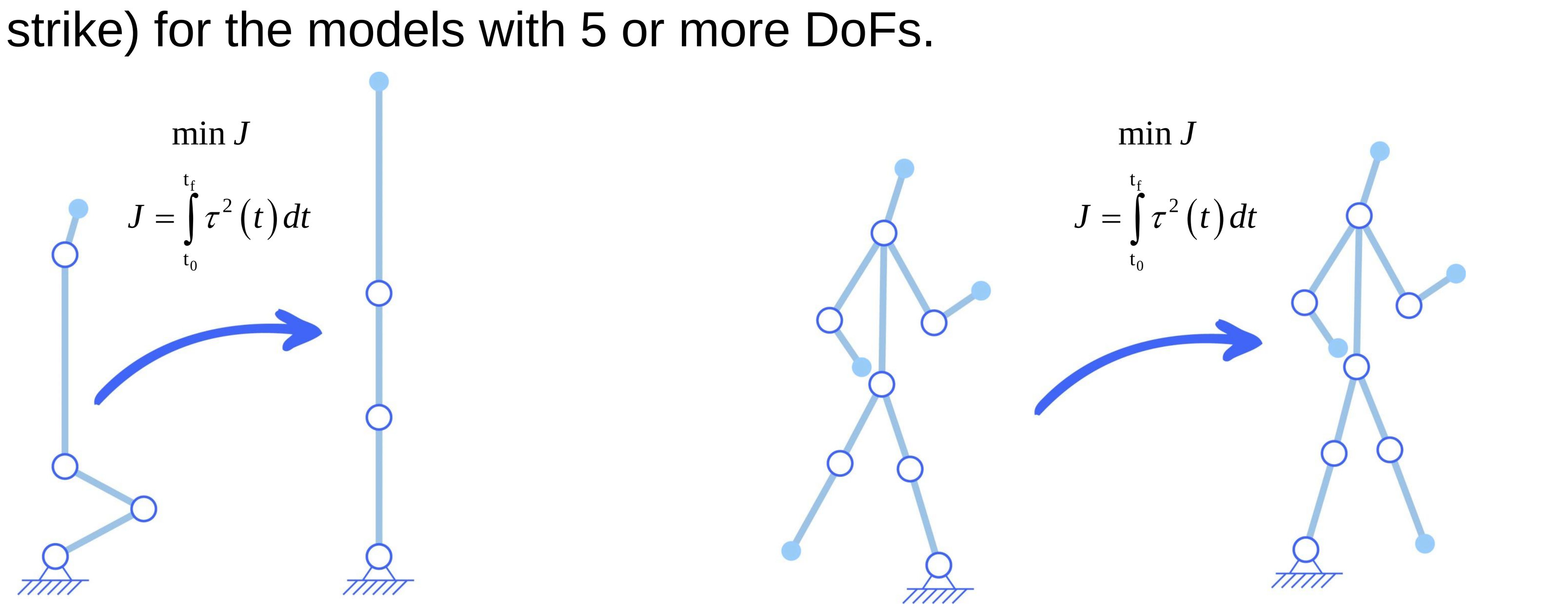
Coordinates



Models



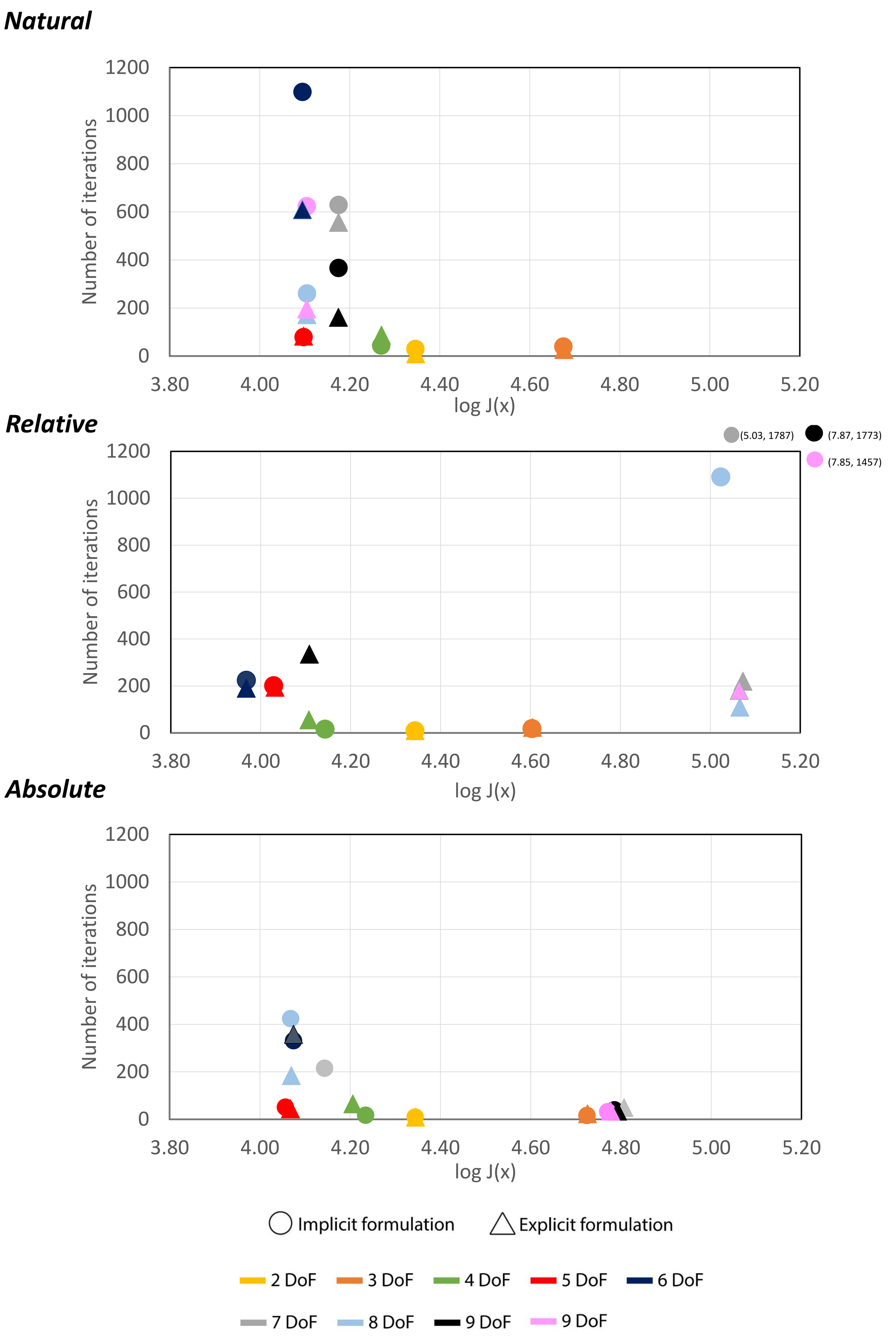
For each model and type of coordinate definition, an optimal control problem was solved twice: using implicit dynamics formulation and using explicit dynamics formulation. Those combinations led to a total of 54 optimal control problems. Each problem consisted in predicting the movement **from an initial to a final state** minimizing the **integral** of squared joint **torque values**. This movement represents a sit-to-stance trial for the models between 2 and 4 DoFs, and a swing phase (from toe-off to heel strike) for the models with 5 or more DoFs.



Results

Explicit dynamic formulations gave better results in terms of number of iterations in more **complex models** (from 7 DoFs) for all three types of coordinates (absolute, relative and natural). Using models with **lower complexity**, optimizations with **implicit dynamic** formulations tended to find optimal solutions earlier.

Overall, the same optimal cost function value was obtained using implicit or explicit dynamic formulations. Using models with **lower complexity** (< 7 DoF), **relative coordinates** gave the **smallest** optimal values of the **cost function**, and the most **complex** ones (>= 7 DoF), **absolute** and **natural** coordinates had the lowest values.



Conclusions

Differences in optimization performance were observed comparing different dynamic formulations and type of coordinates. The fact that we obtained a lower number of iterations when using natural coordinates and explicit formulations in more complex models could be explained by the constant mass matrix [3].

However, an analysis to avoid local minima is required to obtain more robust results and discard disagreements with other studies describing the benefits of using implicit skeletal dynamic formulations [4]. The influence of the mass matrix also needs to be studied, since depending on the point chosen to start the kinematic chain a near-singular matrix could be obtained.

References

[1] Anderson et al. Math. Program. Comput. In press. 2018.
[2] J.G. de Jalón, Multibody Syst Dyn, 18: 15-33, 2007.
[3] Betsch et al. J Comput Nonlin Dyn. DETC2011-47310: 317-326, 2012.
[4] van den Bogert et al. Procedia IUTAM, 2:297-316, 2011