Extremal graph theory, Stability, and Anti-Ramsey theorems

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Extremal graph theory is one of the most developed branches of Discrete Mathematics. Stability methods are very successful in this field. We shall give some illustration of this method for graphs, hypergraphs, and Anti-Ramsey problems. An Anti-Ramsey problem is where a sample graph L is fixed and we colour the edges of , e.g., a complete graph K_n without having a copy of L in which all the edges have distinct colours.

We shall consider Dual Anti-Ramsey problems, coming from Theoretical Computer Science. Burr, Erdős, Graham and T. Sós [1] defined and investigated a *dual* variant of the ANTI-RAMSEY problems.

> **The dual Anti-Ramsey problem.** Fix a sample graph L, and consider a (variable) graph G_n on n vertices, with $e = e(G_n) > \mathbf{ex}(n, L)$ edges. Let $\chi_S(G_n, L)$ denote the *minimum* number of colours needed to colour the edges of G_n so that no $L \subseteq G_n$ has two edges of the same colour. Determine
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> $$\chi_S(n, e, L) := \min \{\chi_S(G_n, L) \ : \ e(G_n) = e\}\,.$$

Here we improve several results of [1] and [2].

This lecture is partly based on a manuscript of Erdős and Simonovits [3] from the late 1980's.

References

[1] S. A. Burr, P. Erdős, R. L. Graham and Vera T. Sós: Maximal antiramsey graphs and the strong chromatic number, Journal of Graph Theory, vol 13(3), (1989) pp 163–182.

[2] S. Burr, P. Erdős, P. Frankl, R. L. Graham, V. T. Sós: Further results on maximal Anti–Ramsey graphs Proc. Kalamazoo Combin. Conf., 1989 pp 193–206

[3] P. Erdős and M. Simonovits: How many colours are needed to colour every pentagon of a graph in five colours? (manuscript, under publication)