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Cooperation of Storage Operation in a Power Network with Renewable Generation

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***Abstract*—In this paper, we seek to properly schedule the operation of multiple storage devices so as to minimize the expected total cost (of conventional generation) in a power network with intermittent renewable generation. Since the power network constraints make it intractable to compute optimal storage operation policies through dynamic programming based approaches, we propose a Lyapunov optimization based online algorithm (LOPN) which makes decisions based only on the current state of the system (i.e., the current demand and renewable generation). The proposed algorithm is computationally simple and achieves asymptotic optimality (as the capacity of energy storage grows large). To improve the performance of the LOPN algorithm for the case with limited storage capacity, we propose a Threshold based Energy Storage Management (TESM) algorithm that utilizes the forecast information (on demand and renewable generation) over the next a few time slots to make storage operation decisions. Numerical experiments are conducted on IEEE 6- and 9-bus test systems to validate the asymptotic optimality of LOPN and compare the performance of LOPN and TESM. Numerical results show that TESM significantly outperforms LOPN when the storage capacity is relatively small.**

***Index terms*—** Energy storage, DC power network, Lyapunov optimization, Look-ahead policy

I. INTRODUCTION

Electricity generation from conventional thermal generators is one of the most important contributors of global green house gas emissions [1]. Renewable energy (e.g. solar, wind etc.) presents a cleaner alternative for electricity generation. However, it is well known that renewable energy sources exhibit significant variability and uncertainty, which make it challenging to integrate renewable generation into power systems. As a result, growing penetration from renewable energy generation may lead to significant increase in the requirement for ramping capacity from conventional generators [2]. Energy storage devices (e.g., batteries) are environmentally friendly candidates that can provide flexibility to the system and mitigate the impact of volatile renewable generations [3].

This work is primarily motivated by the quickly growing adoption of renewable energy generation and energy storage devices in power systems. In 2014, the installed capacity of wind power in the United States constitutes 17.8 % of total electricity generation capacity, which contributes 4.44 % of all generated electrical energy [4]. Indeed, the country is making steady progress towards the goal of supplying 20% of all U.S. electric energy with wind generation by 2030 [5]. Many states of the U.S have also set ambitious targets for energy storage deployments. For example, the state of California has decided to procure 1325 MW utility owned energy storage by the end of 2020, which amounts to 2% of the state's peak demand [6].

In this work, we study the operation of multiple storage devices in a power network with significant generation from intermittent renewable sources (like wind and solar). In each time slot, the system operator dispatches the output of conventional generators and energy storage devices to meet the inelastic demand at different buses, subject to (DC) power network constraints. The system operator faces a complicated sequential decision making problem under uncertainty (of system load and renewable generation), with an objective to minimize the expected total cost of conventional generation.

The topic of energy storage operation has attracted significant attention in the recent years. Herein we restrict our attention to the works that are most relevant to ours. The scheduling of energy storage systems is studied in [7], [8], [9] to maximize the joint profit of wind farms and energy storage systems. As a natural methodology for sequential decision making under uncertainty, dynamic programming (DP) has been adopted to study the optimal operation of a single energy storage device [10], [11], [12], [13], [14], [15], where a variety of optimal threshold based control policies are characterized under different settings (on renewable generation, consumer demand, and electricity prices). The authors of [16], [17] conduct DP based approaches to estimate the capacity value of energy storage. It is worth noting that the aforementioned DP literature has not studied the operation of multiple energy storage devices subject to power network constraints, which is the focus of the present paper.

Closer to the present paper, some recent works have applied the technique of Lyapunov optimization for the management of energy storage in (a single) consumer's demand response [18], [19]. More recently, this technique is applied for the cooperative operation of multiple energy storage devices [20], and our prior work [21] further incorporates power network constraints into the Lyapunov optimization framework. As the storage capacity approaches infinity, the asymptotic optimality of Lyapunov optimization based algorithms is established in the aforementioned works. The key advantage of this

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technique lies in the fact that the control decisions can be easily computed based only on the current state of the system (e.g., the current storage level and net demand), whereas the computation of optimal threshold policies (through a DP approach) could be challenging even for a single storage device [13], [15].

We note, however, that there are two major drawbacks in the existing literature that applies Lyapunov optimization on storage operation. First, existing works [18], [19], [20] ignore the network power flow (NPF) constraints, i.e., the Kirchoff's laws that govern power flows in a network and the capacity constraints of transmission lines. More importantly, although Lyapunov optimization based algorithms perform well when the energy storage capacity is large compared to its maximum charging/discharging rate, they are suboptimal for energy storage devices with relatively small capacity. In this work, we seek to design effective heuristic algorithms that overcome both of these drawbacks.

A. Summary of Results

The main contributions of this paper are twofold. In the first part, we propose a Lyapunov optimization based online algorithm for the operation of multiple energy storage devices in a power network. The proposed algorithm will be referred to as the **LOPN** algorithm in this paper. We show that the time average cost resulting from the LOPN algorithm is within a bounded distance from the optimal value. In particular, LOPN is shown to be asymptotically optimal as the energy storage capacity grows large. A major drawback of LOPN is that its performance is suboptimal when storage capacity is relatively small.

In the second part of the work, we improve the performance of the LOPN algorithm by utilizing forecast information on renewable generation and system load over the next a few time slots. In each time t, the proposed **Threshold based Energy Storage Management (TESM)** algorithm computes a "threshold" such that if possible, the storage level will be preserved above this threshold to meet the potential peak demand (for conventional generation) in the next few time slots. If the storage level is far above this threshold, then the TESM algorithm follows the decision made by LOPN.

The design of TESM algorithm is primarily motivated by the fact that renewable energy and system load can be forecasted with pretty high accuracy over a short time horizon. We note that state-of-the-art load forecasts could achieve high accuracy [22], [10]. For example, the mean absolute percent error of the hour-ahead load forecast of CAISO (California Independent System Operator) stays within 2% of the peak load [23]. Moreover, short term ($1-12$ hour ahead) forecast of wind generation could also be accurate. For example, the root mean square error of the day-ahead forecast on wind generation in Germany has been down to about 6% of the installed capacity since 2011 [24].

We develop two versions of the TESM algorithm. We first design what-we-call the TESM-SB algorithm for a single bus system. We then extend the TESM-SB algorithm to the case with power network constraints by solving simple optimal power flow (OPF) problems. The extended version of TESM algorithm will be referred to as the TESM-PN algorithm in this paper. Different from the LOPN algorithm, the two TESM algorithms use the expected net demand (system load minus renewable generation) over the next a few time slots to make current storage operation decisions. Numerical results show that the TESM-SB and TESM-PN algorithms outperform Lyapunov optimization based online algorithms in both a single bus case and the IEEE 6-bus test system, especially when the storage capacity is relatively small. Our numerical experiments also demonstrate that as the storage capacity grows large, both the TESM-SB and TESM-PN algorithms also achieve asymptotic optimality as they result in lower cost than Lyapunov optimization based online algorithms.

The rest of the paper is organized as follows. We describe the system model and problem formulation in section II. In Section III, we present the Lyapunov optimization framework for energy storage management in power networks. The TESM algorithms are presented and discussed in Section IV, for both the single bus and the power network cases. We summarize our numerical results in section V. Finally, we make some brief concluding remarks in Section VI.

II. System Model and Problem Formulation

We consider a power network consisting of N buses (nodes), denoted by the set $\mathcal{N} = \{1, 2, \ldots, N\}$. Let $\mathcal{L}$ denote the set of transmission lines connecting these buses.

A. Energy Supply and Demand: Every bus i harvests $X_i[t]$ units of energy from renewable generation during time t. In addition, it also generates $G_i[t]$ units of energy from conventional generation. $G_i^{\min}$ and $G_i^{\max}$ represent the minimum and maximum conventional power generation capacities at bus i, respectively,

$$G_i^{\min} \le G_i[t] \le G_i^{\max}. \tag{1}$$

$L_i[t]$ denotes the inelastic demand at bus i in time t. We assume that $L_i[t] \le L_{i,\max} < \infty$ for every i. The future demand and renewable generation can be random. The renewable generation and load vector $\{X_i[t], L_i[t]\}_{i=1}^N$ is assumed to evolve as a finite state irreducible and aperiodic Markov chain. We let $L_{i,\text{net}}[t] = L_i[t] - X_i[t]$ denote the net demand at bus i in time t, i.e., the difference between local system load and renewable generation.

B. Energy Storage: Each bus i is equipped with a storage device with a capacity of $E_i^{\max}$. In time t, the energy level of the storage is denoted by $E_i[t]$, which evolves as

$$E_i[t+1] = E_i[t] + Z_i^{\eta}[t], \qquad \forall\, i \in \mathcal{N}, \tag{2}$$

where $Z_i^{\eta}[t] = \eta_{i,\text{ch}} \max(Y_i[t], 0) + \min(Y_i[t], 0)/\eta_{i,\text{dis}}$ is the change in storage level in time t. Here, $Y_i[t]$ represents the amount of energy charged into the storage in period t ($Y_i[t]$ is negative if energy is withdrawn from the storage), and $\eta_{i,\text{ch}} \in (0, 1]$, $\eta_{i,\text{dis}} \in (0, 1]$ are the charging and discharging efficiency of storage i, respectively. $Y_{i,\text{ch}}^{\max}$ and $Y_{i,\text{dis}}^{\max}$ are the charging/discharging rate limits of storage i, i.e.,

$$-Y_{i,\text{dis}}^{\max} \le Y_i[t] \le Y_{i,\text{ch}}^{\max}. \tag{3}$$

The energy level in storage i is bounded by its capacity $E_i^{\max}$,

$$0 \leq E_i[t] \leq E_i^{\max}, \qquad \forall\, t, \quad \forall\, i \in \mathcal{N}. \tag{4}$$

Further, each storage i cannot be over withdrawn, i.e.,

$$-Y_i[t]/\eta_{i,\text{dis}} \leq E_i[t], \qquad \forall\, t, \quad \forall\, i \in \mathcal{N}. \tag{5}$$

Combining constraints (3) and (5) we have

$$-\min\left(E_i[t]\eta_{i,\text{dis}},\, Y_{i,\text{dis}}^{\max}\right) \leq Y_i[t], \qquad \forall\, t, \quad \forall\, i \in \mathcal{N}. \tag{6}$$

Similarly, from constraints (3) and (4) we have

$$Y_i[t] \leq \min\left(\frac{E_i^{\max} - E_i[t]}{\eta_{i,\text{ch}}}, Y_{i,\text{ch}}^{\max}\right), \quad \forall\, t, \quad \forall\, i \in \mathcal{N}. \tag{7}$$

We naturally have $E_i^{\max} \geq Y_{i,\text{ch}}^{\max}\eta_{i,\text{ch}}$ and $E_i^{\max} \geq Y_{i,\text{dis}}^{\max}/\eta_{i,\text{dis}}$ for every i, i.e., the storage capacity is no less than the maximum charging/discharging rate.

Finally, we define some notation that will be useful later. We denote the set of buses that have no storage devices (with $E_i^{\max} = 0$) by $\mathcal{N}_{\text{n-st}}$. Note that for every $i \in \mathcal{N}_{\text{n-st}}$, we must have $Y_{i,\text{ch}}^{\max} = Y_{i,\text{dis}}^{\max} = 0$. We let

$$\begin{aligned} E^{\sup} &= \max_{i \in \mathcal{N}\setminus\mathcal{N}_{\text{n-st}}} E_i^{\max}, & E^{\inf} &= \min_{i \in \mathcal{N}\setminus\mathcal{N}_{\text{n-st}}} E_i^{\max}, \\ Y_{\text{ch}}^{\sup} &= \min_{i \in \mathcal{N}\setminus\mathcal{N}_{\text{n-st}}} Y_{i,\text{ch}}^{\max}, & Y_{\text{dis}}^{\sup} &= \max_{i \in \mathcal{N}\setminus\mathcal{N}_{\text{n-st}}} Y_{i,\text{dis}}^{\max}, \\ \eta_{\text{ch}}^{\sup} &= \max_{i \in \mathcal{N}\setminus\mathcal{N}_{\text{n-st}}} \eta_{i,\text{ch}}, & \eta_{\text{dis}}^{\inf} &= \min_{i \in \mathcal{N}\setminus\mathcal{N}_{\text{n-st}}} \eta_{i,\text{dis}}, \end{aligned}$$

where $\mathcal{N} \setminus \mathcal{N}_{\text{n-st}}$ is the set of buses equipped with energy storage. Note that $\eta_{\text{dis}}^{\inf} > 0$ is bounded away from zero.

C. Power Network Model: In order to model the power flow $P_{i,j}[t]$ on branch (i, j), we adopt the DC power flow model. We let $B_{i,j}$ denote the negative of the susceptance of transmission line (i, j), and $\theta_i[t]$ denote the voltage phase angle at bus i during the time slot t. In this paper we adopt the following set of *DC power flow equations* [25]:

$$P_{i,j}[t] = B_{i,j}(\theta_i[t] - \theta_j[t]),\ \forall\, (i, j) \in \mathcal{L}, \tag{8}$$

$$L_i[t] - H_i[t] - G_i[t] + Y_i[t] + \sum_{j \neq i} P_{i,j}[t] = 0, \forall\, i \in \mathcal{N}, \tag{9}$$

$$H_i[t] \leq X_i[t], i \in \mathcal{N},\ |P_{i,j}[t]| \leq P_{i,j}^{\max},\ \forall\, (i, j) \in \mathcal{L}, \tag{10}$$

where $P_{i,j}^{\max}$ denotes the thermal limit (power transmission capacity) of the branch (i, j). $H_i[t] \in [0, X_i[t]]$ denotes the amount of renewable energy injected to bus i in time t (here we have assumed free disposal of renewable generation). We will also assume that a feasible dispatch always exists even without energy storage, i.e., the set of equations (1), (8)-(10) have a feasible solution if $Y_i[t]$ is set to be zero for every i.

D. Cost Model and Problem Formulation: The cost of conventional generation is denoted by the function $C_i(G_i[t])$, which is assumed to be a quadratic function given as [25]

$$C_i(G_i[t]) = p_i G_i[t] + q_i G_i^2[t], \tag{11}$$

where p_i and q_i are constants that depend on the generation technology at bus i. The cost of generation of the renewable energy is assumed to be zero. Also, we denote $p_{\max} = \max_{i \in \mathcal{N}} p_i$ and $q_{\max} = \max_{i \in \mathcal{N}} q_i$.

The objective of the controller is to design the system parameters so as to minimize the time average cost of conventional generation within the grid subject to power network and storage operation constraints:

$$\begin{aligned} \overline{f_{\min}} = \quad & \min \quad \lim_{T \to \infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}\left[\sum_{i=1}^{N} C_i(G_i[t])\right] \quad (12) \\ & \text{s.t.} \quad \text{Constraints } (1), (2), (6) - (10), \\ & \text{Variables:} \quad \{H_i[t], Y_i[t], G_i[t], \theta_i[t]\}_{i=1,t=1}^{N,T}. \end{aligned}$$

We denote the minimum time average cost of (12) over all feasible control policies by $\overline{f_{\min}}$. The expectation of the cost function is taken with respect to the randomness of the renewable energy generation. Problem (12) is essentially a stochastic dynamic program. Solving this problem using conventional DP based techniques can be computationally intractable, especially for large-scale power networks. In the next section, we will apply the technique of Lyapunov optimization (cf. [26]) to develop a low complexity online algorithm for Problem (12).

III. Lyapunov Optimization for Power Networks

In this section, we propose the LOPN (Lyapunov Optimization for Power Networks) algorithm that applies Lyapunov Optimization for storage operation in Power Networks. We show in Theorem 1 that LOPN is asymptotically optimal, as the storage capacity increases to infinity.

In order to solve (12) using the technique of Lyapunov optimization [18], we first introduce a relaxed version of this problem where we ignore all the constraints associated with the storage (i.e. the constraints in (4)-(7)):

$$\begin{aligned} \overline{g_{\min}} = \quad & \min \quad \lim_{T \to \infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}\left[\sum\nolimits_{i=1}^{N} C_i(G_i[t])\right] \qquad (13) \\ & \text{s.t.} \quad \lim_{T \to \infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}[Z_i^{\eta}[t]] = 0, \quad i = 1, \ldots, N, \\ & \qquad \text{Constraints } (1), (3), (8), (9), (10). \end{aligned}$$

The optimization problem (13) is a *relaxed problem* of the original problem in (12). In the relaxed problem, the finite sized battery has been relaxed with an infinite sized energy buffer. Further, the first constraint of (13) corresponds to the stability of a virtual energy buffer that evolves according to Eq. (2). The optimal value of Problem (13) is denoted by $\overline{g_{\min}}$. Note that we must have $\overline{g_{\min}} \leq \overline{f_{\min}}$.

In what follows, we solve the relaxed problem (13) using Lyapunov optimization technique. Next we introduce some parameters that will be tuned to ensure that the LOPN algorithm designed for the relaxed problem is feasible for the original problem (12). We first consider the Lyapunov function associated with the virtual energy queues:

$$\Psi[t] = \frac{1}{2} \sum\nolimits_{i=1}^{N} (E_i[t] - E^{\sup})^2. \tag{14}$$

We now examine the Lyapunov drift associated with $\Psi[t]$, which represents the expected change in the Lyapunov function from one time slot to the other:

$$\Delta[t] = \mathbb{E}\left[\Psi[t+1] - \Psi[t] \mid \mathbf{E}[t]\right],$$

where the expectation is with respect to the random processes associated with the system, given the energy queue-length values $\mathbf{E}[t] = [E_1[t], \ldots, E_N[t]]$. Using (2), we prove in Appendix A that the Lyapunov drift is bounded:

$$\Delta[t] \le \tilde{C}_1 + \mathbb{E}\Big[\sum_{i=1}^{N} (E_i[t] - E^{\sup}) Z_i^{\eta}[t] \mid \mathbf{E}[t]\Big], \quad (15)$$

where the expectation is over the possibly randomized action $Z_i^{\eta}[t]$, and $\tilde{C}_1 < \infty$ is a constant. We now add a scaled version of the cost function $V\mathbb{E}[\sum_{i=1}^{N} C_i(G_i[t])]$ (where V is control parameter which will be specified later) to both sides of (15). Denoting $\Delta_V[t] = \Delta[t] + V\mathbb{E}[\sum_{i=1}^{N} C_i(G_i[t])]$, we obtain

$$\Delta_V[t] \le \tilde{C}_1 + \mathbb{E}\Big[\sum_{i=1}^{N} \big[(E_i[t] - E^{\sup}) Z_i^{\eta}[t] + V C_i(G_i[t])\big] \mid \mathbf{E}[t]\Big]. \quad (16)$$

We will refer to $\Delta_V[t]$ as the modified Lyapunov drift.

According to the theory of Lyapunov optimization [26], the control decisions during each time slot is chosen to greedily minimize the bound on the modified Lyapunov drift (i.e., the right hand side of (16)). The modified Lyapunov drift consists of two components, the Lyapunov drift term $\Delta[t]$ and a scaled version of the cost function $V\sum_{i=1}^{N} C_i(G_i[t])$. Intuitively, minimizing the Lyapunov drift term alone pushes the queue-length of the virtual energy queue to a lower value. The second metric $V\sum_{i=1}^{N} C_i(G_i[t])$ can be viewed as a penalty term. The parameter V represents the trade-off between minimizing the queue-length drift and minimizing the penalty function. A higher value of V gives greater priority to minimizing the cost during the current time slot, at the expense of increasing the size of the virtual energy-queue.

We note, however, that the control actions chosen by this method may not be feasible for the original problem in (12). Therefore, instead of directly minimizing the right hand side (RHS) of (16), we minimize its modified version in the following **LOPN** algorithm:

$$\begin{aligned} \min \quad & \sum_{i=1}^{N} (E_i[t] - E^{\sup}) Z_i^{\eta}[t] + V \sum_{i=1}^{N} C_i(G_i[t]) \quad (17) \\ & + \alpha \sum_{i=1}^{N} \mathbb{1}_{E_i[t] + Z_i^{\eta}[t] > E_i^{\max}} + \beta \sum_{i=1}^{N} \mathbb{1}_{E_i[t] + Z_i^{\eta}[t] < 0} \\ \text{s.t.} \quad & \text{Constraints } (1), (3), (8), (9), (10), \end{aligned}$$

where the expression $\mathbb{1}_{\mathcal{A}}$ represents the indicator function:

$$\mathbb{1}_{\mathcal{A}} = \begin{cases} 1 & \text{if } \mathcal{A} \text{ is true}, \\ 0 & \text{otherwise}, \end{cases} \quad (18)$$

and α, β are non-negative perturbation terms that are added to ensure that the LOPN algorithm developed for the relaxed Problem (13) is feasible for the original Problem (12). The values of α, β and V are given in the statement of Lemma 1. We will use the superscript $\widehat{\cdot}$ to denote the solution to the LOPN problem (17).

Lemma 1. *If the parameters α, β and V are chosen as*

$$\alpha = (E^{\sup} - E^{\inf} + \eta_{ch}^{\sup} Y_{ch}^{\sup}) \eta_{ch}^{\sup} Y_{ch}^{\sup}, \quad (19)$$

$$\beta = Y_{dis}^{\sup} \min(Y_{ch}^{\sup}, Y_{dis}^{\sup}) / (\eta_{dis}^{\inf})^2, \quad (20)$$

$$V = \frac{(E^{\sup} - Y_{dis}^{\sup} / \eta_{dis}^{\inf})}{(p_{\max} + q_{\max} Y_{dis}^{\sup})}, \quad (21)$$

then the decisions made by the LOPN algorithm are feasible for Problem (12). *In particular, we have*

$$0 \le \widehat{E_i[t]} \le E_i^{\max}, \quad t = 1, \ldots, T-1, \quad \forall i, \quad (22)$$

where $\{\widehat{E_i[t]}\}_{t=1}^{T-1}$ is a sequence of storage levels resulting from the LOPN algorithm (according to the state evolution equation (2)*), under an arbitrary sample path of realized renewable generation and system load.*

Lemma 1 is proved in Appendix B. Lemma 1 shows that by suitably tuning the control parameters α, β and V, the LOPN algorithm can be made feasible for the original problem (12). In the following theorem we establish the asymptotic optimality of the LOPN algorithm.

Theorem 1. *Suppose that the renewable generation and system load at each bus are independent and identically distributed (i.i.d.) random variables, and are also mutually independent across buses in each time slot. The time average cost function achieved by the LOPN algorithm satisfies*

$$\lim_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}\left[\sum_{i=1}^{N} C_i(\widehat{G_i[t]})\right] \le \overline{f_{\min}} + \frac{\tilde{C}}{V}, \quad (23)$$

where $\tilde{C}$ is a constant. Further, if $\lim_{E^{\sup}\to\infty} E^{\inf}/E^{\sup} = 1$, *then the bound* $\lim_{E^{\sup}\to\infty} \tilde{C}/V \to 0$.

Theorem 1 is proved in Appendix C. Theorem 1 shows that the time average cost resulting from the LOPN algorithm is within a bounded distance from the optimal value. In Appendix D, we generalize Theorem 1 to the case with Markovian renewable generation formulated in Section II. We note from (21) that the parameter V depends on the maximum battery capacity $E^{\sup}$. Theorem 1 shows that if $\lim_{E^{\sup}\to\infty} E^{\inf}/E^{\sup} = 1$, the LOPN algorithm is asymptotically optimal as the storage capacity grows large.

Remark III.1. The LOPN algorithm is computationally simple because its decision making is based on only the current storage level $E[t]$. We have further shown its asymptotic optimality in Theorem 1. However, for energy storage devices with finite capacity, the LOPN algorithm could be sub-optimal. For the rest of the paper we attempt to improve the performance of LOPN by reserving a certain amount of energy in the storage that may be used to meet the energy demand in the next a few stages. In the next section, we propose an alternative online energy storage management algorithm which will be referred to as the Threshold based Energy Storage Management (TESM) algorithm. □

IV. Threshold based Energy Storage Management (TESM) Algorithms

In this section, we will introduce two versions of the TESM algorithm for the case without/with power network constraints.

In order to clearly illustrate the main idea of TESM, we first consider the single bus case in Section IV-A. Then, in Section IV-B we provide an extension of this algorithm in a more general setting with power network constraints.

The key improvement of the TESM algorithms over LOPN is to utilize forecast information on renewable generation and energy load over a short time horizon to determine a "threshold" on the storage level that the TESM algorithms attempt to preserve.[1] The LOPN algorithm, on the other hand, does not utilize any forecast information when making storage operation decisions.

In each time t, the proposed TESM algorithms compute a threshold value (on the storage level) based on the forecast of the net demand over the next a few time slots. The TESM algorithms then seek to preserve the storage level above this threshold to meet the potential peak (net) demand within the next a few time slots. For the single bus case, the TESM-SB algorithm will do the following in each time t: i) if the storage level in time t is below the threshold, charge renewable and conventional generation into the storage up to this threshold; ii) if the storage level in time t is above the threshold, withdraw the storage to meet the net demand in time t while maintaining the storage level above the threshold.

We note that the TESM algorithms have a similar structure as the optimal threshold policies characterized in the literature that applies dynamic programming (DP) on the operation of a single energy storage device [10], [13], [14], [15]. However, unlike DP based approaches, the decisions made by TESM algorithms can be computed by solving simple one-shot optimization problem(s); as a result, TESM algorithms are much less computationally demanding than the optimal threshold policies that have to be computed through backward induction using a DP based approach. Further, the TESM-PN algorithm proposed in Section IV-B could serve as an effective heuristic policy that coordinates the operation of multiple storage devices in a power network with random renewable generation; indeed, no characterization on optimal storage operation policies has been provided for this setting.

A. *TESM-SB Algorithm for Single Bus Systems*

Within this subsection, we focus on the single bus case without the power network constraints (8)-(10). We will therefore drop the subscript i (that was used to denote bus i of a power network) from all notation.

Before describing the TESM-SB algorithm, for the sake of completeness, we briefly review the **LOSB** (Lyapunov Optimization for Single Bus systems) algorithm proposed in [18], [19]. The LOSB algorithm will be used as a benchmark in our numerical results on single-bus systems (see Section V-A). The LOSB algorithm can be derived using an approach similar to that used for the LOPN algorithm, through establishing and minimizing a bound on the modified Lyapunov drift (please refer to [18] for the details). For the rest of the paper we let $(\cdot)^+ = \max\{0, \cdot\}$. In each time slot t, the LOSB solves the following optimization problem to make storage operation decisions:

$$\begin{aligned}
\mathbf{LOSB}: \quad \min \quad & (E[t]-\psi)Z^{\eta}[t] + V_{\text{SB}}C(G[t]) && (24)\\
\text{s.t.} \quad & \max(Y[t],0) - G[t] \le (-L_{\text{net}}[t])^+, \\
& \min(Y[t],0) + G[t] = (L_{\text{net}}[t])^+, \\
& -Y_{\text{dis}}^{\max} \le Y[t] \le Y_{\text{ch}}^{\max}, \\
\text{where} \quad \psi \quad = \quad & E^{\max} - \eta_{\text{ch}} Y_{\text{ch}}^{\max}, && (25)\\
V_{\text{SB}} \quad = \quad & \frac{E^{\max} - (\eta_{\text{ch}} Y_{\text{ch}}^{\max} + Y_{\text{dis}}^{\max}/\eta_{\text{dis}})}{(p + qY_{\text{dis}}^{\max})}. && (26)
\end{aligned}$$

Before proceeding to introduce the TESM-SB (single bus) algorithm, we first provide some intuition behind it.

1) TESM-SB Algorithm - Intuition: Despite the asymptotic optimality result established in [19] and Theorem 1, the LOSB and LOPN algorithms are suboptimal when the storage capacity is comparable to the maximum charging/discharging rate, mainly due to the (possibly highly) suboptimal decisions these two algorithms make when the energy level in the storage is low and is comparable to the maximum charging/discharging rate. The proposed TESM-SB algorithm seeks to improve the performance of LOSB algorithm by modifying the decisions LOSB makes when the storage level is relatively low, based on the forecast information for renewable generation in the next a few periods (which is usually available in practice).

Recall from (11) that the conventional generation cost is a quadratic function of the net demand. As a result, to minimize the total cost it is desirable to have a flat pattern of net demand as the conventional generation cost in each time slot is convex in net demand. In contrast to the LOSB algorithm that makes decisions based only on the current net demand and storage level, the proposed TESM-SB algorithm utilizes the forecast on renewable generation and demand for the next a few (T_{LA}) time slots to determine a level of storage (what-we-call the threshold) that will be preserved to reduce the high net demand over the next T_{LA} time slots.

The TESM-SB algorithm is formulated in Algorithm 1.

Algorithm 1 (TESM-SB Algorithm). *In each time slot t, compute the control decisions by performing the following steps:*

Set $N[t] = 0, R[t] = 0$.
for $n = t+1 : t+T_{LA}$ ***do***
  if $\mathbb{E}[L_{net}[n] \mid L[t], X[t]] \ge (L_{net}[t])^+$ ***then***
    $N[t] = N[t] + 1$.
    $R[t] = R[t] + \mathbb{E}[L_{net}[n] \mid L[t], X[t]] - (L_{net}[t])^+$.
  end if
end for
Define $R_{\min}[t] = \min\{R[t],\ N[t]Y_{dis}^{\max}/\eta_{dis}\}$.
if $0 \le E[t] < R_{\min}[t]$ ***then***

$$Z^{\eta}[t] = \min\Big(\eta_{ch}Y_{ch}^{\max}, E^{\max} - E[t], \max\big(R_{\min}[t] - E[t], -L_{net}[t]\big)\Big),$$

$$G[t] = (Z^{\eta}[t]/\eta_{ch} + L_{net}[t])^+.$$

[1] Note that state-of-the-art load forecasts could achieve high accuracy [22], [10], [23]. Moreover, the accuracy of short term ($1-12$ hour ahead) forecast of wind generation could be high, with only a few percent forecast error [24].

11: ***else if*** $R_{\min}[t] \leq E[t] \leq R_{\min}[t] + Y^{\max}_{dis}/\eta_{dis}$ ***then***
12: ***if*** $L_{net}[t] \leq 0$ ***then***
13: $Z^{\eta}[t] = \min\{\eta_{ch} Y^{\max}_{ch}, E^{\max} - E[t], (-L_{net}[t])^{+}\}$,
14: $G[t] = 0$.
15: ***else***
16: $Z^{\eta}[t] = -\min\left\{Y^{\max}_{dis}/\eta_{dis},\, E[t] - R_{\min}[t],\right.$
17: $\left.(L_{net}[t])^{+}\right\}$,
18: $G[t] = \left(Z^{\eta}[t]\eta_{dis} + L_{net}[t]\right)^{+}$.
19: ***end if***
20: ***else***
21: *Follow the LOSB algorithm.*
22: ***end if***

2) *TESM-SB Algorithm Description:* We now provide some description for the TESM-SB algorithm.

- Steps $1-4$ count the number of time slots from time $t+1$ to $t+T_{\text{LA}}$ during which the forecast of the net demand exceeds the current net demand $L_{\text{net}}[t]$. $N[t]$ denotes number of such time slots within the next T_{LA} time slots. In Step 5, $R[t]$ records the sum of the difference between the forecasted net demand in each of these $N[t]$ time slots and the current net demand $L_{\text{net}}[t]$.
- In Step 8, the algorithm computes the threshold $R_{\min}[t]$, which is the minimum of two quantities: $R[t]$ and $N[t]Y^{\max}_{\text{dis}}/\eta_{\text{dis}}$, the latter of which is the maximum amount of energy that could be withdrawn from the storage in the $N[t]$ time slots. As we will see in the following, the algorithm attempts to reserve $R_{\min}[t]$ amount of energy in the storage at the beginning of time $t+1$ to reduce the (possible) peak net demand in the next T_{LA} time slots.
- Steps $9-10$ consider the case where the storage level in time t is less than the threshold $R_{\min}[t]$. In this case, the algorithm attempts to charge the storage up to the level of $R_{\min}[t]$. In Step 10, the algorithm first uses the excess renewable generation (when $L_{\text{net}}[t]$ is negative), and then the conventional generation to charge the storage. We note that if there is much excess renewable generation, the storage level at the beginning of $t+1$ could be higher than $R_{\min}[t]$.
- Steps $11-18$ consider the case when the storage level is between $R_{\min}[t]$ and $R_{\min}[t] + Y^{\max}_{\text{dis}}/\eta_{\text{dis}}$. In this case, the storage is above the threshold $R_{\min}[t]$ so there is no need to use conventional generation to charge the storage. If the net demand is positive in time t, the storage will be discharged to fulfil the demand, while maintaining a storage level no less than $R_{\min}[t]$.
- Steps $20-21$ consider the case when the storage level is greater than $R_{\min}[t] + Y^{\max}_{\text{dis}}$. In this case, the energy level will be above the threshold $R_{\min}[t]$ regardless of the charging/discharging action taken in time t. The LOSB algorithm is implemented.

B. TESM-PN Algorithm for Power Networks

We extend the TESM-SB algorithm to the case with power network constraints. The new algorithm shares the same foundation as TESM-SB. In what follows we will discuss how the new TESM-PN (power networks) algorithm mimics the TESM-SB algorithm step by step. For notational convenience, we define $\mathbf{L}[t] = \{L_i[t]\}_{i=1}^{N}$ and $\mathbf{X}[t] = \{X_i[t]\}_{i=1}^{N}$.

Steps $1-8$ of Algorithm 1 are mimicked by Algorithm 2, which computes the sum of the aggregate net demand (in excess the current aggregate net demand $\sum_i L_{i,\text{net}}[t]$) across all the buses for the next T_{LA} time slots.

Algorithm 2.
1: $N[t] = 0,\ R[t] = 0$.
2: ***for*** $n = t+1 : t+T_{LA}$ ***do***
3: ***if*** $\sum_i \mathbb{E}[L_{i,net}[n] \mid \mathbf{L}[t], \mathbf{X}[t]] \geq (\sum_i L_{i,net}[t])^{+}$ ***then***
4: $N[t] = N[t] + 1$,
5:
$$R[t] = R[t] + \textstyle\sum_i \mathbb{E}[L_{i,net}[n] \mid \mathbf{L}[t], \mathbf{X}[t]] - (\textstyle\sum_i L_{i,net}[t])^{+}.$$
6: ***end if***
7: ***end for***
8: $R_{\min}[t] = \min(R[t],\ N[t]\sum_i Y^{\max}_{i,dis}/\eta_{i,dis})$.

Next, we show how to generalize Steps $9-21$ of Algorithm 1 to incorporate power network constraints.

- Steps $9-10$ of Algorithm 1 can be mimicked by solving the following problem:

$$\min \quad 0.5\sum_i G_i[t] - \sum_i Y_i[t] \tag{27a}$$
$$\text{s.t.} \quad 0 \leq Y_i[t], \qquad \forall i, \tag{27b}$$
$$\sum_i Y_i[t] \leq \max\Big\{R_{\min}[t] - \sum_i E_i[t], -\sum\nolimits_i L_{i,\text{net}}[t]\Big\}, \tag{27c}$$
$$\text{Constraints } (1), (7)-(10). \tag{27d}$$

We observe from (27a) that the storage charging operation (i.e. $Y_i[t] > 0$) reduces the value of the objective function. As a result, an optimal solution to Problem (27) attempts to charge the storage up to the an aggregate level of $R_{\min}[t]$, subject to the charging rate constraint in (7) and power network constraints in (8)-(10). Note that the co-efficient of the term $\sum_i G_i[t]$ in (27a) is chosen to be in the interval $(0, 1)$ to ensure that[2] i) all excess renewable generation is utilized to charge the storage, and ii) if excess renewable generation is not sufficient to bring the aggregate level of storage to $R_{\min}[t]$, then energy from conventional generation will be used.

- Steps $12-14$ of Algorithm 1 are mimicked by solving the following problem:

$$\min \quad 2\sum_i G_i[t] - \sum_i Y_i[t] \tag{28a}$$
$$\text{s.t.} \quad 0 \leq Y_i[t], \qquad \forall i, \tag{28b}$$
$$\sum\nolimits_i Y_i[t] \leq \Big(-\sum\nolimits_i L_{i,\text{net}}[t]\Big)^{+}, \tag{28c}$$
$$\text{Constraints } (1), (7)-(10). \tag{28d}$$

[2] Although unlikely, the choice the co-efficient associated with the term $\sum_i G_i[t]$ may influence the optimal solution(s) to the Problem (27) because the injection of conventional generation at certain bus(es) may have an impact on the total amount of renewable generation that could be charged into the storage. Our numerical results in Section V show that the performance of TESM-PN is not sensitive to the choice of this co-efficient. Similar arguments apply to the co-efficients in the objective function of Problem (28).

Here we consider the case where the net demand in time t is non-positive at every bus and the aggregate storage level is above the threshold $R_{\min}[t]$. In this case, optimization problem (28) attempts to charge as mush as possible excess renewable generation to the storage devices, subject to the charging rate and power network constraints. The co-efficient of the term $\sum_i G_i[t]$ in (28a) is made larger than 1 to ensure that minimum conventional generation is utilized to charge the storage.

Algorithm 3 (TESM-PN Algorithm). *During each time slot t, compute the control decisions by performing the following steps:*

1: *Follow the steps of Algorithm 2.*
2: ***if*** $0 \leq \sum_i E_i[t] \leq R_{\min}[t]$ ***then***
3:     *Solve* (27) *to obtain the control decisions.*
4: ***else if*** $R_{\min}[t] \leq \sum_i E_i[t] \leq R_{\min}[t] + \sum_i Y^{\max}_{i,dis}/\eta_{i,dis}$ ***then***
5:     ***if*** $L_{i,net}[t] \leq 0$ *for every i* ***then***
6:         *Solve* (28) *to obtain the control decisions.*
7:     ***else***
8:         *Solve* (29) *to obtain the control decisions.*
9:     ***end if***
10: ***else***
11:     *Follow the LOPN algorithm.*
12: ***end if***

• Lastly, steps $15-18$ of Algorithm 1 can be mimicked by solving the following problem:

$$\min \quad \sum_i Y_i[t] + \sum_i G_i[t] - 2\sum_i H_i[t] \tag{29a}$$

$$\text{s.t.} \quad \sum_i Y_i[t] \geq R_{\min}[t] - \sum_i E_i[t] \tag{29b}$$

$$\text{Constraints } (1), (6)-(10). \tag{29c}$$

Here we consider the case where the net demand in time t is positive at some bus(es) and the aggregate storage level is above the threshold $R_{\min}[t]$. In (29a), the co-efficient of the term $\sum_i H_i[t]$ (cf. the definition of $H_i[t]$ in (10)) is made less than -1 to ensure that renewable generation is first used to fulfil the demand. Constraint (29b) ensures that the aggregate storage level does not fall below the threshold $R_{\min}[t]$ after discharging the storage to meet the demand. If there is still positive net demand at some bus(es), conventional generation is deployed to meet the demand.

The TESM-PN algorithm is formulated in Algorithm 3, where in Steps $10-11$ the LOPN algorithm is implemented if the aggregate storage level in time $t+1$ is guaranteed to exceed the threshold $R_{\min}[t]$ regardless of the (discharging) action taken in time t.

V. Numerical Results

In this section, we perform numerical experiments to compare the performance of the proposed algorithms with the LOSB algorithm (originally proposed in [18], [19]). In Section V-A, we numerically compare the performance of LOSB and TESM-SB algorithms in a single bus system. In Sections V-B and V-C, we compare the performance of LOPN and TESM-PN on the IEEE 6-bus test system and the IEEE 9-bus test system, respectively. In Sections V-A and V-B, we let $p = 30$ and $q = 0.2$ in the generation cost function (11); in Section V-C, we consider a different generation cost function with $p = 20$ and $q = 0.25$.

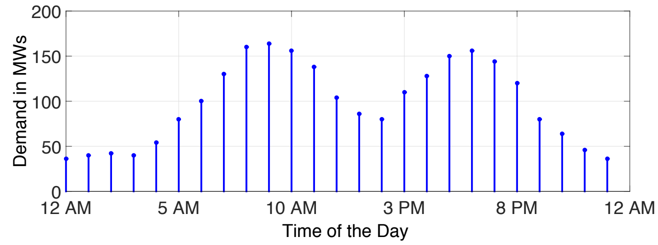


Fig. 1: An example of the hourly system load of a day.

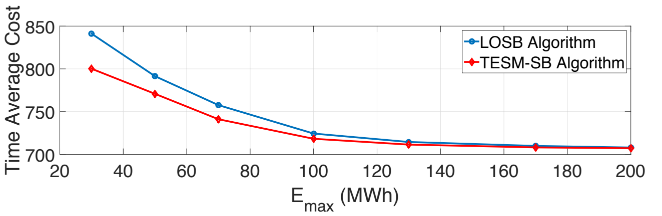
Throughout this section we consider the hourly system load shown in Figure 1, which has a peak demand of 160 MW and an average demand of 100 MW. For simplicity, we ignore the capacity constraint of conventional generation in (1), and let charging/discharging efficiencies $\eta_{i,\text{ch}} = \eta_{i,\text{dis}} = 1$, for every bus i.

For each parameter setting we run the tested algorithm for $T = 10000$ time slots to compute its time average cost. For both the TESM-SB and TESM-PN algorithms, the look ahead interval T_{LA} is set to be 3. In each time t, both the TESM-SB and TESM-PN algorithms use the deterministic demand and the expected renewable generation (in the next 3 time slots) to compute the threshold $R_{\min}[t]$.

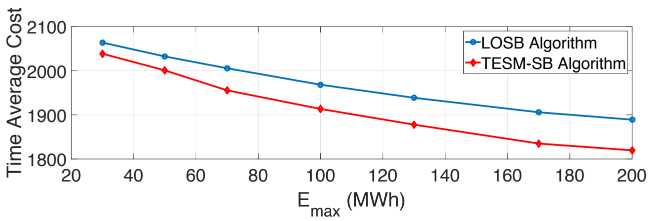
A. Comparison Between TESM-SB and LOSB Algorithms

We compare the performance of the TESM-SB and LOSB algorithms under two different parameter settings. In Setting 1, the average renewable generation equals 100% of the average demand. We generate a sequence of independent and identically distributed (i.i.d.) Gaussian random variables with mean value of 100 and standard deviation of 30 to present the random renewable generation. If the realized random variable happens to be negative, we will modify it to be zero. The storage charging/discharging rate limit is held constant at $Y^{\max}_{\text{ch}} = Y^{\max}_{\text{dis}} = Y^{\max} = 10$ MW.

For the TESM-SB and LOSB algorithms, we plot their time average cost (of conventional generation) as a function of the maximum storage capacity in Figure 2a. We note that TESM-SB always achieves a lower time average cost than LOSB; the gain is more significant when the storage capacity $E^{\max}$ is relatively low. For example, when $E^{\max} = 30$ MWh, there is approximately 5% cost reduction resulting from TESM-SB. As the ratio $E^{\max}/Y^{\max}$ grows large, the time averaged cost of LOSB converges to the optimal value [18], [19]. Numerical results in Fig. 2a show that TESM-SB algorithm also achieves asymptotically optimality and the performance of these two algorithms are nearly identical for large storage sizes. These observations can be explained as follows. With (relatively) low storage capacity, the energy level in the battery is typically low. Hence the energy "preserved" by the TESM-SB algorithm based on the forecast information becomes crucial in reducing peak demand. As a result, the TESM-SB algorithm outperforms the LOSB algorithm in this regime. In contrast, with large storage capacity and high renewable generation penetration, the energy level in the battery is usually high,



(a) Setting 1: Average renewable generation meets 100% of average demand and $Y^{\max} = 10$ MWh.



(b) Setting 2: Average renewable generation meets 60% of average demand and $E^{\max}/Y^{\max} = 6$.

Fig. 2: Time average cost resulting from LOSB and TESM-SB.

and often exceed the threshold levels defined in the TESM-SB algorithm. As a result, the TESM-SB algorithm usually takes the same action as LOSB and achieves similar performance as the LOSB algorithm.

We now consider Setting 2, where the average renewable generation equals 60% of the average demand, and the maximum charging/discharging rate $Y^{\max}$ scales with storage capacity with $E^{\max}/Y^{\max} = 6$. We generate a sequence of i.i.d. Gaussian random variables with mean of 60 and standard deviation of 20 (that are truncated to be non-negative) to present renewable generation. The time average costs of LOSB and TESM-SB are compared in Figure 2b. The performance gain resulting from TESM-SB increases with the storage capacity. Since Setting 2 is in the non-asymptotic regime (note that the ratio $E^{\max}/Y^{\max}$ is fixed to 6), both the LOSB and the TESM-SB algorithms may not achieve asymptotic optimality. Under the TESM-SB algorithm, larger storage capacity (and hence larger discharging capacity) provides more flexibility to flatten the system load profile and helps to reduce the generation cost; for example, when $E_{\max} = 200$ MWh, the TESM-SB leads to more than 5% cost reduction compared to LOSB.

B. Comparison Between TESM-PN and LOPN Algorithms in an IEEE 6-bus Test System

In this subsection, we compare the performance of the LOPN and TESM-PN algorithms in an IEEE 6-bus test system shown in Figure 3 (cf. pp. 104 of [25]). In our simulations, bus 2 is equipped with a conventional generator with no capacity limit. Loads are positioned at buses 4 and 5: the demand at each bus is depicted in Fig. 1. Buses 4 and 6 are equipped with renewable generators and energy storage with the same capacity $E^{\max}$. The renewable generation at bus 4 is independent of that at bus 6. We will consider the two settings (on renewable generation and maximum charging/discharging rates) formulated in Section V-A.

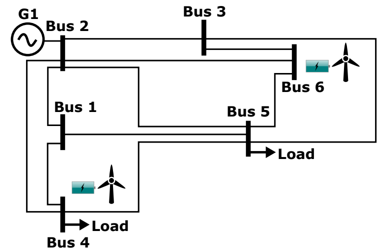
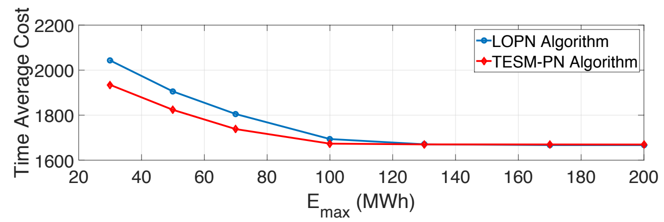
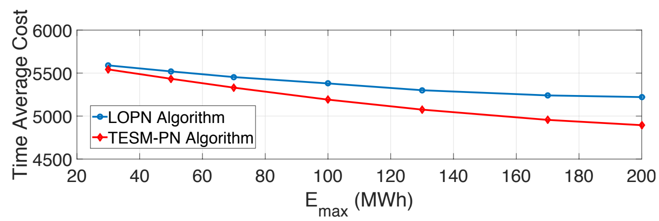


Fig. 3: An IEEE 6-bus test system with renewable generation and energy storage.



(a) Setting 1: Average renewable generation meets 100% of average demand and $Y^{\max} = 10$ MWh.



(b) Setting 2: Average renewable generation meets 60% of average demand and $E^{\max}/Y^{\max} = 6$.

Fig. 4: Time average cost of LOPN and TESM-PN for an IEEE 6-bus test system shown in Fig. 3.

We plot the time average cost (of conventional generation) resulting from LOPN and TESM-PN as a function of the storage capacity $E^{\max}$ in Figures 4a and 4b under the Setting 1 and 2, respectively. Again, we observe from Fig. 4a that the TESM-PN algorithm reduces the cost of LOPN by about 5% when the storage capacity $E^{\max} = 30$ MWh. As the ratio $E^{\max}/Y^{\max}$ grows large, the time average cost of LOPN converges to the optimal value (cf. Theorem 1). It then becomes straightforward from Fig. 4a that TESM-PN algorithm also achieves asymptotically optimality.

Similar to the single-bus case presented in Fig. 2b, we observe from Fig. 4b that the performance gap between the two algorithms increases with the storage capacity. when $E_{\max} = 200$ MWh, the TESM-PN results in about 5% cost reduction compared to LOPN.

C. Comparison Between TESM-PN and LOPN Algorithms in an IEEE 9-bus Test System

In this subsection, we consider an IEEE 9-bus test system shown in Fig. 5. In the considered 9-bus power system, buses 1 and 2 are equipped with two identical conventional generators with $p = 20$, $q = 0.25$. Again, we use the demand profile depicted in Fig. 1 at the load buses 7, 8, and 9. Buses 4, 5 and 9 are equipped with renewable generators and energy storage

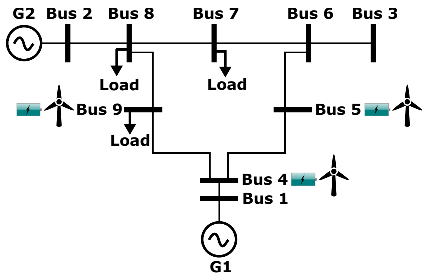
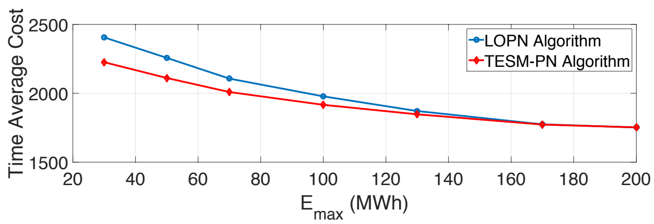
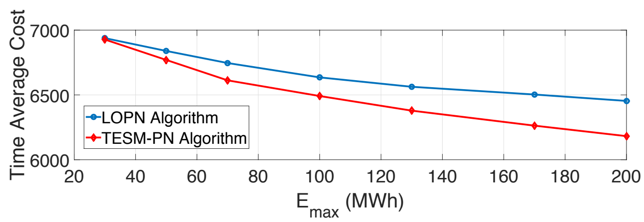


Fig. 5: An IEEE 9-bus test system with renewable generation and energy storage.



(a) Setting 1: Average renewable generation meets 100% of average demand and $Y^{\max} = 10$ MWh.



(b) Setting 2: Average renewable generation meets 60% of average demand and $E^{\max}/Y^{\max} = 6$.

Fig. 6: Time average cost of LOPN and TESM-PN for the IEEE 9-bus test system shown in Fig. 5.

with the same capacity $E^{\max}$. The renewable generation at different buses are assumed to be mutually independent.

The two subplots of Fig. 6 present the numerical results for the two different parameter settings considered in Section V-A. The numerical results presented in Fig. 6 look similar to those shown in Fig. 2 and Fig. 4. For Setting 1, the maximum charging/discharging rates do not scale with the maximum storage capacity, and both the TESM-PN and the LOPN algorithms achieve asymptotic optimality; when the storage capacity is relatively small, on the other hand, TESM-PN outperforms LOPN. For Setting 2, the maximum charging/discharging rates scale with the maximum storage capacity, and the performance gap between TESM-PN and LOPN increases with the storage capacity.

VI. Conclusions

In this work, we studied the operation of multiple storage devices in a power network with significant generation from intermittent renewable sources. We proposed a Lyapunov optimization based online algorithm – the LOPN algorithm. We showed that the LOPN algorithm always makes feasible storage operation decisions subject to DC power network constraints, and achieves asymptotic optimality as the storage capacity grows large. To further improve its performance for the case where the storage capacity is limited, we proposed a look-ahead policy (the TESM algorithm) that uses the forecast information (on renewable generation and system load) over the next $T_{\rm LA}$ time slots to compute a threshold on the storage level. If possible, the algorithm will keep the storage level above this threshold to meet the potential peak demand (for conventional generation) within the next $T_{\rm LA}$ time slots. Numerical results showed that under all the simulation settings considered in this work, the TESM algorithm outperforms the LOSB and LOPN algorithms.

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Appendix A : Proof of (15)

Recall the evolution of the energy level in the battery as specified in (2). Subtracting E^{sup} on both sides of (2), we obtain

$$E_i[t+1] - E^{\text{sup}} = E_i[t] - E^{\text{sup}} + Z_i^{\eta}[t]. \tag{30}$$

Squaring both sides of the equation (30),

$$\begin{aligned}&(E_i[t+1] - E^{\text{sup}})^2\\ &= (E_i[t] - E^{\text{sup}} + Z_i^{\eta}[t])^2\\ &= (E_i[t] - E^{\text{sup}})^2 + (Z_i^{\eta}[t])^2 + 2(E_i[t] - E^{\text{sup}})Z_i^{\eta}[t]. \end{aligned}\tag{31}$$

Dividing both sides of (31) by 2, we obtain

$$\begin{aligned}\frac{1}{2}(E_i[t+1] - E^{\text{sup}})^2 = \frac{1}{2}(E_i[t] - E^{\text{sup}})^2 + \frac{1}{2}(Z_i^{\eta}[t])^2\\ + (E_i[t] - E^{\text{sup}})Z_i^{\eta}[t]. \end{aligned}\tag{32}$$

Using (3), the term $(Z_i^{\eta}[t])^2$ can be bounded as

$$\frac{1}{2}(Z_i^{\eta}[t])^2 \leq \frac{1}{2}\max\left((\eta_{\text{ch}}^{\text{sup}} Y_{\text{ch}}^{\text{sup}})^2, (Y_{\text{dis}}^{\text{sup}}/\eta_{\text{dis}}^{\text{inf}})^2\right) \triangleq \tilde{C}_1. \tag{33}$$

Rearranging (32) and using the bound (33), we obtain

$$\begin{aligned}(E_i[t+1] - E^{\text{sup}})^2 - (E_i[t] - E^{\text{sup}})^2 \leq \tilde{C}_1\\ + (E_i[t] - E^{\text{sup}})Z_i^{\eta}[t]. \end{aligned}\tag{34}$$

Summing (34) over $i = 1, \dots, N$, and taking the conditional expectation on both sides given $\mathbf{E}[t]$, we obtain the bound in (15).

Appendix B: Proof of Lemma 1

Lemma 1 will be proved in the following two main steps.

Step 1: In this step, we establish the following upper bound (on storage level) under the LOPN algorithm, i.e.,

$$\widehat{E_i[t]} \leq E_i^{\max} \qquad \forall\, t, \quad \forall\, i \in \mathcal{N}. \tag{35}$$

A key result in Step 1 is to show that for storages located at bus $i \in \mathcal{N}$ which satisfy

$$E_i^{\max} - \eta_{i,\text{ch}} Y_{i,\text{ch}}^{\max} \leq \widehat{E_i[t]} \leq E_i^{\max},$$

the action taken by the LOPN algorithm can be bounded by

$$0 \leq \widehat{Z_i^{\eta}[t]} \leq E_i^{\max} - \widehat{E_i[t]}.$$

Step 2: In this step, we establish the following lower bound (on storage level) under the LOPN algorithm, i.e.,

$$\widehat{E_i[t]} \geq 0 \qquad \forall\, t, \quad \forall\, i \in \mathcal{N}. \tag{36}$$

A key result in Step 2 is to show that for storages located at bus $i \in \mathcal{N}$ which satisfy

$$0 \leq \widehat{E_i[t]} \leq Y_{i,\text{dis}}^{\max}/\eta_{i,\text{dis}},$$

the action taken by the LOPN algorithm can be bounded by

$$-\widehat{E_i[t]} \leq \widehat{Z_i^{\eta}[t]} \leq 0.$$

Combining Step 1 and Step 2, we conclude that the storage levels (resulting from the LOPN algorithm) satisfy

$$0 \leq \widehat{E_i[t]} \leq E_i^{\max} \qquad \forall\, t, \quad \forall\, i \in \mathcal{N}, \tag{37}$$

which implies that a solution of the LOPN algorithm is feasible for the original problem in (12).

A. *Proof of Step 1:*

Let us partition the set of buses $\mathcal{N}$ as follows:

$$\begin{aligned} &i \in \mathcal{N}_1, \text{ if } E_i^{\max} - \eta_{i,\text{ch}} Y_{i,\text{ch}}^{\max} \leq \widehat{E_i[t]} \leq E_i^{\max},\\ &i \notin \mathcal{N}_1, \text{ if } \widehat{E_i[t]} < E_i^{\max} - \eta_{i,\text{ch}} Y_{i,\text{ch}}^{\max}, \end{aligned}\tag{38}$$

where $\mathcal{N}_1 \subset \mathcal{N}$. For a bus $i \notin \mathcal{N}_1$, the energy level during the next time slot can be bounded as

$$\widehat{E_i[t+1]} \leq \widehat{E_i[t]} + \eta_{i,\text{ch}} Y_{i,\text{ch}}^{\max} < E_i^{\max}, \tag{39}$$

where the first inequality is due to the battery charging constraint and the second one follows from (39). Therefore, we are only concerned with the batteries located at buses $i \in \mathcal{N}_1$, since charging them may violate the upper bound $E_i^{\max}$.

In the rest of this subsection, we argue that for every bus $i \in \mathcal{N}_1$, the control action taken by the LOPN algorithm satisfies $0 \leq \widehat{Z_i^{\eta}[t]} \leq E_i^{\max} - \widehat{E_i[t]}$. This result implies that $\widehat{E_i[t]} \leq E_i^{\max}$ for $i \in \mathcal{N}_1$.

Recall that the LOPN algorithm chooses control actions that minimize (17). We will prove the above claim by showing that for buses $i \in \mathcal{N}_1$, a control action such that $Z_i^{\eta}[t] \geq E_i^{\max} - \widehat{E_i[t]}$ will result in a higher objective value of Eq. (17) than the action $Z_i^{\eta}[t] = 0$.

We first rewrite the objective of the LOPN algorithm in (17) as

$$\begin{aligned}&\sum_{i \in \mathcal{N}} (E_i[t] - E^{\text{sup}}) Z_i^{\eta}[t] + V \sum_{i \in \mathcal{N}} C_i(G_i[t])\\ &\alpha \sum_{i \in \mathcal{N}} \mathbb{1}_{E_i[t] + Z_i^{\eta}[t] > E_i^{\max}} + \beta \sum_{i \in \mathcal{N}} \mathbb{1}_{E_i[t] + Z_i^{\eta}[t] < 0}. \end{aligned}\tag{40}$$

If $Z_i^{\eta}[t] \geq 0$, then the energy required for this operation (charging the storage at bus i) can be supplied by one or more of the following sources:

1. Energy from renewable generation.

2. Energy from conventional generation.
3. Energy from discharging of a battery located at bus $k \neq i$.

Note that source 1 incurs zero cost in (40) whereas sources 2 and 3 incur positive cost in the objective function (40), i.e.,

$$VC_k(G_k[t]) \geq 0, \qquad \text{for } G_k[t] \geq 0,\ k \in \mathcal{N}, \tag{41}$$

$$(\widehat{E_k[t]} - E^{\text{sup}})Z_k^\eta[t] \geq 0, \quad \text{for } Z_k^\eta[t] \leq 0,\ k \in \mathcal{N}. \tag{42}$$

Recall that the LOPN algorithm chooses control actions that minimize (17). Therefore, a necessary condition for the LOPN algorithm to charge the battery located at bus $i \in \mathcal{N}_1$ is

$$(\widehat{E_i[t]} - E^{\text{sup}})Z_i^\eta[t] + \alpha \mathbb{1}_{\widehat{E_i[t]} + Z_i^\eta[t] > E_i^{\max}} \leq 0. \tag{43}$$

In other words, if condition (43) is not satisfied, the control action of the LOPN algorithm will not charge the battery bus i, because such an action incur a higher cost than the action $Z_i^\eta[t] = 0$.

In what follows, we will show that for $i \in \mathcal{N}_1$, the condition (43) is not satisfied whenever $E_i^{\max} - \widehat{E_i[t]} < Z_i^\eta[t] \leq \eta_{i,\text{ch}} Y_{i,\text{ch}}^{\max}$. First, let us rewrite the indicator function in (18) as

$$\begin{aligned} &\mathbb{1}_{\widehat{E_i[t]} + Z_i^\eta[t] > E_i^{\max}} \\ &= \begin{cases} 0 & \text{if } 0 \leq Z_i^\eta[t] \leq E_i^{\max} - \widehat{E_i[t]}, \\ 1 & \text{if } E_i^{\max} - \widehat{E_i[t]} < Z_i^\eta[t] \leq \eta_{i,\text{ch}} Y_{i,\text{ch}}^{\max}. \end{cases} \end{aligned} \tag{44}$$

We consider the first case of (44), i.e., $0 \leq Z_i^\eta[t] \leq E_i^{\max} - \widehat{E_i[t]}$. We will show that this is a feasible action under the LOPN algorithm. Note that when $0 \leq Z_i^\eta[t] \leq E_i^{\max} - \widehat{E_i[t]}$, the expression on the left hand side (LHS) of (43) can be bounded as

$$\begin{aligned} &(\widehat{E_i[t]} - E^{\text{sup}})Z_i^\eta[t] + \alpha \mathbb{1}_{\widehat{E_i[t]} + Z_i^\eta[t] > E_i^{\max}} \\ &= (\widehat{E_i[t]} - E^{\text{sup}})Z_i^\eta[t] \\ &\leq 0, \end{aligned} \tag{45}$$

where the last inequality is true since $\widehat{E_i[t]} \leq E_i^{\max} \leq E^{\text{sup}}$. Therefore, it is feasible for the LOPN algorithm to choose a control action such that $0 \leq Z_i^\eta[t] \leq E_i^{\max} - \widehat{E_i[t]}$ for $i \in \mathcal{N}_1$.

We now consider the second case of (44), i.e., $E_i^{\max} - \widehat{E_i[t]} < Z_i^\eta[t] \leq \eta_{i,\text{ch}} Y_{i,\text{ch}}^{\max}$. We will show that the LOPN algorithm does not choose such an action.

Note that when $E_i^{\max} - \widehat{E_i[t]} < Z_i^\eta[t] \leq \eta_{i,\text{ch}} Y_{i,\text{ch}}^{\max}$, the expression on the LHS of (43) can be bounded as

$$\begin{aligned} &(\widehat{E_i[t]} - E^{\text{sup}})Z_i^\eta[t] + \alpha \mathbb{1}_{\widehat{E_i[t]} + Z_i^\eta[t] > E_i^{\max}} \\ &= (\widehat{E_i[t]} - E^{\text{sup}})Z_i^\eta[t] + \alpha \\ &\overset{(a)}{\geq} (\widehat{E_i[t]} - E^{\text{sup}})\eta_{i,\text{ch}} Y_{i,\text{ch}}^{\max} + \alpha \\ &\overset{(b)}{>} (E_i^{\max} - \eta_{i,\text{ch}} Y_{i,\text{ch}}^{\max} - E^{\text{sup}})\eta_{i,\text{ch}} Y_{i,\text{ch}}^{\max} + \alpha \qquad (46) \\ &\overset{(c)}{\geq} 0. \qquad (47) \end{aligned}$$

where inequality (a) is true since $\widehat{E_i[t]} - E^{\text{sup}} \leq 0$ and $Z_i^\eta[t] \leq \eta_{i,\text{ch}} Y_{i,\text{ch}}^{\max}$, inequality (b) holds because $\widehat{E_i[t]} \geq E_i^{\max} - \eta_{i,\text{ch}} Y_{i,\text{ch}}^{\max}$, and we show inequality (c) in the following. Substituting for the value of α defined in Lemma 1 in the RHS of (46), we obtain

$$\begin{aligned} &(E_i^{\max} - \eta_{i,\text{ch}} Y_{i,\text{ch}}^{\max} - E^{\text{sup}})\eta_{i,\text{ch}} Y_{i,\text{ch}}^{\max} + \alpha \\ &= (E_i^{\max} - \eta_{i,\text{ch}} Y_{i,\text{ch}}^{\max} - E^{\text{sup}})\eta_{i,\text{ch}} Y_{i,\text{ch}}^{\max} \\ &\quad + (E^{\text{sup}} - E^{\text{inf}} + \eta_{\text{ch}}^{\text{sup}} Y_{\text{ch}}^{\text{sup}})\eta_{\text{ch}}^{\text{sup}} Y_{\text{ch}}^{\text{sup}} \\ &\geq (E_i^{\max} - \eta_{i,\text{ch}} Y_{i,\text{ch}}^{\max} - E^{\text{sup}})\eta_{i,\text{ch}} Y_{i,\text{ch}}^{\max} \\ &\quad + (E^{\text{sup}} - E^{\text{inf}} + \eta_{\text{ch}}^{\text{sup}} Y_{\text{ch}}^{\text{sup}})\eta_{i,\text{ch}} Y_{i,\text{ch}}^{\max}, \end{aligned} \tag{48}$$

where in (48), we have used the fact that $\eta_{i,\text{ch}} Y_{i,\text{ch}}^{\max} \leq \eta_{\text{ch}}^{\text{sup}} Y_{\text{ch}}^{\text{sup}}$ and $E^{\text{sup}} - E^{\text{inf}} + \eta_{\text{ch}}^{\text{sup}} Y_{\text{ch}}^{\text{sup}} \geq 0$. Since $E_i^{\max} \geq E^{\text{inf}}$ and $\eta_{\text{ch}}^{\text{sup}} Y_{\text{ch}}^{\text{sup}} \geq \eta_{i,\text{ch}} Y_{i,\text{ch}}^{\max}$, Eq. (48) can be simplified as

$$\begin{aligned} &\text{RHS of (48)} \\ &= (E_i^{\max} - \eta_{i,\text{ch}} Y_{i,\text{ch}}^{\max} - E^{\text{inf}} + \eta_{\text{ch}}^{\text{sup}} Y_{\text{ch}}^{\text{sup}})\eta_{i,\text{ch}} Y_{i,\text{ch}}^{\max} \\ &\geq 0. \end{aligned}$$

This proves inequality (c) in (47).

It follows from (47) that condition (43) is violated. Hence, the LOPN algorithm will not take an action such that $E_i^{\max} - \widehat{E_i[t]} < Z_i^\eta[t] \leq \eta_{i,\text{ch}} Y_{i,\text{ch}}^{\max}$.

From the above arguments, we conclude that for every $i \in \mathcal{N}_1$, we have $0 \leq \widehat{Z_i^\eta[t]} \leq E_i^{\max} - E_i[t]$, and hence

$$\begin{aligned} \widehat{E_i[t+1]} &= \widehat{E_i[t]} + \widehat{Z_i^\eta[t]} \\ &\leq \widehat{E_i[t]} + E_i^{\max} - \widehat{E_i[t]} \\ &= E_i^{\max}. \end{aligned} \tag{49}$$

The bounds derived in (45) and (49) imply the desired result in (35).

B. Proof of Step 2:

We now prove the lower bound, i.e.,

$$\widehat{E_i[t]} \geq 0 \qquad \forall\, t, \quad \forall\, i \in \mathcal{N}. \tag{50}$$

Again, we partition the set of buses $\mathcal{N}$ as

$$i \in \mathcal{N}_2, \ \text{ if } \widehat{E_i[t]} \leq Y_{i,\text{dis}}^{\max}/\eta_{i,\text{dis}}, \tag{51}$$

$$i \notin \mathcal{N}_2, \ \text{ if } \widehat{E_i[t]} > Y_{i,\text{dis}}^{\max}/\eta_{i,\text{dis}}, \tag{52}$$

where $\mathcal{N}_2 \subset \mathcal{N}$. For every $i \notin \mathcal{N}_2$, the storage level in the next time slot $t+1$ can be bounded as

$$\widehat{E_i[t+1]} \geq \widehat{E_i[t]} - Y_{i,\text{dis}}^{\max}/\eta_{i,\text{dis}} > 0, \tag{53}$$

where the first inequality is due to the storage discharging constraint and the second one follows from (52). In the rest of the analysis, we focus on storage at bus $i \in \mathcal{N}_2$.

Next, we will prove the following statement, which will enable us to prove the desired result in this step, i.e., for every $i \in \mathcal{N}_2$, $\widehat{E_i[t+1]} \geq 0$.

For a battery located at $i \in \mathcal{N}_2$, the amount of energy discharged under the LOPN algorithm is bounded by $-\widehat{E_i[t]} \leq \widehat{Z_i^\eta[t]} \leq 0$.

Through a similar approach to that of Subsection A, it can be shown that for every bus $i \in \mathcal{N}_2$, a control action such that $\widehat{Z_i^\eta[t]} \le -\widehat{E_i[t]}$ will result in a higher objective value of Eq. (17) than the action $\widehat{Z_i^\eta[t]} = 0$. The details of the proof are omitted for brevity. Since the LOPN algorithm chooses control actions that minimize (17), we conclude that for $i \in \mathcal{N}_2$, $-\widehat{E_i[t]} \le \widehat{Z_i^\eta[t]} \le 0$.

APPENDIX C: PROOF OF THEOREM 1

Recall that in order to solve the stochastic optimization problem (13) using the Lyapunov optimization technique [26], the control decisions during each time slot is chosen to minimize the bound on the modified Lyapunov drift (i.e. right hand side (RHS) of (16)). However, in order to ensure feasibility of the control decisions for the original problem (12), we introduced the perturbation parameters $\alpha \mathbb{1}_{E_i[t]+Z_i^\eta[t]>E_i^{\max}}$ and $\beta \mathbb{1}_{E_i[t]+Z_i^\eta[t]<0}$ in (17) while computing the control decisions.

To prove Theorem 1, we will compare the time average cost resulting from two algorithms: the first algorithm chooses the action that minimizes the RHS of (16) and the second algorithm is the LOPN algorithm formulated in (17) (with perturbation terms in the objective function).

Let us first consider an algorithm under which the control decisions are chosen to directly minimize the RHS of (16):

$$\textbf{LOPN-R:} \quad \min \quad \sum_{i\in\mathcal{N}} (E_i[t] - E^{\text{sup}}) Z_i^\eta[t] + V \sum_{i\in\mathcal{N}} C_i(G_i[t]) \tag{54}$$
$$\text{s.t.} \quad \text{Constraints } (1), (3), (8), (9), (10).$$

The algorithm is abbreviated as LOPN-R (LOPN-relaxed). Let us denote the solution corresponding to the LOPN-R algorithm by the superscript $\widetilde{\cdot}$. Further, let us define

$$f_{\text{LOPN-R}}[t] = \sum_{i\in\mathcal{N}} (E_i[t] - E^{\text{sup}}) \widetilde{Z_i^\eta[t]} + V \sum_{i\in\mathcal{N}} C_i(\widetilde{G_i[t]}) \tag{55}$$

$$f_{\text{LOPN}}[t] = \sum_{i\in\mathcal{N}} (E_i[t] - E^{\text{sup}}) \widehat{Z_i^\eta[t]} + V \sum_{i\in\mathcal{N}} C_i(\widehat{G_i[t]}). \tag{56}$$

Herein, $f_{\text{LOPN-R}}[t]$ is the minimum value of the objective function of (54) and $f_{\text{LOPN}}[t]$ is the value of the objective function of (54) with the solution of the LOPN algorithm.

Next, we present an intermediate lemma which will be useful in the proof for Theorem 1. The following lemma compares the two cost functions, $f_{\text{LOPN-R}}[t]$ and $f_{\text{LOPN}}[t]$, for each time slot t.

Lemma 2. *The following bound holds for every t:*

$$f_{LOPN}[t] \le f_{LOPN\text{-}R}[t] + \tilde{C}_2 \qquad \forall\, t. \tag{57}$$

where $\tilde{C}_2$ is a bounded constant. Further, as $E^{\text{sup}} \to \infty$, if $\lim_{E^{\text{sup}}\to\infty} E^{\text{inf}}/E^{\text{sup}} = 1$, then $\lim_{E^{\text{sup}}\to\infty} \tilde{C}_2/E^{\text{sup}} = 0$.

Proof. We consider a time t during which both the sets $\mathcal{N}_1$ and $\mathcal{N}_2$ are empty. It is straightforward to check that in this case, the control decisions under both the LOPN and LOPN-R algorithms are exactly the same, and therefore

$$f_{\text{LOPN}}[t] = f_{\text{LOPN-R}}[t]. \tag{58}$$

Suppose that the sets $\mathcal{N}_1$ and $\mathcal{N}_2$ are non-empty. Further, we are only interested in the time slot t during which $\widehat{Z_i^\eta[t]}, \widetilde{Z_i^\eta[t]} \ge 0,\ i \in \mathcal{N}_1$ and $\widehat{Z_i^\eta[t]}, \widetilde{Z_i^\eta[t]} \le 0,\ i \in \mathcal{N}_2$. During these time slots, the control decisions under LOPN and LOPN-R algorithms may be different, because according to Lemma 1 we have,

$$0 \le \widehat{Z_i^\eta[t]} \le E_i^{\max} - E_i[t], \qquad \forall i \in \mathcal{N}_1, \tag{59}$$
$$-E_i[t] \le \widehat{Z_i^\eta[t]} \le 0, \qquad \forall i \in \mathcal{N}_2. \tag{60}$$

whereas

$$0 \le \widetilde{Z_i^\eta[t]} \le \eta_{i,\text{ch}} Y_{i,\text{ch}}^{\max}, \qquad \forall i \in \mathcal{N}_1, \tag{61}$$
$$-Y_{i,\text{dis}}^{\max}/\eta_{i,\text{dis}} \le \widetilde{Z_i^\eta[t]} \le 0, \qquad \forall i \in \mathcal{N}_2. \tag{62}$$

In rest of the cases, the control decisions under the two algorithms will be the same. In what follows, we claim that when the sets $\mathcal{N}_1$ and $\mathcal{N}_2$ are non-empty and $E^{\text{sup}} \to \infty$ and $\lim_{E^{\text{sup}}\to\infty} E^{\text{inf}}/E^{\text{sup}} = 1$, then the control decisions under LOPN and LOPN-R algorithms will only differ in $Z_i^\eta[t],\ i \in \mathcal{N}_1$ and $Z_i^\eta[t],\ i \in \mathcal{N}_2$. The control decisions of the conventional generation $(\widetilde{G_i[t]}, \widehat{G_i[t]}\ i \in \mathcal{N})$ will be the same under the two algorithms. The claim can be justified as follows: Consider the factors that can cause $\widetilde{G_i[t]}$ and $\widehat{G_i[t]}$ to be different:

- The charging limits for $i \in \mathcal{N}_1$ under the LOPN and LOPN-R algorithms in (59) and (61) are different.
  - However, in Lemma 3 (which is presented at the end of this proof), we will show that if $\lim_{E^{\text{sup}}\to\infty} E^{\text{inf}}/E^{\text{sup}} = 1$, then energy from the conventional generation will not be utilized to charge the battery located at bus $i \in \mathcal{N}_1$ under both the LOPN and LOPN-R algorithms. Therefore, this will not result in the decisions $\widetilde{G_i[t]}$ and $\widehat{G_i[t]}$ being different.
- The discharging limits for $i \in \mathcal{N}_2$ under the LOPN and LOPN-R algorithms in (60) and (62) are different.
  - However, recall from the proof of Lemma 1 that energy will be discharged from the batteries located at $i \in \mathcal{N}_2$ only under two conditions: i) The energy that is discharged must be utilized to charge the battery located at bus $k \in \mathcal{N}$. ii) Energy cannot be supplied to bus $k \in \mathcal{N}$ from the renewable/ conventional generation (due to the NPF constraints). There once again, we can conclude that this will not cause the decisions $\widetilde{G_i[t]}$ and $\widehat{G_i[t]}$ to be different.

From these observations, we can conclude that

$$\widehat{G_i[t]} = \widetilde{G_i[t]} \qquad i \in \mathcal{N}. \tag{63}$$

We now examine at the difference between $g_{\min}[t]$ and $f_{\min}[t]$. First consider the difference between the terms corresponding to the battery charging/ discharging decisions for $i \in \mathcal{N}_1$.

Through a similar approach to that of Appendix A, we can show that

$$\begin{aligned} & -|\mathcal{N}_1[t]|(E^{\text{sup}}+\eta_{\text{ch}}^{\text{sup}}Y_{\text{ch}}^{\text{sup}}-E^{\text{inf}})\eta_{\text{ch}}^{\text{sup}}Y_{\text{ch}}^{\text{sup}} \\ & \leq \sum_{i\in\mathcal{N}_1}(E_i[t]-E^{\text{sup}})(\widetilde{Z_i^\eta[t]}-\widehat{Z_i^\eta[t]}) \\ & \leq |\mathcal{N}_1[t]|(E^{\text{sup}}+\eta_{\text{ch}}^{\text{sup}}Y_{\text{ch}}^{\text{sup}}-E^{\text{inf}})\eta_{\text{ch}}^{\text{sup}}Y_{\text{ch}}^{\text{sup}}. \end{aligned} \tag{64}$$

Next consider the difference between the terms corresponding to $i \in \mathcal{N}_2$. Once again, using arguments similar to that of Appendix A, we can show that if $\eta_{\text{dis}}^{\text{sup}} < 1$ and $E^{\text{sup}}(1/\eta_{\text{dis}}^{\text{sup}} - \eta_{\text{ch}}^{\text{sup}}) \geq Y_{\text{dis}}^{\text{sup}}/(\eta_{\text{dis}}^{\text{inf}})^2$, then,

$$\sum_{i\in\mathcal{N}_2}(E_i[t]-E^{\text{sup}})(\widetilde{Z_i^\eta[t]}-\widehat{Z_i^\eta[t]}) = 0. \tag{65}$$

Else if $\eta_{\text{dis}}^{\text{sup}} < 1$ and $E^{\text{sup}}(1/\eta_{\text{dis}}^{\text{sup}} - \eta_{\text{ch}}^{\text{sup}}) \leq Y_{\text{dis}}^{\text{sup}}/(\eta_{\text{dis}}^{\text{inf}})^2$, or if $\eta_{\text{ch}}^{\text{sup}} = \eta_{\text{dis}}^{\text{sup}} = 1$ then,

$$\begin{aligned} & -|\mathcal{N}_2[t]|Y_{\text{dis}}^{\text{sup}}\min(Y_{\text{ch}}^{\text{sup}},Y_{\text{dis}}^{\text{sup}})/(\eta_{\text{dis}}^{\text{inf}})^2 \\ & \leq \sum_{i\in\mathcal{N}_2}(E_i[t]-E^{\text{sup}})(\widetilde{Z_i^\eta[t]}-\widehat{Z_i^\eta[t]}) \\ & \leq |\mathcal{N}_2[t]|Y_{\text{dis}}^{\text{sup}}\min(Y_{\text{ch}}^{\text{sup}},Y_{\text{dis}}^{\text{sup}})/(\eta_{\text{dis}}^{\text{inf}})^2. \end{aligned} \tag{66}$$

Using (63), (64) and (66), we conclude that

$$-\tilde{C}_2 \leq f_{\text{LOPN-R}}[t] - f_{\text{LOPN}}[t] \leq \tilde{C}_2 \qquad \forall\, t, \tag{67}$$

where

$$\begin{aligned} \tilde{C}_2 = {} & |\mathcal{N}_1[t]|(E^{\text{sup}}+\eta_{\text{ch}}^{\text{sup}}Y_{\text{ch}}^{\text{sup}}-E^{\text{inf}})\eta_{\text{ch}}^{\text{sup}}Y_{\text{ch}}^{\text{sup}} \\ & + |\mathcal{N}_2[t]|Y_{\text{dis}}^{\text{sup}}\min(Y_{\text{ch}}^{\text{sup}},Y_{\text{dis}}^{\text{sup}})/(\eta_{\text{dis}}^{\text{inf}})^2. \end{aligned} \tag{68}$$

□

In order to complete the proof of Lemma 2, we present Lemma 3 and its proof in the following:

Lemma 3. *Suppose that* $\lim_{E^{\text{sup}}\to\infty} E^{\text{inf}}/E^{\text{sup}} = 1$. *Under the LOPN and LOPN-R algorithms, energy from the conventional generation will not be utilized to charge the batteries located at bus* $i \in \mathcal{N}_1$ *(cf. the definition of* $\mathcal{N}_1$ *in Eq.* (39)*).*

Proof. Consider a control decision under which $0 < Y_i \leq Y_{i,\text{ch}}^{\max}$ units of energy from conventional generator is used to charge battery located at bus $i \in \mathcal{N}_1$. A necessary condition for the LOPN and the LOPN-R algorithm to choose this control action is given by

$$(E_i[t]-E^{\text{sup}})\eta_{i,\text{ch}}Y_i + VC_k(L_{k,\text{con}}+Y_i) \leq 0, \tag{69}$$

where $k = 1,\ldots,N$ and where $L_{k,\text{con}} \geq 0$ represents the amount of energy drawn from the conventional generator for purposes other than charging the battery located at bus i (e.g. to satisfy the load etc.). Since $E_i[t] \geq E_i^{\max} - \eta_{i,\text{ch}}Y_{i,\text{ch}}^{\max}$ for $i \in \mathcal{N}_1$, the LHS of (69) can be bounded as

$$\begin{aligned} & \text{LHS of (69)} \\ & \geq (E_i^{\max} - \eta_{i,\text{ch}}Y_{i,\text{ch}}^{\max} - E^{\text{sup}})\eta_{i,\text{ch}}Y_i + VC_k(L_{k,\text{con}}+Y_i) \\ & \geq (E^{\text{inf}} - \eta_{i,\text{ch}}Y_{\text{ch}}^{\text{sup}} - E^{\text{sup}})\eta_{i,\text{ch}}Y_i + VC_k(Y_i), \end{aligned} \tag{70}$$

where the second inequality follows from the fact that $C_k(L_{k,\text{con}}+Y_i) \geq C_k(Y_i)$ for $L_{k,\text{con}} \geq 0$. Substituting the expression for the cost function $C_k(\cdot)$ from (11) and V from (21) and using the fact that $E_i^{\max} \geq E^{\text{inf}}$, we obtain

$$\begin{aligned} (70) \geq {} & (E^{\text{inf}} - \eta_{i,\text{ch}}Y_{\text{ch}}^{\text{sup}} - E^{\text{sup}})\eta_{i,\text{ch}}Y_i \\ & + \frac{\left(E^{\text{sup}} - Y_{\text{dis}}^{\text{sup}}/\eta_{\text{dis}}^{\text{inf}}\right)(p_kY_i + q_kY_i^2)}{(p_{\max} + q_{\max}Y_{\text{dis}}^{\text{sup}})}. \end{aligned} \tag{71}$$

We focus on the RHS of (71). As $E^{\text{sup}} \to \infty$, since $\lim_{E^{\text{sup}}\to\infty} E^{\text{inf}}/E^{\text{sup}} = 1$, the RHS of (71) is nonnegative. Hence, the condition in (69) is not satisfied, and this proves the result of the lemma. □

Proof of Theorem 1

We now proceed to state the following result whose result will be used in the proof of Theorem 1. Let us focus on the relaxed problem defined in (13). It can be shown that the optimal solution to the relaxed problem can be obtained by a stationary and randomized policy Π that achieves

$$\mathbb{E}\Big[\sum_{i=1}^{N} C_i(G_i^{\Pi}[t])\Big] = \overline{g_{\min}}, \qquad \forall\, t, \tag{72}$$

and satisfies the following constraints:

$$\begin{aligned} & \mathbb{E}[(Z_i^\eta[t])^{\Pi}] = 0, && \forall\, i \in \mathcal{N}, \quad \forall\, t, \\ & P_{i,j}^{\Pi}[t] = B_{i,j}(\theta_i^{\Pi}[t] - \theta_j^{\Pi}[t]), && \forall\,(i,j)\in\mathcal{L}, \quad \forall\, t, \\ & L_i[t] - H_i^{\Pi}[t] - G_i^{\Pi}[t] + Y_i^{\Pi}[t] + \sum_{j\neq i} P_{i,j}^{\Pi}[t] = 0, && i\in\mathcal{N},\ \forall\, t, \\ & H_i^{\Pi}[t] \leq X_i[t], && \forall\, i \in \mathcal{N}, \quad \forall\, t, \\ & G_i^{\min} \leq G_i^{\Pi}[t] \leq G_i^{\max}, && \forall\, i\in\mathcal{N},\ \forall\, t, \\ & -Y_{i,\text{dis}}^{\max} \leq Y_i^{\Pi}[t] \leq Y_{i,\text{ch}}^{\max}, && \forall\, i\in\mathcal{N},\ \forall\, t, \\ & -P_{i,j}^{\max} \leq P_{i,j}^{\Pi}[t] \leq P_{i,j}^{\max}, && \forall\,(i,j)\in\mathcal{L},\ \forall\, t. \end{aligned} \tag{73}$$

The existence of such a policy can be proved by using the Caratheodory theorem, through a similar approach to that of the arguments in [26]; the details are omitted here for brevity.

We are now ready to prove Theorem 1. Recall the bound on the modified Lyapunov drift derived in (15). Let us rewrite the bound with the control decisions of the LOPN algorithm as follows:

$$\begin{aligned} \Delta_V[t] \leq {} & \tilde{C}_1 \\ & + \mathbb{E}\Big[\sum_{i\in\mathcal{N}}\big[(E_i[t]-E^{\text{sup}})\widehat{Z_i^\eta[t]} + VC_i(\widehat{G_i[t]})\big]\big|\mathbf{E}[t]\Big]. \end{aligned} \tag{74}$$

Using the result of Lemma 2 in the RHS of (74), we obtain

$$\begin{aligned} \Delta_V[t] \leq {} & \tilde{C}_1 + \tilde{C}_2 \\ & + \mathbb{E}\Big[\sum_{i\in\mathcal{N}}\big[(E_i[t]-E^{\text{sup}})\widetilde{Z_i^\eta[t]} + VC_i(\widetilde{G_i[t]})\big]\big|\mathbf{E}[t]\Big], \end{aligned} \tag{75}$$

where $\tilde{C}_2$ is defined in (68). By definition, the control actions $\widetilde{Z_i^\eta[t]}$ and $\widetilde{G_i[t]}$ minimize the RHS of (75) over all feasible control actions. Choosing any other control action must yield a lower value of the final term in (75). Comparing it with the control action chosen according to the stationary and randomized policy of (72), we obtain

$$\begin{aligned}\Delta_V[t] &\le \tilde{C}_1 + \tilde{C}_2 \\ &+ \mathbb{E}\Big[\sum_{i\in\mathcal{N}} \big[(E_i[t] - E^{\sup})(Z_i^{\eta}[t])^{\Pi} + VC_i(G_i^{\Pi}[t])\big]\big|\mathbf{E}[t]\Big] \\ &= \tilde{C}_1 + \tilde{C}_2 + V\overline{g_{\min}}, \end{aligned} \tag{76}$$

where the result of (76) follows from (72) and (73). Taking the expectation on both the sides of (76) over $\mathbf{E}[t]$, and summing from $t = 0, ..., T-1$, we obtain

$$\begin{aligned}&\mathbb{E}\Big[\Psi(T) - \Psi(0)\Big] + \frac{1}{T}\sum_{t=0}^{T-1}\mathbb{E}\left[\sum_{i=1}^{N} C_i(\widehat{G_i[t]})\right] \\ &\le T(\tilde{C}_1 + \tilde{C}_2) + TV\overline{g_{\min}}, \end{aligned} \tag{77}$$

Rearranging the terms and dividing both sides by TV, we obtain

$$\begin{aligned}&\frac{1}{T}\sum_{t=0}^{T-1}\mathbb{E}\left[\sum_{i=1}^{N} C_i(\widehat{G_i[t]})\right] \\ &\le \overline{g_{\min}} + \frac{\tilde{C}_1 + \tilde{C}_2}{V} + \frac{\mathbb{E}\Big[\Psi(0)\Big]}{VT}. \end{aligned} \tag{78}$$

Taking $\lim T \to \infty$, since $\mathbb{E}[\Psi(0)] < \infty$, we have

$$\lim_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\mathbb{E}\left[\sum_{i=1}^{N} C_i(\widehat{G_i[t]})\right] \le \overline{g_{\min}} + \frac{\tilde{C}}{V}, \tag{79}$$

where $\tilde{C} = \tilde{C}_1 + \tilde{C}_2$. Further, since $\overline{f_{\min}} \le \overline{g_{\min}}$, we conclude that

$$\lim_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\mathbb{E}\left[\sum_{i=1}^{N} C_i(\widehat{G_i[t]})\right] \le \overline{f_{\min}} + \frac{\tilde{C}}{V}. \tag{80}$$

Further, from the definition of $\tilde{C}_2$ in (68) and the definition of V in (21), it can be seen that when $E^{\sup} \to \infty$, since $\lim_{E^{\sup}\to\infty} E^{\inf}/E^{\sup} = 1$, we have $\tilde{C}/V = (\tilde{C}_1 + \tilde{C}_2)/V \to 0$. This proves Theorem 1.

Appendix D: LOPN Algorithm Under Markovian System Dynamics

Finally, we generalize the performance bound for the LOPN algorithm (established in Theorem 1) to the case where the renewable energy and system load vector $\{X_i[t], L_i[t]\}_{i=1}^{N}$ is Markovian.

Theorem 2. *Suppose that the renewable energy and system load vector $\{X_i[t], L_i[t]\}_{i=1}^{N}$ evolves according to a finite state irreducible and aperiodic Markov chain. The following results hold for the LOPN algorithm.*

1. *The energy level in the batteries under the LOPN algorithm can be can be bounded as*

$$0 \le \widehat{E_i[t]} \le E_i^{\max}, \qquad i = 1, \dots, N, \tag{81}$$

   and the control decisions of the LOPN algorithm are feasible for problem (12).

2. *The time average cost function achieved by the LOPN algorithm satisfies*

$$\lim_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\mathbb{E}\left[\sum_{i=1}^{N} C_i(\widehat{G_i[t]})\right] \le \overline{f_{\min}} + \tilde{C}\mathcal{O}\left(\frac{1}{V}\right), \tag{82}$$

   where $\tilde{C}$ is the same constant as the one used in Theorem 1, and $\lim_{E^{\sup}\to\infty} V\mathcal{O}(1/V) = K$ with K being a constant. Further, if $\lim_{E^{\sup}\to\infty} E^{\inf}/E^{\sup} = 1$, then the bound $\tilde{C}\mathcal{O}(1/V) \to 0$ as $E^{\sup} \to \infty$.

Proof. Statement 1 of Theorem 2 follows from Lemma 1. Recall that Lemma 1 is a sample path result and does not make any assumptions on the random processes governing the renewable energy and load (whether i.i.d or Markovian process). Statement 2 of Theorem 2 can be proved using the T slot drift technique developed in Theorem 2 of [27]. □

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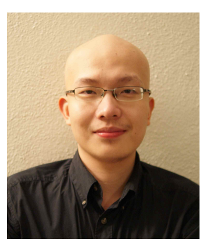

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