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# The nonstationary, nonlinear dynamic interactions in slender continua deployed in high-rise vertical transportation systems in the modern built environment

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**Abstract.** This paper presents a nonlinear mathematical model and numerical results concerning the nonstationary lateral dynamic behaviour of long low tension slender continua deployed and moving at speed in high-rise vertical transportation systems installed in tall structures. The analysis presented in this study involves the identification of conditions for internal lateral resonances that can readily arise in the system when the slowly varying frequencies approach the fundamental or higher frequencies of the structure. The passage through the fundamental resonance leads to dangerously large displacements in the plane of the excitation. Due to the nonlinear (cubic) coupling, interactions between the in-plane modes and the out-of-plane modes occur. These interactions are studied numerically in order to predict and to examine the non-planar motions that may arise due to the autoparametric resonances. In order to suppress the internal resonance interactions higher speed levels and /or cable tension levels should be applied. Alternatively, an active tension control algorithm can be considered.

## 1. Introduction

One-dimensional slender structures such as strings, belts, ropes and cables are deployed as stationary and/or moving structural members in many engineering systems. Their dynamic behaviour is inherently non-linear demonstrating complex dynamic responses [1-3]. In the context of the modern built environment nonlinear resonance phenomena occur in suspension ropes, compensating and overspeed governor cables employed in high-speed elevator installations in high-rise buildings [4-5]. Large whirling motions of ropes and cables in elevator installations may result in damage caused by the impact against the elevator equipment located in the shaft and/or against the shaft walls. The dynamic responses of such systems can be predicted and analyzed by modeling their behaviour and solving the resulting equations of motions. Due to the nonstationary and nonlinear nature of the problems involved solving these equations by applying analytical methods is difficult and the most convenient approach is using numerical simulation [6]. In this paper, the nonstationary linear planar model presented in [7] is developed further to accommodate the nonlinear modal interactions in a system comprising a heavy, low tension vertical cable of varying length moving at speed within a tall host structure subjected to a low frequency sway. The nonstationary nonlinear responses of the cable are studied numerically in order to predict and to examine its non-planar motions that arise due to autoparametric resonances. The results presented in the paper can be applied in the design of compensation systems in high-rise vertical transportation systems.

**2. Dynamic model**

The model of a heavy cable of time-varying length  $L(t)$  is depicted in Figure 1. The cable has mass per unit length  $m$ , elastic modulus  $E$  and an effective cross-sectional area  $A$ . It passes over a sheave at the bottom of the structure with the upper end fixed to a termination at C moving at speed  $V(t)$  while the host structure is subjected to bending in-plane and out-of-plane deformations described by the shape functions  $\Psi_v(z)$  and  $\Psi_w(z)$ , respectively, with  $z$  denoting a coordinate measured from ground level. The bending deformations result in a sway of the structure producing the in-plane and out-of-plane harmonic motions  $v_0(t)$  and  $w_0(t)$  at frequencies  $\Omega_v$ ,  $\Omega_w$  and amplitudes  $v_{0max}$ ,  $w_{0max}$ , respectively, measured at the level defined by the coordinate  $z_0$ . The system is equipped with the actuator F acting at the sheave end of the system to control the tension of the cable.

The sway of the structure leads to an inertial dynamic load acting upon the cable. The dynamic response to this load is represented by the lateral in-plane and out-of-plane displacements denoted as  $v(x,t)$  and  $w(x,t)$ , respectively, where  $x$  is a spatial coordinate measured from the centre of the sheave as shown in Figure 1. The lateral motions are coupled with axial (longitudinal) motions that are denoted as  $u(x,t)$ .

The mean tension of the element is expressed as

$T^i(x,t) = T_0 + m[g + a(t)]x$ , where  $T_0$  represents a constant tension term,  $a(t) = \dot{V}(t)$  is the acceleration of the upper support (an overdot denotes the time derivative) and  $g$  is the acceleration of gravity. By using the large displacement approximation Green's axial strain due to stretching of the element given as  $\epsilon(x,t) = u_x + \frac{1}{2}(v_x^2 + w_x^2)$ , where  $( )_x \equiv \frac{\partial( )}{\partial x}$ , the equations governing the undamped dynamic displacements  $u(x,t)$ ,  $v(x,t)$  and  $w(x,t)$  can be developed by applying Hamilton's principle. It is assumed in the derivation that for tensioned members such as steel wire ropes and metallic cables the lateral frequencies are much lower than the longitudinal frequencies. Furthermore, for structures such as high-rise towers and buildings the excitation frequencies  $\Omega_v$ ,  $\Omega_w$  are much lower than the fundamental longitudinal frequencies of the cable and no interaction will take place between the lateral modes and the longitudinal modes. Thus, the longitudinal inertia of the cable can be neglected and by taking into account the boundary condition  $u(0,t) = u[L(t),t] = 0$  the lateral in-plane and out-of-plane responses are described by the following two equations

$$mv_{tt} - \left\{ T_0 + m \left[ gx - V(t)^2 \right] + EAe(t) \right\} v_{xx} + m[a(t) - g]v_x + 2mV(t)v_{xt} = 0 \tag{1}$$

$$mw_{tt} - \left\{ T_0 + m \left[ gx - V(t)^2 \right] + EAe(t) \right\} w_{xx} + m[a(t) - g]w_x + 2mV(t)w_{xt} = 0 \tag{2}$$

where  $e(t) = \frac{ma(t)L(t)}{2EA} + \frac{1}{2L(t)} \int_0^{L(t)} (v_x^2 + w_x^2) dx$  is the quasi-static axial strain in the cable. The displacements and the spatial coordinate  $x$  are defined in a time-variant domain defined as  $x \in D(t) = \{0 < x < L(t)\}$ . Noting that the lateral displacements at the boundaries are given as

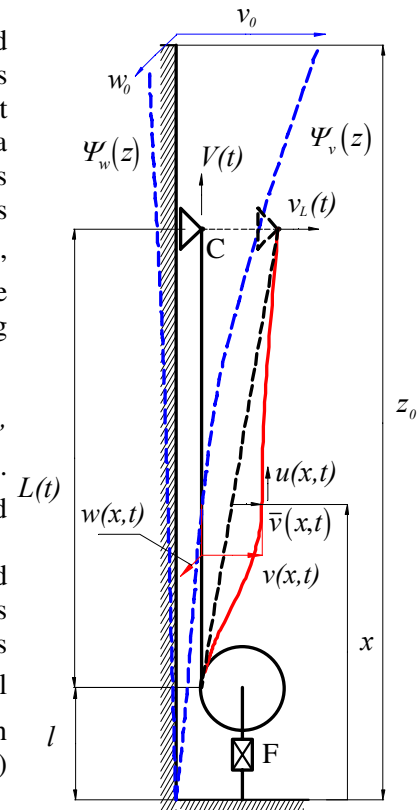


Figure 1. Cable model

$v(0,t) = 0; v[(L(t),t)] = v_L(t); w(0,t) = 0; w[(L(t),t)] = w_L(t)$ , where  $v_L(t)$  and  $w_L(t)$  represent the in-plane and out-of-plane lateral displacements of the structure corresponding to the upper end of the cable, equations (1)–(2) are discretized by expressing the lateral displacements as  $v(x,t) = \bar{v}(x,t) + \frac{x}{L(t)}v_L(t)$  and  $w(x,t) = \bar{w}(x,t) + \frac{x}{L(t)}w_L(t)$ , respectively. In these expressions the displacements  $\bar{v}(x,t)$  and  $\bar{w}(x,t)$  are defined as

$$\bar{v}(x,t) = \sum_{n=1}^N \Phi_n[x;L(t)] p_n(t); \bar{w}(x,t) = \sum_{n=1}^N \Phi_n[x;L(t)] q_n(t), \quad 0 \leq x \leq L(t) \quad (3)$$

These are approximate solutions for the in-plane and out-of-plane modes, respectively, that satisfy homogenous boundary conditions. Functions  $\Phi_n[x;L(t)]$ ,  $n = 1, 2, \dots, N$  represent the trial functions defined as  $\Phi_n[x;L(t)] = \sin \frac{n\pi}{L(t)}x$ , and  $p_n(t)$ ,  $q_n(t)$ ,  $n = 1, 2, \dots, N$  represent the modal coordinates / displacements, with  $N$  denoting the number of modes assumed in the expansions (3).

It can be assumed that the length  $L$  of the cable is a slowly varying parameter, i.e. that its variation is observed on a slow time scale defined as  $\tau = \epsilon t$ , where where  $\epsilon \ll 1$  is a small quantity [6]. Thus, the derivatives of  $L$  with respect to time  $t$  is proportional to  $\epsilon$

$$\frac{dL}{dt} \equiv \dot{L} = \frac{dL(\tau)}{d\tau} \frac{d\tau}{dt} = \epsilon \frac{dL(\tau)}{d\tau}; \ddot{L} = \epsilon^2 \frac{d^2L(\tau)}{d\tau^2} \quad (4)$$

Consequently, the velocity  $V$  and acceleration  $a$  are also assumed to be slowly varying. By using (3) in the equations of motion (1-2), taking into account (4), orthogonalising with respect to the natural modes, when terms  $O(\epsilon)$  and  $O(\epsilon^2)$  are neglected, the following set of  $2N$  ordinary nonlinear differential equations with slowly varying coefficients results

$$\ddot{p}_r(t) + 2\zeta_r \omega_r(\tau) \dot{p}_r(t) + \sum_{n=1}^N C_{rn}(\tau) \dot{p}_n(t) + \lambda_r^2(\tau) \left\{ \bar{c}^2 - V^2(\tau) + \frac{1}{2}a(\tau)L(\tau) + \frac{1}{2}c^2 \left[ \left( \frac{v_L(t)}{L(\tau)} \right)^2 + \left( \frac{w_L(t)}{L(\tau)} \right)^2 \right] \right\} p_r(t) + \quad (5)$$

$$\sum_{n=1}^N K_{rn}(\tau) p_n(t) + \left( \frac{\lambda_r(\tau)}{2} c \right)^2 p_r(t) \sum_{n=1}^N \lambda_n^2(\tau) [p_n^2(t) + q_n^2(t)] = P_r(t, \tau), \quad r = 1, 2, \dots, N$$

$$\ddot{q}_r(t) + 2\zeta_r \omega_r(\tau) \dot{q}_r(t) + \sum_{n=1}^N C_{rn}(\tau) \dot{q}_n(t) + \lambda_r^2(\tau) \left\{ \bar{c}^2 - V^2(\tau) + \frac{1}{2}a(\tau)L(\tau) + \frac{1}{2}c^2 \left[ \left( \frac{v_L(t)}{L(\tau)} \right)^2 + \left( \frac{w_L(t)}{L(\tau)} \right)^2 \right] \right\} q_r(t) + \quad (6)$$

$$\sum_{n=1}^N K_{rn}(\tau) q_n(t) + \left( \frac{\lambda_r(\tau)}{2} c \right)^2 q_r(t) \sum_{n=1}^N \lambda_n^2(\tau) [p_n^2(t) + q_n^2(t)] = Q_r(t, \tau), \quad r = 1, 2, \dots, N$$

where the modal damping represented by the ratios  $\zeta_r$  has been added,  $\omega_r$ ,  $r = 1, 2, \dots, N$ , are slowly varying undamped natural frequencies of the system,  $\lambda_n(\tau) = \frac{n\pi}{L(\tau)}$ ,  $c = \sqrt{EA/m}$  and  $\bar{c} = \sqrt{T_0/m}$

represent the longitudinal wave speed and the lateral wave speed, respectively.  $K_{rn}(\tau)$ ,  $C_{rn}(\tau)$  are linear slow time-variant coefficients and  $P_r(t;\tau)$ ,  $Q_r(t;\tau)$  are the modal excitation functions. Equations (5-6) accommodate the influence of weight of the cable as well as the Coriolis and centrifugal acceleration effects. It is evident that the host structure motion results in modal external excitation terms and also appears in the stiffness coefficients as parametric excitation. The presence of cubic nonlinear coupling terms arising due to the effect of cable stretching promotes conditions for autoparametric interactions which is investigated in the case study in what follows.

### 3. Autoparametric interactions: a case study

Numerical simulation tests are applied to study the lateral dynamics of the system in a scenario where the attachment point C of the cable is moving upwards starting from the lowest position and coming to a stop after the cable is fully extended. The cable is a steel wire rope of mass per unit length  $m = 1.54$  kg/m, having the modulus of elasticity  $E = 0.7 \times 10^5$  N/mm<sup>2</sup> and an effective cross-sectional area  $A = 178$  mm<sup>2</sup>. The modal damping ratios  $\zeta_r$  of the rope are assumed to be 3% across all modes. The rope is being accelerated from rest, when its initial length is  $L(0) = l = 4.9$  m, at the rate of 1.1 m/s<sup>2</sup>. It is moving upwards at the maximum speed and is later being decelerated at the rate of 1.1 m/s<sup>2</sup>, coming to rest reaching the maximum length of  $L_{\max} = 416.2$  m. Two maximum speeds are considered in the simulation tests:  $V_{1\max} = 2$  m/s and  $V_{2\max} = 8$  m/s, respectively. The host structure is subjected to a strong in-plane harmonic excitation and sways in its fundamental mode with large bending motions in the in-plane direction. The deformation shapes of the structure  $\Psi_v(z)$  and  $\Psi_w(z)$  are approximated by cubic polynomial functions [7]. These motions are of frequency 0.0675 Hz ( $\Omega_v = 0.4241$  rad/s) with large displacements reaching the amplitude of  $v_{0\max} = 0.8$  m at the level corresponding to  $z_0 = 422.1$  m. On the other hand, in the out-of-plane direction the fundamental frequency of the structure is assumed to be much higher, it is 0.2 Hz ( $\Omega_w = 1.2566$  rad/s), and only very small bending fundamental mode motions of the structure occur, with the maximum amplitude assumed as  $w_{0\max} = 0.007$  m recorded at the same level (422.1 m). Thus, in this case scenario the excitation can be considered as nearly planar. The numerical simulations are carried out when the cable is subjected to a constant low mean tension of  $T_0 = 61.3$  N. The explicit Runge-Kutta (4,5) formula is used to integrate the equations of motion (5-6) on the fast time scale with the slowly varying coefficients  $C_{rn}$  and  $K_{rn}$  pre-calculated and evaluated at each integration step. The dynamic displacements relative to the stretched configuration of the cable are then determined from the solution using the expansions (3). The tests conducted show that a good convergence in (3) can be achieved by using relatively small number of modes  $N$ . In the case study under consideration  $N = 5$  has been applied.

The natural frequencies  $\omega_r(\tau)$ ,  $r = 1, 2, \dots, N$ , are determined by freezing the slowly varying parameters in the matrix of linear coefficients in the model (5-6) at each time step and calculating the eigenvalues taking into account the influence of weight, speed and acceleration of the cable. The first three natural frequencies of the cable for the two speeds are plotted vs. time in Figure 2(a) and 2(b), respectively, together with the frequencies of the in-plane the out-of plane motions of the host structure. It is evident that a passage through the fundamental resonance in the in-plane direction takes place at the time instant of approximately 90.4 s and 22.1 s for  $V_{1\max} = 2$  m/s and  $V_{2\max} = 8$  m/s, respectively. In the out-of-plane direction the fundamental resonance is reached at the time instant of

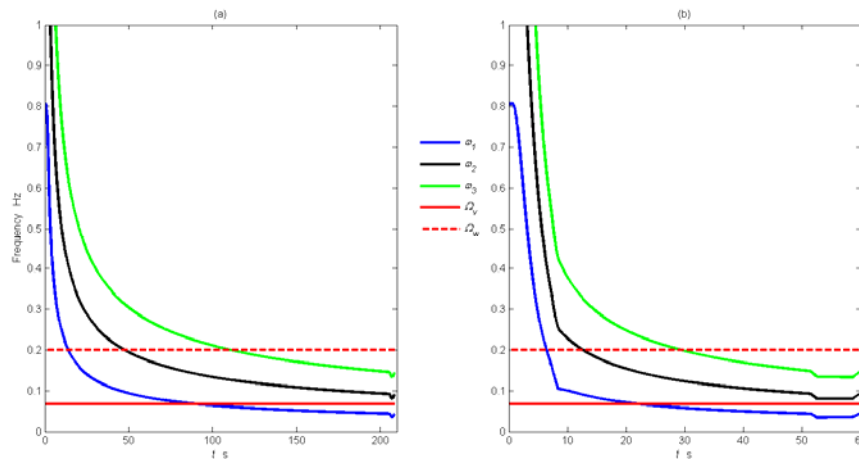


Figure 2. The natural frequencies of the cable shown together with the frequencies of the structure: (a) for  $V_{1max} = 2$  m/s , (b) for  $V_{2max} = 8$  m/s

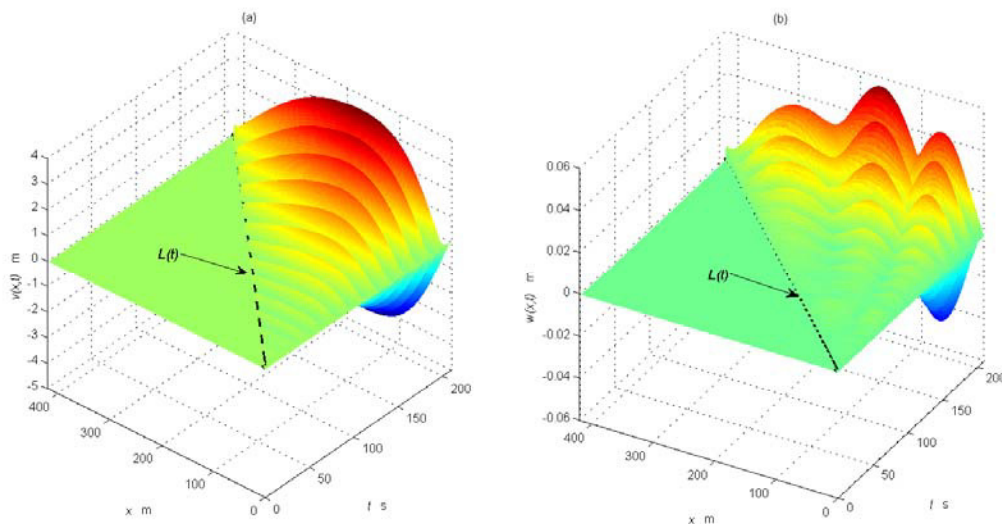


Figure 3. Displacements of the cable corresponding to  $V_{1max} = 2$  m/s : (a)  $v(x,t)$  and (b)  $w(x,t)$

approximately 14 s and 6.4 s, respectively. Then, at the time instants of approximately 47.5 s and 12.6 s the frequency of the excitation becomes equal to the second natural frequency of the cable. More importantly, at the time instants of approximately 110.8 s and 29.3 s the frequency of the excitation becomes equal to the third natural frequency of the rope which results in passage through the third out-of-plane resonance. The analysis of responses shown in Figure 3 and Figure 4 and the examination of the corresponding modal coordinates can be summarized as follows. Both the in-plane and out-of-plane responses grow with time. The in-plane dynamics exhibit responses with large amplitudes and the influence of the fundamental resonance is evident. The out-of-plane displacements are smaller with strong 3<sup>rd</sup> mode contribution being evident. The cubic nonlinearities promote energy exchanges between the modes. The energy from the fundamental in-plane mode is being transferred to the corresponding out-of-plane mode due to autoparametric (internal) 1:1 resonance. This phenomenon is prominent during the final stages of motion when the cable approaches its maximum length. Furthermore, noting that the excitation frequencies of the structure are in the ratio of  $\Omega_w / \Omega_v \simeq 3$ , in the region corresponding to the time instant of 110.8 s for the lower speed, and 29.3 s for the higher speed, the frequency tuning  $\Omega_w = \omega_3 \simeq 3\omega_1$  takes place. Thus, due to a 3:1 internal resonance the energy is being exchanged between the 3<sup>rd</sup> and the fundamental out-of-plane modes. The responses corresponding to the higher speed exhibit smaller amplitudes due to the fact that the passages

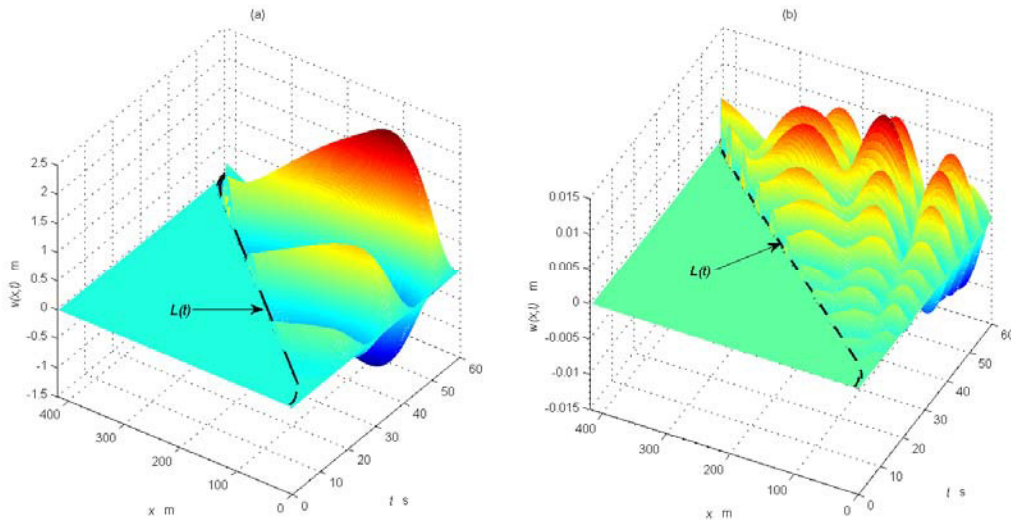


Figure 4. Displacements of the cable corresponding to  $V_{2max} = 8$  m/s : (a)  $v(x,t)$  and (b)  $w(x,t)$

through the resonance regions are faster. However, further simulation tests carried out to investigate the behaviour of the system after the cable has stopped reaching its maximum length reveal that the in-plane displacements continue to grow resulting in full non-planar tubular motions and high tensile stresses in the cable. A possible resonance control strategy would involve using the actuator  $F$  to increase the mean tension in order to shift the natural frequencies above the frequency of the sway excitation. Active methods such as active stiffness control [8] can also be considered.

#### 4. Conclusions

The dynamic behaviour of long, low tension heavy cables and ropes moving at speed in tall civil structures is inherently non-linear. The model presented in this paper accommodates the cubic geometric nonlinearities arising due to the effect of cable stretching and takes into account the influence of slow variation of length, the weight of the cable as well as the Coriolis and centrifugal acceleration effects. The analysis of results presented in the paper shows that due to the nonlinear coupling, important interactions between the in-plane mode and the out-of-plane mode might occur. Ultimately, the planar motions may become unstable resulting in large non-planar tubular motions with the cable being subjected to high tensile stresses. The scenarios discussed in this paper reveal that in order to avoid adverse internal resonance interactions the amplitudes of the excited modes that are involved in 1:1 and 3:1 resonances should not reach their critical threshold levels. This can be achieved if higher travel speeds and/ or cable tensions are applied. Alternatively, a suitable actuator control algorithm can be used to actively suppress the resonance responses.

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