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The 7$^{\text{th}}$ International Conference on Applied Energy – ICAE2015

Preliminary Mean-line Design and Optimization of a Radial Turbo-Expander for Waste Heat Recovery using Organic Rankine Cycle

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Abstract

This study presents an optimized modelling approach for ORC based on radial turbo-expander, where the constant expander efficiency is replaced by dynamic efficiency and is unique for each set of cycle operating conditions and working fluid properties. The model was used to identify the key variables that have significant effects on the turbine overall size. These parameters are then included in the optimization process using genetic algorithm to minimize the turbine overall size for six organic fluids. Results showed that, dynamic efficiency approach predicted considerable differences in the turbine efficiencies of various working fluids at different operating conditions with the maximum difference of 7.3% predicted between the turbine efficiencies of n-pentane and R245fa. Also, the optimization results predicted that minimum turbine overall size was achieved by R236fa with the value of 0.0576m. Such results highlight the potential of the optimized modeling technique to further improve the performance estimation of ORC and minimize the size.

Keywords:Organic Rankine cycle; radial turbo-expander, mean-line modeling, genetic algorithm, optimization

1. Introduction

Low to medium temperature waste heat recovery has received growing attention in the past few years to address the urgent issues of carbon emission and energy consumption. Organic Rankine Cycle (ORC) is a promising candidate for conversion these sources into electricity with the advantages of low capital cost, small size and easy maintenance. There are numerous literature that studied ORC for a broad range of heat sources and applications including biomass heat [1], solar heat [2], geothermal heat [3], waste heat of IC engines [4] and waste heat of the conventional Rankine cycle [5]. In contrast, very few studies paid attention to the detailed modeling of the expander considering that it is a critical component in a relatively efficient ORC system. Among the available expanders, radial turbines offer the advantages of high power to weight ratio and high efficiencies (above 75%) compared to scroll expander that suffers from substantial sealing and lubrication requirements with lower efficiencies of about 67% reported in

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[6]. In contrast to all of the ORC modelling studies [1-8] that assumed a constant expander efficiency for various working fluids and for wide range of operating conditions, this study presents a unified approach for combined modeling of ORC based on radial turbo-expander. In this way it is possible to replace the constant expander efficiency with an interactive efficiency which is unique for each set of cycle operating parameters and fluid properties and unlike the previous literature assures the possibility that the turbine is able to obtain these efficiencies in practice. Moreover, none of the modelling studies in [7-8] conducted the optimization of radial turbine for the optimum geometry. Therefore, parametric studies are carried out using the developed model to investigate the effects of input variables on the turbine overall size. The input variables that have the most remarkable effect on the turbine size are included in the constrained optimization process using genetic algorithm to minimize the turbine overall size for six organic fluids.

Nomenclature

b_4 (m)	Rotor inlet width	W (m/s, W)	Relative flow velocity, power
C (m/s)	Absolute flow velocity, Chord (m)	α	Absolute flow angle with radial
d_{max} (m)	Maximum turbine diameter	β	Relative flow angle with radial
h (J/kg)	Enthalpy	$\eta_{turbine,stage,ts}$	Total-to-static turbine stage efficiency
m (kg/s)	Mass flow rate	$\eta_{thermal,cycle}$	Cycle thermal efficiency
Ma (-)	Mach number	ρ (kg/m^3)	Density
Ns (-)	Specific speed	φ (-)	Flow coefficient
$P_{t,1}$(Pa)	Turbine inlet total pressure	ψ (-)	Loading coefficient
Q_{in} (W)	Heat input	ω (RPM)	Rotational speed
r (m)	Radius	*Subscripts*	
SC (-)	Volute swirl coefficient	1,2…7	Stations across the turbine and ORC
s (m)	Blade spacing		
$T_{t,1}$ (K)	Turbine inlet total temperature		
U (m/s)	Wheel velocity		

2. Methodology for the integrated modeling of ORC with mean-line modeling of the radial turbine

Mean-line modeling approach is based on a one-dimensional assumption that there is a mean streamline through the stage, such that conditions on the mean streamline are the average of the passage conditions [9]. The thermodynamic properties, basic geometry and flow features are determined at key stations across the stage as depicted in Fig. 1. Common fluid dynamics principles such as conservation of mass, momentum and energy and Euler turbomachinery equation together with the inclusion of the turbine losses build the mean-line model. The aerodynamic boundary conditions and non-dimensional parameters that are required for the preliminary design of the radial turbine are listed in Table 1.
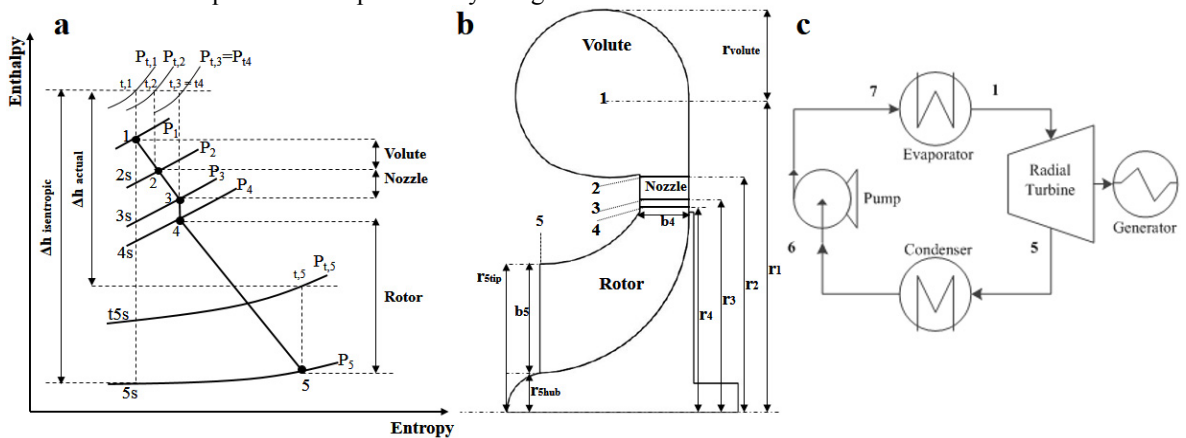

Fig. 1. (a) Enthalpy-entropy expansion diagram; (b) schematic of radial turbine components; (c) schematic of ORC

Table 1. Input data of the turbine mean-line model

Parameter	$T_{t,1}$(K)	$W_{turbine}$(W)	ω(RPM)	ψ(-)	φ(-)	m(kg/s)	α_5(deg)	r_{5hub}/r_4(-)	r_2/r_3(-)	SC(-)
Value/Range	333 - 373	5000	40000- 70000	0.8 - 1.4	0.1 - 0.5	0.2	0	0.2	1.2 - 1.3	0.95

2.1. Rotor, nozzle and volute modeling

Loading (ψ) and flow (ϕ) coefficients are employed to determine the rotor basic geometry and velocity triangles at the rotor inlet and exit following the procedure outlines in [9].

$$\psi = \frac{\Delta h_{actual}}{U_4^2} = \frac{\Delta h_{actual}}{(r_4\omega)^2} \approx \frac{C_{\theta 4}}{U_4} \tag{1}$$

$$\phi = \frac{C_{m5}}{U_4} \tag{2}$$

Unlike the gas turbines or turbochargers that operate at harsh environment of high stresses and high temperatures (up to 1000°C), the ORC operating temperature is significantly lower (up to 200°C) which permits the implementation of back swept blading at rotor inlet. By virtue of the lower temperature, additional stresses due to the back swept blading could be tolerated. This configuration is advantageous compared to conventional rotor with zero inlet blade angle, as shown in Fig. 2. The back swept rotor yields in higher level of inlet tangential velocity ($C_{\theta 4}$) and consequently higher specific work output is obtained as shown by Eq. 3 considering the same values for U_4, U_5 and C_5.

$$\Delta h_{actual} = U_4 C_{\theta 4} - U_5 C_{\theta 5} = \frac{W_{turbine}}{\dot{m}} \tag{3}$$
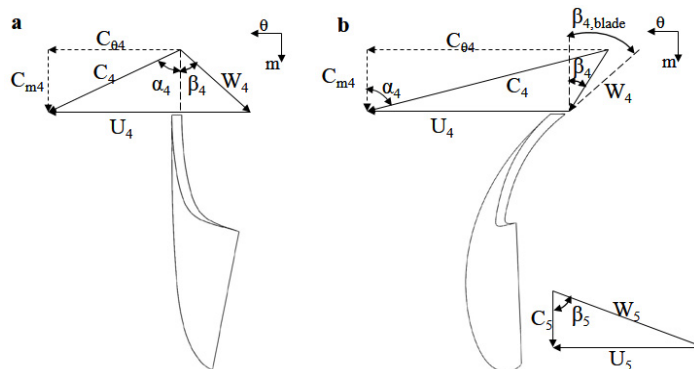
Fig. 2. Schematic of rotor blade profiles and velocity triangles, (a) zero inlet blade angle, (b) back swept inlet blade angle

The rotor inlet and exit total and static thermodynamic properties are determined iteratively with the adiabatic assumption ($h_{t,4} = h_{t,1}$), real gas equation of state, rotor inlet and exit absolute and relative flow velocities (C_4, W_4, C_5, W_5) and equations 4 and 5.

$$P_{t,4} = P_{t,1} - \left(\frac{\rho_{t,1} \cdot \Delta h_{actual} \cdot (1 - \eta_{turbine,stage,ts})}{4\eta_{turbine,stage,ts}} \right) \tag{4}$$

$$h_5 = \left(h_{t,4} - \Delta h_{actual} \right) - \frac{C_5^2}{2} \tag{5}$$

The rotor loss correlations for tip clearance, secondary, friction and exit kinetic are obtained from [9].

The nozzle thermodynamic properties, absolute velocities and radii at inlet and exit are determined iteratively using the conservation of mass, angular momentum, thermodynamic properties at rotor inlet, nozzle radii ratio (r_2/r_3) and Eq. 6.

$$r_3 = r_4 + 2.b_4.\cos(\alpha_4) \tag{6}$$

The nozzle loss coefficient is obtained by Eq. 7.

$$k_{nozzle,loss} = \frac{0.05}{\text{Re}^{0.2}} \left[\frac{3tg\alpha_3}{\frac{s}{c}} + \frac{s\cos\alpha_3}{b_4} \right] \tag{7}$$

Assuming a circular cross section for the volute, shown in Fig.1, the volute inlet radius (r_1), maximum cross section radius (r_{volute}), turbine overall size (d_{max}) and the volute loss are determined iteratively using the input data given in Table 1 and equations 8 to 11 respectively.

$$r_1 = \frac{r_2 C_{\theta 2}}{SC.C_1} \tag{8}$$

$$r_{volute} = \sqrt{\frac{\dot{m}}{\rho_1 C_1 (0.75\pi + 1)}} \tag{9}$$

$$d_{max} = 2(r_1 + r_{volute}) \tag{10}$$

$$\Delta h_{loss,volute} = \frac{(k_{volute} C_2^2)}{2} \tag{11}$$

Where the volute total pressure loss coefficient (k_{volute}) is equal to 0.1 as suggested in [9]. The new estimate of the turbine efficiency is obtained based on the turbine component losses using Eq. 12.

$$(\eta_{turbine,stage,ts})_{new} = \frac{\Delta h_{actual}}{\Delta h_{actual} + \Delta h_{loss,tipclearance} + \Delta h_{loss,secondary} + \Delta h_{loss,friction} + \Delta h_{loss,exit} + \Delta h_{loss,nozzle} + \Delta h_{loss,volute}} \tag{12}$$

2.2. Thermodynamic modeling of the ORC

Modeling of the ORC main parameters is conducted using the calculated turbine thermodynamic properties at inlet and exit states and Eq. 13. It is assumed that the turbine inlet and condenser outlet are at saturated vapour and saturated liquid states respectively and the pump isentropic efficiency is 0.7.

$$\eta_{thermal,cycle} = \frac{W_{net}}{Q_{in}} = \frac{W_{turbine} - W_{pump}}{\dot{m}(h_1 - h_7)} = \frac{W_{turbine} - \frac{\dot{m}(P_7 - P_6)}{\rho_6 \eta_{pump}}}{\dot{m}(h_1 - h_7)} \tag{13}$$

3. Methodology for constrained optimization

With the newly proposed applications for the ORC (i.e. the automotive sector [10, 11]), reducing the turbine overall size and consequently the turbine inertia are critically demanding. The Genetic Algorithm (GA) is employed to minimize the turbine overall size (d_{max}) for six organic fluids as R245fa, R123, R365mfc, R236fa, n-pentane and isobutane. To ensure the practicability of the optimized turbine, GA is constrained by some of the critical geometry parameters and flow features.

- $d_4 > 0.01$ (m). Minimum value for manufacturability.
- $0.05 < b_4/d_4 < 0.15$. To limit the rotor tip clearance losses.
- $r_{5tip}/r_4 < 0.85$. To avoid excessive rotor exit tip curvature and minimize the secondary losses.

- $\beta_{4,blade}$ <70°. Maximum value for manufacturability ($\beta_{4,blade} = \beta_4 - i$ where i is the optimum incidence angle obtained from [12]).
- $Ma_{5tip,rel}$ < 1. To avoid supersonic loss at the rotor exit.

4. Results and discussion

Parametric studies are performed using the developed model to investigate the effects of input variables of Table 1 on the turbine overall size (d_{max}). As shown in Fig. 3, the effect of ψ is remarkable on d_{max}. As ψ increases from 0.8 to 1.4, d_{max} reduces by about 21% for all working fluids. This is directly related to Eq. 1 in which with constant enthalpy drop and rotational speed, r_4 is inversely proportional to ψ and increasing ψ yields to smaller rotor and consequently smaller d_{max}. This highlights the potential of implementing back swept blading at rotor inlet to obtain high loading coefficient and achieve more compact turbine. As illustrated in Fig. 4, d_{max} is not significantly affected by the variation of φ for all investigated fluids. Fig. 5 shows that the effect of ω is considerably high on d_{max}. Increasing RPM from
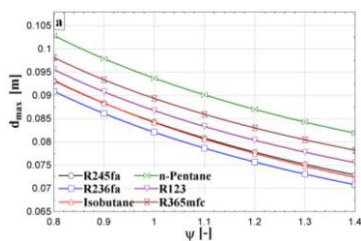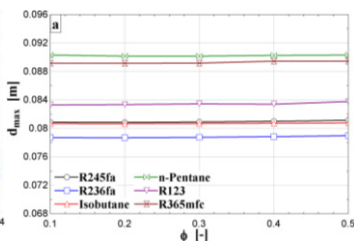

Fig. 3. Variation of ψ for 6 fluids   Fig. 4. Variation of φ for 6 fluids
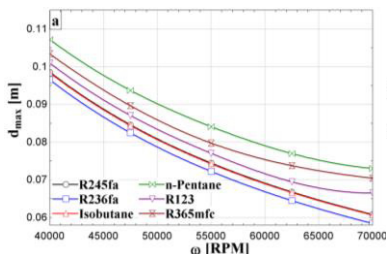

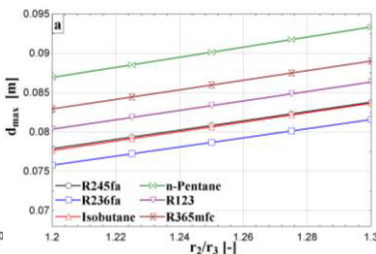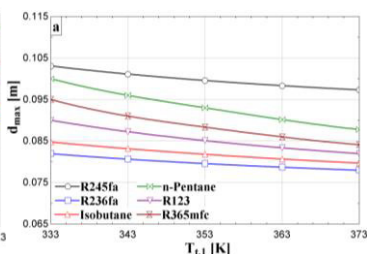Fig. 5. Variation of ω for 6 fluids   Fig. 6. Variation of r_2/r_3 for 6 fluids   Fig. 7. Variation of $T_{t,1}$ for 6 fluids

40000 to 70000 reduces the turbine size by maximum value of 39% for R236fa. Fig. 6 illustrates that d_{max} is linearly proportional to the variations of r_2/r_3. Fig. 7 shows that the effect of $T_{t,1}$ on d_{max} is considerable and it is beneficial to increase $T_{t,1}$. As shown in Figs 3 to 7, only ψ, ω, r_2/r_3 and $T_{t,1}$ have significant effects on d_{max} and are included in the optimization using GA to minimize d_{max} for six organic fluids while the rest of input variables are kept constant. Fig. 8 shows the effects of working fluids on d_{max} at optimized conditions. It is obvious that the choice of working fluid can greatly affect the size of the turbine. For instance the value of d_{max} for n-pentane is about 1.5 times larger than R236fa at inlet temperature of 373K. Furthermore, d_{max} decreases with the rise of $T_{t,1}$ with the maximum value of 44.6% for R365mfc. R236fa exhibits the minimum turbine overall size (d_{max}) of 0.057m at the inlet temperature of 373K. Fig. 9 illustrates the effects of working fluids on the $\eta_{turbine,stage,ts}$ at optimized conditions. It is clear that there exists considerable variations between $\eta_{turbine,stage,ts}$ of investigated fluids.
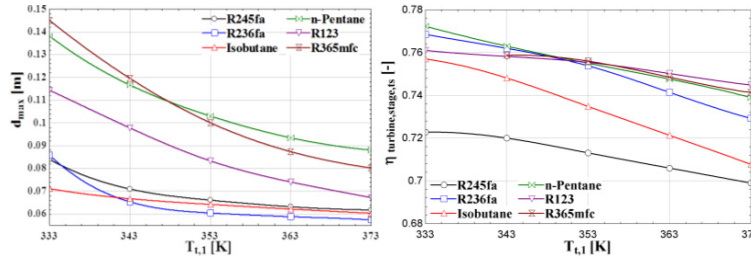
Fig. 8. Optimized d_{max} at various $T_{t,1}$ Fig. 9 $\eta_{turbine,stage,ts}$ at optimized conditions and at various $T_{t,1}$

$\eta_{turbine,stage,ts}$ decreases with the rise of $T_{t,1}$ with the maximum value of 4.9% for isobutane. Also there is a maximum difference of 7.3% between $\eta_{turbine,stage,ts}$ of n-pentane and R245fa. Fig. 10 presents the effects of working fluids and $T_{t,1}$ on the cycle thermal efficiency ($\eta_{thermal,cycle}$) at optimized conditions. It is clear that, there are considerable differences in the $\eta_{thermal,cycle}$ of investigated fluids. It is only for R123, n-pentane and R365mfc that increasing $T_{t,1}$ increases the $\eta_{thermal,cycle}$ steadily. In contrast, for R245fa, isobutane and R236fa increasing $T_{t,1}$ escalates $\eta_{thermal,cycle}$ at first but decreases afterwards. This is due to the fact that the effect of $T_{t,1}$ is dominant in increasing $\eta_{thermal,cycle}$ up to $T_{t,1}$=353K. At the same time $\eta_{turbine,stage,ts}$ of R245fa, isobutane and R236fa are reducing more rapidly compare to the rest of fluids as shown in Fig. 9 and results in reduction of $\eta_{thermal,cycle}$ after the inlet temperature of 353K. The results in Figs 9 and 10 indicate the irrationality of considering a constant expander efficiency that can yield to inaccuracies in estimating the turbine performance and cause errors in the cycle analysis. The optimized input variables of the model that are obtained from the optimization and led to minimum d_{max} together with some of the main turbine and cycle results are shown in Table 2. The values of N_s in Table 2 justify the exclusion of the diffuser from the analysis. This is due to the fact that, for N_s values significantly lower than 0.7 , which is the case for most of the fluids in this study, the gain of using the diffuser is much less and implementation of the diffuser requires an economic analysis [9] which is beyond the scope of this study.
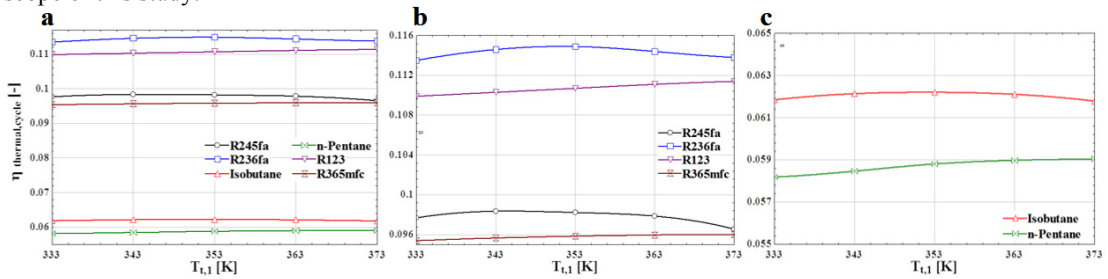

Fig. 10. (a) $\eta_{thermal,cycle}$ at optimized conditions and at various $T_{t,1}$, (b, c) zoom in views

Table 2 Optimized input variables of the turbine-ORC model with summery of the main results at $T_{t,1}$=373K

Parameter	$P_{t,1}$ (kPa)	ψ (-)	ω (rpm)	r_2/r_3 (-)	φ (-)	α_4 (deg)	$\beta_{4,blade}$ (deg)	U_4 (m/s)	d_4 (m)	$d_{5,tip}$ (m)	b_4 (m)	N_s (-)	$Ma_{5tip,rel}$ (-)	W_{net} (kW)	Q_{out} (kW)
R245fa	1265	1.23	68468	1.2	0.4	72	49.6	143	0.039	0.025	0.0013	0.417	0.74	4.78	42.3
R123	784	1.33	68802	1.2	0.3	77.3	67.7	137	0.038	0.032	0.0029	0.524	0.92	4.68	36.9
R365mfc	584	0.8	70000	1.2	0.25	73.1	0	176	0.048	0.037	0.0021	0.609	0.98	4.69	44.2
R236fa	1930	1.28	70000	1.2	0.25	79.1	70	140	0.038	0.026	0.0015	0.383	0.75	4.47	34.4
n-pentane	588	0.8	70000	1.2	0.25	72.7	0	177	0.048	0.033	0.0028	0.539	0.65	4.65	73.4
Isobutane	1979	1.28	70000	1.2	0.25	79.1	69.9	139	0.038	0.021	0.0017	0.3	0.40	4.16	62.8

5. Conclusions

This work presents a novel approach for modelling and optimization of the ORC based on radial turbo-expander to replace the constant turbine efficiency with a dynamic efficiency, obtained by the turbine losses, and conduct constrained optimization based on a wide range of turbine and ORC variables. The results predicted considerable reduction in the turbine efficiencies of all fluids with rise of the turbine inlet temperature. The maximum difference of 7.3% between the turbine efficiencies of n-pentane and R245fa was predicted. Optimization of the turbine overall size using genetic algorithm showed that, the minimum turbine overall size of 0.0576m was achieved by R236fa. Such results highlights the potential of this combined approach to further improve the performance and development of the ORC.

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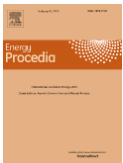
Biography

Kiyarash Rahbar has spent most of his higher education studies at the University of Birmingham, UK. He received his First-Class Honours MEng degree in Mechanical Engineering in 2011. He has been awarded the University of Birmingham scholarship to fund his PhD research programme starting November 2011. He has good background in the fields of thermodynamics, heat transfer and turbomachinery.