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Research Article

$\Upsilon(nS) \to B_c^* \pi, B_c^* K$ Decays with Perturbative QCD Approach

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Besides the traditional strong and electromagnetic decay modes, $\Upsilon(nS)$ meson can also decay through the weak interactions within the standard model of elementary particle. With anticipation of copious $\Upsilon(nS)$ data samples at the running LHC and coming SuperKEKB experiments, the two-body nonleptonic bottom-changing $\Upsilon(nS) \to B_c^* \pi, B_c^* K$ decays ($n = 1, 2, 3$) are investigated with perturbative QCD approach firstly. The absolute branching ratios for $\Upsilon(nS) \to B_c^* \pi$ and $B_c^* K$ decays are estimated to reach up to about 10^{-10} and 10^{-11}, respectively, which might possibly be measured by the future experiments.

1. Introduction

The upsilon $\Upsilon(nS)$ meson is the spin-triplet S-wave state of bottomonium (bound state consisting of bottom quark b and antibottom quark $\bar{b}$) with well-established quantum number of $I^G J^{PC} = 0^- 1^{--}$ [1]. The characteristic narrow decay widths of $\Upsilon(nS)$ mesons for $n = 1$, 2, and 3 provide insight into the study of strong interactions (see Table 1, and note that, for simplicity, $\Upsilon(nS)$ will denote $\Upsilon(1S)$, $\Upsilon(2S)$, and $\Upsilon(3S)$ mesons in the following content if not specified definitely). The mass of $\Upsilon(nS)$ meson is below B meson pair threshold. $\Upsilon(nS)$ meson decays into bottomed hadrons through strong and electromagnetic interactions are forbidden by the law of conservation of flavor number. The bottom-changing $\Upsilon(nS)$ decays can occur only via the weak interactions within the standard model, although with tiny incidence probability. Both constituent quarks of upsilons can decay individually, which provide an alternative system for investigating the weak decay of heavy-flavored hadrons. In this paper, we will study the nonleptonic $\Upsilon(nS) \to B_c^* P$ ($P = \pi$ and K) weak decays with perturbative QCD (pQCD) approach [2–4].

Experimentally, (1) over 10^8 $\Upsilon(nS)$ data samples have been accumulated at Belle and BaBar experiments [5]. More and more upsilon data samples will be collected at the running hadron collider LHC and the forthcoming e^+e^- collider SuperKEKB (the SuperKEKB has started commissioning test run (http://www.kek.jp/en/NewsRoom/Release/)). There seems to exist a realistic possibility to explore $\Upsilon(nS)$ weak decay at future experiments. (2) Signals of $\Upsilon(nS) \to B_c^* \pi, B_c^* K$ decays should be easily distinguished with "charge tag" technique, due to the facts that the back-to-back final states with different electric charges have definite momentum and energy in the rest frame of $\Upsilon(nS)$ meson. (3) B_c^* meson has not been observed experimentally by now. B_c^* meson production via the strong interaction is suppressed due to the simultaneous presence of two heavy quarks with different flavors and higher order in QCD coupling constant α_s. $\Upsilon(nS) \to B_c^* \pi$, $B_c^* K$ decays provide a novel pattern to study B_c^* meson production. The identification of a single explicitly flavored B_c^* meson could be used as an effective selection criterion to detect upsilon weak decays. Moreover, the radiative decay of B_c^* meson provides a useful extra signal and a powerful constraint (the investigation on the radiative decay of B_c^* meson can be found in, e.g., [6], with QCD sum rules). Of course, any discernible evidences of an anomalous production rate of single bottomed meson from upsilon decays might be a hint of new physics.

Theoretically, many attractive QCD-inspired methods have been developed recently to describe the exclusive

Table 1: Summary of mass, decay width, on(off)-peak luminosity, and numbers of $\Upsilon(nS)$.

Meson	Properties [1]		Luminosity (fb^{-1}) [5]		Numbers (10^6) [5]	
	Mass (MeV)	Width (keV)	Belle	BaBar	Belle	BaBar
$\Upsilon(1S)$	9460.30 ± 0.26	54.02 ± 1.25	5.7 (1.8)	$\cdots$	102 ± 2	$\cdots$
$\Upsilon(2S)$	10023.26 ± 0.31	31.98 ± 2.63	24.9 (1.7)	13.6 (1.4)	158 ± 4	98.3 ± 0.9
$\Upsilon(3S)$	10355.2 ± 0.5	20.32 ± 1.85	2.9 (0.2)	28.0 (2.6)	11 ± 0.3	121.3 ± 1.2

nonleptonic decay of heavy-flavored mesons, such as the pQCD approach [2–4], the QCD factorization approach [7–9], and soft and collinear effective theory [10–13], and have been applied widely to vindicate measurements on B meson decays. The upsilon weak decay permits one to further constrain parameters obtained from B meson decay, and cross comparisons provide an opportunity to test various phenomenological models. The upsilon weak decay possesses a unique structure due to the Cabibbo-Kobayashi-Maskawa (CKM) matrix properties which predicts that the channels with one $B_c^{(*)}$ meson are dominant. $\Upsilon(nS) \to B_c^* P$ decay belongs to the favorable $b \to c$ transition, which should, in principle, have relatively large branching ratio among upsilon weak decays. However, there is still no theoretical study devoted to $\Upsilon(nS) \to B_c^* P$ decay for the moment. In this paper, we will present a phenomenological investigation on $\Upsilon(nS) \to B_c^* P$ weak decay with the pQCD approach to supply a ready reference for the future experiments.

This paper is organized as follows. Section 2 focuses on theoretical framework and decay amplitudes for $\Upsilon(nS) \to B_c^*\pi, B_c^* K$ weak decays. Section 3 is devoted to numerical results and discussion. The last section is a summary.

2. Theoretical Framework

2.1. The Effective Hamiltonian. Theoretically, $\Upsilon(nS) \to B_c^*\pi$, $B_c^* K$ weak decays are described by an effective bottom-changing Hamiltonian based on operator product expansion [19]:

$$\mathscr{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{q=d,s} V_{cb} V_{uq}^* \left\{ C_1(\mu) O_1(\mu) + C_2(\mu) O_2(\mu) \right\} + \text{h.c.}, \tag{1}$$

where $G_F \simeq 1.166 \times 10^{-5}\,\mathrm{GeV}^{-2}$ [1] is the Fermi coupling constant; the CKM factors $V_{cb}V_{ud}^*$ and $V_{cb}V_{us}^*$ correspond to $\Upsilon(nS) \to B_c^*\pi$ and $B_c^* K$ decays, respectively; with the Wolfenstein parameterization, the CKM factors are expanded as a power series in a small Wolfenstein parameter $\lambda \sim 0.2$ [1]:

$$\begin{aligned} V_{cb}V_{ud}^* &= A\lambda^2 - \frac{1}{2}A\lambda^4 - \frac{1}{8}A\lambda^6 + \mathcal{O}\left(\lambda^7\right), \\ V_{cb}V_{us}^* &= A\lambda^3 + \mathcal{O}\left(\lambda^7\right). \end{aligned} \tag{2}$$

The local tree operators $Q_{1,2}$ are defined as

$$\begin{aligned} O_1 &= \left[\bar{c}_\alpha \gamma_\mu (1-\gamma_5) b_\alpha\right] \left[\bar{q}_\beta \gamma^\mu (1-\gamma_5) u_\beta\right], \\ O_2 &= \left[\bar{c}_\alpha \gamma_\mu (1-\gamma_5) b_\beta\right] \left[\bar{q}_\beta \gamma^\mu (1-\gamma_5) u_\alpha\right], \end{aligned} \tag{3}$$

where α and β are color indices and the sum over repeated indices is understood.

The scale μ factorizes physics contributions into short- and long-distance dynamics. The Wilson coefficients $C_i(\mu)$ summarize the physics contributions at scale higher than μ and are calculable with the renormalization group improved perturbation theory. The hadronic matrix elements (HME), where the local operators are inserted between initial and final hadron states, embrace the physics contributions below scale of μ. To obtain decay amplitudes, the remaining work is to calculate HME properly by separating from perturbative and nonperturbative contributions.

2.2. Hadronic Matrix Elements. Based on Lepage-Brodsky approach for exclusive processes [20], HME is commonly expressed as a convolution integral of hard scattering subamplitudes containing perturbative contributions with universal wave functions reflecting nonperturbative contributions. In order to effectively regulate endpoint singularities and provide a naturally dynamical cutoff on nonperturbative contributions, transverse momentum of valence quarks is retained and the Sudakov factor is introduced within the pQCD framework [2–4]. Phenomenologically, pQCD's decay amplitude could be divided into three parts: the Wilson coefficients C_i incorporating the hard contributions above typical scale of t, process-dependent rescattering subamplitudes T accounting for the heavy quark decay, and wave functions Φ of all participating hadrons, which is expressed as

$$\int dk C_i(t)\, T(t,k)\, \Phi(k)\, e^{-S}, \tag{4}$$

where k is the momentum of valence quarks and e^{-S} is the Sudakov factor.

2.3. Kinematic Variables. The light cone kinematic variables in $\Upsilon(nS)$ rest frame are defined as follows:

$$\begin{aligned} p_\Upsilon &= p_1 = \frac{m_1}{\sqrt{2}}(1,1,0), \\ p_{B_c^*} &= p_2 = \left(p_2^+, p_2^-, 0\right), \\ p_3 &= \left(p_3^-, p_3^+, 0\right), \\ p_i^\pm &= \frac{(E_i \pm p)}{\sqrt{2}}, \\ k_i &= x_i p_i + \left(0, 0, \vec{k}_{i\perp}\right), \\ \epsilon_1^\parallel &= \frac{p_1}{m_1} - \frac{m_1}{p_1 \cdot n_+} n_+, \end{aligned}$$

$$\begin{aligned}
\epsilon_2^{\parallel} &= \frac{p_2}{m_2} - \frac{m_2}{p_2 \cdot n_-} n_-,\\
\epsilon_{1,2}^{\perp} &= \left(0, 0, \vec{1}\right),\\
n_+ &= (1, 0, 0),\\
n_- &= (0, 1, 0),\\
s &= 2 p_2 \cdot p_3,\\
t &= 2 p_1 \cdot p_2 = 2 m_1 E_2,\\
u &= 2 p_1 \cdot p_3 = 2 m_1 E_3,\\
p &= \frac{\sqrt{\left[m_1^2 - (m_2 + m_3)^2\right]\left[m_1^2 - (m_2 - m_3)^2\right]}}{2 m_1},
\end{aligned} \tag{5}$$

where x_i and $\vec{k}_{i\perp}$ are the longitudinal momentum fraction and transverse momentum of valence quarks, respectively; $\epsilon_i^{\parallel}$ and $\epsilon_i^{\perp}$ are the longitudinal and transverse polarization vectors, respectively, and satisfy relations $\epsilon_i^2 = -1$ and $\epsilon_i \cdot p_i = 0$; the subscript i on variables p_i, E_i, m_i, and ϵ_i corresponds to participating hadrons; namely, $i = 1$ for $\Upsilon(nS)$ meson, $i = 2$ for the recoiled B_c^* meson, and $i = 3$ for the emitted pseudoscalar meson; n_+ and n_- are positive and negative null vectors, respectively; s, t, and u are the Lorentz-invariant variables; p is the common momentum of final states. The notation of momentum is displayed in Figure 2(a).

2.4. Wave Functions. With the notation in [16, 21], wave functions are defined as

$$\begin{aligned}
\left\langle 0 \left| b_i(z) \bar{b}_j(0) \right| \Upsilon\left(p_1, \epsilon_1^{\parallel}\right) \right\rangle &= \frac{f_\Upsilon}{4} \\
&\cdot \int dk_1 e^{-ik_1 \cdot z} \left\{ \not{\epsilon}_1^{\parallel} \left[m_1 \Phi_\Upsilon^v(k_1) - \not{p}_1 \Phi_\Upsilon^t(k_1) \right] \right\}_{ji},\\
\left\langle 0 \left| b_i(z) \bar{b}_j(0) \right| \Upsilon\left(p_1, \epsilon_1^{\perp}\right) \right\rangle &= \frac{f_\Upsilon}{4} \\
&\cdot \int dk_1 e^{-ik_1 \cdot z} \left\{ \not{\epsilon}_1^{\perp} \left[m_1 \Phi_\Upsilon^V(k_1) - \not{p}_1 \Phi_\Upsilon^T(k_1) \right] \right\}_{ji},\\
\left\langle B_c^*\left(p_2, \epsilon_2^{\parallel}\right) \left| \bar{c}_i(z) b_j(0) \right| 0 \right\rangle &= \frac{f_{B_c^*}}{4} \\
&\cdot \int_0^1 dk_3 e^{ik_2 \cdot z} \left\{ \not{\epsilon}_2^{\parallel} \left[m_2 \Phi_{B_c^*}^v(k_2) + \not{p}_2 \Phi_{B_c^*}^t(k_2) \right] \right\}_{ji},\\
\left\langle B_c^*\left(p_2, \epsilon_2^{\perp}\right) \left| \bar{c}_i(z) b_j(0) \right| 0 \right\rangle &= \frac{f_{B_c^*}}{4} \\
&\cdot \int_0^1 dk_2 e^{ik_2 \cdot z} \left\{ \not{\epsilon}_2^{\perp} \left[m_2 \Phi_{B_c^*}^V(k_2) + \not{p}_2 \Phi_{B_c^*}^T(k_2) \right] \right\}_{ji},\\
\left\langle P(p_3) \left| u_i(0) \bar{q}_j(z) \right| 0 \right\rangle &= \frac{i f_P}{4} \\
&\cdot \int dk_3 e^{ik_3 \cdot z} \left\{ \gamma_5 \left[\not{p}_3 \Phi_P^a(k_3) + \mu_P \Phi_P^p(k_3) \right.\right.\\
&\left.\left. + \mu_P \left(\not{n}_- \not{n}_+ - 1 \right) \Phi_P^t(k_3) \right] \right\}_{ji},
\end{aligned} \tag{6}$$

where f_Υ, $f_{B_c^*}$, and f_P are decay constants of $\Upsilon(nS)$, B_c^*, and P mesons, respectively.

Considering mass relations of $m_{\Upsilon(nS)} \simeq 2m_b$ and $m_{B_c^*} \simeq m_b + m_c$, it might assume that the motion of heavy valence quarks in $\Upsilon(nS)$ and B_c^* mesons is nearly nonrelativistic. The wave functions of $\Upsilon(nS)$ and B_c^* mesons could be approximately described with nonrelativistic quantum chromodynamics (NRQCD) [22–24] and time-independent Schrödinger equation. For an isotropic harmonic oscillator potential, the eigenfunctions of stationary state with quantum numbers nL are written as [17]

$$\begin{aligned}
\phi_{1S}\left(\vec{k}\right) &\sim e^{-\vec{k}^2/2\beta^2},\\
\phi_{2S}\left(\vec{k}\right) &\sim e^{-\vec{k}^2/2\beta^2} \left(2\vec{k}^2 - 3\beta^2\right),\\
\phi_{3S}\left(\vec{k}\right) &\sim e^{-\vec{k}^2/2\beta^2} \left(4\vec{k}^4 - 20\vec{k}^2\beta^2 + 15\beta^4\right),
\end{aligned} \tag{7}$$

where parameter β determines the average transverse momentum; that is, $\langle nS | k_\perp^2 | nS \rangle \sim \beta^2$. Employing the substitution ansatz [25],

$$\vec{k}^2 \longrightarrow \frac{1}{4} \sum_i \frac{\vec{k}_{i\perp}^2 + m_{q_i}^2}{x_i}, \tag{8}$$

where x_i and m_{q_i} are the longitudinal momentum fraction and mass of valence quark, respectively; then integrating out $\vec{k}_\perp$ and combining with their asymptotic forms, the distribution amplitudes (DAs) for $\Upsilon(nS)$ and B_c^* mesons can be written as [17]

$$\begin{aligned}
\phi_{\Upsilon(1S)}^{v,T}(x) &= A x \bar{x} \exp\left\{ -\frac{m_b^2}{8\beta_1^2 x \bar{x}} \right\},\\
\phi_{\Upsilon(1S)}^{t}(x) &= B t^2 \exp\left\{ -\frac{m_b^2}{8\beta_1^2 x \bar{x}} \right\},\\
\phi_{\Upsilon(1S)}^{V}(x) &= C \left(1 + t^2\right) \exp\left\{ -\frac{m_b^2}{8\beta_1^2 x \bar{x}} \right\},\\
\phi_{\Upsilon(2S)}^{v,t,V,T}(x) &= D \phi_{\Upsilon(1S)}^{v,t,V,T}(x) \left\{ 1 + \frac{m_b^2}{2\beta_1^2 x \bar{x}} \right\},\\
\phi_{\Upsilon(3S)}^{v,t,V,T}(x) &= E \phi_{\Upsilon(1S)}^{v,t,V,T}(x) \left\{ \left(1 - \frac{m_b^2}{2\beta_1^2 x \bar{x}} \right)^2 + 6 \right\},\\
\phi_{B_c^*}^{v,T}(x) &= F x \bar{x} \exp\left\{ -\frac{\bar{x} m_c^2 + x m_b^2}{8\beta_2^2 x \bar{x}} \right\},
\end{aligned}$$

$$\phi^t_{B_c^*}(x) = Gt^2 \exp\left\{-\frac{\bar{x}m_c^2 + xm_b^2}{8\beta_2^2 x\bar{x}}\right\},$$
$$\phi^V_{B_c^*}(x) = H\left(1-t^2\right)\exp\left\{-\frac{\bar{x}m_c^2 + xm_b^2}{8\beta_2^2 x\bar{x}}\right\}, \tag{9}$$

where $\bar{x} = 1 - x$; $t = x - \bar{x}$. According to NRQCD power counting rules [22], $\beta_i \simeq \xi_i \alpha_s(\xi_i)$ with $\xi_i = m_i/2$ and QCD coupling constant α_s. The exponential function represents $k_\perp$ distribution. Parameters of A, B, C, D, E, F, G, and H are normalization coefficients satisfying the conditions

$$\int_0^1 dx\phi^i_{\Upsilon(nS)}(x) = \int_0^1 dx\phi^i_{B_c^*}(x) = 1 \quad \text{for } i = v, t, V, T. \tag{10}$$

The shape lines of normalized DAs for $\Upsilon(nS)$ and B_c^* mesons are shown in Figure 1. It is clearly seen that (1) DAs for $\Upsilon(nS)$ and B_c^* mesons fall quickly down to zero at endpoint $x, \bar{x} \to 0$ due to suppression from exponential functions; (2) DAs for $\Upsilon(nS)$ meson are symmetric under the interchange of momentum fractions $x \leftrightarrow \bar{x}$, and DAs for B_c^* meson are basically consistent with the feature that valence quarks share momentum fractions according to their masses.

Our study shows that only the leading twist (twist-2) DAs of the emitted light pseudoscalar meson P are involved in decay amplitudes (see Appendix). The twist-2 DAs have the expansion [16],

$$\phi^a_P(x) = 6x\bar{x}\sum_{i=0} a_i C_i^{3/2}(t), \tag{11}$$

and are normalized as

$$\int_0^1 \phi^a_P(x)\,dx = 1, \tag{12}$$

where $C_i^{3/2}(t)$ are Gegenbauer polynomials:

$$C_0^{3/2}(t) = 1,$$
$$C_1^{3/2}(t) = 3t,$$
$$C_2^{3/2}(t) = \frac{3}{2}\left(5t^2 - 1\right), \tag{13}$$
$$\vdots$$

and each term corresponds to a nonperturbative Gegenbauer moment a_i; note that $a_0 = 1$ due to the normalization condition equation (12); G-parity invariance of the pion DAs requires Gegenbauer moment $a_i = 0$ for $i = 1, 3, 5, \ldots$.

2.5. Decay Amplitudes. The Feynman diagrams for $\Upsilon(nS) \to B_c^*\pi$ weak decay are shown in Figure 2. There are two types. One is factorizable emission topology where gluon attaches to quarks in the same meson, and the other is nonfactorizable emission topology where gluon connects to quarks between different mesons.

With the pQCD master formula equation (4), the amplitude for $\Upsilon(nS) \to B_c^* P$ decay can be expressed as [26]

$$\mathscr{A}\left(\Upsilon(nS) \longrightarrow B_c^* P\right) = \mathscr{A}_L\left(\epsilon_1^\parallel, \epsilon_2^\parallel\right) + \mathscr{A}_N\left(\epsilon_1^\perp, \epsilon_2^\perp\right) + i\mathscr{A}_T \epsilon_{\mu\nu\alpha\beta}\epsilon_1^\mu \epsilon_2^\nu p_1^\alpha p_2^\beta, \tag{14}$$

which is conventionally written as the helicity amplitudes [26]:

$$\mathscr{A}_0 = -C_{\mathscr{A}}\sum_j \mathscr{A}_L^j\left(\epsilon_1^\parallel, \epsilon_2^\parallel\right),$$
$$\mathscr{A}_\parallel = \sqrt{2}C_{\mathscr{A}}\sum_j \mathscr{A}_N^j\left(\epsilon_1^\perp, \epsilon_2^\perp\right),$$
$$\mathscr{A}_\perp = \sqrt{2}C_{\mathscr{A}} m_1 p \sum_j \mathscr{A}_T^j, \tag{15}$$
$$C_{\mathscr{A}} = iV_{cb}V_{uq}^* \frac{G_F}{\sqrt{2}}\frac{C_F}{N_c}\pi f_\Upsilon f_{B_c^*} f_P,$$

where $C_F = 4/3$ and the color number $N_c = 3$; the subscript i on A_i^j corresponds to three different helicity amplitudes; that is, $i = L, N, T$; the superscript j on A_i^j denotes indices of Figure 2. The explicit expressions of building blocks $\mathscr{A}_i^j$ are collected in Appendix.

3. Numerical Results and Discussion

In the center-of-mass of $\Upsilon(nS)$ meson, branching ratio $\mathscr{B}r$ for $\Upsilon(nS) \to B_c^* P$ decay is defined as

$$\mathscr{B}r = \frac{1}{12\pi}\frac{p}{m_\Upsilon^2 \Gamma_\Upsilon}\left\{\left|\mathscr{A}_0\right|^2 + \left|\mathscr{A}_\parallel\right|^2 + \left|\mathscr{A}_\perp\right|^2\right\}. \tag{16}$$

The input parameters are listed in Tables 1 and 2. If not specified explicitly, we will take their central values as the default inputs. Our numerical results are collected in Table 3, where the first uncertainty comes from scale $(1 \pm 0.1)t_i$ and the expression of t_i is given in (A.4) and (A.5); the second uncertainty is from mass of m_b and m_c; the third uncertainty is from hadronic parameters including decay constants and Gegenbauer moments; the fourth uncertainty is from CKM parameters. The followings are some comments:

(1) Branching ratio for $\Upsilon(nS) \to B_c^*\pi$ decay is about $\mathcal{O}(10^{-10})$ with pQCD approach, which is well within the measurement potential of LHC and SuperKEKB. For example, experimental studies have showed that production cross sections for $\Upsilon(nS)$ meson in p-p and p-Pb collisions are a few μb at the LHCb [27, 28] and ALICE [29, 30] detectors. Consequently, there will be more than 10^{12} $\Upsilon(nS)$ data samples per ab^{-1} data collected by the LHCb and ALICE, corresponding to a few hundreds of $\Upsilon(nS) \to B_c^*\pi$ events. Branching ratio for $\Upsilon(nS) \to B_c^* K$ decay, $\mathcal{O}(10^{-11})$, is generally

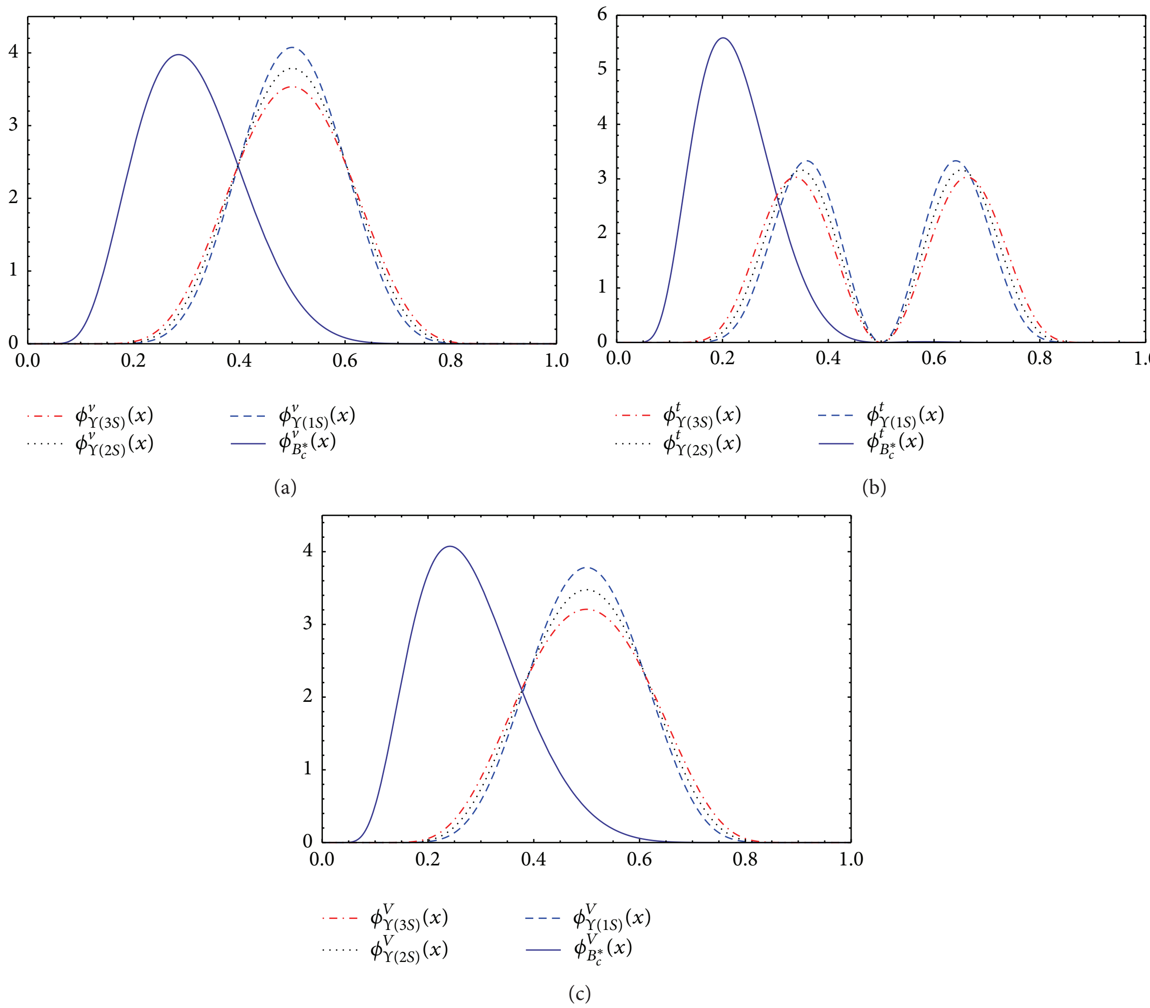

FIGURE 1: The normalized distribution amplitudes for $\Upsilon(nS)$ and B_c^* mesons.

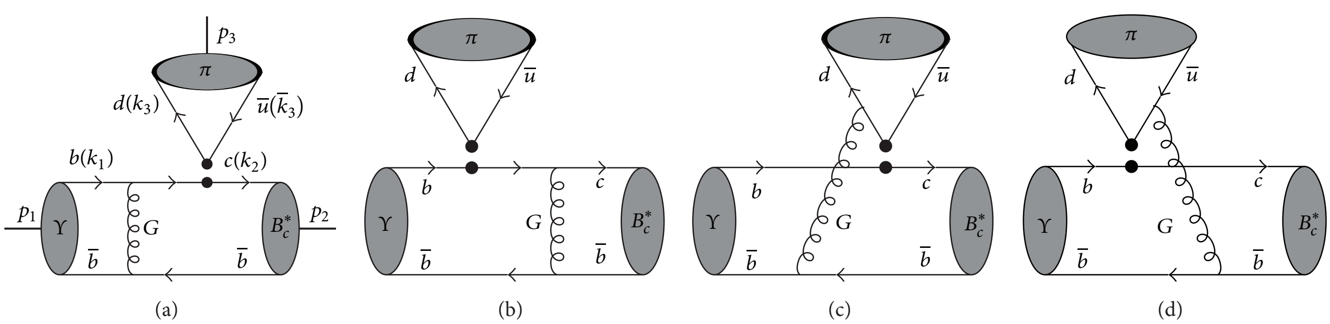

FIGURE 2: Feynman diagrams for $\Upsilon(nS) \to B_c^*\pi$ decay with the pQCD approach, including factorizable emission diagrams (a, b) and nonfactorizable emission diagrams (c, d).

less than that for $\Upsilon(nS) \to B_c^*\pi$ decay by one order of magnitude due to the CKM suppression, $|V_{us}^*/V_{ud}^*|^2 \sim \lambda^2$.

(2) As it is well known, due to the large mass of B_c^*, the momentum transition in $\Upsilon(nS) \to B_c^*P$ decay may be not large enough. One might naturally wonder whether the pQCD approach is applicable and whether the perturbative calculation is reliable. Therefore, it is necessary to check what percentage of the contributions comes from the perturbative region. The contributions to branching ratio for $\Upsilon(nS) \to B_c^*\pi$ decay from different α_s/π region are shown in Figure 3. It can be clearly seen that more than

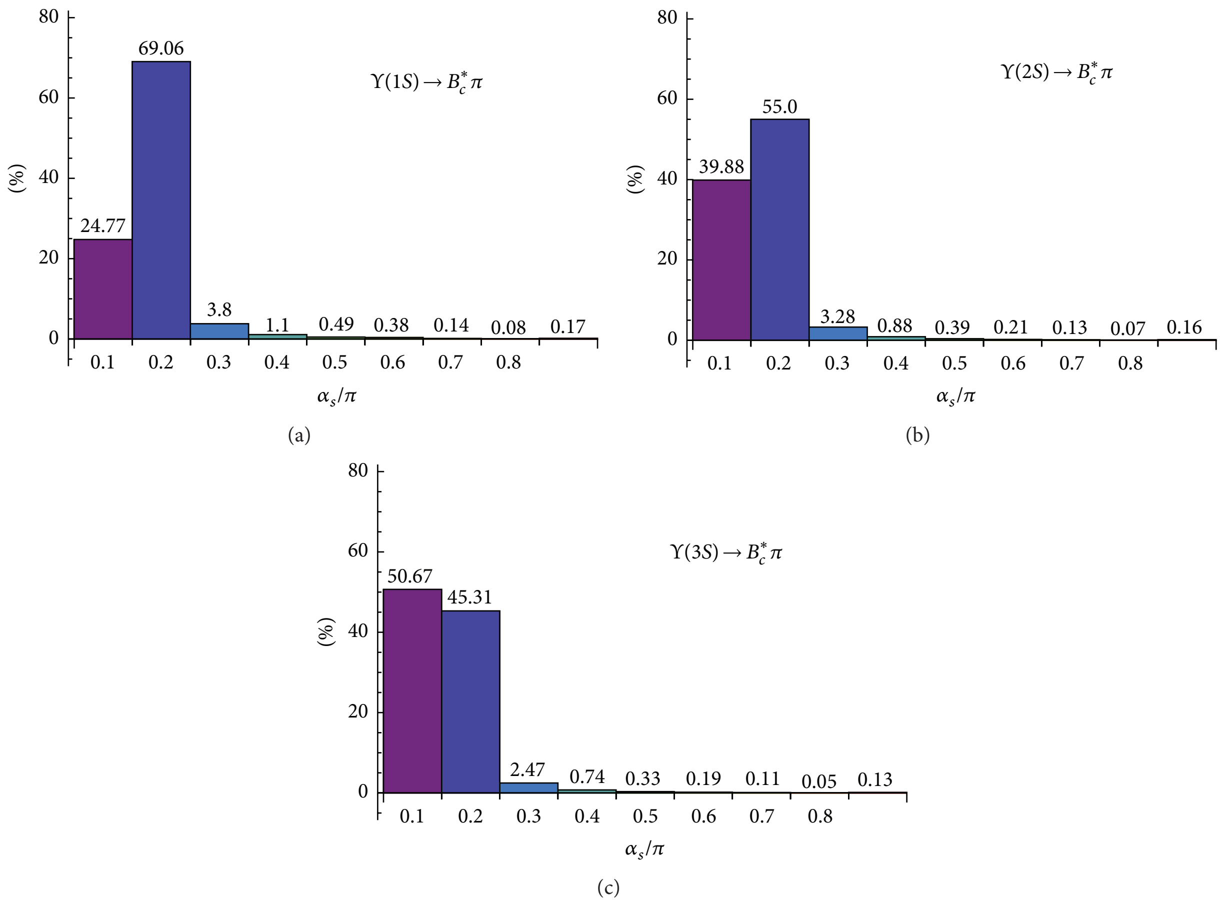

Figure 3: The contributions to branching ratios for $\Upsilon(1S) \to B_c^* \pi$ decay (a), $\Upsilon(2S) \to B_c^* \pi$ decay (b), and $\Upsilon(3S) \to B_c^* \pi$ decay (c) from different region of α_s/π (horizontal axes), where the numbers over histogram denote the percentage of the corresponding contributions.

Table 2: The numerical values of input parameters.

The Wolfenstein parameters	
$A = 0.814^{+0.023}_{-0.024}$ [1],	$\lambda = 0.22537 \pm 0.00061$ [1],
Mass, decay constant, and Gegenbauer moments	
$m_b = 4.78 \pm 0.06$ GeV [1],	$f_\pi = 130.41 \pm 0.20$ MeV [1],
$m_c = 1.67 \pm 0.07$ GeV [1],	$f_K = 156.2 \pm 0.7$ MeV [1],
$m_{B_c^*} = 6332 \pm 9$ MeV [14],	$f_{B_c^*} = 422 \pm 13$ MeV [15][c],
a_1^K (1 GeV) $= -0.06 \pm 0.03$ [16],	$f_{\Upsilon(1S)} = 676.4 \pm 10.7$ MeV [17],
a_2^K (1 GeV) $= 0.25 \pm 0.15$ [16],	$f_{\Upsilon(2S)} = 473.0 \pm 23.7$ MeV [17],
a_2^π (1 GeV) $= 0.25 \pm 0.15$ [16],	$f_{\Upsilon(3S)} = 409.5 \pm 29.4$ MeV [17].

[c]The decay constant $f_{B_c^*}$ cannot be extracted from the experimental data because of no measurement on B_c^* weak decay at the present time. Theoretically, the value of $f_{B_c^*}$ has been estimated, for example, in [18], with the QCD sum rules. From Table 3 of [18], one can see that the value of $f_{B_c^*}$ is model-dependent. In our calculation, we will take the latest value given by the lattice QCD approach [15] just to offer an order of magnitude estimation on branching ratio for $\Upsilon(nS) \to B_c^* P$ decays.

Table 3: Branching ratio for $\Upsilon(nS) \to B_c^* P$ decays.

Modes	$\Upsilon(1S) \to B_c^* \pi$	$\Upsilon(2S) \to B_c^* \pi$	$\Upsilon(3S) \to B_c^* \pi$
$10^{10} \times \mathscr{B}r$	$4.35^{+0.29+0.19+0.44+0.17}_{-0.24-0.41-0.31-0.30}$	$2.28^{+0.13+0.26+0.40+0.09}_{-0.03-0.35-0.16-0.15}$	$2.14^{+0.12+0.09+0.48+0.07}_{-0.12-0.41-0.15-0.15}$
Modes	$\Upsilon(1S) \to B_c^* K$	$\Upsilon(2S) \to B_c^* K$	$\Upsilon(3S) \to B_c^* K$
$10^{11} \times \mathscr{B}r$	$3.45^{+0.23+0.13+0.38+0.13}_{-0.21-0.35-0.27-0.25}$	$1.91^{+0.11+0.07+0.36+0.07}_{-0.09-0.31-0.15-0.14}$	$1.65^{+0.09+0.08+0.40+0.05}_{-0.21-0.33-0.13-0.12}$

93% (97%) contributions come from $\alpha_s/\pi \le 0.2$ (0.3) region, implying that $\Upsilon(nS) \to B_c^*\pi$ decay is computable with the pQCD approach. As the discussion in [2–4], there are many factors for this, for example, the choice of the typical scale, retaining the quark transverse moment and introducing the Sudakov factor to suppress the nonperturbative contributions, which deserve much attention and further investigation.

(3) Because of the relations among masses $m_{\Upsilon(3S)} > m_{\Upsilon(2S)} > m_{\Upsilon(1S)}$ resulting in the fact that phase space increases with the radial quantum number n in addition to the relations among decay widths $\Gamma_{\Upsilon(3S)} < \Gamma_{\Upsilon(2S)} < \Gamma_{\Upsilon(1S)}$, in principle, there should be relations among branching ratios $\mathcal{B}r(\Upsilon(3S) \to B_c^*P) > \mathcal{B}r(\Upsilon(2S) \to B_c^*P) > \mathcal{B}r(\Upsilon(1S) \to B_c^*P)$ for the same pseudoscalar meson P. But the numerical results in Table 3 are beyond such expectation. Why? The reason is that the factor of $p/m_{\Upsilon(nS)}^2$ in (16) has almost the same value for $n \le 3$, so branching ratio is proportional to factor $f_{\Upsilon(nS)}^2/\Gamma_{\Upsilon(nS)}$ with the maximal value $f_{\Upsilon(1S)}^2/\Gamma_{\Upsilon(1S)}$ for $n \le 3$. Besides, contributions from $\alpha_s/\pi \in [0.2, 0.3]$ regions decrease with n (see Figure 3), which enhance the decay amplitudes.

(4) Besides the uncertainties listed in Table 3, other factors, such as the models of wave functions, contributions of higher order corrections to HME, and relativistic effects, deserve the dedicated study. Our results just provide an order of magnitude estimation.

4. Summary

$\Upsilon(nS)$ decay via the weak interaction, as a complementary to strong and electromagnetic decay mechanism, is allowable within the standard model. Based on the potential prospects of $\Upsilon(nS)$ physics at high-luminosity collider experiment, $\Upsilon(nS)$ decay into $B_c^*\pi$ and B_c^*K final states is investigated with the pQCD approach firstly. It is found that (1) the dominant contributions come from perturbative regions $\alpha_s/\pi \le 0.3$, which might imply that the pQCD calculation is practicable and workable; (2) there is a promiseful possibility of searching for $\Upsilon(nS) \to B_c^*\pi$ (B_c^*K) decay with branching ratio about 10^{-10} (10^{-11}) at the future experiments.

Appendix

Building Blocks for $\Upsilon \to B_c^*P$ Decays

The building blocks $\mathcal{A}_i^j$, where the superscript j corresponds to indices of Figure 2 and the subscript i relates to different helicity amplitudes, are expressed as follows:

$$\begin{aligned}
\mathcal{A}_L^a &= \int_0^1 dx_1 \int_0^1 dx_2 \int_0^\infty b_1 db_1 \\
&\cdot \int_0^\infty b_2 db_2 H_{ab}(\alpha_e, \beta_a, b_1, b_2) E_{ab}(t_a) \phi_\Upsilon^v(x_1) \\
&\cdot \alpha_s(t_a) a_1(t_a) \\
&\cdot \left\{\phi_{B_c^*}^v(x_2)\left[m_1^2 s - \left(4m_1^2p^2 + m_2^2u\right)\bar{x}_2\right]\right. \\
&\left. + \phi_{B_c^*}^t(x_2) m_2 m_b u\right\}, \\
\mathcal{A}_N^a &= \int_0^1 dx_1 \int_0^1 dx_2 \int_0^\infty b_1 db_1 \\
&\cdot \int_0^\infty b_2 db_2 H_{ab}(\alpha_e, \beta_a, b_1, b_2) E_{ab}(t_a) \phi_\Upsilon^V(x_1) \\
&\cdot \alpha_s(t_a) a_1(t_a) m_1 \left\{\phi_{B_c^*}^V(x_2) m_2 (u - s\bar{x}_2)\right. \\
&\left. + \phi_{B_c^*}^T(x_2) m_b s\right\}, \\
\mathcal{A}_T^a &= -2m_1 \int_0^1 dx_1 \int_0^1 dx_2 \int_0^\infty b_1 db_1 \\
&\cdot \int_0^\infty b_2 db_2 H_{ab}(\alpha_e, \beta_a, b_1, b_2) E_{ab}(t_a) \phi_\Upsilon^V(x_1) \\
&\cdot \alpha_s(t_a) a_1(t_a) \left\{\phi_{B_c^*}^T(x_2) m_b + \phi_{B_c^*}^V(x_2) m_2 x_2\right\}, \\
\mathcal{A}_L^b &= \int_0^1 dx_1 \int_0^1 dx_2 \int_0^\infty b_1 db_1 \\
&\cdot \int_0^\infty b_2 db_2 H_{ab}(\alpha_e, \beta_b, b_2, b_1) E_{ab}(t_b) \phi_{B_c^*}^v(x_2) \\
&\cdot \alpha_s(t_b) a_1(t_b) \left\{\phi_\Upsilon^v(x_1)\left[m_2^2u - m_1^2\left(s - 4p^2\right)\bar{x}_1\right]\right. \\
&\left. + \phi_\Upsilon^t(x_1) m_1 m_c s\right\}, \\
\mathcal{A}_N^b &= \int_0^1 dx_1 \int_0^1 dx_2 \int_0^\infty b_1 db_1 \\
&\cdot \int_0^\infty b_2 db_2 H_{ab}(\alpha_e, \beta_b, b_2, b_1) E_{ab}(t_b) \phi_{B_c^*}^V(x_2) \\
&\cdot \alpha_s(t_b) a_1(t_b) m_2 \left\{\phi_\Upsilon^V(x_1) m_1 (s - u\bar{x}_1)\right. \\
&\left. + \phi_\Upsilon^T(x_1) m_c u\right\}, \\
\mathcal{A}_T^b &= -2m_2 \int_0^1 dx_1 \int_0^1 dx_2 \int_0^\infty b_1 db_1 \\
&\cdot \int_0^\infty b_2 db_2 H_{ab}(\alpha_e, \beta_b, b_2, b_1) E_{ab}(t_b) \phi_{B_c^*}^V(x_2) \\
&\cdot \alpha_s(t_b) a_1(t_b) \left\{\phi_\Upsilon^V(x_1) m_1 x_1 + \phi_\Upsilon^T(x_1) m_c\right\}, \\
\mathcal{A}_L^c &= \frac{1}{N_c} \int_0^1 dx_1 \int_0^1 dx_2 \int_0^1 dx_3 \int_0^\infty db_1 \int_0^\infty b_2 db_2 \\
&\cdot \int_0^\infty b_3 db_3 H_{cd}(\alpha_e, \beta_c, b_2, b_3) E_{cd}(t_c) \phi_P^a(x_3) \\
&\cdot \alpha_s(t_c) C_2(t_c) \left\{\phi_\Upsilon^v(x_1) \phi_{B_c^*}^v(x_2) 4m_1^2p^2(x_1 - \bar{x}_3)\right. \\
&\left. + \phi_\Upsilon^t(x_1) \phi_{B_c^*}^t(x_2) m_1 m_2 \left(ux_1 - sx_2 - 2m_3^2\bar{x}_3\right)\right\} \\
&\cdot \delta(b_1 - b_2),
\end{aligned}$$

$$\mathcal{A}_N^c = \frac{1}{N_c}\int_0^1 dx_1 \int_0^1 dx_2 \int_0^1 dx_3 \int_0^\infty db_1 \int_0^\infty b_2 db_2 \cdot \int_0^\infty b_3 db_3 H_{cd}\left(\alpha_e, \beta_c, b_2, b_3\right) E_{cd}\left(t_c\right) C_2\left(t_c\right) \alpha_s\left(t_c\right) \cdot \delta\left(b_1 - b_2\right) \phi_\Upsilon^T\left(x_1\right) \phi_{B_c^*}^T\left(x_2\right) \phi_P^a\left(x_3\right) \cdot \left\{m_1^2 s\left(x_1 - \overline{x}_3\right) + m_2^2 u\left(\overline{x}_3 - x_2\right)\right\},$$

$$\mathcal{A}_T^c = \frac{2}{N_c}\int_0^1 dx_1 \int_0^1 dx_2 \int_0^1 dx_3 \int_0^\infty db_1 \int_0^\infty b_2 db_2 \cdot \int_0^\infty b_3 db_3 H_{cd}\left(\alpha_e, \beta_c, b_2, b_3\right) E_{cd}\left(t_c\right) C_2\left(t_c\right) \alpha_s\left(t_c\right) \cdot \delta\left(b_1 - b_2\right) \phi_\Upsilon^T\left(x_1\right) \phi_{B_c^*}^T\left(x_2\right) \phi_P^a\left(x_3\right) \left\{m_1^2\left(\overline{x}_3 - x_1\right) + m_2^2\left(x_2 - \overline{x}_3\right)\right\},$$

$$\mathcal{A}_L^d = \frac{1}{N_c}\int_0^1 dx_1 \int_0^1 dx_2 \int_0^1 dx_3 \int_0^\infty db_1 \int_0^\infty b_2 db_2 \cdot \int_0^\infty b_3 db_3 H_{cd}\left(\alpha_e, \beta_d, b_2, b_3\right) E_{cd}\left(t_d\right) \phi_P^a\left(x_3\right) \cdot \alpha_s\left(t_d\right) C_2\left(t_d\right) \left\{\phi_\Upsilon^v\left(x_1\right) \phi_{B_c^*}^v\left(x_2\right) 4 m_1^2 p^2\left(x_3 - x_2\right) + \phi_\Upsilon^t\left(x_1\right) \phi_{B_c^*}^t\left(x_2\right) m_1 m_2\left(s x_2 + 2 m_3^2 x_3 - u x_1\right)\right\} \cdot \delta\left(b_1 - b_2\right),$$

$$\mathcal{A}_N^d = \frac{1}{N_c}\int_0^1 dx_1 \int_0^1 dx_2 \int_0^1 dx_3 \int_0^\infty db_1 \int_0^\infty b_2 db_2 \cdot \int_0^\infty b_3 db_3 H_{cd}\left(\alpha_e, \beta_d, b_2, b_3\right) E_{cd}\left(t_d\right) C_2\left(t_d\right) \cdot \alpha_s\left(t_d\right) \delta\left(b_1 - b_2\right) \phi_\Upsilon^T\left(x_1\right) \phi_{B_c^*}^T\left(x_2\right) \phi_P^a\left(x_3\right) \cdot \left\{m_1^2 s\left(x_3 - x_1\right) + m_2^2 u\left(x_2 - x_3\right)\right\},$$

$$\mathcal{A}_T^d = \frac{2}{N_c}\int_0^1 dx_1 \int_0^1 dx_2 \int_0^1 dx_3 \int_0^\infty db_1 \int_0^\infty b_2 db_2 \cdot \int_0^\infty b_3 db_3 H_{cd}\left(\alpha_e, \beta_d, b_2, b_3\right) E_{cd}\left(t_d\right) C_2\left(t_d\right) \cdot \alpha_s\left(t_d\right) \delta\left(b_1 - b_2\right) \phi_\Upsilon^T\left(x_1\right) \phi_{B_c^*}^T\left(x_2\right) \phi_P^a\left(x_3\right) \cdot \left\{m_1^2\left(x_1 - x_3\right) - m_2^2\left(x_2 - x_3\right)\right\}, \tag{A.1}$$

where $\overline{x}_i = 1 - x_i$; variable x_i is the longitudinal momentum fraction of the valence quark; b_i is the conjugate variable of the transverse momentum $k_{i\perp}$; and $\alpha_s(t)$ is the QCD coupling at the scale of t; $a_1 = C_1 + C_2/N_c$.

The function H_i is defined as follows [17]:

$$H_{ab}\left(\alpha_e, \beta, b_i, b_j\right) = K_0\left(\sqrt{-\alpha} b_i\right) \cdot \left\{\theta\left(b_i - b_j\right) K_0\left(\sqrt{-\beta} b_i\right) I_0\left(\sqrt{-\beta} b_j\right) + \left(b_i \longleftrightarrow b_j\right)\right\},$$

$$H_{cd}\left(\alpha_e, \beta, b_2, b_3\right) = \left\{\theta(-\beta) K_0\left(\sqrt{-\beta} b_3\right) + \frac{\pi}{2}\theta(\beta)\left[i J_0\left(\sqrt{\beta} b_3\right) - Y_0\left(\sqrt{\beta} b_3\right)\right]\right\} \cdot \left\{\theta\left(b_2 - b_3\right) K_0\left(\sqrt{-\alpha} b_2\right) I_0\left(\sqrt{-\alpha} b_3\right) + \left(b_2 \longleftrightarrow b_3\right)\right\}, \tag{A.2}$$

where J_0 and Y_0 (I_0 and K_0) are the (modified) Bessel function of the first and second kind, respectively; α_e (α_a) is the gluon virtuality of the emission (annihilation) topological diagrams; the subscript of the quark virtuality β_i corresponds to the indices of Figure 2. The definition of the particle virtuality is listed as follows [17]:

$$\begin{aligned}
\alpha &= \overline{x}_1^2 m_1^2 + \overline{x}_2^2 m_2^2 - \overline{x}_1 \overline{x}_2 t, \\
\beta_a &= m_1^2 - m_b^2 + \overline{x}_2^2 m_2^2 - \overline{x}_2 t, \\
\beta_b &= m_2^2 - m_c^2 + \overline{x}_1^2 m_1^2 - \overline{x}_1 t, \\
\beta_c &= x_1^2 m_1^2 + x_2^2 m_2^2 + \overline{x}_3^2 m_3^2 - x_1 x_2 t - x_1 \overline{x}_3 u \\
&\quad + x_2 \overline{x}_3 s, \\
\beta_d &= x_1^2 m_1^2 + x_2^2 m_2^2 + x_3^2 m_3^2 - x_1 x_2 t - x_1 x_3 u \\
&\quad + x_2 x_3 s.
\end{aligned} \tag{A.3}$$

The typical scale t_i and the Sudakov factor E_i are defined as follows, where the subscript i corresponds to the indices of Figure 2:

$$t_{a(b)} = \max\left(\sqrt{-\alpha}, \sqrt{-\beta_{a(b)}}, \frac{1}{b_1}, \frac{1}{b_2}\right), \tag{A.4}$$

$$t_{c(d)} = \max\left(\sqrt{-\alpha}, \sqrt{\left|\beta_{c(d)}\right|}, \frac{1}{b_2}, \frac{1}{b_3}\right), \tag{A.5}$$

$$E_{ab}(t) = \exp\left\{-S_\Upsilon(t) - S_{B_c^*}(t)\right\}, \tag{A.6}$$

$$E_{cd}(t) = \exp\left\{-S_\Upsilon(t) - S_{B_c^*}(t) - S_P(t)\right\}, \tag{A.7}$$

$$S_\Upsilon(t) = s\left(x_1, p_1^+, \frac{1}{b_1}\right) + 2\int_{1/b_1}^t \frac{d\mu}{\mu}\gamma_q, \tag{A.8}$$

$$S_{B_c^*}(t) = s\left(x_2, p_2^+, \frac{1}{b_2}\right) + 2\int_{1/b_2}^t \frac{d\mu}{\mu}\gamma_q, \tag{A.9}$$

$$S_{\pi,K}(t) = s\left(x_3, p_3^+, \frac{1}{b_3}\right) + s\left(\bar{x}_3, p_3^+, \frac{1}{b_3}\right) + 2\int_{1/b_3}^{t} \frac{d\mu}{\mu}\gamma_q, \quad \text{(A.10)}$$

where $\gamma_q = -\alpha_s/\pi$ is the quark anomalous dimension; the explicit expression of $s(x, Q, 1/b)$ can be found in the appendix of [2].

Competing Interests

The authors declare that there are no competing interests related to this paper.

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References

[1] K. Olive, K. Agashe, C. Amsler et al., "Review of particle physics," *Chinese Physics C*, vol. 38, no. 9, Article ID 090001, 2014.

[2] H. Li, "Applicability of perturbative QCD to $B \to D$ decays," *Physical Review D*, vol. 52, no. 7, pp. 3958–3965, 1995.

[3] C. V. Chang and H. Li, "Three-scale factorization theorem and effective field theory: analysis of nonleptonic heavy meson decays," *Physical Review D*, vol. 55, no. 9, pp. 5577–5580, 1997.

[4] T. Yeh and H. Li, "Factorization theorems, effective field theory, and nonleptonic heavy meson decays," *Physical Review D*, vol. 56, no. 3, pp. 1615–1631, 1997.

[5] E. A. Bevan, B. Golob, T. Mannel et al., "The physics of the B factories," *The European Physical Journal C*, vol. 74, article 3026, 2014.

[6] Z. Wang, "The radiative decays $B_c^{*\pm} \to B_c^{\pm}\gamma$ with QCD sum rules," *The European Physical Journal C*, vol. 73, no. 9, article 2559, 2013.

[7] M. Beneke, G. Buchalla, M. Neubert, and C. T. Sachrajda, "QCD factorization for $B \to \pi\pi$ decays: strong phases and CP violation in the heavy quark limit," *Physical Review Letters*, vol. 83, no. 10, pp. 1914–1917, 1999.

[8] M. Beneke et al., "QCD factorization for exclusive non-leptonic B-meson decays: general arguments and the case of heavy-light final states," *Nuclear Physics B*, vol. 591, no. 1-2, pp. 313–418, 2000.

[9] M. Beneke, G. Buchalla, M. Neubert, and C. T. Sachrajda, "QCD factorization in $B \to \pi K, \pi\pi$ decays and extraction of Wolfenstein parameters," *Nuclear Physics B*, vol. 606, no. 1-2, pp. 245–321, 2001.

[10] C. W. Bauer, S. Fleming, D. Pirjol, and I. W. Stewart, "An effective field theory for collinear and soft gluons: heavy to light decays," *Physical Review D*, vol. 63, no. 11, Article ID 114020, 17 pages, 2001.

[11] C. Bauer, D. Pirjol, and I. Stewart, "Soft-collinear factorization in effective field theory," *Physical Review D*, vol. 65, Article ID 054022, 2002.

[12] C. Bauer, S. Fleming, D. Pirjol, I. Z. Rothstein, and I. W. Stewart, "Hard scattering factorization from effective field theory," *Physical Review D*, vol. 66, no. 1, Article ID 014017, 23 pages, 2002.

[13] M. Beneke, A. P. Chapovsky, M. Diehl, and Th. Feldmann, "Soft-collinear effective theory and heavy-to-light currents beyond leading power," *Nuclear Physics B*, vol. 643, no. 1–3, pp. 431–476, 2002.

[14] R. Dowdall, C. T. H. Davies, T. C. Hammant, and R. R. Horgan, "Precise heavy-light meson masses and hyperfine splittings from lattice QCD including charm quarks in the sea," *Physical Review D*, vol. 86, no. 9, Article ID 094510, 19 pages, 2012.

[15] B. Colquhoun, C. T. H. Davies, J. Kettle et al., "B-meson decay constants: a more complete picture from full lattice QCD," *Physical Review D*, vol. 91, no. 11, Article ID 114509, 2015.

[16] P. Ball, V. Braun, and A. Lenz, "Higher-twist distribution amplitudes of the K meson in QCD," *Journal of High Energy Physics*, vol. 2006, no. 5, article 004, 2006.

[17] Y. Yang, J. Sun, Y. Guo, Q. Li, J. Huang, and Q. Chang, "Study of $\Upsilon(nS) \to B_c P$ decays with perturbative QCD approach," *Physics Letters B*, vol. 751, pp. 171–176, 2015.

[18] Z. Wang, "Analysis of the vector and axialvector B_c mesons with QCD sum rules," *The European Physical Journal A*, vol. 49, article 131, 2013.

[19] G. Buchalla, A. J. Buras, and M. E. Lautenbacher, "Weak decays beyond leading logarithms," *Reviews of Modern Physics*, vol. 68, no. 4, pp. 1125–1244, 1996.

[20] G. P. Lepage and S. J. Brodsky, "Exclusive processes in perturbative quantum chromodynamics," *Physical Review D*, vol. 22, no. 9, pp. 2157–2198, 1980.

[21] P. Ball and G. Jones, "Twist-3 distribution amplitudes of K^* and ϕ mesons," *Journal of High Energy Physics*, vol. 2007, no. 3, article 069, 2007.

[22] G. P. Lepage, L. Magnea, C. Nakhleh, U. Magnea, and K. Hornbostel, "Improved nonrelativistic QCD for heavy-quark physics," *Physical Review D*, vol. 46, no. 9, pp. 4052–4067, 1992.

[23] G. T. Bodwin, E. Braaten, and G. P. Lepage, "Rigorous QCD analysis of inclusive annihilation and production of heavy quarkonium," *Physical Review D*, vol. 51, no. 3, pp. 1125–1171, 1995.

[24] N. Brambilla, A. Pineda, J. Soto, and A. Vairo, "Effective-field theories for heavy quarkonium," *Reviews of Modern Physics*, vol. 77, no. 4, pp. 1423–1496, 2005.

[25] B. Xiao, X. Qian, and B. Ma, "The kaon form factor in the light-cone quark model," *The European Physical Journal A*, vol. 15, no. 4, pp. 523–527, 2002.

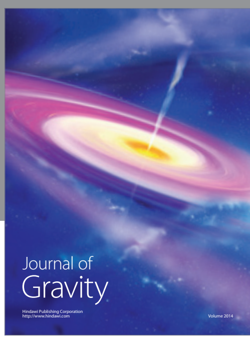
[26] C. Chen, Y. Keum, and H. Li, "Perturbative QCD analysis of $\vec{B}\varphi K^*$ decays," *Physical Review D*, vol. 66, Article ID 054013, 2002.

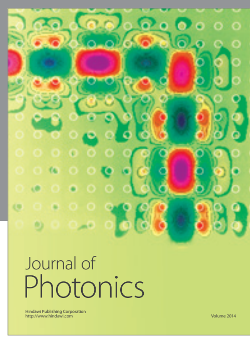
[27] R. Aaij, B. Adeva, M. Adinolfi et al., "Measurement of Υ production in pp collisions at $\sqrt{s} = 2.76$ TeV," *The European Physical Journal C*, vol. 74, article 2835, 2014.

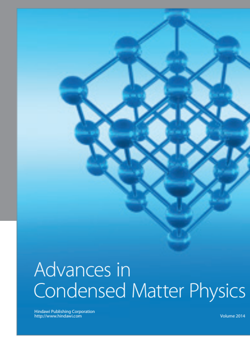
[28] R. Aaij, B. Adeva, M. Adinolfi et al., "Study of Υ production and cold nuclear matter effects in pPb collisions at $\sqrt{s_{NN}} = 5$ TeV," *Journal of High Energy Physics*, vol. 2014, no. 7, article 094, 2014.

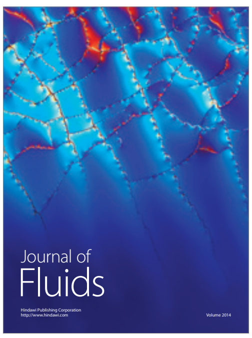
[29] G. Aad, T. Abajyan, B. Abbott et al., "Measurement of upsilon production in 7 TeV pp collisions at ATLAS," *Physical Review D*, vol. 87, Article ID 052004, 2013.

[30] B. Abelev, J. Adam, D. Adamová et al., "Production of inclusive $\Upsilon(1S)$ and $\Upsilon(2S)$ in p–Pb collisions at $\sqrt{s_{\rm NN}} = 5.02$ TeV," *Physics Letters B*, vol. 740, pp. 105–117, 2015.









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