Towards an ultra-local model control of two-tank-system

Hajer Thabet • Mounir Ayadi • Frédéric Rotella

Abstract This paper deals with the design of an ultra-local model control. The proposed approach is based on the estimation of the ultra-local model parameters using least squares resolution technique instead of numerical derivation technique. The closed-loop control is implemented through an adaptive PI in order to reject the influences of the disturbance and noise output signais. Its main advantages are: its simplicity and its robustness with respect to the parameter uncertainties of system. In this paper, it is processed to test the efficiency of the parameter estimation method compared with the performance of numerical derivation technique. The method is applied to the water level control of a two-tank-system. Numerical simulations show that the generated desired trajectory is followed in an efficient way even with severe operating conditions.

Keywords Ultra-local model control • Least squares method • Robustness analysis • Adaptive PI controller • Two-tank-system

1 Introduction

Today, the complex systems control remains an open problem. Their implemented solutions are often partial and of significant complexity because of the need to find an accurate model of the system. In this case, instead of relying on a more accurate knowledge structure of the controlled system model, the ultra-local model control (called also free-model control) is based on a simple local modeling decorrelated from the physical reality. This approach, recently introduced by Fliess and Join [1- 3], is also based on the rapid estimation techniques [4]. It does not estimate the unknown parameters. Instead, it estimates a variable composed of error model (coming from the difference between the unknown real model and the simple used model) and disturbances.

This new control approach has many advantages. First it is easy to implement, it is also highly robust. Moreover, the time of implementation is reduced thanks to the low parameter number of design. The advantages of ultra-local mode] control and of the corresponding adaptive PID controllers led to a number of exciting applications in various fields [1,4- 14]. For example, in [1], numerical derivation (ND) techniques have been employed to estimate noisy signais. This estimator can be easily implemented in the form of discrete-time linear filter to mitigate the measurements noises. However, the estimation of a single variable by the numerical derivation technique is insufficient to obtain the desired performance, particularly, if the estimation of the second parameter is required. For this reason and in order to improve these performances, we propose in this paper a new ultra-local model control approach based on the linear least squares (LS) technique to estimate the ultra-local model variables.

The comparison between the numerical derivation technique and the linear least squares method estimating the ultra-local model parameters, is kept here to clarify the performance improvement and effectiveness of proposed controller design.

The present work deals with the automatic water level control in the flow channels which was developed by an abundant

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literature (see [10, 15- 17]). The water level control constitutes a desired trajectory tracking problem with rejection of flow disturbances with nonlinear dynamics. In this paper, the ultra-local model control is applied to two-tank water system which is considered as a nonlinear system of first-order.

The paper is organised as follows: the concept of ultra-local model control is presented in the second section. Two different methods of parameter estimation (the least squares and the numerical derivation methods) are elaborated in Sect. 3. The two-tank-system model, the control design and the simulation results are given in the Sect. 4. Finally, some concluding remarks are presented in the Sect. 5.

2 Ultra-local model control

2.1 Basic idea

The ultra-local model control is based on local modeling, constantly updated, from the solely knowledge of input-output behavior. For the unknown differential equation:

$$E(y, \dot{y}, \ldots, y^{(l)}, u, \dot{u}, \ldots, u^{(K)}) = 0 \quad (1)$$

which can be linear or not, where u is the system input, y is the system output and E is a sufficiently smooth fonction of its arguments, we assume that for an integer v, $0 < v < L$, $\frac{\partial E}{\partial y^{(v)}} \neq 0$, the implicit fonction theorem then allows to write the following equation:

$$y^{(v)} = \varphi(t, y, \dot{y}, \ldots, y^{(v-1)}, y^{(v+1)}, \ldots, y^{(y)}, u, \dot{u}, \ldots, u^{(K)}) \quad (2)$$

approximatively describing the input-output behavior. Ultra-local model control consists in trying to estimate via the input and the output measurements what can be compensated by control in order to achieve a good output trajectory tracking. This implies the construction of a purely numerical model also called ultra-local model of the system that can be written as:

$$y^{(v)}(t) = F(t) + \alpha(t)\, u(t) \quad (3)$$

The two quantities $F(t)$ and $\alpha(t)$ contain the whole structural information which should be identified in real time.

Remark 1 The order of derivation v of y in (3) is strictly less in general than y in (1), when this latter is known [1].

In many works, Fliess and Join indicate that in practice it is appropriate to consider an ultra-local model (3) in the cases of $v = 1$ or $v = 2$.

2.2 Adaptive PI controller

If $v = 1$ in (3), the desired closed-loop behavior is obtained thanks to an adaptive PI or, in abbreviated, a-PI [1]. The control signal is given by:

$$u(t) = \frac{-F(t) + \dot{y}_d(t) + K_p e(t) + K_I \int e(t)}{\alpha(t)} \quad (4)$$

where

- $y_d(t)$ is the desired output trajectory, obtained by flatness properties [18- 20) that is well adapted to solve the trajectory planning problems,
- $e(t) = y_d(t) - y(t)$ is the tracking error.
- K_p, K_I are suitable gains, the tuning of which is quite straightforward.

Remark 2 The nonlinear system $\dot{x} = f(x, u)$ is differentially fiat if we can find fiat outputs:

$$z = h_f(x, u, \dot{u}, \ldots, u^{(\rho)}) \quad (5)$$

such that:

$$\begin{aligned} x &= \phi(z, \dot{z}, \ddot{z}, \ldots, z^{(r)}) \\ u &= \psi(z, \dot{z}, \ddot{z}, \ldots, z^{(r+1)}) \end{aligned} \quad (6)$$

z is called the fiat output which can be or not the system output. As represented above in the Eq. (6), the states and the input variables of the system are expressed in terms of the fiat outputs and their higher derivatives.

The a-PI controller of the Eq. (4) compensates the unknown term $F(t)$ and $\alpha(t)$.

Remark 3 If $v = 1$, we brought back with (4) to a pure integrator stabilization. Therefore, the two gains setting K_p and K_I become very simple in contrast to classical PI (see [21) for more details).

Combining Eqs. (3) and (4) yields to the fonctional equation 0:

$$\delta(e) = \ddot{e}(t) + K_p \dot{e}(t) + K_I e(t) = 0 \quad (7)$$

which the roots should be stable and a good tracking is asymtotically ensured, i.e.

$$\lim_{t \to +\infty} e(t) = 0 \quad (8)$$

We obtain then a linear differential equation with constant coefficients of order 2. The tuning of K_p and K_I becomes therefore straightforward for obtaining a good tracking of y_d. This is a major benefit when compared to the tuning of classic Pis.

3 Ultra-local model identification methods

3.1 Numerical derivation (ND) method

In several works as [1, 2], a fast identification technique, based on the numerical differentiations, is applied to estimate the time-varying fonction $F(t)$ thanks to the knowledge of $u(t)$ and y (t). Numerical derivation, which is a classic field of investigation in engineering and in applied mathematics, is a key ingredient for implementing the feedback loop (3). This solution has already played an important role in model-based nonlinear contrai and in signal processing (see [4] for further details and related references).

Important theoretical developments, which are of utmost importance for the computer implementation, may be found in [12].

The estimate of the first order derivative of a noisy signal y is defined as follows (see [22]):

$$y = - T^{3} \int_{0}^{T} (T - 2t)\, y(t)\, dt \tag{9}$$

where [O, T], $T > 0$, is a quite short time window. This window is sliding in order to get this estimate at each time instant. Denoising of y leads to the following estimate:

$$y = \frac{2}{T^{2}} \int_{0}^{T} (2T-3t)\, y(t)\, dt \tag{10}$$

In these works [1], the quantity F (t) in (3) is updated at each sampling time from the measurement of the output and the knowledge of the input. At sampling time k (i.e. $t = kTe$, where Te denotes the sampling period), the estimation of F is written as:

$$A = Y_k - \alpha u_{k-1} \tag{11}$$

where Y_k is the estimation of first derivation of the output that can be laid at time k, α is a non-physical constant design parameter, and U_{k-1} is the control input that has be applied to the system during the previous sampling period. Based on the estimation of F, the control is calculated on (3) as follows:

$$u(t) = \frac{-ft(t) + yd(t) + Kpe(t) + Kr\, Je(t)}{a}, \tag{12}$$

where a is chosen by the practitioner.

The identification of the two parameters $F(t)$ and a (t) is a difficult task via the algebraic derivation technique, particularly, if the estimation of the second parameter $a(t)$ is required. For this reason and in order to improve the performances, a new parameter estimation method is proposed in the following to solve the problem of the both estimation of F (t) and $\alpha(t)$.

3.2 Least squares (LS) method

The estimation of the two parameters $\hat{F}(t)$ and $\hat{a}(t)$ leads to the following principle of ultra-local model control:

$$u(t) = \frac{\dot{y}_d(t) - \hat{F}(t) + R\left(p, p^{-1}\right)\left(y_d(t) - y(t)\right)}{\hat{a}(t)}, \tag{13}$$

where $R(p, p^{-1})$ is a polynomial matrix with the operators of derivation p and integration p^{-1}. In the case of adaptive PI, the polynomial R is written as follows:

$$R\left(p, p^{-1}\right) = K_P + K_I p^{-1}. \tag{14}$$

In the present approach, the application of control input requires the knowledge of the system output y and the desired trajectory yd, as well as the real-time estimation of the two quantities F and α in (3).

Assuming the numerical control with constant sampling period Te which allows to dispose on the system of constant control U_{k-1} between the instants $(k - 1)\, Te$ and kTe and available information until the instant kTe, unless U_k.

The main aim of using a reduced ultra-local model (i.e. choosing an order of derivation $v = 1$ or $v = 2$) lies actually in the increasing of accuracy. In fact, the reduction of the order of derivation v involves a little sampling time Te of numerical control so a minimal time of calculation. This increases accuracy and minimizes uncertainties.

From the simple model $y(t) = F(t) + \alpha(t)\, u(t)$, the integration between two sampling instants gives:

$$\begin{aligned} Y_k &= Y_{k-1} + \int_{k-1}^{k} F(t)\, dt + \int_{k-1}^{k} a(t)\, u(t)\, dt \\ &= y_{k-1} + \int_{k-1}^{k} F(t)\, dt + \left[\int_{k-1}^{k} \alpha(t)\, dt\right] u_{k-1} \end{aligned} \tag{15}$$

Let $\hat{F}_k$ and $\hat{a}_k$ the mean values, in $[(k - 1)\, Te, kTe]$, of F (t) and α (t), finally we get:

$$y_k = y_{k-1} + \hat{F}_k T_e + \hat{\alpha}_k T_e u_{k-1} \tag{16}$$

Considering the following notations:

$$y_k^T = \frac{y_k - y_{k-1}}{Te},$$

$$H_k^T = [\,1 \quad u_{k-1}\,], \qquad (17)$$

$$\theta_k^T = \left[\hat{F}_k \ \hat{\alpha}_k \right],$$

the previous relation (16) can be written in the following form:

$$Y_k = H_k \theta_k \qquad (18)$$

Since the regression matrix $H_k = [\,1 \ u_{k-1}\,]$ has a default rank then, this system is always consistent (i.e., rank $[H_k]$ = rank $[\,H_k \ Y_k\,]$). The aim is to seek at each instant kTe to estimate θ_k. According to the linear system resolution technique detailed in [23], the general expression of estimation is:

$$\theta_k = H_k^{\{1\}} Y_k + \left(I_{m+1} - H_k^{\{1\}} H_k \right) \Lambda_k \qquad (19)$$

where:

- $H_k^{\{1\}}$ denotes the Moore-Penrose generalized inverse of H_k, that is mean the matrix X such as $AXA = A$ [24],
- Λ_k is an arbitrary matrix of size $(m \times 1)$.

The coefficients of matrix Λ_k appear as degrees of freedom that can be used to satisfy other relating constraints to the system control. However, these degrees of freedom are equal to the rank of $I_{m+1} - H_k^{\{1\}} H_k$.

Based on the numerical knowledge of F and α, the control input is calculated in (3) as a closed-loop tracking of a reference trajectory $t \longmapsto y_d(t)$, and a simple cancellation of the nonlinear terms F and α.

The application ofthis new approach of ultra-local model control is considered in the case of a two-tank-system presented in the following section.

4 Case study: two-tank-system

4.1 Model description

Consider the two-tank-system described in the Fig. 1 which is constituted by two identical water tanks that have the same section S. Denote by $h_1(t)$ the water level in the upper tank, which also represents the system output, $h_2(t)$ the water level in the lower tank, $q_1(t)$ the input flow of the upper tank, $q_2(t)$ the output flow of the upper tank and $q_3(t)$ the output flow

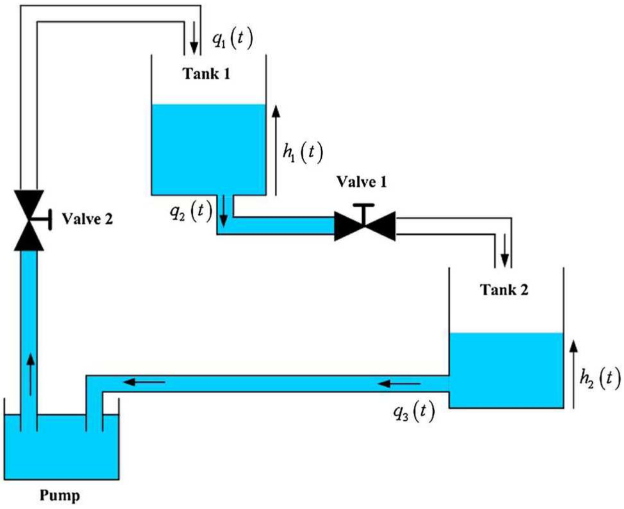

Fig. 1 Two-tank-system

of the lower tank. In the steady state, the conservation of the total volume of water leads to $q_1(t) = q_3(t)$.

The nonlinear model of the considered system is as follows:

$$\begin{aligned} S\dot{h}_1(t) &= q_1(t) - q_2(t) \\ S\dot{h}_2(t) &= q_2(t) - q_3(t) \end{aligned} \qquad (20)$$

with $q_2(t) = k_1\sqrt{h_1(t)}$ and $q_3(t) = k_2\sqrt{h_2(t)}$.

The term $k_i\sqrt{h_i(t)}$, $i = 1, 2$, comes from the turbulent regime of the water evacuation by the valves. The two parameters k_1 and k_2 represent the coefficients of the canalization restriction.

We obtain then the following model:

$$\begin{aligned} \dot{h}_1(t) &= -\frac{k_1}{S}\sqrt{h_1(t)} + \frac{1}{S}q_1(t) \\ \dot{h}_2(t) &= \frac{k_1}{S}\sqrt{h_1(t)} - \frac{k_2}{S}\sqrt{h_2(t)} \end{aligned} \qquad (21)$$

These two equations are nonlinear due to the presence of the term $\sqrt{h(t)}$, hence the most difficult task in the control of this considered system will be the control of the water level $h_1(t)$ in different operating conditions.

4.2 Control design

In this work, we choose to generate a desired trajectory, $h_1^*(t)$ of system output, satisfying the constraints of the two-tank-system. Moreover, the trajectory generally satisfies the constraints in terms of response time and rise time. Our reference trajectory ensures a transition between the initial water level $h_1^*(t_0) = 2$ cm and the final water level $h_1^*(t_f) = 10$ cm.

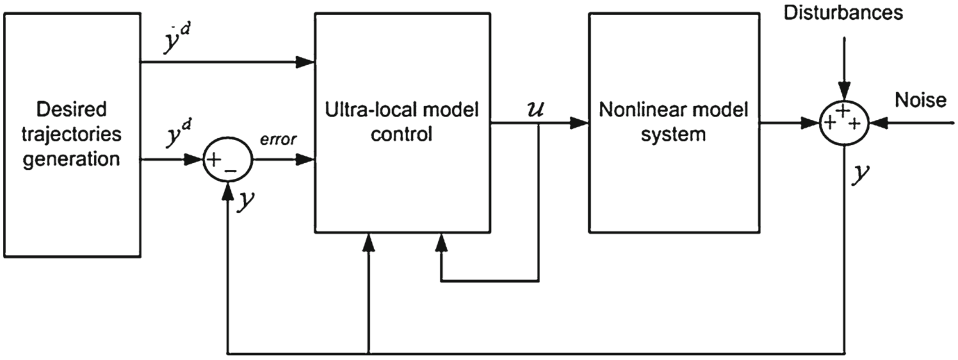

Fig. 2 General struct ure of an ult ra-lo c al mode! control

In the numerical simulations, taking the two transition instants $to = 50$ s and $t_f = 150$ s, and calculating a fifth order polynomial as reference trajectory between these two instants . This trajectory satisfies the differentiability and continuity conditions at the in stants of set-point change.

The principle of an ultra-local model control of the nonlinear considered system is summarised in Fig. 2 which represents the block diagram of the two-tank-system control in closed-loop. The numerical values of two-tank-system parameters are given in the Table 1. The desired trajectories are generated based on the concept of flatness [19,20]. We note that the adding of noises and dis tur bances output aims to test the robustness of the a-PI controller.

For comparison purpose, we have firstly implemented a classical PI control. The controller parameters have been manually tuned and are given in the Table 2. So, the tuning of the classical PI gains has been done in two steps: Firstly, tuning of the proportional gain on the un disturbed process. Secondly, tuning of the integral gain in order to achieve a good perturbation rejection. Then, we have also implemented an adaptive PI control using numerical derivation method. After a few attempts, we set $T = 25Te$ in Eq. (9). The two parameter K_p and K_I of a-PI controller, given in the Table 2, are chosen according to a classical second order dynamics ($p^2 + K_p p + K_I = 0$). For our control approach, the two gains Kp and K_I of adaptive Pl are chosen (see Table 2) in order to stabilize the tracking error. The tuning of a-PI gains is trivial by applied the functional equation (7).

Table 1 Parameter value s

Parameter	Value
Te	0 .1 s
S	332.5 cm^2
k1	42.1 $cm^5 l^2$/s
$k2$	42.1 $cm^5 l^2$/s

Table 2 Controller parameters

Gain	Classical PI	Adaptive PI (ND method)	Adaptive PI (LS meth od)
Kp	3	5 x 10- 1	10
Kt	9 x 10- 1	6 x 10- 2	3
a		1	

4.3 Numerical simulations

The simulation results are summ arized in the following figures where we have studied the ultra-local mode! control with two-tank-system in the presence of disturbances and noise s. A centred white noise (normal law N(0,0.001)) is added to the system output, prese nted in the Fig. 3, in order to test the robustness of numerical simu lations of this work. At $t = 180$ s, a level water disturbance of 0.8 cm, which is due to a problem in the sensor, is applied to the system.

When the system is controlled by a classical PI controller, we observe in Fig. 4 that the system output reaches the desired trajectory in approximately 40 s with 0.5 cm overshoot. Howeve r, a 12.75 s response time with 0.42 cm overshoot is obtained when the system is controlled with our a-PI controller. The parameters in this case are estimated using le ast squares resolution method. It is then clear that the new control strategy is better than the classic al PL

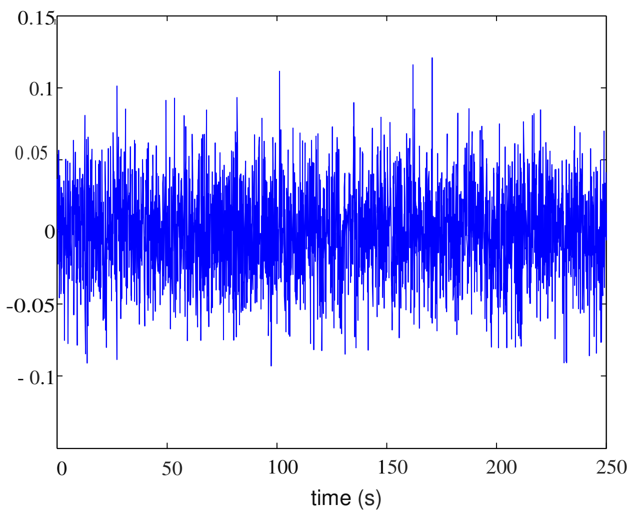

Fig. 3 Centered white no ise of the output

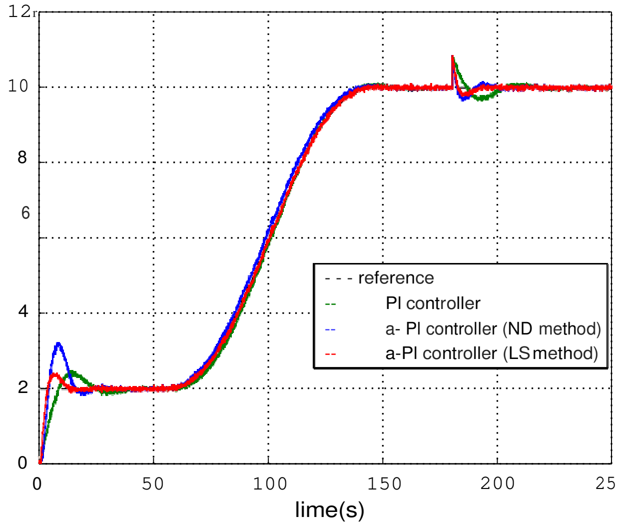

Fig. 4 Reference and noisy system outputs in the case of the three different methods

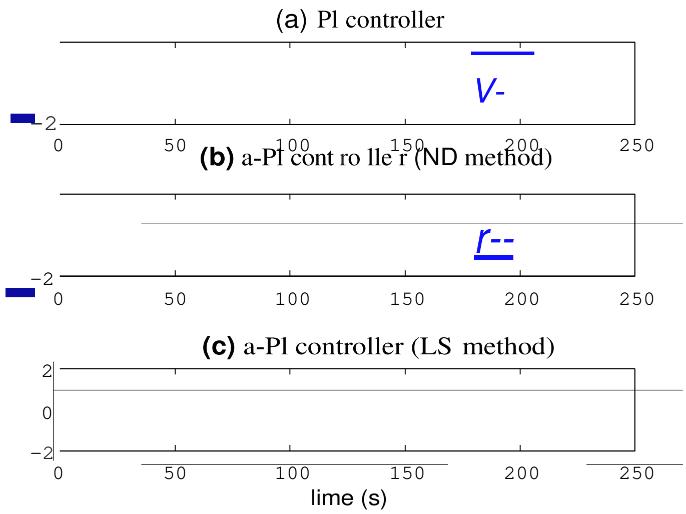

Fig. 5 Tracking errors in the case of the three different methods

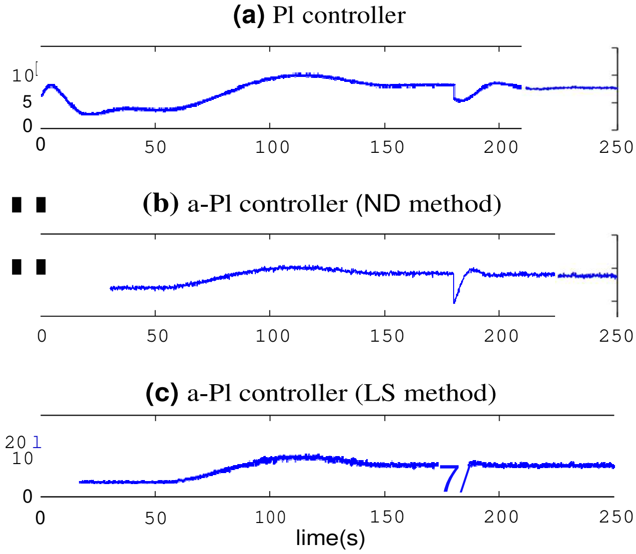

Fig. 6 Control inputs in the case of the three different methods

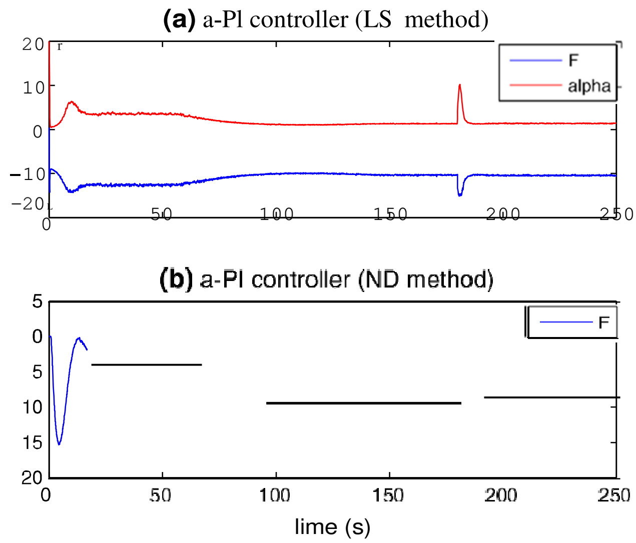

Fig. 7 Parameter estimation in the case of LS and ND methods

In the Fig. 4, the a-PI control using least squares resolution has ensured better tracking of desired trajectory despite the addition of centered white noise to response system (see tracking errors given in Fig. 5). However, the tracking performances are unsatisfactory for the a-PI control using numerical derivation technique (Fig. 6). The response time (17.5 s) and the overshoot (1.1 cm) of the latter technique are greater in comparison with those of our proposed technique. We can observe **in** Fig. 4 that the consequence of level water disturbance is smaller and rejected faster by the **LS** method than the numerical derivation method. A better robustness of the new control strategy with respect to external disturbances is then given thanks to the adaptive **PI** and the both on-line parameter estimation (see Figs. 4, 5, 6, 7).

To properly compare the robustness performances of the two adaptive controllers, we have tested the system dynamics in the case of parameter uncertainties. In the numerical simulations, we have treated the uncertainty of parameter S which is equal to 550 cm^2 if 30 s $< t <$ 170 s instead of 332.5 cm^2. It is clear that the tracking of desired trajectory is better when the ultra-local model parameters are estimated by LS method (see Fig. 9). In the Fig. 8, we can see that the proposed method is able to reject perturbations faster than the numerical derivation technique. Due to the parameter uncertainties, we can observe that the control input of ND method achieved negative values (see Fig. 10). Thus, the response time of system output becomes more important (45 s). All these drawbacks show that the numerical derivation method

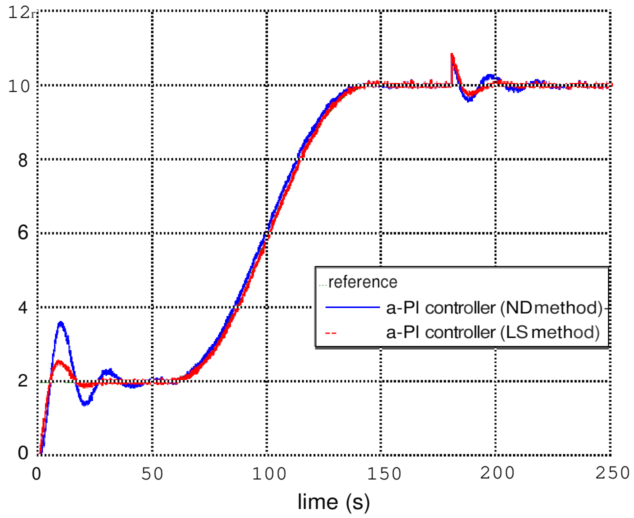

Fig. 8 Reference and noisy system outputs in the case of LS and ND methods-parameter uncertainties 50 % of S

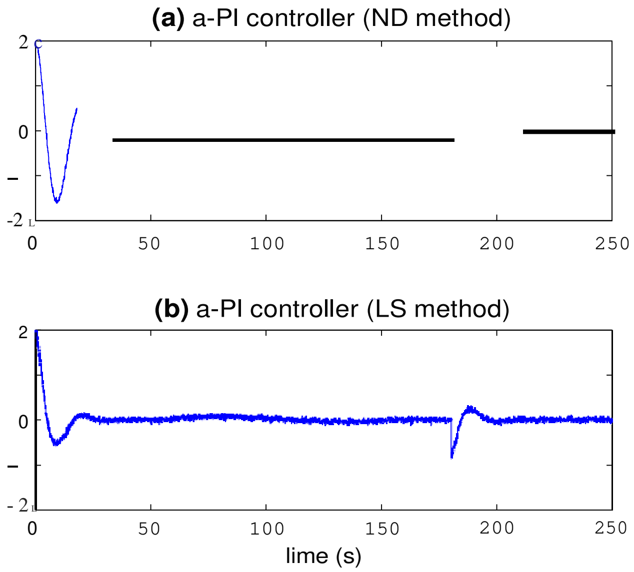

Fig. 9 Tracking errors in the case ofLS and ND method s- parameter uncertainties 50 % of S

is more sensitive to parameter uncertainties, than the new estimation method presented in this paper. The both parameter estimation plays a very important role in the robustness of the proposed ultra-local model control with respect toperturbation rejection and parameter uncertainties.

5 Conclusions

The contribution of the paper has allowed the design of a new water level controller, which is able to insure good trajectory tracking performance even in severe operating conditions. An improvement of performance is obtained with the proposed

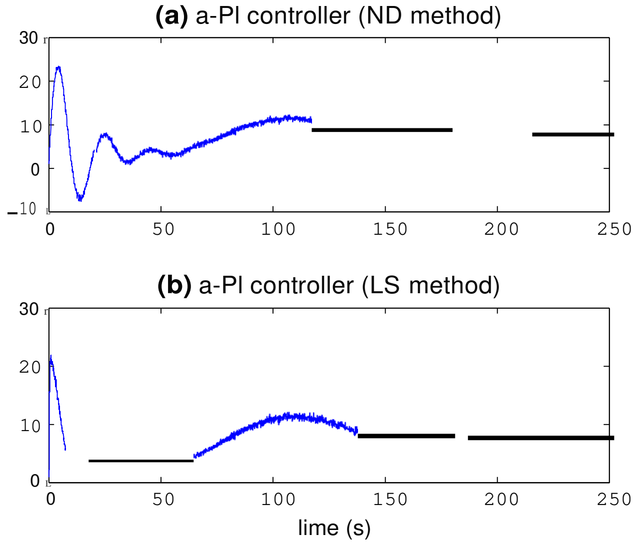

Fig. 10 Control inputs in the case ofLS and ND methods-parameter uncertainties 50 % of S

approach of parameter estimation, compared to the numerical derivation method proposed in [1].

The main advantages of the proposed control method which is appreciable for industrial applications are as follows:

Allowing to bypass the difficult task of mathematical modeling and therefore complex identification procedures,

Leading to a straightforward gain tuning and a time reduction of commissioning tests,

Providing a good robustness towards external dist urbances and parameter variations of process.

Due toits properties of robustness, adaptability and simplicity, the ultra-local mode] control provides outstanding performance with a very short time of implementation.92671474.

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