How Much is Location Information Worth? A Competitive Analysis of the Online Traveling Salesman Problem with Two Disclosure Dates

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ERIM REPORT SERIES *RESEARCH IN MANAGEMENT*	
ERIM Report Series reference number	ERS-2008-075-LIS
Publication	November 2008
Number of pages	23
Persistent paper URL	http://hdl.handle.net/1765/13837
Email address corresponding author	jsrour@rsm.nl
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REPORT SERIES
RESEARCH IN MANAGEMENT

ABSTRACT AND KEYWORDS	
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Free Keywords	traveling salesman, advanced information, competitive ratio, worst-case ratio, online routing
Availability	The ERIM Report Series is distributed through the following platforms: Academic Repository at Erasmus University (DEAR), DEAR ERIM Series Portal Social Science Research Network (SSRN), SSRN ERIM Series Webpage Research Papers in Economics (REPEC), REPEC ERIM Series Webpage
Classifications	The electronic versions of the papers in the ERIM report Series contain bibliographic metadata by the following classification systems: Library of Congress Classification, (LCC) LCC Webpage Journal of Economic Literature, (JEL), JEL Webpage ACM Computing Classification System CCS Webpage Inspec Classification scheme (ICS), ICS Webpage

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Abstract

In this paper we derive the worst-case ratio of an online algorithm for the Traveling Salesman Problem (TSP) with two disclosure dates. This problem, a variant of the online TSP with release dates, is characterized by the disclosure of a job's location at one point in time followed by the disclosure of that job's release date at a later point in time. We present an online algorithm for this problem restricted to the positive real number line. We then derive the worst-case ratio of our algorithm and show that it is best-possible in two contexts – the first, one in which the amount of time between the disclosure events and release time are fixed and equal for all jobs; and a second in which the time between disclosure events varies for each job. We conclude that the value of advanced information can be attributed to the location information alone – yielding an optimal solution in favorable instances.

Keywords: Traveling Salesman; advanced information; competitive ratio; worst-case ratio; online routing

1 Introduction

Intensely, widely, and well-studied – not to mention important – are all adjectives used to describe the well-known Traveling Salesman Problem (TSP). In short the TSP addresses the problem of finding the shortest tour (beginning and ending at a depot or origin city) through a set of jobs or cities in a given metric space. If the salesman is traveling at constant speed, finding the shortest path is equivalent to minimizing the time the salesman returns to the depot. The literature on this problem begins with the seminal papers by Dantzig et al. (1954) and Flood (1956), includes at least four books (Lawler et al., 1985; Reinelt, 1994; Gutin and Punnen, 2002; Applegate et al., 2007), multiple survey papers (Bellmore and Nemhauser, 1968; Burkard et al., 1995; Jünger et al., 1995, 1997), and a myriad of articles.

The TSP literature contains a variety of extensions to the basic problem formulation. The extension we are interested in is known as the "TSP with release dates". In this variation, the salesman may visit each job only on or after a specified release date. One method of solving this problem is to assume that all of the job locations and their release dates are known before a solution algorithm is implemented. This method, referred to as the offline optimization approach, is not particularly realistic. In the majority of real-world applications, jobs (or cities) and their release times are revealed over time – often after the salesman has already left the city of origin (or depot). Methods designed to handle the arrival of new information during execution are termed online algorithms.

We may add a further level of realism by assuming that the exact location of each job is also revealed over time. Specifically, the location of each job may be revealed in advance of information on the release date, which is revealed in advance of the actual release date. For example, consider a dray company, such as that documented in Mahr et al. (2008), that must pick up containers from several port terminals. In the morning, the dray provider learns the location of the terminals that will release containers for transport. Later in the day, the company learns the exact time at which those containers will be released from customs for pick-up; for some terminals this information may come early, for others this information may arrive much later in the day. While the dray company could wait until all information is known, the containers will certainly be served sooner if the company can cleverly exploit each piece of information when it arrives. For this the dray provider needs an online algorithm.

In this small example, the subproblem of finding the best ordering of the jobs for a single truck is equivalent to the TSP. For this reason we examine, in this paper, a problem we term the online TSP with two disclosure dates. For the ease of analysis we restrict the metric space to the non-negative real number line with the depot located at the origin, $\mathbb{R}_0^+$. We are thus able to indicate the value of each piece of information via the worst-case ratio of the online algorithm cost to the offline optimal algorithm cost.

1.1 Literature Review

The offline TSP with release dates on $\mathbb{R}_0^+$ is not new. Psaraftis et al. (1990) introduced this problem as one of routing and scheduling along a shoreline. They examine both path and tour versions of the problem and demonstrate that in the tour version on such a restricted metric space these problems are trivially solved in polynomial time.

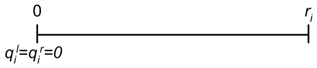
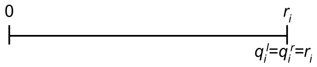
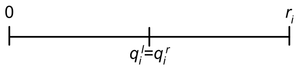
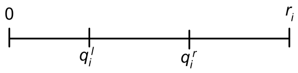
Blom et al. (2001) provide an algorithm with a worst-case ratio of $\frac{3}{2}$ for the online variant of this problem; they term this version of the problem the *online TSP* (OLTSP). They prove that their algorithm, Move-Right-If-Necessary (MRIN), is best-possible for the OLTSP with release dates. MRIN is a zealous algorithm that sends the salesman immediately to any job on the right and back to the origin (left) if there are no other jobs to the right. Jaillet and Wagner (2006) and Wagner (2006), however, note that the result of Blom et al. (2001) is dependent on the assumption that the disclosure time of a job's location and release time occurs at the moment of release. In this way, Jaillet and Wagner (2006) formulate a TSP scenario with advanced information and demonstrate the benefit of that advanced information.

Specifically, Jaillet and Wagner (2006) introduce a disclosure time, at which both the location and the release time are announced. If this disclosure time is equal to the release time then we are in the case where MRIN yields a solution with worst-case ratio of $\frac{3}{2}$. If, however, the disclosure time occurs a fixed amount of time in advance of the release date then the worst-case ratio for an arbitrary homing (or tour) online algorithm improves to at least $(\frac{3}{2} - \frac{a}{2l_{\max}}) \in [1, \frac{3}{2}]$ where a is the fixed amount of advanced notice time and $l_{\max}$ is the location of the job farthest from the origin on $\mathbb{R}_0^+$. Note, we use the expression homing in a manner similar to Ausiello et al. (2001) in order to indicate that the algorithm must return to the depot or origin at the point in time when all known jobs have been served.

1.2 Our Contribution

We position our work as depicted in Table 1. In this table the name of the problem examined appears in the far left column. The second column provides a graphical depiction of information arrival over time that characterizes the associated problem; note, q_i^l represents the time the location of a job $i \in N = \{1, \ldots n\}$ is disclosed, q_i^r represents the time the release time of job i is disclosed, and r_i represents the release time of the job. The third and fourth columns indicate the main result and reference for the associated problem, respectively. This table emphasizes the focus of our work on the impact of early location disclosure in the context of the TSP on $\mathbb{R}_0^+$.

Table 1: Overview of work to date and our contribution.

Problem	Depiction of Info. Arrival (Time)	Main Result on $\mathbb{R}_0^+$	Reference
Offline TSP with Release Dates		Optimal algorithm in $O(n)$ time.	(Psaraftis et al., 1990)
Online TSP with Release Dates		Best-possible algorithm with worst-case ratio of $\frac{3}{2}$.	(Blom et al., 2001)
Online TSP with Disclosure Dates		Disclosure dates give advantage over release dates; worst-case ratio for both fixed and variable advanced notice is dependent on time between disclosure and release, but bounded by $\frac{3}{2}$.	(Jaillet and Wagner, 2006)
Online TSP with Two Disclosure Dates		Advanced location information gives an advantage over simultaneous disclosure dates; worst-case ratio for both fixed and variable advanced notice is dependent on time between both disclosure dates and the release time, but bounded by $\frac{3}{2}$.	This paper

In our case, we have two disclosure dates – the disclosure of the job location and the disclosure of the release time. This split information arrival serves to give our online algorithm a greater advantage over other online algorithms in comparison to the optimal offline strategy. This case is also more realistic as there are many real-world instances in which the job locations are known early in execution, but the release times come later. We begin by introducing an online algorithm for $\mathbb{R}_0^+$ designed to exploit both pieces of information as they are made available. Our online algorithm is a homing algorithm as the salesman must return to the origin following the completion of all known jobs.

We prove that our online algorithm is best-possible with a worst-case ratio of $\max\left\{1, \frac{3}{2} - \frac{(a+b)}{2l_{\max}}\right\}$, where a and b represent fixed amounts of time between the disclosure events and release of the job.

We also address the case of variable amounts of advanced notice (i.e. the case where a and b vary by job taking any positive real value). In this case we obtain a ratio of

$$1 \leq 1 + \min\left\{\left(\frac{\max_{i\in N}\left\{q_i^l + 2l_i\right\}}{\max_{i\in N}\left\{r_i + l_i\right\}} - 1\right)^+, \left(\frac{\max_{i\in N}\left\{q_i^l + 2l_i\right\}}{\max_{i\in N}\left\{2l_i\right\}} - 1\right)\right\} \leq \frac{3}{2}$$

We show that this ratio is the best possible in this setting.

The remainder of this paper is organized as follows: in Section 2 we state the problem of interest in mathematical terms and define the necessary notation; we also present in greater detail the optimal offline algorithm and online algorithms for the TSP with release dates and TSP with disclosure dates. In Section 3 we present our algorithm, Move-Right-Early-Left-Late (MRELL), for the OLTSP on $\mathbb{R}_0^+$ with two disclosure dates. In Section 4 we derive the worst-case ratio for the case in which the amount of time between disclosure events is fixed; we also demonstrate that MRELL is best possible in this case. In Section 5 we study the case in which the amount of time between disclosure events varies across jobs; we demonstrate that MRELL is best possible for that case as well. Finally, we conclude with a discussion of these results and statement of future research in Section 6.

2 Assumptions, Notation, and Preliminaries

To facilitate an understanding of the exact nature of the problem under consideration, we begin by stating some assumptions and describing the notation we will use throughout this paper.

1. All job locations are along the positive real number line, $\mathbb{R}^+$.
2. The origin is at the point, 0, on $\mathbb{R}^+$ which is where the salesman begins at the start of each problem at time 0 and must return to after visiting all jobs.
3. The location of a job, i, is only revealed to the salesman at a time in advance of its release time (and the disclosure of that time); this location disclosure time is denoted q_i^l.
4. A job's release time is only revealed to the salesman at a time after the disclosure of its location, but before the time of release; this release disclosure time is denoted q_i^r.
5. The salesman always travels at unit speed along $\mathbb{R}^+$; otherwise he is idle.
6. The objective of this online TSP is to minimize the time required to serve all jobs and return to the origin.
7. In the online problem, the salesman does not know in advance how many jobs are in a single problem instance. In the offline problem, all jobs and their release times are known *a priori*.
8. A problem instance, N, is a collection of n jobs, numbered $1, \ldots, n$.

Note, we can completely describe a job $i \in N$ by the following vector: (q_i^l, q_i^r, r_i, l_i) where l_i represents the location of job i on $\mathbb{R}_0^+$; q_i^l is the point in time at which l_i is revealed; and q_i^r is the point in time at which r_i is revealed, where r_i represents the release time of job i. In our variation of the online TSP, the information arrives such that $0 \leq q_i^l \leq q_i^r \leq r_i$.

We further specify $l_{\max}$ to represent the job that is farthest from the origin; that is, $l_{\max} = \max_{i \in N} \{l_i\}$. Similarly, $r_{\max} = \max_{i \in N} \{r_i\}$ represents the job that is released the latest. The job at $l_{\max}$ is not necessarily the same job with release time $r_{\max}$. The notation $(x)^+$ is used as a short hand for $\max\{x, 0\}$.

As the remainder of this document focusses on competitive analysis, we use the notation $C_A(N)$ to represent the cost of an algorithm, A, on an instance, N, of n jobs. Furthermore, we define the performance ratio of an algorithm A on an instance N as $\frac{C_A(N)}{C_{OPT}(N)}$. The value ρ_A is defined as the infimum over all performance ratios, which implies that $C_A(N) \leq \rho_A C_{OPT}(N)$ for any instance N. In the literature, ρ_A is referred to as both the *competitive ratio* and the *worst-case ratio*; we will use the term worst-case ratio. A best possible algorithm is thus defined as an algorithm guaranteed to achieve a performance ratio less than or equal to the infimum over all algorithms of ρ_A. Finally, throughout this paper we use the language of Jaillet and Wagner (2006) when writing out the relevant algorithms and affiliated costs.

The remainder of this section is divided into two subsections – the first in which we describe the optimal offline algorithm of Psaraftis et al. (1990) and the second in which we describe the online algorithms of Blom et al. (2001) and Jaillet and Wagner (2006).

2.1 Optimal Offline Algorithm for the TSP on $\mathbb{R}^+$ with Release Dates

The offline version of the TSP with release dates on $\mathbb{R}_0^+$ was first introduced in the context of routing and scheduling on a shoreline by Psaraftis et al. (1990). They propose an optimal offline algorithm entitled TRAVERSE and prove that it solves the problem exactly in $O(n)$ time. The formal steps of the algorithm are repeated here, for convenience. TRAVERSE works by going to the farthest job from the origin, waiting at that job until the point in time where a smooth (i.e. no waiting) return to the origin can be made.

Algorithm 1 TRAVERSE or OPT

1. Go directly to job $l_{\max}$.
2. Wait at $l_{\max}$ for $\max_{i \in N} \{\max\{0, r_i - (2l_{\max} - l_i)\}\}$ units of time.
3. Proceed directly back to the origin.

It is clear that the cost of this algorithm (that is the earliest point in time the salesman will return to the origin) is the time required to travel to the farthest location and back to the origin plus any waiting time incurred at that farthest location. Thus, the following is a closed form expression for $C_{TRAVERSE}(N)$ (termed $C_{OPT}(N)$ for future reference):

$$C_{OPT}(N) = \max_{i \in N} \{\max\{2l_i, r_i + l_i\}\} \tag{1}$$

2.2 Online TSP Algorithms

In this subsection, we review two different cases of advanced information arrival; for each we present the best-possible online algorithms. The first case is one in which a job's location and release time are disclosed at the moment of release, that is $q_i^l = q_i^r = r_i$. This first case is identical to that of the "OLTSP with release dates" originally proposed and studied by Blom et al. (2001). The second case is one in which the location and release time are disclosed simultaneously at a time in advance of the release time, that is $q_i^l = q_i^r < r_i$. This second case is identical to the "OLTSP with disclosure dates" originally proposed by Jaillet and Wagner (2006).

2.2.1 OLTSP with Release Dates

In their study of zealous algorithms and fair adversaries for the OLTSP with release dates, Blom et al. (2001) specify the Move-Right-if-Necessary (MRIN) algorithm as a strategy in the $\mathbb{R}_0^+$ metric space. MRIN is a zealous algorithm in which the salesman moves to jobs on his right as soon as they are released and returns to the origin if there are no more jobs on the right.

Algorithm 2 MRIN

1. If there is an unserved job to the right of the salesman, he moves toward it at unit speed.
2. If there are no unserved jobs to the right of the salesman, he moves back toward the origin at unit speed.
3. Upon reaching the origin, the salesman becomes idle.

Blom et al. (2001) show that MRIN is a best-possible online algorithm for the OLTSP with release dates on $\mathbb{R}_0^+$ with a worst-case ratio of $\frac{3}{2}$. Therefore, in the case where $q_i^l = q_i^r = r_i$, MRIN is the best possible strategy.

2.2.2 OLTSP with Disclosure Dates

In their study of online routing problems, Jaillet and Wagner (2006) introduce the OLTSP with disclosure dates and specify the Move-Left-If-Beneficial (MLIB) algorithm as a strategy in the $\mathbb{R}_0^+$ metric space. MLIB is based on the idea that with prior knowledge of jobs to the left of the salesman it is better to wait as far right for as long as possible. In this way, MLIB represents a compromise strategy between the optimal offline, TRAVERSE algorithm and the online MRIN strategy.

Algorithm 3 MLIB

1. If there is an unserved job to the right of the salesman, he moves toward it at unit speed.
2. If there are no unserved jobs to the right of the salesman, he moves back toward the origin if and only if the return trajectory reaches all unserved jobs on or after their release date; otherwise the salesman remains idle at his current location.
3. Upon reaching the origin, the salesman becomes idle.

Jaillet and Wagner (2006) show that MLIB is a best-possible online algorithm for the OLTSP with disclosure dates on $\mathbb{R}_0^+$ when the amount of advanced notice (i.e. the time between disclosure and release) is fixed. We extend their results slightly to show that MLIB is also best-possible when the amount of advanced notice is variable (see Section 5). In both settings (fixed and variable advanced notice) the worst-case ratio of MLIB is not constant and instead varies based on the amount of advanced notice; nevertheless the worst-case ratio never exceeds $\frac{3}{2}$. Therefore, in the case where $q_i^l = q_i^r \leq r_i$, MLIB is the best possible strategy.

3 OLTSP with Two Disclosure Dates

The primary focus of this paper is one in which the location is disclosed earlier than the release time which is disclosed earlier than the release itself, that is $q_i^l \leq q_i^r \leq r_i$. In this instance, we can construct an algorithm that not only exploits the advanced release information but also the earlier disclosed location information. The Move-Right-Early-Left-Late (MRELL) algorithm is based on the idea that it is better to wait as far in the field as long as possible than hastily return to the origin.

Note that if this algorithm is applied to a case where $q_i^l = q_i^r = r_i$ then MRELL is

Algorithm 4 MRELL

1. If there is a job for which the location has been revealed to the right of the salesman he moves towards it at unit speed.
2. If there are no jobs to the right of the salesman, he moves back to the origin according to the following rules:
   (a) If the salesman knows the release time of all the jobs to his left, whose locations have been disclosed, then the salesman returns to the origin at the point in time that allows him to pass all jobs on or after their release time.
   (b) If the salesman knows the release time of only some of all jobs to his left, whose locations have been disclosed, then the salesman remains idle until the time that allows him to pass all release time disclosed jobs on or after their release time, but the salesman must stop along this trajectory and wait at any job for which only the location is known.
   (c) If the salesman knows none of the release times for all the location-disclosed jobs to his left, then he moves toward the nearest job waiting there until its release.
3. Upon reaching the origin, the salesman remains idle.

equivalent to MRIN (Blom et al., 2001). Furthermore, if this algorithm is applied to a case where $q_i^l = q_i^r < r_i$ then MRELL is equivalent to MLIB (Jaillet and Wagner, 2006). Additionally, note that if $0 = q_i^l = q_i^r < r_i, \forall i \in N$ then this algorithm is indistinguishable from the optimal offline algorithm (see Psaraftis et al., 1990).

Lemma 1. *The cost of MRELL is bounded as follows:*

$$C_{MRELL}(N) \leq \max_{i \in N} \left\{ \max \left\{ q_i^l + 2l_i, r_i + l_i \right\} \right\} \tag{2}$$

Proof Using logic similar to Jaillet and Wagner (2006), we derive the cost of MRELL by analyzing the final segment of the salesman's journey. That is, the segment of the salesman's journey in which he leaves a job to return directly to the origin without stopping to wait at any other job along the way. We say that this final segment will begin at a time, t_0 with the salesman arriving at the origin at time $z = C_{MRELL}(N)$. According to the algorithm, MRELL, the salesman may begin his final segment to the origin from any job (a job we will term the *final departure job*) to the right of the origin, on the condition that all jobs in between the final departure job and the origin will be passed on or after their release time. We proceed by analyzing two cases.

1. Salesman leaves final departure job as soon as he arrives.

   This represents the case where the salesman arrives to the final departure job after the release time of that job and at a point in time at which all jobs between that final departure job and the origin can be passed on or after their release times. Note that in this case, the salesman was traveling away from the origin just before turning back for the final segment at the final departure job. Thus, the salesman begins his return segment immediately after arriving to the final departure job, k. This gives us that $t_0 = \textit{arrival to } k \leq q_k^l + l_k$. Note that $q_k^l + l_k$ represents departure from the origin and hence the worst case. Thus, $z \leq q_k^l + l_k + l_k = q_k^l + 2l_k$.

2. Salesman leaves final departure job after waiting.

   This case represents a situation where the salesman must wait for the release of some job between the final departure job and the origin (possibly the final departure job itself). In this second case the salesman will already have spent some time at the final departure job before returning to the origin - thus he may have come to that final departure job from either the left or the right. In this case the final segment is timed to pass through some job, m, at a time $t > t_0$ such that $t = r_m$ and r_m is the latest release time remaining. Thus, the salesman will finish the final segment at $z = r_m + l_m$.

Because the last segment of the salesman's trajectory can only be of one case type, we may say that $z \leq \max\left\{q_k^l + 2l_k, r_m + l_m\right\}$. Furthermore, because these cases represent the latest event in the trajectory of the salesman we can write, $C_{MRELL}(N) = z \leq \max_{i \in N}\left\{\max\left\{q_i^l + 2l_i, r_i + l_i\right\}\right\}$. □

The following corollary further illustrates the relationship between $C_{MRELL}(N)$ and $C_{OPT}(N)$. Corollary 1 will also be used in proving Theorem 2.

Corollary 1. *If $q_i^l = 0$ and $q_i^r \leq r_i, \forall i \in N$, then $C_{MRELL}(N) = C_{OPT}(N)$.*

Related to Corollary 1 we have Lemma 2 that will be used in the proof of both Theorem 2 and Theorem 4.

Lemma 2. *For any instance, N, of the online TSP with release dates on $\mathbb{R}^+$, we can construct a related instance, $\tilde{N}$, in which all jobs in the set $Q = \left\{i \in N \mid q_i^l = 0\right\}$ are excluded. The performance ratio for instance $\tilde{N}$ will not be less than the performance ratio for instance N.*

Proof Let $C_{MRELL}(\tilde{N})$ be the cost of MRELL on the instance $\tilde{N} = N \setminus Q$; similarly let $C_{OPT}(\tilde{N})$ be the cost of OPT on the instance $\tilde{N} = N \setminus Q$.

If $\max_{i \in Q} \{r_i + l_i, 2l_i\} \geq C_{MRELL}(\tilde{N})$ then $C_{MRELL}(N) = \max_{i \in Q} \{r_i + l_i, 2l_i\} = C_{OPT}(N)$; which gives us a performance ratio of 1. As the performance ratio for instance $\tilde{N}$ must be greater than or equal to 1 we have that the performance ratio for instance $\tilde{N}$ will not be less than the performance ratio for instance N.

If instead, $\max_{i \in Q} \{r_i + l_i, 2l_i\} < C_{MRELL}(\tilde{N})$ then $C_{MRELL}(\tilde{N}) = C_{MRELL}(N)$. Since, $C_{OPT}(\tilde{N}) \leq C_{OPT}(N)$, the performance ratio for instance N is less than or equal to the performance ratio of $\tilde{N}$. □

4 Fixed Amounts of Advanced Notice

In this case, we imagine that the salesman is told the location of each job at a point in time $(a + b)$ units of time before the release of the job. Similarly, the release time of each job is announced a units of time before the release of the job. We may also write this as follows. For each job in a problem instance, there exist constants a and b such that $(a + b) \in [0, r_{\max}]$, yielding $q_i^r = (r_i - a)^+, \forall i \in N$ and $q_i^l = (r_i - a - b)^+, \forall i \in N$. Given this notation and noting that $2l_{\max}$ is a lower bound on the length of the optimal TSP tour through all jobs, we have the following theorem.

Theorem 1. *Let A be an arbitrary homing online algorithm with cost $C_A(N)$ on an instance of n jobs. Then for all $n \geq 2$ there exists an instance N of size n where the performance ratio is at least* $\left[\frac{3}{2} - (\frac{a+b}{2l_{\max}})\right] \in [1, \frac{3}{2}]$.

Proof Using logic similar to Jaillet and Wagner (2006), we begin by establishing an arbitrary instance N' of $n - 1$ jobs. Given this instance, the time at which the salesman finishes serving all $n-1$ jobs and returns to the origin is given by our arbitrary algorithm, A, as $C_A(N')$. We now designate an n^{th} job which is further out on $\mathbb{R}^+$ than any of the previous $n - 1$ jobs. Thus, $l_n = l_{\max}$. To specify the exact location of $l_{\max}$, we note that $C_A(N') \geq C_{OPT}(N') \geq 2l_i, \forall i \in N'$. Thus, by setting l_n equal to $C_A(N')$ plus some constant term, we are assured that l_n is $l_{\max}$ for this instance of n jobs. We therefore select $l_n = (a + b) + C_A(N')$. Note, if $(a + b) = 0$ then $q_n^l = r_n$ and the analysis of Blom et al. (2001) applies thus completing our proof. However, if $(a + b) > 0$, we obtain the following description of job n:
$(q_n^l, q_n^r, r_n, l_n) = (C_A(N'), a + C_A(N'), (a + b) + C_A(N'), (a + b) + C_A(N'))$.

Given this job and the knowledge that the salesman is at the origin at $q_n^l = C_A(N')$, we obtain the following:
$C_A(N) \geq q_n^l + 2l_n = 3C_A(N') + 2(a+b)$.

Turning our attention to the optimal offline algorithm, we have:

$$C_{OPT}(N) = \max\{\max\{2l_i, r_i + l_i\}\} = 2C_A(N') + 2(a+b)$$

As $(a+b) > 0$ then $C_{OPT}(N) > 0$. We now obtain the desired result:

$$\begin{aligned} \frac{C_A(N)}{C_{OPT}(N)} &\geq \frac{3C_A(N') + 2(a+b)}{2C_A(N') + 2(a+b)} \\ &= 1 + \frac{C_A(N')}{2l_{\max}} \\ &= 1 + \frac{l_{\max} - (a+b)}{2l_{\max}} \\ &= \frac{3}{2} - (\frac{a+b}{2l_{\max}}) \end{aligned}$$

Given that $(a+b) \leq l_{\max}$, we conclude that $\frac{3}{2} - (\frac{a+b}{2l_{\max}}) \in [1, \frac{3}{2}]$. □

Theorem 2. *When the amount of advanced notice is fixed such that,* $q_i^r = (r_i - a)^+$ *and* $q_i^l = (r_i - a - b)^+, \forall i \in N$, *then MRELL is a best-possible algorithm.*

Proof Define $\mathfrak{L} = \left\{i \in N | q_i^l > 0\right\}$. Note that if $\mathfrak{L} = \emptyset$ then the location of all jobs are known at the start of the day. Thus, by Lemma 2, we obtain $C_{MRELL}(N) = C_{OPT}(N)$. However, if $\mathfrak{L}$ is not empty, then we rewrite inequality 2 as:

$$C_{MRELL}(N) \leq \max\left\{\max_{i \in \mathfrak{L}}\left\{\max\left\{q_i^l + 2l_i, r_i + l_i\right\}\right\}, \max_{i \in N \setminus \mathfrak{L}}\left\{\max\left\{2l_i, r_i + l_i\right\}\right\}\right\}$$

Now, by Lemma 2 we can ignore all the jobs not in $\mathfrak{L}$ without risk of reducing the competitive ratio. Thus we obtain:

$$C_{MRELL}(N) \leq \max_{i \in \mathfrak{L}}\left\{\max\left\{q_i^l + 2l_i, r_i + l_i\right\}\right\}$$

Using the definition of q_i^l we do the following algebra:

$$\begin{aligned} C_{MRELL}(N) &\leq \max_{i\in\mathfrak{L}} \left\{\max\left\{q_i^l + 2l_i, r_i + l_i\right\}\right\} \\ &= \max_{i\in\mathfrak{L}} \{\max\{r_i - a - b + 2l_i, r_i + l_i\}\} \\ &= \max_{i\in\mathfrak{L}} \{r_i + l_i + \max\{l_i - a - b, 0\}\} \\ &\leq \max_{i\in\mathfrak{L}} \{r_i + l_i + \max\{l_{\max} - a - b, 0\}\} \end{aligned} \tag{3}$$

We now analyze two cases:

Case 1: $l_{\max} - a - b < 0 \Leftrightarrow l_{\max} < a + b$. This case implies that $C_{MRELL}(N) \leq \max_{i\in\mathfrak{L}} \{r_i + l_i\} \leq C_{OPT}(N)$ which implies that $C_{MRELL}(N) = C_{OPT}(N)$.

Case 2: $l_{\max} - a - b \geq 0 \Leftrightarrow l_{\max} \geq a + b$. In this case we may rewrite inequality 3 in the following way.

$$\begin{aligned} C_{MRELL}(N) &\leq \max_{i\in\mathfrak{L}} \{r_i + l_i + l_{\max} - a - b\} \\ &= \max_{i\in\mathfrak{L}} \{r_i + l_i\} + l_{\max} - a - b \\ &\leq C_{OPT}(N) + l_{\max} - a - b \end{aligned}$$

Rewriting $l_{\max} - a - b$ as $\frac{l_{\max}-a-b}{l_{\max}} l_{\max}$, we obtain the desired result.

$$\begin{aligned} C_{MRELL}(N) &\leq C_{OPT}(N) + \frac{l_{\max} - a - b}{2l_{\max}} 2l_{\max} \\ &\leq C_{OPT}(N) + \frac{l_{\max} - a - b}{2l_{\max}} C_{OPT}(N) \\ &= \left[\frac{3}{2} - \frac{(a+b)}{2l_{\max}}\right] C_{OPT}(N) \end{aligned}$$

Thus, recognizing that these cases are disjoint we may state,

$$C_{MRELL}(N) \leq \max\left\{1, \frac{3}{2} - (\frac{a+b}{2l_{\max}})\right\} C_{OPT}(N)$$

. As Theorem 1 gives us that $\max\left\{1, \frac{3}{2} - (\frac{a+b}{2l_{\max}})\right\}$ is the lowest possible performance ratio for any algorithm we may conclude that MRELL is a best-possible algorithm.

□

5 Variable Amounts of Advanced Notice

In this subsection, we explore the worst-case ratio of MRELL in the context of variable amounts of advanced notice time for both the location and release time disclosures. In examining Lemma, 1 we note that the job driving the cost of MRELL (we will call this job d) can be one of two types: (1) the job may be such such that $\max_{i\in N}\left\{\max\left\{q_i^l+2l_i, r_i+l_i\right\}\right\} = r_d + l_d$ or (2) the job may be such that $\max_{i\in N}\left\{\max\left\{q_i^l+2l_i, r_i+l_i\right\}\right\} = q_d^l+2l_d$. If job d is of type one, then the cost of MRELL will be equal to the cost of the optimal offline algorithm. Given this phenomenon, the worst-case ratio is primarily determined by the value of $\max_{i\in N}\left\{q_i^l+2l_i\right\}$.

Theorem 3. *Let A be an arbitrary homing online algorithm with cost $C_A(N)$ on an instance, N, of n jobs. Then for all $n \geq 2$ there exists an instance of size n where the performance ratio is at least*

$$1+\min\left\{\left(\frac{\max_{i\in N}\left\{q_i^l+2l_i\right\}}{\max_{i\in N}\left\{r_i+l_i\right\}}-1\right)^+, \left(\frac{\max_{i\in N}\left\{q_i^l+2l_i\right\}}{\max_{i\in N}\left\{2l_i\right\}}-1\right)\right\} \in \left[1,\frac{3}{2}\right].$$

Proof Applying the same logic as in Theorem 1, we specify an arbitrary instance N' of $n-1$ jobs that the salesman serves and then returns to the origin. Thus, the salesman is at the origin at time $C_A(N')$. We now specify the n^{th} job at a location on $\mathbb{R}^+$ that is further from the origin than any other of the $n-1$ jobs with a release time later than all others. We may therefore describe the n^{th} job fully as follows.

$$\begin{aligned}\left(q_n^l, q_n^r, r_n, l_n\right) &= \left(q_{\max}^l, q_{\max}^r, r_{\max}, l_{\max}\right)\\ &= \left(C_A(N'), C_A(N')+\max_{i\in N\backslash n}\left\{r_i-q_i^r\right\}, C_A(N')+\delta, C_A(N')+\delta\right)\end{aligned}$$

Note that $\delta=\max_{i\in N\backslash n}\left\{r_i-q_i^l\right\}$.

This plus the knowledge that the salesman is at the origin at time $C_A(N')$ yields:

$$C_A(N) \geq q_n^l+2l_n = 3C_A(N')+2\delta$$

Turning our attention to the cost of the optimal offline algorithm we have:

$$C_{OPT}(N) = \max_{i\in N}\left\{\max\left\{2l_i, r_i+l_i\right\}\right\} = 2C_A(N')+2\delta$$

This gives us the following:

$$\begin{aligned}
\frac{C_A(N)}{C_{OPT}(N)} &\geq \frac{3C_A(N')+2\delta}{2C_A(N')+2\delta} \qquad (4)\\
&= 1+\frac{C_A(N')}{2C_A(N')+2\delta}\\
&= 1+\frac{C_A(N')+2\left(C_A(N')+\delta\right)-2\left(C_A(N')+\delta\right)}{2\left(C_A(N')+\delta\right)}\\
&\geq 1+\min\left\{\frac{q_n^l+2l_n-(r_n+l_n)}{r_n+l_n},\frac{q_n^l+2l_n-2l_n}{2l_n}\right\}\\
&= 1+\min\left\{\left(\frac{\max_{i\in N}\left\{q_i^l+2l_i\right\}}{\max_{i\in N}\left\{r_i+l_i\right\}}-1\right)^+,\left(\frac{\max_{i\in N}\left\{q_i^l+2l_i\right\}}{\max_{i\in N}\left\{2l_i\right\}}-1\right)\right\}
\end{aligned}$$

We now note that if we let δ decrease to 0 in equation (4), then this fraction increases to $\frac{3}{2}$; alternately if we take the limit of δ approaching ∞, then this fraction decreases to 1. Therefore,

$$1+\min\left\{\left(\frac{\max_{i\in N}\left\{q_i^l+2l_i\right\}}{\max_{i\in N}\left\{r_i+l_i\right\}}-1\right)^+,\left(\frac{\max_{i\in N}\left\{q_i^l+2l_i\right\}}{\max_{i\in N}\left\{2l_i\right\}}-1\right)\right\}\in\left[1,\frac{3}{2}\right].$$

□

The following theorem establishes that MRELL is also a best-possible algorithm in the context of variable notice.

Theorem 4. *When the amount of advanced notice varies for each job, $i\in N$, then $\frac{C_{MRELL}(N)}{C_{OPT}(N)}\leq 1+\min\left\{\left(\frac{\max_{i\in N}\left\{q_i^l+2l_i\right\}}{\max_{i\in N}\left\{r_i+l_i\right\}}-1\right)^+,\left(\frac{\max_{i\in N}\left\{q_i^l+2l_i\right\}}{\max_{i\in N}\left\{2l_i\right\}}-1\right)\right\}\leq\frac{3}{2}$ and MRELL is a best-possible algorithm.*

Proof Let $q_m^l+2l_m=\max_{i\in N}\left\{q_i^l+2l_i\right\}$, $r_p+l_p=\max_{i\in N}\left\{r_i+l_i\right\}$, and $2l_{\max}=\max_{i\in N}\left\{2l_i\right\}$. Note that jobs m, p, and max in the case of $l_{\max}$ may actually represent the same job depending on the instance. We further define two sets:

$$\begin{aligned}
\mathcal{L}(N) &= \left\{j\in N\mid q_j^l+2l_j=\max\left\{\max_{i\in N}\left\{q_i^l+2l_i,r_i+l_i\right\}\right\}\right\}\\
\mathcal{R}(N) &= \left\{k\in N\mid r_k+l_k=\max\left\{\max_{i\in N}\left\{q_i^l+2l_i,r_i+l_i\right\}\right\}\right\}
\end{aligned}$$

Given these two sets we proceed with the proof by examining three cases: (1) $\mathcal{L}(N)=$

$\emptyset, \mathcal{R}(N) \neq \emptyset$, (2) $\mathcal{L}(N) \neq \emptyset, \mathcal{R}(N) \neq \emptyset$, and (3) $\mathcal{L}(N) \neq \emptyset, \mathcal{R}(N) = \emptyset$. Note if $\mathcal{L}(N) = \mathcal{R}(N) = \emptyset$ then there are no jobs in the problem instance.

Case 1: $\mathcal{L}(N) = \emptyset, \mathcal{R}(N) \neq \emptyset$ In this case $p \in \mathcal{R}(N)$. Thus,

$$r_p + l_p = \max\left\{\max_{i\in N}\left\{q_i^l + 2l_i, r_i + l_i\right\}\right\} \geq 2l_{\max}$$

which implies that $C_{MRELL}(N) \leq r_p + l_p \leq C_{OPT}(N)$. Thus, $\frac{C_{MRELL}(N)}{C_{OPT}(N)} \leq 1 \leq \frac{3}{2}$. We further note, that in this case $r_p + l_p > q_m^l + 2l_m$. Hence $\frac{q_m^l+2l_m}{r_p+l_p} < 1$ which yields

$$\left(\frac{\max_{i\in N}\left\{q_i^l + 2l_i\right\}}{\max_{i\in N}\left\{r_i + l_i\right\}} - 1\right)^+ = 0.$$

Therefore, in this case,

$$\min\left\{\left(\frac{\max_{i\in N}\left\{q_i^l + 2l_i\right\}}{\max_{i\in N}\left\{r_i + l_i\right\}} - 1\right)^+, \left(\frac{\max_{i\in N}\left\{q_i^l + 2l_i\right\}}{\max_{i\in N}\left\{2l_i\right\}} - 1\right)\right\} = 0.$$

As a result we can conclude that in this case,

$$\begin{aligned}\frac{C_{MRELL}(N)}{C_{OPT}(N)} &\leq 1 + \min\left\{\left(\frac{\max_{i\in N}\left\{q_i^l + 2l_i\right\}}{\max_{i\in N}\left\{r_i + l_i\right\}} - 1\right)^+, \left(\frac{\max_{i\in N}\left\{q_i^l + 2l_i\right\}}{\max_{i\in N}\left\{2l_i\right\}} - 1\right)\right\} \\ &\leq \frac{3}{2}.\end{aligned}$$

Case 2: $\mathcal{L}(N) \neq \emptyset, \mathcal{R}(N) \neq \emptyset$ In this case $m \in \mathcal{L}(N)$ and $p \in \mathcal{R}(N)$ which gives us that $q_m^l + 2l_m = q_j^l + 2l_j = \max\left\{\max_{i\in N}\left\{q_i^l + 2l_i, r_i + l_i\right\}\right\} = r_k + l_k = r_p + l_p$. Thus, $C_{MRELL}(N) \leq r_p + l_p \leq C_{OPT}(N)$ yielding $\frac{C_{MRELL}(N)}{C_{OPT}(N)} \leq 1 \leq \frac{3}{2}$. In this case we also note that $r_p + l_p = q_m^l + 2l_m$ which gives that $\left(\frac{\max_{i\in N}\left\{q_i^l+2l_i\right\}}{\max_{i\in N}\left\{r_i+l_i\right\}} - 1\right)^+ = 0$. Therefore,

$$\min\left\{\left(\frac{\max_{i\in N}\left\{q_i^l + 2l_i\right\}}{\max_{i\in N}\left\{r_i + l_i\right\}} - 1\right)^+, \left(\frac{\max_{i\in N}\left\{q_i^l + 2l_i\right\}}{\max_{i\in N}\left\{2l_i\right\}} - 1\right)\right\} = 0.$$

As a result we conclude that in this case,

$$\begin{aligned}\frac{C_{MRELL}(N)}{C_{OPT}(N)} &\leq 1+\min\left\{\left(\frac{\max_{i\in N}\left\{q_i^l+2l_i\right\}}{\max_{i\in N}\left\{r_i+l_i\right\}}-1\right)^+,\left(\frac{\max_{i\in N}\left\{q_i^l+2l_i\right\}}{\max_{i\in N}\left\{2l_i\right\}}-1\right)\right\}\\ &\leq \frac{3}{2}.\end{aligned}$$

Case 3: $\mathcal{L}(N)\neq\emptyset,\mathcal{R}(N)=\emptyset$ In this case, $m\in\mathcal{L}(N)$. Thus,

$$q_m^l+2l_m=\max\left\{\max_{i\in N}\left\{q_i^l+2l_i,r_i+l_i\right\}\right\}.$$

So we may conclude that $q_m^l+2l_m>r_p+l_p$. We may also conclude that $q_m^l+2l_m>2l_{\max}$, because Lemma 2 allows us to ignore all jobs for which $q_i^l=0$. We thus examine two cases, (1) $q_m^l+2l_m>r_p+l_p>2l_{\max}$ and (2) $q_m^l+2l_m>2l_{\max}>r_p+l_p$.

Case 3.1: $q_m^l+2l_m>r_p+l_p>2l_{\max}$

$$\begin{aligned}C_{MRELL}(N) &\leq q_m^l+2l_m\\ &= \frac{q_m^l+2l_m}{r_p+l_p}(r_p+l_p)\\ &\leq \left[\left(\frac{q_m^l+2l_m}{2l_{\max}}-1\right)+1\right]C_{OPT}(N)\end{aligned}$$

Thus, $\frac{C_{MRELL}(N)}{C_{OPT}(N)}\leq 1+\left(\frac{q_m^l+2l_m}{r_p+l_p}-1\right)^+\leq 1+\left(\frac{q_m^l+2l_m}{2l_{\max}}-1\right)$. Which gives us the result that $\frac{C_{MRELL}(N)}{C_{OPT}(N)}\leq 1+\min\left\{\left(\frac{\max_{i\in N}\left\{q_i^l+2l_i\right\}}{\max_{i\in N}\{r_i+l_i\}}-1\right)^+,\left(\frac{\max_{i\in N}\left\{q_i^l+2l_i\right\}}{\max_{i\in N}\{2l_i\}}-1\right)\right\}$.

Case 3.2: $q_m^l+2l_m>2l_{\max}>r_p+l_p$

$$\begin{aligned}C_{MRELL}(N) &\leq q_m^l+2l_m\\ &= \frac{q_m^l+2l_m}{2l_{\max}}(2l_{\max})\\ &\leq \left[\left(\frac{q_m^l+2l_m}{r_p+l_p}-1\right)+1\right]C_{OPT}(N)\end{aligned}$$

Thus, $\frac{C_{MRELL}(N)}{C_{OPT}(N)}\leq 1+\left(\frac{q_m^l+2l_m}{2l_{\max}}-1\right)\leq 1+\left(\frac{q_m^l+2l_m}{r_p+l_p}-1\right)^+$, which gives us the result that $\frac{C_{MRELL}(N)}{C_{OPT}(N)} \leq 1+\min\left\{\left(\frac{\max_{i\in N}\left\{q_i^l+2l_i\right\}}{\max_{i\in N}\{r_i+l_i\}}-1\right)^+,\left(\frac{\max_{i\in N}\left\{q_i^l+2l_i\right\}}{\max_{i\in N}\{2l_i\}}-1\right)\right\}$.

We conclude Case 3 by proving that

$$1+\min\left\{\left(\frac{\max_{i\in N}\left\{q_i^l+2l_i\right\}}{\max_{i\in N}\left\{r_i+l_i\right\}}-1\right)^+,\left(\frac{\max_{i\in N}\left\{q_i^l+2l_i\right\}}{\max_{i\in N}\left\{2l_i\right\}}-1\right)\right\}\leq\frac{3}{2}.$$

Proving this statement is done via contradiction. Assume that

$$1+\min\left\{\left(\frac{\max_{i\in N}\left\{q_i^l+2l_i\right\}}{\max_{i\in N}\left\{r_i+l_i\right\}}-1\right)^+,\left(\frac{\max_{i\in N}\left\{q_i^l+2l_i\right\}}{\max_{i\in N}\left\{2l_i\right\}}-1\right)\right\}>\frac{3}{2}.$$

This implies:

$$\left(\frac{\max_{i\in N}\left\{q_i^l+2l_i\right\}}{\max_{i\in N}\left\{r_i+l_i\right\}}-1\right)^+>\frac{1}{2}\wedge\left(\frac{\max_{i\in N}\left\{q_i^l+2l_i\right\}}{\max_{i\in N}\left\{2l_i\right\}}-1\right)>\frac{1}{2}$$

$$\begin{aligned}
&\Rightarrow\quad \frac{\max_{i\in N}\left\{q_i^l+2l_i\right\}}{\max_{i\in N}\left\{r_i+l_i\right\}}>\frac{3}{2}\\
&\Rightarrow\quad 2\left(q_m^l+2l_m\right)>3\left(r_p+l_p\right)>3\left(r_m+l_m\right)>3\left(q_m^l+l_m\right)\\
&\Rightarrow\quad l_m>q_m^l\\
&\Rightarrow\quad 2\left(3l_m\right)>2\left(q_m^l+2l_m\right)>3(2l_{\max})\\
&\Rightarrow\quad 2l_m>2l_{\max}=\max_{i\in N}\left\{2l_i\right\}
\end{aligned}$$

Which is a contradiction. Thus,

$$1+\min\left\{\left(\frac{\max_{i\in N}\left\{q_i^l+2l_i\right\}}{\max_{i\in N}\left\{r_i+l_i\right\}}-1\right)^+,\left(\frac{\max_{i\in N}\left\{q_i^l+2l_i\right\}}{\max_{i\in N}\left\{2l_i\right\}}-1\right)\right\}\leq\frac{3}{2}.$$

As these three cases cover all possible situations we obtain the desired result:

$$\begin{aligned}
\frac{C_{MRELL}(N)}{C_{OPT}(N)} &\leq 1+\min\left\{\left(\frac{\max_{i\in N}\left\{q_i^l+2l_i\right\}}{\max_{i\in N}\left\{r_i+l_i\right\}}-1\right)^+,\left(\frac{\max_{i\in N}\left\{q_i^l+2l_i\right\}}{\max_{i\in N}\left\{2l_i\right\}}-1\right)\right\}\\
&\leq \frac{3}{2}.
\end{aligned}$$

□

This analysis also serves to further the results of Jaillet and Wagner (2006). In their paper, Jaillet and Wagner (2006) give a rather complex worst-case ratio of MLIB under conditions of variable advanced notice. However, by noting that when $q_i^l = q_i^r$ the two algorithms, MRELL and MLIB, are equivalent, we may give the following expression as the competitive ratio of MLIB under conditions of variable advanced notice.

$$\begin{aligned}\frac{C_{MLIB}(N)}{C_{OPT}(N)} &\leq 1 + \min\left\{\left(\frac{\max_{i\in N}\{q_i^r + 2l_i\}}{\max_{i\in N}\{r_i + l_i\}} - 1\right)^+, \left(\frac{\max_{i\in N}\{q_i^r + 2l_i\}}{\max_{i\in N}\{2l_i\}} - 1\right)\right\} \\ &\leq \frac{3}{2}.\end{aligned}$$

Furthermore, by following a similar set of arguments as outlined in Theorems 3 and 4, it is possible to prove that MLIB is a best-possible algorithm when $q_i^l = q_i^r \forall i \in N$ and conditions of variable amounts of advanced notice prevail.

6 Discussion of Results

Given these elaborate worst-case ratios for MRELL under conditions of fixed and variable advanced notice, what can be said about the value of location information? We begin by noting that MRIN is an algorithm that uses no advanced information; all actions are take at r_i. MRELL on the other hand uses advanced location and release time information; actions are taken at both q_i^l and q_i^r. Therefore by comparing these two extreme algorithms we may specify a value for the advanced location information.

In previous papers (see e.g. Jaillet and Wagner (2006)) the comparison between different algorithms was undertaken by subtracting the worst-case ratios of the two algorithms. We too will begin our comparison between MRIN and MRELL using this methodology. We then show that this method may yield a deceptive value for the location information. As a final result we specify a realistic range of values and describe policies that give MRELL a consistent improvement over MRIN.

We begin our comparison by studying the difference $\rho_{MRIN} - \rho_{MRELL}$. If we calculate this value directly we obtain:

$$\rho_{MRIN} - \rho_{MRELL} = \frac{1}{2} - \min\left\{ \left(\frac{\max_{i\in N}\left\{q_i^l + 2l_i\right\}}{\max_{i\in N}\left\{r_i + l_i\right\}} - 1 \right)^+, \left(\frac{\max_{i\in N}\left\{q_i^l + 2l_i\right\}}{\max_{i\in N}\left\{2l_i\right\}} - 1 \right) \right\} \tag{5}$$

As this expression is strictly positive, we may be inclined to conclude that advanced location information is similarly strictly beneficial. However, if we recall that $1 \leq \frac{C_{MRIN}(N)}{C_{OPT}(N)} \leq \rho_{MRIN} \leq \frac{3}{2}$ and $1 \leq \frac{C_{MRELL}(N)}{C_{OPT}(N)} \leq \rho_{MRELL} \leq \frac{3}{2}$. Then, we may say:

$$-\frac{1}{2} \leq 1 - \rho_{MRELL} \leq \frac{C_{MRIN}(N) - C_{MRELL}(N)}{C_{OPT}(N)} \leq \rho_{MRIN} - 1 \leq \frac{1}{2} \tag{6}$$

Equation (6) implies that in some instances advanced location information can be detrimental. Given these conflicting observations, stemming from the broad range in which $\frac{C_{MRIN}(N)-C_{MRELL}(N)}{C_{OPT}(N)}$ can fall, we cannot immediately specify a value for advanced location information. We therefore explore the full implications of this range in more detail.

We begin our more complete comparison of MRIN and MRELL by examining the extreme left of the range. It appears from the analysis in equation (6) that $\frac{C_{MRIN}(N)-C_{MRELL}(N)}{C_{OPT}(N)}$ can be as low as $-\frac{1}{2}$. This is, however, not true as there are no instances such that $\frac{C_{MRIN}(N)}{C_{OPT}(N)} = 1$ at the same time that $\frac{C_{MRELL}(N)}{C_{OPT}(N)} = \frac{3}{2}$. Instead, we put forth the following conjecture.

Conjecture 1. *$\frac{C_{MRIN}(N)-C_{MRELL}(N)}{C_{OPT}(N)} \geq -\frac{1}{3}$ for all instances N.*

An example of one such instance that drives the difference in the algorithms' costs to its lowest value of $-\frac{1}{3}$ is: $l_1 = l_2 = 2$, $r_1 = 2$, $q_1^l = 0$, $q_1^r = 1$, $r_2 = q_2^l = q_2^r = 4$.

We now explore the extreme positive end of the range for $\frac{C_{MRIN}(N)-C_{MRELL}(N)}{C_{OPT}(N)}$. We can immediately see that there exist instances such that $\frac{C_{MRIN}(N)-C_{MRELL}(N)}{C_{OPT}(N)} = \frac{1}{2}$. Take for example the instance where $q_1^l = 0$ and $q_1^r = r_1 = 1$. We, therefore, conclude that:

$$-\frac{1}{3} \leq \frac{C_{MRIN}(N) - C_{MRELL}(N)}{C_{OPT}(N)} \leq \frac{1}{2}. \tag{7}$$

If we assume a uniform distribution of instances across this range then we can say that on average using MRELL to exploit advanced location information will yield a cost improvement of $\frac{1}{12}$. Of course, if the instances are distributed differently the benefit of advanced location information may be drastically reduced. We therefore turn our attention toward policies that can improve the value of advanced location information.

We first note that the instances rendering advanced location information detrimental are those for which an earlier job drives the cost of MRIN while a later job with no advanced notice drives the cost of MRELL. Thus, the best policy strategy is one that requires all job locations to be announced at some point in advance of their release date. In fact this is the reasoning behind the analysis of fixed information in section 4. It is important to note that in instances of fixed advanced notice, where $a > 0$ and $b > 0$, $C_{MRIN}(N) \geq C_{MRELL}(N)$. This is because given the point in time that the location is revealed, the release time can be computed. As both a and b are positive this information can be computed in advance of the actual release time thereby avoiding the types of detrimental instances examined above.

An alternate strategy is to introduce a job pricing scheme that charges a premium for those jobs not willing or able to announce the location until a time close to the job's release date. This premium can be set dynamically to cover any costs originating from acting too soon for a previous job. For example, by specifying a price per job equal to the time the location is revealed plus the round trip distance of the job (i.e. $q_i^l + 2l_i$), then customers will have an incentive to provide the job location information early. If a job location is revealed late then such a fee would cover the cost of service regardless of the situation created by a previous job. Admittedly, while this scheme is theoretically sufficient to cover the cost of jobs revealed too late it may be confusing to customers who are likely to prefer fixed rates based solely on distance. Nevertheless this still provides some benefit to the customer as they do not need to reveal the release time any earlier – only the location of the job.

This observation yields the following question, does providing information on the release time early yield any benefit? We answer this question by noting that the earliest that the release time may be disclosed is $q_i^r = q_i^l$. If this is done for all jobs $i \in I$, then $C_{MLIB}(N) = C_{MRELL}(N)$; thus, $\frac{C_{MLIB}(N) - C_{MRELL}(N)}{C_{OPT}(N)} = 0$. From this analysis, we may conclude that the value of location information is immense. The revelation of location information alone brings all the benefit or detriment. This value ranges, dependent on the problem instance, from $-\frac{1}{3}$ to $\frac{1}{2}$ in terms of the difference in the cost of these algorithms as compared to the optimal solution.

These results represent only a first step towards a meaningful analysis of the vehicle routing problem presented as an example in Section 1. A clear first extension to this work is an analysis of the same problem in more realistic metric spaces, such as a general metric space or $\mathbb{R}^2$. A second extension of interest is the design of an online job selection algorithm. For example, by rejecting jobs based on a comparison of their disclosed locations to already accepted job locations might yield significant performance gains. Finally, we recommend studying other versions of the TSP – such as the TSP with pick-up and delivery or the TSP with multiple salesmen.

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