Integer Programming

Global Impact

George Nemhauser
Georgia Institute of Technology
Atlanta, GA, USA

EURO, INFORMS
Rome, Italy
July 2013

Integer Programming

Optimization models with

integer variables
mostly binary

Integer Programming

Binary variables used to model

- Decisions
- Logical relations
- Nonlinearities
- Nonconvexities

Agenda for today:

- Past, present and future
- Real applications
  - Transportation
  - Supply chain
  - Energy
  - Sports
  - Health
  - Finance

A bit of history

CUTTING PLANES

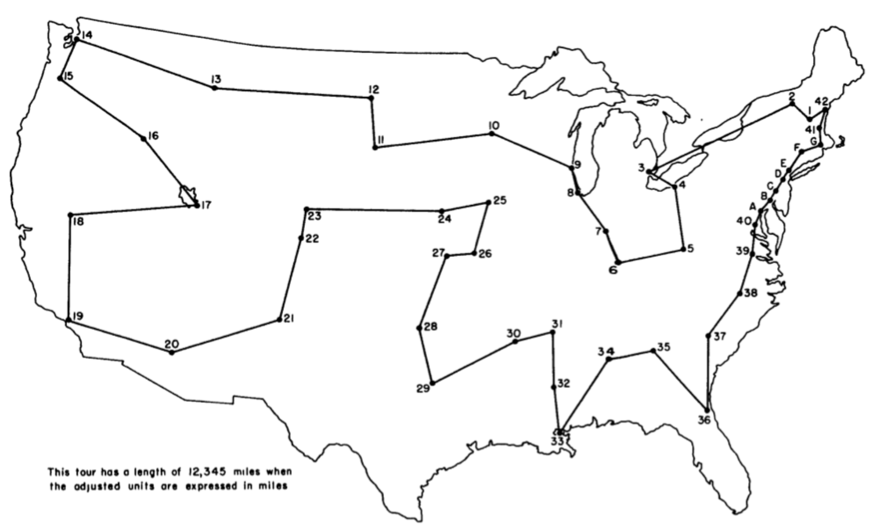
49-city TSP solved by linear programming and cutting planes

Dantzig, Fulkerson, Johnson (1954)

General IP, finite cutting plane algorithm

Implementation in FORTRAN

Gomory (1958)



This tour has a length of 12,345 miles when the adjusted units are expressed in miles

Fig 16 The optimal tour of 49 cities

BRANCH-AND-BOUND

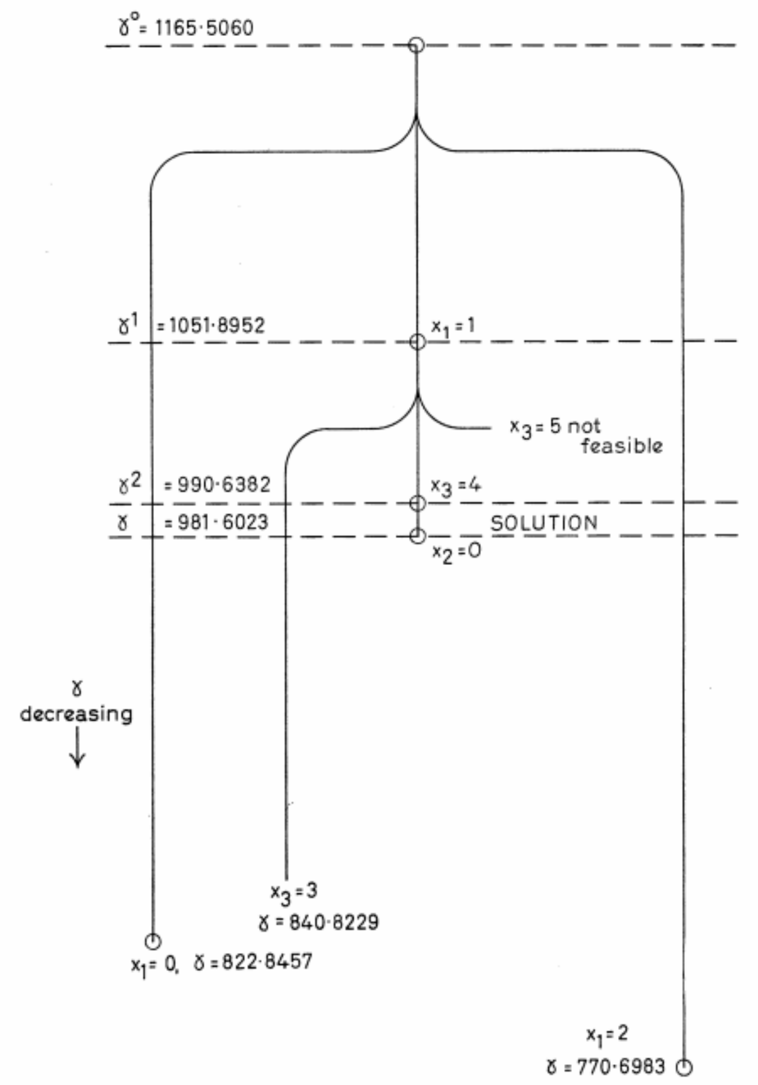
LP + enumeration for general MIP

Land and Doig (1960)

Assignment problem + enumeration for TSP

Coined the term,
successful computation

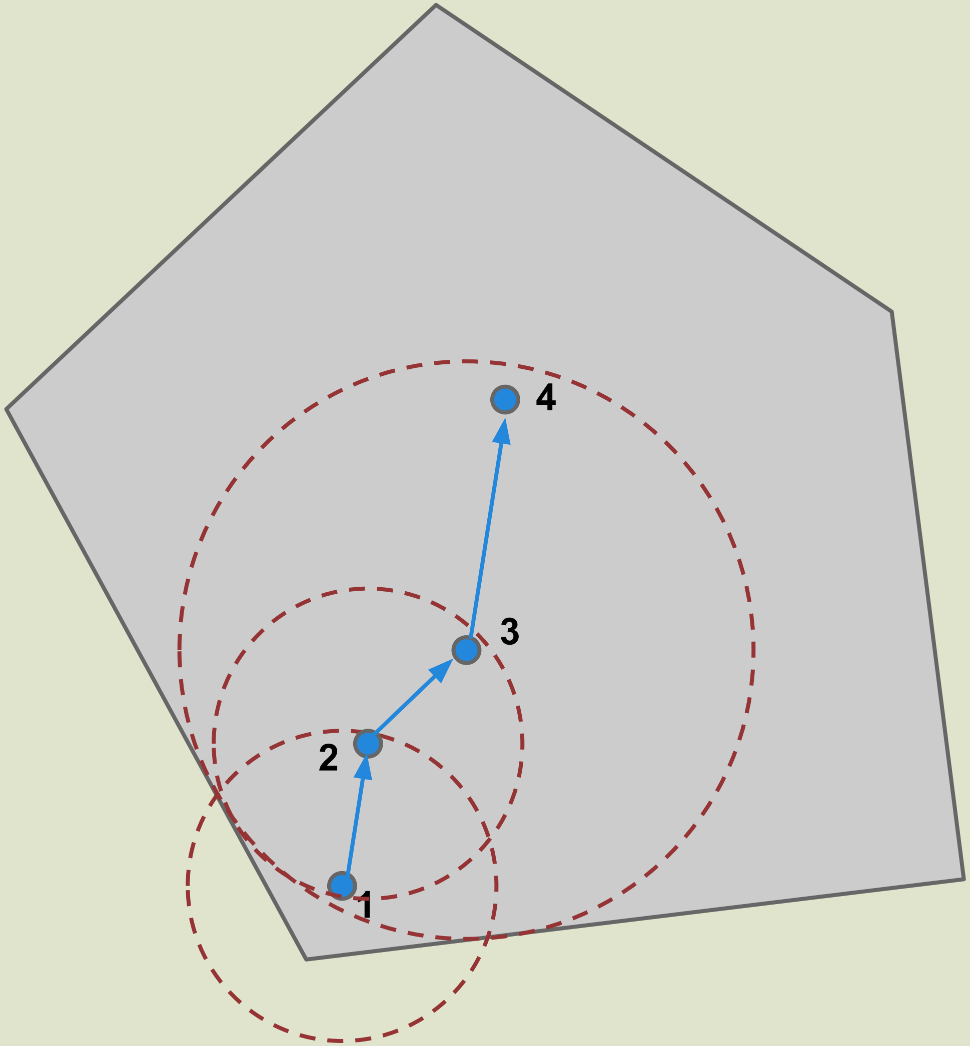
Little et al. (1963)



HEURISTICS

Local search

Reiter and Sherman (1965)

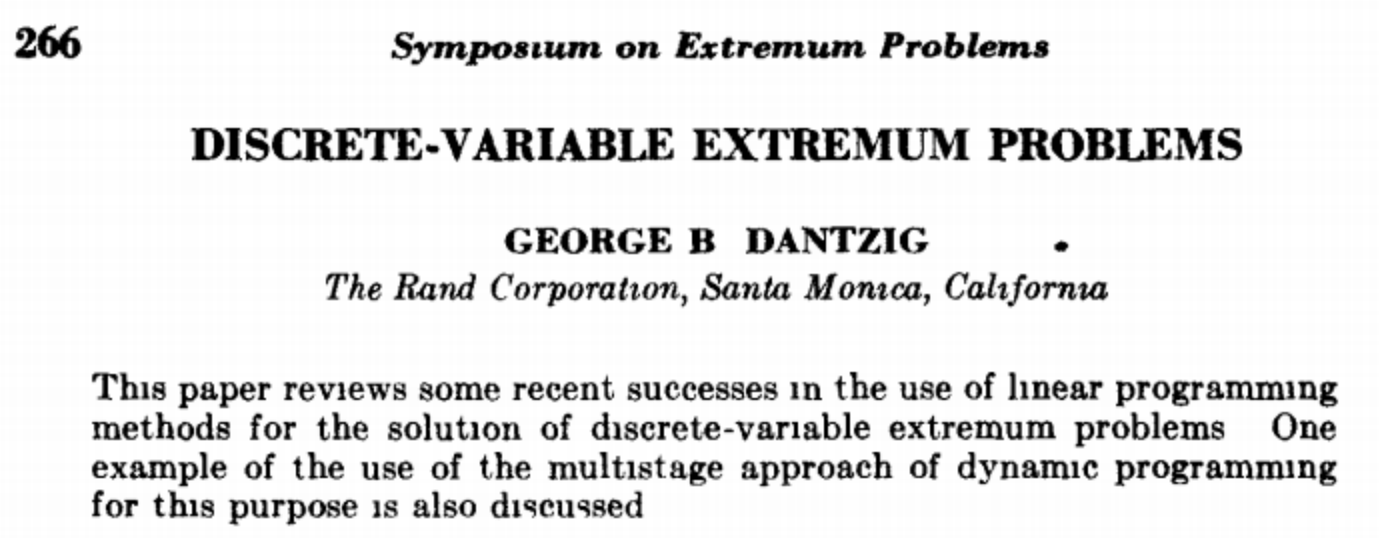


FORMULATIONS

Dantzig (1957)

Many proposed models, but solutions ???

Scheduling problem, more cutting planes than enumerating all possibilities



266 *Symposium on Extremum Problems*

DISCRETE-VARIABLE EXTREMUM PROBLEMS

GEORGE B DANTZIG

The Rand Corporation, Santa Monica, California

This paper reviews some recent successes in the use of linear programming methods for the solution of discrete-variable extremum problems One example of the use of the multistage approach of dynamic programming for this purpose is also discussed

Real applications

Almost a decade to go from methods and models to solving industrial scale problems

Documentation of early applications very difficult to find

Mostly personal communication

- The petro-chemical industry provided some of the first motivation
  (Land and Doig were supported by BP
  to work on maritime routing)

- The first code to successfully "solve" real MIPs was CEIR's LP 90/94 (late 60's)
  Branch-and-bound code by Martin Beale
  (together with Forrest, Shaw, Small and Tomlin)

First applications (from Max Shaw)

- Philips Electronics - Location of factories in Spain.
- British Petroleum - Oil refineries and transport.

First published application

- UK Military - Allocation of ships and airplanes (1968)

1970's

- Transition to more powerful MIP codes

UMPIRE ↓ SICONIC

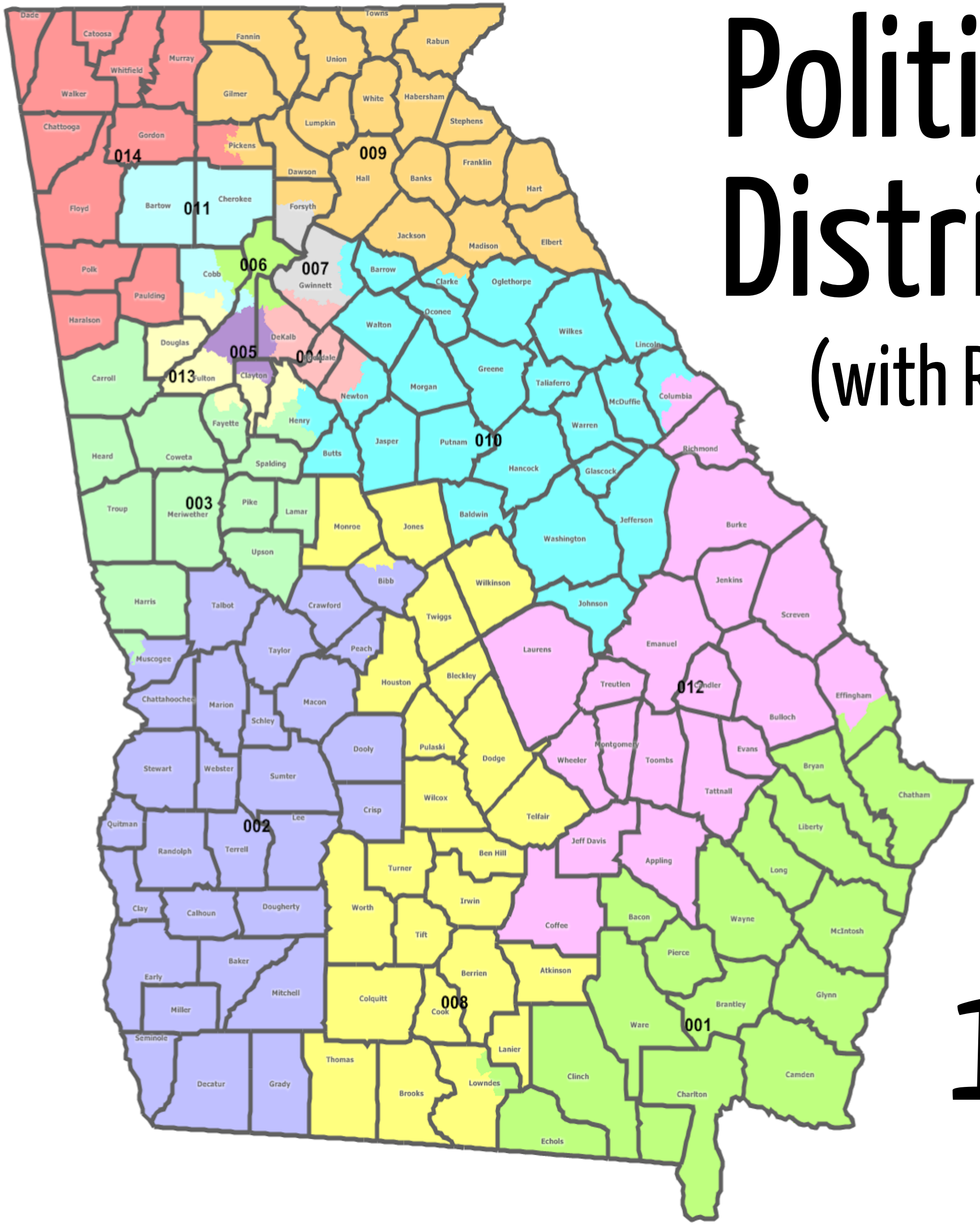
MPS X ↓ MPS X 370

- More industrial applications

My start in IP

Political Districting

(with R. Garfinkel)



Late 1960's

Political districting

Divide a state into districts

- Roughly equal population (one man, one vote)
- Contiguity, compactness, natural boundaries
- Safe districts + other political considerations

Set partitioning problem

- Generate all feasible districts and their respective "costs".

- Choose districts such that each population unit is in exactly one of them and cost is minimized.

Our algorithm

Implicit enumeration (no LP) with fathoming rules similar to Constraint Programming

With very careful assembly language programming we could solve problems with

50 populations units, 5-10 districts

Current practice

Huge problems solved with branch-and-price

One man one vote is achieved, but results can significantly favor the political party in power

Transition period

1970's - 1980's

Lots of theory developed

- Complexity theory
- Polynomial time LP algorithms
- Practical barrier method to compete with Simplex
- Specialized cuts for solving MIPs

  Knapsack cover, flow cover, clique, mixed-integer rounding

But still...

- Basic LP-based branch-and-bound
- Application of MIP very limited

An exception:
Airline crew scheduling

Hundreds of thousands of variables

Hundreds of constraints

Cooperation between airlines and academia

Modern Integer Programming

1990's to present

Ability to solve real problems

Speedups:

CPLEX 1.2 (1991)

29530x

CPLEX 11 (2007)

Gurobi 1.0 (2009)

20.5x

Gurobi 5.5 (2013)

Speedups together:

1991

2013

256000x

Speedups together:

Assuming (a modest) 1000x machine speedup

It took	It takes
>4 months (early 90's)	1 second (2007)
>7 years (early 90's)	1 second (now)

What made the incredible improvements possible?

Branch-and-bound

Cuts

Better LP

Heuristics

Preprocessing

What made the incredible improvements possible?

1. Steepest edge dual Simplex for LP

2. Preprocessing

Fixing variables, eliminating constraints, reducing coefficients

3. Cutting planes

Gomory cuts and many others to tighten bounds

4. Heuristics

Solving sub MIPs to get better solutions

5. Disjunction selection for branching (still open)

Modern Integer Programming Codes

Commercial	Non-commercial (open source)
CPLEX	MINTO
XPRESS-MP	COIN-OR
Gurobi	SCIP

Impact of modern Integer Programming

How do we measure this impact?

Franz Edelman prize finalists (6 per year)

Since 2000

53% of them used MIP or some kind of discrete optimization

Improving Performance and Flexibility at Jeppesen
Jeppesen Sanderson, Inc.

Optimized Crew Scheduling
Air New Zealand

Optimizing Customer Mail Streams
Fingerhut

Rightsizing and Management of Prototype Vehicle Testing
Ford Motor Company

Pricing Analysis
Merrill Lynch

Crew recovery
Continental Airlines

Combinatorial and Quantity-Discount Procurement Auctions
Mars - IBM

Optimizing Periodic Maintenance Operations

Schindler Elevator Corporation

Transforming Operations to Develop Operating Plans

Canadian Pacific Railway

Optimizing Air Network

UPS

Optimizing Network Routing

Menlo Worldwide Forwarding

Accelerating the Profitability of Supply Chains

Hewlett-Packard

Reinventing the Supplier Negotiation Process

Motorola

Routing Optimization

Waste Management

Improving Fractional Aircraft Ownership Operations

Bombardier Flexjet

Gaining Elastic Capacity Using a Decision-Support System

Honkong international Terminals

An OR/MS Approach to Managing an Educational Complex

Nanzan Gakuen

Expressive Competition Enabled by Optimization

Procter & Gamble

Optimizing Supply Chain for Delivering Calcium Carb. Slurry

Omya Hustadmarmor

Becoming a Travel Retailer

Travelocity

Operations Research Advances Cancer Therapeutics

Memorial Sloan-Kettering Cancer Center

The New Dutch Timetable

Netherlands Railways

Scheduling of home care to assist elderly and disabled

City of Stockholm

Optimizing Natural Gas Production and Transport

Gassco / StatoilHydro

Reduce Contamination Risks in Drinking Water

US Environmental Protection Agency

Cashing In on Optimized Equipment Distribution

CSX Railway

Transforming Product Portfolio Management

HP

Improving Global Profitability

Norske Skog

New Operating and Settlement System

INDEVAL

Inventory Optimization

Procter & Gamble

Achieving Transportation Asset Management Excellence

New Brunswick Department of Transportation

A Strategic Empty Container Logistics Optimization

CSAV

Branch Reconfiguration Practice

Industrial and Commercial Bank of China Limited

Retail Price Optimization

IHG

Application of OR for Energy and Ancillary Services Markets

Midwest Independent Transmission System Operator

Supply Chain–Wide Optimization

TNT Express

Advancing Public Health and Medical Preparedness

US Centers for Disease Control and Prevention

Reengineering Global Distribution Process

Zara

Some impact areas

Supply chain

Energy

Finance

Health

Transportation

Natural resources

Sports

Transportation

Airline optimization

Pioneers in really using optimization in practice

A huge impact on development of integer programming methodology

Airline optimization

- Network planning
- Fleet assignment
- Crew planning and rostering
- Gate assignment
- Robust planning and operations recovery

Crew scheduling

How to partition a set of flights by crews

Set partitioning problem

with constraints for every flight and 0-1 variables for every subset of flights that a specific crew could fly over a duty of 4-5 days.

Crew scheduling

1960's: Airlines explore IP to solve the problem

1980's: Sub MIPs were solved, precursor of the primal methods used today

1980's: Special branching rules (Led to branching needed to solve IPs with exponential number of variables by Branch-and-Price)

1990's: Column generation to try to deal with billions of variables

Crew scheduling

Today:

LPs with 10^{12} - 10^{14} variables solved by column generation to produce IPs with about 20,000 - 30,000 variables and 1,000 constraints

Fleet assignment

What capacity should be assigned to each flight to maximize revenue

Fleet assignment

- MIP introduced as tool in early 1990's (only basic models could be solved)
- Weekly models can be solved with 5,000 - 6,000 daily flights and 12-15 subfleets
- Models include crew, maintenance, airport operations and flight retiming constraints
- Challenges in moving from leg to itinerary based fleet assignment

Robust scheduling:

Integrated planning and operations recovery

- Newer areas that point to the demand to deal with uncertainty and producing online solutions
- Much academic work over the last decade
- Implementation has just begun and proceeds slowly (limited data, unclear objectives and huge models)
- Network planning (markets, frequency, code sharing)

Supply chain

Maritime inventory routing

Combines inventory and supply at demand ports with routing of vessels that move the inventory

Maritime transportation

Sectors served

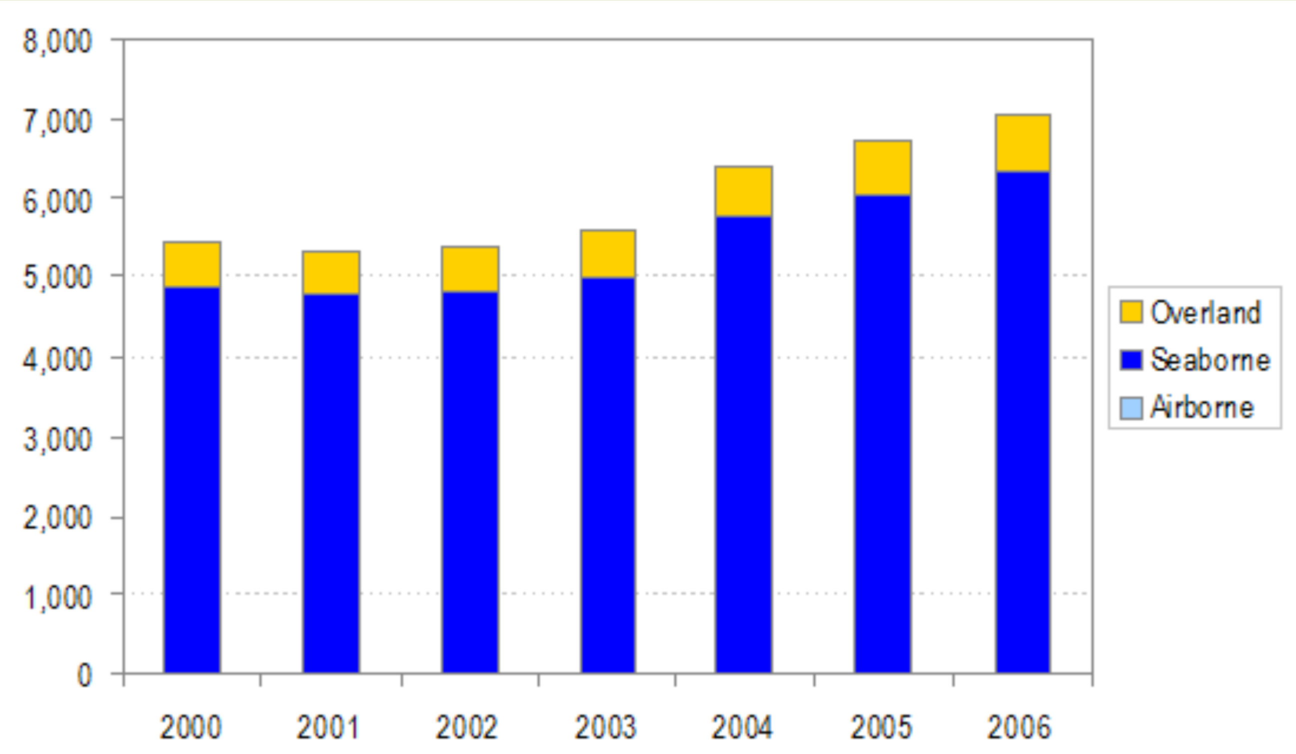
- Oil & gas
- Agriculture
- Many others

Ships travel the globe

- Tankers
- Bulk carriers
- Container ships
- Liquefied gas carriers
- Roll-on roll-off ships for cars

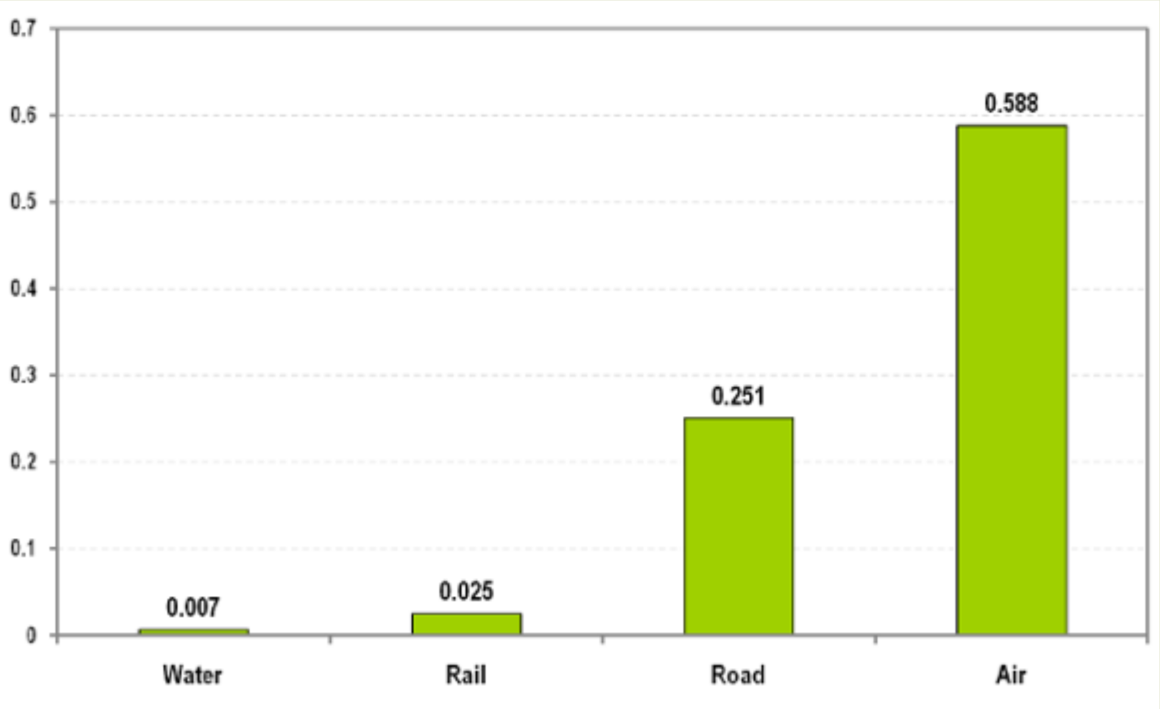
Modal split

by million metric tons



Modal transport cost

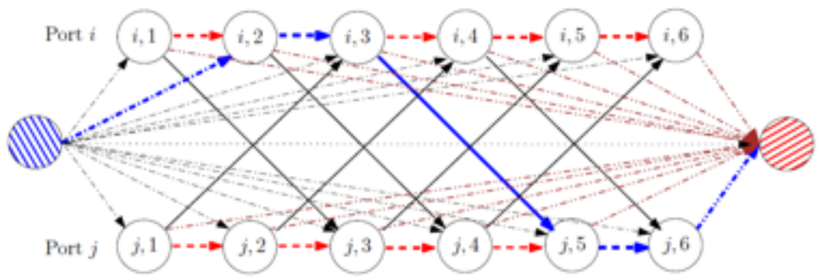
per ton-mile



Maritime transportation

leverages the power of MIP

- MIP used for strategic and tactical planning
- Split pickup & split delivery inventory routing models
- Large-scale instances involve routing many ships over a long planning horizon



2M variables

500k constraints

MIP Model

Current practice for a mega oil company

Strategic planning problem

Time horizon: 365 periods (days)
Ports: 5
Ships: 15

Time to 5% gap (vs. optimal): 1 - 5 hours
Time to 1% gap (relative to bound): days

Need to solve bigger problems, reduce gap and improve times (enormous costs. e.g., demurrage)

Energy

The unit commitment problem

Solving the economic dispatch of power more efficiently and reliably

World gross production (2009)

20,000 Terawatt hr

at 100 megawatt hr: Cost $2,000 billion/year

1% savings = $20 billion per year

US System

- Operated by about 10 regional organizations
- Coordinate, control and monitor electricity transmission by nodal pricing
- Electricity cannot be stored

US System

Auctions

- Real-time for efficient dispatch
- Day-ahead for efficient unit scheduling

Electric network optimization

10^6 nodes

10^6 transmission constraints

10^5 binary variables

MIP paradigm change

In the last 15 years optimization switched from Lagrangian to MIP

Pre-1999

- MIP too slow
- LR - simpler models, no commercial solvers

Now MIP is prefered

- Ease of development and maintenance
- Exact models of complex functionality
- Ability to specify solution accuracy
- Continuous improvements from multiple vendors

MIP paradigm change

2011 - MIP creates savings > 500 million annually in the US

2015 - Savings predicted to be > 1 billion annually and 10-30% more savings possible

Sports scheduling

Photo: Don Davis

Is this really an impact area?

Sports: $300 billion annually

=

2x automobile industry

=

7x movie industry

TV scheduling constraints are key to revenue

Major league baseball in the US

Sports Scheduling Group

GN, Mike Trick and Kelly Easton

Why is baseball scheduling hard?

Huge size

- 30 teams, 2 leagues of 15 each, 3 divisions of 5 each per league
- Each play 162 games over 180 days
- Many hard and soft constraints

Why is baseball scheduling hard? Huge size

- Balanced home and away games
- Travel is limited
- Stadium constraints
- Time between 2 teams playing each other
- Can't be away for several consecutive weekends
- Television

Sports scheduling is mainly a

feasibility problem

overconstrained and with many soft constraints put into the objective with weights adjusted over several iterations.

The full problem is too big

100,000 binary variables

200,000 constraints

broken into subproblems with

15,000 binary variables

20,000 constraints

pieced together heuristically

Process takes 3 - 6 months

Is there any hope of optimizing with one MIP?

Health

Integer programming in the operating room

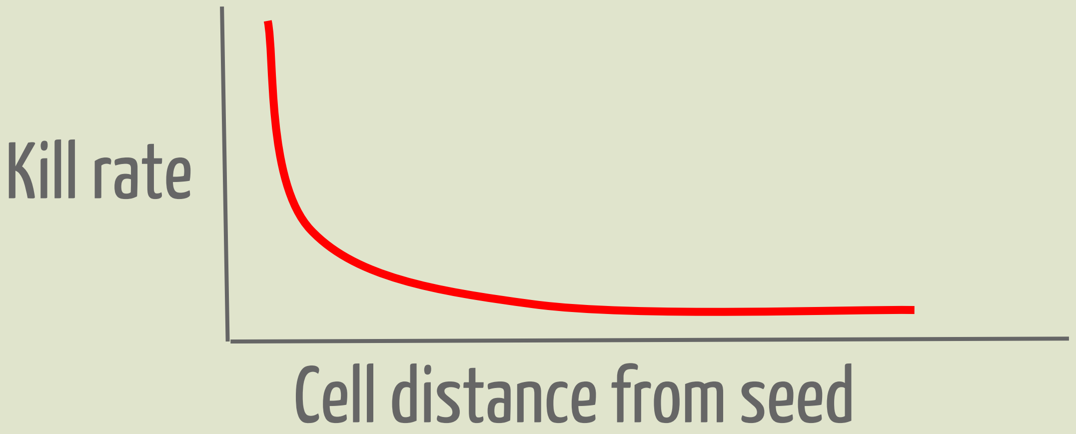
Treatment of prostate cancer with brachytherapy (placement of radioactive seeds inside a tumor)

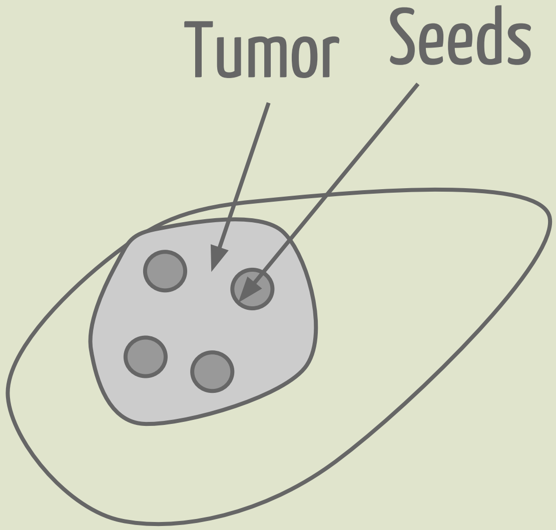
Over 500,000 new cases per year

Over 30% mortality rate

3-D location problem

More cells are killed as seed gets closer





Objective: maximize dose to tumor, avoiding (insofar as possible) exposure to normal tissue

Technical challenge:

Very dense constraint matrices

Results

- Safer and more reliable treatments (45-60%) reduction in complications
- 20-30% reduction in number of seeds
- 15% reduction in needles
- Average treatment cost in the US reduced by $5,000
- Similar application to radiation beam treatment of other types of cancer

Finance

Photo: http://www.flickr.com/photos/68751915@N05/

Managing funds

- $800 billion to $1 trillion of active funds managed with optimization using estimate of expected returns
- Probably another $8 trillion use optimization for passive management

Add assets to a portfolio to improve its risk characteristics

Basic models

Min risk,
constraint
on return

or

Max return,
constraint
on risk

Inputs: Investment universe, expected returns, covariance of returns, budget, max acceptable risk or min acceptable return

Output: Portfolio weights

Need quadratic constraints or objective

Binary variables are needed for

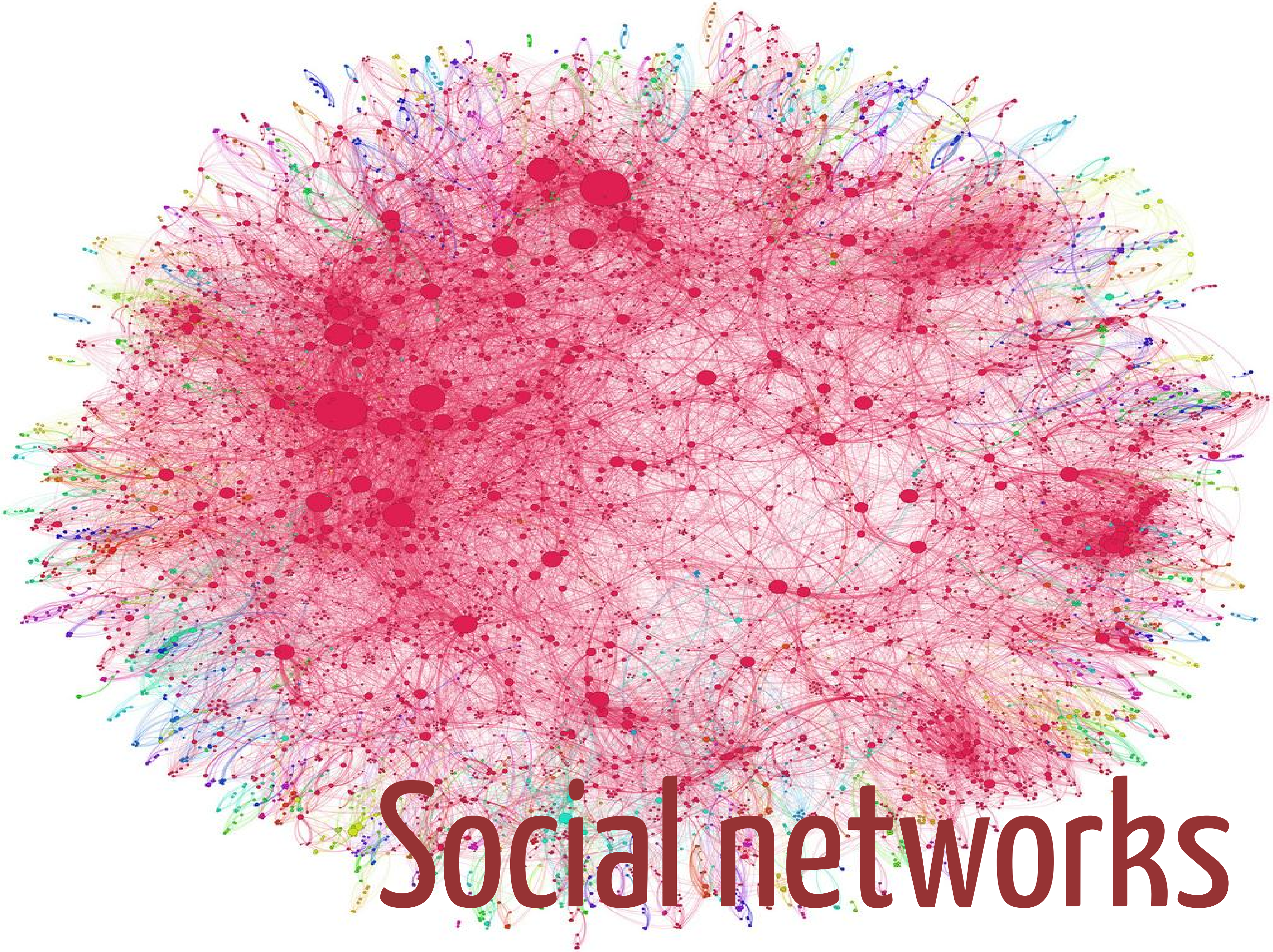
- Fixed transaction costs
- Threshold holdings ($x_i = 0$ or $x_i \geq c$)
- If-then constraints
- Tax considerations

Typical model size:

up to **10,000 assets**

- 1 to 4 0-1 variables per asset
- 1 or 2 continuous variables per asset
- Many constraints with various types of exposure limits

Optimizing over multiperiods and multiple portfolios still too large to do with MIP



Social networks

Social networks

Optimization on graphs

- Terrorist networks
- Intelligent data
- Privacy
- Data mining

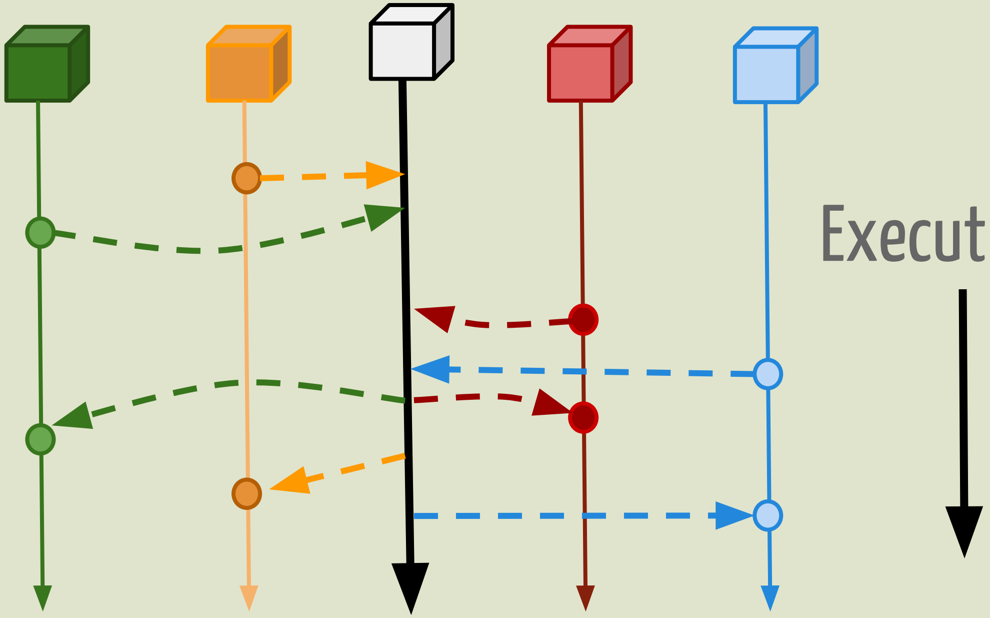
The future

Almost in every area discussed there is a need for:

- Dealing with uncertainty
- Bigger models (multiple periods, combined systems)
- Solving much faster (real-time applications)

Technology innovations

- Parallel
- Learning
- What else?



Thanks to:

Max Shaw (retired) and John Tomlin (Yahoo) - Early computational systems and applications

Bob Bixby and Zhongao Gu (Gurobi) - CPLEX and Gurobi

Sergey Shebalov (Sabre) - Airline optimization

Kelly Easton (Sports Scheduling Group) - Major league baseball scheduling

Dimitri Papageorgiou (ExxonMobil) - Maritime inventory routing

Dick O'Neill (US Department of Energy) - Energy optimization

Pam Vance (Axioma) - Finance

Rodolfo Carvajal (Georgia Tech) - General help