Disequilibrium Growth Theory in an International Perspective

Author(s): Charles van Marrewijk and Jos Verbeek

Source: *Oxford Economic Papers*, New Series, Vol. 45, No. 2 (Apr., 1993), pp. 311-331
Published by: Oxford University Press
Stable URL: http://www.jstor.org/stable/2663641
Accessed: 27/08/2008 05:19

Oxford Economic Papers 45 (1993), 311–331

DISEQUILIBRIUM GROWTH THEORY IN AN INTERNATIONAL PERSPECTIVE

By CHARLES VAN MARREWIJK* *and* JOS VERBEEK†

1. Introduction

THE economics profession has strongly turned to reviving two strands of literature over the past six years: growth theory and neo-Keynesian disequilibrium economics. This renewed interest, which tries to remedy some shortcomings of the existing literature, is quite understandable (growth effects are ultimately more important than level effects, while the ubiquitous nature of market disequilibrium is almost undisputed) but does not integrate the two approaches, nor does it address two problems in particular.

The Neo-Keynesian theories of market disequilibrium are static in nature because they lack an endogenous capital accumulation process (which in general will influence the type of short-run equilibrium studied in these models). This holds (in open and closed economies) not only for the 'old' Neo-Keynesian literature,[1] but also for the 'Neo-Neo (or New)-Keynesians'.[2] Growth theory, on the other hand, analyzes the impact of (human and/or physical) capital accumulation over time, but fails to analyze the consequences of market disequilibrium. Again this holds (in open and closed economies) both for the 'old' growth theory,[3] and for the 'new' growth theory.[4] Disequilibrium growth theory, introduced by Ito (1978a, 1980), overcomes these specific shortcomings of the two strands of literature by studying both short-run market disequilibrium and endogenous capital accumulation over time that will affect the short-run equilibrium.

Progress in the theory of disequilibrium growth has been steady, but slow.[5] Honkapohja and Ito (1982), Picard (1983) and Schittko and Eckwert (1985) investigate monetary versions of the Neo-Keynesian model. Ginsburgh, Henin and Michel (1985) study endogenous savings behavior in a Ramsey framework, while Neary and Stiglitz (1983) and van Wijnbergen (1985, 1987) examine discrete time two-period models. In this paper we extend Ito's (1980) continuous time model to two sectors (as termed desirable by Ito, 1980, p. 399) by

[1] For example Clower (1965), Leijonhufvud (1968), Barro and Grossman (1971, 1976), Dixit (1978), Malinvaud (1977), Benassy (1975, 1982), Drèze (1975), Böhm (1978), or Honkapohja (1980).

[2] For example Blanchard and Kiyotaki (1987), Cooper and John (1988), Ball and Romer (1989, 1990), Neary (1990), or Cooper (1990).

[3] For example Ramsey (1928), Solow (1956), Uzawa (1962, 1963), Bardhan (1965), Oniki and Uzawa (1965), Inada (1968), Kemp (1968), Burmeister and Dobell (1970) or Abel and Blanchard (1983).

[4] For example Romer (1986, 1990), Barro (1990), Grossman and Helpman (1991), Alogoskoufis and van der Ploeg (1990), van Marrewijk and Verbeek (1991), or van Marrewijk, de Vries and Withagen (1992).

[5] The slow speed of progress is partly caused by some technical dynamic problems that naturally arise in disequilibrium growth theory, see below.

recognizing the fact that consumption goods and investment goods are inherently different commodities produced in distinct sectors of the economy. This enables us to analyze for the first time spill-over effects and simultaneous adjustment of prices and quantities in the long run as a sequence of short-run fixed price equilibria in a small open economy. It will be shown that the existence of short-run disequilibrium and income distribution may have long-run effects. The model will be applied to examine the dynamic implications of a change in the terms of trade and of real wage rigidity.

The source of short term disequilibrium in our model will be sluggish real wage rate adjustment. Bean, Layard and Nickell (1986) present annual wage equations for 18 OECD countries and find plenty of evidence for real wage inertia, see also Branson and Rotemberg (1980) and Newell and Symons (1986). Numerous theories have been put forward to explain real or nominal stickiness. Blinder (1991), for example, recently reported about ongoing research (using interview studies) on 12 theories of sluggish adjustment.[6] His preliminary conclusion is that adjustment is indeed sluggish: the mean response lag between a change in demand or cost and an adjustment is three to four months and less than 15% of GNP is repriced more frequently than quarterly.

The production functions in the two sectors of the economy, the consumption good sector and the investment good sector, are characterized by constant returns to scale, as supported by recent empirical studies by Benhabib and Jovanovich (1991) and Mankiw, Romer and Weil (1992), which enables the model to be written in per capita terms. The presence of two goods enables countries to reap the benefits of international trade by specializing (to some extent or completely) in the production of the good for which they have a comparative advantage. More specifically, trade in investment goods comprised some 35% of total world trade in 1990 (Oldersma, 1991).

Ideally, we would analyze the short term disequilibrium interaction between demand and supply at home and abroad, determine the equilibrium price ratio of the two goods and analyze the dynamics of capital accumulation and wages in both countries over time. There are, however, three regimes in the labor market (excess-demand, unemployment and full employment) and three production possibilities (production of investment goods, consumption goods or both goods). Simultaneous investigation of these possibilities for both countries would require the analysis of 81 different regimes and four differential equations, excluding the possibility of graphical illustration and analysis. Therefore, we adopt the popular 'small country' assumption, in which the international terms of trade cannot be influenced. The second economy becomes a 'garbage-can' economy where we can take out and put in anything necessary or superfluous in the home economy. The effect of a change in the terms of trade will be investigated at the end of the paper.

[6] These twelve theories, ranked by Blinder in order of importance, are: delivery lags/service, coordination failure, cost-based pricing, implicit contracts, explicit nominal contracts, costs of price adjustment, procyclical elasticity, pricing points, inventories, constant marginal cost, hierarchies and judging quality by price.

We retain the Solow (1956) assumption of fixed savings rates, but allow for these rates to be different for capital-owners and laborers, see Kaldor (1956), Pasinetti (1962) and Ito (1980).

Analyzing a two-sector disequilibrium model entails the introduction of a rationing scheme to distribute the available supply of labor over the two sectors if the demand for labor is rationed.[7] Our scheme will have the two basic properties, voluntary exchange and market efficiency, stressed in the literature.[8] More specifically, the short side rule holds and in the excess-demand regime labor will be allocated to the two sectors (to the extent possible, i.e. up to the point where complete specialization occurs) so as to equalize the value marginal product of labour. Malinvaud (1977, p. 50) defends the use of an efficient rationing scheme (equalizing the marginal product of labor between different producers in the good sector) arguing that

> the most productive firms will be able to attract labour (... when there is a shortage ...) because they will provide safer employment and better prospects, faster wage increases and perhaps other advantages; (... while with rationed sales ...) the most productive firms will perform (... be able to ...) afford advertising campaigns etc.

As a result of efficient rationing in our two-sector model the profit per worker, and hence the marginal incentive for a producer to employ a worker in his company, will be equal in the two sectors. In a companion paper, van Marrewijk and Verbeek (forthcoming), we study the effects of a 'priority' or 'sector specific' rationing scheme, possibly installed and supervised by a central planner. Suppose the investment good industry can, at the going below-equilibrium wage rate, employ as many laborers as it wants such that the labour shortage shows up exclusively in the consumption goods sector. Then the natural outcome of this rationing scheme is that, other things being equal and given sufficiently high elasticities of substitution, the speed at which capital accumulates increases. Similarly, if the consumption goods sector gets priority, the speed at which capital accumulates declines, see van Marrewijk and Verbeek (forthcoming) for details.

A technical problem arises in disequilibrium growth theory because different short term disequilibrium regimes give rise to different dynamic systems. The dynamic analysis, therefore, has to take into consideration the possibility of regime switching. Various solutions have been put forward to overcome this problem, but they generally do not give unique solutions. Here we use the method originating from Filippov (1960), developed further by Honkapohja and Ito (1983) and Schittko and Eckwert (1985), which gives a unique solution coinciding with the 'normal' solution to differential equations in the interior of the regimes.

[7] We are especially grateful to Pierre Dehez for discussions on this topic.

[8] See for example Clower (1960, 1965), Hahn and Negishi (1962), Barro and Grossman (1971, 1976), Grossman (1971), and Benassy (1982).

2. The full employment model

We investigate a model with investment goods (the numéraire, sub-index 1) and consumption goods (sub-index 2), produced in distinct sectors of the economy using capital (K_i) and labor (L_i) as inputs in neoclassical production functions that satisfy the Inada conditions. Let $\bar{p}$ be the relative price of consumption goods, determined on the world market, that cannot be influenced by this small country for the period under investigation. The consequences of changes in the terms of trade are examined in Section 5. Define $k_i \equiv K_i/L_i$, $k \equiv K/L$ and $l_i \equiv L_i/L$, where K is total capital supply and L is total (perfectly inelastic) labor supply. Then we can write the production process in intensive form

$$y_1 = l_1 f_1(k_1) + x_1 \tag{1}$$

$$y_2 = l_2 f_2(k_2) - x_1/\bar{p} \tag{2}$$

where y_1 (y_2) is per capita investment (consumption), which can be produced at home, $l_1 f_1(k_1)$ $[l_2 f_2(k_2)]$, or imported from abroad, x_1 $(-x_1/\bar{p})$.[9] Throughout the rest of this paper we exclude the possibility of factor intensity reversal. For ease of exposition, and without loss of generality, the investment good industry will be assumed to be relatively capital intensive ($k_1 > k_2$). Let w be the wage rate and r the rental rate of capital, then perfect competition on the factor markets ensure

$$\left.\begin{aligned} &f'_1(k_1) \leqslant r,\ f_1(k_1) - k_1 f'_1(k_1) \leqslant w \text{ with equality if } f_1(k_1) > 0 \\ &\bar{p}f'_2(k_2) \leqslant r,\ \bar{p}\{f_2(k_2) - k_2 f'_2(k_2)\} \leqslant w \text{ with equality if } f_2(k_2) > 0 \end{aligned}\right\} \tag{3}$$

Full employment of capital and labor give

$$k_1 l_1 + k_2 l_2 = k \tag{4}$$

$$l_1 + l_2 = 1 \tag{5}$$

Labourers save a fraction s_w of their income wL, while the capital-owners save a fraction s_r out of their income rK, i.e.

$$y_1 = s_w w + s_r r k \tag{6}$$

The initial stock of capital is given, the labor force grows at the rate n, and the depreciation rate is μ. This gives the capital accumulation process.

$$\dot{k} = y_1 - (\mu + n)k \tag{7}$$

[9] Hence trade is balanced and exchange rates are fully flexible. An alternative specification would be with fixed exchange rates and spending proportional to real money balances, see Dornbusch (1980, chs 7 and 8).

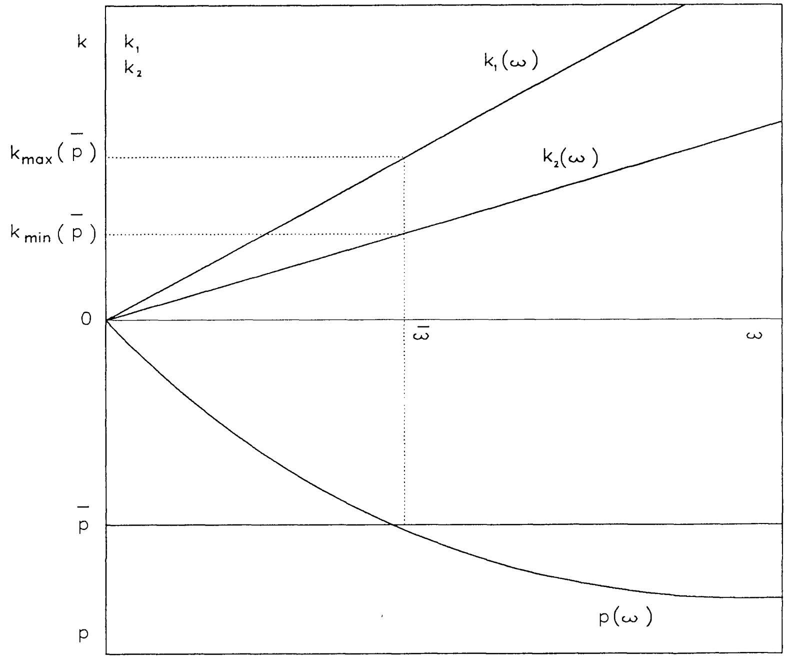

FIG. 1. The Harrod-Johnson diagram if investment goods are capital intensive

Define the ratio of marginal productivities in sector i, ω_i, and ω as

$$\omega_i \equiv [f_i(k_i)/(f'_i(k_i)] - k_i,\ i = 1, 2 \tag{8}$$

$$\omega \equiv \begin{cases} \omega_1, & \text{if good 1 is produced} \\ \omega_1 = \omega_2, & \text{if both goods are produced} \\ \omega_2, & \text{if good 2 is produced} \end{cases}$$

The ratio of marginal productivities in each sector determines the capital labor ratio $k_i = k_i(\omega_i)$, with $k_i'(\omega_i) = -(f_i')^2/[f_i f_i''] > 0$. If both goods are to be produced marginal productivity in both sectors has to be equal, $\omega_1 = \omega_2 = \omega$, and we get from equation (3)

$$\bar{p} = p(\omega) \equiv f'_1(k_1(\omega))/f'_2(k_2(\omega)) \tag{9}$$

with $p'(\omega)/p(\omega) = [k_2(\omega) + \omega]^{-1} - [k_1(\omega) + \omega]^{-1} > 0$ as $k_1(\omega) > k_2(\omega)$ which is illustrated in the familiar Harrod-Johnson diagram, see Fig. 1.

The given world price $\bar{p}$ determines $\bar{\omega}$ which determines the capital–labor ratios $\bar{k}_1$ and $\bar{k}_2$. If the actual economy-wide capital-labor ratio k is between the two extremes $\bar{k}_1$ and $\bar{k}_2$, then the economy will produce both goods. If k is below $\bar{k}_2$ the economy will only produce good 2, while if k is above $\bar{k}_1$ the economy produces only good 1. Clearly, $\bar{k}_1$ and $\bar{k}_2$ depend on $\bar{\omega}$ and hence on $\bar{p}$ and so does the area in which incomplete specialization occurs. Therefore, we define $k_{\max}(\bar{p}) \equiv \bar{k}_1$ and $k_{\min}(\bar{p}) \equiv \bar{k}_2$ and conclude

$$0 < k < k_{\min}(\bar{p}) \Rightarrow \text{produce good 2; } w = w_2(k)$$

$$k_{\min}(\bar{p}) \leqslant \mathrm{k} \leqslant k_{\max}(\bar{p}) \Rightarrow \text{produce both goods; } w = \bar{w}$$

$$k_{\max}(\bar{p}) < k \Rightarrow \text{produce good 1; } w = w_1(k)$$

The wage rate is a continuous function of the capital–labor ratio, $w = w^*(k)$ say, but is not differentiable at $k_{\min}(\bar{p})$ and at $k_{\max}(\bar{p})$. The production pattern affects the dynamic behavior of the economy.

Suppose both goods are produced. Given $\bar{p}$, the equations (3) and (8) determine $\bar{k}_1$, $\bar{k}_2$, $\bar{w}$. $\bar{r}$ and $\bar{\omega}$, see above. These variables, identified by an upper bar, will be fixed until Section 5 of the paper. Capital accumulates according to

$$\dot{k} = \{f'_1(\bar{k}_1)[s_w\bar{\omega} + s_r k]/k - (\mu + n)\}k \equiv q_0(k)k \tag{10}$$

Suppose only investment goods are produced. This implies $l_1 = 1$ and $k = k_1$, hence $\omega = \omega_1(k) \equiv \{f_1(k) - kf'_1(k)\}/f'_1(k)$ and $w = w_1(k) \equiv f_1(k) - kf'_1(k)$. Therefore capital accumulates according to

$$\dot{k} = \{f'_1(k)[s_w\omega_1(k) + s_r k]/k - (\mu + n)\}k \equiv q_1(k)k \tag{10$'$}$$

Suppose only consumption goods are produced. This implies $l_2 = 1$ and $k = k_2$, hence $\omega = \omega_2(k) \equiv \{f_2(k) - kf'_2(k)\}/f'_2(k)$ and $w = w_2(k) \equiv \bar{p}[f_2(k) - kf'_2(k)]$. Hence capital accumulates according to:

$$\dot{k} = \{\bar{p}f'_2(k)[s_w\omega_2(k) + s_r k]/k - (\mu + n)\}k \equiv q_2(k)k \tag{10$''$}$$

In short, at any point in time capital may accumulate according to equation (10), (10′) or (10″). Let σ_i be the elasticity of substitution in sector i, that is $\sigma_i \equiv \omega_i/k_i\omega'_i(k_i)$. Since

$$q'_0(k) = -f'_1(\bar{k}_1)s_w\bar{\omega}/k^2 < 0$$

$$q'_1(k) = f''_1(k)[s_w\omega_1(k) + s_r k]/k + s_w f_1(k)\omega_1(k)(1 - \sigma_1)/\sigma_1 k^2$$

$$q'_2(k) = \bar{p}f''_2(k)[s_w\omega_2(k) + s_r k]/k + s_w\bar{p}f'_2(k)\omega_2(k)(1 - \sigma_2)/\sigma_2 k^2$$

a sufficient condition[10] for $q'_1(k) < 0$ and $q'_2(k) < 0$ is $\sigma_1 \geqslant 1$ and $\sigma_2 \geqslant 1$ respectively. From $\omega_2(k_{\min}(\bar{p})) = \bar{\omega}$ and $k_{\min}(\bar{p}) = \bar{k}_2$ it follows that $q_0(k_{\min}(\bar{p})) = q_2(k_{\min}(\bar{p}))$. Similarly, $q_0(k_{\max}(\bar{p})) = q_1(k_{\max}(\bar{p}))$. This establishes both uniqueness and stability of the steady state, k^* say, provided we assume σ_1 and σ_2 are bigger than or equal to one. At the steady state k^* we may either completely specialize in the production of one good or incompletely specialize and produce both goods. The wage rate adjusts instantaneously along $w = w^*(k)$ until

[10] For alternative conditions see Burmeister and Dobell (1970) or Ito (1980).

the long run equilibrium is reached. For ease of exposition we will henceforth assume that the long run equilibrium k^* is in the incomplete specialization region, but clearly indicate in the theorems below if the results are different when the economy completely specializes in the production of one good in the steady state.[11]

Proposition 1. In the neoclassical full employment model with instantaneous adjustment the long-run steady state k^* is unique and stable if $\sigma_1 \geqslant 1$ and $\sigma_2 \geqslant 1$.

3. The disequilibrium regimes

In the full employment regime of Section 2 the wage rate adjusts instantaneously to pressure in the labor market to clear this market. Now, we introduce sluggish real wage rate adjustment such that there may be unemployment or excess-demand in the labor market. The short-run wage rate is then exogenously given. As will be shown in Section 5, starting from an initial stationary state, a simple change in the terms of trade can lead the economy into a disequilibrium regime.[12] The actual quantity traded is determined by the short side rule, $L = \min\{L^d, L^s\}$, where L^d is the quantity of labor demanded and L^s the quantity supplied. We distinguish

$$\begin{array}{ll} \text{Unemployment}\,(U) & L^d < L^s \\ \text{Excess-Demand}\,(E) & L^d > L^s \end{array}$$

The full employment regime, analyzed in the previous section, is the boundary between the two regimes. It only occurs if the short run exogenous wage rate equals $w^*(k)$. The different regimes are described by the two state variables k and w. It is the exogeneity of w in the short run that causes the possibility of disequilibrium in the labor market. The analysis of the unemployment regime and the excess-demand regime should distinguish between variables obtained by dividing by the amount of labor demanded (indicated by a superscript ^{d}) on the one hand and variables obtained by dividing by the amount of labor supplied (no superscript) on the other hand. As usual we assume that the wage rate reacts to pressure in the labor market, i.e.

$$\dot{w} = \zeta_j[(k/k^d) - 1], \quad \text{with } \zeta_j > 0, \text{ for } j = E, U \tag{11}$$

[11] Detailed calculations are available from the authors upon request.

[12] Various other possibilities for the economy to enter a disequilibrium regime arise, see e.g. Benassy (1982, 1986). Note, however, that the introduction of some form of government intervention, e.g. introduction of subsidies or tariffs, which could lead the economy into a disequilibrium regime, in general affects the dynamic behavior of the economy. Such policy measures should therefore be carefully studied on a case-by-case basis. This is not the purpose of the present paper.

3.1 The unemployment regime

In the unemployment regime the wage rate is too high, $w > w^*(k)$, to offer everyone a job, i.e. $L^d < L^s$. The producers are on their demand curve for labor, employing people up to the point where the marginal product of labor equals the wage rate.[13] Wages determine the amount of labor hired and therefore the demanded capital–labor ratio (k^d). If both goods are to be produced the ratio of marginal productivities in each sector must be equal and the domestic price is given by $p(\omega)$. The economy, however, is a price taker in the output market, so we need $p(\omega) = \bar{p}$, which implies $\omega = \bar{\omega}$ and $w = \bar{w}$. The economy, therefore, only produces both goods in the unemployment regime if $w = \bar{w} > w^*(k)$. If the wage rate exceeds $\bar{w}$ and $w^*(k)$ we will specialize in the production of investment goods, good 1. The desired capital–labor ratio, k^d, will solve the equation $w = w^*(k^d) = w_1(k^d)$. Capital accumulation takes place according to $\cdot = q_1(k^d)k$. Otherwise, if $w^*(k) < w < \bar{w}$, the economy will specialize in the production of consumption goods, good 2. The desired capital–labor ratio then solves $w = w^*(k^d) = w_2(k^d)$. Capital accumulation takes place according to $k = q_2(k^d)k$. This leaves one indeterminacy to be settled. If $w = \bar{w} > w^*(k)$ the desired capital–labor ratio k^d could be anywhere in between $k_{\min}(\bar{p})$ and $k_{\max}(\bar{p})$ and we can choose from a continuum of equilibria.[14] Any of these k^d would exceed k. We could, as a matter of convention, pick an arbitrary k^d between $k_{\min}(\bar{p})$ and $k_{\max}(\bar{p})$. However, we do not have to make an *ad hoc* choice on this boundary set of measure zero as it will not affect the dynamic behavior of the economy studied in Section 4 (the Filippov solution ignores vectors of direction on an arbitrary set of measure zero; this is precisely one of the nice properties of the Filippov solution).

3.2 The excess-demand regime

In the excess-demand regime labor demand exceeds labor supply, $L^d > L^s$, because the wage rate is below the market clearing level, $w < w^*(k)$. The producers are not on their demand curve for labor, causing the marginal product of labor to exceed the wage rate. This, and the assumption of linear homogeneity enables the firms to make a profit per worker, π, equal to the difference between the value marginal product per worker and the actual wage rate paid. As explained in the introduction we will focus attention on an efficient rationing scheme that equalizes the value marginal product of labor between the two sectors if both goods are produced up to the point where the economy allocates all workers to one sector (at $k_{\min}(\bar{p})$ and $k_{\max}(\bar{p})$), after which complete specialization occurs. This ensures that the economy produces

[13] A static version of this where the real wage is the minimum wage (due to custom, law or labor union) is investigated in Brecher (1974). For the dynamic implications of his model see Section 5.
[14] Something similar holds in excess-demand.

according to its comparative advantage, i.e.

$$w < w^*(k) \text{ and } 0 < k < k_{\min}(\bar{p}) \Rightarrow \text{excess-demand; produce good 2}$$

$$w < w^*(k) \text{ and } k_{\min}(\bar{p}) \leqslant k \leqslant k_{\max}(\bar{p}) \Rightarrow \text{excess-demand; produce both goods}$$

$$w < w^*(k) \text{ and } k_{\max}(\bar{p}) < k \Rightarrow \text{excess-demand; produce good 1}$$

We will investigate two benchmark cases of profit distribution, (*i*) profits go to the laborers or (*ii*) profits go to the capital-owners, with s_w and s_r as their respective savings rates. Investment in capital is given by

$$y_1 = s_w w + s_r rk + s_\pi \pi, \text{ for } s_\pi = s_r, s_w \tag{12}$$

Suppose both goods are produced, then the ratio of marginal productivities in both sectors must be equal, that is $\omega_1 = \omega_2$. But both factors of production are completely utilized, while we must compete on the world market, hence $p(\omega) = \bar{p}$. This implies $\omega_1 = \omega_2 = \bar{\omega}$, and hence $k_1 = \bar{k}_1$ and $k_2 = \bar{k}_2$. In the region of incomplete specialization, then, l_1 and l_2 must be chosen such that $k = \bar{k}_1 l_1 + \bar{k}_2 l_2$. This, in turn, means that both sectors produce the same amount of investment goods and consumption goods in the excess-demand regime as in the concomitant full employment regime and that k_i and l_i are independent of w. This obviously also holds for the two regions of specialization in the excess-demand regime. It is only the distribution of income between the laborers and the capital-owner which distinguishes the full employment regime from the excess demand regime. This contrasts with efficient rationing in a closed economy where there are both distribution and production effects, see van Marrewijk and Verbeek (forthcoming). Income distribution affects the capital accumulation process, i.e.

$$\dot{k} = s_w w + s_r rk + s_\pi \pi - (\mu + n)k \tag{13}$$

Benchmark case. Laborers receive the profits. If the laborers receive the profits, $s_\pi = s_w$. Profits per worker equal the value of the marginal product of labor, v_w say, minus the wage rate paid. If good 1 is produced $v_w = f_1(k) - kf'_1(k)$, if good 2 is produced $v_w = \bar{p}[f_2(k) - kf'_2(k)]$, while if both goods are produced $v_w = f_1(\bar{k}_1) - \bar{k}_1 f'_1(\bar{k}_1) = \bar{p}[f_2(\bar{k}_2) - \bar{k}_2 f'_2[\bar{k}_2)]$. In any case, given k, the profits are given by $\pi = v_w - w$ and are decreasing in w. We get

$$\dot{k} = s_w w + s_r rk + s_w(v_w - w) - (\mu + n)k = s_w v_w + s_r rk - (\mu + n)k \tag{13'}$$

Equation (13′) is the same capital accumulation equation as in the full employment regime (equations 10, 10′ and 10″). This is easy to understand as distributing profits (generated by not paying laborers the value marginal product of labor) to laborers amounts to a roundabout way of paying them the value marginal product of labor. It follows from the analysis in Section 2, therefore, that if $\sigma_1 \geqslant 1$ and $\sigma_2 \geqslant 1$, then the $\dot{k} = 0$ curve is a vertical line at k^*, see Fig. 2.

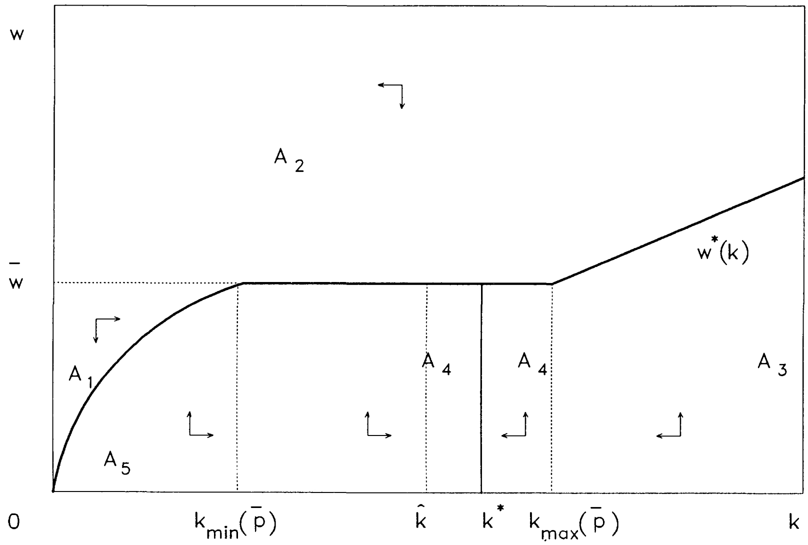

FIG. 2. A complete phase diagram if laborers receive the profits. The areas A_i are defined in Section 4

Distribution effects. Capital-owners receive the profits. If the profits go to the capital-owners capital accumulates according to

$$\dot{k} = s_w w + s_r r k + s_r(v_w - w) - (\mu + n)k \tag{13''}$$

Let $w = \phi(k)$ give the combinations of k and w in the excess demand regime for which the capital-labor ratio does not change, that is for which k in equation (13″) equals zero. That ϕ is indeed a function for its domain will become apparent in the sequel. In the region of incomplete specialization both $r = \bar{r}$ and $v_w = \bar{w}$ in equation (13″) are constant, hence $w = \phi(k)$ is a straight line with positive slope if laborers save more than capital-owner ($s_w > s_r$) and negative slope if they save less ($s_w < s_r$). Some inspection and a trite calculation (see Ito, 1980) shows that $w = \phi(k)$ is continuous, but not differentiable, at $k_{\min}(\bar{p})$ (if $s_w > s_r$) or at $k_{\max}(\bar{p})$ (if $s_w > s_r$), provided ϕ is defined there. Figure 3a (3b) illustrates the above if laborers save more (less) than capital owners.

4. Stability

The 'usual' analysis for stability of a system of differential equations cannot be applied here without restrictions and modifications. This is caused, as already mentioned in the introduction, by the fact that the system consists of different sets of differential equations with the possibility of switching regimes. On the boundary of two regimes, which has Lebesgue measure zero, there is a

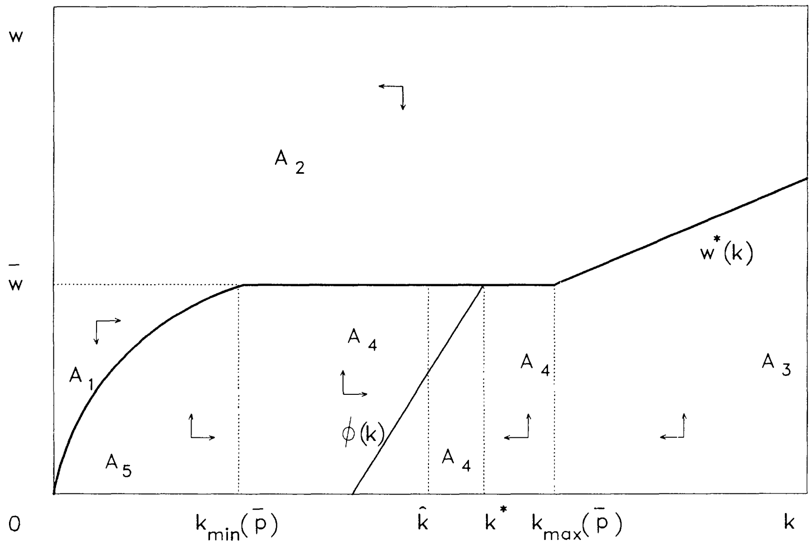

FIG. 3*a*. A complete phase diagram if the capital-owners receive the profits and save less ($s_r < s_w$) than laborers

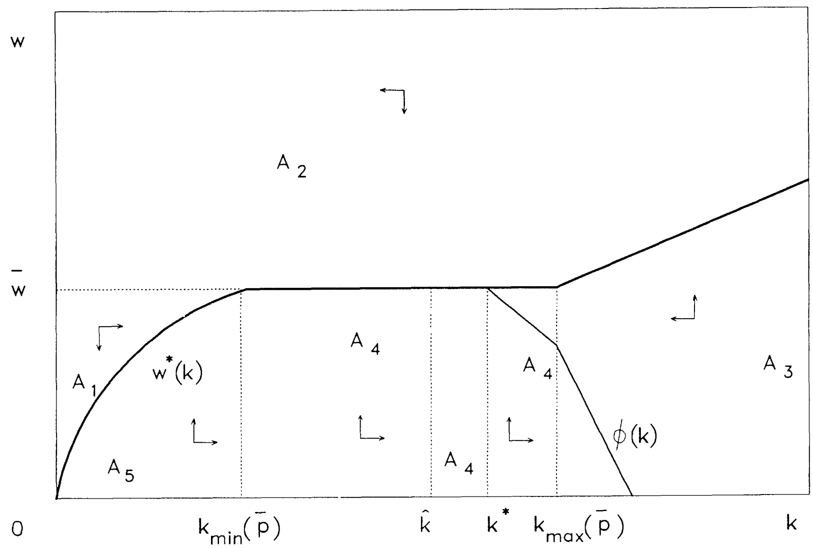

FIG. 3*b*. A complete phase diagram if the capital-owners receive the profits and save more ($s_r > s_w$) than laborers

discontinuity in the differential equations.[15] Here we follow Honkapohja and Ito (1983) and Ito (1980) by employing the Filippov solution, see Filippov (1960), which ignores vectors of direction on an arbitrary set of measure zero near a point of discontinuity.[16] There are five (open) areas with positive Lebesgue measure

$$A_1 \equiv \{(k, w) \in R_+^2 \mid w^*(k) < w < \bar{w}\}$$

$$A_2 \equiv \{(k, w) \in R_+^2 \mid w > w^*(k) \text{ and } w > \bar{w}\}$$

$$A_3 \equiv \{(k, w) \in R_+^2 \mid w < w^*(k) \text{ and } k_{\max}(\bar{p}) < k\}$$

$$A_4 \equiv \{(k, w) \in R_+^2 \mid w < w^*(k) \text{ and } k_{\min}(\bar{p}) < k < k_{\max}(\bar{p})\}$$

$$A_5 \equiv \{(k, w) \in R_+^2 \mid w < w^*(k) \text{ and } k < k_{\min}(\bar{p})\}$$

on which the differential equations are continuously defined. Let them be denoted $\theta_j(k, w)$ and $\eta_j(k, w)$, $j = 1, \ldots, 5$, for the wage rate and the capital-labor ratio respectively, that is

$$\begin{bmatrix} \dot{k} \\ \dot{w} \end{bmatrix} = \begin{bmatrix} \eta_j(k, w) \\ \theta_j(k, w) \end{bmatrix}, \text{ for } (k, w) \in A_j, j = 1, \ldots, 5. \tag{14}$$

A_1 is the unemployment area where the economy produces good 2

A_2 is the unemployment area where the economy produces good 1

A_3 is the excess-demand area where the economy produces good 1

A_4 is the excess-demand area where the economy produces both goods

A_5 is the excess-demand area where the economy produces good 2

The location of the areas $A_1, \ldots, A_5$ is indicated in Figs 2, 3a and 3b. The Filippov solution to system (14) ensures that trajectories will cross the boundaries between two areas, say between A_1 and A_5, if the vectors of direction both point into the same area. This is true for all boundaries, as the reader may verify, except for the boundary between A_2 and A_4 which is given by $w = \bar{w}$ for $k_{\min}(\bar{p}) < k < k_{\max}(\bar{p})$. On this boundary vectors of direction from A_2 point into A_4, whereas those from A_4 point into A_2. The boundary then becomes a 'sliding trajectory', see Filippov (1960), that is the solution does not leave the boundary but it continues in a certain way. This is illustrated in Fig. 4. Point M represents an arbitrary point on the boundary. The vector a_2 gives the limit of the direction of change if we approach point M from the unemployment area A_2, whereas the vector a_4 gives the limit of the direction of change if we approach M from the excess-demand area A_4. The endpoints of the two vectors are connected by a dotted line. If we are at point M, then, we know from the

[15] For a general introduction into measurement theory see Royden (1968).

[16] It is possible that the 'patched up' system is unstable even though the systems themselves are stable. For an example see Honkapohja and Ito (1983).

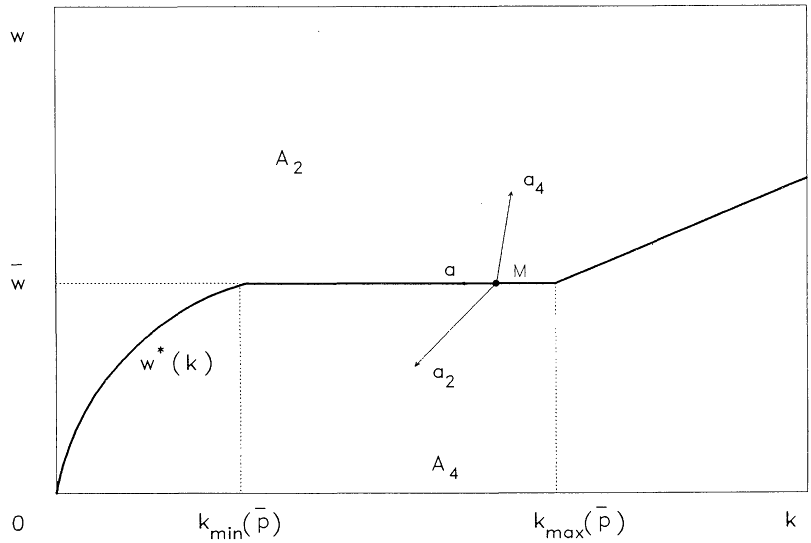

FIG. 4. The behavior of a point M on the boundary between the areas A_2 and A_4 according to the Filippov solution

Filippov solution that (i) the trajectory does not leave the boundary and (ii) the endpoint of the direction of change from point M must be on the dotted line. This leaves one soluton, namely vector a in Fig. 4, where we move to the left on the boundary between A_2 and A_4. We are now in a position to give theorem 1 which holds both for complete and incomplete specialization in the steady state.

Theorem 1. System (14) is globally asymptotically stable if $\sigma_1 \geqslant 1$, $\sigma_2 \geqslant 1$ and

(i) $s_\pi = s_w$ (labourers receive the profits), or
(ii) $s_\pi = s_r$ and $s_w > s_r$ (capital-owners receive the profits and save less than laborers).

Proof. The situation is illustrated in Fig. 2 (hypothesis i) and 3b (hypothesis ii). The function $\phi(k)$ is, because of hypothesis (ii), upward sloping.[17] Once a trajectory reaches the excess-demand regime from the left or above the solution path gets 'locked in' between the $w^*(k)$ curve and the $\phi(k)$ curve and will move to the steady state over time. It then suffices to prove that the disequilibrium growth path does not reach the vertical axis if $w > \bar{w}$ in the unemployment regime. Sufficient conditions, see Ito (1978), derived from

[17] Under hypothesis (i) $\dot{k} = 0$ in excess demand is given by a vertical line from $(k^*, 0)$ to $(k^*, \bar{w})$. The arguments given in the text still hold.

Olech's stability conditions are (for $w > \bar{w}$)

(i) $\partial\eta_2/\partial k - \eta_2(k, w)/k + \partial\eta_2/\partial w < 0$
(ii) $(\partial\eta_2/\partial k - \eta_2(k, w)/k)\, \partial\theta_2/\partial w - (\partial\eta_2/\partial w)(\partial\theta_2/\partial k) > 0$
(iii) $(\partial\eta_2/\partial w)(\partial\theta_2/\partial k) \neq 0$

Now, $\eta_2(k, w) = q_1(k^d(w))k$, therefore $\partial\eta_2(k, w)/\partial k - \eta_2(k, w)/k = 0$ and $\partial\eta_2(k, w)/\partial w = q_1'(k^d(w))/w^{*\prime}(k^d(w)) < 0$, so condition (i) is satisfied provided $\sigma_1 \geqslant 1$. Condition (ii) reduces to $-(\partial\eta_2/\partial w)(\partial\theta_2/\partial k) > 0$. Since $\partial\theta_2/\partial k = \xi_U/k > 0$, and $\partial\eta_2/\partial w < 0$, see above, condition (ii) is satisfied. It is straightforward to see that condition (iii) is satisfied, because both terms are non-zero.

Under the conditions of theorem 1 global stability is assured, but it is important to note that the Filippov steady state may be different from the neoclassical steady state. Hence the existence of short-run disequilibrium in the labor market and sluggish real wage adjustment can have long-run effects. To be precise, theorem 2 shows that this holds (does not hold) if the neoclassical economy incompletely (completely) specializes. The divergence between the neoclassical and Filippov steady state is caused by the Filippov trajectory at a point on the boundary of A_2 and A_4 being a weighted average of the directions of change from the unemployment and excess-demand areas, whereas the neoclassical trajectory is only determined by the capital accumulation equation, $\dot{k} = q_0(k)k$.

Theorem 2. Let $\hat{k}$ be the Filippov steady state capital–labor ratio and k^* the neoclassical steady state capital–labor ratio, then

(i) $k_{\min}(\bar{p}) < k^* < k_{\max}(\bar{p})$ implies $k_{\min}(\bar{p}) < \hat{k} < k^*$, else
(ii) $k^* = \hat{k}$

Proof. As indicated before we only prove part (i) and leave part (ii) to the reader. Suppose, then, that k^* is in the region of incomplete specialization, that is $k_{\min}(\bar{p}) < k^* < k_{\min}(\bar{p})$, see Fig. 3a. At the point $(k^*, \bar{w})$ we have $\eta_4(k^*, \bar{w}) = 0$. Hence a construction of the direction of change along the lines of Fig. 4 at this point leads us to the left. Similarly, a point on the boundary between A_2 and A_4 close to $(k_{\min}(\bar{p}), \bar{w})$ leads us to the right. The Filippov steady state k must be somewhere between $k_{\min}(\bar{p})$ and k^* and is easily seen to be unique.

Global stability is ensured if laborers receive the profits, or if capital-owners receive the profits and save less than laborers. What can we say if capital-owners receive the profits and save more than the laborers?

Theorem 3a. System (14) is locally asymptotically stable if the steady state is in a region of specialization, $s_\pi = s_r > s_w$, $\sigma_1 \geqslant 1$ and $\sigma_2 \geqslant 1$.

Proof. See Ito (1980).

A somewhat stronger result can be obtained if the steady state is in the region of incomplete specialization.

Theorem 3b. Suppose k^* is in the region of incomplete specialization, $s_\pi = s_r > s_w$, $\sigma_1 \geqslant 1$ and $\sigma_2 \geqslant 1$. Then system (14) is globally asymptotically stable, approaching $\hat{k}$, if

(i) $\phi(k) = 0$ for some $k < k_{\max}(\bar{p})$, or
(ii) if for some point in time $(k, w) \in A_4$, while $w \geqslant \phi(k_{\max}(\bar{p}))$.

Proof. Condition (ii) is illustrated in Fig. 3b. If the trajectory at some point in time reaches the rectangle, R_1 say, with corners $(k_{\min}(\bar{p}), \phi(k_{\max}(\bar{p})))$, $(k_{\min}(\bar{p}), \bar{w})$, $(k_{\max}(\bar{p}), \bar{w})$ and $(k_{\max}(\bar{p}), \phi(k_{\max}(\bar{p})))$, we can again use the 'locked in' argument from the proof of theorem 1. Under condition (i) this rectangle will always be reached.

Here $\phi(k_{\max}(\bar{p}))$ is understood to be zero if it is not defined. The only possibility for instability is caused by the solution trajectory following circular-like motions around the rectangle described in theorem 3b. The reader may wonder whether it is possible for the trajectories to go to infinity if $s_\pi = s_r > s_w$. It is not. The function ϕ is then downward sloping and reaches the k-axis at a certain point, k' say. Let the rectangle R_2 have corners $(0, 0)$, $(k', 0)$, $(k', w^*(k'))$ and $(0, w^*(k'))$. Careful examination of the various possibilities and realizing that the 'crossing' of two trajectories is impossible[18] leads to the conclusion that we will ultimately reach and not leave rectangle R_2. This could be termed 'box stability'.

5. Some implications

The framework developed in the previous sections can be used to investigate several implications, for example: long-run real wage rigidity, a change in the terms of trade, a tariff on the import good, a subsidy on the export good, the possibility of business cycles, a difference in technological progress between the home country and the rest of the world, etc. In this section we will discuss the first two options above and leave the other subjects for future research (see also footnote 13). In line with the exposition in the previous sections, we take $k_1(\omega) > k_2(\omega)$, that is investment goods are relatively capital intensive, and assume that the neoclassical steady state, and therefore the Filippov steady state, is in the region of incomplete specialization.

[18] If two trajectories starting at two different points (not the stationary state) were to cross at the point P, then starting at the point P we would have two solutions, which is not possible since the Filippov solution is unique.

5.1 *Real wage rigidity*

Any point in Figs 2, 3a and 3b is viewed as a quantity constrained equilibrium given the fixed real wage in the short run. It is interesting to see what happens if the wages are fixed in the long run as well as in the short run, say due to government intervention or strong unions. Only the capital–labor ratio can adjust over time. Suppose the real wage is fixed at too high a level, i.e. $w(t) = \bar{\bar{w}} > \bar{w}$ for all t. The first thing to note is that, starting from any initial capital–labor ratio, this implies that the economy completely specializes in the production of one good (in this case the investment good). Secondly, capital will decumulate indefinitely over time (unemployment will increase) until complete decapitalization occurs. In this respect it is a dangerous and rather catastrophic policy. On the other hand, suppose the real wage is fixed at too low a level, i.e. $w(t) = \bar{\bar{w}} < \bar{w}$ for all t. Then, starting from any initial capital–labor ratio, we will move over time (possibly starting in unemployment) into the excess-demand regime until the capital–labor ratio does not change, i.e. until $k = \phi^{-1}(\bar{\bar{w}})$. Hence the long-term steady state is given by $(\phi^{-1}(\bar{\bar{w}}), \bar{\bar{w}})$ and the long-run degree of capital accumulation depends on (i) whether laborers or capital-owners get the profits and (ii) whether laborers save more or less than capital-owners. An interesting special case arises when the real wage is set at the long-run equilibrium 'prematurely', i.e. $w(t) = \bar{w}$ for all t and $k(0) \leqslant k_{\min}(\bar{p})$. It then follows that the economy is stuck at the current capital per capita and there is a continuum of quantity constrained (underemployed) equilibria. This is explicitly stated in theorem 4 (if the neoclassical steady state is at complete specialization substitute k^* for $k_{\min}(\bar{p})$).

Theorem 4. There are unemployment long-run equilibria if the real wage rate is fixed at the steady state rate and if the capital per capita is below $k_{\min}(\bar{p})$.

5.2 *A change in the terms of trade*

So far, we have kept the world price $\bar{p}$ fixed for the period under investigation. Now, we discuss the consequences of a change in the terms of trade, if capital-owners receive the profits and save more than the laborers. The situation is illustrated in Fig. 5. Initially, we are at the Filippov equilibrium S_0, where we produce both goods. Suppose, then, that the world relative price of consumption goods decreases, that is $\bar{p}$ decreases. This causes a fall in the ratio of marginal productivities where we produce both goods $\bar{\omega}$ (see Section 2 and Fig. 1), and hence a fall in the wage rate $\bar{w}$, so the boundary between regions A_2 and A_4 (see Fig. 3b) moves down. The fall in $\bar{p}$ also causes a decrease in the minimum and maximum capital-labor ratios $k_{\min}(\bar{p})$ and $k_{\max}(\bar{p})$ in between which both goods are produced in the excess-demand and full employment regime. Finally, the fall in $\bar{p}$ causes a downward rotation of the $w_2(k)$ curve. All of this is illustrated in Fig. 5, where the new Filippov steady state is at S_1. The change in the terms of trade, then, causes the economy to end up in a

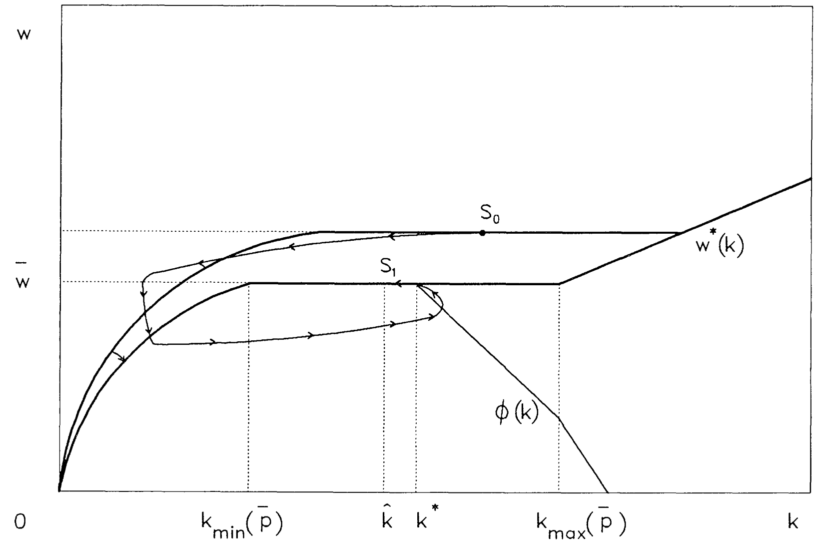

FIG. 5. Adjustment path of the Filippov solution form initial equilibrium S_0 to the new equilibrium S_1 after a change in the terms of trade (a decrease in the world price of consumption goods)

state of disequilibrium in the labor market at the initial equilibrium point S_0, due to sluggish real wage rate adjustment. A possible (k, w)-trajectory from S_0 to S_1 is drawn in Fig. 5. We distinguish several adjustment phases.[19]

(i) First, we specialize in the production of investment goods. There is unemployment in the economy. Both the wage rate and the capital–labor ratio are decreasing in this stage of the adjustment process.

(ii) Second, we specialize in the production of consumption goods. There is still unemployment in the economy and the wage rate is still declining, but the capital–labor ratio starts to increase.

(iii) Third, we still specialize in the production of consumption goods and the capital–labor ratio is still increasing, but the economy is now in excess-demand, so the wage rate starts to rise.

(iv) Fourth, we start to produce both goods with rising capital–labor ratio, rising wage rate and excess-demand.

(v) Fifth, both goods are produced in excess-demand, so the wage rate rises, but the capital–labor ratio starts to decline.

(vi) Sixth, we have reached full employment, so the wage rate does not change and we produce both goods. We are moving along the 'sliding trajectory' (between areas A_2 and A_4) to the left, with decreasing capital–labor ratio, until the new Filippov steady state at S_1 is reached.

[19] Other trajectories with different adjustment phases are of course also possible. The parameters ζ_E and ζ_U play a crucial role here.

In this example the adjustment process is quite intricate, with repeated overshooting, going both through a phase of unemployment and excess-demand while specializing in turn in the production of investment goods and consumption goods to finally end up producing again both goods. Other possible adjustment paths may be somewhat simpler, but it is clear that the model can explain all the phenomena mentioned above by a simple change in the terms of trade. The analysis can, of course, be replicated for various other cases. For example, if we specialize in the production of one of the goods in the steady state, or if consumption goods are capital intensive, or if the world price of consumption goods increases, or if laborers save more than capital-owners, etc.

6. Conclusions

We examine a two-good small open economy characterized by sluggish real wage rate adjustment. This causes short-run unemployment or excess-demand on the labor market, which in turn affects the speed of capital accumulation. Global stability is ensured if laborers receive the profits or if capital-owners receive the profits and save less than laborers. Otherwise, if capital-owners receive the profits and save more than laborers, local stability and 'box' stability hold but endless business cycles cannot be excluded. Both short-run disequilibrium ('sliding trajectory') and income redistribution (in conjunction with sticky real wages) may have long-run effects, i.e. may affect the long-run capital–labor ratio. In excess-demand an efficient rationing scheme is used and short-run disequilibrium has, in contrast to a two-sector closed economy, only income distribution and no production effects. The model is applied to examine the dynamic implications of a change in the terms of trade and of real wage rigidity. Fixed real wages may cause complete specialization and complete decapitalization over time or the economy may move to a steady state that differs from the neoclassical steady state and is influenced by income redistribution and the difference in savings rates of laborers and capital-owners. Long-run underemployment equilibria are also possible. A simple change in the terms of trade will put the economy in disequilibrium and may cause several phases of unemployment and excess-demand, complete specialization in different goods in different periods and repeated overshooting.

* *Erasmus University Rotterdam*
† *Tinbergen Institute and World Bank*

ACKNOWLEDGEMENTS

We would like to thank, without implicating, seminar participants at the Universities of Amsterdam, Antwerp, Eindhoven, Maryland, Montreal and Rotterdam as well as two anonymous referees, an editor, Peter van Bergeijk, Andries Brandsma, Russel Cooper, Pierre Dehez, Eric Drissen, Sanjeev Goyal, Martijn Herrmann, Sheng Cheng Hu, Maarten Janssen, Marcel Peeters, Rick van der Ploeg, Jean-Marie Viaene, Casper de Vries, Claus Weddepohl and Sweder van Wijnbergen for their useful suggestions and perceptive comments. An earlier version was presented at the Econometric Society European Meeting, Cambridge 1991. This paper is part of a larger study on disequilibrium growth theory, involving *a.o.* profit distribution, different rationing schemes, sector-specific capital and

government policies. The views expressed in the paper are those of the authors and not necessarily those of the World Bank.

REFERENCES

ABEL, A. and BLANCHARD, O. (1983). 'An intertemporal model of saving and investment', *Econometrica*, **51**, 675–92.

ALOGOSKOUFIS, G. and VAN DER PLOEG, F. (1990). 'Endogenous growth and overlapping generations', working paper no. 495, Center Tilburg University.

BALL, L. and ROMER, D. (1989). 'The equilibrium and optimal timing of price changes', *Review of Economic Studies*, **56**, 179–98.

BALL, L. and ROMER, D. (1990). 'Real rigidities and the non-neutrality of money', *Review of Economic Studies*, **57**, 183–203.

BARDHAN, P. K. (1965). 'Equilibrium growth in the international economy', *Quarterly Journal of Economics*, **79**, 455–64.

BARRO, R. J. (1990). 'Government spending in a simple model of endogenous growth', *Journal of Political Economy*, **98**, s103–25.

BARRO, R. J. and GROSSMAN, H. I. (1971). 'A general disequilibrium model of income and unemployment', *American Economic Review*, **61**, 82–93.

BARRO, R. J. and GROSSMAN, H. I. (1976). *Money, employment and inflation*, Cambridge University Press, Cambridge.

BEAN, C. R., LAYARD, P. R. G. and NICKELL, S. J. (1986). 'The rise in unemployment: A multi-country study', *Economica* (supplement), **53**, s1–22.

BENASSY, J.-P. (1975). 'Neo-Keynesian disequilibrium in a monetary economy', *Review of Economic Studies*, **42**, 503–24.

BENASSY, J.-P. (1982). *The economics of market disequilibrium*, Academic Press, New York.

BENASSY, J.-P. (1986). *Macroeconomics: an introduction to the non-Walrasian approach*, Academic Press, New York.

BENHABIB, J. and JOVANOVICH, B. (1991). 'Externalities and growth accounting', *American Economic Review*, **81**, 82–113.

BLANCHARD, O. J. and KIYOTAKI, N. (1987). 'Monopolistic competition and the effects of aggregate demand', *American Economic Review*, **77**, 647–66.

BLINDER, A. S. (1991). 'Why are prices sticky? Preliminary results from an interview study', *American Economic Review*, Papers and Proceedings **81**, 89–96.

BÖHM, V. (1978). 'Disequilibrium dynamics in a simple macroeconomic model', *Journal of Economic Theory*, **17**, 179–99.

BRANSON, W. and ROTEMBERG, J. (1980). 'International adjustment with wage rigidities', *European Economic Review*, **13**, 309–32.

BRECHER, R. A. (1974). 'Minimum wage rates and the pure theory of international trade', *Quarterly Journal of Economics*, **88**, 98–116.

BURMEISTER, E. and DOBELL, A. R. (1970). *Mathematical Theories of Economic Growth*, Macmillan, London.

CLOWER, R. W. (1960). 'Keynes and the classics: a dynamical perspective', *Quarterly Journal of Economics*, **74**, 318–23.

CLOWER, R. W. (1965). 'The Keynesian counter-revolution: a theoretical appraisal', in F. H. Jahn and F. P. R. Brechling (eds), *The theory of interest rates*, Macmillan & Co., London.

COOPER, R. (1990). 'Predetermined wages and prices and the impact of expansionary government policy', *Review of Economic Studies*, **57**, 205–14.

COOPER, R. and JOHN, A. (1988). 'Coordinating coordination failures in Keynesian models', *Quarterly Journal of Economics*, **103**, 441–63.

DIXIT, A. V. (1978). 'The balance of trade in a model of temporary equilibrium with rationing', *Review of Economic Studies*, **45**, 393–404.

DORNBUSCH, R. (1980). *Open Economy Macroeconomics*, Basic Books, New York.

DRÈZE, J. H. (1975). 'Existence of an exchange equilibrium under price rigidities', *International Economic Review*, **16**, 301–20.

FILIPPOV, A. F. (1960). 'Differential equations with discontinuous right-hand side', *Matematicheskii Sbornik*, **51**, 99–128. [English translation, American Mathematical Society Translations, Series 2, **42**, 1964, 199–231].

GINSBURGH, V., HENIN, P. Y., and MICHEL, Ph. (1985). 'A dual decision approach to disequilibrium growth', *Oxford Economic Papers*, **37**, 353–61.

GROSSMAN, H. I. (1971). 'Money, interest and prices in market disequilibrium', *Journal of Political Economy*, **79**, 943–61.

GROSSMAN, H. and HELPMAN, E. (1991). 'Quality ladders and product cycles', *Quarterly Journal of Economics*, **106**, 557–86.

HAHN, F. H. and NEGISHI, T. (1962). 'A theorem on non-tatonnement stability', *Econometrica*, **30**, 463–69.

HONKAPOHJA, S. (1980). 'The employment multiplier after disequilibrium dynamics', *Scandinavian Journal of Economics*, **82**, 1–14.

HONKAPOHJA, S. and ITO, T. (1982). 'Disequilibrium dynamics with monetarist price expectations', *Economics Letters*, **9**, 69–75.

HONKAPOHJA, S. and ITO, T. (1983). 'Stability with regime switching', *Journal of Economic Theory*, **29**, 22–48.

INADA, K. (1968). 'International trade, capital accumulation and factor price equalization', *Economic Record*, **44**, 322–41.

ITO, T. (1978). 'A note on the positivity constraint in Olech's Theorem', *Journal of Economic Theory*, **17**, 312–18.

ITO, T. (1980). 'Disequilibrium growth theory', *Journal of Economic Theory*, **23**, 380–409.

KALDOR, N. (1956). 'Alternative theories of distribution', *Review of Economic Studies*, **23**, 83–100.

KEMP, M. C. (1968). 'International trade and investment in a context of growth', *Economic Record*, **44**, 211–23.

LEIJONHUFVUD, A. (1968). *On Keynesian economics and the economics of Keynes*, Oxford University Press, London.

MALINVAUD, E. (1977). *The theory of unemployment reconsidered*, Basil Blackwell, Oxford.

MANKIW, N. G., ROMER, D. and WEIL, D. N. (1992). 'A contribution to the empirics of economic growth', *Quarterly Journal of Economics*, **107**, 407–37.

MARREWIJK, C. VAN and VERBEEK, J. (1991). 'Growth, budget deficits and fiscal policies in an overlapping generations model', *Journal of Economics*, **53**, 185–203.

MARREWIJK, C. VAN, DE VRIES, C. G., and WITHAGEN, C. (1992). 'Optimal localized production experience and schooling', *International Economic Review*, **33**, 91–110.

MARREWIJK, C. VAN and VERBEEK, J. (1992). 'Two-sector disequilibrium growth', *European Journal of Political Economy*, forthcoming.

NEARY, J. P. (1990). 'Neo-Keynesian macroeconomics in an open economy', in F. van der Ploeg (ed.), *Advanced lectures in quantitative economics*, Academic Press, New York.

NEARY, J. P. and STIGLITZ, J. (1983). 'Towards a reconstruction of Keynesian economics: expectations and constrained equilibria', *Quarterly Journal of Economics*, **95**, 403–29.

NEWELL, A. and SYMONS, J. S. V. (1986). 'The Phillips curve is a real wage equation', Discussion Paper no. 246, Centre for Labour Economics, London School of Economics.

OLDERSMA, H. (1991). 'De export van kapitaalgoederen', *Economisch Statistische Berichten*, **76**, 1180–83.

ONIKI, H. and UZAWA, H. (1965). 'Patterns of Trade and Investment in a Dynamic Model of International Trade', *Review of Economic Studies*, 15–38.

PASINETTI, L. L. (1962). 'The rate of profit and income distribution in relation to the rate of economic growth', *Review of Economic Studies*, **29**, 267–79.

PICARD, P. (1983). 'Inflation and growth in a disequilibrium macroeconomic model', *Journal of Economic Theory*, **30**, 266–95.

RAMSEY, F. (1928). 'A mathematical theory of saving', *Economic Journal*, **38**, 543–59.

ROMER, P. (1986). 'Increasing returns and long run growth', *Journal of Political Economy*, **94**, 1002–37.

ROMER, P. (1990). 'Endogenous Technological Change', *Journal of Political Economy*, **98**, s71–102.

ROYDEN, H. L. (1968). *Real Analysis*, Macmillan, New York.

SCHITTKO, U. K. and ECKWERT, B. (1985). 'Dynamic aspects in a temporary equilibrium model of international trade with quantity rationing', *European Journal of Political Economy*, **1**, 189–220.

SOLOW, R. M. (1956). 'A contribution to the theory of economic growth', *Quarterly Journal of Economics*, **70**, 65–94.

UZAWA, H. (1962). 'On a two-sector model of economic growth', *Review of Economic Studies*, **29**, 40–47.

UZAWA, H. (1963). 'On a two-sector model of economic growth II', *Review of Economic Studies*, **30**, 105–18.

WIJNBERGEN, S. VAN (1985). 'Oil price shocks, private investment, employment and the current account: an intertemporal disequilibrium analysis', *Review of Economic Studies*, **52**, 627–45.

WIJNBERGEN, S. VAN (1987). 'Government deficits, private investment and the current account: an intertemporal disequilibrium analysis', *The Economic Journal*, **97**, 596–615.