

## Research Article

# Particle Ratios from Strongly Interacting Hadronic Matter

Waseem Bashir, Saeed Uddin, and Hamid Nanda

Department of Physics, Jamia Millia Islamia (A Central University), New Delhi, India

Correspondence should be addressed to Waseem Bashir; bashir\_waseem@yahoo.com

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We calculate the particle ratios  $K^+/\pi^+$ ,  $K^-/\pi^-$ , and  $\Lambda/\pi^-$  for a strongly interacting hadronic matter using nonlinear Walecka model (NLWM) in relativistic mean field (RMF) approximation. It is found that interactions among hadrons modify  $K^+/\pi^+$  and  $\Lambda/\pi^-$  particle ratios, while  $K^-/\pi^-$  is found to be insensitive to these interactions.

## 1. Introduction

Since the discovery of asymptotic freedom [1] in case of nonabelian gauge field theories, it was postulated that a phase transition from nuclear state of matter to quark matter is possible. It was further argued that this phase transition can take place at sufficiently high temperature and/or densities and can result in the transformations of hadrons into a new state of matter dubbed as quark-gluon plasma (QGP). Since then, a considerable effort has been put forward to create and understand the properties of this new state of matter (QGP) and the corresponding phase transition. In order to study the dynamics of any phase transition in general, a complete description of a given state of matter on the basis of some underlying theory is required. To understand the dynamics of quark-hadron phase transition, the equation of state for both QGP phase and the hadronic phase is required. The QGP phase so far has been fairly described using Lattice Gauge theory in case of vanishing or low baryon chemical potential. However, the description of strongly interacting hadronic phase in terms of fundamental theory of strong interactions has proven to be far from being trivial. This is primarily due to strong coupling among hadrons, due to which the conventional methods of quantum field theory, for example, perturbative analysis, do not remain valid for the description of such strongly interacting hadronic phase. Therefore, one has to rely on alternate methods to describe the properties of hadronic phase, for example, hadron resonance gas models, chiral models, and quasi-particle models.

However, one can use another approach to determine the dynamics of strongly interacting hadronic phase and consequently of quark-hadron phase transition. By studying the spectra of hadrons, one can in principle comment on some of the properties of the strongly interacting hadronic matter. For example, by studying  $\bar{p}/p$  ratio, it has been argued that transparency effects in case of high energy heavy-ion collisions may become operative [2]. Recently, it has been found that particle ratios of some of the hadrons, for example,  $K^+/\pi^+$ ,  $K^-/\pi^-$ , and  $\Lambda/\pi^-$ , show a sudden rise for a specified range of center of mass energy  $\sqrt{S_{NN}}$  in case of heavy-ion collisions [3–6]. Taking into account the dependence of baryon chemical potential  $\mu_B$  and temperature  $T$  on the variable  $\sqrt{S_{NN}}$ , one can infer that the behaviour of these particle ratios may be sensitive to the critical region of quark-hadron phase transition. In this article, we therefore evaluate the particle ratios  $K^+/\pi^+$ ,  $K^-/\pi^-$ , and  $\Lambda/\pi^-$  for a strongly interacting hadronic matter and analyse their behaviour near first-order quark-hadron phase transition. For hadronic phase, we use nonlinear Walecka model within relativistic mean field (RMF) approximation. RMF theory has been widely and successfully used to describe the properties of the nuclear matter and finite nuclei. Further RMF theory has been also used to describe the equation of state for strongly interacting dense hadronic matter for the application in supernova and neutron stars [7–15]. In RMF theory, hadrons interact via the exchange of scalar and vector mesons and the interaction strength or coupling among hadrons is determined by different methods. For example, the nucleon-meson coupling constants are determined by reproducing the

ground state properties of the finite nuclei or by using nuclear matter properties, which is discussed in Section 2. To describe the quark-gluon plasma (QGP) phase, we use a Bag model equation of state.

## 2. Model

**2.1. Hadronic Phase: Baryons.** The equation of state for asymmetric baryonic matter is presented in this section. To describe baryonic matter, we use relativistic nonlinear Walecka model (NLWM). In this model, the interaction between baryons is governed by the exchange of various mesons. We include in this model baryons ( $p, n, \Sigma^+, \Sigma^0, \Sigma^-, \Xi^-, \Xi^0, \Lambda$ ) along with their antiparticles. The interaction between baryons is carried out by the exchange of neutral  $\sigma$ , isoscalar-vector  $\omega$ , isovector-vector  $\rho$ , and two additional hidden strangeness mesons  $\sigma^*$  and  $\phi$ . In this model, the Lagrangian density for baryons reads

$$\begin{aligned} \mathcal{L}_{\text{Bary}} = & \sum_{\alpha} \bar{\psi}_{\alpha} \\ & \cdot \left[ \gamma_{\mu} \left( i\partial^{\mu} - g_{\omega\alpha}\omega^{\mu} - g_{\phi\alpha}\phi^{\mu} - g_{\rho\alpha}\vec{\tau}_{\alpha} \cdot \vec{\rho}^{\mu} \right) - M_{\alpha}^{*} \right] \psi_{\alpha} \\ & + \frac{1}{2} \partial_{\mu}\sigma\partial^{\mu}\sigma - U(\sigma) - \frac{1}{4} W_{\mu\nu}W^{\mu\nu} + \frac{1}{2} m_{\omega}^2 \omega_{\mu}\omega^{\mu} + \frac{1}{4} \\ & \cdot c_3 (\omega_{\mu}\omega^{\mu})^2 - \frac{1}{4} \vec{R}_{\mu\nu} \cdot \vec{R}^{\mu\nu} + \frac{1}{2} m_{\rho}^2 \vec{\rho}_{\mu} \cdot \vec{\rho}^{\mu} + \frac{1}{2} \\ & \cdot \partial_{\mu}\sigma^* \partial^{\mu}\sigma^* - \frac{1}{2} m_{\sigma^*}^2 \sigma^{*2} - \frac{1}{4} S_{\mu\nu} S^{\mu\nu} + \frac{1}{2} m_{\phi}^2 \phi_{\mu}\phi^{\mu}, \end{aligned} \quad (1)$$

where  $\psi_{\alpha}$  is the fermionic field corresponding to baryon  $\alpha$ . The interaction between baryons is carried out by the exchange of neutral  $\sigma$ , isoscalar-vector  $\omega$ , isovector-vector  $\rho$ , and two additional hidden strangeness mesons  $\sigma^*$  and  $\phi$ .  $U(\sigma) = (1/2)m_{\sigma}^2\sigma^2 + (1/2)g_2\sigma^3 + (1/2)g_3\sigma^4$  is the scalar self-interaction term for  $\sigma$  field. Also  $S^{\mu\nu} = \partial^{\mu}\phi^{\nu} - \partial^{\nu}\phi^{\mu}$ ,  $\vec{R}^{\mu\nu} = \partial^{\mu}\vec{\rho}^{\nu} - \partial^{\nu}\vec{\rho}^{\mu} + g_{\rho}(\vec{\rho}^{\mu} \times \vec{\rho}^{\nu})$ ,  $W^{\mu\nu} = \partial^{\mu}\omega^{\nu} - \partial^{\nu}\omega^{\mu}$ , and  $(1/4)c_3(\omega_{\mu}\omega^{\mu})^2$  is the Bodmer correction or self-interaction term for the vector field  $\omega^{\mu}$  and  $g_{i\alpha}$  are the coupling constants that characterise the strength of interaction between mesons  $i = \omega, \sigma, \sigma^*, \rho, \phi$  and baryons  $\alpha = p, n, \Lambda, \Sigma, \Xi$ . Here,  $M_{\alpha}^{*} = M_{\alpha} + g_{\sigma\alpha}\sigma + g_{\sigma^*\alpha}\sigma^*$  is in-medium mass of baryon, where  $M_{\alpha}$  is the bare mass of baryon. Also  $m_i$  is the mass of exchange mesons and  $\vec{\tau}$  is the isospin operator. Using relativistic mean field (RMF) approximation under which the field variables are replaced by their space-time independent classical expectation values, that is,  $\sigma \rightarrow \langle \sigma \rangle = \sigma_0$ ,  $\omega_{\mu} \rightarrow \langle \omega_{\mu} \rangle = \delta_{\mu 0}\omega_0$ ,  $\vec{\rho}_{\mu} \rightarrow \langle \vec{\rho}_{\mu} \rangle = \delta_{\mu 0}\delta^{i3}\rho_{03}$ , and  $\phi_{\mu} \rightarrow \langle \phi_{\mu} \rangle = \delta_{\mu 0}\phi_0$ , the thermodynamic potential per unit volume corresponding to Lagrangian density (1) can be written as

$$\begin{aligned} \left( \frac{\Omega}{V} \right)_{\text{Bary}} = & \frac{1}{2} m_{\sigma}^2 \sigma_0^2 + \frac{1}{3} g_2 \sigma_0^3 + \frac{1}{4} g_3 \sigma_0^4 - \frac{1}{2} m_{\omega}^2 \omega_0^2 - \frac{1}{4} \\ & \cdot c_3 \omega_0^4 - \frac{1}{2} m_{\rho}^2 \rho_{03}^2 + \frac{1}{2} m_{\sigma^*}^2 \sigma_0^{*2} - \frac{1}{2} m_{\phi}^2 \phi_0^2 \end{aligned}$$

$$\begin{aligned} & - 2T \sum_{\alpha} \int \frac{d^3k}{(2\pi)^3} \\ & \cdot \left\{ \ln \left[ 1 + e^{-\beta(E_{\alpha}^{*} - \nu_{\alpha})} \right] + \ln \left[ 1 + e^{-\beta(E_{\alpha}^{*} + \nu_{\alpha})} \right] \right\}, \end{aligned} \quad (2)$$

where effective baryon energy is  $E_{\alpha}^{*} = (k_{\alpha}^2 + M_{\alpha}^{*2})^{1/2}$  and effective baryon chemical potential is  $\nu_{\alpha} = \mu_{\alpha} - g_{\omega\alpha}\omega_0 - g_{\phi\alpha}\phi_0 - g_{\rho\alpha}\tau_{3\alpha}\rho_{03}$ . Also parameter  $\beta$  is  $\beta = 1/T$ , where  $T$  is the temperature.

**2.2. Hadronic Phase: Bosons (Pions + Kaons).** To incorporate bosons (pions + kaons) in our model, we use an approach similar to the one used to model baryonic phase; that is, we use a meson-exchange type of Lagrangian for bosons as well. The Lagrangian density in a minimal-coupling scheme is [18, 19]

$$\mathcal{L}_{\text{Bosons}} = \sum_b D_{\mu}^{*} \Phi_b^{*} D^{\mu} \Phi_b - m_b^{*2} \Phi_b^{*} \Phi_b, \quad (3)$$

where  $\Phi_b$  is the bosonic field with summation carried over bosons  $b$ . Here covariant derivative is

$$D_{\mu} = \partial_{\mu} + iX_{\mu}, \quad (4)$$

with the four-vector  $X_{\mu}$  defined as

$$X_{\mu} \equiv g_{\omega b}\omega_{\mu} + g_{\rho b}\vec{\tau}_b \cdot \vec{\rho}_{\mu} + g_{\phi b}\phi_{\mu}, \quad (5)$$

and  $m_b^{*} = m_b + g_{\sigma b}\sigma + g_{\sigma^* b}\sigma^*$  is the effective mass of bosons. Also  $g_{ib}$  are the coupling constants that characterise the strength of interaction between exchange mesons  $i = \sigma, \omega, \rho, \phi, \sigma^*$  and bosons (pions + kaons). Here,  $\vec{\tau}_b$  is the isospin operator with its third component defined as

$$\begin{aligned} \tau_{3\pi} = & +1(\pi^{+}), 0(\pi^{0}), -1(\pi^{-}); \\ \tau_{3K} = & \frac{+1}{2(K^{+}, K^0)}, \frac{-1}{2(K^{-}, \bar{K}^0)}. \end{aligned} \quad (6)$$

It has to be mentioned that one can use even chiral perturbation theory [20] to describe bosons in the hadronic matter. In an earlier work [21], kaons were incorporated using chiral perturbation theory, whereas baryons were incorporated using Walecka model. However, in [22], it was put forward that this approach of modelling baryonic phase with Walecka model and bosonic phase with chiral Lagrangian has some inconsistency that may influence the final results. In our approach, baryons and bosons are incorporated using similar methodology, that is, using meson-exchange type Lagrangian, and therefore this approach is expected to be more consistent. In RMF approximation, the thermodynamic potential for Lagrangian density (3) can be written as

$$\begin{aligned} \left( \frac{\Omega}{V} \right)_{\text{Bosons}} = & T \sum_b \Gamma_b \int \frac{d^3k}{(2\pi)^3} \\ & \cdot \left\{ \ln \left[ 1 - e^{-\beta(\omega_b + \mu_b)} \right] + \ln \left[ 1 - e^{-\beta(\omega_b - \mu_b)} \right] \right\}, \end{aligned} \quad (7)$$

where  $\omega_{b\pm} = E_b^* \pm X_0$ , with effective energy defined as  $E_b^* = \sqrt{k^2 + m_b^{*2}}$ , and  $X_0$  is the temporal component of four-vector  $X_\mu \equiv g_{\omega b} \omega_\mu + g_{\rho b} \vec{\tau}_b \cdot \vec{\rho}_\mu + g_{\phi b} \phi_\mu$ . Also  $\mu_b$  is the boson chemical potential and  $\Gamma_b$  is the spin-isospin degeneracy factor of boson  $b$ .

**2.3. Hadronic Phase: Field Equations.** The thermodynamic potential per unit volume for hadronic medium  $(\Omega/V)_H$  can be therefore written as

$$\left(\frac{\Omega}{V}\right)_H = \left(\frac{\Omega}{V}\right)_{\text{Bary}} + \left(\frac{\Omega}{V}\right)_{\text{Bosons}}, \quad (8)$$

where  $(\Omega/V)_{\text{Bary}}$  and  $(\Omega/V)_{\text{Bosons}}$  are as defined in (2) and (7), respectively.

The different thermodynamic observables of the hadronic system, for example, entropy, pressure, and number density, can be evaluated as follows:

$$\begin{aligned} S(T, V, \mu_H) &= - \left. \frac{\partial \Omega_H}{\partial T} \right|_{V, \mu_H} \\ P(T, V, \mu_H) &= - \left. \frac{\partial \Omega_H}{\partial V} \right|_{T, \mu_H} \\ N(T, V, \mu_H) &= - \left. \frac{\partial \Omega_H}{\partial \mu_H} \right|_{T, V}, \end{aligned} \quad (9)$$

provided the expectation values of the exchange mesons field variables  $(\sigma_0, \sigma_0^*, \omega_0, \rho_0, \phi_0)$  are known.

To evaluate the expectation value of exchange meson field variables, one can solve the following set of coupled equations of motion for different field variables that are obtained after minimising the action integral  $S = \int (\mathcal{L}_{\text{Bary}} + \mathcal{L}_{\text{bosons}}) d^4x$  with respect to different exchange meson field variables; that is,

$$\begin{aligned} m_\sigma^2 \sigma_0 &= -g_2 \sigma_0^2 - g_3 \sigma_0^3 \\ &\quad - \sum_\alpha g_{\sigma\alpha} \frac{\Gamma_\alpha}{(2\pi)^3} \int \frac{d^3k}{\sqrt{(k^2 + M_\alpha^{*2})}} M_\alpha^* (F_\alpha^{(+)} + F_\alpha^{(-)}) \\ &\quad - \frac{1}{2} \sum_b m_b g_{\sigma b} \frac{\Gamma_b}{(2\pi)^3} \int d^3k \times (\omega_{b+}^{-1} F_b^{(+)} + \omega_{b-}^{-1} F_b^{(-)}), \end{aligned}$$

$$\begin{aligned} m_\omega^2 \omega_0 &= -c_3 \omega_0^3 + \sum_\alpha g_{\omega\alpha} n_\alpha + \frac{1}{2} \sum_b g_{\omega b} n_b \\ &\quad - \frac{1}{2} \sum_b g_{\omega b} X_0 \frac{\Gamma_b}{(2\pi)^3} \int d^3k (\omega_{b+}^{-1} F_b^{(+)} + \omega_{b-}^{-1} F_b^{(-)}), \end{aligned}$$

$$\begin{aligned} m_\rho^2 \rho_0 &= \sum_\alpha g_{\rho\alpha} \tau_{3\alpha} n_\alpha + \frac{1}{2} \sum_b g_{\rho b} \tau_{3b} n_b \end{aligned}$$

$$\begin{aligned} &\quad - \frac{1}{2} \sum_b g_{\rho b} \tau_{3b} X_0 \frac{\Gamma_b}{(2\pi)^3} \int d^3k (\omega_{b+}^{-1} F_b^{(+)} + \omega_{b-}^{-1} F_b^{(-)}), \\ m_{\sigma^*}^2 \sigma_0^* &= \sum_\alpha g_{\sigma^* \alpha} \frac{\Gamma_\alpha}{(2\pi)^3} \int \frac{d^3k}{\sqrt{(k^2 + M_\alpha^{*2})}} M_\alpha^* (F_\alpha^{(+)} + F_\alpha^{(-)}) \\ &\quad - \frac{1}{2} \sum_b m_b g_{\sigma^* b} \frac{\Gamma_b}{(2\pi)^3} \int d^3k (\omega_{b+}^{-1} F_b^{(+)} + \omega_{b-}^{-1} F_b^{(-)}), \end{aligned} \quad (10)$$

$$\begin{aligned} m_\phi^2 \phi_0 &= \sum_\alpha g_{\phi\alpha} n_\alpha + \frac{1}{2} \sum_b g_{\phi b} n_b \\ &\quad - \frac{1}{2} \sum_b g_{\phi b} \frac{\Gamma_b}{(2\pi)^3} \int d^3k (\omega_{b+}^{-1} F_b^{(+)} + \omega_{b-}^{-1} F_b^{(-)}), \end{aligned} \quad (11)$$

where distribution functions for baryons  $F_\alpha^{(+)}$  and antibaryons  $F_\alpha^{(-)}$  are given by

$$F_\alpha^{(\pm)} = \frac{1}{e^{\beta(E_\alpha^* \mp \nu_\alpha)} + 1} \quad (12)$$

and net-baryon density is

$$n_{\text{Bary}} = \sum_\alpha n_\alpha = \sum_\alpha \frac{2}{(2\pi)^3} \int (F_\alpha^{(+)} - F_\alpha^{(-)}) d^3k. \quad (13)$$

Similarly, the distribution function of bosons  $F_b^{(+)}$  and their antiparticles  $F_b^{(-)}$  is

$$\begin{aligned} F_b^{(\pm)} &= \frac{1}{e^{\beta(\omega_{b\pm} \mp \mu_b)} - 1} = \frac{1}{e^{\beta[(\epsilon_b^* \pm X_0) \mp \mu_b]} - 1} \\ &= \frac{1}{e^{\beta(\epsilon_b^* \mp \nu_b)} - 1}, \end{aligned} \quad (14)$$

where  $\nu_b = \mu_b - X_0$  is the effective chemical potential of boson  $b$ . Also the boson density is

$$n_{\text{bosons}} = \sum_b n_b = \sum_b \frac{\Gamma_b}{(2\pi)^3} \int d^3k (F_b^{(+)} - F_b^{(-)}). \quad (15)$$

**2.4. Hadronic Phase: The Coupling Constants.** To fix baryon-meson coupling constants, we use two very successful parameter sets of RMF model, namely, parameter sets TM1 and NL3. These parameter sets are listed in Table 1. These parameters have been obtained by evaluating the ground state properties of finite nuclei [16, 23]. For meson-hyperon

TABLE 1: TMI parameter set used in our calculation.

	$m_N$	$m_\sigma$	$m_\omega$	$m_\rho$	$g_{\sigma N}$	$g_{\omega N}$	$g_{\rho N}$	$g_2 (fm^{-1})$	$g_3$	$c_3$
TM1	938.0	511.198	783.0	770.0	10.029	12.614	4.632	-7.2323	0.618	71.308
NL3	939.0	508.194	782.501	763.0	10.217	12.868	4.474	-10.431	-28.885	—

TABLE 2: Meson-hyperon coupling constants.

	$g_{\sigma\Lambda}$	$g_{\sigma\Sigma}$	$g_{\sigma\Xi}$	$g_{\sigma^*\Lambda}$	$g_{\sigma^*\Xi}$
TM1	6.170	4.472	3.202	7.018	12.600
NL3	6.269	4.709	3.242	7.138	12.809

coupling constants, we use quark model values of vector couplings. These are given by

$$\begin{aligned}
\frac{1}{3}g_{\omega N} &= \frac{1}{2}g_{\omega\Lambda} = \frac{1}{2}g_{\omega\Sigma} = g_{\omega\Xi}, \\
g_{\rho N} &= \frac{1}{2}g_{\rho\Sigma} = g_{\rho\Xi}, \\
g_{\rho\Lambda} &= 0, \\
2g_{\phi\Lambda} &= 2g_{\phi\Sigma} = g_{\phi\Xi} = -\frac{2\sqrt{2}}{3}g_{\omega N}, \\
g_{\phi N} &= 0.
\end{aligned} \tag{16}$$

The potential depth for hyperons in baryonic matter is fixed as follows. Representing the potential depth of hyperon  $h$  in baryonic matter  $B$  as  $U_h^{(B)}$ , we use  $U_\Lambda^{(N)} = -28$  MeV,  $U_\Sigma^{(N)} = +30$  MeV, and  $U_\Xi^{(N)} = -18$  MeV to determine the value of scalar coupling constants  $g_{\sigma\Lambda}$ ,  $g_{\sigma\Sigma}$ , and  $g_{\sigma\Xi}$  respectively [24–26]. The hyperon couplings with strange mesons are restricted by the relation  $U_\Xi^{(\Xi)} \simeq U_\Lambda^{(\Xi)} \simeq 2U_\Xi^{(\Lambda)} \simeq 2U_\Lambda^{(\Lambda)}$  obtained in [27]. For the hyperon-hyperon interactions, we use the square well potential with depth  $U_\Lambda^{(\Lambda)} = -20$  MeV [28]. In Table 2, we list the values of the coupling constants determined from these hyperon potentials. Next, in Tables 3 and 4, we give kaon-meson and pion-meson coupling constants that are used in our calculation.

Regarding antibaryon-meson couplings [29], there is no reliable information particularly for high-density matter. Therefore, we will use in our work the values of the antibaryon-meson couplings that are derived using G-parity transformation. The G-parity transformation is analogous to ordinary parity transformation in configurational space, which inverts the direction of three vectors. The G-parity transformation is defined as the combination of charge conjugation and rotation. The  $\pi$  degree of rotation is done around the second axis of isospin space.

It is already known that exchange mesons  $\sigma$  and  $\rho$  have positive G-parity and  $\omega$  and  $\phi$  have negative G-parity. Therefore, by applying G-parity transformation to nucleon potentials, one can obtain the corresponding potential for

TABLE 3: Kaon-meson coupling constants.

Set	TM2	GL85
	UK[S]	UK[W]
Reference	[16]	[11]
$g_{\sigma K}$	2.27	1.27
$g_{\omega K}$	3.02	3.02
$g_{\rho K}$	3.02	3.02
$g_{\sigma^* K}$	2.65	2.65
$g_{\phi K}$	4.27	4.27

TABLE 4: Pion-meson coupling constants [17].

$g_{\sigma\pi}$	$g_{\omega\pi}$	$g_{\rho\pi}$	$g_{\sigma^*\pi}$	$g_{\phi\pi}$
-0.170	-0.001	0.506	0.0	0.0

antinucleons. The result of G-parity transformation can be written as

$$\begin{aligned}
g_{\sigma\bar{\alpha}} &= g_{\sigma\alpha}, \\
g_{\omega\bar{\alpha}} &= -g_{\omega\alpha}, \\
g_{\rho\bar{\alpha}} &= g_{\rho\alpha}, \\
g_{\phi\bar{\alpha}} &= -g_{\phi\alpha},
\end{aligned} \tag{17}$$

where  $\alpha$  and  $\bar{\alpha}$  denote baryons and antibaryons, respectively. It is worthwhile to mention here that kaon-meson couplings can be fixed for two different kaonic potentials, namely, strongly attractive potential UK[S] and weakly attractive potential UK[W]. These are given in Table 3. Finally, the pion-meson couplings are given in Table 4. Since the pion is nonstrange particle, its coupling with strange mesons  $\sigma^*$  and  $\phi$  is essentially zero.

### 3. Results and Discussions

In this article, our main aim is to evaluate the properties of strongly interacting hadronic matter at finite temperature. Therefore, we make an attempt to analyse the properties of particle ratios  $K^+/\pi^+$ ,  $K^-/\pi^-$ ,  $\Lambda/\pi^-$  for this matter. Using the values of baryon-meson and boson-meson coupling constants as defined in the previous section, one can solve the coupled integral equations for the field variables  $(\sigma_0, \omega_0, \rho_0, \sigma_0^*, \phi_0)$  and consequently one can obtain the thermodynamic observables of the hadronic system for given values of temperature  $T$  and baryon chemical potential  $\mu_B$ . In the discussion to follow, we use following parameter sets for fixing baryon-meson and boson-meson coupling constants used in this model. For baryon-meson couplings, we use parameter set TMI as listed in Tables 1 and 2.

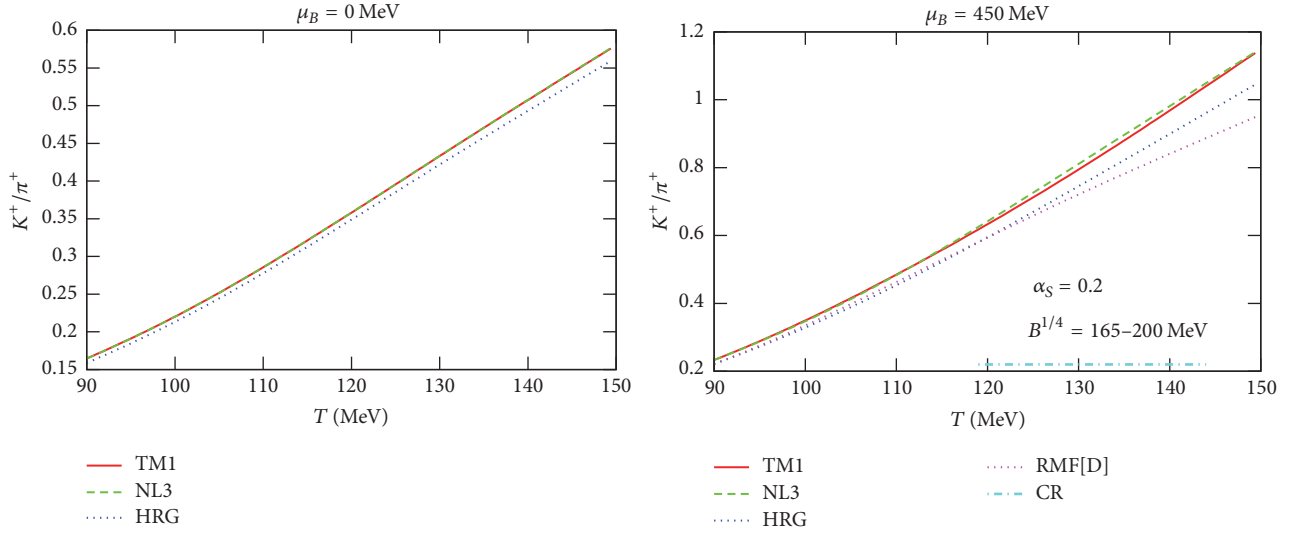


FIGURE 1:  $K^+/\pi^+$  ratio as a function of temperature  $T$  for fixed value of baryon chemical potential  $\mu_B = 0$  MeV and  $\mu_B = 450$  MeV for strongly interacting hadronic matter with coupled baryons and bosons with two parameter sets TM1 and NL3 for kaonic potentials UK[S] and UK[W], respectively, for strongly interacting hadronic matter with decoupled baryons and bosons RMF[D], and for hadron resonance gas (HRG) model. CR is the critical region of first-order quark-hadron phase transition calculated with Bag value  $B^{1/4} = 165\text{--}200$  MeV.  $\alpha_S$  is the coupling strength for quarks.

For kaon-meson coupling constants, we use parameter sets TM2 and GL85 that correspond to strongly attractive and weakly attractive kaonic potentials, respectively, and are listed in Table 3. The pion-meson coupling constants are listed in Table 4. In the following analysis, we will impose the strangeness conservation criteria  $S - \bar{S} = 0$ , where  $S$  and  $\bar{S}$  are total strangeness and antistrangeness of the system under consideration.

In Figure 1, we plot the variation of  $K^+/\pi^+$  with temperature for fixed values of baryon chemical potential. For  $\mu_B = 0$  MeV, we show  $K^+/\pi^+$  for hadronic matter with coupled baryons and bosons for two parameter sets TM1 and NL3 with kaonic potentials UK[S] and UK[W], respectively. The result obtained for noninteracting hadrons is calculated using hadron resonance gas (HRG) model. The effect of chiral symmetry is analysed by decoupling Nambu-Goldstone modes (pions and kaons) from baryons and result is denoted by RMF[D]. This is obtained by setting  $g_{\sigma b} = g_{\sigma^* b} = g_{\omega b} = g_{\phi b} = g_{\rho b} = 0$ . It is worthwhile to mention here that, with this choice of boson-meson couplings, baryons can still interact strongly via the exchange of mesons ( $\sigma, \sigma^*, \omega, \rho, \phi$ ), while bosons (pions + kaons) get decoupled from baryons and hence remain in the system as free particle with no interaction. One can see that, for  $\mu_B = 0$  MeV, the effect of interaction is negligible on  $K^+/\pi^+$  ratio even up to relatively high temperature of about  $T = 150$  MeV. For higher baryon chemical potential, that is,  $\mu_B = 450$  MeV, one can see that the interactions modify  $K^+/\pi^+$  ratio. For hadronic phase with coupled baryons and bosons, the effect of interaction is to increase  $K^+/\pi^+$  ratio. Further, one can see that the rise of  $K^+/\pi^+$  ratio is significant only in the critical region (CR) of first-order quark-hadron phase transition, which corresponds to large value of Bag constant  $B$ . Here, we

have calculated critical region with equation of state for quark-gluon plasma which is consistent with Lattice QCD (see Appendix). The Bag value was fixed in the range  $B^{1/4} = 165\text{--}200$  MeV. However, on decoupling baryons from bosons, one can see that the ratio  $K^+/\pi^+$  drops and in fact becomes slightly lower than HRG model's result.

In Figure 2, we plot the variation of  $K^+/\pi^+$  with baryon chemical potential  $\mu_B$  at fixed value of temperature  $T = 50$  MeV and  $T = 70$  MeV. At lower temperature,  $K^+/\pi^+$  ratio for NL3 parameter set is the same as that for HRG model, while, for TM1 parameter set,  $K^+/\pi^+$  ratio is less compared to that of HRG model. However, at higher temperature, the particle ratio  $K^+/\pi^+$  for both parameter sets, that is, TM1 and NL3, is less compared to HRG model.

In Figure 3, we next plot the variation of particle ratio  $\Lambda/\pi^-$  with temperature for fixed values of baryon chemical potential  $\mu_B$ . At lower baryon chemical potential, that is,  $\mu_B = 0$  MeV, the effect of interaction among hadrons is negligible on particle ratio  $\Lambda/\pi^-$  even for relatively high temperature of  $T = 150$  MeV. However, for higher baryon chemical potential, that is,  $\mu_B = 450$  MeV, the effect of interaction among hadrons increases the ratio  $\Lambda/\pi^-$ . One can again see that the rise of  $\Lambda/\pi^-$  is somewhat significant in the critical region of phase transition which corresponds to a large value of Bag constant  $B$ . Here, CR denotes the critical region of first-order quark-hadron phase transition which is calculated with Bag value  $B^{1/4} = 165\text{--}200$  MeV and lattice motivated equation of state (see Appendix). Interestingly, one can see that the effect of interaction on  $\Lambda/\pi^-$  vanishes if bosons decouple from baryons.  $\Lambda/\pi^-$  ratio for a system of strongly interacting hadrons with baryons decoupled from bosons is represented by RMF[D].

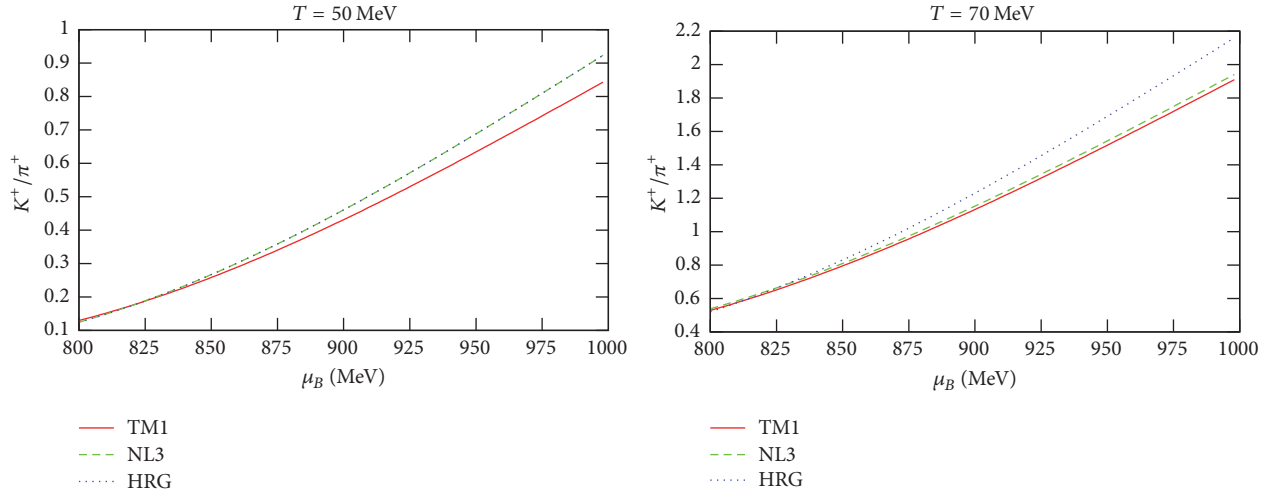


FIGURE 2:  $K^+/\pi^+$  as a function of baryon chemical potential  $\mu_B$  at fixed values of temperature  $T = 50$  MeV and  $T = 70$  MeV for strongly interacting hadronic matter with coupled baryons and bosons with two parameter sets TM1 and NL3 for kaonic potentials UK[S] and UK[W], respectively, and for noninteracting hadrons using hadron resonance gas (HRG) model.

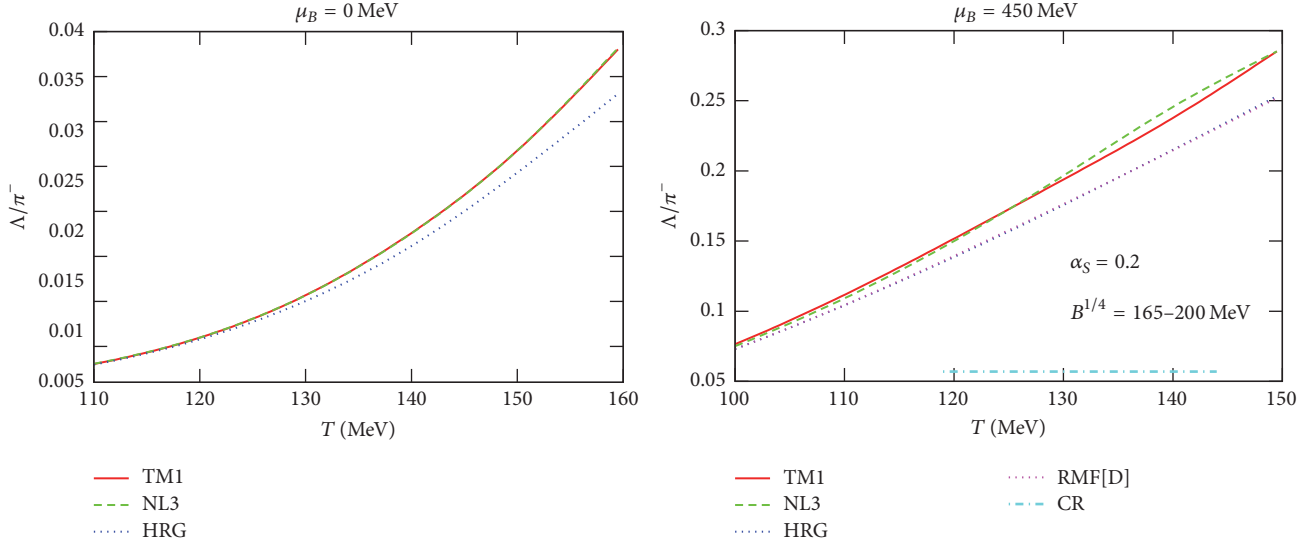


FIGURE 3:  $\Lambda/\pi^-$  as a function of temperature  $T$  for fixed values of baryon chemical potential  $\mu_B = 0$  MeV and  $\mu_B = 450$  MeV for strongly interacting hadronic matter with coupled baryons and bosons with two parameter sets TM1 and NL3 for kaonic potentials UK[S] and UK[W], respectively, for strongly interacting hadronic matter with decoupled baryons and bosons RMF[D], and for hadron resonance gas (HRG) model. CR is the critical region of first-order quark-hadron phase transition calculated with Bag value  $B^{1/4} = 165\text{--}200$  MeV.  $\alpha_s$  is the coupling constant for quarks.

In Figure 4, we next plot the variation of particle ratio  $\Lambda/\pi^-$  with baryon chemical potential at fixed values of temperature  $T = 50$  MeV and  $T = 70$  MeV. At lower temperature, that is,  $T = 50$  MeV, the particle ratio  $\Lambda/\pi^-$  behaves differently under parameter sets TM1 and NL3. However, at large temperature, that is,  $T = 70$  MeV, the particle ratio  $\Lambda/\pi^-$  for parameter sets TM1 and NL3 coincides with noninteracting systems even up to very large values of baryon chemical potential.

In Figure 5, we next plot the variation of particle ratio  $K^-/\pi^-$  as a function of temperature for fixed baryon chemical potential, that is,  $\mu_B = 0$  MeV and  $\mu_B = 450$  MeV. For

lower value of baryon chemical potential, that is,  $\mu_B = 0$  MeV, the effect of interactions among hadrons with coupled baryons and bosons is negligible on the particle ratio  $K^-/\pi^-$ . Interestingly, even for higher baryon chemical potential, that is,  $\mu_B = 450$  MeV, the effect of interaction is still negligible on  $K^-/\pi^-$  ratio. However, on decoupling bosons from baryons, the  $\Lambda/\pi^-$  ratio becomes large at higher values of temperature. For a reference, we have again shown the critical region of first-order quark-hadron phase transition CR. Here, we have used equation of state for quark-gluon plasma phase with Bag value  $B^{1/4} = 165\text{--}200$  MeV and strong coupling constant  $\alpha_s = 0.2$ .

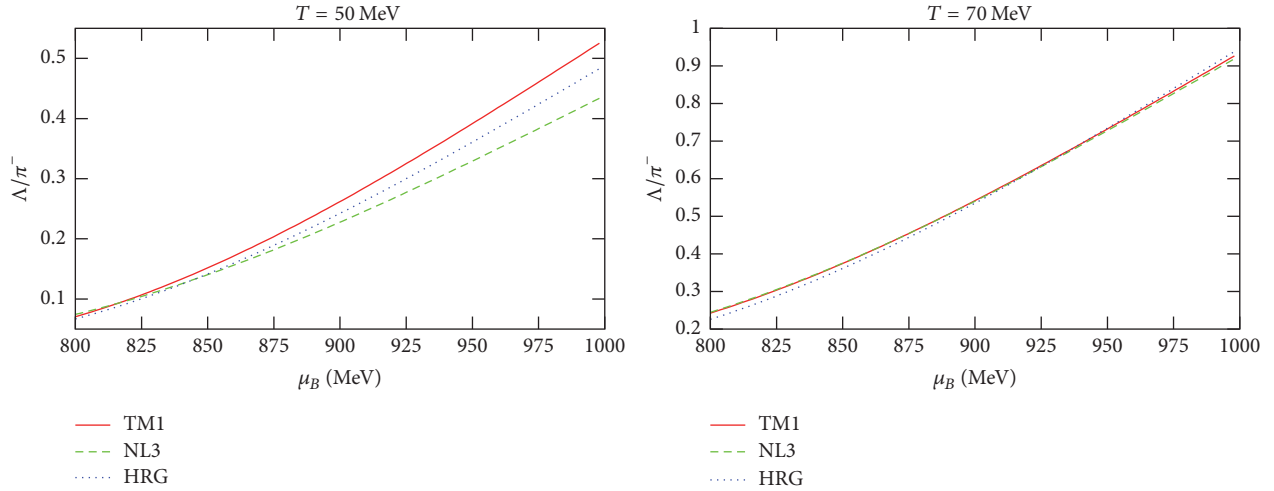


FIGURE 4:  $\Lambda/\pi^-$  as a function of baryon chemical potential  $\mu_B$  at fixed values of temperature  $T = 50$  MeV and  $T = 70$  MeV for strongly interacting hadronic matter with coupled baryons and bosons with two parameter sets TM1 and NL3 for kaonic potentials UK[S] and UK[W], respectively, and for system of noninteracting hadrons modelled using hadron resonance gas (HRG) model.

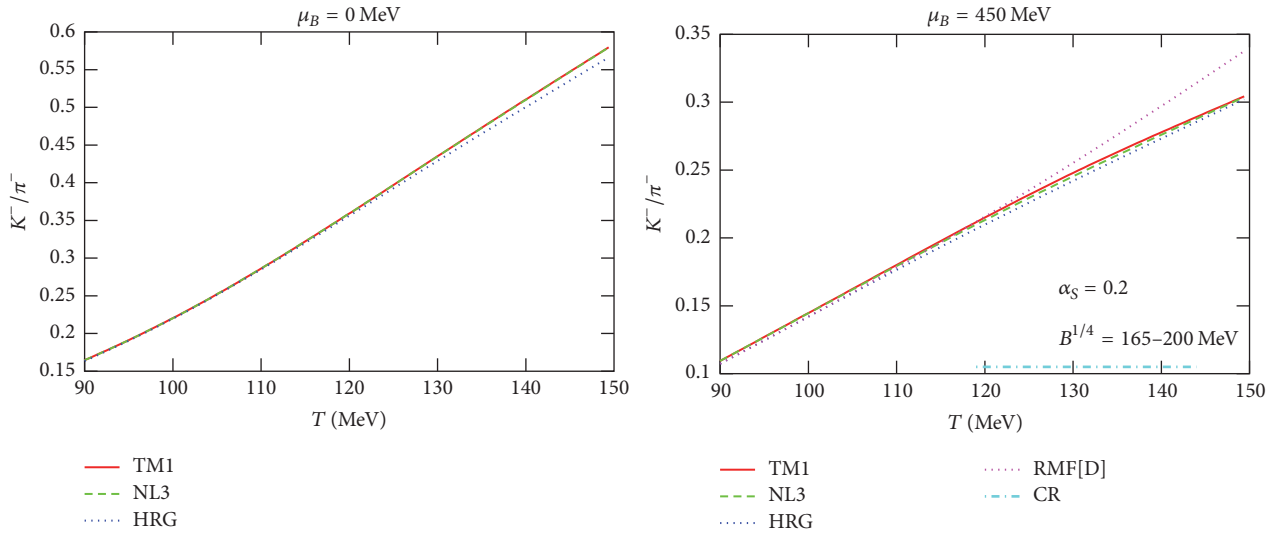


FIGURE 5: Particle ratio  $K^-/\pi^-$  as a function of temperature  $T$  for different values of baryon chemical potential, that is,  $\mu_B = 0$  MeV and  $\mu_B = 450$  MeV, for strongly interacting hadronic matter with coupled baryons and bosons with two model parameter sets TM1 and NL3 for kaonic potentials UK[S] and UK[W], respectively, for strongly interacting hadronic matter with decoupled baryons and bosons RMF[D], and for a system of noninteracting hadrons modelled using hadron resonance gas (HRG) model. CR is the critical region of first-order quark-hadron phase transition calculated with Bag value  $B^{1/4} = 165\text{--}200$  MeV.  $\alpha_S$  is the strong coupling constant for quarks.

Finally, in Figure 6, we show the variation of particle ratio  $K^-/\pi^-$  with baryon chemical potential  $\mu_B$  for fixed values of temperature  $T = 50$  MeV and  $T = 70$  MeV. For the case of strongly interacting hadronic matter with coupled baryons and bosons, the particle ratio  $K^-/\pi^-$  is the same as that of a system of noninteracting hadrons even up to very large baryon chemical potential values.

In Figure 7, we plot the variation of effective chemical potential  $\nu_{\text{Hadrons}}$  and effective mass ( $m^*$ ) to bare mass ( $m$ ) ratio  $m^*/m$  of hadrons as a function of temperature for fixed values of baryon chemical potential  $\mu_B = 0$  MeV and  $\mu_B = 450$  MeV. Next, in Figure 8, we have shown the

variation of these observables, that is,  $\nu_{\text{Hadrons}}$  and  $m^*/m$ , as a function of baryon chemical potential  $\mu_B$  at fixed values of temperature  $T = 50$  MeV and  $T = 70$  MeV. These observables have been calculated for a system of strongly interacting hadrons with coupled baryons and bosons with parameter set TM1. For kaon-meson coupling, we use parameter set TM2 which corresponds to strongly attractive kaonic potential UK[S]. To complete our discussion, we next plot in Figure 9 the variation of strange chemical potential  $\mu_S$  as functions of baryon chemical potential  $\mu_B$  and temperature  $T$  at fixed values of temperature and baryon chemical potential, respectively. The strangeness chemical potential is fixed by

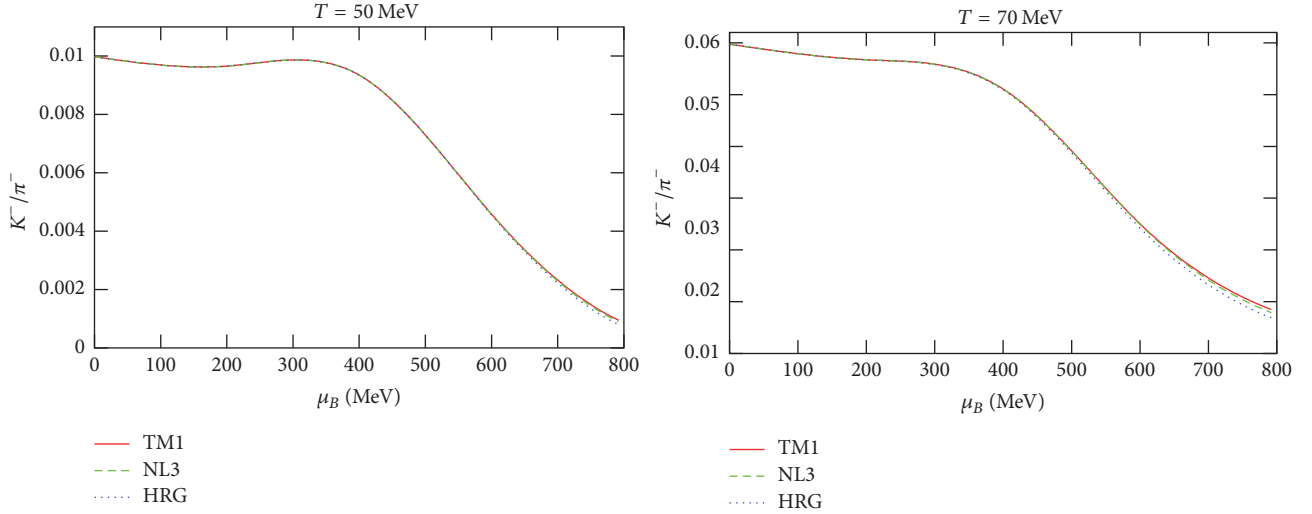


FIGURE 6: Particle ratio  $K^-/\pi^-$  as a function of baryon chemical potential  $\mu_B$  for fixed values of temperature  $T = 50$  MeV and  $T = 70$  MeV for strongly interacting hadronic matter with coupled baryons and bosons with two parameter sets TM1 and NL3 for kaonic potentials UK[S] and UK[W], respectively, and for system of noninteracting hadrons modelled using hadron resonance gas (HRG) model.

imposing the constraint of strangeness conservation ( $S - \bar{S} = 0$ ) in the hadronic medium. The values of strangeness chemical potential  $\mu_s$  show a decrease with increasing baryon chemical potential with increasing temperature. Further, we also observe that the strangeness chemical potential decreases with the increase of temperature for increasing values of baryon chemical potential. This is consistent with the hadron resonance gas model [30].

#### 4. Summary

In this article, we have calculated particle ratios  $K^+/\pi^+$ ,  $K^-/\pi^-$ ,  $\Lambda/\pi^-$  for a strongly interacting hadronic matter using nonlinear Walecka model in relativistic mean field (RMF) approximation. In the hadronic medium, we incorporate baryons and bosons (pions + kaons). To describe baryons and bosons, we use a meson-exchange type of Lagrangian and evaluate thermodynamic observables of hadronic matter in RMF approximation. It is found that the interaction among hadrons which in the present model is mediated by the exchange of  $\sigma, \sigma^*, \omega, \rho$  and  $\phi$  mesons can result in the modification of  $K^+/\pi^+$  and  $\Lambda/\pi^-$  ratios, while the particle ratio  $K^-/\pi^-$  is found to be independent of the interaction among hadrons.

#### Appendix

##### Quark-Gluon Plasma Phase

In this section, we present equation of state for quark-gluon plasma (QGP) phase. We consider three quark flavours, up (u), down (d), and strange (s), and gluons. We use a Bag model equation of state with perturbative corrections of the

order of  $\alpha_s$  which is consistent with Lattice data [31, 32]. The pressure  $P$  and energy density  $\epsilon$  are given by

$$\begin{aligned}
 P_{\text{QGP}} &= \left( \tilde{N}_g + \frac{21}{2} \tilde{N}_f \right) \frac{\pi^2 T^4}{90} + \tilde{N}_f \left( \frac{\mu^2 T^2}{18} + \frac{\mu^4}{324\pi^2} \right) \\
 &+ \frac{1-\zeta}{\pi^2} \int_{m_s}^{\infty} dE (E^2 - m_s^2)^{3/2} (F_q^{(+)} + F_q^{(-)}) - B, \\
 \epsilon_{\text{QGP}} &= \left( \tilde{N}_g + \frac{21}{2} \tilde{N}_f \right) \frac{\pi^2 T^4}{30} + 3\tilde{N}_f \left( \frac{\mu^2 T^2}{18} + \frac{\mu^4}{324\pi^2} \right) \\
 &+ 3 \left( \frac{1-\zeta}{\pi^2} \right) \int_{m_s}^{\infty} dE (E^2 - m_s^2)^{3/2} (F_q^{(+)} + F_q^{(-)}) \\
 &+ B,
 \end{aligned} \tag{A.1}$$

where  $\tilde{N}_g = 16(1 - (4/5)\zeta)$  and  $\tilde{N}_f = 2(1 - \zeta)$  are the effective numbers of gluons and fermions, respectively. The quark chemical potential is  $\mu_q$  and  $\zeta = \alpha_s$  is the coupling constant. For the determination of coupling constant  $\alpha_s$ , see, for example, [33]. Here,  $F_q^{(+)}$  and  $F_q^{(-)}$  are the Fermi-Dirac distribution functions for quarks and antiquarks, respectively, and  $B$  is the Bag constant. To see the possible effect of Bag constant on various observables, see, for example, [34]. In the above calculation, up (u) and down (d) quarks are considered massless, while strange quark (s) is of finite mass  $m_s = 150$  MeV. In the present discussion, heavy quark flavours have not been considered.



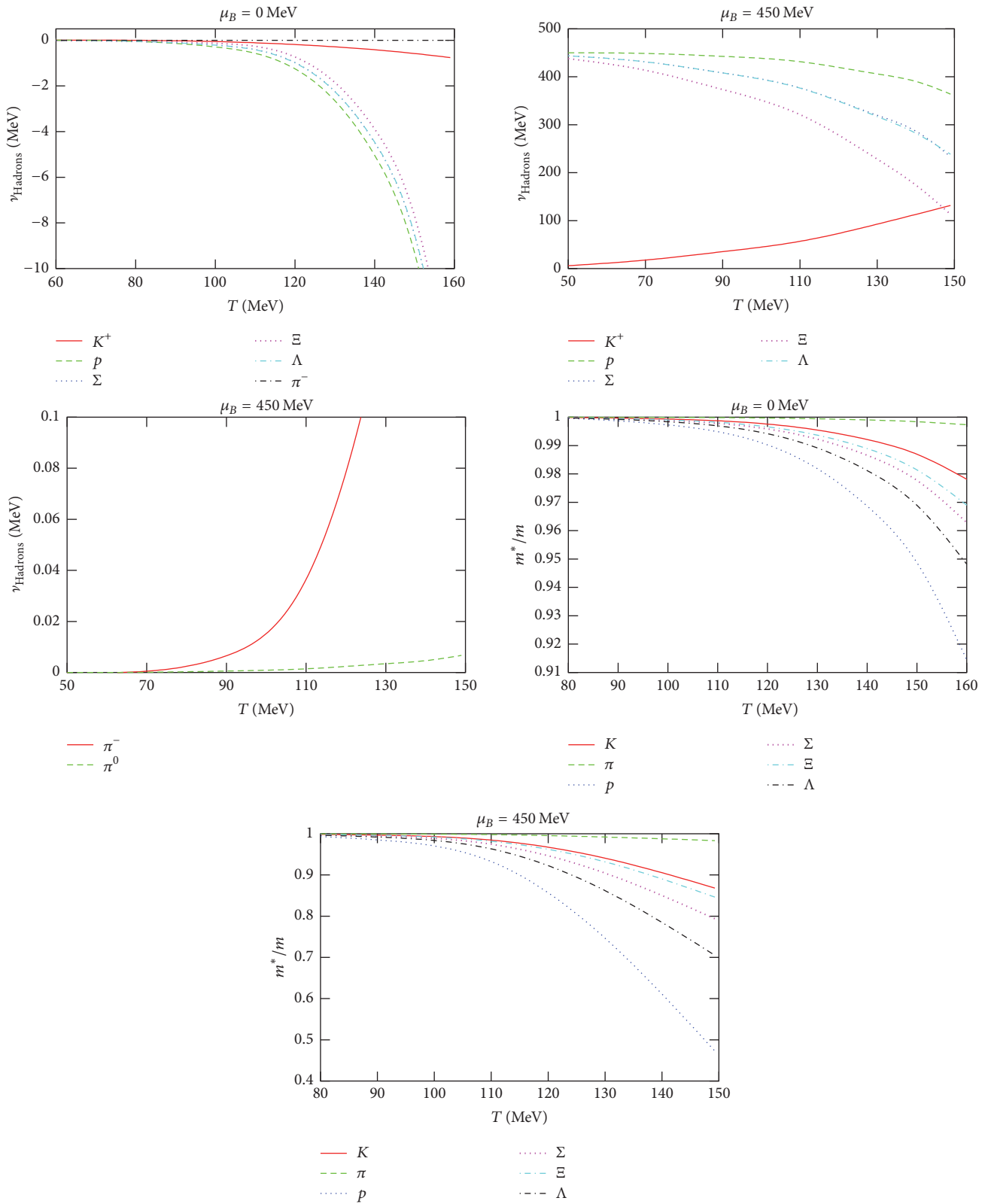


FIGURE 7: Effective chemical potential  $\nu_{\text{Hadrons}}$  and mass ratio  $m^*/m$  of hadrons as a function of temperature  $T$  for fixed values of baryon chemical potential  $\mu_B = 0$  MeV and  $\mu_B = 450$  MeV in case of strongly interacting hadronic matter with coupled baryons and bosons. Here, parameter set TM1 is used.

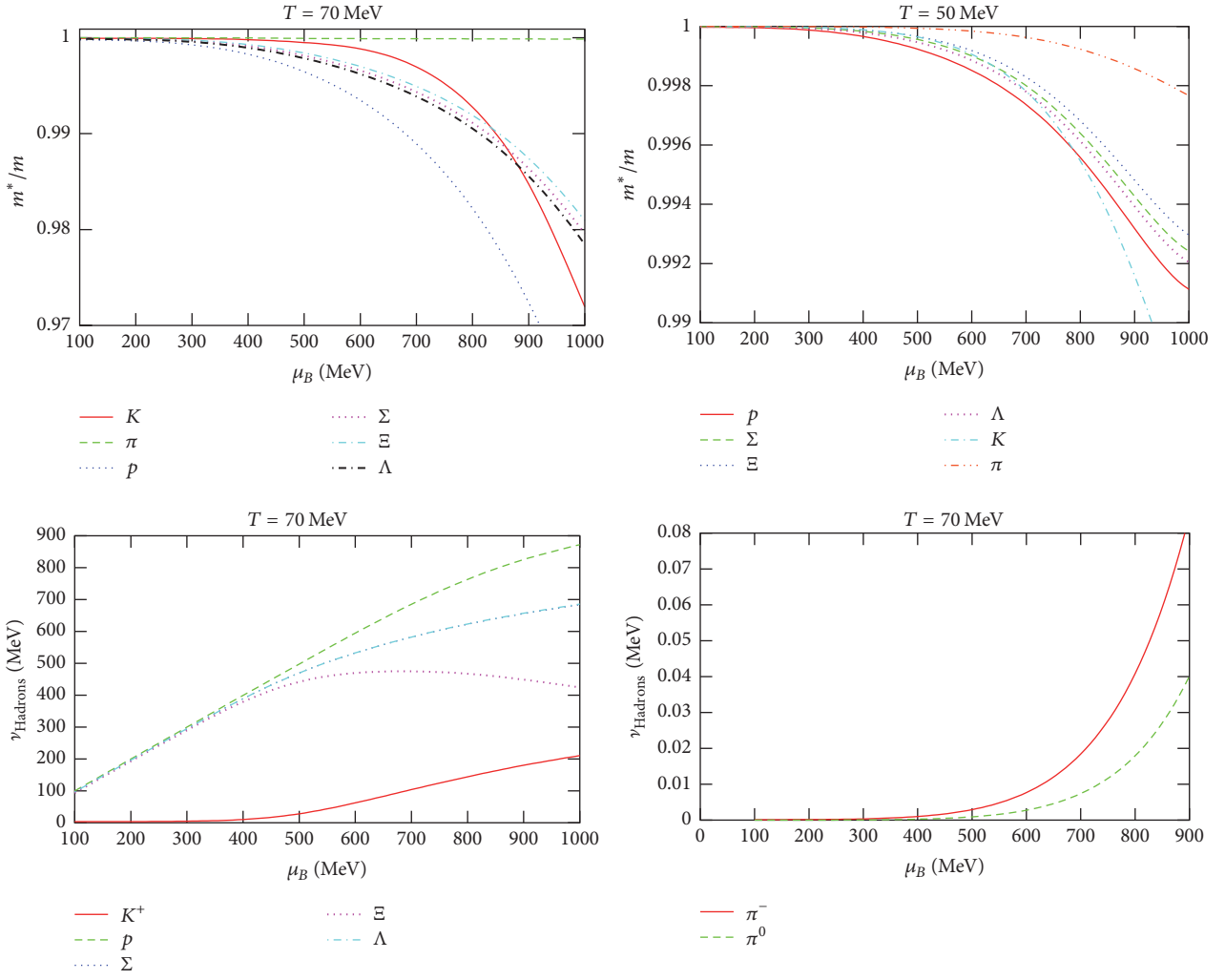


FIGURE 8: Effective chemical potential  $\nu_{\text{Hadrons}}$  and mass ratio  $m^*/m$  of hadrons as a function of baryon chemical potential  $\mu_B$  for fixed values of temperature  $T = 50$  MeV and  $T = 70$  MeV in case of strongly interacting hadronic matter with coupled baryons and bosons. Here, parameter set TM1 is used.

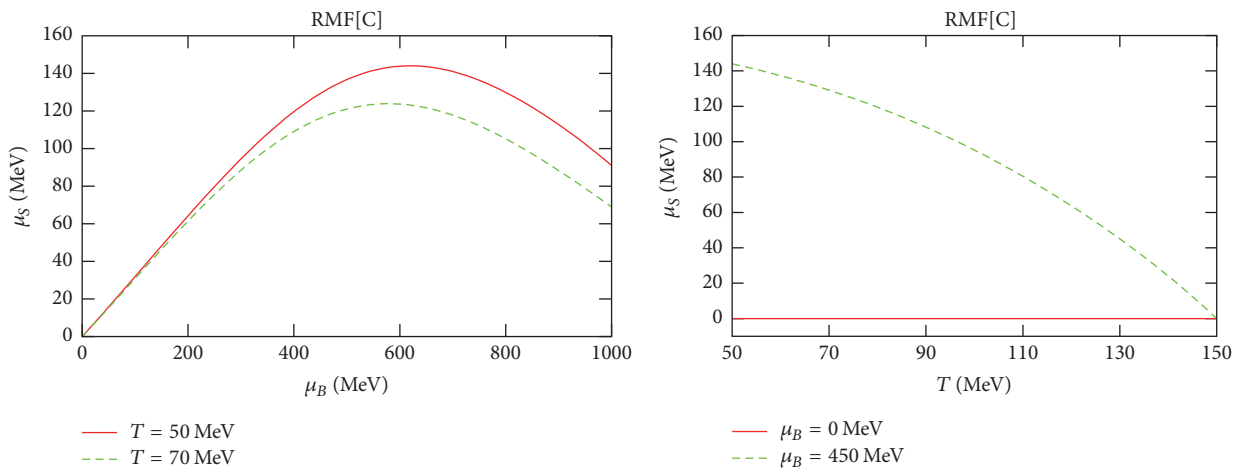


FIGURE 9: Strange chemical potential  $\mu_S$  as a function of baryon chemical potential  $\mu_B$  and temperature  $T$  for fixed values of temperature and baryon chemical potential, respectively, in case of a strongly interacting hadronic system with coupled baryons and bosons RMF[C]. Here, parameter set TM1 is used.

## Conflicts of Interest

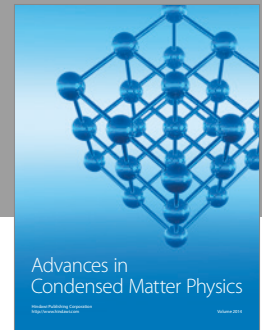
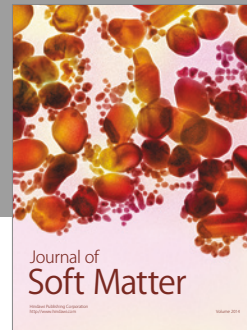
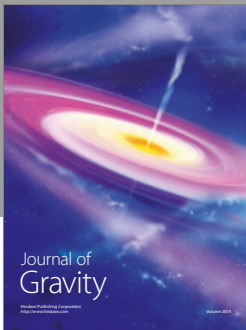
The authors declare that they have no conflicts of interest.

## Acknowledgments

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