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2015-12-23

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# How classical are TeV-scale black holes?

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We show that the Hawking temperature and the entropy of black holes are subject to corrections from two sources: the generalized uncertainty principle and thermal fluctuations. Both effects increase the temperature and decrease the entropy, resulting in faster decay and “less classical” black holes. We discuss the implications of these results for TeV-scale black holes that are expected to be produced at future colliders.

## I. INTRODUCTION

The possibility of the existence of large extra dimensions has recently opened up new and exciting avenues of research in quantum gravity [1, 2]. In particular, a host of interesting work is being done on different aspects of low-energy scale *quantum gravity phenomenology*. One of the most significant sub-fields is the study of black hole (BH) [3] and brane [4] production at particle colliders, such as the (Very) Large Hadron Collider [V(LHC)] [5] and the muon collider [6], as well as in ultrahigh energy cosmic ray (UHECR) airshowers [7]. (For recent reviews, see Refs. [8].) Several scenarios predict the fundamental Planck scale to be of order of the TeV. The simplest model postulates a number  $n$  of toroidally compactified extra dimensions with length ranging from a few microns ( $n = 2$ ) to a Fermi ( $n = 7$ ) [1]. Extra dimensions of infinite size and non-trivial “warp-factor” may also lead to similar predictions [2]. In either case, particle collisions with center-of-mass (c.m.) energy above the fundamental Planck scale, and impact parameter smaller than the horizon radius corresponding to that energy, should produce BHs and branes [9]. (For criticisms, however, see Refs. [10].) Since the c.m. energy of next-generation particle colliders and UHECR primaries is as high as tens or hundreds of TeV, BH and brane production is likely to be observed. For this kind of event, the initial mass of the gravitational object is expected to be of the order of a few Planck masses.

Newly formed BHs first lose hair associated with multipole and angular momenta, then approach classically stable Schwarzschild solutions, and finally evaporate via Hawking radiation [11]. Decay time and entropy completely determine the observables of the process. BH formation and decay can be described semiclassically, provided that the entropy is sufficiently large. The timescale for the complete decay of a BH to up to its supposed final Planck-sized remnant is expected to be of order of the  $\text{TeV}^{-1}$ .

BH thermodynamic quantities depend on the Hawking temperature  $T_H$  via the usual thermodynamic relations (Stefan-Boltzmann law). The Hawking temperature undergoes corrections from many sources, and these corrections are particularly relevant for BHs with mass of the order of the Planck mass. Therefore, the study of TeV-scale BHs in UHECR and particle colliders requires a careful investigation of how temperature corrections affect BH thermodynamics. In this article, we concentrate on the corrections due to the generalized uncertainty principle (GUP) and thermal fluctuations of thermodynamic systems. These corrections are not tied down to any specific model of quantum gravity; the GUP can be derived using arguments from string theory [12] as well as other approaches to quantum gravity [13, 14]. Similarly, corrections from thermal effects do not depend on the underlying quantum gravity theory since thermal fluctuations are present in any system. This generality provides in fact a strong motivation in studying GUP and thermal fluctuation effects.

We show below that the BH decay rate is increased by GUP and thermal fluctuation corrections, resulting in shorter decay times. Thus the BHs may not behave like well-defined resonances. We also show that a diminished entropy leads to smaller particle emission during the evaporation phase and “less classical” BHs. The paper is organised as follows. In the next section, we review the connection between the uncertainty principle and Hawking radiation

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[15, 16]. In section III, we derive corrections to the Hawking decay rate, entropy, and multiplicity due to the GUP. We show that for BHs with mass  $M = 5 - 10M_{Pl}$ , the decay time and the entropy (or multiplicity) dramatically decrease if the GUP parameter is nonvanishing. In section IV we review the corrections to thermodynamic quantities due to thermal fluctuations. We apply these results to BHs in section V, where we show that BH decay time and entropy also decrease compared to their semiclassical value. We conclude with a summary of our results and a brief discussion of open questions in section VI.

## II. UNCERTAINTY PRINCIPLE AND HAWKING RADIATION

In this section we review and generalize to  $d$  dimensions the derivation of the Hawking radiation of Adler *et al.* [15]. A  $d$ -dimensional spherically symmetric BH of mass  $M$  (to which the collider BHs will settle into before radiating) is described by the metric

$$ds^2 = - \left( 1 - \frac{16\pi G_d M}{(d-2)\Omega_{d-2} c^2 r^{d-3}} \right) c^2 dt^2 + \left( 1 - \frac{16\pi G_d M}{(d-2)\Omega_{d-2} c^2 r^{d-3}} \right)^{-1} dr^2 + r^2 d\Omega_{d-2}^2, \quad (1)$$

where  $\Omega_{d-2}$  is the metric of the unit  $S^{d-2}$  and  $G_d$  is the  $d$ -dimensional Newton's constant. Since the Hawking radiation is a quantum process, the emitted quanta must satisfy the Heisenberg uncertainty principle

$$\Delta x_i \Delta p_j \gtrsim \hbar \delta_{ij}, \quad (2)$$

where  $x_i$  and  $p_j$ ,  $i, j = 1 \dots d-1$ , are the spatial coordinates and momenta, respectively. By modelling a BH as a  $(d-1)$ -dimensional cube of size equal to twice its Schwarzschild radius  $r_s$ , the uncertainty in the position of a Hawking particle at the emission is

$$\Delta x \approx 2r_s = 2\lambda_d \left[ \frac{G_d M}{c^2} \right]^{1/(d-3)}, \quad (3)$$

where  $\lambda_d = [16\pi/((d-2)\Omega_{d-2})]^{1/(d-3)}$ . Using Eq. (2), the uncertainty in the energy of the emitted particle is

$$\Delta E \approx c\Delta p \approx \frac{M_{Pl} c^2}{2\lambda_d} m^{-1/(d-3)}, \quad (4)$$

where  $m = M/M_{Pl}$  is the mass in Planck units and  $M_{Pl} = [\hbar^{d-3}/c^{d-5}G_d]^{1/(d-2)}$  is the  $d$ -dimensional Planck mass.  $\Delta E$  can be identified with the characteristic temperature of the BH emission, i.e. the Hawking temperature. Setting the constant of proportionality to  $(d-3)/2\pi$  we get

$$T_H = \frac{d-3}{4\pi\lambda_d} M_{Pl} c^2 m^{-1/(d-3)}. \quad (5)$$

The energy radiated per unit time is governed by the Stefan-Boltzmann law. The surface gravity is constant over the horizon. Thus the Hawking temperature of the higher-dimensional BH and the temperature of the induced BH on the brane are identical. The BH temperature  $T_H$  can be used in the calculation of the emission rate. Neglecting thermal emission in the bulk, and assuming that the brane has  $\mathcal{D}$  spacetime dimensions (we will substitute  $\mathcal{D} = 4$  at the end, since BHs are supposed to radiate mainly on the brane [11]), the emission rate for a massless scalar particle on the brane is

$$\frac{dM}{dt} = -c\sigma_{\mathcal{D}} A_{\mathcal{D}} T^{\mathcal{D}}, \quad (6)$$

where  $A_{\mathcal{D}} = \Omega_{\mathcal{D}-2} r_c^{\mathcal{D}-2}$  is the horizon area of the induced BH with radius  $r_c = [(d-1)/2]^{1/(d-3)} [(d-1)/(d-3)]^{1/2} r_s$ , and

$$\sigma_{\mathcal{D}} = \frac{\Omega_{\mathcal{D}-3} \Gamma(\mathcal{D}) \zeta(\mathcal{D})}{(\mathcal{D}-2)(2\pi\hbar c)^{\mathcal{D}-1}} \equiv \frac{\bar{\sigma}_{\mathcal{D}}}{(\hbar c)^{\mathcal{D}-1}} \quad (7)$$

is the Stefan-Boltzmann constant in  $\mathcal{D}$ -spacetime dimensions. If the BH evaporates into different particle species on the brane, the Stefan-Boltzmann constant has to be multiplied by the factor

$$\sum_i c_i(\mathcal{D}) \Gamma_{s_i}(\mathcal{D}) f_i(\mathcal{D}), \quad (8)$$

where the sum is over all particle flavors,  $c_i$  are the degrees of freedom of the species  $i$ ,  $\Gamma_{s_i}$  are the greybody factors for spin  $s_i$  and  $f_i = 1$  ( $f_i = 1 - 2^{1-D}$ ) for bosons (fermions). (We neglect the energy dependence of the greybody factors. See, e.g., Refs. [11].) Expressing Eq. (6) in terms of  $m$ , we obtain

$$\frac{dm}{dt} = -\frac{\mu}{t_{Pl}} m^{-2/(d-3)}, \quad (9)$$

where  $t_{Pl} = (\hbar G_d/c^{d+1})^{1/(d-2)}$  is the Planck time, and

$$\mu = \left(\frac{r_c}{r_s}\right)^{D-2} \left(\frac{d-3}{4\pi}\right)^D \frac{\bar{\sigma}_D \Omega_{D-2}}{\lambda_d^2}. \quad (10)$$

Integration over  $t$  yields the decay time

$$\tau_0 = \mu^{-1} \left(\frac{d-3}{d-1}\right) m_i^{(d-1)/(d-3)} t_{Pl}, \quad (11)$$

where  $m_i \equiv M_i/M_{Pl}$ , and  $M_i$  is the initial BH mass. The decay time  $\tau_0$  is finite. Equation (9) implies that the end stage of Hawking evaporation is catastrophic, with infinite radiation rate and infinite temperature. However, a heuristic argument suggests that the final temperature and radiation rate are finite. At the last stage of evaporation,  $\Delta E$  in Eq. (4) must be of the order of the BH mass, and  $\Delta E = \Delta M c^2 \approx M_{end} c^2$ . This implies a minimum BH mass  $M_{end} \approx M_{Pl}$ , a maximum Hawking temperature  $T_{max} = \mathcal{O}(M_{Pl})$ , and a smaller decay time.

The thermodynamic properties of the BH can be computed via the usual thermodynamic relations. The entropy and the BH specific heat are

$$S_0 = \frac{4\pi\lambda_d}{d-2} m^{(d-2)/(d-3)} = \frac{d-3}{d-2} \frac{M c^2}{T_H}, \quad (12)$$

and

$$\mathcal{C}_0 = -4\pi\lambda_d m^{(d-2)/(d-3)}, \quad (13)$$

respectively. The statistical total number of quanta emitted during the evaporation is proportional to the initial entropy of the BH. The exact relation is

$$N = S_0 \frac{\zeta(D-1)}{(D-1)\zeta(D)} \frac{\sum_i c_i(D) \Gamma_{s_i}(D) f_i(D-1)}{\sum_j c_j(D) \Gamma_{s_j}(D) f_j(D)}. \quad (14)$$

The flavor multiplicity is

$$N_i = N \frac{c_i(D) \Gamma_{s_i}(D) f_i(D-1)}{\sum_j c_j(D) \Gamma_{s_j}(D) f_j(D-1)}. \quad (15)$$

Equation (15) gives the statistical number of particles per species produced during the evaporation process.

### III. CORRECTIONS TO BH THERMODYNAMICS FROM THE GENERALIZED UNCERTAINTY PRINCIPLE

We now determine the corrections to the above results due to the GUP. The general form of the GUP is

$$\Delta x_i \geq \frac{\hbar}{\Delta p_i} + \alpha^2 \ell_{Pl}^2 \frac{\Delta p_i}{\hbar}, \quad (16)$$

where  $\ell_{Pl} = (\hbar G_d/c^3)^{1/(d-2)}$  is the Planck length and  $\alpha$  is a dimensionless constant of order one. There are many derivations of the GUP, some heuristic and some more rigorous. Equation (16) can be derived in the context of string theory [12], non-commutative quantum mechanics [13], and from minimum length [17] considerations [14]. The exact value of  $\alpha$  depends on the specific model. The second term in r.h.s. of Eq. (16) becomes effective when momentum

and length scales are of the order of Planck mass and of the Planck length, respectively. This limit is usually called “quantum regime”. Inverting Eq. (16), we obtain

$$\frac{\Delta x_i}{2\alpha^2 \ell_{Pl}^2} \left[ 1 - \sqrt{1 - \frac{4\alpha^2 \ell_{Pl}^2}{\Delta x_i^2}} \right] \leq \frac{\Delta p_i}{\hbar} \leq \frac{\Delta x_i}{2\alpha^2 \ell_{Pl}^2} \left[ 1 + \sqrt{1 - \frac{4\alpha^2 \ell_{Pl}^2}{\Delta x_i^2}} \right]. \quad (17)$$

The left-inequality gives the correct  $\ell_{Pl}/\Delta x_i \rightarrow 0$  limit and will be considered henceforth. The GUP implies the existence of a minimum length  $L_{min} \approx \Delta x = 2\alpha \ell_{Pl}$ . The string regime and the classical regimes are recovered by setting  $\Delta x_i \approx 2\alpha \ell_{Pl}$  and  $\Delta x_i \gg \ell_{Pl}$  in Eq. (16), respectively.

BHs with horizon radius smaller than  $L_{min}$  do not exist. Therefore, the minimum length implies the existence of a minimum BH mass

$$M_{min} = \frac{d-2}{8\Gamma\left(\frac{d-1}{2}\right)} (\alpha\sqrt{\pi})^{d-3} M_{Pl}. \quad (18)$$

The minimum BH mass is a rapidly increasing function of the unknown parameter  $\alpha$  for  $d \geq 6$ ; a value of  $\alpha$  larger than unity may lead to a minimum BH mass  $M_{min} \gg M_{Pl}$ .

The corrections to the BH thermodynamic quantities can be calculated by repeating the argument of the previous section. Setting  $\Delta x = 2r_s$  the GUP-corrected Hawking temperature is

$$T'_H = \frac{(d-3)\lambda_d}{2\pi\alpha^2} m^{1/(d-3)} \left[ 1 - \sqrt{1 - \frac{\alpha^2}{\lambda_d^2 m^{2/(d-3)}}} \right] M_{Pl} c^2. \quad (19)$$

Equation (19) may be Taylor expanded around  $\alpha = 0$ :

$$T'_H = \frac{(d-3)}{4\pi\lambda_d} m^{-1/(d-3)} \left[ 1 + \frac{\alpha^2}{4\lambda_d^2 m^{2/(d-3)}} + \dots \right] M_{Pl} c^2. \quad (20)$$

The GUP-corrected Hawking temperature is higher than the semiclassical Hawking temperature  $T_H$  of Eq. (5). The first-order correction is

$$\Delta T_{GUP} \equiv T'_H - T_H = \frac{d-3}{16\pi} \frac{\alpha^2}{\lambda_d^3 m^{3/(d-3)}} M_{Pl} c^2. \quad (21)$$

From the first law of BH thermodynamics the first-order correction to the BH entropy is

$$\begin{aligned} \Delta S_{GUP} &= -\frac{\pi\alpha^2 m^{(d-4)/(d-3)}}{(d-4)\lambda_d} \quad d > 4, \\ &= -\frac{\pi\alpha^2}{2} \ln(m) \quad d = 4. \end{aligned} \quad (22)$$

This follows from the exact expression:

$$S_{GUP} = 2\pi\lambda_d \left( \frac{\alpha}{\lambda_d} \right)^{d-2} I(1, d-4, \lambda_d m^{1/(d-3)}/\alpha), \quad (23)$$

where

$$I(p, q, x) = \int_1^x dz z^q \left( z + \sqrt{z^2 - 1} \right)^p. \quad (24)$$

From Eq. (22) and Eq. (23) it follows that the GUP-corrected entropy is smaller than the semiclassical Bekenstein-Hawking. The GUP-corrected Stefan-Boltzmann law is

$$\frac{dm}{dt} = -2^{\mathcal{D}} \frac{\mu}{t_{Pl}} m^{-2/(d-3)} \left[ 1 + \sqrt{1 - \frac{\alpha^2}{\lambda_d^2 m^{2/(d-3)}}} \right]^{-\mathcal{D}}. \quad (25)$$

Taylor expanding Eq. (25) we have

$$\frac{dm}{dt} = -\frac{\mu}{t_{Pl} m^{2/(d-3)}} \left[ 1 + \frac{\alpha^2 \mathcal{D}}{4\lambda_d^2 m^{2/(d-3)}} + \dots \right]. \quad (26)$$

The relative GUP first-order correction to the Stefan-Boltzmann law is positive:

$$\Delta \left( \frac{dm}{dt} \right) / \left( \frac{dm}{dt} \right)_0 = \frac{\alpha^2 \mathcal{D}}{4\lambda_d^2} m^{-2/(d-3)}, \quad (27)$$

where  $(dm/dt)_0$  is defined in Eq. (9). The Hawking evaporation ends at  $m_{min} = M_{min}/M_{Pl} = (\alpha/\lambda_d)^{d-3}$ , where the emission rate becomes imaginary. The emission rate is finite at the end:

$$\left( \frac{dm}{dt} \right)_{m_{min}} = -2^{\mathcal{D}} \frac{\mu}{t_{Pl}} \left( \frac{\lambda_d}{\alpha} \right)^2. \quad (28)$$

This means that the end-point of Hawking radiation is not catastrophic. Since the final emission rate is finite, it might be argued that once the final stage has been reached, the BH evaporates completely by emitting a hard Planck-mass quantum in a finite time  $\mathcal{O}(t_{Pl})$ . However, the BH specific heat

$$C \equiv T \frac{\partial S}{\partial T} = -2\pi\lambda_d m^{(d-2)/(d-3)} \sqrt{1 - \frac{\alpha^2}{\lambda_d^2 m^{2/(d-3)}}} \cdot \left( 1 + \sqrt{1 - \frac{\alpha^2}{\lambda_d^2 m^{2/(d-3)}}} \right) \quad (29)$$

vanishes at the endpoint. Therefore, the BH cannot exchange heat with the surrounding space. The endpoint of Hawking evaporation in the GUP scenario is characterized by a Planck-size remnant with maximum temperature

$$T_{max} = 2T_0 \Big|_{M=M_{min}}. \quad (30)$$

The GUP prevents BHs from evaporating completely, just like the standard uncertainty principle prevents the hydrogen atom from collapsing. The existence of BH remnants as a consequence of the GUP was pointed in Refs. [15] in the context of primordial BHs in cosmology [18]. BH remnants have also been predicted in string and quantum gravity models [19] and could play an important role in cosmology. (See, e.g., Refs. [20].)

The GUP implies a faster BH decay. The first-order decay time is

$$\tau_1 = \mu^{-1} \left( \frac{d-3}{d-1} \right) \left\{ \left[ m_i^{(d-1)/(d-3)} - \frac{\mathcal{D}(d-1)\alpha^2}{4(d-3)\lambda_d^2} m_i \right] - \left[ 1 - \frac{\mathcal{D}(d-1)}{4(d-3)} \right] \left( \frac{\alpha}{\lambda_d} \right)^{d-1} \right\} t_{Pl}. \quad (31)$$

If the initial mass far exceeds the Planck mass, i.e.  $m_i \gg 1$ , the last term inside the curly brackets can be ignored. Using Eq. (11), we find

$$\frac{\Delta\tau_1}{\tau_0} \equiv \frac{\tau_1 - \tau_0}{\tau_0} = -\frac{\mathcal{D}(d-1)\alpha^2}{4(d-3)\lambda_d^2} m_i^{-2/(d-3)}. \quad (32)$$

The GUP-corrected decay time is smaller than the semiclassical decay time. The GUP-corrected multiplicity is obtained from Eq. (14) with  $S_0 = S_{GUP}$ . Table I shows the GUP-corrected parameters for two typical BHs produced at particle colliders or in UHECRs, with initial mass equal to  $5M_{Pl}$  and  $10M_{Pl}$ . The first row gives the standard Hawking parameters. The GUP effects on the thermodynamic parameters increase as the minimum BH mass becomes larger. It is interesting to note that decay time, entropy, and multiplicity are drastically reduced when  $\alpha$  approaches unity. In the limiting case of a BH with initial mass  $5M_{Pl}$  and GUP parameter  $\alpha = 1$ , the minimum BH mass coincides essentially with the initial BH mass, and the BH does not evaporate. In contrast to the standard theory, the GUP-corrected entropy shows that a BH with a mass five times the fundamental Planck scale is not a classical object; quantum effects become manifest at an earlier stage of the BH evaporation phase than was predicted by the semiclassical Hawking analysis [8]. Therefore, GUP corrections have important consequences on the BH phenomenology in particle colliders and in UHECR airshowers.

#### IV. ENTROPY CORRECTIONS DUE TO THERMODYNAMIC FLUCTUATIONS

Thermodynamic systems (including BHs) undergo small thermal fluctuations from equilibrium which affect thermodynamic quantities. In this section we calculate the corrections to entropy and Hawking temperature. We have seen that the GUP corrections affect the Hawking temperature of the BH while the BH energy remains constant. Therefore, we consider fluctuations around the equilibrium temperature instead of the equilibrium energy. This leads

TABLE I: GUP-corrected thermodynamic quantities for two ten-dimensional BHs with mass  $m = 5$  and 10 (in fundamental units). The values in brackets give the percentage deviation from standard Hawking quantities.

$m = 5$						
	Minimum mass	Initial Temperature	Final Temperature	Decay time	Entropy	Multiplicity
$\alpha = 0$	–	.553	$\infty$	.334	7.92	3
$\alpha = 0.5$	.037	.591 (+7%)	2.23	.233 (-30%)	7.18 (-9%)	2 (-33%)
$\alpha = 1.0$	4.73	.981 (+77%)	1.11	.002 (-99%)	.269 (-97%)	0 (-100%)

  

$m = 10$						
	Minimum mass	Initial Temperature	Final Temperature	Decay time	Entropy	Multiplicity
$\alpha = 0$	–	.500	$\infty$	.814	17.5	6
$\alpha = 0.5$	.037	.529 (+6%)	2.23	.610 (-25%)	16.2 (-7%)	5 (-17%)
$\alpha = 1.0$	4.73	.696 (+39%)	1.11	.100 (-88%)	6.66 (-62%)	2 (-66%)

to a decrease in the black hole entropy. Note that the Bekenstein-Hawking entropy is identified with the canonical entropy of the system<sup>1</sup>. Let us consider a canonical ensemble with partition function [22, 23]:

$$Z(\beta) = \int_0^\infty \rho(E) e^{-\beta E} dE, \quad (33)$$

where  $\beta = 1/T$  is the inverse of the temperature. The density of states can be obtained from Eq. (33) by the inverse Laplace transform (at fixed  $E$ )

$$\rho(E) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} Z(\beta) e^{\beta E} d\beta = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} e^{S(\beta)} d\beta, \quad (34)$$

where

$$S(\beta) = \ln Z(\beta) + \beta E. \quad (35)$$

To evaluate the complex integral in Eq. (34) by the method of steepest descent, we expand  $S(\beta)$  around the saddle point  $\beta_0 (= 1/T_0)$ , where  $T_0$  is the equilibrium temperature. Also using the fact that  $S'_0 = (\partial S(\beta)/\partial \beta)_{\beta=\beta_0}$ , we get [22, 24]:

$$S = S_0 + \frac{1}{2} (\beta - \beta_0)^2 S''_0 + \dots, \quad (36)$$

where  $S_0 = S(\beta_0)$  and  $S''_0 = (\partial^2 S(\beta)/\partial \beta^2)_{\beta=\beta_0}$ . Substituting Eq. (36) in Eq. (34), the density of states is

$$\begin{aligned} \rho(E) &= \frac{e^{S_0}}{2\pi i} \int_{c-i\infty}^{c+i\infty} e^{(\beta-\beta_0)^2 S''_0/2} d\beta \\ &= \frac{e^{S_0}}{\sqrt{2\pi S''_0}}. \end{aligned} \quad (37)$$

The corrected entropy is given by the logarithm of the density of states  $\rho(E)$ :

$$S = \ln \rho(E) = S_0 - \frac{1}{2} \ln S''_0 + (\text{higher order terms}). \quad (38)$$

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<sup>1</sup> If fluctuations around the equilibrium energy were considered, the Bekenstein-Hawking entropy could be identified with the microcanonical entropy [21].

From  $E = -(\partial \ln Z(\beta)/\partial \beta)_{\beta_0}$  and the definition of specific heat,  $C = (\partial E/\partial T)_{\beta_0}$ ,  $S''(\beta)$  can be written as

$$\begin{aligned} S''(\beta) &= \frac{1}{Z} \left( \frac{\partial^2 Z(\beta)}{\partial \beta^2} \right) - \frac{1}{Z^2} \left( \frac{\partial Z}{\partial \beta} \right)^2 \\ &= \langle E^2 \rangle - \langle E \rangle^2 \\ &= C T^2. \end{aligned} \tag{39}$$

Equation (39) shows that a non-vanishing  $S''$  is a consequence of thermal fluctuations. Substituting Eq. (39) in Eq. (38), we obtain

$$S = \ln \rho = S_0 - \frac{1}{2} \ln (C T^2) + \dots \tag{40}$$

The above formula applies to any thermodynamic system in equilibrium. In particular, when applied to BHs,  $T$  is the Hawking temperature. For example, for a non-rotating three-dimensional BTZ BH, both  $T_H$  and  $C$  are proportional to the BH entropy  $S_0$ . In this case, Eq. (40) reads

$$S = \ln \rho = S_0 - \frac{3}{2} \ln S_0 + \dots \tag{41}$$

Similarly, for an AdS-Schwarzschild BH in  $d$ -dimensions, it can be shown that (see Ref. [22])

$$S = S_0 - \frac{d}{2(d-2)} \ln S_0 + \dots \tag{42}$$

Although Eq. (40) is not applicable directly to the Schwarzschild BHs because of its negative specific heat, the entropy corrections can be shown to be logarithmic by either assuming a small cosmological constant, or by putting the BH into a finite box. The result is:

$$S_{Thermo} = S_0 - k \ln S_0, \tag{43}$$

where  $k$  is a positive constant of order unity. We will apply these results to brane world BHs in the next section.

## V. CORRECTIONS TO BH DECAY RATE DUE TO THERMODYNAMIC FLUCTUATIONS

The corrected Hawking temperature is obtained from the first law of BH thermodynamics. The result is:

$$T_H'' = \frac{(d-3)}{4\pi\lambda_d} m^{-1/(d-3)} \left[ 1 + \frac{k(d-2)}{4\pi\lambda_d} m^{-(d-2)/(d-3)} + \dots \right] M_{Pl} c^2. \tag{44}$$

Equation (44) gives the first-order correction to the Hawking temperature

$$\Delta T_{Thermo} \equiv T_H'' - T_H = \frac{k(d-2)(d-3)}{16\pi^2\lambda_d^2} m^{-(d-1)/(d-3)} M_{Pl} c^2. \tag{45}$$

The correction to the BH entropy follows from Eq. (43):

$$\Delta S_{Thermo} = -k \ln S_0. \tag{46}$$

The multiplicity is also reduced, and is now given by Eq. (14) with  $S_0 \rightarrow S_{Thermo}$ . Similarly to the GUP corrections, thermodynamic fluctuations reduce the number of degrees of freedom of the BH. Taking the ratio of Eq. (21) and Eq. (45), we find

$$\frac{\Delta T_{GUP}}{\Delta T_{thermo}} = \left[ \frac{\pi\alpha^2}{\lambda_d k(d-2)} \right] m^{(d-4)/(d-3)}. \tag{47}$$

The GUP and the thermal fluctuation corrections are of the same order for  $d = 4$ . In  $d > 4$ , the situation is more complicated. If  $m \gg 1$ , the GUP corrections far exceed the corrections due to thermodynamic fluctuations. However,



when  $m \approx 1$ , i.e. near the end stage of evaporation, the rates are comparable. The first-order corrected specific heat is:

$$\mathcal{C} = \mathcal{C}_0 \left[ 1 - \frac{k(d-1)(d-2)}{4\pi\lambda_d} m^{-(d-2)/(d-3)} \right], \quad (48)$$

where  $\mathcal{C}_0$  is given by Eq.(13). The first-order specific heat vanishes for the non-zero value of  $m$

$$m_0 = \left[ \frac{k(d-1)(d-2)}{4\pi\lambda_d} \right]^{(d-3)/(d-2)}. \quad (49)$$

This suggests that the BH becomes thermodynamically stable when the BH mass reaches  $m_0$ . However, this conclusion should be interpreted with care; the thermodynamic fluctuations of a BH with mass  $m \sim m_0$  are large and the first-order approximation (43) breaks down. The Stefan-Boltzmann law is obtained from Eq. (44):

$$\frac{dm}{dt} = -\frac{\mu}{t_{Pl}m^{2/(d-3)}} \left[ 1 + \frac{k\mathcal{D}(d-2)}{4\pi\lambda_d} m^{-(d-2)/(d-3)} + \dots \right]. \quad (50)$$

The first-order correction to the Stefan-Boltzmann law due to thermal fluctuations is positive:

$$\Delta \left( \frac{dm}{dt} \right) / \left( \frac{dm}{dt} \right)_0 = \frac{k\mathcal{D}(d-2)}{4\pi\lambda_d} m^{-(d-2)/(d-3)}. \quad (51)$$

Integrating Eq. (50) we obtain the expression for the time decay

$$\tau_2 = \mu^{-1} \left( \frac{d-3}{d-1} \right) m_i^{(d-1)/(d-3)} \left[ 1 - \frac{k\mathcal{D}(d-1)(d-2)}{4\pi\lambda_d} m_i^{-(d-2)/(d-3)} + \dots \right] t_{Pl}. \quad (52)$$

Similarly to the GUP case, it can be easily verified that the BH takes less time to decay when fluctuation corrections are taken into account:

$$\frac{\Delta\tau_2}{\tau_0} \equiv \frac{\tau_2 - \tau_0}{\tau_0} = -\frac{k\mathcal{D}(d-1)(d-2)}{4\pi\lambda_d} m_i^{-(d-2)/(d-3)}. \quad (53)$$

The decay time is dramatically reduced by thermal fluctuations. Equation (52) implies a relation between the thermal fluctuation threshold  $m_0$  and the BH initial mass. By imposing  $\tau_2 > 0$  we have

$$m_i > \mathcal{D}^{(d-3)/(d-2)} m_0 \equiv m'_0. \quad (54)$$

BH with initial mass smaller than  $m'_0$  form in a regime where thermal fluctuations dominate. A careful study of their thermodynamic properties should include higher-order terms in the expansion (36). Since the thermodynamic fluctuations prevent the analytical evaluation of the integral in Eq. (37), numerical techniques may have to be used to get accurate estimates of the thermodynamic quantities. In any case, our analysis shows that semiclassical Hawking theory is inadequate for the description of these black holes. Note that for  $k = 0.5$  (1),  $m'_0 = 10.25$  (18.8). Thermal fluctuations cannot be neglected in particle collider or UHECR BH events.

## VI. DISCUSSION

We have examined the effects of the GUP and small thermal fluctuations on temperature, decay rate, and entropy of microscopic BHs. Although these effects are small under most circumstances, they can be significant in BH production at the TeV scale, where the BH mass is expected to be of the order of the fundamental Planck mass. The GUP and the thermal fluctuation corrections increase the BH temperature, and decrease decay time, entropy, and multiplicity of the evaporation phase: Quantum BHs are hotter, shorter-lived, and tend to evaporate less than classical BHs. The results described here are applicable to the ADD as well as the RS brane world scenarios [1, 2].

Under the most favorable circumstances, the semiclassical cross section for BH formation at the TeV scale reaches hundreds of pb for proton-proton collision at the LHC, and millions of pb for neutrino-nucleon collision in the atmosphere. According to the semiclassical scenario, the Hawking evaporation mechanism will allow detection of microscopic BHs with mass of the order of a few Planck masses in next-generation particle colliders and UHECR detectors. However, our results seem to suggest that the semiclassical description could be inaccurate for this kind of events.

Firstly, a shorter lifetime implies that the quantum BH may not behave like a well-defined resonance. Secondly, the classical picture breaks down if the degrees of freedom of the BH, i.e. its entropy, is small. The semiclassical entropy has been widely used in the literature to measure the validity of the semiclassical approximation. When the entropy is sufficiently large, the BH can be considered as a classical object [8]. If this is the case, i) the BH evaporation phase is described by a thermal spectrum with Hawking temperature; ii) the BH cross section for elementary particles is well approximated by the geometrical cross section; and iii) the total cross section for composite particles (e.g. nucleons) is obtained by integrating the geometrical cross section over the structure functions. A BH with mass equal to few Planck masses is usually assumed to have entropy above the threshold of validity of the classical description. However, the reduction in entropy by GUP and thermal fluctuation effects increases this threshold. Therefore, it may not be appropriate to treat these BHs as classical objects. (See also Ref. [10]). Thirdly, GUP physics implies the existence of a minimum BH mass given by Eq. (18). The existence of a minimum mass increases the lower cutoff for BH formation, thus reducing the rate of BH events. Even if a detectable signal is produced during the BH decay phase, it could prove very difficult to distinguish it from the background. Finally, GUP and thermodynamic fluctuations further decrease the already-weak lower bounds on the fundamental Planck scale that follow from the nonobservation of BH events up to date [25].

Let us conclude with a list of open problems and possible future research topics. It would be interesting to compute the effects of small residual charge  $Q$  and angular momentum  $J$  on the above results. In presence of nonzero charge and angular momentum, the corrections to the thermodynamic quantities are expected to depend on  $J$  and  $Q$ . However, the precise form of the corrections is yet to be determined. It would also be interesting to examine the relation of the GUP and thermodynamic corrections to other corrections that have been predicted for TeV-scale BHs (see e.g. Refs. [8], [10]). A detailed single-event analysis of particle collider and UHECR events when GUP and thermodynamic corrections are present, is also missing. We plan to report on these and other issues elsewhere.

### Acknowledgements:

We thank R. Maartens and M. Maggiore for interesting discussions. S.D. would like to thank R. K. Bhaduri, A. Dasgupta, J. Gegenberg, V. Husain, P. Majumdar and M. Walton for interesting discussions and comments. This work was supported in part by the Natural Sciences and Engineering Research Council of Canada and funds of the University of Lethbridge.

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