# Form

Fine, Kit

*DOI:*
10.5840/jphil20171141036

*License:*
Other (please specify with Rights Statement)

*Document Version*
Peer reviewed version

*Citation for published version (Harvard):*
Fine, K 2017, 'Form', Journal of Philosophy, vol. 114, no. 10, pp. 509-535.
https://doi.org/10.5840/jphil20171141036

# Form

This paper is a belated sequel to my paper on Cantorian abstraction (Fine [1998]). In the earlier paper, I attempted to defend Cantor's account of cardinal numbers as sets of units, using the theory of arbitrary objects, previously developed in (Fine [1986]), to explain what the units were. Of course, no one now adopts Cantor's own account of cardinal number, preferring instead von Neumann's elegant treatment of cardinal numbers as initial ordinals; and this may have led some readers - or potential readers - of my earlier paper to dismiss it as being of purely scholarly interest. But as I had already mentioned (Fine [1998], p. 602), 'the Cantorian theory can be extended to provide a more general theory of types - covering not merely the abstract formal types of mathematics but also the more concrete types of ordinary and scientific discourse'; and, in the present paper, I wish to consider the extension of the account to these other kinds of types (or what I now also wish to call forms).[1]

There is a related application to structural universals, which I shall also consider. This was a topic that was much discussed in the 1980's but that has since fallen out of fashion.[2] If I am right, structural universals are - or are merely a special case - of forms; and so the theory of the one is - or is merely a special case - of the theory of the other. I hope, in other work, to consider some further applications and to consider, in particular, the application of the theory of arbitrary objects to the construction of models in science (as described in Godfrey-Smith [2006], for example).

Although my previous work on arbitrary objects and Cantorian abstraction provide the backdrop to the present paper, I have tried, as far as possible, to make the present paper self-contained. The reader may find it helpful to consult the previous work on arbitrary objects for formal detail and the previous work on Cantorian abstraction for philosophical motivation.

## §1 Form

The notion of form that I am interested in is well illustrated by the case of logical form, so familiar in discussions of logical truth. To fix ideas, suppose we have a formal language for sentential logic, in which formulas are formed from a given stock of sentence letters $p_1$, $p_2$, ... through the formation of negations, disjunctions and conjunctions. Thus when A is a formula so is $\neg A$; when A and B are formulas then so are $(A \vee B)$ and $(A \wedge B)$; and all formulas of the language can be obtained by starting with the sentence letters and successively forming negations, disjunctions and conjunctions. We shall follow common practice of using p, q, r, and s, respectively, as abbreviations for $p_1$, $p_2$, $p_3$, and $p_4$.

Each formula has something which we might call its form. This form is unique; there is just one object which is *the* form of the formula. But a formula may have a number of other

---

[1] I am grateful to Steve Kuhn. It was a number of helpful discussions with him on the nature of forms that led to my renewed interest in the topic; and he also made a number of interesting comments on an earlier draft of the paper.

[2] Though see Forrest [2016] for an attempt to rehabilitate something akin to the original ideas of Armstrong [1978].

forms which are, in some sense, less fine-grained than its unique form. Thus *the* form of the formula $(p \vee (q \wedge r))$ is given by the fact that it is a disjunction, whose left disjunct is a sentence letter and whose right disjunct is a conjunction of sentence letters, each distinct from one another and from the left disjunct, but it also have the form of a disjunction and even the form of an arbitrary formula. Other formulas may share the form of a given formula. Thus *the* form of the formula $(q \vee (s \wedge r))$ is the same as *the* form of the formula $(p \vee (q \wedge r))$, and the formula $(s \vee r)$ has *a* form in common with $(p \vee (q \wedge r))$, that of a disjunction, and a form, that of an arbitrary formula, in common with every other formula.

I should make clear that my concern is with what one might call the *logical* form of formulas, as given by the logical connectives $\neg$, $\vee$ and $\wedge$. A formula might also, perhaps, be taken to have various form qua string of symbols. For example, $(p \vee q)$ might, one sense, be said to have the same form as $(q \wedge r)$ since the two strings have the same number of symbols and symbols of the same type in corresponding positions, or it might, in another sense, be said to have the same form as $))p\wedge\wedge$ since the number of symbols in each string is the same. In any case, our focus, in what follows, will be exclusively on their logical form.

Our task is to give a rigorous and systematic account of what forms are and of how they behave. More ambitiously, we might think of ourselves as attempting to get at the essence of forms, of what they are by their very nature though, to a large extent, it will be possible to understand what we are doing without taking on this more ambitious aim.

Although I have focused on the case of the logical form of the formulas of sentential logic, it is important to bear that this is merely by way of illustration. Talk of form is common, both in ordinary parlance and in many branches of scientific enquiry. We may talk, in more or less the same sense, of the logical form of concrete sentences, as opposed to formulas, or of grammatical form as opposed to logical form. We may also talk of the form of natural objects, such as molecules. Thus methane molecules will have a molecular form, as given by the chemical formula $CH_4$, while methylene molecules will have a molecular form given by the formula $CH_2$. Or again, we may talk of the form of musical works - as in sonata form, or rondo form, or simple ABA form. The reader may also note an affinity - especially in regard to the issue of self-predication - between these common ways of talking about form and Plato's own treatment of the topic; and so there is some hope that an account of the one may help shed some light and even provide some kind of vindication for the Platonic position.

## §2 Some Methodological Remarks

I should perhaps say a few words about my methodological stance, since it differs greatly from what is to be found in much of the literature and is likely to be misunderstood in the absence of further explanation.

I have distinguished between *naive* and *foundational* metaphysics (Fine [1982], p. 99; Fine [2017]). Naive metaphysics is concerned with appearance, with how things present themselves to us, while foundational metaphysics is concerned with reality, with how things are in themselves. The present project is one in naive metaphysics; it is concerned with how forms present themselves to us. I am entirely uninterested in the reality of forms. Perhaps at the end of the day, we will be strict nominalists and reject the reality of forms altogether or perhaps we will be strict Platonists and not accept the reality of anything beyond the forms. These questions, whatever their ultimate interest, have no interest to me in the present context. I am simply willing to accept the existence of forms and, under this supposition, to consider what they are and how they behave.

This means, in particular, that I do not take seriously the idea that talk of forms is a mere facon de parler. It might be thought that when I talk of the form of a formula I am not genuinely referring to something, the form, but that such talk, when taken in context, is to be seen as a way of talking about formulas that in itself makes no essential reference or appeal to forms. Thus if I say that the formula $(p \vee q)$ has the form of a disjunction $(A \vee B)$, then all that I am really saying is that $(p \vee q)$ is a disjunctive formula, i.e the result of disjoining two formulas.

I myself doubt that any such hypothesis - considered as a hypothesis about our actual use of language - is in fact correct. It perhaps has some plausibility when we look at simple constructions like '$(p \vee q)$ has the form of a disjunction', but it is much harder to sustain in constructions such as 'the form of $(p \vee (q \wedge r))$ is more complex than the form of $(p \vee q)$' or 'the form of $(p \vee (q \wedge p))$ involves two occurrences of one sentence letter and one occurrence of another'.

But, of course, philosophers have been very ingenious in coming up with more and more complicated forms of paraphrase; and so let it be granted that talk of form is indeed a facon de parler. Even so, we seem to talk as if there are forms - adopting most, if not all, of the usual idioms of referential discourse. And so we might ask: how successful have we been in this regard? Even if we are not in fact talking about forms, is our discourse perfectly consonant with the supposition that we are talking about forms, so that the simulacrum of referential discourse is as good as it could be? And if it is, then what does that say about the kind of object we might have been talking about even though we are not in fact talking about any such object? What, so to speak, is the referential fiction we are engaged in?

These are interesting questions in their own right and I suspect that philosophers, in their haste to get at how things really are, have ignored these basic questions, that lie on the very surface of the phenomena of interest to them and whose answers might even have helped them achieve their anti-realist aims, in so far as they would have helped make clear what a successful paraphrase or reduction must achieve. But be that as it may, our current interest will be in developing an account of forms as objects in their own right rather than as an elaborate referential fiction.

I also have no explanatory aims beyond wanting to explain the nature of forms themselves. Many philosophers - going all the way back to Plato - have wanted forms or 'structural universals' or the like to do all kinds of explanatory work. They are meant to explain how things resemble one another; or how laws of nature work; or how the world is structured. Some of these philosophers may even have thought that the existence of forms is to be justified by the explanatory tasks that they perform, so that it is only if we can show that they are indispensable in the performance of these tasks that we have any reason to believe that they exist.

I have no such lofty aims and, indeed, I regard them as a distraction from the task at hand. I am simply concerned to come up with a reasonable account of forms. If it can perform further work, then so much the better. But it is no part of the initial task to ensure that they are capable of doing this further work; and even if they were not then that, in my opinion, would in no way detract from the reasons we have for their existence.

I am perhaps like the mountaineer who climbs a mountain because it is there. I want to understand forms because they are there. There is no further aim beyond reaching the summit or achieving the desired understanding. Of course, in climbing the mountain, one might learn things that one did not know before and which perhaps one could not learn in any other way; and once one has reached the summit, one might see things that one had never seen before and perhaps could not see in any other way. But the aim is not to learn or see new things but simply to reach the summit.

These remarks will apply to almost any other object of our naive ontology - to beliefs and desires, chairs and tables, numbers and shapes. I mention them here, not so much because of their special relevance to forms, but because so much of the recent literature on structural universals and the like has been tied to the issue of Platonism versus nominalism and to the question of whether there are any larger explanatory purposes which these objects may serve and by which their existence may be justified. If the present paper is to be properly understood, then it must be disassociated from these larger metaphysical aims.

## §3 The Assumptions

There appear to be enormous difficulties in giving a satisfactory account of form. However, it is hard to state these difficulties with complete precision since the difficulties depend to some extent on how certain key terms are to be understood. But let us at least make a start.

Given any formula, there is a single object which we may call *the* form of the formula. Thus the formula $(p \vee q)$ has the form of a disjunction, $(p \vee p)$ the form of an identical disjunction (with identical disjuncts), $\neg(p \vee q)$ the form of the negation of a disjunction.

It is natural to represent the form of a formula by means of meta-formulas. These are like ordinary formulas but constructed from the meta-letters A, B, C, ..., standing in for formulas, instead of from the sentence-letters $p_1, p_2, \ldots$ . The form of $(p \vee q)$ will then be represented by the meta-formula $(A \vee B)$, the form of $(p \vee p)$ by $(A \vee A)$, and the form of $\neg(p \vee q)$ by $\neg(A \vee B)$. We might in this way talk of a formula $(p \vee q)$ being 'of the form $(A \vee B)$'.

However, the form of a formula should not be *identified* with the meta-formula by which it is represented. For the form of the formula $(p \vee q)$ might equally well be represented by the meta-formulas $(B \vee A)$ or $(B \vee C)$, say, and these cannot both be identical to the form. From this point of view, the meta-formulas are no better able than the formulas themselves to represent the forms; their one advantage, it seems, is to indicate that the form is, in some way, something over and above the formulas of that form.

Given any formula, there are a number of objects each of which is *a* form of the formula. Thus the formula $\neg(p \vee q)$ has the form $\neg A$ of a negation as well as the form A of an arbitrary formula. When **F** is a form of a formula A, we may also say that A is an *instance of* the form. Thus $\neg(p \vee q)$ will be an instance of the form $\neg A$ and also of the form A. *The* form of a formula is, of course, *a* form of the formula and what distinguishes it from the other forms is that it is the most fine-grained of them; no further articulation of the form of the formula can be given.

There is some room for debate as to what we should take the instances of a form to be. One issue concerns the recognition of difference. Should we allow $(p \vee p)$, for example, to be an instance of the form $(A \vee B)$ or should we insist that the disjuncts in any instance of the form be distinct? We end up with an *homomorphic* conception of form in the first case and an *isomorphic* conception in the second case. A separate issue concerns the identity of the sentence letters. Should we allow any formula to be of the form of p or should we insist that only other sentence letters are of that form? We obtain a *simplicity-insensitive* conception of form in the first case and a *simplicity-sensitive* conception in the second case. All of these different conceptions of form can be accommodated in the account to be given below although, for the sake of definiteness, I shall usually presuppose that we are operating within an homomorphic and simplicity-insensitive conception of form.[3]

We are now in a position to set up the puzzle. To this end, let us recall that two formulas are *alphabetic variants* if they differ merely in the identity of their sentence letters. Thus $(p \vee q)$ is an alphabetic variant of $(q \vee p)$ and of $(r \vee s)$, though not of $(p \vee p)$, say, or of $\neg(p \vee q)$. We would like the notion of form to satisfy the following three requirements:

<u>Existence</u> For each formula there is an object which is the form of the formula

<u>Identity</u> The forms of two formulas are the same if and only if the formulas are alphabetic variants

<u>Structual Similarity</u> The form of a negative formula is the 'negation' of a form; and, similarly, the form of a disjunctive formula the 'disjunction' of two forms and the form of a conjunctive formula the 'conjunction' of two forms.

The reason for the scare quotes in the formulation of Structual Similarity is that we do not want to insist that the form of a disjunction, say, literally is a disjunction of two other forms. Indeed, given that the operation of forming a disjunction only applies to formulas and is only capable of yielding formulas, the only way for the disjunctive form to be a disjunction of two forms would be for the form to be itself a disjunctive formula that was formed from forms that were themselves formulas. However, it *is* required that there be an operation on forms corresponding to the operation of disjunction on formulas and that the disjunction form should then be the result of applying the corresponding operation on two forms.

We might represent the form of a disjunctive formula by:

$$A \vee B.$$

We might then represent the form of the disjunctive form by:

$$\mathbf{F} \circledvee \mathbf{G}$$

where '$\circledvee$' signifies the corresponding operation on forms. The common 'archetype' or 'meta-form' of these two forms might then be represented by:

$$\alpha \circ \beta$$

What is then required is that the disjunctive form and a disjunctive formula should have, in this way, a common archetype.

Although we do not wish to require that the corresponding operation on the forms and its

---

[3]Under our liberal conception of form, the instances of the form of a formula A will be given by {*s*(A): for some substitution function *s*}. Under a simplicity-sensitive conception of structure, we should restrict the substitutions *s* to those which always map a sentence letter onto a sentence letter; and under the isomorphic conception of structure, we should restrict the substitutions *s* to those which always map distinct sentence letters onto distinct formulas. The distinction between the homomorphic and isomorphic conceptions of form is discussed in Fine [2005], 48-50, in connection with the Quinean problem of quantifying in.

instances be the same, nor do we wish to rule this out. We might talk of *autonomous instantiation* and of an *autonymous* form when the operations are the same, so that the form itself possesses the given structural feature and hence is literally an instance of itself; and we might talk of *homonymous instantiation* and of an *homonymous* form when the operations are not the same and the form only possesses an analogous structural feature and hence may not literally be an instance of itself.

There are some additional requirements, of both a formal and an informal character, that might be imposed upon forms, although only the ones we have stated will be needed to set up the puzzle. Consider Structual Similarity again. According to this requirement, the form **F** of $(p \vee (q \wedge r))$, will be a 'disjunction' of forms **G** and **H**. But we might insist that the structure of the constituent forms should be iterative: the form **G** should, in its turn, be a 'conjunction' of forms; and similarly for other, more complex, formulas. The structural features of the formula should be mirrored, all the way down, by the form. We might also want to insist on 'Unique Readability'. Not only should **F** be a 'disjunction' of the forms **G** and **H** but it should not also be the 'disjunction' of any other pair of forms; if **F** is a a 'disjunction' of **G**′ and **H**′, then **G** = **G**′ and **H** = **H**′ .

On the more informal side, we might require that any form should serve as some kind of template for its instances; the instances must in some significant sense be *formed* from the form. We might also require that the forms not be arbitrary; they should not involve an arbitrary choice among competing conceptions of form that are equally capable of satisfying the more formal requirements.

Although these latter two requirements are not at all precise, they do serve to rule out some of the more standard approaches to what the forms might be. We might, for example, identify a form with the class of its instances. Thus the disjunctive form would then be the class of all disjunctive formulas. This conception of form is perhaps not arbitrary but there is no reasonable sense in which the instances are *formed* from the form. Or again, we might take a form to be a representative instance of that form. The form of a disjunction, for example, might be taken to be the formula $(p \vee q)$. Instances of the form could then perhaps be taken to be formed from the form through substitution of different formulas for p and q. However, the choice of the formula $(p \vee q)$ is completely arbitrary. There is no good reason why the form should be $(p \vee q)$, say, rather than $(q \vee p)$, or $(q \vee r)$ or any other disjunction of distinct sentence letters.

Although I have tailored the formulation of these requirements to the case of formulas, it is my intention that analogues of them should hold, across the board, to any case in which we may relevantly talk of the form or structure of the objects within some given domain. In such cases, there will be a number of structural operations on the objects of the domain (in analogy to the operations of negation, disjunction and conjunction on formulas) by which the structure of the objects is defined. Often it may be evident from the domain what we should take the relevant structure to be. But not always; and sometimes there may be different options. For example, in the case of formulas, we could take their structure to be given by their constitution as strings of symbols. The relevant structural operation will then be concatenation on symbols and the status of the formulas as as negations or disjunctions or conjunctions will be irrelevant to their structure.

We also suppose that, within the domain, there are certain 'atoms', which do not result from applying any of the structural operations to other objects, and that all of the objects of the domain can be generated from the atoms through successive application of the structural operations. Using the structural operations in place of the logical operations, we may then formulate analogues of the previous requirements. For example, two objects can be said to be isomorphs (the analogue of alphabetic variants) if they have an isomorphic construction from the atoms; and Identity becomes the principle that the form of two objects from the domain is the same just in case they are isomorphs.[4]

§4 The Puzzle

The puzzle can now be stated. By Existence, there is a form **D** which is the form of the disjunctive formula $(p \vee q)$. By Structual Similarity, **D** is itself a 'disjunction' **G** ⓥ **H** of two forms **G** and **H**. Similarly, there is, by Existence, a form **D**′ which is the form of the formula $(p \vee p)$. By Structual Similarity again, **D**′ is the 'disjunction' **G**′ ⓥ **H**′ of two forms **G**′ and **H**′. By Identity (left- to-right), the forms **D** and **D**′ are distinct (since $(p \vee q)$ and $(p \vee p)$ are not alphabetic variants). It follows that the forms **G**, **H**, **G**′ and **H**′ are not all the same. But the problem now is to explain how this might be so. For what could the forms **G** and **H** be other than the respective forms of p and q and what could the forms of **G**′ and **H**′ be other than the form of p? But p and q are alphabetic variants and so, by Identity (right-to-left), the form of p is the same as the form of q.

We may state the puzzle more starkly as follows. Let us use |A| for the form of the formula A. Then we want the form of $|p \vee q|$ to be a disjunction of two forms; and this would appear to require that:

$$|p \vee q| = |p| ⓥ |q|$$

Similarly,

$$|p \vee p| = |p| ⓥ |p|$$

But $|p| = |q|$ and hence:

$$|p \vee q| = |p \vee p|,$$

which is not so.

This is a perfectly general problem and does not depend upon embracing any particular conception of form. Suppose, for example, that, we take the form of a formula A to be the class of its substitution-instances. Thus the form of p would be the class of all formulas and the form of $(p \vee q)$ would be the class of all disjunctive formulas. We might then define the disjunction $\Delta$ ⓥ $\Gamma$ of two classes of formulas $\Delta$ and $\Gamma$ to be the class $\{A \vee B: A \in \Delta \text{ and } B \in \Gamma\}$ of their disjunctions (and similarly for negation and conjunction). If we take Fml to be the class of all formulas, then we may define the disjunctive form (the class of all disjunctive formulas) to be Fml ⓥ Fml, as required. But we then lack any account of the identical disjunctive form (the class of identical disjunctions). Or again, suppose we identify the form with a representative formula with that form, perhaps singled out by taking it to be the formula of that form whose

[4]For the intended applications, some form of 'Unique Readability' should also be assumed. I do not know if it is possible to extend the theory to accommodate ill-founded objects, which do not 'bottom out' in atoms.

subscripts from left to right are as small as possible. Thus the straight disjunctive form will be the formula $(p_1 \vee p_2)$, for example (rather than $(p_2 \vee p_3)$, say, or $(p_2 \vee p_1)$) and the identical disjunctive form will be $(p_1 \vee p_1)$ (rather than $(p_2 \vee p_2)$, say). If we define the disjunction of two forms simply to be their disjunction qua formulas, then the identical disjunctive form will be the disjunction of the form $p_1$ with itself but there will be no account of the straight disjunctive form $(p_1 \vee p_2)$ since $p_2$ is not a form.

## §5 Arbitrary Objects

I shall attempt to solve the puzzle above by appealing to arbitrary objects and, in this section, I shall outline the relevant parts of the theory of arbitrary objects that I have developed in previous work.[5]

I shall think of variable or arbitrary objects as a kind of objectual analogue of variable signs (in fact, 'variable' was originally used in mathematics to designate something non-symbolic). Like variable signs, these objects will take other objects as values. But unlike variable signs, they will not be linguistic in character; they will not be conventional symbols for their values, but abstract objects which assume those values by way of their intrinsic nature.

The variable signs of logic take values independently of one another; what value one variable sign takes is not constrained by the values that other variable signs take. However, I wish to allow for the possibility that the value of one arbitrary object (A-object, for short) may depend upon the value taken by others. I will allow, for example, that there are A-objects $\boldsymbol{x}$ and $\boldsymbol{y}$ whose values are any real numbers $x$ and $y$ for which $x = -y$. (I use bold face italic for variable objects, light face italic for their values, and plain lower case for variable signs.)

There are two different ways in which we may think of one A-object as depending upon another. On the one hand, we can think of the dependence as two-way, with the values of the A-objects being simultaneously constrained. On other hand, we can think of it as one-way, with the values of one A-object being given in terms of the value of the other. Thus we may either think of the variable $\boldsymbol{x}$ and $\boldsymbol{y}$ above as being simultaneously constrained by the requirement that the value of one should be the negative of the value of the others; or we may think of the value of one of the variables, say $\boldsymbol{y}$, as being given as the negative of the value of the other.

A *system* is a set of A-objects closed under dependence: if one A-object belongs to the set then so does any A-object upon which it depends. We shall assume that isomorphic systems are the same (there is an alternative development of the theory in which this assumption is relaxed). But rather than provide a formal definition of an isomorphic system, let me explain by way of example what it means.[6]

Suppose we have an A-object **A** that is independent, i.e. does not depend upon any other A-objects, and which takes exactly the formulas of our sentential language as values. Then the set {**A**} constitutes a system and any other independent A-object **B** which takes exactly the

---

[5] Fine [85], [86], and §3 of [1998].

[6] Some formal details are to found in the addendum to Fine [1998]. It should be noted that we also make a disjointness assumption to the effect that there are no 'accidental' identities between objects which belong to different systems of A-objects.

formulas of our sentential language as values will be identical to **A**. For {**B**} is a system isomorphic to {**A**}; so, by the identity assumption, {**B**} = {**A**}; and so **B** = **A**.

Suppose now that we have two A-objects $\mathbf{A}_1$ and $\mathbf{A}_2$ that depend upon one another and nothing else and that take any pair of distinct formulas $A_1$ and $A_2$ as values. Then the set {$\mathbf{A}_1$, $\mathbf{A}_2$} constitutes a system and any isomorphic system {$\mathbf{B}_1$, $\mathbf{B}_2$} consisting of A-objects that depend only upon one another and take any pair of distinct formulas as values will be identical to {$\mathbf{A}_1$, $\mathbf{A}_2$}. Note that the A-objects $\mathbf{A}_1$ and $\mathbf{A}_2$ are themselves indistinguishable. The identity assumption rules out distinct indistinguishable systems but not indistinguishable A-objects *within* a system.

The identity assumption is also capable of ruling out systems of A-objects that might otherwise be taken to exist. Consider the putative system {**E**, $\mathbf{E}_L$, $\mathbf{E}_R$} where: **E** is an independent A-object taking all identical disjunctions as values; $\mathbf{E}_L$ is the A-object dependent upon **E** and **E** alone, which, for any value E of **E**, takes as its value the left disjunct of E; and $\mathbf{E}_R$ is a *distinct* A-object dependent upon **E** and **E** alone, which, for any value E of **E**, takes as its value the right disjunct of E:

$$\begin{array}{ccccc} & & \mathbf{E} & & \\ & / & & \backslash & \\ \text{left disjunct} & \mathbf{E}_L & & \mathbf{E}_R & \text{right disjunct} \end{array}$$

Then the system {**E**, $\mathbf{E}_L$, $\mathbf{E}_R$} cannot exist for {**E**, $\mathbf{E}_L$} and {**E**, $\mathbf{E}_R$} are two isomorphic subsystems and hence $\mathbf{E}_L = \mathbf{E}_R$.[7]

§6 The Solution

I shall now suggest how the previous puzzle concerning forms can be solved by appeal to arbitrary objects. If we simply confine our attention to independent arbitrary objects then no solution is possible. For in that case, there is only one form **F** corresponding to the structural feature of being a formula, the independent A-object whose values are exactly the formulas. But if the form of a disjunction and the form of an identical disjunction are both to be disjunctions in some sense, then the only choice for the disjuncts would appear to be **F** and so the two forms cannot be distinguished.

However, the theory of arbitrary objects allows us to countenance A-objects which depend upon other A-objects. So consider again the independent A-object **D** whose values are exactly the disjunctive formulas. In analogy to the putative case above, there will be two dependent A-objects $\mathbf{D}_L$ and $\mathbf{D}_R$, both dependent upon **D** and **D** alone, the first taking as its value the left disjunct of D, for a given value D of **D**, and the second taking as its value the right

---

[7]Under a less extensional conception of a system, we might distinguish between the systems {**E**, $\mathbf{E}_L$} and {**E**, $\mathbf{E}_R$}, since the way $\mathbf{E}_L$ depends upon **E** is intensionally different from the way $\mathbf{E}_R$ depends upon **E**, one way being the restriction of a relation *left disjunct* which is extensionally different from the relation *right disjunct* of which the other way is a restriction.

disjunct of D, for a given value D of **D**. The sets {**D**, $\mathbf{D}_L$} and {**D**, $\mathbf{D}_R$} will both now constitute systems but we will no longer have the difficulty we had with the sets {**E**, $\mathbf{E}_L$} and {**E**, $\mathbf{E}_R$} above, since the systems {**D**, $\mathbf{D}_L$} and {**D**, $\mathbf{D}_R$} are not isomorphic; for when the value of **D** is a non-identical disjunction D = ($D_1 \vee D_2$) (with $D_1 \neq D_2$), the value of $\mathbf{D}_L$ will be $D_1$ while the value of $\mathbf{D}_R$ will be $D_2$.

Given the A-objects $\mathbf{D}_L$ and $\mathbf{D}_R$, we may now define another A-object $\mathbf{D}^*$, immediately dependent upon $\mathbf{D}_L$ and $\mathbf{D}_R$ alone, whose value, for given respective values $D_1$ and $D_2$ of $\mathbf{D}_L$ and $\mathbf{D}_R$ is their disjunction ($D_1 \vee D_2$). The case is illustrated below:

$$
\begin{array}{lccl}
 & & \mathbf{D} & \text{given disjunction} \\
 & / & & \backslash \\
\text{left disjunct} & \mathbf{D}_L & & \mathbf{D}_R \quad \text{right disjunct} \\
 & \backslash & & / \\
 & & \mathbf{D}^* & \text{dependent disjunction}
\end{array}
$$

Our proposal is to take the disjunctive form to be the object $\mathbf{D}^*$.

Let us consider a couple of other cases, which should help illustrate the general method. Consider first the form of an identical disjunction. As before, we let **E** be the independent A-object whose values are exactly the identical disjunctions. For reasons given before, we cannot now define two separate dependent A-objects $\mathbf{E}_L$ and $\mathbf{E}_R$ for the left and right disjunct, in analogy to the definition of $\mathbf{D}_L$ and $\mathbf{D}_R$. However, we can take there to be a single A-objects $\mathbf{E}_{L,R}$, solely dependent upon **E** and each other, which, for a given value E of **E**, indifferently takes the left or right disunct of E as its value. We can then take the form of the identical disjunction to be the A-object **E***, immediately dependent upon $\mathbf{E}_{L,R}$, which, for a given value A of $\mathbf{E}_{L,R}$, takes (A $\vee$ A) as its value[8]:

$$
\begin{array}{cl}
\mathbf{E} & \text{given identical disjunction} \\
| & \\
\mathbf{E}_{L,R} & \text{left/right disjunct} \\
| & \\
\mathbf{E}^* & \text{dependent identical disjunction}
\end{array}
$$

Let us now consider the form of a 'centered' disjunction ((A $\vee$ B) $\vee$ (B $\vee$ C)), where the left disjunct is itself a disjunction and the right disjunct is a disjunction whose left disjunct is identical to the right disjunct of the left disjunct. Let **G** be the independent A-object whose values are the formulas ((A $\vee$ B) $\vee$ (B $\vee$ C)). Let $\mathbf{G}_L$ be the left disjunct of **G**, i.e. the A-object

---

[8]We might also consider replacing $\mathbf{E}_{L,R}$ with two A-objects $\mathbf{E}_L$ and $\mathbf{E}_R$ , dependent upon one another and upon **E**, which for a given value (A $\vee$ A) of **E**, take A as their values. $\mathbf{E}^*$ will then take the disjunction of the (identical) values of $\mathbf{E}_L$ and $\mathbf{E}_R$ as it value. However, given that each identical disjunction is the disjunction of a formula with itself, we would like its $\mathbf{E}^*$ to be the disjunction of its form to be the disjunction of a form with itself.

dependent upon **G** whose value, for a given value ((A ∨ B) ∨ (B ∨ C)) of **G** is (A ∨ B). Similarly, let $\mathbf{G}_R$ be the right disjunct of **G**, $\mathbf{G}_{LL}$ and $\mathbf{G}_{LR}$ the left and right disjuncts of $\mathbf{G}_L$, and $\mathbf{G}_{RL}$ and $\mathbf{G}_{RR}$ the left and right disjuncts of $\mathbf{G}_R$. Also let $\mathbf{G}^*_L$ be the disjunction of $\mathbf{G}_{LL}$ and $\mathbf{G}_{LR}$ and $\mathbf{G}^*_R$ be the disjunction of $\mathbf{G}_{RL}$ and $\mathbf{G}_{RR}$. We then take the form of a disjunction ((A ∨ B) ∨ (B ∨ C)) be the disjunction of $\mathbf{G}^*_L$ and $\mathbf{G}^*_R$:

$$\begin{array}{lccccccccl}
 & & & & \mathbf{G} & & & & & \text{given centered disjunction}\\
 & & & / & & \backslash & & & & \\
\text{left disjunct} & & \mathbf{G}_L & & & & \mathbf{G}_R & & & \text{right disjunct}\\
 & / & & \backslash & & / & & \backslash & & \\
\text{left-left/right disjunct} & \mathbf{G}_{L,L} & & \mathbf{G}_{L,R} & & \mathbf{G}_{R,L} & & \mathbf{G}_{R,R} & & \text{right-left/right disjunct}\\
 & \backslash & & / & & \backslash & & / & & \\
\text{left disjunction} & & \mathbf{G}^*_L & & & & \mathbf{G}^*_R & & & \text{right disjunction}\\
 & & & \backslash & & / & & & & \\
 & & & & \mathbf{G}^* & & & & & \text{dependent centered disjunction}
\end{array}$$

Note the symmetry in the system of A-objects in each of these cases (and most apparent in the last case). The initial segment of the system breaks down the given formula into its components:

$$\begin{array}{ccccccc}
 & & & \mathbf{G} & & & \\
 & & / & & \backslash & & \\
 & \mathbf{G}_L & & & & \mathbf{G}_R & \\
/ & & \backslash & & / & & \backslash \\
\mathbf{G}_{L,L} & & \mathbf{G}_{L,R} & & \mathbf{G}_{R,L} & & \mathbf{G}_{R,R}
\end{array}$$

The concluding segment of the system then reverses the process and builds the formula back up:

$$\begin{array}{ccccccc}
\mathbf{G}_{L,L} & & \mathbf{G}_{L,R} & & \mathbf{G}_{R,L} & & \mathbf{G}_{R,R}\\
\backslash & & / & & \backslash & & / \\
 & \mathbf{G}^*_L & & & & \mathbf{G}^*_R & \\
 & & \backslash & & / & & \\
 & & & \mathbf{G}^* & & &
\end{array}$$

We might regard the whole system as the result of attaching the concluding segment of the system to the initial segment.[9]

---

[9]More accurately, what we attach is a system corresponding to the concluding segment in which the initial objects are taken to be arbitrary formulas. This may, of course, be in violation of our identity criteria for A-objects. But in attaching the one system to the other, we reinterpret the initial objects of the one system to be the same as the corresponding terminal objects of the

What the symmetry in the system of A-objects brings out is that we must first identify the components from which the form is constructed, where the process of identifying these components is the reverse of the process by which the form is constructed. To go up we must first go down.

We might think of $\mathbf{G}_L$, somewhat loosely, as the form of the occurrence of the left disjunct, in regard to its position within the whole formula. And we might think of $\mathbf{G}_L^*$ also as the form of the left disjunct, but now in regard to is construction from other occurrences. Just as there are more or less fine-grained ways of delineating the form of a formula so, on this view, there will be more or less fine-grained ways of delineating the form of an occurrence, either upwards in terms of its position or also downwards in terms of its construction.

§7 Defense

I shall show that the proposed account of form satisfies the requirements of Existence, Identity and Structural Similarity, laid out above; and I shall also show, along the way, that it meets the additional formal and informal requirements that we considered. Of course, this does not amount to a proof that the present account is correct. But I know of no better account nor of any serious deficiencies in the present account.

I shall focus, as before, on the case of the forms of formulas, assuming that the discussion will generalize to other cases. To this end, I shall take the forms in question to be the upwardly starred A-objects that are yielded by the method described above. Given a formula A, we suppose given an appropriate independent A-object $\mathbf{A}_*$, whose values are all the substitution-instances of A; we then introduce various dependent A-objects corresponding to the successively simpler subformulas of A; and, finally, we introduce A-objects corresponding to the successively more complex subformulas of A, culminating in the object $\mathbf{A}^*$, which is taken to be the form of A.

To establish Existence, we need to show that, for any formula A, there is an A-object $\mathbf{A}^*$. This does not strictly follow from the theory of arbitrary objects, as we have set it out, since the theory makes no assumptions about which A-objects exist. However, it should be clear that there is no impediment to assuming the existence of $\mathbf{A}^*$ (in particular, the existence of $\mathbf{A}^*$ is not in conflict with the proposed identity criterion for A-objects) and so any reasonably complete theory of A-objects should allow the object $\mathbf{A}^*$ to exist.

Identity has two directions. From right-to-left, it says that if two formulas A and B are alphabetic variants then $\mathbf{A}^* = \mathbf{B}^*$. But if A and B are alphabetic variants they will result in essentially the same 'construction', i.e. in isomorphic systems of A-objects; and so, by the identity criterion for A-objects, the two systems of A-objects will be the same and hence $\mathbf{A}^*$ and $\mathbf{B}^*$ will be identical. From left-to-right, Identity says that if the two formulas A and B are not alphabetic variants then $\mathbf{A}^* \neq \mathbf{B}^*$. But if A and B are not alphabetic variants, one of them will fail to be a substitution instance of the other. Suppose, without loss of generality, that B is not a substitution instance of A. Then $\mathbf{B}_*$ will take on a value B that $\mathbf{A}_*$ does not. So $\mathbf{A}_* \neq \mathbf{B}_*$. But it then follows that the corresponding 'terminal' objects $\mathbf{A}^*$ and $\mathbf{B}^*$ of the two systems of A-objects

other system.

generated by $\mathbf{A}^*$ and $\mathbf{B}^*$ are also not the same, since it is only if the two systems start out in the same way that an object of one can be identical to an object of the other.

The verification of Structual Similarity is less straightforward since we need to define the sense in which one A-object may be the 'disjunction' of two others (and similarly for negation and conjunction). Suppose that **B** and **C** are A-objects whose values are formulas. I would then like to suggest that **A** is the 'disjunction' of **B** and **C** if (i) **A** immediately depends upon **B** and **C** (and upon **B** and **C** alone) and (ii) given any formulas B and C as values for **B** and **C**, the value of **A** is $(B \vee C)$. It should now be evident from the construction that the form $\mathbf{A}^*$ of $A = (B \vee C)$ will, in this sense, be a disjunction of two other A-objects **B** and **C**, corresponding to the subformulas B and C.

We should also note that the forms will be 'cumulative'. If $\mathbf{A}^*$ is the form of $(p \vee (q \wedge r))$, for example, then it will be the 'disjunction' of a form **B** and a form **C**, where **C** itself is a 'conjunction' of forms. The forms will also conform to the requirement of Unique Readability. For by the identity criteria for A-objects, it will follow, given that **A** is the 'disjunction' of **B** and **C** and $\mathbf{A}'$ the 'disjunction' of $\mathbf{B}'$ and $\mathbf{C}'$ that $\mathbf{B} = \mathbf{B}'$ and $\mathbf{C} = \mathbf{C}'$.[10]

However, a problem remains. For by what right do we say that **B** and **C** are *forms*? Perhaps, in a way, it does not matter since, even if they are not, we can still take the form $\mathbf{A}^*$ to be a disjunction of other objects. All the same, it would be desirable if we could maintain that **B** and **C** are also forms, since we would like the structure of the forms as far as possible to mirror, at the level of the forms, the structure of the formulas: if complex formulas come from formulas then complex forms should come from forms.

Now it is clear that the A-objects **B** and **C** are not themselves the respective forms $\mathbf{B}^*$ and $\mathbf{C}^*$ of B and C. For the objects $\mathbf{B}^*$ and $\mathbf{C}^*$ belong to systems whose initial objects are $\mathbf{B}_*$ and $\mathbf{C}_*$, whereas **B** and **C** belong to a system whose initial object is $\mathbf{A}_*$. But even though **B** and **C** cannot be regarded as the forms of the *formulas* B and C, they can still be regarded as the forms of the *subformulas* B and C of A, conceived now not as formulas in their own right but as subformulas of A. **B** still is, in a sense, the form of the formula B but considered in situ, so to speak, as a subformula of A. Where A is a disjunction we might represent its form, qua disjunction, by:

$(B \vee C)$

But where B is a disjunct of A, we might represent its form, qua disjunct, by:

$(\underline{B} \vee C)$

thereby indicating that the formula is to be considered in the context of a larger formula.

We therefore see that there is a distinction among two kinds of form. There are the regular, synthetic or downward looking, forms which are constructed from constituent forms and which may be described by how they are constituted. But there are also the novel, analytic or upward looking, forms which are extracted from more comprehensive forms and which may be described by how they constitute. The one form is given by its inward structure, while the other is given by its outward structure or role within a large structure.[11]

---

[10]In the case of an identical disjunction, the pair of disjuncts will be unique but not their order.

[11]A related distinction is drawn in §10 of Fine [2010]

The error in the reasoning leading to the puzzle can now be traced to the failure to recognize the distinction between these two kinds of form. For we wanted the form of $(p \vee q)$ to be itself a 'disjunction' of two forms; and we could not see what these other forms could be except the form of p and the identical form of q, thereby preventing us from seeing how the form of $(p \vee q)$ could differ from the form of $(p \vee p)$. But we now see that the form of $(p \vee q)$ is indeed a 'disjunction' of forms of p and q, but in their role as disjuncts and not as free-standing formulas. Thus what enables us to avoid the problem is a more expansive view of what the forms of the formula might be taken to be.

One final problem remains. It will be noted that, for any formula A, the initial A-object $\mathbf{A}_*$ will have the same formulas as values as the terminal A-object $\mathbf{A}^*$. Indeed, the values will be coordinated: $\mathbf{A}_*$ will take the formula B as a value just in case $\mathbf{A}^*$ takes B as a value. Moreover, if we adopt a weaker definition of disjunction, according to which **D** is a disjunction of **B** and **C** just in case, for given values B and C of **B** and **C**, the value of **D** is $(B \vee C)$, then $\mathbf{A}_*$ will be a disjunction of two A-objects **B** and **C** whenever $\mathbf{A}_*$ is a disjunction of **B** and **C** (and a disjunction of many other pairs of A-objects besides). Thus from a perspective in which we simply take the values of the A-objects into account and not their dependency upon other A-objects, there will be nothing to choose between them. So why not take the simpler A-object $\mathbf{A}_*$ to be the form of A rather than $\mathbf{A}^*$?

One reason is evident from our previous discussion. For taking the forms of formulas to be independent A-objects runs us afoul of the puzzle: we can take the form of $(p \vee q)$ to be a disjunction of the form of p and the form of q but we cannot take the form of $(p \vee p)$ to be a disjunction of the form of p and the form of p.

But the independent A-objects are also unsuited in themselves to serve as the forms. For the form of a formula should, in some sense, be *given* as something with the same structure as the formula. It should wear the structure on its sleeve, so to speak. This is true of the terminal A-objects $\mathbf{A}^*$. For the identity of any A-object will be given by the other A-objects it depends upon and by how it depends on those other A-objects. So it follows, in particular, that the identity of the form of the disjunctive form $\mathbf{D}^*$ will be constituted by its being a disjunction, in the strong dependence-theoretic sense, of two other A-objects; and similarly for other complex forms. The identity of the initial A-object $\mathbf{D}_*$, by contrast, will be constituted by its being an independent A-object that takes the disjunctive formulas as values. If we were willing to engage in essentialist talk, then we might say that it is essential or intrinsic to $\mathbf{D}^*$ that it be the (strong) disjunction of **B** and **C**, while the sense in which $\mathbf{D}_*$ is a disjunction of **B** and **C** is inessential or extrinsic.

We should also note that the sense in which $\mathbf{D}^*$ is the disjunction of **B** and **C** is one that arises very naturally within the theory of A-objects and is not merely some artifact of our construction. For A-objects are canonically defined by let-clauses.[12] Consider the following two clauses, for example:

Let x be a real

Let y = 2x.

Then under the most natural interpretation of these clauses as definitions of A-objects, the first

[12] Also discussed in Fine [1998], §3 and Fine [1986].

clause will define ‘x’ as a term for an independent A-object **x** that takes the reals as its value, while the second clause will define ‘y’ as a term for an A-object immediately dependent upon **x** that takes the value 2r when **x** takes the value r. Suppose now $\mathbf{D}^*$ is defined by the following let-clauses (where it is understood that B and C have previously been defined):

Let D be the disjunction of B and C.

Then D will stand for the A-object **D** that is, in the required strong dependence-theoretic sense, the disjunction of the A-objects **B** and **C** signified by B and C. We should note that the notion of disjunction that figures in the clause is the ordinary notion of disjunction, as defined on formulas. The general interpretation of let-clauses then provides us with the means to ‘lift’ this operation on formulas to A-objects. Thus the sense in which we have a disjunction of A-objects is not some ad hoc stipulation but falls out from a general account of how operations on A-objects are to be defined.

I would not want to deny that the initial A-object $\mathbf{D}_*$ is a form but it is a relatively undifferentiated form and not something that can properly be called *the* form of a given formula. We might think of it as a proto-type or proto-form, not yet endowed with any particular structure (as in §4 of Fine[1998]) . The terminal A-object $\mathbf{D}^*$ then provides the means by which the proto-type is converted into a fully fledged form, genuinely endowed with structure. Indeed, the disjunctive form, as embodied in $\mathbf{D}^*$, is, in a quite literal sense, the form of an arbitrary disjunction, as given by $\mathbf{D}_*$.

So let it be granted that the form of a disjunction, say, should be an A-object that, like $\mathbf{D}^*$, is, in the strong dependence-theoretic sense, the disjunction of two other A-objects **B** and **C**. Why, it may now be asked, should the form be the disjunction of the two particular A-objects we have chosen? We may define $\mathbf{D}^*$ by means of the following three clauses:

Let $D_*$ be a disjunction;

Let B be the left disjunct of $D_*$ and C the right disjunct of $D_*$;

Let $D^*$ be the disjunction of B and C.

But then why should we have defined the terms ‘B’ and ‘C’ in the way that we have, as the left and right disjuncts of an arbitrary disjunction, rather than in some other way?

I have no answer to this question except to say that the definition we have given seems very natural and that, minor details aside, there seems to be no reasonable alternative. One might, for example, take $\mathbf{D}^*$ to be the disjunction of two arbitrary formulas **B** and **C**, as defined by the following two clauses:

Let B and C be two formulas

Let $D^*$ be the disjunction of B and C

But however exactly the first clause is understood (as either giving us two independent or two co-dependent A-objects), the objects **B** and **C** will be indistinguishable. The second clause is then improper or, at least, ambiguous, since if there is a disjunction **D** of **B** and **C** (in that order) then there will also be a distinct disjunction $\mathbf{D}'$ of **C** and **B** (in that order). **D** and $\mathbf{D}'$ will therefore be indistinguishable and so neither can properly be regarded as the form of a disjunction.

One might also wonder why we should take the form to be an arbitrary object in our sense rather than some other kind of object, perhaps even something sui generis. After all, we might identify a general strategy behind our approach, which is to admit, in addition to the regular synthetic forms the novel analytic forms, so that the disjunctive form, for example, can be regarded as a disjunction of two disjunct forms rather than of two formula forms; and we might then attempt to explain the distinction between these two kinds of forms in some other way.

I have no objection in principle to considering alternative accounts of this sort though, in the present case, I am at a loss to know what they might be. Nor do I have any objection in principle to regarding the forms as sui generis though again, in the present case, it is hard to know what it is about the forms, as opposed to the corresponding A-objects, that would make one want to insist that they are a different kind of object.

In the preceding discussion, I have simply attempted to develop an account of the *ontology* of forms, of what they might be. It would also be desirable to develop a *theory* of forms. Employing suitable form-theoretic primitives, such as 'the form of' or 'a form of', and perhaps also taking for granted a stock of primitive structural operations, one would then lay down general principles governing the behavior of the forms. Hopefully, the ontology and the theory would mesh, so that the theory could be regarded as describing - even as *completely* describing - the ontology.

Of special interest in this regard is the distinction between synthetic and analytic forms. The theory of synthetic forms is relatively straightforward. In many contexts, for example, we may adopt a principle of extensionality according to which two forms will be identical if their instances are the same, and we may define *the* form of an object to be that one of its forms whose instances are instances of any other of its forms. But these general principles must be relinquished when we expand the ontology to include analytic forms. Any disjunct form, for example, will have the same instances (viz. all formulas) as the form of an arbitrary formula, so that extensionality will fail as will the definition of *the* form in terms of *a* form. Some other way of developing a comparable theory must therefore be found.

## §8 Abstraction

I would like briefly to discuss the relationship between the present account of forms and my previous account of Cantorian/Dedekindian abstraction in Fine [1998].

Cantor took the number 2 to be the set consisting of two units $u_1$ and $u_2$. But what are these units? In my reconstruction of Cantor, I took the units belonging to the number 2 to constitute a system of two A-objects each dependent upon the other and taking any pair of distinct objects as their values.

There are two main differences from the account of form given above. The first is that the disjunctive form for me is an homonymous form; it is not literally an instance of itself, i.e. a disjunction. Cantorian 2, by contrast, is an autonomous form; it is literally an instance of itself, i.e. a two-membered set.

The reason for this difference is that the set-builder, in contrast to the operation of disjunction, is a universal operation; it applies to any objects whatever. There is therefore nothing to prevent it from applying to the two units $u_1$ and $u_2$ by which the number 2 is defined. On the other hand, the operation of disjunction is not universal, it only applies to formulas, and since the forms from which the disjunctive form are defined are not themselves formulas, some other way must be found to say what it is.

The other difference is that in the Cantorian case, there is no 'host' or proto-type by which the two units are defined. Thus we do not suppose that there is an independent A-object,

$2_*$, whose values are all two-membered sets, and then define the two units $u_1$ and $u_2$ to be two A-objects which depend upon $2_*$ (and upon themselves) and which, for a given set $\{a, b\}$ as its value, take the members $a$ and $b$ of the set as their values. The two units are just given exogenously.

The reason for this difference is that the set builder does not privilege any of its members; sets have no 'friends'. More precisely, suppose that we have a set $\{a, b\}$ consisting of the individuals $a$ and $b$ and that we map $a$ onto $b$ and $b$ onto $a$. We then end up with the very same set $\{b, a\} = \{a, b\}$. This means that we do not face the embarrassment of having two indistinguishable objects $\{u_1, u_2\}$ and $\{u_2, u_1\}$, which might, with equal right, be identified with the number 2.

Turn now to the order-type of the two-element linear order. For purposes of convenience, we identify a two-element linear order with a set of the form $\{<a, b>\}$. In this case, the units are defined by reference to a proto-type. Thus if we let $\bar{2}_*$ be the independent A-object whose values are all two-element linear orders, then we may let the units $v_1$ and $v_2$ be the A-objects dependent upon $\bar{2}_*$ which take the values $a$ and $b$ when $\bar{2}_*$ takes the value $\bar{2}_*$; the order-type $\bar{2}^*$ can then be identified with the set $\{<v_1, v_2>\}$.

There is now only one difference from our treatment of the forms of formulas. For the order-type $\bar{2}_*$, like the forms of formulas, is defined by reference to a proto-type. However, $\bar{2}_*$ is an autonomous type, it is itself a two-element linear order, in contrast to the form of formulas.

It is somewhat embarrassing that we have these different treatments of the form or type in these different cases. But as I have previous suggested (§4 of Fine[1998]), there is a way of assimilating the different cases. In regard to the issue of autonomy, we might suggest in general that the form or type should be a dependent A-object but then hold as a substantive principle that each of the sets $\{u_1, u_2\}$ and $\{<v_1, v_2>\}$ is identical to the corresponding A-object. And in regard to the issue of the proto-type, we may say, in general, that a proto-type is required but that, in certain cases, it will do no work and may therefore, on grounds of simplicity, be omitted. We thereby obtain something close to a general theory of form.

## §9 Tokens, Types and Structural Universals

The cases I have focused on are ones in which the instances of a form are themselves abstract entities, such as formulas or sets or linear orders. But I would also like to deal with the case in which the instances of the form are concrete entities, such as token expressions or particular molecules. Thus just as a formula has a certain form, so a token expression or particular molecule may be of a certain type (in such cases, the term 'type' seems somewhat more appropriate than 'form').

The difference in application comes with some corresponding formal differences. One is that, in the abstract case, we can usually suppose that the structural operations are always defined on the domain of application. Thus, given two formulas, we can always form their disjunction or, given two objects, we can always form their doubleton. Not so in the concrete case. Thus, given two token symbols far apart, there may be no expression which is their juxtaposition or, given an isolated carbon atom and an isolated oxygen atom, there may be no molecule of carbon monoxide which is the result of them being bonded.

The other difference is perhaps even more significant. Recurrent application of the structural operations is possible in the abstract case. Thus we may form the disjunction of a formula with itself or we may form the ordered pair of an object with itself. But usually recurrent application is not possible in the concrete case. Two token symbols can be juxtaposed, but a token symbol cannot be juxtaposed with itself; or again, a carbon atom can be bonded with an oxygen atom but not with itself.

The first difference makes it harder to provide an extensional treatment of the types. Suppose two particular types of symbol have never been juxtaposed, say オ and η (I was careful to separate them by 'and'!). Then how are we to account for the type of their juxtaposition in terms of their instances and how are we to distinguish the type of their juxtaposition from other types that are also not instantiated? Perhaps we can appeal to possible instead of actual instances. Thus within the context of our theory of arbitrary objects, we might specify the values taken by an A-object or its value-dependence on other A-objects in intensional, rather than extensional terms and distinguish the A-objects accordingly.

The second difference, by contrast, makes it somewhat easier to adopt an extensional stance. Recall that our previous puzzle over disjunction depended on the possibility or recurrent application; we needed to distinguish the form of $(p \vee q)$ from that of $(p \vee p)$. If there is no recurrent application, then the problem does not arise. Indeed, if we ignore the first issue (concerning uninstantiated types), we can simply identify a type with the class of its instances. The concatentation of two symbolic types σ and τ, regarded as classes of token, can then be defined as the concatentation of a token from σ with a token of τ whenever the concatentation exists. The concatenation of σ and σ will then unambiguously give us the concatentations of a token of σ with *another* token of σ, since a token can never be concatenated with itself.

Even so, there are good reasons to consider an alternative account. One concerns generality; one would like to adopt a uniform approach to all varieties of form or type. Another concerns identity; one would like to respect our intuitions as to what the types or forms are. In particular, one would like to respect the intuition that the expression type 'aba', for example, is itself a concatenation, in that order, of a type 'a' expression, a type 'b' expression, and a type 'a' expression.

We can satisfy these desiderata if we adopt our previous account of form or type. In this particular case, the proto-type $\mathbf{E}_*$ of the expression 'aba' will be the independent A-object whose values are all the (actual and possible) tokens of the form 'aba'. There will be three intermediate A-objects $\mathbf{E}_1$ $\mathbf{E}_2$ and $\mathbf{E}_3$, dependent upon $\mathbf{E}_*$, which, for a given token value $\mathbf{e}$ of $\mathbf{E}_*$ , take, as their respective values, the first, second and third tokens of $\mathbf{e}$. The type $\mathbf{E}^*$ of 'aba' will then be the A-object, dependent upon $\mathbf{E}_1$, $\mathbf{E}_2$, and $\mathbf{E}_3$, which, for given values $\mathbf{e}_1$, $\mathbf{e}_2$, and $\mathbf{e}_3$ of $\mathbf{E}_1$, $\mathbf{E}_2$ and $\mathbf{E}_3$, takes their concatenation as its value. We thereby achieve uniformity with the previous account of form; and the type $\mathbf{E}^*$ of 'aba' will, by definition, be the concatenation of the 'a' type $\mathbf{E}_1$, the 'b' type $\mathbf{E}_2$, and the 'a' type $\mathbf{E}_3$.[13]

---

[13]A related suggestion is to be found in Davis [2014]. He wants to take the two occurrences of 'Macavity' in the type 'Macavity, Macavity' to be two subtypes of the type 'Macavity', just as I would want to say that there are two dependent A-objects whose values are tokens of 'Macavity'. But our concerns are somewhat different - he wishes to give a reductive account of occurrences, whereas I wish to give a structural account of type; and he provides no

The present account of types bears comparison with the discussion of structural universals in Lewis [1986], a paper which helped shape subsequent discussion of the topic. His running example involves molecules rather than expressions and so let us go along with him in this regard. Let us also focus on his discussion of what he calls the 'pictorial conception' of universals (p. 33), since this is closest in kind to my own account.

There are two things I find odd in Lewis' discussion, both having to do with the underlying mereology. The first is that he accepts a standard mereological framework in which the only wholes are fusions. As he then points out, this makes it difficult to see how the universal or type methane ($CH_4$), for example, could contain the type hydrogen four times over, as opposed to just once. However, it is also difficult to see how a particular methane molecule could properly be regarded as the fusion of a particular carbon atom and four particular hydrogen atoms. For the molecule only exists when the atoms are suitably bonded while the fusion will exist regardless of whether the atoms are bonded. Thus it is not even clear from the particulars of interest to him that we should be working within a standard mereological framework; and the same goes, of course, for the case of expressions.

I myself would be inclined to think of a molecule as a rigid embodiment (in the sense of Fine [1999]). Thus if $c$ is the particular carbon atom and $h_1$, $h_2$, $h_3$ and $h_4$ are the particular hydrogen atoms then the methane molecule will be the bonding $c, h_1, h_2, h_3, h_4 /B$ of $c, h_1, h_2, h_3, h_4$ , an object that will have $c, h_1, h_2, h_3, h_4$ as its parts and that will only exist when they are suitably bonded. Be that as it may, we should suppose that there is some way in which these atoms come together so that the molecule is an appropriate bonding $B(c, h_1, h_2, h_3, h_4)$ of the respective atoms.

The other oddity concerns Lewis' characterization of the pictorial conception. At the intuitive level, the pictorial conception requires some kind of isomorphism between the type or universal and its instances. Lewis understands this isomorphism in mereological terms: the instance of the universal is a whole built up from parts and the universal should then be built up from corresponding parts (correspondingly related). As he puts it:

'On the pictorial conception, a structural universal is isomorphic to its instances. The methane atom [sic] consists of one carbon atom and four hydrogen atoms, with the carbon bonded to each of the four hydrogens; the structural universal methane likewise consists of several parts, one for each of the five atoms, and one for each of the four bonds'. (p. 33)

But why characterize the isomorphism in mereological terms? Presumably the answer is that there is no reasonable alternative conception of the structure of the instance or of the universal to be had. After all, we would not want to say that there is an isomorphism between you and me because we both have parents who are correspondingly related (as biological parents).

However, once we are prepared to appeal to the notion of essence, another answer is available to us. For the universal will essentially have a certain structure, whether mereological

---

background theory - such as our theory of arbitrary objects - which might explain how indistinguishable subtypes are possible. I also doubt that his notion of subtype would be of much help in providing an account of the structure of abstract types (such as the form of the formula ($p \vee p$)).

or not, and we can then require that this structure of the universal should correspond to the structure of the particular. Thus we may take the type $\mathbf{M}^*$ of methane to be that A-object which, by definition, is the particular 'bonding' $\mathbf{B}(\mathbf{C}, \mathbf{H}_1, \mathbf{H}_2, \mathbf{H}_3, \mathbf{H}_4)$ of a carbon type $\mathbf{C}$ with four hydrogen types $\mathbf{H}_1$, $\mathbf{H}_2$, $\mathbf{H}_3$, and $\mathbf{H}_4$, each dependent upon one another and the proto-type $\mathbf{M}_*$, as displayed below[14]:

$$\begin{array}{ccccccccc} & \mathbf{M}_* & & & & & & & \\ / & & \backslash & & & & & & \\ \mathbf{C} & & \mathbf{H}_1 & — & \mathbf{H}_2 & — & \mathbf{H}_3 & — & \mathbf{H}_4 \\ \backslash & & / & & & & & & \\ & \mathbf{M}^* & & & & & & & \end{array}$$

Whether we should regard the type $\mathbf{B}(\mathbf{C}, \mathbf{H}_1, \mathbf{H}_2, \mathbf{H}_3, \mathbf{H}_4)$ as a whole whose parts are $\mathbf{C}$, $\mathbf{H}_1$, $\mathbf{H}_2$, $\mathbf{H}_3$, $\mathbf{H}_4$ is a difficult question about which I am not sure to say. But regardless of whether it is such a whole, we can still maintain that there is an isomorphism between the definition $\mathbf{B}(\mathbf{C}, \mathbf{H}_1, \mathbf{H}_2, \mathbf{H}_3, \mathbf{H}_4)$ of the methane type and the definition $\mathrm{B}(c, h_1, h_2, h_3, h_4)$ of a particular instance of methane.

There is a way in which Lewis himself comes close to endorsing a theory of this sort. At one point he consider a view according to which a structural universal might contain 'amphibians', which are like universals but capable of occurring repeatedly. Thus 'we need four hydrogen amphibians as parts of the universal methane, one for each of the four hydrogen atoms in the molecules that instantiate it' (p. 39).

In considering how such a view might be developed, he writes:

> We face some fascinating questions. (1) What becomes of our original monadic universals, such as the one universal *hydrogen?* Do we have them as well as their amphibians, perhaps instantiated by their amphibians? 2) Does the same amphibian ever occur as part of two different structural universals? (3) If we have two hydrogen atoms in two different methane molecules, is there indeed a distinction between the case in which they instantiate the same amphibian of the structural universal *methane* and the case in which they instantiate different ones?

to which he responds:

> I do not mean to put these questions forward as unanswerable. I might even suggest answers. But I shall not. I shall suggest indeed that the questions are too bizarre to take seriously. The theory that asks them just has to be barking up the wrong tree. There comes a time not to go on following where the argument leads! (p. 40)

But if I am right these questions are capable of receiving perfectly straightforward answers. (1) The universal or type *hydrogen* (considered as an independent A-object whose

---

[14]Here, in contrast to the case of an identical disjunction, we would like the intermediary forms $\mathbf{H}_1$, $\mathbf{H}_2$, $\mathbf{H}_3$, $\mathbf{H}_4$ to be distinct from one another. For the identical disjunction is formed from two identical disjuncts while a methane molecule is formed from four distinct hydrogen atoms.

To take account of isomers, we should allow for different operations of bonding.

values are all the hydrogen atoms) will still exist along with all the amphibians (considered as intermediate A-objects) but it is only instantiated by normal particulars not by the amphibians themselves. (2) The same amphibian will not occur in two different structural universals, given a natural disjointness condition on systems of A-objects. (3) Given two hydrogen atoms in two methane molecules, there will be no distinction in regard to their instantiating one hydrogen amphibian as opposed to another. Thus in the case above, it will be a matter of indifference whether we regard the hydrogen atoms $h_1$, $h_2$, $h_3$, $h_4$ in a particular molecule of methane as instancing $\mathbf{H}_1$, $\mathbf{H}_2$, $\mathbf{H}_3$, $\mathbf{H}_4$, let us say, as opposed to $\mathbf{H}_2$, $\mathbf{H}_2$, $\mathbf{H}_4$, $\mathbf{H}_3$. What led to bewilderment on Lewis' part is the lack of a systematic basis upon which these questions might have been answered. But once given an account of types or universals in terms of A-objects, we have such a basis and the questions he asks turn out to be neither bizarre nor difficult to answer.

## References

D. M. Armstrong [1978] 'Universals and Scientific Realism: I & II', Cambridge: Cambridge University Press.

Davis W. [2014] 'Of Occurrences of Types in Types', Australasian Journal of Philosophy 92.2, 349-63.

Fine K., [1982] 'The Problem of Non-Existents I- Internalism', Topoi', vol. 1, 97-140

Fine K., [1985] 'Natural Deduction and Arbitrary Objects', Journal of Philosophical Logic 14, 57-107; reprinted in Philosopher's Annual, vol VIII (1985), Ridgeway: California.

Fine K., [1986] 'Reasoning with Arbitrary Objects', Oxford: Blackwell.

Fine K., [1998] 'Cantorian Abstraction: A Reconstruction and Defense', Journal of Philosophy 95(12), 599-634.

Fine K., [1999] 'Things and Their Parts', Mid-west Studies XXIII (ed. French & Wettstein), pp. 61-74.

Fine K., [2005] 'The Problem of De Re Modality', chapter 2 of 'Modality and Tense: Philosophical Papers', Oxford: Clarendon Press.

Fine K., [2009] 'The Question of Ontology', in 'Metametaphysics: New Essays on the Foundations of Ontology' (eds. D. Chalmers, D. Manley, R. Wassermann), Oxford: Oxford University Press, 157-77 (2009).

Fine K., [2010] 'Towards a Theory of Part', Journal of Philosophy CV11, no. 11, 559-89.

Fine K., [2017] 'Naive Metaphysics' (ed. Jonathon Schaffer), Oxford: Oxford University Press.

Forrest P., [2016] 'The Mereology of Structural Universals', Logic and Logical Philosophy, v. 25, 259-83.

Godfrey-Smith P., [2006] 'The Strategy of Model-Based Science', Biology and Philosophy 21.5, 725-740

Lewis D. [1986] 'Against Structural Universals', Australasian Journal of Philosophy 64.1, 25-46.