EXPLORING YOUNG STUDENTS CREATIVITY: THE EFFECT OF MODEL ELICITING ACTIVITIES

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The aim of this paper is to show how engaging students in real-life mathematical situations can stimulate their mathematical creative thinking. We analyzed the mathematical modeling of two girls, aged 10 and 13 years, as they worked on an authentic task involving the selection of a track team. The girls displayed several modeling cycles that revealed their thinking processes, as well as cognitive and affective features that may serve as the foundation for a methodology that uses model-eliciting activities to promote the mathematical creative process.

Keywords: Creative thinking process; Mathematical creativity; Model eliciting activities; Real-life problems

Exploración de la creatividad de jóvenes estudiantes: el efecto de actividades que suscitan modelos

El objetivo de este artículo es mostrar cómo involucrar a los estudiantes en situaciones matemáticas de la vida real puede estimular su pensamiento matemático creativo. Analizamos la modelización matemática de dos chicas, de 10 y 13 años, cuando trabajaban en una tarea auténtica que involucraba la selección de un equipo de atletismo. Las chicas mostraron varios ciclos de modelización que revelaron sus procesos de pensamiento, así como las características cognitivas y afectivas que pueden servir como fundamento para una metodología que usa actividades que suscitan modelos para promover los procesos matemáticos creativos.

Términos clave: Actividades que suscitan modelos; Creatividad matemática; Problemas de la vida real; Procesos de pensamiento creativo

For the past few years, there has been an increasing demand for new ways of structuring mathematics. The Organization for Economic Cooperation and Development (2008) stated that mathematics “curricula should reflect the reality… [and] should stress innovative applications of mathematics” (p. 18).

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Finding or developing diverse dimensions of mathematical education is not enough; one has to consider the rapid progress in science and technology, which has characterized the 21st century, and its effects. This accelerating progress has become a part of almost every aspect of our changing world, requiring the development of certain abilities and skills among students; among these, adaptability, the ability to solve non-routine real-life problems, creativity and systems skills have become crucial factors (Hilton, 2008; Jerald, 2009). Therefore introducing new methods of learning and teaching mathematics should reflect this rapid progress, enabling our students to successfully integrate into the 21st century. Mathematical Model-Eliciting Activities (MEAs) provide the student with opportunities to deal with non-routine real-life challenges. These authentic challenges encourage them to ask questions, and to be sensitive to the complexity of mathematically structured situations, as part of developing, creating and inventing significant mathematical ideas. However, the development of students' mathematical creative thinking through MEAs has only been addressed in a few studies to date (Chamberlin & Moon, 2005). The study reported herein examines the role of affective and cognitive elements (Goldin, Epstein, Schorr, & Warner, 2011) in facilitating the development of students' mathematical creativity through MEAs.

Mathematical Modeling

Mathematical models are conceptual systems consisting of elements, relations, operations, and rules governing interactions; these are expressed with external notation systems, which are used to construct, describe, explain or predict the behaviors of other systems. Model-development processes usually involve a series of recursive cycles consisting of developing, testing, and revising phases in which a variety of different ways of thinking are repeatedly expressed, tested, and revised or rejected (Lesh & Doerr, 2003; Lesh & Thomas, 2010). Mathematical-modeling activities are based on "real-life" problem situations in which students are given the opportunity to construct powerful ideas relating to interdisciplinary data (Lesh & Sriraman, 2005). These activities are open-ended in nature. The ambiguity of the problem statement and data representation suggests that various responses may be appropriate and that there are likely various levels of correctness, depending on students' interpretations, mathematical abilities, general knowledge and skills (Chamberlin & Moon, 2005). MEAs are designed according to six principles: reality, construction, self-evaluation, shareability, model documentation, and effective prototype. These principles emphasize the importance of stimulating students' competence to extend their own personal knowledge and apply their real-life sense-making abilities to the creation of original mathematical models (Lesh, Amit, & Schorr, 1997; Lesh, Hoover, Hole, Kelly, & Post, 2000).

Mathematical Creativity

Many researchers see the potential for mathematical creativity as a dynamic ability that can be developed in students. They associate students' creative ability with cognitive problem-solving abilities and suggest several ways of stimulating and assessing it (Amit, 2010; Haylock, 1997; Sriraman, 2008). Haylock (1997) suggests that breaking mental set or as he described it "overcoming fixation" is a crucial factor in creativity. In his study he demonstrated creative responses, which allowed students to overcome fixations and solve complex problems. Sriraman (2008) defines mathematical creativity as the ability to produce novel or original work. He claims "in order for mathematical creativity to manifest itself in the classroom, students should be given the opportunity to tackle non-routine problems with complexity and structure problems which require not only motivation and persistence but also considerable reflection" (p. 32). According to Kruteskii (1976), mathematical creativity appears as flexible mathematical thinking which is "switching from one mental operation to another qualitatively different one" (p. 282), which depends on openness to free thinking and exploration of diverse approaches to a problem. Pólya (1957) provides heuristics to tackle mathematical problems in his book *How to Solve It* and defines some cognitive characteristics of the ingenious solver that might lead him/her to the discovery of an original solution. He claims that analogous objects agree in certain relations of their respective parts, and explain "all sort of analogy may play a role in discovery of the solution…" (p. 38). Leikin (2009) suggests observing and evaluating mathematical creativity through the lens of multiple solution tasks and states "solving mathematical problems in multiple ways is closely related to personal mathematical creativity" (p. 133). Some researchers have examined the connection between mathematical problem posing and creativity (Silver, 1997; Yuan & Sriraman, 2010). Yuan and Sriraman (2010) compare problem posing and creative abilities of mathematically gifted Chinese and American students. Silver (1997) demonstrates an approach of fostering mathematical creativity through problem posing and problem solving in terms of fluency, flexibility and novelty.

Methodology

In this section we present the methodological characteristics of the empirical study, focusing on the research design, the participants, and the data sources.

Research Design

The study reported herein was based on two tasks: a warm-up activity and the MEA. The warm-up activity was aimed at preparing the girls for the modeling task and took about an hour and a half. Each girl received a newspaper article about Usain St. Leo Bolt, the Jamaican sprinter and Olympic gold medalist. The article contained Bolt's records and a qualitative description of his run in which he set the new world record in the 100 meter dash. After reading the article, each girl had to answer questions about it, constituting the basis for a discussion held between the researcher and the two girls. During that discussion, questions were raised regarding the definitions of the article's concepts (speed, rate, etc.) and their implications. The main purpose of this activity was to stimulate the girls' interest and motivation, and to familiarize them with the context of the modeling task, including factual knowledge, and cognitive and technical skills, so that their solution would stem from their own experience (Lesh et al., 2000). The modeling task (Figure 1) was designed according to the afore-listed six principles (Chamberlin & Moon, 2005) and based on English and Watters' modeling activity "The Olympic Team" (English & Watters, 2005). It was a non-routine, real-life challenge, which allowed formulation of several (mathematically justified) solutions, depending on each girls mathematical abilities, general knowledge and skills (Sriraman, 2008). The modeling task was based on a situation that could exist in the girls' daily lives and required a "real" solution (Chamberlin & Moon, 2005). The MEA consisted of the three following sessions.

- ♦ Model development: Each girl worked by herself (75 minutes).
- ♦ Presentation and discussion: Each girl presented her solution (30 minutes).
- ♦ Interviews with each girl.

Assigning team members for 100 x 4 boys' and girls' relay race

Tables 1 and 2 contain the records of 4 boys and 4 girls who won silver or gold medals in 60 meter or 100 meter runs that took place in autumn, winter, spring and summer of 2010. A relay race for 6th graders is going to take place 2 weeks from now. For the first time, boys and girls will compete together in a mixed race between all of the schools in the city. Due to the short notice, the head of the sports committee at your school needs your help: he has to decide which two boys and two girls to assign to the relay team based on their accomplishments in the 2010 races. Your task is to construct a guide that will help the head of the sports committee choose the best team members for the 100 x 4 relay race.

Sample from Tables I and II:

Ali –(name)	**Autumn**	**Winter**	**Spring**	**Summer**
Table I 60m	9.5 s (Gold-medal)	9.7 s	9.5s	9.3 s (Gold-medal)
Table II 200m	38 s	37.5 s (Silver-medal)	39 s (Silver-medal)	38 s

Figure 1. Relay race modeling task

Participants

The participants in this case study were two girls, 10-year-old (Rotem) and 13-year-old (Shir). The girls had high achievements in mathematics and were participating in a special enrichment class for excellent students at their school. In addition, the two girls were enthusiastic about sports and took part in running races at school.

Data Sources

The study was based on recorded interviews with each of the girls, their written material collected at the end of both tasks, the researcher's notes taken during the task solving and recordings of conversations during the activities and of the final discussions at the end of each task. It should be emphasized that the girls were asked to write down everything, so that drafts, sketches and final solutions could be collected. During the interviews and the conversations, the researcher did not accept simple or standard answers. Each answer was discussed with the girls in order to understand their way of thinking. Attention was paid to their body language and the vocabulary they used, in order to understand their experience, and its meaning and importance from each girl's perspective.

Findings and Results

Analysis of the findings revealed two types of characteristics involved in the mathematical-modeling process: cognitive and affective. These features influenced the progress of the creative process and the creativity of both girls' conceptual tools. The girls' mathematical models contained unusual criteria for ranking scores to grade all runners.

The Mathematical-Modeling Process

During the MEA, the girls went through several modeling cycles. In each cycle, the girls creatively developed mathematics that were new to them. Shir's model-eliciting process consisted of four cycles. In the first cycle, criterion selection, Shir chose three criteria, based on her notion of fairness. She said, "I need criteria to decide, who the best is. I need at least three criteria no less. I have to provide an equal opportunity for all runners". After quantifying the data, she found that three criteria were not enough, as two runners received identical values. She moved to the second cycle, record improvement, where she added a fourth criterion to resolve the problematic situation: She decided to quantify each runner's improvement over the course of the year. In the third cycle, scoring system, she realized that quantifying the fourth criterion and comparing runners' results was inconvenient. She therefore ranked the results for every criterion and set up a scoring system: "I have to weigh all of the data to know who the best is. Scoring is much more convenient than comparing each and every one". In the fourth cycle, generalization, Shir tried generalizing her solution. For each criterion, she added a mathematical formulation along with a written explanation that clarified scoring calculation and ranking weight and could be adapted to, and transformed, for other, similar situations (e.g., establishing other sports teams).

As an example, Figure 2 presents part of Shir's letter to the head of the committee explaining how to use the improvement criterion.

III Record improvement: Select the competition that all nominees participated in twice, at least half year apart, and compare. Check how the results improved and set score criterion. For example: 1 second of improvement equals 1 point.
Note: At the end of grading, sum all scores for each criterion and select those with the highest scores

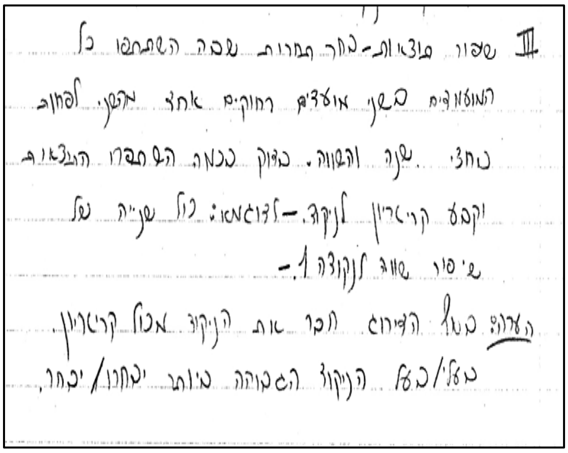

Figure 2. Records improvement

Cognitive Characteristics

We focus on three aspects as cognitive characteristics: flexibility, combination, and analogy.

Flexibility. In the first modeling cycle, Rotem chose three criteria to distinguish between all runners. In the second cycle, she realized that one of these—tallying medals for each runner—was not a sufficient good criterion because two runners got the same score. She found a different way to solve the problem: "… to run in summer is much more difficult, so winning a medal in the summertime is worth more." She used seasons as weighting variables to formulate a weighted sum for her medal criterion.

Combination. In the second cycle, Shir added one more criterion by bringing together the runners' complete record and the season in which they competed in an original mathematical combination. Figure 2 shows part of Shir's letter to the head of the sports committee, generally explaining how to apply this criterion.

Analogy. During the warm-up activity, Rotem compared Bolt's records. She drew an analogy between getting tired and slowing down: "A 400 meter run is much more tiring than a 100 meter run, so you run much slower because you are getting tired and it takes more time." She continued her solution idea and said "400 is four times 100, but 400 meters he [Bolt] runs in 45 seconds and 100 meters he runs in 10 seconds (approximately) so 400 took him 45 instead of 40." Rotem discovered a new mathematical formulation for the concept speed—the number of seconds taken to pass a fixed distance—, which suited her intuitive everyday thinking.

Affective Characteristics

We focus on motivation and interest, self-efficacy and persistence, and metacognition and persistence and self-reflection as affective characteristics.

Motivation and interest. During the activities, the girls showed intense involvement, which was reflected in the level of interest, curiosity and meaning they found in the modeling task. Rotem explained: "In all mathematical exercises you only need to calculate and solve, but when 'real life' is involved it is much easier and fun to think, because you don't think about the mathematics you think about life."

Self-efficacy and persistence. The girls' understanding and recognition of the necessity of the task affected their persistence and will to continue, even though it was sometimes difficult and complex. Rotem said: "It wasn't easy but I hardly thought about it, I knew and understood that I can find the solution and help him [the head of the sports committee] find the best runner."

Metacognition and self-reflection. Throughout the course of the task activities and each of its phases, the girls were aware of their own thinking in a way that affected and regulated their activities. Rotem, at the end of her first cycle said: "I didn't think well enough about my criteria… to know who the best runner is… I have to think about some more criteria and how to formulate them." During the interview, Shir described her work: "I didn't know how to apply to the task; I had to think in a different way, to think more real thinking, there was no single right solution and it made me think about other solutions, which is the best one, and not to think in a rigid way."

Conclusions

In the presented case study we examined how teaching for creativity through MEAs encourages the development of students' creative mathematical thinking. The findings clearly show some cognitive and affective characteristics that could establish the foundations for creative process development methodology using MEAs. The participants were two girls aged 10 and 13 years. The modeling task was based on meaningful situations that could occur in their real lives in order to stimulate their motivation and engagement. The results exhibit the essential role of the affective (Goldin et al., 2011) and cognitive aspects in the development of creative performance during the mathematical-modeling process. The practical implications of the current case study suggest that engaging students with non-routine mathematical problems (Sriraman, 2008) through MEAs can encourage them to develop, create or invent significant mathematical artifacts or tools (Lesh & Thomas, 2010).

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