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Abstract: In a multiperiod setting, decision-makers can learn about the consequences of their decisions through experimentation. In this paper we examine how in a two-party system polarization and political instability affect learning through experimentation. We distinguish two cases: (1) the decision to be made is not salient and does not affect the outcome of the following elections (exogenous elections) and (2) the decision is salient and the election outcome depends on it (endogenous elections). We show that while the possibility of learning increases activism, the existence of political instability distorts learning. Furthermore, in contrast to the existing literature, we demonstrate that, when elections are exogenous, polarization between political parties does not always decrease active learning. In the case with endogenous elections we find that electoral concerns may induce candidates not to experiment, even if the majority of voters prefers activism.

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1. Introduction

The consequences of many decisions are surrounded by uncertainty. In a multi-period setting, decision-makers can learn about the consequences of decisions through experimentation. Two types of learning through experimentation can be distinguished: passive learning and active learning. In case of passive learning, the decision-maker's action is not induced by the possibility of learning. In case of active learning, experimentation reduces the current utility of the decision-maker, but it leads to information that can be used to improve future decision making. There exists a small but interesting literature on active learning. Early papers are Prescott (1972) and Rothschild (1974). The concept of active learning can be applied to several situations. For example, it can help to understand how consumers make decisions about buying goods with uncertain quality (Grossman, Kihlstrom and Mirman, 1977) or how firms come to know the demand curves of their products (Rothschild, 1974). Concerning economic policy, economists have studied the normative implications of active learning. Bertocchi and Spagat (1997) argue that the possibility of learning through experimentation provides a rationale for shock therapy in transition economies. In another paper (Bertocchi and Spagat, 1993), they study the implications of active learning for the optimality of money supply rules. So far, little attention has been paid to the implications of active learning for positive approaches to the making of policy.[1]

In this paper, we apply the concept of learning to decision making in a political setting. More specifically, we address the question how in a two-party system polarization and political instability affect learning through experimentation. To answer this question, we construct a highly stylized, two-period model of the behavior of a policy maker who has to make a decision about a public project. This project can be either implemented or rejected. Initially, the consequences of the project are uncertain. However, if the policy maker implements the project, she learns its consequences. In the next section, we show that when the policy maker is certain to stay in office, the possibility of learning increases activism. This result is completely in line with the existing literature on active learning. In Section 3, we introduce elections into the model. The policy maker in period 1 faces an exogenous probability that in period 2 she is replaced with a policy maker with different preferences. We expected that uncertainty about the future preferences of the policy maker would decrease activism (as in Bertocchi and Spagat, 1997). However, this appeared not to be the case generally. When there is a large probability that a policy maker, who is biased against implementation, is succeeded by a policy maker, who is biased towards implementation, learning considerations increase activism. The reason is that by implementing the project the policy maker in period 1 can persuade her successor not to implement the project in period 2. Experimentation is thus sometimes induced by a "let the other learn" effect. In Section 4, we endogenize elections. Endogenizing elections has two implications. First, provided that the two candidates choose different policies, voters can choose the candidate whose policy is in line with their preferences. Second, policy choice in period 1 may have an effect on the probability that the incumbent will be reelected. We show that electoral concerns may induce candidates not to experiment, even if a majority of voters prefers activism.

The motivation for this paper is twofold. First, we want to understand how electoral competition impacts information collection about policy consequences. This is important, because the quality of policy often depends on the information the policy decision has been based on. Second, in several countries reforms are considered aimed at facilitating learning. The Netherlands, for example, intends to adopt a "performance" accounting system. The hope is that continuous evaluation of public policies leads to better policy decisions. Studies, like the present one, may give insight into the possible obstacles to such desirable reforms.

[1] An exception is Bertocchi (1993), who uses results from the theory of active learning to provide an explanation for existing systems of public debt management.

2. Basic model

This section discusses a simple two-period model, in which a policy maker must make a decision about a public project. In each period *t*, the policy maker can choose between two alternatives: implementation, denoted by $x_t = 1$, and rejection, denoted by $x_t = 0$. When the policy maker chooses $x_t = 1$, her payoff is

$$U_t(x_t = 1|\mu) = p + \mu \tag{1}$$

where *p* denotes the policy maker's predisposition towards the project, and μ is a stochastic term, reflecting that the consequences of the project are surrounded by uncertainty. We assume that μ is uniformly distributed on [-*h*, *h*]. When the policy maker rejects the project, her payoff is $U_t(x_t = 0) = 0$ by normalization. Under full information, the policy maker would choose $x_t = 1$ if $\mu > -p$. However, at the beginning of period 1, the policy maker does not observe μ. Throughout this paper it is assumed that $|p| < h$. This assumption ensures that the realization of μ determines whether or not the policy maker benefits from undertaking the project. As a consequence, the policy maker benefits from information about μ. For notational simplicity, we assume that the policy maker does not discount the future. Her total utility is thus given by $\sum_{t=1}^{2} U_t$. The policy maker can learn the value of μ by implementing the project in the first period. If learning takes place, the decision about the project in period 2 is made under certainty.

Formally, the stages of the game can be described as follows:

1) Nature draws μ from a uniform distribution with range $[-h, h]$.

2) The policy maker makes a decision about the project in period 1: $x_1 = 1$ or $x_1 = 0$.

3) If $x_1 = 1$, nature reveals the value of μ to the policy maker.

4) The policy maker makes a decision about the project in period 2: $x_2 = 1$ or $x_2 = 0$.

Solution of the basic model

In this subsection we show how the opportunity to learn affects the policy decision in period 1. More specifically, we derive the value of p for which the policy maker is indifferent between $x_1 = 0$ and $x_1 = 1$. To ensure a time consistent solution, we start with analyzing the second period.

In period 2 the decision about the project depends on the decision the policy maker has made in period 1. When $x_1 = 0$, the policy maker has not obtained information about μ, and chooses $x_2 = 1$ if and only if $p > 0$.[2] When $x_1 = 1$, the policy maker knows μ and chooses $x_2 = 1$ if and only if $\mu > -p$.

Now consider the policy maker's decision about the project in period 1. Anticipating her decision about the project in period 2, the expected payoff to the policy maker when she chooses $x_1 = 1$ is

$$\begin{aligned} & p + \Pr(\mu > -p)[p + E(\mu \mid \mu > -p)] = p + \frac{1}{2h}(h+p)[p + \frac{1}{2}(h-p)] \\ & = \frac{1}{4h}(p^2 + 6hp + h^2) \end{aligned} \qquad (2)$$

When the policymaker chooses $x_1 = 0$, her expected payoff is 0, if $p \le 0$, and her expected payoff is *p*, if $p > 0$. Because the second term of the RHS of the first row in (2) is positive (expected utility in period 2 is greater than zero), $x_1 = 1$ yields a higher expected payoff than $x_1 = 0$, if $p > 0$. Hence, if $p > 0$, then $x_1 = 1$. When $p \leq 0$, $x_1 = 1$ yields a higher expected payoff than $x_1 = 0$ if the last expression in (2) is positive, implying:[3]

$$p > p^I = (-3 + 2\sqrt{2})h \tag{3}$$

where p^I denotes the predisposition of a policy maker who is indifferent between $x_1 = 0$ and $x_1 = 1$. There are two alternative ways of interpreting p^I. First, we can interpret p^I as giving the type of policy makers, in terms of their predisposition towards a **given project**, who choose $x_1 = 1$. Second, we can interpret p^I as giving the type of projects, in terms of their attractiveness, which are implemented by a **given policy maker**. The first interpretation implies that a decrease in p^I means that more policy makers choose $x_1 = 1$. The second interpretation implies that a decrease in p^I means that more projects are implemented by the policy maker.

It is easy to see from (3) that $p^I < 0$. The implication is that a policy maker who is biased against implementation may choose $x_1 = 1$. On the basis of (3), we can make a clear distinction between two well-known concepts in the literature on learning. Passive learning takes place if $p > 0$: the opportunity to learn affects the policy decision in period 2, but not the policy decision in period 1. Active learning takes place if the opportunity to learn affects

[2] Without loss of generality, we assume that when the policy maker is indifferent between $x_t = 0$ and $x_t = 1$, she chooses $x_t = 0$.

[3] The last expression in (2) is also positive if $p < (-3 - 2\sqrt{2})h$. However, our assumptions concerning the values of *p* and *h* exclude this solution.

the policy in period 1. This occurs if $p \in \left((-3+2\sqrt{2h},0\right]$. When there is no scope for learning (for instance, because a project cannot be repealed), $x_1 = 0$ if $p < 0$.

Equation (3) implies that p^I decreases with *h*. Thus, the higher is the level of uncertainty about the consequences of the project, the less restrictive is (3). The intuition behind this result is that the benefits of learning increase with uncertainty. When policymakers are risk-averse, the benefits of active learning are even higher. However, with risk-aversion, more uncertainty directly reduces the attractiveness of implementation.

3. Uncertainty about the Preferences of the Future Policy Maker

In this section we introduce elections into the basic model in order to analyze the effect of elections on the condition for active learning. In the augmented model, elections are held at the end of period 1. Two policy makers compete for office: policy maker *P*, whose preferences are described by (1), and policy maker *R*, whose preferences are described by $U_t^r(x_t = 1|\mu) = r + \mu$ and $U_t^r(x_t = 0)$. The parameter r $(|r| < h)$ denotes policy maker R 's predisposition towards the project. The deviation of *p* from *r* can be interpreted as a measure of polarization. In period 1, policy maker *P* is in office. The probability that in period 2 *P* stays in office is denoted by π. The probability that *R* wins the elections is therefore given by $1-\pi$. In this section, we make the strong assumption that π is exogenous. The motivation for this assumption is that policymakers make numerous decisions that do not receive attention from the media. It is unlikely that those decisions affect voter behavior. Concerning salient policy decisions, the assumption that π is exogenous seems implausible. For this reason, we relax this assumption in the next section.

Solution of the Augmented Model

In period 2 the decision about the project depends on the decision policy maker *P* has made in period 1 and on the election outcome. Suppose $x_1 = 1$. Clearly, when policy maker *P* wins the elections, she chooses $x_2 = 1$ if and only if $\mu > -p$. Likewise, when policy maker *R* wins the elections, he chooses $x_2 = 1$ if and only if $\mu > -r$. Now suppose $x_1 = 0$. In this case, the policy maker, who has won the elections, chooses $x_2 = 1$ if her predisposition towards the project is higher than zero.

In period 1, policy maker *P*'s expected total utility is

$$\begin{aligned} & p + \pi \Pr(\mu > -p)[p + E(\mu | \mu > -p)] + (1-\pi)\Pr(\mu > -r)[p + E(\mu | \mu > -r)] \\ & = \frac{1}{4h}\left(\pi p^2 + [6h + 2(1-\pi)r]p + h^2 - (1-\pi)r^2\right) \end{aligned} \tag{4}$$

when she chooses $x_1 = 1$. When *P* chooses $x_1 = 0$, her total expected utility depends on the signs of p and *r* [total expected utility is zero when $p, r \leq 0$; it is πp when $p > 0$ and $r \leq 0$; it is $(1-\pi)p$ when $p \leq 0$ and $r > 0$; it is *p* when $p, r > 0$]. Using (4), it is easy to show that it is optimal for *P* to choose $x_1 = 1$, when $p > 0$. When $p \leq 0$ and $r \leq 0$, $x_1 = 1$ yields a higher total utility than $x_1 = 0$ if (4) is higher than zero, implying:

$$\pi p^2 + [6h + 2(1-\pi)r]p + h^2 - (1-\pi)r^2 > 0. \tag{5}$$

When $p \leq 0$ and $r > 0$, $x_1 = 1$ yields a higher total utility than $x_1 = 0$ if (4) is higher than $(1-\pi)p$, implying:

$$\pi p^2 + [(1+2\pi)2h + 2(1-\pi)r]p + h^2 - (1-\pi)r^2 > 0. \tag{6}$$

To examine the conditions under which policy maker *P* chooses $x_1 = 1$, we again analyze the value of p for which *P* is indifferent between choosing $x_1 = 0$ and choosing $x_1 = 1$. Let $p^I_{r \le 0}$ denote this value of *p*, given that $r \le 0$. Equation (5) implicitly defines $p^I_{r \le 0}$ as a function of *r*, *h* and π. Application of the implicit function theorem shows that (see Appendix A):

$$\frac{\partial p^I_{r \le 0}}{\partial r} > 0 \quad if \quad p < r$$
$$\frac{\partial p^I_{r \le 0}}{\partial r} < 0 \quad if \quad p > r \qquad (7)$$

$$\frac{\partial p^I_{r \le 0}}{\partial \pi} < 0 \qquad (8)$$

$$\frac{\partial p^I_{r \le 0}}{\partial h} < 0 \qquad (9)$$

The partial derivatives in (7) say that $p^I_{r \le 0}$ increases with $|p - r|$. Hence, when $r \le 0$, polarization reduces the incentive for learning actively. The partial derivative in (8) shows that a higher probability of reelection increases the incentive for learning actively. The intuition behind these two results is straightforward. Uncertainty about the preferences of the future policy maker reduces the benefits of learning. As a consequence, more polarization and a higher probability of losing office weaken the incentive to learn. In our model, learning requires implementation of projects. Hence, more polarization and a lower probability of reelection imply that implementation of projects becomes less attractive. The partial derivative in (9) confirms our earlier result that a higher degree of uncertainty encourages learning.

In Appendix B, we repeat the analysis for the case $r > 0$. Unsurprisingly, the results concerning the effects of *r* on $p^I_{r>0}$ are qualitatively the same as implied by (7). There remains the analysis of the effect of an increase in *r* on p^I, when $r = 0$. When *r* goes to zero, (5) reduces to $\pi p^2 + 6hp + h^2 > 0$ and (6) reduces to $\pi p^2 + 2(1+2\pi)hp + h^2 > 0$, so that

$$p^I_{r\leq 0} = \frac{(-3+\sqrt{9-\pi^2})h}{\pi} \qquad (10)$$

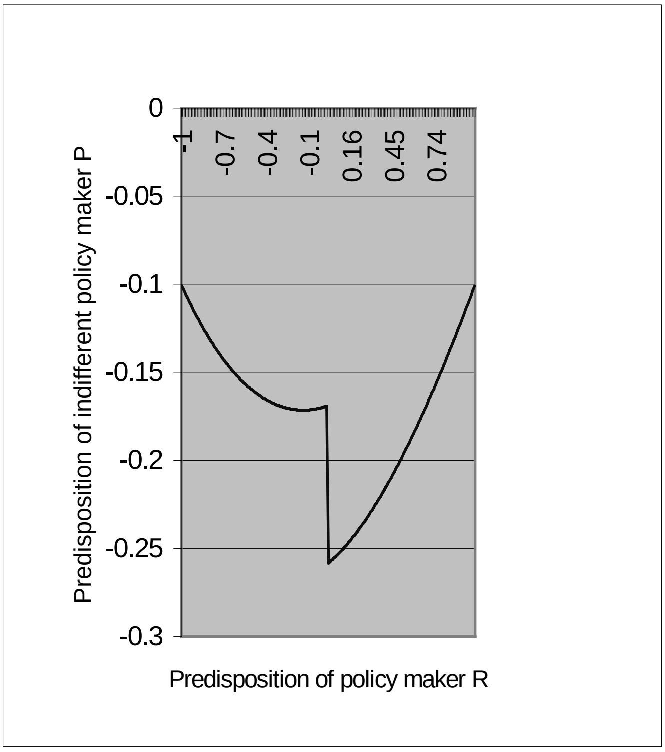
and

$$p^I_{r>0} = \frac{(-1-2\pi+\sqrt{1+3\pi+4\pi^2})h}{\pi} \quad . \qquad (11)$$

It is easy to show that, for $0 < \pi < 1$, the expression in (11) is smaller than the expression in (10). The incentive for active learning is thus stronger when policy maker *R* is marginally biased towards implementation of the project than when he is marginally biased towards rejection of the project. The reason is that, when $r > 0$, $x_1 = 0$ induces policy maker *R* to choose $x_2 = 1$ if in office. By choosing $x_1 = 1$, policy maker *P* reduces the probability that policy maker *R* chooses $x_2 = 1$. This increases her utility, because $p < 0$. In addition to this "let the other learn" effect, policy maker *P* learns μ herself by choosing $x_1 = 1$. A comparison between (3) and (11) shows that polarization may encourage policy maker *P* to choose $x_1 = 1$ rather than $x_1 = 0$. This illustrates the potential importance of the "let the other learn" effect.

Figure 1 summarizes our result about the effect of polarization on active learning. In general, a marginal increase in the degree of polarization reduces policy maker *P*'s willingness to choose $x_1 = 1$. However, because p^I experiences a negative jump at $r = 0$, there exists a range of values of $r > 0$, for which active learning is more likely in a polarized system than in a system where policy maker P is certain to stay in office.

Figure 1. Effect of polarization on active learning



The graph is drawn for $h = 1$, $\pi = \frac{1}{2}$.

4. Endogenous elections

In this section we relax the assumption that the election outcome is exogenous. Moreover, we allow voters to choose the policy maker in period 1. Thus, two elections are held. The first election determines the policy maker in period 1. The second determines the policy maker in period 2. At the elections, each voter casts her ballot for the candidate whose policy yields highest expected utility. Voter v's preferences are described by $u_t^v(x_t = 1 | \mu) = v + \mu$ and $u_t^v(x_t = 0) = 0$. We assume a continuum of voters in terms of v. It is easy to see that in this setting the median voter's vote is decisive. Throughout, we assume that $\pi = \frac{1}{2}$ if the median voter is indifferent between P and R. Let m denote the median voter's predisposition towards the project. We assume that m is common knowledge and that $p < (-3 + 2\sqrt{2})h < r < 0$[4] and $p < m < r$. The implication of the first condition is that in a setting without elections the policy makers would have made different policy choices: R would have chosen $x_1 = 1$, while P would have chosen $x_1 = 0$. Notice that we assume that, like both policy makers, the median voter is negatively predisposed towards the project. The reason is that we focus on active learning.

As we will show below, with endogenous elections, the election outcome in period 2 depends on policy in period 1. As a consequence, the policy maker in period 1 can influence her probability of re-election. We assume that policy makers receive (ego) rents from holding office. More specifically, we add λdum_t^i to the utility function of each policy maker, where dum_t^i is a dummy variable, which takes the value one if policy maker i holds office in period t and takes the value zero otherwise. The parameter λ is a measure of how much value the policy maker attaches to holding office.

Solution of the model with endogenous elections

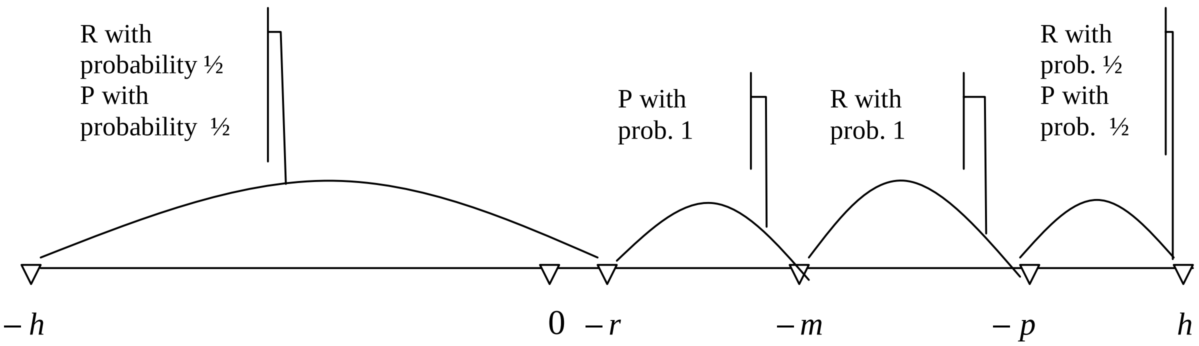
Suppose that policy maker P chooses $x_1 = 0$ and policy maker R chooses $x_1 = 1$. In this case policy is always in line with the policy preferred by the median voter. When the median voter prefers policy $x_1 = 1$ to $x_1 = 0$, she votes for R. In period 2 the election outcome

[4] The second term in the expression is equal to the predisposition of the policy maker, who is indifferent between

depends on μ. If $\mu > -m$ the median voter votes for R and if $\mu \le -m$ she votes for P. If the median voter prefers $x_1 = 0$ to $x_1 = 1$, she votes for P in the first elections.

Let us now identify the conditions under which R chooses $x_1 = 1$, given that P chooses $x_1 = 0$ and $m > (-3 + 2\sqrt{2})h$. When R chooses $x_1 = 0$, then $dum_2^R = 1$ with probability $\frac{1}{2}$ and $dum_2^R = 0$ with probability $\frac{1}{2}$. When R chooses $x_1 = 1$, then $dum_2^R = 1$ if $\mu \in (-m, -p]$, $dum_2^R = 0$ if $\mu \in (-r, -m]$ and $dum_2^R = 1$ with probability $\frac{1}{2}$ if $\mu \in [-h, -r]$ and if $\mu \in (-p, h]$ (see figure 2 below).

Figure 2. Winners of the elections depending on the value of μ



Hence, when policy maker R chooses $x_1 = 1$, his expected total utility is

$$
\begin{aligned}
&r + \lambda + \Pr(\mu \in (-m, h])\left(r + E(\mu \mid \mu \in (-m, h])\right) \\
&+ \lambda[\Pr(\mu \in (-m, -p]) + \frac{1}{2}\Pr(\mu \in [-h, -r]) + \frac{1}{2}\Pr(\mu \in (-p, h])] \quad (12) \\
&= r + \lambda + \frac{1}{2h}(h+m)(r + \frac{1}{2}(h-m)) + \frac{1}{2h}\lambda\left[h + m - \frac{1}{2}(r+p)\right]
\end{aligned}
$$

undertaking learning and not doing that in the basic case without the elections.

When R chooses $x_1 = 0$, his expected utility is equal to $\lambda + \frac{1}{2}\lambda$. Consequently, R prefers $x_1 = 1$ to $x_1 = 0$ if expression (12) is larger than $\frac{3}{2}\lambda$. It is easy to see that for a large enough value of λ, R chooses $x_1 = 0$ if $m < \frac{1}{2}(r + p)$, i.e. if the preferences of the median voter are closer to those of P than to those of R. Thus, although the median voter prefers $x_1 = 1$ to $x_1 = 0$ neither R nor P chooses $x_1 = 1$. The possible choice of R for $x_1 = 0$ is induced by electoral considerations: $x_1 = 1$ would reduce his probability of winning the second elections.

Turn now to the conditions under which P chooses $x_1 = 0$ given that R chooses $x_1 = 1$. Following the same steps as above, it is straightforward to derive that a median voter, who prefers $x_1 = 0$ to $x_1 = 1$, may not be able to choose her policy when neither R nor P chooses $x_1 = 0$. This occurs for a large enough utility of holding office λ and if the preferences of the median voter are closer to those of R than to those of *P*.

From the above analysis two results emerge. First, when polarization induces the two candidates to choose different policies voters are always able to choose their most preferred policy. In this case, polarization does not distort active learning, in the sense that society engages in active learning when a majority of voters prefers active learning to no learning. Second, electoral considerations may induce candidates to choose the same policy. Then, too little or too much learning may occur from the median voter's point of view.

5. Concluding Remarks

Learning through experimentation –or active learning- occurs when a decision maker makes a decision that reduces current utility, but leads to information that is expected to improve future decision making. In this paper, we have addressed the question how polarization and political instability affect learning through experimentation in a two-party system where a policy maker with different preferences may succeed the incumbent. Two cases have been distinguished.

First, we have analyzed a model in which a policymaker must make a binary decision about a project, which does not affect her chances of reelection. We have shown that in this case a higher degree of polarization generally reduces the incumbent's incentive to learn through experimentation. However, the relationship between polarization and learning through experimentation is not continuous. It matters whether the successor is biased against implementation or is biased towards implementation. When the successor is biased towards implementation, the incumbent may have a stronger incentive to implement the project than when the successor is biased against the project.

Next, we have analyzed a model in which elections revolve around the project under consideration. We have shown that when the two candidates choose different policies, policy always accords with the policy most preferred by a majority of voters. However, electoral considerations may induce parties to choose the same policy. The reason is that a candidate, who prefers learning through experimentation from an ideological point of view, may reduce her chances of reelection by implementing the project. We have argued that when the two candidates choose the same policy, policy may differ from the policy most preferred by a majority of voters.

Our analysis is based on several restrictive assumptions. Some of them are made for simplification and are innocuous. For example, we have assumed that policy makers do not discount the future and that once the project has been implemented its consequences are known. Relaxing these assumptions does not affect our results qualitatively. Two other assumptions are less innocuous. First, our model revolves around a single project. As a consequence, voters evaluate candidates on the basis of a single issue. In reality, policy makers make decisions about numerous projects, old and new ones. It is easy to show that when policy makers must make decisions about more than one project, voters sometimes face a trade-off between learning about new projects and repealing unfavorable, old projects.

Second, our model focuses on projects that can be either implemented or not. Often policy makers must make binary decisions. However, it is unclear that our results generalize to decisions about continuous variables.

Though our results are derived from a highly stylized model, we believe that they are important for two reasons. First, our analysis gives insights into the way polarization and political instability affect policy making under uncertainty. Second, our results have normative implications. Nowadays, several scientists are thinking about ways to transform the public sector into a learning organization (OECD, 1999). Our analysis points out that polarization and political instability are potential obstacles to such a transformation.

References:

Bertocchi, G. (1993). A theory of public debt management with unobservable demand. *Economic Journal* 103: 960-974

Bertocchi, G. and M. Spagat (1993). Learning, experimentation, and monetary policy. *Journal of Monetary Economics* 32: 169-183

Bertocchi, G. and M. Spagat (1997). Structural uncertainty and subsidy removal for economies in transition. *European Economic Review* 41:1709-1733

Grossman, S.J., Kihlstrom, R.E. and L.J.Mirman (1977). A Bayesian approach to the production of information and learning by doing. *Review of Economic Studies* 44: 533-547

OECD (1999). *Improving Evaluation Practices*. PUMA/PAC(99)1.

Prescott, E.C. (1972). The multi-period control problem under uncertainty. *Econometrica* 40, N6: 1043–1058

Rothschild, M. (1974). A two-armed bandit theory of market pricing. *Journal of Economic Theory* 9: 185-202

Appendix A.

In this appendix we derive (7) - (9). Straightforward differentiation of (5) with respect to r, π and h results in the following expressions:

(i) $\frac{\partial p^I_{r\leq 0}}{\partial r} = \frac{(1-\pi)[r - p^I_{r\leq 0}]}{3h + (1-\pi)r + \pi p^I_{r\leq 0}}$

Since $|r| < h$, $|p^I_{r\leq 0}| < h$ and $\pi \in (0,1)$, $\frac{\partial p^I_{r\leq 0}}{\partial r} > 0 \quad if\ r > p^I_{r\leq 0}$ and $\frac{\partial p^I_{r\leq 0}}{\partial r} \leq 0 \quad if\ r \leq p^I_{r\leq 0}$.

(ii) $\frac{\partial p^I_{r\leq 0}}{\partial \pi} = -\frac{[r - p^I_{r\leq 0}]^2}{2[3h + (1-\pi)r + \pi p^I_{r\leq 0}]} \leq 0$ since the numerator in the ratio is non-negative and the denominator is positive (see (i) in this appendix).

(iii) $\frac{\partial p^I_{r\leq 0}}{\partial h} = -\frac{h + 3p^I_{r\leq 0}}{3h + (1-\pi)r + \pi p^I_{r\leq 0}} < 0$ since the numerator in the ratio is positive ($p^I_{r\leq 0} > (-3+\sqrt{2})h > -\frac{h}{3}$), and so is the denominator (see (i) in this appendix).

Appendix B

In this appendix we derive the analogue of expression (7) for the case $r > 0$. Let $p^I_{r>0} > 0$ denote the value of p, for which P is indifferent between choosing $x_1 = 1$ and choosing $x_1 = 0$ given that $r > 0$. Equation (6) implicitly defines $p^I_{r>0}$ as a function of r, π and h. Straightforward differentiation of (6) with respect to r, taking account of the implicit function theorem, results in the following expression:

$$\frac{\partial p^I_{r>0}}{\partial r} = \frac{(1-\pi)[r - p^I_{r>0}]}{h(1+2\pi)+(1-\pi)r+\pi p^I_{r>0}} > 0$$ since $r > p^I_{r>0}$ and

$-\pi\, p^I_{r>0} \le -p^I_{r>0} < h < (1+2\pi)h \le (1+2\pi)h + (1-\pi)r$ for $\pi \in (0,1)$.