MADPH-95-872, UG-FT-51/95

CP violation in the lepton sector with Majorana neutrinos

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February 7, 2008

Abstract

We study CP violation in the lepton sector in extended models with right-handed neutrinos, without and with left-right symmetry, and with arbitrary mass terms. We find the conditions which must be satisfied by the neutrino and charged lepton mass matrices for CP conservation. These constraints, which are independent of the choice of weak basis, are proven to be also sufficient in simple cases. This invariant formulation makes apparent the necessary requirements for CP violation, as well as the size of CP violating effects. As an example, we show that CP violation can be much larger in left-right symmetric models than in models with only additional right-handed neutrinos, *i.e.*, without right-handed currents.

1. Introduction

The origin of CP violation is still an open problem in particle physics. In the standard model CP violation is related to the mixing between flavour and mass eigenstates (Cabbibo-Kobayashi-Maskawa (CKM) mechanism) [1]. In this case three standard families of quarks with non-degenerate masses must exist. This is known to happen, and small CP violating effects has been observed in the $K^0 - \bar{K}^0$ system.

Why these effects must be small is also understood. Physical quantities are independent of the choice of (weak) quark basis. Hence, only weak basis invariants enter in measurable quantities like cross sections or decay widths. Two sets of CP symmetry breaking invariants have been constructed [2,3]. For three families there is only one independent invariant, which can be chosen to be the determinant of the commutator of $M_u M_u^\dagger$ and $M_d M_d^\dagger$ [2],

$$\begin{array}{c} Det[M_u M_u^\dagger, M_d M_d^\dagger] = -2i\ (m_t^2 - m_c^2)(m_t^2 - m_u^2)(m_c^2 - m_u^2) \\ (m_b^2 - m_s^2)(m_b^2 - m_d^2)(m_s^2 - m_d^2)\ Im(V_{ud} V_{cs} V_{us}^* V_{cd}^*), \end{array} \tag{1}$$

where $M_{u(d)}$ is the up (down) quark mass matrix in a weak current eigenstate basis, m_i is the mass of the quark i, and V_{ij} is the ij entry of the CKM matrix; or the trace of the triple product of the commutator [3],

$$Tr[M_u M_u^\dagger, M_d M_d^\dagger]^3 = 3\ Det[M_u M_u^\dagger, M_d M_d^\dagger]. \tag{2}$$

As a consequence, in the standard model all CP violating effects are proportional to

$$\delta_{KM} = Im(V_{ud} V_{cs} V_{us}^* V_{cd}^*). \tag{3}$$

Using the unitarity of the CKM matrix we can write

$$|\delta_{KM}| = |Im(V_{ub} V_{cs} V_{us}^* V_{cb}^*)|, \tag{4}$$

and substituting the experimental values of $|V_{ij}|$ in Eq. (4) it can be shown that

$$|\delta_{KM}| \leq 10^{-4}; \tag{5}$$

and then that all CP violating effects are small [4]. This is usually summarized saying that CP violation has been only observed in the $K^0 - \bar{K}^0$ system and that it is small due to the small mixing angles $|V_{ij}|$ in Eq. (4).

No CP violation has been observed in the lepton sector. This may be related to the smallness (or vanishing) of the masses of the electron, muon and tau neutrinos. LEP results exclude the existence of more than three left-handed neutrinos with masses practically up to the Z^0 mass. Neutrinos with larger masses can exist and are predicted in many models beyond the standard model. How large can CP violation be in the lepton sector with heavy (Majorana) neutrinos has not been fully analysed to our knowledge. Only the number of CP violating phases for different neutrino contents has been calculated [5].

As for quarks, the construction of a set of CP violating invariants for lepton mass matrices shall allow for:

- deciding more easily in any weak lepton basis if CP is conserved;
- understanding the origin of CP violation if for any (physical) reason a definite (class of) model(s) is distinguished;
- motivating model building; and
- obtaining the size of CP violating effects, because all physical quantities are proportional to weak basis invariants and then, knowing a set of neccesary and sufficient invariant constraints for CP conservation stands for knowing the possible factors suppressing the CP violating observables.

In this paper we discuss extended models with an arbitrary number of right-handed neutrinos and standard lepton families, without (Section 2) and with (Section 3) left-right symmetry,. We find necessary conditions for CP conservation for arbitrary lepton mass matrices. These, which are independent of the choice of weak basis, are proven to be sufficient in simple cases. Using these invariant conditions we address the question of how large can CP violation effects be for models with and without left-right symmetry (Section 4). (We do not discuss other possible CP violating effects mediated by Higgses.)

2. Extended models with right-handed neutrinos

We proceed analogously to the quark case [3]: we state the conditions for CP conservation in the lepton sector satisfied in any weak basis, we count the number of CP breaking phases in a particular basis [5], and we construct invariant conditions for CP conservation, proving in some simple cases that these are necessary and sufficient, what constitutes our main result.

2.1. CP invariance

The gauge interactions in extended electroweak models with n_R right-handed neutrinos (and n_L standard lepton families) are written in any weak basis as in the standard model [4]:

$$\mathcal{L}_{CC} = -\frac{g}{\sqrt{2}}\bar{\nu}_L\gamma^\mu l_L W^+_\mu + h.c., \tag{6}$$

and

$$\begin{array}{c}\mathcal{L}_{NC} = e\ (\bar{l}_L\gamma^\mu l_L + \bar{l}_R\gamma^\mu l_R)\ A_\mu \\ +\ \frac{g}{2cos\theta_W}\ \left(-\bar{\nu}_L\gamma^\mu\nu_L + (1-2sin^2\theta_W)\ \bar{l}_L\gamma^\mu l_L - 2sin^2\theta_W\ \bar{l}_R\gamma^\mu l_R\right)\ Z_\mu,\end{array} \tag{7}$$

where $l_{L,R}$ are two n_L component vectors in flavour space describing the n_L left-handed and n_L right-handed charged leptons, and ν_L is a n_L component vector describing the n_L left-handed neutrinos. $\mathcal{L}_{CC,NC}$ are left invariant by any CP transformation

$$\begin{array}{c}\nu_L \longrightarrow V_L C\nu^*_L, \\ l_L \longrightarrow V_L C l^*_L, \\ l_R \longrightarrow V^l_R C l^*_R,\end{array} \tag{8}$$

where V_L and V^l_R are $n_L \times n_L$ unitary matrices, and C is the Dirac charge conjugation matrix. Similarly a general CP transformation on the right-handed neutrinos reads

$$\nu_R \longrightarrow V^\nu_R C\nu^*_R, \tag{9}$$

where V^ν_R is a $n_R \times n_R$ unitary matrix. Then CP is conserved if there exist transformations (8) and (9) that left invariant the mass terms [6]

$$\begin{array}{c}\mathcal{L}_{mass} = -(\bar{l}_L M_l l_R + \bar{l}_R M^\dagger_l l_L) \\ -\frac{1}{2}(\bar{\psi}_L M_\nu \psi_R + \bar{\psi}_R M^*_\nu \psi_L),\end{array} \tag{10}$$

where M_l is a $n_L \times n_L$ complex matrix, $\psi_{L,R}$ are $n_L + n_R$ component vectors describing the n_L left-handed and n_R right-handed neutrinos,

$$\psi_L = \begin{pmatrix}\nu_L \\ \nu^c_L = i\gamma^2\nu^*_R\end{pmatrix},\ \psi_R = \begin{pmatrix}\nu^c_R = i\gamma^2\nu^*_L \\ \nu_R\end{pmatrix},$$

and M_ν is a $(n_L + n_R) \times (n_L + n_R)$ complex symmetric matrix,

$$M_\nu = \begin{matrix} n_L\{ \\ n_R\{ \end{matrix} \begin{pmatrix} \overbrace{M_L}^{n_L} & \overbrace{M_D}^{n_R} \\ M_D^T & M_R \end{pmatrix}. \tag{11}$$

The invariance of (10) under the transformations (8) and (9) implies

$$\begin{aligned} V_L^\dagger M_l V_R^l &= M_l^*, \\ V_L^\dagger M_L V_L^* &= M_L^*, \\ V_L^\dagger M_D V_R^\nu &= M_D^*, \\ V_R^{\nu\ T} M_R V_R^\nu &= M_R^*. \end{aligned} \tag{12}$$

Thus, CP is conserved in an extended electroweak model with Dirac and Majorana mass matrices $M_{l,D}, M_{L,R}$ if there exist (unitary) matrices $V_L, V_R^{l,\nu}$ satisfying (12). The converse is also true, if CP is conserved, such (unitary) matrices do exist.

We assume that the Yukawa couplings can be related to the mass matrices and that they satisfy analogous equations. This is often the case for minimal models. We concentrate on these models, then neglecting possible CP violating effects mediated by Higgs bosons.

2.2. Weak basis independence

Eqs. (12) are weak basis independent. In any other weak basis

$$\begin{aligned} {\nu'}_L &= W_L \nu_L, \\ {l'}_L &= W_L l_L, \\ {l'}_R &= W_R^l l_R, \\ {\nu'}_R &= W_R^\nu \nu_R, \end{aligned} \tag{13}$$

the mass matrices M_l, M_ν can be written

$$\begin{aligned} {M'}_l &= W_L M_l W_R^{l\ \dagger}, \\ {M'}_L &= W_L M_L W_L^T, \\ {M'}_D &= W_L M_D W_R^{\nu\ \dagger}, \\ {M'}_R &= W_R^{\nu\ *} M_R W_R^{\nu\ \dagger}, \end{aligned} \tag{14}$$

and satisfy Eqs. (12) but with unitary matrices

$$
\begin{array}{c}
{V'}_L = W_L V_L W_L^T, \\
{V'}_R^{\,l} = W_R^l V_R^l W_R^{l\ T}, \\
{V'}_R^{\,\nu} = W_R^\nu V_R^\nu W_R^{\nu\ T}.
\end{array}
\tag{15}
$$

Although Eqs. (12) are the necessary and sufficient conditions for CP conservation, they are of little practical use. However, they suggest, as we show below, how to construct CP invariant constraints which do not depend explicitly on the unitary matrices $V_L, V_R^{l,\nu}$, and are then more useful.

First, we introduce a convenient basis to parametrize CP violation. In this basis we will prove in simple cases afterwards that the more useful CP invariant constraints are not only necessary but sufficient.

2.3. CP conserving gauge interactions in the mass eigenstate basis and counting of CP breaking phases in the lepton sector

Using the freedom to choose the weak basis we can assume M_l and M_R diagonal with real positive elements (see Eqs. (14)). In this basis Eqs. (12) for M_l and M_R imply (for non-degenerate charged lepton masses and non-degenerate diagonal M_R elements)

$$
\begin{array}{c}
(V_L)_{ij} = (V_R^l)_{ij} = e^{i\delta_i}\delta_{ij}, \\
(V_R^\nu)_{ij} = e^{i\alpha_i}\delta_{ij},
\end{array}
\tag{16}
$$

where α_i is equal to 0 or π (arbitrary) for non-zero (vanishing) $(M_R)_{ii}$. Then Eqs. (12) for M_L and M_D are satisfied and CP conserved if and only if

$$
\begin{array}{rl}
(M_L)_{ij} = & (M_L^r)_{ij}\ e^{i\frac{1}{2}(\delta_i+\delta_j)}, \\
(M_D)_{ij} = & (M_D^r)_{ij}\ e^{i\frac{1}{2}(\delta_i-\alpha_j)},
\end{array}
\tag{17}
$$

with $(M_L^r)_{ij}$ and $(M_D^r)_{ij}$ real. Any extra phase violates CP. In order to count the CP breaking phases, let us see how look like the CP conserving gauge interactions in the mass eigenstate basis. In the weak basis above, if CP is conserved (see Eqs. (17)),

$$
M_\nu = \begin{pmatrix} \Sigma & 0 \\ 0 & \Pi \end{pmatrix} \begin{pmatrix} M_L^r & M_D^r \\ M_D^{r\ T} & M_R^r \end{pmatrix} \begin{pmatrix} \Sigma & 0 \\ 0 & \Pi \end{pmatrix},
\tag{18}
$$

with

$$\begin{aligned} \Sigma_{ij} &= e^{i\frac{\delta_i}{2}}\delta_{ij}, \\ \Pi_{ij} &= e^{-i\frac{\alpha_i}{2}}\delta_{ij}, \end{aligned} \tag{19}$$

and $M^r_{L,D}$ real and $M^r_R = \Pi^2 M_R$, where M_R is a diagonal matrix with real positive elements. In the mass eigenstate basis with m_i the (positive) neutrino masses,

$$\begin{pmatrix} m_1 & & & \\ & m_2 & & \\ & & \ddots & \\ & & & m_{n_L+n_R} \end{pmatrix} = U^T M_\nu U, \tag{20}$$

where U is the unitary matrix diagonalizing M_ν. For M_ν in Eqs. (18,19) U can be written

$$U = \begin{pmatrix} U_L^* \\ U_R \end{pmatrix} = \begin{pmatrix} \Sigma^* & 0 \\ 0 & \Pi^* \end{pmatrix} O\Lambda, \tag{21}$$

where $O = \binom{O_L}{O_R}$ is the real orthogonal $(n_L + n_R) \times (n_L + n_R)$ matrix diagonalizing

$$\begin{pmatrix} M_L^r & M_D^r \\ M_D^{r\,T} & M_R^r \end{pmatrix} = O \begin{pmatrix} \epsilon_1 m_1 & & & \\ & \epsilon_2 m_2 & & \\ & & \ddots & \\ & & & \epsilon_{n_L+n_R} m_{n_L+n_R} \end{pmatrix} O^T, \tag{22}$$

with $\epsilon_i = 1$ or -1, and Λ the diagonal matrix guaranteeing positive neutrino masses:

$$\Lambda_{ij} = \delta_{ij} e^{i\frac{\pi}{4}(1-\epsilon_i)}. \tag{23}$$

Then, Eq. (6) reads in the mass eigenstate basis (calling the neutrino mass eigenstates $N_L = U^T \psi_L$)

$$\begin{aligned} \mathcal{L}_{CC} &= -\frac{g}{\sqrt{2}} \bar{N}_L U_L^\dagger \gamma^\mu l_L W_\mu^+ + h.c. \\ &= -\frac{g}{\sqrt{2}} \bar{N}_L \Lambda O_L^T \gamma^\mu \Sigma^* l_L W_\mu^+ + h.c., \end{aligned} \tag{24}$$

where $U_L^* = \Sigma^* O_L \Lambda$ and O_L above are rectangular matrices corresponding to the first n_L rows of U and O, respectively. The Σ^* phases can be absorbed in the charged lepton mass eigenstates defining them equal to $e^{-i\frac{\delta_i}{2}} l_{Li}$. Thus these phases do not violate CP because they are unphysical. The mixing of the charged current reduces then to the real matrix O_L^T. (The Λ phases can be associated to the neutrino eigenstate definition as we argue below.) The neutral current lagrangian in Eq. (7) remains unchanged for the charged lepton mass eigenstates; whereas for the neutrinos reads

$$\begin{array}{l} \mathcal{L}_{NC} = -\frac{g}{2cos\theta_W}\ \bar{N}_L U_L^\dagger \gamma^\mu U_L N_L\ Z_\mu \\ = -\frac{g}{2cos\theta_W}\ \bar{N}_L \Lambda O_L^T O_L \gamma^\mu \Lambda^* N_L\ Z_\mu . \end{array} \tag{25}$$

In conclusion, the only possible complex factors in the gauge interactions are the elements of the diagonal matrix Λ:

$$\Lambda_{kk} = 1(i) \text{ for } \epsilon_k = 1(-1). \tag{26}$$

At any rate CP is conserved if we define the CP parity of the Majorana neutrino mass eigenstate k

$$\eta_{CP}(k) = \epsilon_k i. \tag{27}$$

In this case all Majorana neutrinos satisfy

$$N^C \equiv C\bar{N}^T = N. \tag{28}$$

Other authors prefer to get rid of the Λ phases in the lagrangian, defining the Majorana neutrino mass eigenstates equal to $\Lambda^* N_L$ [7], and considering Λ^* as creation phase factors [8]. In this case the right-hand side of Eq. (28) is equal to $-N$ for Majorana neutrinos with $\eta_{CP} = -i$. We prefer to keep (28) generic and the i factors in the gauge interactions for Majorana neutrinos with $\eta_{CP} = -i$.

If U_L is of the form in Eq. (21), the gauge interactions in Eqs. (24,25) conserve CP. Any extra phase gives rise to CP violation. Hence, the number of CP violating phases is equal to the number of independent phases in n_L rows of a $(n_L + n_R) \times (n_L + n_R)$ unitary matrix ($n_L(n_L + n_R) - \frac{n_L(n_L-1)}{2}$) minus the number of phases which can be absorbed in the charged lepton mass eigenstate definition (n_L) [5]:

$$n_L(n_L + n_R) - \frac{n_L(n_L - 1)}{2} - n_L = \frac{n_L(n_L + 2n_R - 1)}{2}. \tag{29}$$

2.4. CP invariant constraints on the lepton mass matrices

In this Section we derive from Eqs. (12) neccesary conditions for CP conservation which are independent of the weak basis and do not require to know the unitary matrices involved in the definition of the CP transformation. If these conditions, which are simple functions of the mass matrices, are not satisfied, CP is violated. That they are not only necessary but sufficient has to be proven case by case. It looks feasible (and is proven) in (the) simple(st) cases only. This formulation provides (and we obtain) the factors suppressing CP violating observables. Their knowledge will allow for a discussion of the size of CP violating effects.

Motivated by Eqs. (12) and the quark case we classify the products of the mass matrices $M_{l,L,D,R}$, as well as their sums, in three classes G_L, G^l_R, G^ν_R, depending under which unitary matrix V_L, V^l_R, V^ν_R they transform, respectively:

$$\begin{array}{c} V_L^\dagger G_L V_L = G_L^*, \\ V_R^{l\dagger} G_R^l V_R^l = G_R^{l*}, \\ V_R^{\nu\dagger} G_R^\nu V_R^\nu = G_R^{\nu*}. \end{array} \tag{30}$$

To these classes belong:

$$\begin{array}{rcc} \{G_L\} & = & \{A_{L1} = M_l M_l^\dagger; A_{L2} = M_L M_L^\dagger; A_{L3} = M_D M_D^\dagger; \\ & & A_{Li}A_{Lj}, i,j = 1,2,3; M_L M_l^* M_l^T M_L^\dagger; \\ & & M_L M_D^* M_D^T M_L^\dagger; M_L M_D^* M_R M_D^\dagger; M_D M_R^\dagger M_R M_D^\dagger; \\ & & \text{and higher order products; and sums}\}, \\ \{G_R^l\} & = & \{A_l = M_l^\dagger M_l; A_l^2; M_l^\dagger M_L M_L^\dagger M_l; M_l^\dagger M_D M_D^\dagger M_l; \\ & & \text{and higher order products; and sums}\}, \\ \{G_R^\nu\} & = & \{A_{\nu 1} = M_D^\dagger M_D; A_{\nu 2} = M_R^\dagger M_R; A_{\nu i}A_{\nu j}, i,j = 1,2; \\ & & M_D^\dagger M_L M_L^\dagger M_D; M_D^\dagger M_L M_D^* M_R; M_R^\dagger M_D^T M_D^* M_R; \\ & & \text{and higher order products; and sums}\}. \end{array} \tag{31}$$

Now observing that the trace and the determinant of any element of these classes is invariant under unitary transformations, and then under weak basis transformations, and that Eqs. (30), which follow from CP conservation, implies that these traces and determinants are real, we can write a set of necessary conditions for CP conservation which are weak basis invariant:

$$\begin{array}{c} ImTr(G) = 0, \\ ImDet(G) = 0, \end{array} \tag{32}$$

where G is any element of $\{G_L\}, \{G_R^l\}$ or $\{G_R^\nu\}$ in Eqs. (31).

Eqs. (32) are the corner stone of our analysis. They apply to any number of standard families n_L and right-handed neutrinos n_R. Although these conditions are all necessary, they are not all independent. To find a set of conditions which are also sufficient seems to be difficult in general. We shall obtain such a subset of necessary and sufficient CP conserving conditions for some simple (lowest n_L and n_R) cases only.

We will find a set of necessary and sufficient conditions for $n_L + n_R < 3$. In each model we state the number of CP violating phases, Eq. (29); the set of necessary (and sufficient) CP constraints; a parametrization of M_ν in the convenient weak basis where M_l and M_R are diagonal with real (and positive M_l) elements; the expressions of the CP constraints for this parametrization; the proof that if these invariants vanish CP is conserved (M_l and M_ν can be made real); and the expressions of these conditions as functions of physical observables (masses and mixing angles). The cases with $n_L + n_R = 3$ are also discussed.

- $n_L = 0, n_R;\ n_L = 1, n_R = 0$:

There is no CP violation (and, of course, no CP violating phase) in these models.

- $n_L = 1, n_R = 1$:

In this case there is one CP violating parameter. A necessary and sufficient condition for CP conservation is

$$\Delta_{11} = ImTr(M_D^\dagger M_L M_D^* M_R) = 0. \tag{33}$$

$M_{D,L,R}$ are one-dimensional in this model and we can write

$$M_\nu = \begin{pmatrix} m_L & ae^{i\alpha} \\ ae^{i\alpha} & m_R \end{pmatrix}, \tag{34}$$

in the convenient weak basis, which is completely specified requiring $a \geq 0,\ \alpha \in [0, \frac{\pi}{2})$. In this parametrization

$$\Delta_{11} = -m_L m_R a^2 sin(2\alpha). \tag{35}$$

The vanishing of Δ_{11} is apparently a sufficient condition for CP conservation: M_ν is real for a or $\alpha = 0$, and it can be made real by a field redefinition for m_L or $m_R = 0$, choosing $W_L = e^{-i\alpha}$ or $W_R^\nu = e^{i\alpha}$, respectively, in Eq. (14). As a function of physical observables

$$\Delta_{11} = m_1 m_2 (m_2^2 - m_1^2) Im(U_{11}^{*\,2} U_{12}^2). \tag{36}$$

Thus, CP is conserved if there is a massless neutrino (m_1 or $m_2 = 0$), or the neutrinos are degenerate ($m_1 = m_2$). On the other hand, any CP violating effect is proportional to $Im(U_{11}^{*\,2} U_{12}^2)$, where $U_{11}(U_{12})$ is the element in the mixing matrix in Eq. (24) fixing the charged coupling of the charged lepton to the neutrino of mass $m_1(m_2)$.

- $n_L = 2, n_R = 0$:

This model has deserved some attention [9]. It has one CP violating parameter and one necessary and sufficient condition for CP conservation is

$$\Delta_{20} = ImDet(M_L M_l^* M_l^T M_L^\dagger - M_L M_L^\dagger M_l M_l^\dagger) = 0. \tag{37}$$

In the basis where M_l is diagonal with charged lepton masses m_e, m_μ and

$$M_\nu = M_L = \begin{pmatrix} a & be^{i\beta} \\ be^{i\beta} & c \end{pmatrix}, \; a, b \geq 0, \; \beta \in [0, \frac{\pi}{2}), \tag{38}$$

$$\Delta_{20} = -(m_\mu^2 - m_e^2)^2 acb^2 sin(2\beta). \tag{39}$$

It is easy to prove that $\Delta_{20} = 0$ is a sufficient condition for CP conservation: M_ν is real for b or $\beta = 0$, and it can be made real for $a = 0$; and $c = 0$, choosing in Eqs. (13-15)

$$W_L = W_R^l = \begin{pmatrix} e^{-i\beta} & 0 \\ 0 & 1 \end{pmatrix}; \text{ and } W_L = W_R^l = \begin{pmatrix} 1 & 0 \\ 0 & e^{-i\beta} \end{pmatrix}, \tag{40}$$

respectively. Δ_{20} also vanishes for $m_e = m_\mu$, but in this case M_l can be diagonal and $W_L = W_R^l$ be still an arbitrary unitary matrix. Hence, M_ν can be made not only real but diagonal and positive by an appropriate choice of this unitary transformation:

$$W_L M_\nu W_L^T = \begin{pmatrix} m_1 & \\ & m_2 \end{pmatrix}, \tag{41}$$

where $m_{1,2}$ are the neutrino masses. As a function of physical observables

$$\Delta_{20} = m_1 m_2 (m_2^2 - m_1^2)(m_\mu^2 - m_e^2)^2 Im(U_{11}^{*\,2} U_{12}^2). \tag{42}$$

Hence, CP violation requires two massive ($m_{1,2} \neq 0$) and non-degenerate ($m_1 \neq m_2$) neutrinos, as well as two non-degenerate charged leptons ($m_e \neq m_\mu$). As for $n_L = n_R = 1$ any CP violating effect is proportional to $Im(U_{11}^{*\,2} U_{12}^2)$, where $U_{11}(U_{12})$ is the element in the mixing matrix in Eq. (24) fixing the charged coupling of one charged lepton, let say e, to the neutrino of mass $m_1(m_2)$. (Notice that $Im(U_{11}^{*\,2} U_{12}^2) = Im(U_{21}^{*\,2} U_{22}^2)$.)

- $n_L + n_R \geq 3$:

There are two CP violating phases for $n_L = 1, n_R = 2$. Two necessary and sufficient conditions for CP conservation are

$$\begin{gathered} \Delta_{12}^{(1)} = ImTr(M_D^\dagger M_L M_D^* M_R) = 0, \\ \Delta_{12}^{(2)} = ImDet(M_R^\dagger M_D^T M_D^* M_R - M_D^\dagger M_D M_R^\dagger M_R) = 0. \end{gathered} \tag{43}$$

There are three CP violating phases for $n_L = 2, n_R = 1$. It can be proved, after some work, that

$$\begin{gathered} \Delta_{21}^{(1)} = ImDet(M_L M_l^* M_l^T M_L^\dagger - M_L M_L^\dagger M_l M_l^\dagger) = 0, \\ \Delta_{21}^{(2)} = ImDet(M_D M_D^\dagger M_L M_L^\dagger - M_L M_D^* M_D^T M_L^\dagger) = 0, \\ \Delta_{21}^{(3)} = ImDet(M_D M_D^\dagger M_l M_l^\dagger - M_L M_D^* M_D^T M_L^\dagger) = 0, \\ \Delta_{21}^{(4)} = ImDet(M_D M_D^\dagger M_l M_l^\dagger - M_l M_l^\dagger M_L M_L^\dagger) = 0, \\ \Delta_{21}^{(5)} = ImDet(M_L M_l^* M_l^T M_L^\dagger - M_D M_D^\dagger M_l M_l^\dagger) = 0, \\ \Delta_{21}^{(6)} = ImTr(M_D^\dagger M_L M_D^* M_R) = 0, \\ \Delta_{21}^{(7)} = ImTr(M_l M_l^\dagger M_D M_R^* M_D^T M_l^* M_l^T M_L^\dagger) = 0, \\ \Delta_{21}^{(8)} = ImTr((M_l M_l^\dagger)^2 M_D M_R^* M_D^T (M_l^* M_l^T)^2 M_L^\dagger) = 0, \end{gathered} \tag{44}$$

form a set of necessary and sufficient conditions for CP conservation.

For $n_L = 3, n_R = 0$, although there are three CP breaking phases as for $n_L = 2, n_R = 1$, it seems difficult to find a subset of sufficient conditions for CP conservation. Analogously to the quark case [3] the obstacle, which is generic for large(r) n_L, n_R, is the non-linearity of conditions (32) on the CP breaking phases. We expect to handle this case, as well as the models with

$n_L = n_R = 2, 3$, with a computer.

3. Extended models with left-right symmetry

The analysis of left-right models parallels the analysis of extended models with extra right-handed neutrinos only. The number of left-handed and right-handed neutrinos is now the same $n_L = n_R = n$. There are also extra charged and neutral currents which further constrain the CP transformations.

3.1. CP invariance

The gauge interactions can be written in this case [10]:

$$\mathcal{L}_{CC} = -\frac{g}{\sqrt{2}}(\bar{\nu}_L \gamma^\mu l_L W^+_{L\mu} + \kappa \bar{\nu}_R \gamma^\mu l_R W^+_{R\mu}) + h.c., \tag{45}$$

and

$$\begin{aligned} \mathcal{L}_{NC} = e\ (\bar{l}_L\gamma^\mu l_L + \bar{l}_R\gamma^\mu l_R)\ A_\mu \\ + \tfrac{g}{2cos\theta_W}\ \left(-\bar{\nu}_L\gamma^\mu\nu_L + (1-2sin^2\theta_W)\ \bar{l}_L\gamma^\mu l_L - 2sin^2\theta_W\ \bar{l}_R\gamma^\mu l_R\right)\ Z_\mu \\ - \tfrac{e}{2cos\theta_W}\tfrac{1}{\alpha}\ (\bar{\nu}_L\gamma^\mu\nu_L + (1+\alpha^2)\bar{\nu}_R\gamma^\mu\nu_R + \bar{l}_L\gamma^\mu l_L + (1-\alpha^2)\bar{l}_R\gamma^\mu l_R)\ Z_{LR\ \mu}, \end{aligned} \tag{46}$$

with $\alpha = \sqrt{\kappa^2 cot^2\theta_W - 1}$ real. $W^\pm_{L,R}$ are the charged gauge bosons associated to $SU(2)_{L,R}$; and $Z, Z_{L,R}$ are the two heavy neutral gauge bosons associated to the standard model and its left-right (LR) extension, respectively. Whereas $l_{L,R}$ and $\nu_{L,R}$ are $n_L = n_R$ component vectors in flavour space describing left-, right-handed charged leptons and left-, right-handed neutrinos. $\mathcal{L}_{CC,NC}$ are now left invariant by any CP transformation

$$\begin{aligned} \nu_L &\longrightarrow V_L C \nu_L^*, \\ l_L &\longrightarrow V_L C l_L^*, \\ l_R &\longrightarrow V_R C l_R^*, \\ \nu_R &\longrightarrow V_R C \nu_R^*, \end{aligned} \tag{47}$$

where $V_{L,R}$ are $n \times n$ unitary matrices. On the other hand, the mass matrices $M_{l,L,D,R}$ are all $n \times n$. The invariance of the mass terms in Eq. (10) under the CP transformations in Eq. (47) implies

$$\begin{aligned} V_L^\dagger M_l V_R &= M_l^*, \\ V_L^\dagger M_L V_L^* &= M_L^*, \\ V_L^\dagger M_D V_R &= M_D^*, \\ V_R^T M_R V_R &= M_R^*. \end{aligned} \tag{48}$$

These conditions are necessary and sufficient for CP conservation in left-right symmetric models.

3.2. Weak basis independence

As before conditions (48) are weak basis independent: Eqs. (13-15) apply but with $W_R^l = W_R^\nu = W_R,\ V_R^l = V_R^\nu = V_R$ and ${V'}_R^l = {V'}_R^\nu = {V'}_R$.

3.3. CP conserving gauge interactions in the mass eigenstate basis and counting of CP breaking phases in the lepton sector

The freedom to choose the weak basis allows to assume that M_l is diagonal. In this basis Eq. (48) for M_l (for non-degenerate charged lepton masses) implies

$$(V_L)_{ij} = (V_R)_{ij} = e^{i\delta_i}\delta_{ij}. \tag{49}$$

Then Eqs. (48) for $M_{L,D,R}$ are satisfied and CP conserved if and only if

$$\begin{aligned} (M_L)_{ij} = &\ (M_L^r)_{ij}\ e^{i\frac{1}{2}(\delta_i+\delta_j)}, \\ (M_D)_{ij} = &\ (M_D^r)_{ij}\ e^{i\frac{1}{2}(\delta_i-\delta_j)}, \\ (M_R)_{ij} = &\ (M_R^r)_{ij}\ e^{-i\frac{1}{2}(\delta_i+\delta_j)}, \end{aligned} \tag{50}$$

with $(M_{L,D,R}^r)_{ij}$ real. The number of possible CP violating phases is larger in left-right models because the definition of a CP transformation is less general. To write the CP conserving gauge interactions for left-right models in the mass eigenstate basis, we follow the same steps as in Section 2.3. Now $(\Pi = \Sigma^*)$ [11]

$$U = \begin{pmatrix} \Sigma^* & 0 \\ 0 & \Sigma \end{pmatrix} O\Lambda, \tag{51}$$

and

$$\begin{aligned} \mathcal{L}_{CC} &= -\frac{g}{\sqrt{2}}(\bar{N}_L U_L^\dagger \gamma^\mu l_L W_{L\ \mu}^+ + \kappa \bar{N}_R U_R^\dagger \gamma^\mu l_R W_{R\ \mu}^+) + h.c. \\ &= -\frac{g}{\sqrt{2}}(\bar{N}_L \Lambda O_L^T \gamma^\mu \Sigma^* l_L W_{L\ \mu}^+ + \kappa \bar{N}_R \Lambda^* O_R^T \gamma^\mu \Sigma^* l_R W_{R\ \mu}^+) + h.c., \end{aligned} \tag{52}$$

where $N_{L,R} = U^T \psi_{L,R}$ are the neutrino mass eigenstates. As in Eq. (24) the Σ^* phases can be absorbed in the charged lepton mass eigenstates defining then equal to $e^{-i\frac{\delta_i}{2}} l_{L,R\ i}$. The neutral current lagrangian in Eq. (46) remains unchanged for the charged lepton mass eigenstates; whereas for the neutrinos can be written

$$\begin{array}{c} \mathcal{L}_{NC} = -\frac{g}{2cos\theta_W} \bar{N}_L U_L^\dagger \gamma^\mu U_L N_L Z_\mu \\ -\frac{e}{2cos\theta_W} \frac{1}{\alpha} (\bar{N}_L U_L^\dagger \gamma^\mu U_L N_L + (1+\alpha^2) \bar{N}_R U_R^\dagger \gamma^\mu U_R N_R) Z_{LR\ \mu} \\ = -\frac{g}{2cos\theta_W} \bar{N}_L \Lambda O_L^T O_L \gamma^\mu \Lambda^* N_L Z_\mu \\ -\frac{e}{2cos\theta_W} \frac{1}{\alpha} (\bar{N}_L \Lambda O_L^T O_L \gamma^\mu \Lambda^* N_L + (1+\alpha^2) \bar{N}_R \Lambda^* O_R^T O_R \gamma^\mu \Lambda N_R) Z_{LR\ \mu}. \end{array} \tag{53}$$

The number of possible CP violating phases in $\mathcal{L}_{CC,NC}$ (Eqs. (52,53)) for a general $2n \times 2n$ unitary matrix $U = \binom{U_L^*}{U_R}$ is equal to the number of independent phases in U ($\frac{2n(2n+1)}{2}$) minus the number of phases which can be absorbed in the charged lepton mass eigenstate definition (n) [5]:

$$n(2n+1) - n = 2n^2. \tag{54}$$

3.4. CP invariant constraints on the lepton mass matrices

As in Section 2.4. we shall derive necessary conditions for CP conservation which are independent of the weak basis and do not require to know the unitary matrices involved in the definition of the CP transformation. That a subset of them is also sufficient has to be proven case by case.

The products of the mass matrices $M_{l,L,D,R}$, as well as their sums, can be classified in two classes $G_{L,R}$, which under $V_{L,R}$ transform, respectively:

$$\begin{array}{c} V_L^\dagger G_L V_L = G_L^*, \\ V_R^\dagger G_R V_R = G_R^*. \end{array} \tag{55}$$

To these classes belong:

$$\begin{array}{rcl} \{G_L\} & = & \{A_{L1} = M_l M_l^\dagger; A_{L2} = M_L M_L^\dagger; A_{L3} = M_D M_D^\dagger; \\ & & A_{L4} = M_l M_D^\dagger; A_{L5} = M_D M_l^\dagger; A_{Li} A_{Lj}, \\ & & i,j = 1,2,...,5; B_i B_j^\dagger, i,j = 1,2,...,4; \\ & & \text{and higher order products; and sums}\}, \\ \{G_R\} & = & \{A_{R1} = M_l^\dagger M_l; A_{R2} = M_D^\dagger M_D; A_{R3} = M_R^\dagger M_R; \\ & & A_{R4} = M_l^\dagger M_D; A_{R5} = M_D^\dagger M_l; A_{Ri} A_{Rj}, \\ & & i,j = 1,2,...,5; B_i^T B_j^*, i,j = 1,2,...,4; \\ & & \text{and higher order products; and sums}\}, \end{array} \tag{56}$$

where $B_1 = M_l M_R^\dagger;\ B_2 = M_L M_l^*;\ B_3 = M_L M_D^*;\ B_4 = M_D M_R^\dagger$. As before, Eqs. (32) but with G any element of $\{G_L\}$ or $\{G_R\}$ in Eq. (56), are a set of necessary conditions for CP conservation which are weak basis invariant. They follow from Eqs. (55) and the invariance of the trace and the determinant under unitary transformations. We shall obtain a subset of sufficient conditions for $n = 1$. For $n = 2$ there are eight CP violating phases. A set of necessary and sufficient conditions will include the constraints in Eqs. (43,44) and many more, requiring its analysis a long casuistry. Besides, for larger n there is the same difficulty to find a subset of sufficient constraints as for extended models with only extra right-handed neutrinos: conditions (32) are non-linear on the CP breaking phases. We expect to handle these cases with the help of a computer.

- $n = 1$:

There are two CP violating parameters in this case (which is the simplest one). Two necessary and sufficient conditions for CP conservation are

$$\begin{aligned} \Delta_1^{(1)} &= ImTr(M_l M_D^\dagger) = 0, \\ \Delta_1^{(2)} &= ImTr(M_l M_R^\dagger M_l^T M_L^\dagger - M_L M_D^* M_R M_D^\dagger) = 0. \end{aligned} \tag{57}$$

The neutrino mass matrix can be written in the convenient weak basis where $M_{l,L}$, which are one-dimensional, are real and M_l is also positive

$$M_\nu = \begin{pmatrix} m_L & ae^{i\alpha} \\ ae^{i\alpha} & m_R e^{i\phi} \end{pmatrix},\ a \geq 0,\ \alpha, \phi \in [0, \pi). \tag{58}$$

In this parametrization

$$\begin{aligned} \Delta_1^{(1)} &= -m_e a sin(\alpha), \\ \Delta_1^{(2)} &= -m_L m_R (m_e^2 sin(\phi) + a^2 sin(\phi - 2\alpha)), \end{aligned} \tag{59}$$

with m_e the charged lepton mass. The vanishing of $\Delta_1^{(1,2)}$ is sufficient for CP conservation: M_ν is real for $a, m_R = 0$; $\alpha, m_R = 0$; and $\alpha, \phi = 0$, and it can be made real by a field redefinition for $m_e, a = 0$ or $m_e = 0, \phi - 2\alpha = 0, -\pi$; $m_L, a = 0$ or $m_L, \alpha = 0$; $m_e, m_L = 0$; and $m_e, m_R = 0$, choosing $W_R = e^{i\frac{\phi}{2}}$; $W_L = W_R = e^{i\frac{\phi}{2}}$; $W_L = e^{i(\frac{\phi}{2} - \alpha)},\ W_R = e^{i\frac{\phi}{2}}$; and $W_R = e^{i\alpha}$, respectively. As a function of physical observables

$$\begin{aligned} \Delta_1^{(1)} &= m_e Im(m_1 U_{11} U_{21} + m_2 U_{12} U_{22}), \\ \Delta_1^{(2)} &= m_e^2 Im((m_1 U_{11}^2 + m_2 U_{12}^2)(m_1 U_{21}^2 + m_2 U_{22}^2)) \\ &\quad - m_1 m_2 (m_2^2 - m_1^2) Im({U_{11}^*}^2 U_{12}^2), \end{aligned} \tag{60}$$

where $m_{1,2}$ are the neutrino masses and U_{ij} are the $U = \binom{U_L^*}{U_R}$ matrix elements fixing the charged coupling of the left- (right-) handed charged lepton $i = 1(2)$ to the neutrino of mass m_j (see Eq. (52)). This model has two (independent) physical phases, which we associate to $P = m_1 U_{11} U_{21} + m_2 U_{12} U_{22}$ and $Q = (m_1 U_{11}^2 + m_2 U_{12}^2)(m_1 U_{21}^2 + m_2 U_{22}^2)$ in Eq. (60), respectively. Then, any CP violating effect can be written as a function of P and/or Q. In particular, the second term of $\Delta_1^{(2)}$ in (60) is equal to $-Im(P^2 Q^*)$.

4. CP violating effects and conclusions

In this paper we study the formulation of CP violation in the lepton sector in extended models with right-handed neutrinos, without and with left-right symmetry. We obtain necessary conditions for CP conservation for any number of standard families n_L and right-handed neutrinos n_R (in left-right symmetric models $n_L = n_R = n$). These are independent of the weak basis used to express the charged lepton and the neutrino mass matrices. However, they are not all independent, and in general it appears difficult to find a subset of sufficient conditions . Proceeding case by case we do find such a set of necessary and sufficient conditions for the simplest cases (lowest n_L, n_R): $n_L = 0, n_R$; $n_L = 1, n_R = 0$; $n_L = 1, n_R = 1$; $n_L = 2, n_R = 0$; $n_L = 1, n_R = 2$; $n_L = 2, n_R = 1$, and $n = 1$. The proof takes profit of the freedom to choose the weak basis. This is fixed requiring that the charged lepton mass matrix be diagonal with real positive eigenvalues and the neutrino mass matrix have as many diagonal blocks and real (positive) diagonal elements as possible. For cases with larger n_L, n_R we have to rely on a computer.

This invariant formulation allows for deciding more easily if CP is conserved because the CP conserving constraints can be simply calculated in any weak basis. It may also help to understand the origin of CP violation if a pattern of leptonic mass matrices is distinguished for some physical reason. It can be used as a guide for model buildig. However, the main practical application of constructing such a set of necessary and sufficient conditions for CP conservation follows from observing that any CP violating effect is proportional to weak basis invariants, and then to the invariant factors appearing in these constraints. This permits to discuss the possible suppression factors and then the size of the CP violating observables.

As an example we can compare the cases $n_L = n_R = 1$, without, and $n = 1$, with left-right symmetry. CP is violated for $n_L = n_R = 1$ if and only if (see Section 2.4)

$$Im((m_1 U_{11} U_{21} + m_2 U_{12} U_{22})^2 ((m_1 U_{11}^2 + m_2 U_{12}^2)(m_1 U_{21}^2 + m_2 U_{22}^2))^*) \neq 0,$$

and for $n = 1$ if and only if (see Section 3.4)

$$m_e Im(m_1 U_{11} U_{21} + m_2 U_{12} U_{22}) \neq 0, \text{ and/or}$$

$$m_e^2 Im((m_1 U_{11}^2 + m_2 U_{12}^2)(m_1 U_{21}^2 + m_2 U_{22}^2)) - m_1 m_2 (m_2^2 - m_1^2) Im({U_{11}^*}^2 U_{12}^2) \neq 0.$$

Therefore, CP violation can be larger in left-right models because any CP violating effect in the presence of right-handed currents is a function of $P = m_1 U_{11} U_{21} + m_2 U_{12} U_{22}$ and/or $Q = (m_1 U_{11}^2 + m_2 U_{12}^2)(m_1 U_{21}^2 + m_2 U_{22}^2)$; whereas a CP violating observable in their absence is a function of the product $P^2 Q^*$ only, whose imaginary part above is equal to $m_1 m_2 (m_2^2 - m_1^2) Im({U_{11}^*}^2 U_{12}^2)$. This is generic. In left-right models CP can be violated in left-handed as well as in right-handed currents. That CP violation can be larger in left-right models can be seen when producing two heavy neutrinos $N_1 N_2$ at $e^+ e^-$ [11].

Acknowledgements

This work was partially supported by CICYT under contract AEN94-0936. F. A. was also supported by the Junta de Andalucía and by the European Union under contract CHRX-CT92-0004 and M. Z. by the Curie Sklodowska grant MEN/NSF 93-145.

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