TST/PSS 2017;3:8-17
DOI: 10.21846/TST.2017.4.1
2. évfolyam, 4. szám / Year 2, Issue 4
2017.

A 3D computer simulator to examine the effect of wind and altitude on a soccer ball trajectory

A szél és a magasság futball labda röppályájára gyakorolt hatásának vizsgálata három dimenziós számítógépes szimulációval

Alexander Egoyan, Ilia Khipashvili, Karlo Moistsrapishvili

Georgian State Teaching University of Physical Education and Sport, Tbilisi, Georgia

Abstract - It is well known that wind, altitude and air resistance can affect the trajectory of a soccer ball and cannot be neglected by players and coaches. The theoretical model of our ball kicking simulator is based on a system of differential equations and describes the complex effect of wind, altitude, temperature and Magnus force on a soccer ball trajectory. Our program together with calculated diagrams is supposed to help coaches and players to predict the necessary velocity vector to kick the ball into the goal. The main features of our simulator are its simplicity, the intuitive graphical design, the three-dimensionality, the possibility of adding virtual players and to view shots in the real-time regime. The program's interface represents a first-person's point of view where a virtual football player can move in the directions perpendicular and parallel to the opponent's goal. Simply by pressing the keyboard buttons, the user can easily change parameters and generate three-dimensional trajectories of a soccer ball. Our program allows us to calculate a soccer ball trajectory for different wind speeds and different altitudes and make rough approximations. Knowing the strength of their kicks, the players will be able to select the best angle for a kick taking into account the wind and altitude.
Keywords: projectile aerodynamics, Magnus force, computer modeling, air resistance

Absztrakt - Közismert, hogy a szél, a magasság és a légellenállás egyaránt befolyásolhatja a futball labda röppályáját, ezt pedig a játékosok és az edzők sem hagyhatják figyelmen kívül. Az általunk alkalmazott, a labda elrúgásának vizsgálatára készített szimulátor differenciálegyenletek rendszerén alapul, és megmutatja a szél, a magasság, a hőmérséklet és a Magnus-effektus bonyolult hatásmechanizmusát, mely a labda röppályáját meghatározza. A programunk, illetve a számításokat bemutató diagramok segítségével az edzők és játékosok megjósolhatják a szükséges sebességvektort, amellyel a labda a kapuba juttatható. Szimulátorunk legfontosabb tulajdonságai az egyszerűség, az intuitív grafikai dizájn, a háromdimenziósság, valamint a virtuális játékosok hozzáadásának, illetve a felvételek valós idejű megtekintésének lehetősége. A program kezelőfelülete egyes szám első személyű nézőpontból mutatja meg a helyzetet, amelyben egy virtuális labdarúgó az ellenfél kapujához képest párhuzamosan és merőlegesen mozgatható. A számítógép billentyűinek egyszerű lenyomásával a felhasználó könnyen alakíthat a paramétereken és létrehozhatja a futball labda különböző háromdimenziós röppályáit. A program segítségével kiszámíthatóvá és megbecsülhetővé válik a labda röppályája különböző szélerősség, vagy magasság esetén. Lövőerejükhöz mérten a játékosok képesek lehetnek a szél és a magasság figyelembevételével kiválasztani az ideális szöget a rúgás elvégzéséhez.
Kulcsszavak: aerodinamika, Magnus-effektus, számítógépes modellezés, légellenállás

Alexander Egoyan
Georgian State Teaching University of Physical Education and Sport
49, Chavchavadze Avenue, 0162 Tbilisi, Georgia
+995-591-144-531 | alexegoyan@gmail.com

Introduction

The motion of a ball though the air is one of the most complex problems in sports science, and it is still not completely understood to this day. One of the reasons why this problem is so challenging is that, in general, there are many different forces acting on the ball, including gravity, drag and Magnus force, which in their turn depend on the ball's mass, cross-sectional area, form and shape, as well as on the external factors such as air density and the air velocity (De Mestre, 1990; Dupeux et al., 2010; Mehta, 1985; White, 2010).

It is possible to classify the different ball sports in three categories: the ones dominated by gravity, the ones dominated by aerodynamics and the ones where both effects can be used. For example, table tennis, golf and tennis are dominated by aerodynamics: basketball and handball by gravity; and, finally, soccer, volleyball and baseball belong to the third group of sports where both gravity and aerodynamics play comparable roles (Dupeux et al., 2011).

The forces on projectiles moving through the air have been discussed in many articles [De Mestre, 1990; Mehta, 1985; Myers & Mitchell, 2013; Wesson, 2002; White, 2010). The trajectory of a soccer ball is governed by three different forces: gravity, drag and Magnus force.

Simulation software helps us to understand how elite soccer players like Beckham and Carlos do what they do in a free kick (Bray & Kerwin, 2003; Carre´ et al., 2002; Passmore et al., 2008). Thanks to simulation software, researchers at the University of Sheffield, Yamagata University, and Fluent have been able to figure out the complex physics of soccer and solve the mystery of "bending" free kicks (Asai et al., 2007; Barber et al., 2009; Goff & Carre, 2009; Goff & Carre, 2010). Researchers study how the motion of soccer balls depends on the shape and design of the balls by using computer modeling (Alam et al., 2012; Alam et al., 2014; Asai & Seo, 2013; Carre´ et al., 2005; Oggiano & Sætran, 2010).

Since there are many football kick simulators for modeling how the same kick behaves under different external conditions, it is often difficult to decide which one to choose. Main factors which put obstacles in the way of the use of many existing simulators are the following:

- Data published in the scientific literature often contradict each other. A good model should have enough complexity and be compatible with the reliable scientific data.
- Almost all simulators which model the effect of altitude and Magnus force are not programmed to model the effect of wind.
- We failed to find simulators modeling the wind effect for arbitrary directions of the wind and not only for head or tail winds or crosswinds.
- Most simulators are two-dimensional or use wrong proportions when displaying the field in 3D.
- Many simulators have complicated interfaces and are non-intuitive.

The purpose of our research is to create a reliable computational approach for modeling a soccer ball trajectory taking into account the complex effect of such factors as wind and altitude. The research also has educational purposes to make the process of modeling of the motion of a soccer ball simpler and more comprehensible.

The Method of Computer Modeling

To calculate the trajectory of the ball in the air the following model is used. The force acting on the ball could be calculated as (Myers & Mitchell, 2013): $F = F_G + F_D + F_S + F_L$, where F_G is the downward force experienced by gravity, F_S is the sideways component of the Magnus force and F_L is the lifting component of the Magnus force. Fig. 1 shows the various forces on the ball: the gravitational force F_G points downward; the drag force F_D is opposite to the ball's velocity v; the lift force F_L is perpendicular to the ball's velocity v and lies in the plane formed by the velocity v and the ball's weight; and the sideward force F_S (not shown) is toward the page.

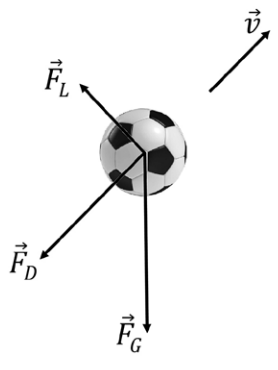

Figure 1. Forces acting on a soccer ball.

To further explain F_S and F_L imagine a ball that is rotating strictly with topspin or backspin. The ball will have no sideward rotation hence $F_S = 0$. Likewise, consider a ball that has strictly a sideward spin. The Magnus force now has no component in the z direction and thus $F_L = 0$. However, when a ball is rotating in more than one axis, F_S and F_L must be considered.

According to the accepted theoretical model, magnitudes of forces F_D, F_S, F_L are proportional to the air density, the ball's cross-sectional area and the corresponding aerodynamical coefficient, and inversely proportional to the mass of the ball:

$F_D=C_D*(\rho*A/2.0/m)$, $F_L=C_L*(\rho*A/2.0/m)$, $F_S=C_S*(\rho*A/2.0/m)$,

where ρ is air density, A is the cross-sectional area of the ball, m is the mass of the ball and C_D, C_L, C_S are aerodynamical coefficients which depend on air density, wind, ball's velocity, spin, form and shape.

Air density ρ decreases with increasing altitude. An approximate relation gives about a 3% reduction in air density for every 305m increase in altitude. This relation can be described using a simple formula:

$\rho=\rho_0\cdot(100-(h\cdot3/305))/100$, where $\rho_0=1.225$ kg/m^3 is air density at sea level at 15°C.

For describing the motion of the ball in the air the system of differential equations (8)-(10) from (Myers & Mitchell, 2013) was used. The system doesn't include wind, so we had to add the wind component. This has been done according to the method described in (Wesson, 2002).

The typical parameter values used in our model are shown in Table 1 (see (Myers & Mitchell, 2013; Wesson, 2002)).

Table 1. Typical parameter values.

Parameter	Value
Ball's mass m	0.430 kg
Ball's diameter D	0.22 m
Ball's cross-sectional area A	0.039 m^2
Air density ρ	1.225 kg/m^3
Temperature T	288.15 K
Drag coefficient C_D	0.2
Drag coefficient C_S	0.0
Drag coefficient C_L	0.0

The main features of our ball kicking simulator that distinguish it from other simulators are:

1. Its simplicity: only by pressing the arrow buttons in combination with Ctrl, Alt and Shift keys the user can easily change the position of the virtual player on the virtual field, as well as the initial velocity, direction and angular deviance of the ball's velocity. We use left, right, up and down arrows for motion, Ctrl + arrows for rotations of the velocity vector; Shift + arrows for changing the wind's components along the directions parallel and perpendicular to the sides of the football field, Alt + arrows (left, right) for changing altitude; Alt + arrows (up, down) for changing the initial velocity of the ball, Ctrl + Alt + arrows for changing topspin and backspin and, finally, we use Enter for kicking. Pressing the arrows, we can increase or decrease the value of the selected parameter.

2. The intuitive graphical design (see Fig. 2): the program's interface represents the first-person's point of view where a virtual football player can move in the directions perpendicular and parallel to the opponent's goal. The interface has two parts: the upper part displays the football field and players, while the lower part displays the values of the main parameters and the hints containing key combinations for changing those values. Main parameters such as the ones that coordinate the virtual player on the field, the horizontal and vertical deviations of the velocity vector, the values of the initial velocity of the ball, its topspin and backspin, as well as the values of altitude and wind components are shown in the table together with the key combinations for changing those parameters. There is also an instruction explaining how to use functional keys F1, F2, F3, F4 and F8 for working with calculated trajectories at the bottom of the screen. The program also displays the coordinates of the final position of the ball when it passes through the goal or lands on the ground.

3. Three-dimensionality: the football field on the screen has a perspective, it changes its size when the user changes the shooter's position on the field and all its proportions correspond to the real football field proportions.

4. The possibility to add virtual players: by entering their coordinates on the field the user can add virtual players. This allows us to predict whether the ball will strike the opponent's players or not.

5. The possibility to view shots in the real-time regime: soccer ball trajectories are synchronized with the time variable making the simulation more realistic.

6. The option to compare two different trajectories: the computer program allows the user to save the calculated trajectory and display it later together with another trajectory.

7. The option to display the projection of the ball's trajectory on the field in order to create the feeling of three-dimensionality: this option allows the user to estimate two angles - the angle of vertical deviation and the angle of horizontal azimuthal deviation of the ball's trajectory.

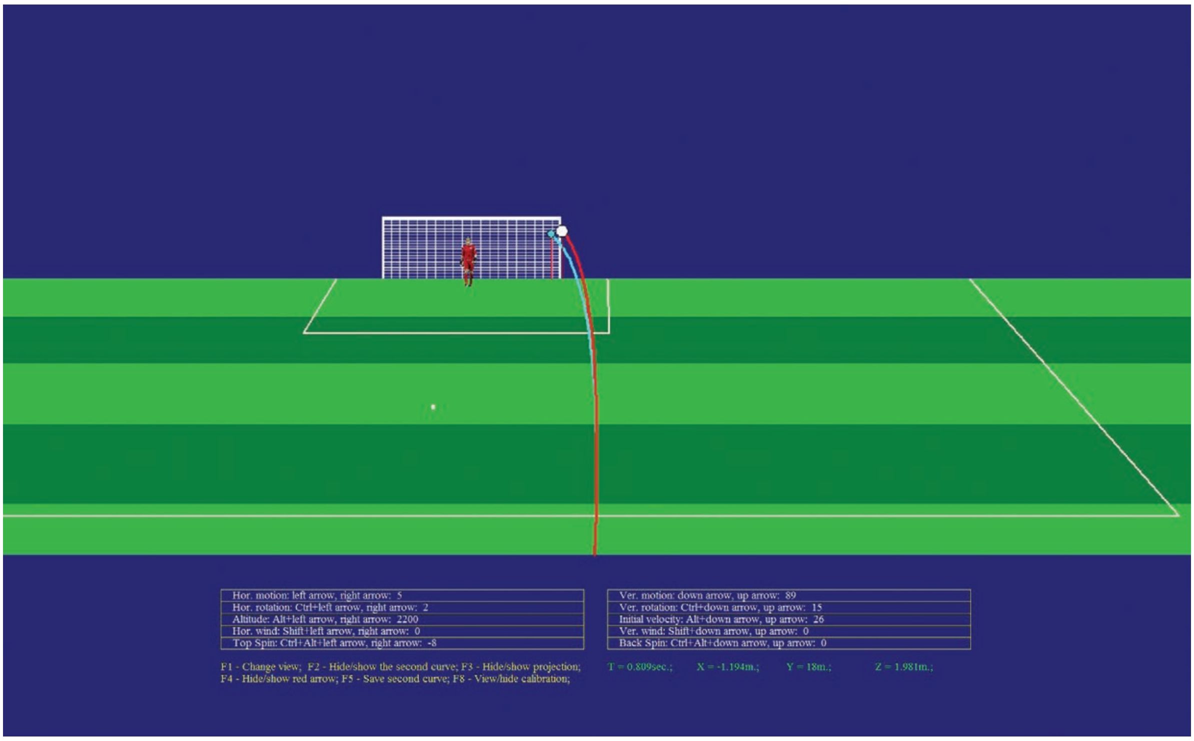

Figure 2. Interface of the computer program

The user can easily change the wind components: the orientation of the football field on the screen doesn't change and the wind may be represented as a superposition of two winds - vertical and horizontal, parallel and perpendicular to the sides of the field, respectively. The program allows switching views between three-dimensional to two-dimensional, including the top, front and side projections of the field.

Results of Computer Modeling and Discussion

Imagine a soccer ball that flies through still air. We assume that the ball has no spin. We know that the flight of a ball through a vacuum is governed by gravity and its trajectory is a parabola. However, when the projectile is launched through the air there is an aerodynamic drag force F_D which acts in the opposite direction to the instantaneous velocity of the ball and decreases the speed of the ball. The magnitude of the drag force depends on the speed of the ball through the air, the cross-sectional area of the ball, and the shape and surface characteristics of the ball. The shape and surface characteristics are accounted for by the drag coefficient C_D which is usually about 0.2 for non-spinning soccer balls (Asai et al., 2007; Myers & Mitchell, 2013; Wesson, 2002). Fig. 3 shows that drag can significantly change the ball's trajectory depending on the starting angle of a kick.

Trajectories of a free kick and a goal kick started at angles 15° and 45°, respectively, with the same velocities equal to 30 m/s. Dash lines show trajectories calculated without drag.

Using our program, we have calculated how the range of a goal kick launched at 45° depends on the starting velocity of the kick and the altitude. Fig. 4 shows that the calculated graphs have almost linear character: for V_0=15 m/s the range increases only

DOI: 10.21846/TST.2017.4.1

by 0.9 m when the altitude increases from 0 m to 3000 m, while for V_0=30 m/s such an increase is about 7 m and it is about 9.7 m for V_0=35 m/s. A rough approximation for goal kicks is, that a ball that travels about 50 m through the air at sea level will travel about 2.0-2.5 m further with each 1000 m increasing of altitude.

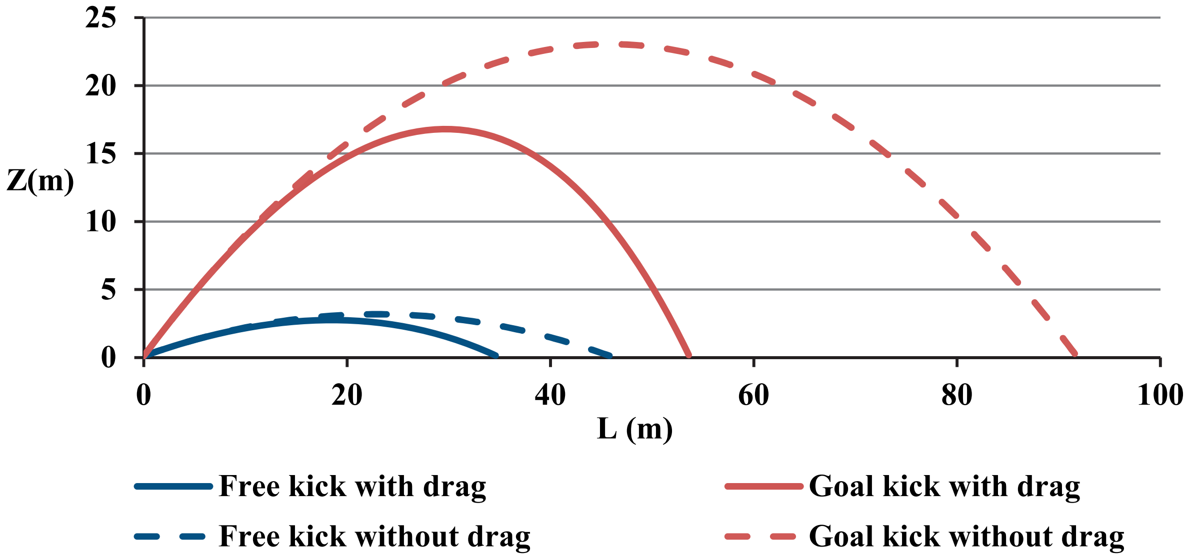

Figure 3. Trajectories of a free kick and a goal kick with and without drag.

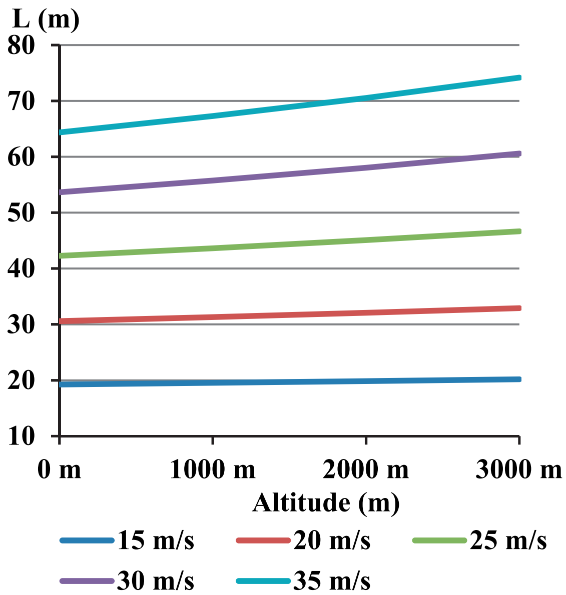

Figure 4. Dependencies of the range of a goal kick on the altitude.

When there is a wind, the speed of the air over the ball is changed and there is an additional force on a ball. This force depends on the speed of the ball and is approximately proportional to the speed of the wind [19]. It is clear that a tail wind will increase the range of a kick and head wind will decrease it. Fig. 5 shows how the range of a goal kick depends on the ball's starting velocity and the speed of the wind.

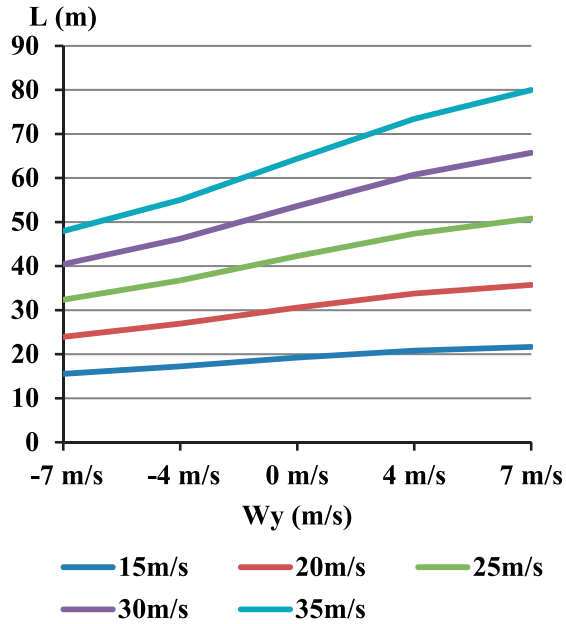

Figure 5. Dependencies of the range of a goal kick on the speed of a tail/head wind.

Here we also see almost linear graphs, where the effect of a wind increases when the initial velocity of the ball increases. For a goal kick at sea level with V_0=30 m/s, a rough approximation is that the range is increased or decreased by 2 m for each meter per second of the wind. For example, a goal kick which without a wind would reach the back of the center circle would be carried by a 13.5 m/s (30 mph) tail wind into the penalty area.

Crosswinds of moderate strength or less do not affect the range of soccer balls very much. However, they can deflect their trajectory quite considerably. The deviation traces out the curved path because of natural drag effects which reduce the duration of the flight (White, 2010).

Our calculations show that each meter per second of a side wind displaces the flight of a goal kick launched at 45° with V_0=30 m/s approximately by 1 m (see Fig. 6).

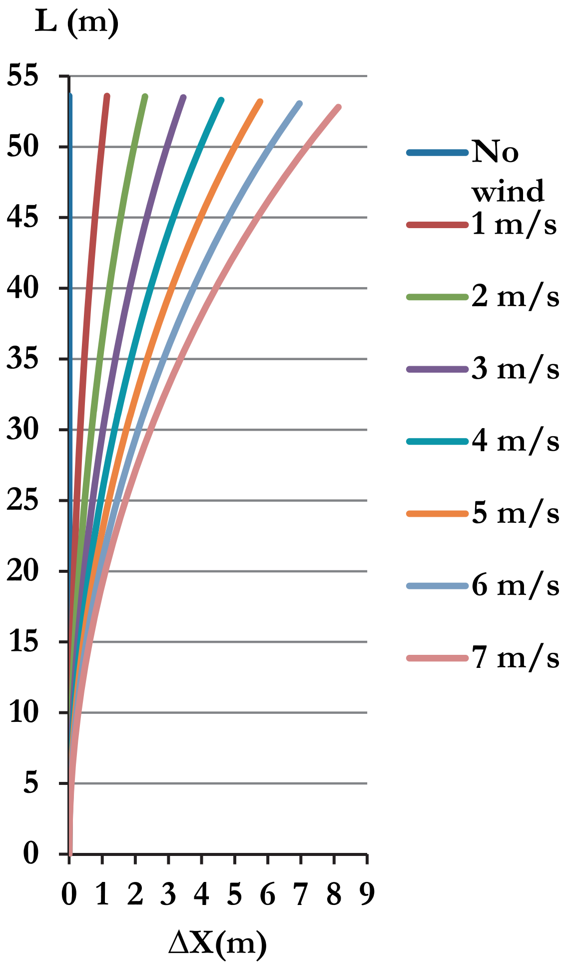

Figure 6. Top view of trajectories of a goal kick deflected by a side wind.

As we would expect, the deflection caused by a side wind increases with the wind speed and with the time of flight. A 7 m/s side wind displaces the flight of a penalty kick by 18 cm, and a 10 m/s side wind by about 0.26 m. For free kicks and goal kicks the deflection caused by a side wind increases. For example, a 7 m/s side wind would deflect a 20 m free kick by about 60 cm and a goal kick by about 8 meters.

These trajectories are calculated for a goal kick with the starting parameters V_0=30 m/s and α=45°

The side wind's velocity takes all integer values from the interval between 0 m/s and 7 m/s. ΔX is the displacement of the ball along the OX axis. The deflection $D=|\Delta X|$.

In Fig. 7 goal kick trajectories are shown (top view) for different starting velocities when the side wind is constant and equals to 7 m/s. Starting velocities take integer values from the interval between 15 m/s and 35 m/s. We can see that both displacements increase when the starting speed increases and are connected by the linear equation $L \approx 5 \cdot \Delta X + 10$ (see Fig. 7).

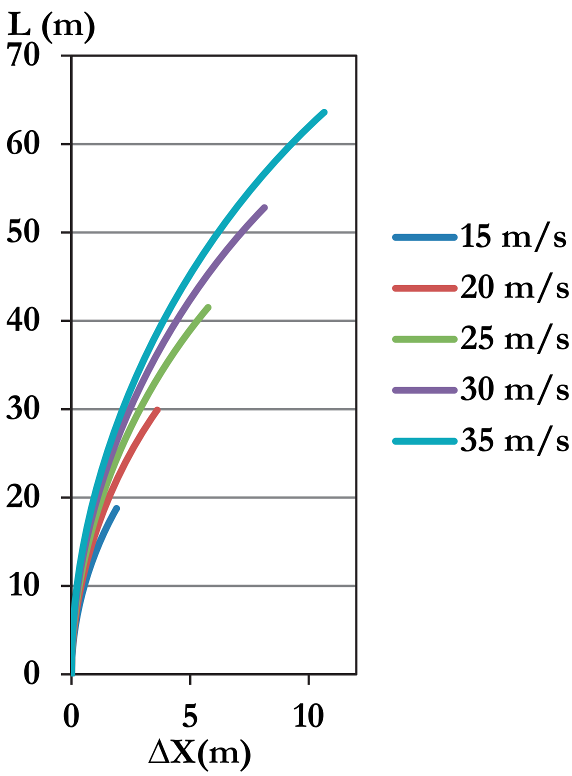

Figure 7. Top view of trajectories of goal kicks deflected by a 7 m/s side wind.

These goal kicks are launched at the same angle α=45° with different velocities: 15 m/s, 20 m/s, 25 m/s, 30 m/s and 35 m/s.

DOI: 10.21846/TST.2017.4.1

At high altitudes wind effects become weaker: for example, the deflection of a goal kick with V_0=30 m/s and α=45° caused by a crosswind with Wx=7 m/s at 3000 m altitude becomes 6.924 m against 8.06 m deflection at sea level.

Next to goal kicks we study free kicks which are very important in the soccer game. Let's consider a free kick launched at the angle α=15° and velocity V_0=26 m/s. There is no spin applied to the ball. The distance to the goal is 19 m and the starting point lies on the axis OY (see Fig. 5). The coordinates of the starting point are (0, 19m, 0). The horizontal (azimuthal) deviation of the starting velocity vector is zero. This means that the initial velocity vector lies in the YOZ plane.

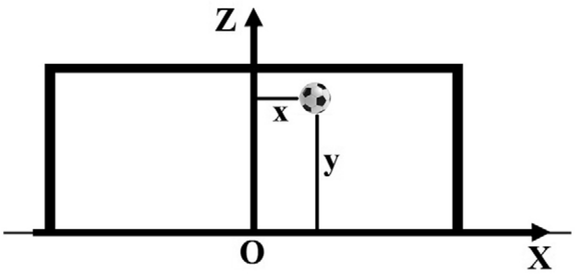

Figure 8. A reference frame associated with the goal.

The center of the reference frame is placed on the ground in the center of the soccer goal. The coordinates of the top left corner of the goal are (-3.66, 2.44).

Using our program, we calculate X and Z coordinates of the ball on the point where it crosses the XOZ plane and also calculate the ball's possible displacements ΔX and L along OX and OY axes before reaching the ground. We consider the cases for still air with no wind, tail wind 7 m/s, head wind 7 m/s, crosswinds 4 m/s and 7 m/s. Altitude takes four different values: 0 m, 1000 m, 2000 m and 3000 m. Then we compare the calculated cases with the first case from Table 2 calculated at sea level with no wind. Our calculations show that at sea level a 7 m/s tail wind increases z coordinate of the ball crossing the goal's plane by 16 cm, while the head wind decreases it by 25.8 cm. The deflections caused by 4 m/s and 7 m/s crosswinds at sea level are 0.353 m and 0.635 m, respectively. The calculations clearly demonstrate that at higher amplitudes the ball flies higher and farther, but the wind's effects become weaker: for example, at 3000 m altitude the displacement along z coordinate caused by the 7 m/s tail wind and -7 m/s head wind are 10.04 cm and -14.9 cm, respectively. Crosswinds at 3000 m altitude also makes the ball's trajectory weaker and deflect the ball's trajectory by 0.244 m in case of the 4 m/s crosswind and by 0.438 m in the 7 m/s crosswind. We see that crosswinds affect the position of the point where the ball crosses the goal's plane stronger than tail and head winds of the same strength. Tail winds significantly affect the whole range of kick L increasing it by 2.5-3.5 m depending on the altitude. The effect of head winds becomes significantly weaker at higher altitude: under the influence of the -7 m/s head wind when the ball flies at 26.989 m at the altitude 3000 m, which is only by 0.684 m less than the range of the same kick in the still air at sea level.

Table 2. Displacements of a free kick with no spin for different wind speeds and altitudes.

#	W_y(m/s)	W_x(m/s)	Altitude(m)	X(m)	Z(m)	ΔX(m)	L (m)
1	0	0	0	0	1.944	0	27.673
2	7	0	0	0	2.104	0	30.218
3	-7	0	0	0	1.686	0	24.77
4	0	4	0	0.353	1.938	0.77	27.61
5	0	7	0	0.635	1.925	1.372	27.483
6	0	0	1000	0	1.992	0	28.195
7	7	0	1000	0	2.133	0	30.568
8	-7	0	1000	0	1.775	0	25.467
9	0	4	1000	0.316	1.987	0.715	28.136

10	0	7	1000	0.568	1.976	1.273	28.015
11	0	0	2000	0	2.04	0	28.742
12	7	0	2000	0	2.162	0	30.93
13	-7	0	2000	0	1.859	0	26.205
14	0	4	2000	0.28	2.035	0.656	28.686
15	0	7	2000	0.502	2.026	1.169	28.57
16	0	0	3000	0	2.087	0	29.316
17	7	0	3000	0	2.191	0	31.304
18	-7	0	3000	0	1.938	0	26.989
19	0	4	3000	0.244	2.083	0.593	29.265
20	0	7	3000	0.438	2.075	1.058	29.161

X, Z are the ball coordinates in the point where the trajectory crosses the goal's plane, ΔX is the horizontal displacement of the ball and L is its range when the ball reaches the ground. The kick's launching angle is 15° and starting velocity is 26 m/s.

At the next stage, we consider the same kick with topspin=-8 rev/sec applied to the ball. The results of computer modeling are shown in Table 3 and Fig. 9. In general, the aerodynamical coefficients depend on the spin of the ball and are calculated according to the diagrams presented in the reliable scientific papers ([1], [2]). In our case with topspin=-8 rev/sec, C_D≈0.242 and C_S≈-0.242. The initial velocity V_0=26 m/s and its vector lies in the YOZ plane.

Table 3. Displacements of a free kick with topspin for different wind speeds and altitudes.

#	W_y(m/s)	W_x(m/s)	Altitude(m)	X(m)	Z(m)	ΔX(m)	L (m)
1	0	0	0	-2.851	1.731	-5.209	25.25
2	7	0	0	-1.616	1.936	-3.468	28.019
3	-7	0	0	-4.89	1.321	-7.116	22.14
4	0	7	0	-2.118	1.565	-3.36	23.637
5	0	-7	0	-3.571	1.817	-7.162	26.57
6	0	0	1000	-2.547	1.814	-4.914	25.981
7	7	0	1000	-1.46	1.985	-3.251	28.553
8	-7	0	1000	-4.263	1.5	-6.75	23.076
9	0	7	1000	-1.892	1.685	-3.211	24.458
10	0	-7	1000	-3.203	1.882	-6.722	27.226
11	0	0	2000	-2.251	1.892	-4.584	26.744
12	7	0	2000	-1.303	2.033	-3.014	29.107
13	-7	0	2000	-3.687	1.652	-6.334	24.06
14	0	7	2000	-1.671	1.792	-3.035	25.322
15	0	-7	2000	-2.841	1.945	-6.072	27.47
16	0	0	3000	-1.96	1.965	-4.218	27.542
17	7	0	3000	-1.146	2.08	-2.754	29.68

DOI: 10.21846/TST.2017.4.1

18	-7	0	3000	-3.151	1.783	-5.864	25.098
19	0	7	3000	-1.455	1.889	-2.829	26.231
20	0	-7	3000	-2.484	2.005	-5.712	28.612

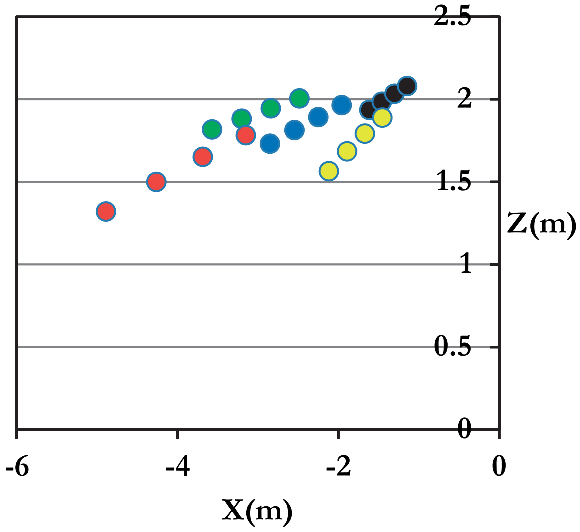

Figure 9. Coordinates of spinning ball trajectories crossing the goal's plane calculated for different altitudes and winds.

X, Z are the ball's coordinates in the point where the trajectory crosses the goal's plane, ΔX is the horizontal displacement of the ball and L is its range when the ball reaches the ground. The kick's launching angle is 15° starting velocity is 26 m/s and topspin=-8 rev/sec

First, we consider the ball's motion at sea level and start with kick#1 from Table 3 and then we compare kicks 2, 3, 4, 5 to that kick. Under the influence of Magnus force the ball's trajectory starts to curve to the left just like in Fig. 1. This curving leads to the final displacement ΔX=-5.209 m (first blue marker from the left in Fig. 9). The position of the ball in the goal is (-2.851, 1.731) against (0, 1.944) calculated without spin. Kicks from Table 3 are more widely distributed than kicks from Table 2. A 7 m/s tail wind increases the range of the kick almost by 3 m and decreases the deflection by 1.74 m, and the position of the ball in the goal is (-1.616, 1.936) (first black marker from the left in Fig. 9). A -7 m/s head wind decreases the range of kick #1 by 3.11 m and increases the deflection by 1.9 m, and the position of the ball in the goal is (-4.89, 1.321) (first red marker from the left in Fig. 9). Side winds significantly change the deflection of the ball's trajectory when passing through the goal: a 7 m/s side wind decreases it by 1.849 m (first yellow marker in Fig. 9), and a -7 m/s side wind increases it by 1.953 m (first green marker in Fig. 9). The final deflections of these two kicks are 3.36 m and 7.162 m, respectively (see Table 3).

The blue markers correspond to the still air with no wind, the red - to a -7 m/s head wind, the green - to a -7 m/s side wind, the yellow - to a 7 m/s side wind and the black - to a 7 m/s tail wind. The kick starts from the point (0, 19 m, 0) lying on the OY axis.

At higher altitudes balls fly faster, farther, higher and curve less: Fig. 9 shows that increase of altitude moves the markers of the same color to the right and higher. Even without wind the increase of altitude from sea level to 1800 m leads to the horizontal displacement of the ball's trajectory in the goal's plane by about 0.55 m. This result is in good agreement with the data published in (Hörzer et al., 2010).

Besides altitude there is another factor affecting the air density - the temperature. All our trajectories were calculated for the temperature T=15° C which corresponds to sea level standard temperature 288.15 K.

Air density ρ depends on temperature T and may be calculated using the following formulas:

$\rho=p/(R_{specific}\cdot T)$, where $p\approx p_0\cdot\exp(-gMh/R_0/T_0)$ - absolute pressure, $R_{specific}$ - 287.058 J/(kg·K), T - absolute temperature (K), p_0=101.325 kPa - sea level standard atmospheric pressure, g=0.9806 m/s^2 - earth-surface gravitational acceleration, M=0.0289644 kg/mol - molar mass of dry air, h(m) - altitude, R_0=8.31447 J/(mol·K) - ideal (universal) gas constant, T_0=288.15 K - sea level standard temperature.

In our program the temperature together with the ball's mass, the ball's cross-sectional area, the ball's diameter and aerodynamical coefficients is an external parameter and may be changed using a special interface. But there is also a rough approximation: the air density decreases when temperature increases and a 5° C temperature increase is approximately equivalent to a 150 m increase of altitude. This means, that the increase of temperature by 15° C from 15° C to 30° C is equivalent to the increase of altitude by 450 m which cannot be neglected especially if there is a side wind.

Conclusions

In this paper, a computational study of the three-dimensional equations describing the motion of a soccer ball through the air is presented. Excellent agreement is demonstrated between our results and numerical, analytical and experimental results published in the reliable scientific papers. The analytical solution shows explicitly how the ball's motion depends on parameters such as the ball size, mass and velocity and atmospheric conditions. The importance of applying three-dimensional models, rather than two-dimensional approximations, is demonstrated.

Based on our analysis we make rough approximations and recommendations for athletes and coaches:

For a goal-kick at sea level a rough approximation is that the range is increased or decreased by 2 m for each meter per second of the tail/head wind.

A rough approximation for goal kicks is, that a ball that travels about 50 m through the air at sea level will travel about 2.0-2.5 m farther with each 1000 m increasing of altitude.

Our calculations show that each meter per second of a side wind increases the deflection of the flight of a goal kick approximately by 1 m.

In case of free kicks crosswinds produce 2-3 times greater displacements in the goal's plane than tail/head winds.

At higher altitudes, it is more effective to use straight and fast free kicks, while at lower altitudes curved free kicks may be more resultative.

At higher altitudes balls fly faster, farther, higher and curve less.

Air density decreases when temperature increases. The increase of temperature by 5° C at sea level is approximately equivalent to the increase of altitude by 150 m.

Players should be aware of possible external effects on the ball trajectory. If they understand what happens in certain situations, that improves their performance. Our program helps players and coaches to understand and predict the effects of wind and altitude on the trajectory of a soccer ball. Players who are aware of altitude's and wind's effect on aerodynamics could have an advantage over those who don't. Knowing the average initial velocity of their kicks players can model how the ball will fly under effects of wind and altitude and determine the right angle for kicking. They will be able to better predict a trajectory of a soccer ball during a game and occupy better positions.

References

1. Alam, F., Chowdhury, H., Stemmer, M., Wang, Z., Yang, J. (2012). Effects of surface structure on soccer ball aerodynamics. Procedia Engineering 34, 146 – 151.
2. Alam, F., Chowdhury, H., Loganathan, B., Mustary, I., Watkins, S. (2014). Aerodynamic Drag of Contemporary Soccer Balls. 19th Australasian Proceedings of Fluid Mechanics Conference, Melbourne, Australia.
3. Asai, T., Seo, K., Kobayashi, O., Sakashita, R. (2007). Fundamental aerodynamics of the soccer ball. Sports Eng 10, 101–110.
4. Asai, T., Seo, K. (2013). Aerodynamic drag of modern soccer balls. SpringerPlus 2, 171-178. http:/dx.doi.org/10.1186/2193-1801-2-171
5. Barber, S., Chin, S. B., Carré, M. J. (2009). Sports ball aerodynamics: A numerical study of the erratic motion of soccer balls. Computers & Fluids 38(6), 1091–1100.
6. Bray, K., Kerwin, D. (2003). Modelling the flight of a soccer ball in a direct free kick. J Sports Sci 21(2), 75–85.
7. Carre´, M. J., Asai, T., Akatsuka, T., Hakke, S. J. (2002). The curve kick of a football ll: flight through the air. Sports Eng 5, 193–200.
8. Carre´, M. J., Goodwill, S. R., Haake, S. J. (2005) Understanding the effect of seams on the aerodynamics of an association football. J Mech Eng Sci 219, 657–666.
9. De Mestre, N. (1990). The Mathematics of Projectiles in Sports, Cambridge U. P., Cambridge.
10. Dupeux, G., Le Goff, A., Qur, D., Clanet, C. (2010). The spinning ball spiral. New J Phys 12, 1-12. http:/dx.doi.org/10.1088/1367-2630/12/9/093004
11. Dupeux, G., Cohen, C., Le Goff, A., Quere, D., Clanet, C. (2011). Football curves. Journal of Fluids and Structures 27, 659–667.
12. Goff, J. E., Carre´, M. J. (2009). Trajectory analysis of a soccer ball. Am J Phys 77(11),

DOI: 10.21846/TST.2017.4.1

1020–1027.
13. Goff, J. E., Carre´, M. J. (2010). Soccer ball lift coefficients via trajectory analysis. Eur J Phys 31, 775–784.
14. Hörzer, S., Fuchs, C., Gastinger, R., Sabo, A., Mehnem, L., Martinek, J., Reichel, M. (2010). Simulation of spinning soccer ball trajectories influenced by altitude, 8th conference of the International Sports Engineering Association (ISEA). Procedia Eng 2, 2461–2466.
15. Mehta, R. D. (1985). Aerodynamics of sports balls, Annu. Rev. Fluid Mech. 17, 151–189.
16. Myers, T. G., Mitchell, S. L. (2013). A mathematical analysis of the motion of an inflight soccer ball. Sports Eng 16, 29-41. http:/dx.doi.org/10.1007/s12283-012-0105-8
17. Oggiano, L., Sætran, L. (2010). Aerodynamics of modern soccer balls. Procedia Eng 2, 2473–2479.
18. Passmore, M., Tuplin, S., Spencer, A., Jones, R. (2008). Experimental studies of the aerodynamics of spinning and stationary footballs. Proc Inst Mech Eng Part C J Mech Eng Sci 222(2), 195–205.
19. Wesson, J. (2002). The science of soccer. IoP Pub. Ltd., Bristol.
20. White, C. (2010). Projectile Dynamics in Sport: Principles and Applications. Routledge.