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# Quantum Gravity Corrections and Entropy at the Planck time

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We investigate the effects of Quantum Gravity on the Planck era of the universe. In particular, using different versions of the Generalized Uncertainty Principle and under specific conditions we find that the main Planck quantities such as the Planck time, length, mass and energy become larger by a factor of order  $10-10^4$  compared to those quantities which result from the Heisenberg Uncertainty Principle. However, we prove that the dimensionless entropy enclosed in the cosmological horizon at the Planck time remains unchanged. These results, though preliminary, indicate that we should anticipate modifications in the set-up of cosmology since changes in the Planck era will be inherited even to the late universe through the framework of Quantum Gravity (or Quantum Field Theory) which utilizes the Planck scale as a fundamental one. More importantly, these corrections will not affect the entropic content of the universe at the Planck time which is a crucial element for one of the basic principles of Quantum Gravity named Holographic Principle.

## I. INTRODUCTION

Gravity is a universal and fundamental force. Anything which has energy creates gravity and is affected by it, although the smallness of Newton's constant  $G$  often means that the associated classical effects are too weak to be measurable.

An important prediction of various theories of quantum gravity (such as String Theory) and black hole physics is the existence of a minimum measurable length [1]. The prediction is largely model-independent, and can be understood as follows: the Heisenberg Uncertainty Principle (HUP),  $\Delta x \sim \hbar/\Delta p$ , breaks down for energies close to the Planck scale, when the corresponding Schwarzschild radius is comparable to the Compton wavelength (both being approximately equal to the Planck length). Higher energies result in a further increase of the Schwarzschild radius, resulting in  $\Delta x \approx \ell_{Pl}^2 \Delta p / \hbar$ .

At this point, it should be stressed that limits on the measurement of spacetime distances as well as on the synchronization of clocks were put in much earlier studies [2]. These limitations showed up when Quantum Mechanics (QM) and General Relativity (GR) were put together under simple arguments. It is more than obvious that in this context where one attempts to reconcile the principles of QM with those of GR there are several and even diverging paths to follow [3]. In this framework, two of the authors (SD and ECV) tracked a new path and showed that certain effects of Quantum Gravity are universal, and can influence almost any system with a well-defined Hamiltonian [4]. Although the resultant quantum effects are generically quite small, with current and future experiments, bounds may be set on certain parameters relevant to quantum gravity, and improved accuracies could even make them measurable [4, 5].

One of the formulations, among those existing in the literature, of the *Generalized Uncertainty Principle* (GUP) and which holds at all scales, is represented by [1]

$$\Delta x_i \Delta p_i \geq \frac{\hbar}{2} [1 + \beta ((\Delta p)^2 + \langle p \rangle^2) + 2\beta (\Delta p_i^2 + \langle p_i \rangle^2)] , \quad i = 1, 2, 3 \quad (1)$$

where  $p^2 = \sum_{j=1}^3 p_j p_j$ ,  $\beta = \beta_0 / (M_{Pl} c)^2 = \beta_0 \ell_{Pl}^2 / \hbar^2$ ,  $M_{Pl}$  = Planck mass, and  $M_{Pl} c^2$  = Planck energy  $\approx 1.2 \times 10^{19}$  GeV.

It is normally assumed that the dimensionless parameter  $\beta_0$  is of the order of unity. However, this choice renders Quantum Gravity effects too small to be measurable. On the other hand, if one does not impose the above condition *a priori*, current experiments predict large upper bounds on it, which are compatible with current observations, and may signal the existence of a new length scale. Note that such an intermediate length scale,  $\ell_{inter} \equiv \sqrt{\beta_0} \ell_{Pl}$  cannot exceed

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the electroweak length scale  $\sim 10^{17} \ell_{Pl}$  (as otherwise it would have been observed). This implies  $\beta_0 \leq 10^{34}$ . Therefore, as stated above, Quantum Gravity effects influence all quantum Hamiltonians [4]. Moreover, some phenomenological implications of this interesting result were presented in [5]<sup>1</sup>.

The recently proposed *Doubly Special Relativity* (or DSR) theories on the other hand (which predict maximum observable momenta), also suggest a similar modification of commutators [7–9]. The commutators which are consistent with String Theory, Black Holes Physics, DSR, and which ensure  $[x_i, x_j] = 0 = [p_i, p_j]$  (via the Jacobi identity) under specific assumptions lead to the following form [10]

$$[x_i, p_j] = i\hbar \left[ \delta_{ij} - \alpha \left( p \delta_{ij} + \frac{p_i p_j}{p} \right) + \alpha^2 (p^2 \delta_{ij} + 3p_i p_j) \right] \quad (2)$$

where  $\alpha = \alpha_0 / M_{Pl} c = \alpha_0 \ell_{Pl} / \hbar$ .

Equation (2) yields, in 1-dimension, to  $\mathcal{O}(\alpha^2)$

$$\Delta x \Delta p \geq \frac{\hbar}{2} [1 - 2\alpha \langle p \rangle + 4\alpha^2 \langle p^2 \rangle] \quad (3)$$

where the dimensional constant  $\alpha$  is related to  $\beta$  that appears in equation (1) through dimensional analysis with the expression  $[\beta] = [\alpha^2]$ . However, it should be pointed out that it does not suffice to connect the two constants  $\alpha$  and  $\beta$  through a relation of the form  $\beta \sim \alpha^2$  in order to reproduce equation (1) from (3), or vice versa. Equations (1) and (3) are quite different and, in particular, the most significant difference is that in equation (1) all terms appear to be quadratic in momentum while in equation (3) there is a linear term in momentum. Commutators and inequalities similar to (2) and (3) were proposed and derived respectively in [11–13]. These in turn imply a minimum measurable length and a maximum measurable momentum (to the best of our knowledge, (2) and (3) are the only forms which imply both)

$$\Delta x \geq (\Delta x)_{min} \approx \alpha_0 \ell_{Pl} \quad (4)$$

$$\Delta p \leq (\Delta p)_{max} \approx \frac{M_{Pl} c}{\alpha_0} . \quad (5)$$

It is normally assumed as in the case of  $\beta_0$  that the dimensionless parameter  $\alpha_0$  is of the order of unity, in which case the  $\alpha$  dependent terms are important only when energies (momenta) are comparable to the Planck energy (momentum), and lengths are comparable to the Planck length. However, if one does not impose this condition *a priori*, then using the fact that all quantum Hamiltonians are affected by the Quantum Gravity corrections as was shown in [4] and applying this formalism to measure a single particle in a box, one deduces that all measurable lengths have to be quantized in units of  $\alpha_0 \ell_{Pl}$  [10].

In order to derive the energy-time uncertainty principle, we employ the equations

$$\begin{aligned} \Delta x &\sim c \tau \\ \Delta p &\sim \frac{\Delta E}{c} \end{aligned}$$

where  $\tau$  is a characteristic time of the system under study, and it is straightforward to get

$$\Delta x \Delta p \approx \Delta E \tau . \quad (6)$$

Substituting equation (6) in the standard HUP, one gets the energy-time uncertainty principle

$$\Delta E \tau \geq \frac{\hbar}{2} . \quad (7)$$

It should be stressed that the characteristic time  $\tau$  is usually selected to be equal to the Planck time  $t_{Pl}$  in the context of cosmology.

The scope of the present work is to investigate in a cosmological setup what corrections, if any, are assigned to physical quantities such as the mass and energy of the universe at the Planck time. In particular, our present approach, regarding the Quantum Gravity Corrections at the Planck time, has been based on a methodology that presented in the book of Coles and Lucchin [14]. Simply, in our phenomenological formulation instead of using the

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<sup>1</sup> For a brief presentation of the results in [4] and [5] see [6].

standard HUP for deriving the Planck time (as done in [11]), we let ourselves to utilize various versions of the GUP and basically apply the methodology presented in the previously mentioned book.

The significance of Planck time *per se* is due to the fact that it is really a “turning point” because from the birth of the universe till the Planck time Quantum Gravity corrections are significant (classical General Relativity does not work at all) while after that General Relativity seems to work properly.

## II. GUP AND ENTROPY AT PLANCK TIME

In this section we investigate the effects of Quantum Gravity on the Planck era of the universe. By employing the different versions of GUP presented before, we evaluate the modifications to several quantities that characterize the Planck era, i.e. Planck time, length, mass, energy, density, effective number density and entropy. This will enhance our understanding of the consequences of the Quantum Gravity in the universe during the Planck epoch and afterwards.

### A. HUP and the standard Planck era

Without wanting to appear too pedagogical, we briefly present how one can derive some physical quantities at the Planck epoch starting from the HUP. Following the methodology of [14] (see page 110), we first write the HUP in the form

$$\Delta E \tau \simeq \hbar \quad (8)$$

and we adopt as characteristic time  $\tau$  of the system under study the Planck time, i.e.  $t_{Pl}$ , for which the quantum fluctuations exist on the scale of Planck length, i.e.  $\ell_{Pl} = ct_{Pl}$ . In addition, the uncertainty in energy can be identified with the Planck energy and thus  $\Delta E = E_{Pl} = M_{Pl}c^2$ . Thus, the HUP as given in equation (8) is now written as

$$M_{Pl}c^2 t_{Pl} \simeq \hbar . \quad (9)$$

Since the universe at Planck time can be seen as a system of radius  $\ell_{Pl}$ , the Planck mass can be written as

$$M_{Pl} \simeq \rho_{Pl} \ell_{Pl}^3 \quad (10)$$

where by employing the first Friedmann equation the Planck density, on dimensional grounds, reads

$$\rho_{Pl} \simeq \frac{1}{Gt_{Pl}^2} . \quad (11)$$

Substituting equations (10) and (11) in equation (9), one gets

$$\frac{1}{Gt_{Pl}^2} \ell_{Pl}^3 c^2 t_{Pl} \simeq \hbar \quad (12)$$

and therefore one can easily prove that the Planck time is

$$t_{Pl} \simeq \sqrt{\frac{\hbar G}{c^5}} \simeq 10^{-43} \text{ sec} . \quad (13)$$

All the other parameters are defined in terms of the Planck time modulus some constants. Indeed, the Planck length, density, mass, energy, temperature and effective number density are given by the following expressions

$$\ell_{Pl} \simeq \sqrt{\frac{\hbar G}{c^3}} \simeq 1.7 \times 10^{-33} \text{ cm} , \quad \rho_{Pl} \simeq \frac{1}{Gt_{Pl}^2} \simeq \frac{c^5}{\hbar G^2} \simeq 4 \times 10^{93} \text{ g cm}^{-3} , \quad (14)$$

$$M_{Pl} \simeq \rho_{Pl} \ell_{Pl}^3 \simeq \sqrt{\frac{\hbar c}{G}} \simeq 2.5 \times 10^{-5} \text{ g} , \quad E_{Pl} \simeq M_{Pl} c^2 \simeq \sqrt{\frac{\hbar c^5}{G}} \simeq 1.2 \times 10^{19} \text{ GeV} , \quad (15)$$

$$T_{Pl} \simeq \frac{E_{Pl}}{k_B} \simeq \sqrt{\frac{\hbar c^5}{G}} k_B^{-1} \simeq 1.4 \times 10^{32} \text{ K} , \quad n_{Pl} \simeq \ell_{Pl}^{-3} \simeq \left( \frac{c^3}{G\hbar} \right)^{3/2} \simeq 10^{98} \text{ cm}^{-3} . \quad (16)$$

### B. GUP quadratic in $\Delta E$

The corresponding energy-time GUP of equation (1)

$$\Delta E \tilde{t}_{Pl} \geq \frac{\hbar}{2} \left[ 1 + \beta_0 \frac{\tilde{\ell}_{Pl}^2 (\Delta E)^2}{\hbar^2 c^2} \right] \quad (17)$$

where we have kept only the first GUP-induced term of order  $O(\beta_0)$ . Note that the tilde denotes quantities with respect to the GUP. As expected, for  $\beta_0 = 0$  the GUP boils down to the standard form dictated by the Heisenberg result ( $\Delta E \tau \simeq \hbar$ ).

From the previously presented formalism (see section I), the uncertainty in energy  $\Delta E$  at the Planck time is of order of the modified Planck energy, i.e.  $\tilde{E}_{Pl} = \tilde{M}_{Pl} c^2$ , where the modified Planck mass lies inside the universe's horizon of scale of the modified Planck length, i.e.  $\tilde{\ell}_{Pl}$ , and expands as

$$\tilde{M}_{Pl} = \tilde{\rho}_{Pl} \tilde{\ell}_{Pl}^3. \quad (18)$$

The modified Planck density can be easily derived from the first Friedmann equation (or, from the definition of the dynamical time scale) and be given by

$$\tilde{\rho}_{Pl} \simeq \frac{1}{G \tilde{t}_{Pl}^2} \quad (19)$$

modulus some constants. Substituting equations (18) and (19) in equation (17), one gets an equation for the Planck time which now has been affected by the Quantum Gravity corrections, namely

$$\tilde{\rho}_{Pl} \tilde{\ell}_{Pl}^3 c^2 \tilde{t}_{Pl} \simeq \frac{\hbar}{2} \left[ 1 + \beta_0 \frac{c^2 \tilde{t}_{Pl}^2}{\hbar^2} \tilde{\rho}_{Pl}^2 c^8 \tilde{t}_{Pl}^6 \right] \quad (20)$$

$$\frac{\tilde{\ell}_{Pl}^3}{G \tilde{t}_{Pl}^2} c^2 \tilde{t}_{Pl} \simeq \frac{\hbar}{2} \left[ 1 + \beta_0 \frac{c^2 \tilde{t}_{Pl}^2}{\hbar^2} \frac{c^8}{G^2 \tilde{t}_{Pl}^4} \tilde{t}_{Pl}^6 \right] \quad (21)$$

$$\frac{c^5}{G} \tilde{t}_{Pl}^2 \simeq \frac{\hbar}{2} \left[ 1 + \beta_0 \frac{c^{10}}{\hbar^2 G^2} \tilde{t}_{Pl}^4 \right]. \quad (22)$$

Therefore, after some simple algebra, one gets the following equation

$$\frac{\beta_0}{2} \left( \frac{c^5}{\hbar G} \right)^2 \tilde{t}_{Pl}^4 - \left( \frac{c^5}{\hbar G} \right) \tilde{t}_{Pl}^2 + \frac{1}{2} \simeq 0. \quad (23)$$

It is easily seen that if we choose  $\beta_0$  to be strictly equal to zero then the current solution of the above equation reduces practically to that of the standard Planck time (cf. (13)). On the other hand,  $\forall \beta_0 \in (0, 1]$  equation (23) has two real solutions of the form

$$\begin{aligned} \tilde{t}_{Pl} &= \sqrt{\frac{G\hbar}{c^5}} f_{\pm}(\beta_0) \\ &= t_{Pl} f_{\pm}(\beta_0) \end{aligned} \quad (24)$$

where

$$f_{\pm}(\beta_0) = \left[ \frac{1 \pm \sqrt{1 - \beta_0}}{\beta_0} \right]^{1/2}. \quad (25)$$

It is worth noting that for  $\beta_0 = 1$  [or, equivalently,  $f_{\pm}(1) = 1$ ] we find  $\tilde{t}_{Pl} = t_{Pl}$ , despite the fact that we have started from a completely different Uncertainty Principle. This implies that in this specific case, i.e.  $\beta_0 = 1$ , the GUP-induced effects in equation (24) cannot be observed.

Now we must first decide which is the important term when  $\beta_0$  takes values in the set  $(0, 1]$ ; is it  $f_-(\beta_0)$  or  $f_+(\beta_0)$ ? Using some basic elements from calculus, one can prove that the function  $f_-(\beta_0)$  is continuous and increases strictly in the range of  $0 < \beta_0 \leq 1$  which implies that  $f_-(\beta_0) \in (\lim_{\beta_0 \rightarrow 0} f_-(\beta_0), f_-(1)]$ , where  $\lim_{\beta_0 \rightarrow 0} f_-(\beta_0) = \sqrt{2}/2$ . Therefore, the modified Planck time lies in the range

$$\frac{\sqrt{2}}{2} t_{Pl} \leq \tilde{t}_{Pl} \leq t_{Pl}. \quad (26)$$

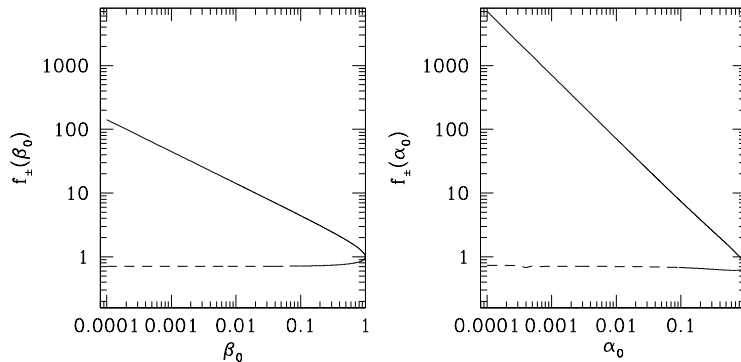


FIG. 1: *Left Panel:* The predicted Quantum Gravity corrections using the GUP of equation (17). *Right Panel:* The corresponding corrections utilizing the GUP of equation (30). The solid and the dashed lines correspond to  $f_+$  and  $f_-$ , respectively.

In the left panel of figure 1, we present in a logarithmic scale the  $f_-(\beta_0)$  (dashed line) as a function of  $\beta_0$ . Practically speaking, the  $f_-(\beta_0)$  term has no effect on the Planck time. On the other hand, following the latter analysis, we find that the  $f_+(\beta_0)$  function (solid line) decreases strictly in the range  $0 < \beta_0 \leq 1$  which means that  $f_+(\beta_0) \in [f_+(1), \lim_{\beta_0 \rightarrow 0} f_+(\beta_0))$ , where  $\lim_{\beta_0 \rightarrow 0} f_+(\beta_0) = +\infty$ . It becomes evident that as long as  $\beta_0 \ll 1$  [or, equivalently,  $f_+(\beta_0) \simeq (2/\beta_0)^{1/2}$ ] the current GUP can affect the Planck quantities via the function  $f_+(\beta_0)$ . For example, in the case where  $\beta_0 = \mathcal{O}(10^{-2} - 10^{-4})$  we find that  $f_+ \simeq \tilde{t}_{Pl}/t_{Pl} \simeq 10 - 10^2$ .

From a cosmological point of view, the ratio of the modified Planck density, i.e.  $\tilde{\rho}_{Pl}$ , versus the measured dark energy, i.e.  $\rho_{DE} \simeq (1 - \Omega_m) \times 10^{-29} g cm^{-3}$  with  $\Omega_m \simeq 0.27$  (for details see [15]), is given as

$$\frac{\tilde{\rho}_{Pl}}{\rho_{DE}} \simeq \frac{10^{123}}{f_+^2} \simeq \mathcal{O}(10^{119} - 10^{121}). \quad (27)$$

One now is interested to investigate if and how the main Planck quantities related to the Planck time are affected by the above Quantum Gravity corrections. The corresponding relations are

$$\frac{\tilde{\ell}_{Pl}}{\ell_{Pl}} = \frac{\tilde{M}_{Pl}}{M_{Pl}} = \frac{\tilde{E}_{Pl}}{E_{Pl}} = \left( \frac{\rho_{Pl}}{\tilde{\rho}_{Pl}} \right)^{1/2} = \left( \frac{n_{Pl}}{\tilde{n}_{Pl}} \right)^{1/3} = f_+. \quad (28)$$

Finally, it should be stressed that the dimensionless entropy enclosed in the cosmological horizon of size  $\tilde{\ell}_{Pl}$  now reads

$$\tilde{\sigma}_{Pl} \simeq \frac{\tilde{\rho}_{Pl} c^2 \tilde{\ell}_{Pl}^3}{k_B \tilde{T}_{Pl}} \simeq \sigma_{Pl}. \quad (29)$$

It is evident that the entropic content of the universe behind the cosmological horizon at the Planck time is unaltered when Quantum Gravity corrections are taken into account. Therefore, the information remains unchanged: one ‘‘particle’’ of Planck mass is ‘‘stored’’ in the Planck volume of the universe at the Planck time behind the cosmological horizon of size  $\tilde{\ell}_{Pl}$ .

### C. GUP versus all terms of $\Delta E$

The corresponding energy-time GUP of equation (3) becomes

$$\Delta E \tilde{t}_{Pl} \geq \frac{\hbar}{2} \left[ 1 - 2\alpha \frac{(\Delta E)}{c} + 4\alpha^2 \frac{(\Delta E)^2}{c^2} \right]. \quad (30)$$

As it is anticipated, for  $a = 0$  (or, equivalently,  $a_0 = 0$  since  $\alpha = \alpha_0 \frac{c \tilde{t}_{Pl}}{\hbar}$ ) the GUP boils down to the standard form dictated by HUP. Evidently, performing the same methodology as before (see subsection B), we obtain the following

equation

$$2\alpha_0^2 \left( \frac{c^5}{\hbar G} \right)^2 \tilde{t}_{Pl}^4 - \frac{c^5}{\hbar G} (1 + \alpha_0) \tilde{t}_{Pl}^2 + \frac{1}{2} \simeq 0. \quad (31)$$

In deriving equation (31) we have substituted the various terms as  $\Delta E \simeq \tilde{E}_{Pl}$ ,  $\tau \simeq \tilde{t}_{Pl}$ ,  $\tilde{\ell}_{Pl} \simeq c\tilde{t}_{Pl}$ , and  $\alpha = \alpha_0 \frac{c\tilde{t}_{Pl}}{\hbar}$ . In this framework, equation (31) has two real solutions  $\forall \alpha_0 \in [-1/3, 0) \cup (0, 1]$ . These are

$$\tilde{t}_{Pl} = t_{Pl} f_{\pm}(\alpha_0) \quad (32)$$

where

$$f_{\pm}(\alpha_0) = \left[ \frac{(1 + \alpha_0) \pm \sqrt{(1 - \alpha_0)(1 + 3\alpha_0)}}{4\alpha_0^2} \right]^{1/2}. \quad (33)$$

Again it is routine to estimate the limiting values of  $f_{\pm}(\alpha_0)$

$$f_{\pm}(1) = \frac{\sqrt{2}}{2} \quad f_{\pm}\left(-\frac{1}{3}\right) = \sqrt{\frac{3}{2}} \quad \lim_{\alpha_0 \rightarrow 0} f_{-}(\alpha_0) = \frac{\sqrt{2}}{2}.$$

Therefore, the function  $f_{-}(\alpha_0)$  does not play a significant role (see the dashed line in the right panel of figure 1), since the modified Planck time tends to the usual value ( $\tilde{t}_{Pl} \approx t_{Pl}$ ). On the contrary, if we consider the case of  $f_{+}(\alpha_0)$  (see the solid line in the right panel of figure 1), then it becomes evident that for small values of  $\alpha_0$  the function  $f_{+}(\alpha_0) \approx 1/2\alpha_0$  goes rapidly to infinity. As an example for  $\alpha_0 = \mathcal{O}(10^{-2} - 10^{-4})$  we find that  $f_{+} \simeq \tilde{t}_{Pl}/t_{Pl} \simeq 10^2 - 10^4$ . Thus the ratio of the modified Planck density versus the measured dark energy now reads

$$\frac{\tilde{\rho}_{Pl}}{\rho_{DE}} \sim \mathcal{O}(10^{115} - 10^{119}). \quad (34)$$

The main Planck quantities related to the Planck time are affected by the above Quantum Gravity corrections exactly in the same way as shown in equation (28) employing the current form of  $f_{+}(\alpha_0)$  defined in equation (33).

Furthermore, it is interesting to point out that the dimensionless entropy enclosed in the cosmological horizon of size  $\tilde{\ell}_{Pl}$  remains unaltered  $\tilde{\sigma}_{Pl} \simeq \sigma_{Pl}$ . This result is in accordance with the result derived in previous subsection (see equation (29)). Therefore, the information in the Planck volume remains unchanged even if one takes into account the Quantum Gravity effects, i.e. one ‘‘particle’’ of Planck mass is ‘‘stored’’ in the Planck volume of the universe at the Planck time behind the cosmological horizon of size  $\tilde{\ell}_{Pl}$ .

### III. CONCLUSIONS

In this work we have investigated analytically the Quantum Gravity corrections at the Planck time by employing a methodology that was introduced in the book of Coles and Lucchin [14]. Specifically, in this work instead of using the standard HUP for deriving the Planck time (as done in [11]), we let ourselves to utilize various versions of the GUP. From our analysis, it becomes evident that the Planck quantities, predicted by the Generalized Uncertainty Principle GUP, extends nicely to those of the usual Heisenberg Uncertainty Principle (HUP) and connects smoothly to them. We also find that under of specific circumstances the modified Planck quantities defined in the framework of the GUP are larger by a factor of  $f_{+} \sim (10 - 10^4)$  with respect to those found using the standard HUP.

These results indicate that we anticipate modifications in the framework of cosmology since changes in the Planck epoch will be inherited to late universe through Quantum gravity (or Quantum Field Theory). As an example, the calculation of the density fluctuations at the epoch of inflation sets an important limit on the potential of inflation. Indeed, in the context of slow-roll approximation, one can prove that the density fluctuations are of the form  $\delta_H \sim H^2/\dot{\phi}^2 = 3H^3/V'$  where  $\phi$  is the scalar field called inflaton,  $H$  stands for the Hubble parameter and  $V(\phi)$  is the potential energy of the scalar field. Assuming that  $V \simeq m^2\phi^2$  where  $m$  is the inflaton mass, and  $H \simeq \sqrt{V}/M_{Pl}$ , we obtain  $\delta_H \simeq m\phi^2/M_{Pl}^3$ . In order to achieve inflation the scalar field has to satisfy the inequality  $\phi \geq M_{Pl}$  [16]. Combining the above equations and utilizing the observational value  $\delta_H \approx 10^{-5}$ , one gets  $m \leq 10^{-5}M_{Pl}$ . Employing the Quantum Gravity corrections, the latter condition becomes  $\tilde{m} \leq 10^{-5}f_{+}M_{Pl}$ .

From the latter calculations it becomes evident that the Quantum Gravity corrections affect directly the main cosmological quantities (such as the inflaton mass) in the early universe, via the  $f_{+} = \tilde{t}_{Pl}/t_{Pl}$  factor. Due to the fact

that the  $f_+$  factor is the consequence of the Quantum Gravity *per se* (based on the GUP) implies that it can not be re-absorbed by a redefinition of units.

From the observational point of view one can study the latter corrections in the context of the primordial gravitational waves which can be detected with very sensitive measurements of the polarization of the CMB (see page 5 in [17]). It is interesting to mention here that the polarization of the CMB will be one of the main scientific targets of the next generation of the CMB data based on the Planck satellite. Therefore, if in the near future the observers measure such an effect in the CMB data then we may open an avenue in order to understand the transition from the mainly-quantum gravitational regime to the mainly-classical regime. We have already started to investigate theoretically the above possibility and we are going to present our results in a forthcoming paper.

Furthermore, it was also shown that the dimensionless entropy enclosed in the cosmological horizon does not “feel” the Quantum Gravity corrections and thus the information remains unaltered. Therefore, the entropic content of the universe at the Planck time remains the same. This is quite important since entropy is the cornerstone for one of the basic principles of Quantum Gravity named Holographic Principle and for its incarnation known as AdS/CFT.

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