EVALUATING STYLE ANALYSIS

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Evaluating Style Analysis*

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Abstract

In this paper we evaluate applications of (return based) style analysis. The portfolio and positivity constraints imposed by style analysis are useful in constructing mimicking portfolios without short positions. Such mimicking portfolios can be used, e.g., to construct efficient portfolios of mutual funds with desired factor loadings if the factor loadings in the underlying factor model are positively weighted portfolios. Under these conditions style analysis may also be used to determine a benchmark portfolio for performance measurement. Attribution of the returns on portfolios of which the actual composition is unobserved to specific asset classes on the basis of return based style analysis is attractive if moreover there are no additional cross exposures between the asset classes and if fund managers hold securities that on average have a beta of one relative to their own asset class. If such restrictions are not met, and in particular if the factor loadings do not generate a positively weighted portfolio, the restrictions inherent in return based style analysis distort the outcomes of standard regression approaches rather than that the analysis is improved. The size of the distortions is illustrated by considering empirical results on style analysis of US mutual funds.

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1 Introduction

In recent years return based style analysis, as introduced by Sharpe (1992) has become a very popular tool for analyzing mutual fund returns. Essentially, in return based style analysis a factor model is used to explain fund returns. The factors are taken to be the returns on several benchmark portfolios, such as value, growth, small cap, momentum, country, or sector portfolios. Standard style analysis imposes the factor loadings to be positive and to sum to one. Therefore, these factor loadings constitute a positively weighted portfolio and mutual fund returns can be decomposed in the return on the style portfolio and an idiosynchratic fund return.

Return based style analysis determines the mimicking portfolio of mutual funds or other investment opportunities with positive portfolio weights, i.e., the positively weighted style portfolio that is closest to the mutual fund in a least squares sense. These mimicking portfolios are used in various applications. In this paper we evaluate several of these applications in order to examine whether these style portfolios are appropriate for answering the underlying questions. One possible application of this style portfolio is to examine whether the style portfolio is available at lower costs than the mutual fund itself (either because of smaller load fees or because of smaller expense ratios). If this is the case, the style portfolio might be a more attractive investment alternative. Secondly, the style portfolio can also be used as a benchmark in evaluating the performance of the mutual fund. We will discuss this application in some detail and show how it is related to the more traditional Jensen measure. A third application of style portfolios is the construction of efficient portfolios of mutual funds that have desired factor loadings (see e.g. Lucas and Riepe (1996)). In addition to the factor exposures, return based style analysis yields the idiosyncratic fund returns which are essential ingredients in constructing the optimal portfolios. When using return based style analysis to determine the relevant factor exposures, the portfolio and positivity constraints will, in general, result in inconsistent estimates unless the factor loadings are positivily weighted portfolios. Finally, return based style analysis may be used to determine the actual asset allocation of a mutual fund using return data only. In this application the aim of return based style analysis coincides with the aim of holdings based style analysis where observations on the portfolio holdings of the fund are used to allocate every asset to a specific asset class. We show that return based style analysis is only able to generate the actual asset allocation of a mutual fund if some additional assumptions are satisfied.

In this paper we argue that the answer to the question whether or not it is useful to impose the portfolio and positivity constraints that characterize return based style analysis depends on the aim of the analysis. The case when no constraints are imposed on the factor loadings, will be refered to as *weak* style analysis. The case where only the portfolio constraint is imposed will be referred to as *semi-strong* style analysis and the case where both the portfolio and the positivity constraints are imposed will be referred to as *strong* style analysis, or style analysis as proposed by Sharpe (1992). In Section 2 we discuss the relation between unrestricted factor loadings and (positively weighted) mimicking portfolios, i.e., between weak, semi-strong, and strong style-analysis. In Section 3 we show how the estimated factor exposures can be used to construct optimal factor portfolios from the available mutual fund returns. In Section 4 we consider the relation between style analysis and performance measurement. Section 5 subsequently considers the assumptions under which return based style analysis reduces to the determination of the actual portfolio composition of a fund, as in holdings based style analysis. Section 6 illustrates the various applications of style analysis using data for US-based internationally diversified mutual funds and shows how imposing the restrictions that define style analysis improve the analysis. Section 7 concludes.

2 From unrestricted factor loadings to positively weighted mimicking portfolios

Before analyzing the possible applications of style analysis, we start by evaluating the effects of the portfolio and positivity constraints in style analysis. Suppose that K factor (mimicking) portfolios with return vector R_t drive the asset returns. In addition, there are N mutual funds with return vector r_t, for which we have the linear factor model

$$r_t = a + BR_t + \varepsilon_t, \tag{1}$$

where $E[\varepsilon_t] = E[\varepsilon_t R_{i,t}] = 0$ for $i = 1, ..., K$. In this case $B = \Sigma_{rR}\Sigma_{RR}^{-1}$, and $a = \mu_r - B\mu_R$, where Σ is a covariance matrix and μ is an expected return vector. When using (1) as a factor model, we do not impose any constraints on a and B. In particular, the rows of B do not necessarily constitute positively weighted portfolios. On the other hand, in style analysis, it is common to refer to the regression in (1) as the style regression, where we impose the constraints that the rows of B are positively weighted portfolios. In the sequel, if there are no restrictions on B, we refer to this as *weak* style analysis and to $a+\varepsilon_t$ as the weak idiosyncratic returns. If we define a_i as the ith element of a and b_i as the ith row of B, then a_i and b_i are the solutions to the problem

$$\min_{\alpha,\beta} E\left[(r_{i,t}-\alpha-\beta' R_t)^2\right]. \tag{2}$$

The vector b reflects the fund mimicking positions or the minimum variance hedge positions for the mutual fund.

To see the effect of the portfolio constraint $\sum_j \beta_j = 1$, let $\widetilde{a}_i$ and $\widetilde{b}_i$ be the solutions of the problem

$$\begin{aligned} & \min_{\alpha,\beta} E\left[(r_{i,t}-\alpha-\beta' R_t)^2\right], \\ \text{s.t. } \beta'\iota_K &= 1 \end{aligned} \tag{3}$$

where ι_K is a K-dimensional vector of ones. Thus, $\widetilde{b}_i$ are the factor exposures which are constrained to sum to one, i.e., they characterize a portfolio. The case where only the portfolio constraint is imposed, will be referred to as *semi-strong* style analysis. Using standard least squares results, it is straightforward to show that the coefficients $\widetilde{b}_i$ can be written as

$$\widetilde{b}_i = b_i + (1-b_i'\iota_K)\Sigma_{RR}^{-1}\iota_K(\iota_K'\Sigma_{RR}^{-1}\iota_K)^{-1}. \tag{4}$$

Notice that the last part of this expression equals the Global Minimum Variance (GMV) Portfolio of the factor portfolios: $w_{GMV} = \Sigma_{RR}^{-1}\iota_K(\iota_K'\Sigma_{RR}^{-1}\iota_K)^{-1}$. Defining $c_i = b_i'\iota_K$, the ith row of $\widetilde{b}_i$ reads

$$\begin{aligned} \widetilde{b}_i &= b_i + (1-c_i)w_{GMV} \\ &= c_i\left(\frac{b_i}{b_i'\iota_K}\right) + (1-c_i)w_{GMV}. \end{aligned} \tag{5}$$

Thus, for each mutual fund, the semi-strong style coefficients, or portfolio restricted exposures $\widetilde{b}_i$ are equal to a weighted average of the GMV portfolio and a hedge portfolio $b_i/b_i'\iota_K$. It follows immediately from (5) that $\widetilde{b}_i$ only coincides with the unrestricted exposures b_i if $c_i = 1$ which is the case if the weak style coefficients already are a portfolio.

In a similar fashion, it is straightforward to show that the portfolio constraint implies that the intercepts $\widetilde{a}$ equal

$$\widetilde{a}_i = a_i + (b_i'\iota_K - 1)E[R_t^{GMV}]. \tag{6}$$

The semi-strong style coefficients in (5) yield the style portfolio that is closest to the mutual fund in a least squares sense, i.e., it is the best mimicking portfolio. Since the difference between the mutual fund return $r_{i,t}$ and the return on the mimicking portfolio $\widetilde{b}_i' R_t$ is simply the tracking error, $e_{i,t} = r_{i,t} - \widetilde{b}_i' R_t$, the mimicking portfolio is the portfolio that yields the lowest tracking error variance. It is also obvious from (4) and (6) that if the portfolio restriction is not valid, these mimicking portfolio weights and the resulting intercept give inconsistent estimates of the actual factor loadings B, and the associated intercept a, where the inconsistency is proportional to the GMV portfolio, w_{GMV}.

In addition to the portfolio constraint, it is common in style analysis to impose positivity constraints on the estimated factor exposures. The style portfolios $\widehat{b}_i$ and the associated intercepts $\widehat{a}_i$ are then the solution to the problem

$$\begin{aligned} \min_{\alpha,\beta} E\left[(r_{i,t} - \alpha - \beta' R_t)^2\right], \qquad (7) \\ \text{s.t. } \beta'\iota_K &= 1, \\ \beta &\geq 0, \end{aligned}$$

where the inequality sign applies componentwise. We refer to this case as *strong* style analysis. If we order the benchmarks as $R_t' = (R_{1t}' \; R_{2t}')$ such that the positivity constraints are not binding for R_{1t} and binding for R_{2t} (implying that $\beta' = (\beta_1' \; 0_2')$, where 0_2 is a vector of zeros with the same dimension as R_{2t}), then the coefficients $\widehat{b}_{1i}$ coincide with the portfolio constrained coefficients in a regression of the mutual fund return on the benchmarks R_{1t} only. It follows that the coefficients b_{1i} can be written as

$$\begin{aligned} \widehat{b}_{1i} &= b_i^{(1)} + (1 - c_i^{(11)}) w_{GMV}^{(1)} \qquad (8) \\ &= c_i^{(1)} \left(\frac{b_i^{(1)}}{b_i^{(1)\prime}\iota_1} \right) + (1 - c_i^{(1)}) w_{GMV}^{(1)}, \end{aligned}$$

where

$$\begin{aligned} c_i^{(1)} &= b_i^{(1)\prime}\iota_1, \\ w_{GMV}^{(1)} &= \frac{\Sigma_{11}^{-1}\iota_1}{\iota_1'\Sigma_{11}^{-1}\iota_1}, \end{aligned}$$

and the coefficients $b_i^{(1)}$ result from the regression

$$r_{i,t} = a_i^{(1)} + b_i^{(1)\prime} R_{1t} + \varepsilon_{i,t}^{(1)}. \tag{9}$$

It is well known that the coefficients $b_i^{(1)}$ can be expressed in terms of b_i as

$$b_i^{(1)} = b_{1i} + \Sigma_{11}^{-1}\Sigma_{12} b_{2i}. \tag{10}$$

Similarly, the intercept $\widehat{a}_i$ can be written as

$$\widehat{a}_i = a_i^{(1)} + (b_i^{(1)\prime}\iota_1 - 1)E[R_{1t}^{GMV}]. \tag{11}$$

Again, we get that the strong style portfolio is a weighted average of the GMV portfolio $w_{GMV}^{(1)}$ and a hedge portfolio $b_i^{(1)}/b_i^{(1)\prime}\iota_1$, but now these portfolios are based on the subset of benchmarks, R_{1t}, for which the positivity constraints are not binding.

The strong style coefficients as given in (8) reflect the positively weighted portfolio of the benchmarks that mimics the mutual fund. Although it is the best positively weighted mimicking portfolio, there is an additional potential inconsistency in the estimated coefficients relative to the actual factor exposures in (1), because of the positivity constraints. In estimating the strong style portfolio $\widehat{b}_i$, two inconsistencies occur relative to the weak portfolio b. The first one arises because of the portfolio constraint and is proportional to the GMV-portfolio of either R_t or R_{1t}. The second inconsistency arises because of the positivity constraints, which result in estimated coefficients that are based on the subset R_{1t} only, rather than the entire set of benchmarks R_t, and this inconsistency is related to (10). Of course, if the factor exposures b are all positive and sum to one, then the coefficients $\widehat{b}_i$ and b_i coincide and there is no inconsistency. In this case, imposing the portfolio and positivity constraints will actually lead to consistent and more efficient estimates.

3 Using factor exposures and estimated styles in constructing optimal portfolios

A first application of style analysis is the construction of efficient portfolios from the mutual funds that have desirable factor or risk exposures, as shown, for instance, in Lucas & Riepe (1996). If the factor-model in (1) generates asset returns, then the coefficients a and B together with the covariance matrix of the residuals ε_t, $\Sigma_{\varepsilon\varepsilon}$, provide the necessary input in constructing efficient portfolios for investors. Given that the returns R_t represent the K relevant risk factors for investors, an investor may want to have specific exposures to those risk factors, which are given by the K-dimensional vector ρ (see, e.g., Fama (1996) and Cochrane (1999)). For instance, the investor may desire a specific exposure to stocks, bonds, and real estate, in which case the relevant risk factors are given by those asset classes. Alternatively, asset returns may be driven by factors such as market wide risk and recession risk, where investors may need a specific hedge against recession risk, which then determines the desired exposure. We assume that the investor chooses his portfolio from the N mutual funds that are available. The portfolio he chooses is denoted by w_r and the portfolio return by $r_t^p = w_r' r_t$. If the investor wants to obtain a specific expected return m^p and wants to minimize the portfolio variance, then the portfolio problem he faces can be written as

$$\begin{aligned} \min_{w_r} Var[r_t^p] &= w_r' \Sigma_{rr} w_r, \\ \text{s.t. } E[r_t^p] &= w_r' \mu_r = m^p, \\ w_r' \iota_N &= 1, \\ B w_r &= \rho. \end{aligned} \tag{12}$$

Fama (1996) denotes portfolios that solve (12) as multifactor efficient (MFE). If the factor model in (1) is valid, then the variance of the portfolio return can be written as

$$\begin{aligned} Var[r_t^p] &= w_r' Var\left[a + BR_t + \varepsilon_t\right] w_r \\ &= w_r' B \Sigma_{RR} B' w_r + w_r \Sigma_{\varepsilon\varepsilon} w_r \\ &= \rho' \Sigma_{RR} \rho + w_r \Sigma_{\varepsilon\varepsilon} w_r, \end{aligned} \tag{13}$$

and the expected portfolio return equals

$$\begin{aligned} E[r_t^p] &= w_r' E\left[a + BR_t + \varepsilon_t\right] \\ &= w_r' a + \rho' \mu_R. \end{aligned} \tag{14}$$

This implies that the portfolio variance and expected return depend on $\Sigma_{\varepsilon\varepsilon}$ and a only, since the exposures ρ are predetermined.

Given this setup and defining $m = m^p - \rho' \mu_R$, the portfolio problem in (12) can be rewritten as

$$\min_{w_r} w_r' \Sigma_{\varepsilon\varepsilon} w_r, \tag{15}$$

$$\begin{aligned} \text{s.t. } w_r' a &= m, \\ w_r' \iota_N &= 1, \\ B w_r &= \rho. \end{aligned}$$

This setup shows that style analysis provides the necessary input to solve the portfolio problem, because style analysis determines B and $\Sigma_{\varepsilon\varepsilon}$ as well as a. It also shows that, given the desired factor exposures ρ, the relevant characteristics of the mutual funds are given by their factor exposures, or weak style return, and the mean and variance of their weak idiosyncratic return, a and $\Sigma_{\varepsilon\varepsilon}$. It is important to note that in (13) we explicitly used the fact that $E[\varepsilon_t R_t] = 0$. If strong style analysis is used to determine a, B, and $\Sigma_{\varepsilon\varepsilon}$, then this requirement is generally not met which shows the limits of using style analysis for the portfolio problem in (15). Imposing portfolio and positivity constraints on the style coefficients when the actual factor loadings are no positively weighted portfolios, leads to error terms e_t that are not necessarily orthogonal to the benchmark returns and to inconsistent estimates of a and B.

The restrictions on the portfolio weights w_r in (15) can be written in short hand as

$$A' w_r = \theta,$$

where

$$\begin{aligned} A &= \begin{bmatrix} a & \iota_N & B' \end{bmatrix}, \\ \text{and } \theta &= \begin{pmatrix} m \\ 1 \\ \rho \end{pmatrix}. \end{aligned}$$

It is now straightforward to show (see, e.g. Cochrane (1999)) that the optimal portfolio w_r equals

$$w_r = \Sigma_{\varepsilon\varepsilon}^{-1} A \left(A' \Sigma_{\varepsilon\varepsilon}^{-1} A \right)^{-1} \theta. \tag{16}$$

From this optimal portfolio it follows that if there are K relevant risk factors, then all possible choices of expected returns and exposures can be attained when there are at least $K + 2$ different mutual funds, the returns of which are linearly independent, because there are $K + 2$ portfolio constraints that have to be satisfied.

Having a factor model like (1) is useful in determining efficient portfolios that have the desired factor exposures. If strong style analysis is used to determine the relevant coefficients from the factor model, then the real factor loadings B must satisfy the portfolio and positivity constraints. If this is not the case, then style analysis yields inconsistent coefficients $\widehat{a}$ and $\widehat{B}$, and residuals e_t which are not orthogonal to the benchmark returns R_t and which therefore yield misleading results when substituted in the portfolio problem in (15). Thus, unless one is willing to assume that the factor loadings B constitute positively weighted portfolios, one should not impose those constraints in style analysis if the results are to be used in constructing efficient portfolios.

4 Style analysis and performance measurement

A second way in which the style portfolio $\widehat{b}_i$ is often used, is to provide a benchmark to evaluate the performance of the mutual fund. Since $\widehat{b}_i$ reflects the best positively weighted mimicking portfolio, it seems natural to compare the mutual fund returns r_t with the returns on the mimicking portfolio $\widehat{b}_i' R_t$. The intercept $\widehat{a}_i$ in the style regression

$$r_{i,t} = \widehat{a}_i + \widehat{b}_i' R_t + e_{i,t}, \tag{17}$$

gives the expected excess return of the mutual fund relative to the mimicking portfolio. If it is possible to find a perfect mimicking portfolio $\widehat{b}_i$, implying that $Var[e_{i,t}] = 0$, then a positive value of $\widehat{a}_i$ implies that the fund return can only be obtained at higher cost when using the benchmarks, and that investors will strictly prefer the mutual fund over the mimicking portfolio. If $Var[e_{i,t}] > 0$, a positive value of $\widehat{a}_i$ does not necessarily mean that the fund outperforms the mimicking portfolio though, since the mutual fund may also be riskier than the mimicking portfolio. If the choice is to invest either in the mimicking portfolio or in the mutual fund, the performance can therefore best be measured by the Sharpe ratio, which gives the excess expected return of the portfolio (or fund) relative to its standard deviation:

$$Sh_i = \frac{E[r_{i,t}] - R^f}{\sigma(r_{i,t})}.$$

Since the difference in expected returns between the mutual fund and the mimicking portfolio is the style intercept, $\widehat{a}_i$, a positive value of $\widehat{a}_i$ will induce a higher Sharpe ratio, unless this is offset by a higher standard deviation of the mutual fund, $\sigma(r_{i,t})$.

The variance of the mutual fund return can be written as

$$\begin{aligned} Var[r_{i,t}] &= Var[\widehat{b}_i' R_t + e_{i,t}] \\ &= Var[\widehat{b}_i' R_t] + Var[e_{i,t}] + 2Cov[\widehat{b}_i' R_t, e_{i,t}] \\ &= Var[\widehat{b}_i' R_t] + Var[e_{i,t}] + 2 \times \frac{1 - b_i' \iota_K}{\iota_K' \Sigma_{RR}^{-1} \iota_K}, \end{aligned}$$

where the last term arises because the error term $e_{i,t}$ may be correlated with $\widehat{b}_i' R_t$ because of the portfolio constraint, as follows from (5). Thus, the variance of the mutual fund return will exceed that of the mimicking portfolio return, if

$$\begin{aligned} \frac{1 - b_i' \iota_K}{\iota_K' \Sigma_{RR}^{-1} \iota_K} &> -\frac{1}{2} \times Var[e_{i,t}] \Longleftrightarrow \\ 1 - b_i' \iota_K &> -\frac{1}{2} \frac{Var[e_{i,t}]}{Var[R_t^{GMV}]}. \end{aligned} \tag{18}$$

Similarly, the variance of the mutual fund return is smaller if the inequality is reversed. Notice that a necessary condition for a smaller variance of the mutual fund return is that $b_i' \iota_K > 1$, implying that - without the portfolio constraint - the mimicking portfolio would require a bigger investment than the mutual fund. In addition to this, it follows from (18) that in terms of variance, the mutual fund becomes more attractive than the mimicking portfolio if $Var[R_t^{GMV}]$ increases and if $Var[e_{i,t}]$ decreases.

Obviously, a sufficient condition for the mutual fund to be more (less) attractive than the mimicking portfolio is that $\widehat{a}_i \geq (\leq) 0$ and $(1 - b_i' \iota_K)$ is smaller (bigger) than the right hand side of (18). In these cases, the Sharpe ratio of the mutual fund will clearly be bigger (smaller) than the Sharpe ratio of the mimicking portfolio. For other cases, there is always a trade off between a higher (lower) expected return of the mutual fund and a bigger (smaller) variance.

An alternative way of analyzing the mutual fund performance is by using the Jensen measure, which is the intercept in a regression of the mutual fund excess returns on the benchmark excess returns:

$$r_{i,t} - \eta = \alpha_{J,i} + B(R_t - \eta \iota_K) + \varepsilon_{i,t}. \tag{19}$$

Here η is the zero-beta rate associated with a mean-variance efficient portfolio, which may be replaced by the risk free rate. A high value of the Jensen measure indicates that the maximum obtainable Sharpe ratio from the benchmark assets R_t only can be improved upon if the investor also includes the mutual fund in his investment portfolio. Thus, whereas the Sharpe ratio can be used to make a choice between two investment alternatives, the mutual fund and the benchmark portfolio, the Jensen measure gives the improvement in the Sharpe ratio that can be obtained if the mutual fund is added to the benchmark assets (see, e.g., DeRoon and Nijman (1999)). From Equation (6) it follows that the portfolio restricted intercept $\widetilde{a}_i$ equals a special case of the generalized Jensen measure, since $\widetilde{a}_i$ equals the intercept α_{Ji} in the regression

$$r_{i,t} - E[R_t^{GMV}] = \alpha_{Ji} + \beta \left(R_t - E[R_t^{GMV}]\iota_K \right) + u_{i,t}. \tag{20}$$

Thus, for investors such that the zero-beta rate equals the expected return on the GMV portfolio, we obtain the Jensen measure as the portfolio restricted intercept in a regression of the fund returns on the benchmark returns. In a similar fashion, the intercept $\widehat{a}_i$ in the style analysis, which includes both the portfolio and the positivity constraints is also a special case of the Jensen measure as in (20), but based on the subset R_{1t} only, for which the positivity constraints are not binding.

In summary, the performance measurement of the mutual fund relative to the mimicking portfolio should not be based on the intercept $\widehat{a}_i$ only, since the mutual fund may be also be riskier than the mimicking portfolio which may actually result in a lower Sharpe ratio even though the intercept $\widehat{a}_i$ is positive. In addition, although the $\widehat{a}_i$ may be interpreted in terms of the Jensen measure, it should be noted that $\widehat{a}_i$ is the Jensen measure for a very specific group of investors.

5 Style analysis and mutual fund portfolio weights

A third application of style analysis, apart from finding the best mimicking portfolio or benchmark, or from providing the necessary input for constructing efficient portfolios, is to estimate the portfolio holdings of mutual fund managers (see, e.g., (Brown & Goetzmann (1997)). In this case, the portfolio and positivity constraints on the factor portfolios are imposed to reflect the portfolio and short sales constraints faced by mutual fund managers. In this section we consider only one mutual fund, $N = 1$, and analyze the use of style analysis given the fund's portfolio holdings, in order to see if style analysis can indeed be used to estimate the portfolio holdings.

Notice that R_t contains the returns on K benchmark or factor *portfolios* which themselves consist of individual assets. Most fund managers typically invest in a subset of the assets underlying an index only and, moreover, give the assets in their portfolio different weights than the index. Denote the vector of the stock returns that are present in benchmark index i as $R_t^{(i)}$, where $R_t^{(i)}$ has $K^{(i)}$ elements. The index return $R_{i,t}$ itself is defined by a particular index portfolio $x^{(i)}$, i.e.,

$$R_{i,t} = x^{(i)\prime} R_t^{(i)}.$$

The fund manager chooses a portfolio $v^{(i)}$ from $R_t^{(i)}$ for which in general $v^{(i)} \neq x^{(i)}$. Assuming that the manager chooses portfolios $v^{(i)}$, from K asset classes, he also has to determine the weights w_i assigned to each asset class. Thus, we have that $\Sigma_j v_j^{(i)} = 1, \forall i$, $\Sigma_i w_i = 1$, and $v_j^{(i)} \geq 0$ and $w_i \geq 0$, $\forall i, j$. The manager's return on asset class i is equal to

$$r_t^{(i)} = v^{(i)\prime} R_t^{(i)}. \tag{21}$$

The return on the fund is now equal to

$$r_t = \Sigma_{i=1}^K w_i r_t^{(i)} = \Sigma_{i=1}^K w_i \Sigma_{j=1}^{K^{(i)}} v_j^{(i)} R_{j,t}^{(i)}.$$

Now consider a regression of $r_t^{(i)}$ on index i, $R_{i,t}$ and a constant:

$$r_t^{(i)} = a^{(i)} + b^{(i)} R_{i,t} + u_t^{(i)},$$

then

$$b^{(i)} = \frac{Cov[v^{(i)\prime} R_t^{(i)}, R_{i,t}]}{Var[R_{i,t}]} = v^{(i)\prime} \beta^{(i)},$$

where $\beta^{(i)}$ is the vector of beta's of the assets in subset i relative to their own index. Notice that $b^{(i)}$ is not necessarily equal to one, since it may well be the case that the manager has selected high-beta stocks in his portfolio for instance. Also, a priori there is no need for $a^{(i)}$ to be equal to zero.

Assuming that $E[u_t^{(i)} R_{j,t}] = 0$, $\forall i, j$ (i.e., the residual for asset class i is uncorrelated with *all* available indices - which is a rather strong assumption of course), it follows that, without imposing any constraints on the regression in (1), the coefficients B and a are given by

$$\begin{aligned} B &= [w_1 b^{(1)} \; w_2 b^{(2)} \; ... \; w_K b^{(K)}] \\ &= [w_1 v^{(1)\prime}\beta^{(1)} \; w_2 v^{(2)\prime}\beta^{(2)} \; ... \; w_K v^{(K)\prime}\beta^{(K)}], \end{aligned} \tag{22}$$

and

$$a = \Sigma_{i=1}^{K} w_i a^{(i)}.$$

Notice that since $b^{(1)}...b^{(K)}$ are not necessarily equal to one, even in this case the elements of B do not have to sum to one and do not have to be positive, even though there may be short sales and portfolio restrictions for the managers' actual portfolio holdings. Also, since the elements of B are combinations of w_i, $v^{(i)}$ and $\beta^{(i)}$, these elements do not represent the portfolio weights $v^{(i)}$ of the mutual fund. If the mutual fund manager holds stocks with high (low) $\beta^{(i)}$'s for instance, then the elements of B will overestimate (underestimate) the portfolio factor weights w_i. The coefficients B do represent the general style of the mutual fund though. For instance, if the fund manager invests a certain fraction w_1 of his wealth in asset class 1, while selecting stocks that have a high (low) $\beta^{(1)}$, then this will be reflected by higher (lower) values of b_1, reflecting a higher sensitivity of the fund for factor 1. This would then also show up in a relatively higher weight for factor portfolio 1 in the mimicking portfolio for the fund. Thus, although the weak style coefficients do not need to reflect the actual portfolio holdings, they do provide the sensitivities or exposures of the mutual fund relative to the factor or benchmark portfolios.

To see the implications for style analysis, suppose a fund manager is only investing in the stocks of index i and in no stocks underlying all the other indices. To simplify notation, assume $i = 1$. Thus, $v^{(1)} \neq 0$, while $v^{(j)} = 0$, for $j \neq 1$. Partition the covariance matrix of the indices as

$$Var[R_t] = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{1K} \\ \sigma_{21} & \sigma_{22} & \cdots & \sigma_{2K} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{K1} & \sigma_{K2} & \cdots & \sigma_{KK} \end{pmatrix} = \begin{pmatrix} \sigma_{11} & \Sigma_{1j} \\ \Sigma_{j1} & \Sigma_{jj} \end{pmatrix},$$

where $j \neq 1$. Similarly, the covariance vector of the mutual fund with the benchmark indices is partitioned as

$$Cov[R_t, r_t] = \begin{pmatrix} \sigma_{1r} \\ \Sigma_{jr} \end{pmatrix}.$$

Using partitioned inverses and defining $D \equiv (\sigma_{11} - \Sigma_{1j}\Sigma_{jj}^{-1}\Sigma_{j1})$ the weak or unrestricted style coefficients can be written as

$$B = \begin{pmatrix} D^{-1}(\sigma_{1r} - \Sigma_{1j}\Sigma_{jj}^{-1}\Sigma_{1j}) \\ \Sigma_{jj}^{-1}\Sigma_{jr} - \Sigma_{jj}^{-1}\Sigma_{j1}D^{-1}(\sigma_{1r} - \Sigma_{1j}\Sigma_{jj}^{-1}\Sigma_{jr}) \end{pmatrix}.$$

It's obvious that if the fund mimics index 1 perfectly, i.e., $r_t = R_{1,t}$, then $\sigma_{1r} = \sigma_{11}$ and $\Sigma_{jr} = \Sigma_{j1}$, implying that

$$B = \begin{pmatrix} 1 \\ 0_{K-1} \end{pmatrix}.$$

However, in general,

$$r_t = a_1 + \beta_{r1}R_{1,t} + u_t,$$

implying

$$\begin{aligned} \sigma_{1r} &= \sigma_{11}\beta_{1r} \text{ and} \\ \Sigma_{jr} &= \Sigma_{j1}\beta_{1r} + \Sigma_{ju}. \end{aligned}$$

From this, we have that

$$B = \begin{pmatrix} \beta_{1r} - D^{-1}\Sigma_{1j}\Sigma_{jj}^{-1}\Sigma_{ju} \\ \Sigma_{jj}^{-1}\Sigma_{ju} + \Sigma_{jj}^{-1}\Sigma_{j1}D^{-1}\Sigma_{1j}\Sigma_{jj}^{-1}\Sigma_{ju} \end{pmatrix}.$$

This shows that there are two reasons why B_1 may deviate from one: The fund manager may invest in a portfolio that has a β_{1r} that deviates from one, or the residual u_t may be correlated with $R_{j,t}$, $j \neq 1$. Similarly, B_j will deviate from zero because $\Sigma_{ju} \neq 0$. This latter situation is likely to arise because the indices used in style analysis need not be mutually exclusive and the stocks underlying index i may be correlated with index j.

It is clear from this analysis that given that the factor portfolios or indices are constructed from individual stock holdings and that the individual stock positions of mutual fund managers will in general be different from the weights in the factor portfolios, the factor loadings of the mutual funds, B, will in general not constitute positively weighted portfolios. Using the results in Section 2, it then follows that imposing these constraints in style analysis will yield inconsistent estimates of B and a. In addition, since the factor loadings are a combination of the mutual fund stock holdings and the beta's of the stocks relative to the factor portfolios, even the weak style coefficients

B will not represent the actual weights assigned to the factor classes by the mutual fund manager. However, because the elements of B represent the weights assigned to the factor classes adjusted for the $\beta^{(i)}$'s of the individual stocks, B does provide a more accurate picture of the exposure of the fund to the asset classes than the portfolio holdings themselves.

6 Data and empirical analysis

In the previous sections we showed that the portfolio and positivity constraints in return based style analysis might lead to inconsistent estimation results, which subsequently will lead to inappropriate results in various applications of style analysis. In order to illustrate the potential consequences, we use style analysis in a number of the applications mentioned before. We employ a sample of eighteen US-based internationally investing mutual funds over the period January 1982 through April 1999. The mutual fund data are obtained from Morningstar's Principia Pro database and have as reported investment style 'foreign' or 'world', where the distinction between the two styles is that in case of 'foreign' it is not allowed to hold US-stocks. Our sample is comparable with the sample of Cumby and Glenn (1991) and DeRoon, Nijman and TerHorst (1998), studying the performance of, respectively, fifteen funds over the period January 1982 through June 1988 and eighteen funds over the period January 1982 through December 1994.

In Table 1 we present some summary statistics for the sample of funds that we employ. It appears that New Perspective realized the highest average return, i.e. 1.39% (16.7% annually), with the lowest standard deviation. The fund charges an initial load fee of 5.75%, and is by far the largest fund in size. The worldwide diversified fund First Invest Global charges the highest load fee of 6.25%, while six funds in the sample do not charge an initial load fee. The Vanguard International Growth fund can be characterized as a passively managed fund, while the other funds in the sample follow an active selection strategy.

6.1 Style analysis and performance measurement

As mentioned in the analysis of Section 4, style analysis is often used to provide a benchmark in order to evaluate the performance of mutual funds (see, e.g. Sharpe (1992) and Fung and Hsieh (1997)). A question that receives

Table 1: The table reports the average monthly fund return over the period January 1982 through April 1999, and the corresponding standard deviation of the fund return. The column labeled 'style' presents the reported investment F(oreign) or W(orld). The column 'Net Assets' reports the size of the fund as measured at the end of 1998, while the column 'Front Load' reports the load fee that the fund charges for a position in the fund.

Mutual Fund	Style	Average Return	Stand. Dev.	Net Assets	Front Load
		(%)	(%)	(mln $)	(%)
Alliance Global Sm	W	0.94	5.80	74.4	4.25
Alliance Intl	F	1.09	5.04	76.7	4.25
Bailard, Biehl Intl	F	0.99	4.95	113.4	0.00
Evergreen Intl Gr	F	0.94	4.37	66.0	0.00
First Invest Global	W	1.17	4.92	312.4	6.25
Kemper Intl	F	1.10	4.45	398.4	5.75
Nations Intl Gr	F	1.05	4.81	22.4	0.00
New Perspective	W	1.39	3.86	23061.1	5.75
Oppenheimer Global	W	1.33	4.99	3580.5	5.75
Phoenix-Aberdeen	W	1.00	5.67	185.3	4.75
Putnam Global Gr	W	1.35	4.23	3518.3	5.75
Scudder Intl	F	1.24	4.44	3103.7	0.00
T.Rowe Price Intl	F	1.26	4.56	10006.7	0.00
Templeton Global Sm	W	1.14	4.36	1095.8	5.75
Templeton Gr	W	1.28	4.08	12319.5	5.75
Templeton World	W	1.31	4.10	8589.9	5.75
United Intl Gr	F	1.27	4.29	1236.4	5.75
Vanguard Intl Gr	F	1.32	4.77	7601.6	0.00

considerable attention in the performance evaluation literature is why people invest in actively managed mutual funds (see, e.g. Gruber (1996)). Actively managed mutual funds are characterised by active stock selection strategies and market timing strategies in order to beat the return on a benchmark. In contrast, passively managed mutual funds mainly follow buy and hold strategies, where the investment objective is to replicate as close as possible a certain benchmark or market index. Consequently, due to the higher trading activity, actively managed mutual funds usually have much higher operating expenses than passively managed funds, i.e. on average respectively 1% vs 0.2% per year. Since these operating expenses are deducted from a mutual fund's gross income, investors might be interested in a potentially cheaper alternative. Most studies report that actively managed funds provide lower net returns than the passively managed funds (see, e.g. Wermers (2000)).

In order to examine whether it is cheaper to invest in a combination of passively managed funds or in one of the seventeen actively managed funds in our sample, we report in Table 2 the estimation results of the following weak and strong style analysis

$$r_{i,t} = \hat{a}_i + \hat{b}_1 R_t^{(VangUSA)} + \hat{b}_2 R_t^{(VangWorld)} + e_{i,t}$$

where $R_t^{(VangUSA)}$ and $R_t^{(VangWorld)}$ denote the returns in period t on the passively managed funds Vanguard 500 index and Vanguard International Growth, both offered by The Vanguard Group. The Vanguard 500 index fund seeks to match the performance of a benchmark that measures the investment return of large-capitalization US stocks, while the Vanguard International Growth seeks to provide long-term capital growth and primarily invests in stocks of seasoned companies outside the United States. The table also reports the average tracking error $\hat{a}_i$ of the strong style analysis. This tracking error can be interpreted as the average relative under or outperformance of the mutual fund with respect to the passive benchmark consisting of the Vanguard 500 index fund and the Vanguard International Growth fund.

It appears that the actively managed funds in the sample relatively underperform their corresponding mimicking portfolio that is a combination of the Vanguard 500 index fund and the Vanguard International Growth fund. The underperformance varies between 0.54% (i.e. 6.48% annually) for Alliance Global Small Companies fund and 0.04% (i.e. 0.48% annually) for New Perspective. However, as discussed in Section 4, a negative average tracking error does not necessarily indicate that investors should invest in the mimicking portfolio if the choice is restricted to invest in either the mutual fund or

Table 2: The table reports the estimation results of a weak and strong style analysis with benchmark assets the Vanguard 500 index fund and the Vanguard International Growth fund. The column labeled ' average tracking error' reports the fund's relative performance on a monthly basis to the mimicking portfolio of Vanguard funds.

Mutual Fund	strong style analysis			weak style analysis	
	average	exp. to Vanguard		exp. to Vanguard	
	track. error	500 index	Int. Growth	500 index	Int. Growth
Alliance Global Sm	-0.54	0.92	0.08	1.06	0.16
Alliance Intl	-0.26	0.15	0.85	0.18	0.86
Bailard, Biehl Intl	-0.33	0.03	0.97	0.00	0.95
Evergreen Intl Gr	-0.42	0.22	0.78	0.14	0.74
First Invest Global	-0.22	0.37	0.63	0.36	0.62
Kemper Intl	-0.24	0.14	0.86	0.09	0.82
Nations Intl Gr	-0.29	0.11	0.89	0.11	0.89
New Perspective	-0.04	0.58	0.42	0.53	0.39
Oppenheimer Global	-0.06	0.40	0.60	0.45	0.63
Phoenix-Aberdeen	-0.47	0.84	0.16	0.94	0.22
Putnam Global Gr	-0.05	0.40	0.60	0.39	0.60
Scudder Intl	-0.12	0.17	0.83	0.15	0.81
T.Rowe Price Intl	-0.09	0.13	0.87	0.11	0.86
Templeton Global Sm	-0.31	0.70	0.30	0.64	0.26
Templeton Gr	-0.17	0.70	0.30	0.65	0.26
Templeton World	-0.14	0.73	0.27	0.69	0.25
United Intl Gr	-0.11	0.31	0.69	0.24	0.64

the mimicking portfolio, since all mimicking portfolios contain some residual risk relative to the actively managed mutual fund. In order to answer that question we report in Table 3 the Sharpe ratios of the mutual funds and the corresponding mimicking portfolios. Since from the strong style analysis it followed that the style intercept $\hat{a}_i$ is negative for all the actively managed funds in the sample, a higher Sharpe ratio of the fund can only be caused by a lower standard deviation of the mutual fund compared to the mimicking portfolio.

If the choice for the investor is to invest either in the actively managed mutual fund or in the passively managed mimicking portfolio then from Table 3 it follows that the investor should choose the mimicking portfolio in all cases except one: the Sharpe ratio of New Perspective is greater than the Sharpe ratio corresponding to the mimicking portfolio. For all other cases, the investor can obtain a higher expected return per unit of risk by investing in the fund mimicking portfolio.

As shown by Jobson and Korkie (1989), the Jensen measure is the relevant one if the investor already holds a portfolio. Therefore, we propose to use the Jensen measure, as given in (19), as an alternative performance measure that answers the question whether an investor can improve the maximum obtainable Sharpe ratio of his initial portfolio by also investing in an actively managed internationally investing mutual fund. From the weak style analysis results reported in Table 2, it appears that for none of the seventeen mutual funds the positivity constraint is binding. Therefore, as mentioned in Section 4, the intercept $\hat{a}_i$ in the style analysis with portfolio and positivity constraints can be considered as a special case of the Jensen measure in case the zero beta rate equals the expected return on the GMV portfolio. In Table 3 we report the Jensen measure as in (20), where the benchmark assets R_t are the Vanguard 500 index fund and the Vanguard International Growth, while we assume that the zero beta rate is equal to 1.43% at a monthly basis. This zero beta rate corresponds to the average return on the Global Minimum Variance portfolio of the Vanguard 500 index fund and the Vanguard International Growth fund. It appears that six of the seventeen actively managed mutual funds have a Jensen measure that is significantly smaller than zero, indicating that the investor can improve the maximum obtainable Sharpe ratio of his initial portfolio by taking a short position in the mutual fund under consideration (see, e.g. Elton, Gruber and Blake (1996), DeRoon, Nijman and TerHorst (1998), DeRoon, Nijman and Werker (2000)).

Table 3: The table reports the Sharpe ratios for the mutual funds and their corresponding mimicking portfolios determined with strong style analysis using the Vanguard 500 index fund and the Vanguard International Growth fund as benchmark assets. Furthermore, the table reports the generalized Jensen measure for a zero beta rate of 1.43 %, i.e. the average rate of the GMV portfolio. The investors' initial portfolio contains the two Vanguard funds. A * behind the Generalized Jensen Measure indicates a value that is significant at the 5 % level.

Mutual Fund	Jensen Measure	Sharpe Ratio	
	$\eta = 1.43\%$	mutual fund	mim. portfolio
Alliance Global Sm	-0.543*	0.163	0.355
Alliance Intl	-0.256	0.216	0.303
Bailard, Biehl Intl	-0.335*	0.200	0.282
Evergreen Intl Gr	-0.419*	0.215	0.314
First Invest Global	-0.218	0.238	0.338
Kemper Intl	-0.244*	0.248	0.302
Nations Intl Gr	-0.291*	0.218	0.296
New Perspective	-0.036	0.360	0.358
Oppenheimer Global	-0.059	0.267	0.341
Phoenix-Aberdeen	-0.474*	0.176	0.360
Putnam Global Gr	-0.046	0.318	0.341
Scudder Intl	-0.117	0.278	0.307
T.Rowe Price Intl	-0.087	0.276	0.299
Templeton Global Sm	-0.308	0.261	0.362
Templeton Gr	-0.170	0.313	0.362
Templeton World	-0.139	0.320	0.363
United Intl Gr	-0.111	0.295	0.329

6.2 Style analysis and portfolio weights

Usually, we do not observe the underlying portfolio holdings of the mutual funds. Although style analysis was not initially developed to determine the underlying holdings of the mutual fund, strong style analysis is often suggested as a method to estimate the portfolio holdings of the fund managers. As already mentioned by Sharpe (1992), and more extensively analyzed in Section 5, the portfolio and positivity constraints in style analysis might generally lead to inconsistent estimates of the fund exposures. Therefore, semi-strong and strong style analysis is not an appropriate method for the purpose of determining underlying fund holdings if these restrictions are not satisfied. As discussed in Section 5, even without he portfolio and positivity constraints in style analysis, the estimates will still not in general reflect the actual holdings since the fund manager could e.g. hold stocks with high beta's with respect to their own index. Nevertheless, the weak style estimates will reflect the sensitivity of the fund for certain factor or benchmark portfolios, i.e. the fund exposures.

In order to illustrate the potential consequences of imposing the portfolio and positivity constraints, in this subsection we will apply style analysis on the sample of eighteen internationally investing mutual funds, and compare it with the actual portfolio holdings over the sample period January 1991 through April 1999. Note that the sample period is different from the previous analysis, which is due to the fact that from the mutual funds in the sample we observe the reported holdings at an annual frequency for the investment regions North America, Europe and Pacific only over this shorter sample period. In Table 4 we report the average returns and corresponding standard deviations for the set of benchmark indices that we employ in return based style analysis. For each region or country index that we use the table reports summary statistics for growth and value stock indices. The data are obtained from Datastream International.

It appears that the MSCI Hong Kong Value index has the highest average monthly return of 2.22% on a monthly basis (i.e. about 26.6% annually), but also the highest risk as measured by the standard deviation, i.e. about 38.2% annually, over the sample period. The lowest average monthly return is for MSCI Japan Growth index, i.e. 0.08% on a monthly basis (about 0.95% annually). Corresponding to what is usually found in the literature (see, e.g. Fama and French (1998)), on average, value stocks outperform growth stocks at the country level, except in Canada and France where the MSCI Growth

Table 4: The table reports average monthly returns and corresponding standard deviations for a set of Morgan Stanley Capital Indices (MSCI) over the period January 1991 through April 1999. For each country or region, the table reports a Growth MSCI index and a Value MSCI index. The column labeled $\beta^{(i)}$ reports the beta's of the country indices relative to their regional index. Returns are measured in dollars.

Region	Country	Growth			Value		
		avg	std	$\beta^{(i)}$	avg	std	$\beta^{(i)}$
North America		1.68	3.63		1.59	3.58	
	United States	1.83	4.13	1.08	1.63	3.58	0.93
	Canada	0.95	5.08	0.97	0.87	5.00	0.98
Europe		1.24	3.92		1.44	4.16	
	France	1.29	5.32	1.06	1.27	5.16	1.09
	Germany	1.01	5.17	1.00	1.40	4.92	0.98
	United Kingdom	1.23	4.19	0.91	1.34	4.56	0.96
	Italy	1.09	7.28	0.87	1.42	8.23	1.00
Pacific		0.18	6.13		0.56	6.32	
	Australia	0.86	5.47	0.46	1.41	5.09	0.48
	Hong Kong	1.75	8.05	0.52	2.22	11.04	0.74
	Japan	0.08	6.86	1.08	0.48	6.94	1.09

index outperforms the MSCI Value index.

In order to illustrate that style analysis does not necessarily accurately estimate the portfolio holdings of fund managers, we first of all apply strong style analysis using three asset classes, i.e. regional indices of North America, Europe and Pacific. Table 5 reports the estimated exposures for these style indices over the period January 1991 through April 1999, and subsequently compares them with the average reported holdings over the same period.

It is interesting to note that for most of the funds in Table 5 the order of the estimated exposures corresponds to the order of the average reported holding over the period 1991 - 1998. The only two exceptions are Evergreen Intl Growth and Kemper Intl, where the order between the exposure to North America and Pacific has changed. The bottom three rows of Table 5 give an indication of the difference between the estimated strong style exposures and the reported actual holdings. On average the estimated style exposures exceed the reported holdings for North America (14.2%) and Europe (12.4%), whereas the style exposures are lower than the reported holdings for the Pacific index (-7.0%). For all three indices, we find that the estimated exposures and reported holdings are highly correlated (approximately 0.90).

It follows from the analysis in Section 5 that there are two reasons why the style estimates may be different from the actual holdings. The first reason is that fund managers usually invest in a subset of assets underlying an index, and assign different portfolio weights to these assets than in the index. In case the fund manager invests in stocks with low $\beta^{(i)}$'s, the style coefficients will underestimate the portfolio holding w_i. The second reason is that the residual in the style analysis may be correlated with the benchmark assets in the style regression. This reason is not unlikely since the indices in the style analysis usually are not mutually exclusive and, moreover, the stocks underlying a certain index may be correlated with another index.

In order to analyze what causes the differences between the estimated style exposures and the reported holdings, Table 6 reports some results of strong style analysis using benchmarks at a more disaggregated level. For instance, in case of North America, we now use four different indices: US Growth and Value indices and Canadian Growth and Value indices. If the beta's of these subindices relative to the aggregate North America index are not equal to one and if the weights of these subindices in the aggregate index differ from the weights assigned to them by the fund manager, then this will cause a difference between the estimated style exposures and the reported holdings, as follows from Section 5.

Table 5: The table reports the estimated exposures to regional indices based on return-based style analysis over the period January 1991 through April 1999, and the average reported holdings over the corresponding sample period.

Mutual Fund	Style	estimated exposures (average reported holdings)			
		North America	Europe	Pacific	Other
Alliance Global Sm	W	59.1 (52.6)	27.5 (20.2)	13.4 (9.9)	(2.0)
Alliance Intl	F	7.1 (1.7)	58.9 (47.5)	34.0 (38.0)	(2.0)
Bailard, Biehl Intl	F	16.0 (2.6)	64.0 (53.3)	20.0 (30.0)	(3.2)
Evergreen Intl Gr	F	29.1 (4.8)	56.0 (38.7)	14.9 (24.6)	(5.8)
First Invest Global	W	39.6 (28.8)	44.3 (35.4)	16.1 (24.8)	(3.2)
Kemper Intl	F	20.7 (3.5)	63.0 (51.7)	16.3 (29.6)	(5.2)
Nations Intl Gr	F	20.0 (0.7)	59.1 (47.6)	20.1 (33.0)	(6.0)
New Perspective	W	52.5 (32.7)	39.8 (29.6)	7.7 (11.1)	(2.6)
Oppenheimer Global	W	40.8 (23.9)	45.7 (35.2)	13.5 (15.1)	(6.1)
Phoenix-Aberdeen	W	39.2 (31.3)	48.6 (33.7)	12.2 (18.2)	(4.9)
Putnam Global Gr	W	43.3 (26.3)	43.8 (34.8)	13.0 (24.3)	(2.5)
Scudder Intl	F	17.4 (3.1)	58.2 (46.8)	24.4 (33.8)	(2.1)
T.Rowe Price Intl	F	14.0 (1.5)	61.5 (49.1)	24.5 (32.9)	(4.8)
Templeton Global Sm	W	49.1 (32.0)	43.8 (29.0)	7.1 (13.4)	(6.1)
Templeton Gr	W	51.5 (30.2)	35.5 (27.0)	13.1 (14.7)	(5.2)
Templeton World	W	47.9 (32.6)	39.4 (28.1)	12.7 (14.7)	(5.4)
United Intl Gr	F	10.7 (3.1)	82.9 (57.5)	6.4 (13.5)	(8.4)
Vanguard Intl Gr	F	9.3 (0.5)	65.7 (49.9)	25.0 (39.0)	(2.3)
mean difference		14.2	12.4	-7.0	
stdev difference		5.5	4.2	4.8	
correlation		0.95	0.96	0.88	
mean beta		0.74	0.80	0.40	
GMV-portfolio		0.60	0.35	0.05	

For each fund, Table 6 first of all reports the sum of the estimated style exposures, $\Sigma_i \widehat{b}_i$. If style analysis provides consistent estimates of the actual portfolio holdings, then these summes exposures should be close to the estimated exposures to the aggregate indices in Table 5. For North America, although the summed exposures are in the same order of magnitude as the aggregate exposures in Table 5, they are certainly not equal. Also, the difference between the summed exposures and the reported holdings is not smaller than the difference between the aggregate exposures and the reported holdings as can be found in Table 5. The bottom three rows of the table summarize the relation between the summed style exposures and the actual reported weights. Comparing the mean and standard deviation of the difference with the ones reported in Table 5, it can be seen that the use of subindices does not give any improvement for the North American case. Also, the correlation between the summed style exposures and the actual reported weights in Table 6 is almost identical to the one reported in Table 5, which is based on the aggregate index.

This picture changes if we focus on the European indices. For the European indices, the summed exposures in Table 6 are much closer to the actual reported holdings than the estimated exposures in Table 5. The average difference decreases from 12.4% in Table 5 to 2.2% in Table 6, and the standard deviation of the differences likewise decreases. For the Pacific region on the other hand, the differences between the summed style exposures and the actual reported holdings is substantial, and does not show any improvement relative to the differences reported in Table 5. The average difference changes from -7.0% in Table 5 to +3.1% in Table 6.

From Section 5, the summed exposures need not be equal to the reported holdings if the beta's of the subindices relative to the aggregate indices are different from one. To correct for this, Table 6 also reports the sum of the estimated style exposures for each region, weighted by the $\beta_j^{(i)}$ of each subindex j relative to the aggregate regional index i. If the style exposure for the disaggregated indices reflect the actual portfolio weights assigned by the fund manager, then this weighted sum should be closer to the reported holdings. Comparing the two columns for each region in Table 6, we see that the two summed exposures are very close in case of North America and Europe, but not for the Pacific case. This reflects the fact that the β's of the subindices relative to the aggregate index are close to one in case of North America and Europe, whereas in the Pacific case they can be as low as 0.46, as can be seen in Table 4. However, even though for the Pacific case the β's are clearly different from one, the weighted summed exposures do not explain the difference between the estimated aggregate exposure and the reported holdings in Table 5. On the contrary, the average difference between the summed exposure and the actual reported holdings increase from 3.1% to 18.6% and the correlation between the summed style exposures and the reported holdings even decreases.

Although the analysis is limited by the availability of the data, Table 6 indicates that the differences between estimated exposures and reported holdings is not likely to be explained by the fact that fund managers hold on average high or low beta stocks relative to the index. It follows then that the difference between reported holdings and estimated exposures is more likely to be caused by the correlations between the different indices. Thus, if style analysis is to be used to estimate the actual portfolio holdings, then it will be important to use indices which are mutually exclusive and which have low (zero) correlations with each other. The results in this section suggest that if the benchmark indices are not uncorrelated, then the estimated style exposures do not reflect the actual portfolio holdings of the fund manager.

6.3 Constructing efficient portfolios using different forms of style analysis

Asset allocation is one of the main issues in the investment decision. It is often the case that investors want to allocate a fixed percentage of their portfolio to e.g. international growth stocks. Obviously, the investor could simply choose an internationally investing mutual fund with reported investment style growth. However, the actual investment style does not necesarilly correspond to the reported investment style (see, e.g. Sharpe (1992), Brown and Goetzmann (1997) and Lucas and Riepe (1996)). Consequently, choosing a fund with the desired style does not imply that a fixed percentage of the investors' portfolio is allocated to it. Therefore, return based style analysis can be implemented in the the investors' asset allocation decision to properly find the optimal mix, as suggested by Lucas and Riepe (1996).

As shown in Section 3, the application of style analysis in its strong form will only lead to efficient portfolio weights with desired factor exposures if the actual factor loadings are already a positively weighted portfolio. Otherwise, the residuals e_t of this style analysis are not necessarily uncorrelated with the benchmark assets and this might lead to inconsistent style estimates.

Table 6: The colums '$\sum \hat{b}_i$' in the table reports the estimated exposures aggregated at a regional level of a strong style analysis, using as asset classes the growth and value indices of each country underlying a regional index. The countries underlying a certain regional index are reported in Table 4. The columns '$\sum \hat{b}_i/\beta^{(i)}$' report a weighted estimated exposure.

Mutual Fund	estimated exposure					
	North America		Europe		Pacific	
	$\sum \hat{b}_i$	$\sum \hat{b}_i/\beta^{(i)}$	$\sum \hat{b}_i$	$\sum \hat{b}_i/\beta^{(i)}$	$\sum \hat{b}_i$	$\sum \hat{b}_i/\beta^{(i)}$
Alliance Global Sm	0.656	0.665	0.193	0.202	0.152	0.191
Alliance Intl	0.111	0.115	0.489	0.499	0.400	0.518
Bailard, Biehl Intl	0.173	0.176	0.553	0.561	0.274	0.400
Evergreen Intl Gr	0.321	0.324	0.474	0.490	0.206	0.299
First Invest Global	0.392	0.389	0.378	0.382	0.230	0.335
Kemper Intl	0.216	0.228	0.500	0.513	0.284	0.483
Nations Intl Gr	0.200	0.206	0.453	0.460	0.347	0.581
New Perspective	0.467	0.460	0.346	0.356	0.187	0.328
Oppenheimer Global	0.400	0.389	0.392	0.407	0.207	0.309
Phoenix-Aberdeen	0.360	0.343	0.402	0.413	0.238	0.412
Putnam Global Gr	0.370	0.353	0.394	0.410	0.236	0.381
Scudder Intl	0.179	0.184	0.472	0.486	0.349	0.503
T.Rowe Price Intl	0.109	0.105	0.514	0.522	0.377	0.584
Templeton Global Sm	0.546	0.579	0.288	0.301	0.166	0.328
Templeton Gr	0.482	0.515	0.251	0.257	0.267	0.485
Templeton World	0.455	0.483	0.282	0.286	0.263	0.459
United Intl Gr	0.168	0.172	0.626	0.650	0.207	0.400
Vanguard Intl Gr	0.092	0.098	0.534	0.554	0.374	0.563
mean difference	0.143	0.148	0.022	0.033	0.031	0.186
stdev difference	0.054	0.063	0.031	0.034	0.047	0.079
correlation	0.95	0.93	0.96	0.96	0.88	0.71

Consequently, the use of inconsistent style estimates might lead to optimal portfolios with undesired factor exposure. In order to illustrate the potential consequences of using strong style analysis in constructing efficient portfolios we will look at the following problem.

We assume that an investor chooses his mean-variance efficient portfolio from the $N = 18$ mutual funds that are available, and that he desires an international asset allocation with fixed exposure to certain investment regions and types of stock. We consider the following two combinations of desired factor exposure: portfolio A : 25% to North American growth stocks, 50% to European growth stocks and 25% to Pacific growth stocks, portfolio B: 65% to North American value stocks, 25% to European value stocks and 10% to Pacific value stocks. In Table 7 we report for each desired exposure the optimal portfolio for a required expected return of 1.25% on a monthly basis. We apply the three forms of style analysis to determine the exposure of the funds for the factors under consideration.

It appears that for an efficient portfolio with desired exposure 'A' and expected return 1.25% monthly, a short position has to be taken in nine out of the eighteen mutual funds in the sample in case that we apply the weak form of style analysis. The portfolio has a short position of more than 46% in Kemper International in combination with a long position of more than 67% in T.Rowe Price International. The determined portfolio with desired factor exposures has a standard deviation of 4.50% on a monthly basis (i.e. 15.6% annually). While keeping the expected return on the portfolio fixed, it is interesting to note that in case of semi-strong and strong style analysis the corresponding risk on the portfolio decreases. Apparently, the use of portfolio and the positivity restriction in style analysis leads to inconsistent style estimates which subsequently lead to inconsistent portfolio weight estimates. The consequence of the inconsistent portfolio weights is a reduction in the risk of the portfolio, while imposing extra restrictions should have led to a higher risk. If we use the weak style estimates in combination with the optimal weights determined with semi-strong and strong style analysis, then it appears that the portfolio does not have the desired factor exposure, i.e. 22.7% to North American growth stocks, 48.7% to European growth stocks and 24.7% to Pacific growth stocks, indicating that the necessary requirement in (13) that $E[\varepsilon_t R_t] = 0$ does not hold in semi-strong and strong style analysis. For 'Combination B', where certain exposures for international value indices were desired, similar results are found.

As mentioned, when applying the weak form of style analysis in the asset

Table 7: The table reports the optimal weights for two combinations of desired factor exposure. Combination A reflects a desired exposure of 25 % North American, 50 % European and 25 % Pacific Growth stocks, while Combination B reflects a desired exposure of 65 % American, 25 % European and 10 % Pacific Value stocks.

	Combination A			Combination B		
Mutual Fund	weak	semi	strong	weak	semi	strong
Alliance Global Sm	1.19	-4.15	-4.30	7.92	-0.47	-0.47
Alliance Intl	6.98	3.53	3.53	8.81	-0.04	-0.04
Bailard, Biehl Intl	33.44	34.11	34.14	11.04	11.23	11.23
Evergreen Intl Gr	17.79	20.71	20.66	-2.61	5.66	5.65
First Invest Global	15.45	13.77	13.79	13.16	9.28	9.28
Kemper Intl	-46.20	-41.17	-41.22	-3.37	4.97	4.97
Nations Intl Gr	-9.03	-9.54	-9.57	3.43	3.16	3.17
New Perspective	8.16	14.00	13.98	18.29	31.83	31.83
Oppenheimer Global	-1.71	-6.54	-6.58	-0.00	-8.06	-8.07
Phoenix-Aberdeen	-3.46	-3.94	-3.71	0.87	-0.08	-0.08
Putnam Global Gr	32.88	28.42	28.39	21.60	18.01	18.01
Scudder Intl	-4.53	0.35	0.31	-10.90	-9.51	-9.51
T.Rowe Price Intl	67.37	62.18	62.32	6.83	6.28	6.26
Templeton Global Sm	-16.27	-11.40	-11.42	-4.86	2.82	2.82
Templeton Gr	-14.96	-12.05	-12.00	4.16	5.06	5.06
Templeton World	31.12	30.57	30.51	50.30	47.26	47.26
United Intl Gr	-12.50	-11.67	-11.69	-8.17	-8.39	-8.39
Vanguard Intl Gr	-5.73	-7.19	-7.15	-16.49	-21.92	-21.92
expected return (%)	1.25	1.25	1.25	1.25	1.25	1.25
stand. deviation (%)	4.50	4.35	4.36	4.09	3.82	3.82

allocation decision with fixed exposure, the optimal portfolio contains short positions in a number of funds. Obviously, it is hard to take short positions in a mutual fund. Therefore we impose short sell restrictions on the mutual funds under consideration. As shown by e.g. Markowitz (1991) this leads to a segmented mean-variance frontier with on each segment those mutual funds with non-binding short sell restrictions. In Table 8 we report for two combinations of desired exposure the optimal portfolio for a required expected return of 1.25% on a monthly basis. As before, we apply the three forms of style analysis to determine the exposure of the funds for the factors under consideration.

First of all, it appears that in case of short sell restrictions, the desired 'Combination A' in combination with a required return of 1.25% is not attainable. Furthermore, similar to the results without short sell restrictions, it appears that the optimal weights of the portfolios with desired exposure are inconsistent when semi-strong or strong style analysis is used. This follows from the reduced risk, although higher risk has to be expected because of additional restrictions. Therefore, we propose not to impose the portfolio and positivity constraint in style analysis, when style analysis is used for asset allocation decisions with fixed exposure to certain factors. In case of weak style analysis, it appears that for the desired 'Combination B', a considerable position has to be taken in the Templeton World fund, while for ten funds the short sell constraint is binding. The fact that the positivity and portfolio constraints in semi-strong and strong style analysis leads to inconsistent weight estimates illustrates that in order to determine a portfolio with fixed exposure to certain asset classes, the use of the weak form of style analysis is recommended.

7 Summary and conclusions

The portfolio and positivity constraints that are usually imposed in return based style analysis can lead to inconsistent estimates. The aim of the application where style analysis is used for, determines whether the constraints are desired. Strong style analysis is only recommended in case of relative performance evaluation, while in all other analyzed applications the constraints can have serious impacts on the results of the underlying question.

In relative performance evaluation the aim of style analysis is to determine a benchmark portfolio that mimicks the fund under consideration. In

Table 8: The table reports the optimal weights for two combinations of desired factor exposure in case it is not allowed to take short positions in the mutual funds. A '-' in the table indicates that there is a binding short sell constraint. Combination A reflects a desired exposure of 25 % North American, 50 % European and 25 % Pacific Growth stocks, while Combination B reflects a desired exposure of 65 % American, 25 % European and 10 % Pacific Value stocks.

	Combination A			Combination B		
Mutual Fund	weak	semi	strong	weak	semi	strong
Alliance Global Sm	-	-	-	15.66	7.05	7.11
Alliance Intl	-	-	-	0.03	-	-
Bailard, Biehl Intl	-	-	-	1.25	2.26	2.28
Evergreen Intl Gr	-	-	-	-	5.42	5.42
First Invest Global	-	-	-	7.87	7.68	7.63
Kemper Intl	-	-	-	-	-	-
Nations Intl Gr	-	-	-	-	-	-
New Perspective	-	-	-	10.19	21.26	21.27
Oppenheimer Global	-	-	-	-	-	-
Phoenix-Aberdeen	-	-	-	-	0.99	0.94
Putnam Global Gr	-	-	-	13.73	-	-
Scudder Intl	-	-	-	-	-	-
T.Rowe Price Intl	-	-	-	3.50	-	-
Templeton Global Sm	-	-	-	-	4.25	4.22
Templeton Gr	-	-	-	-	11.11	11.10
Templeton World	-	-	-	47.77	39.97	40.02
United Intl Gr	-	-	-	-	-	-
Vanguard Intl Gr	-	-	-	-	-	-
expected return (%)	-	-	-	1.25	1.25	1.25
stand. deviation (%)	-	-	-	4.11	3.96	3.96

this case, the portfolio and positivity constraints are required since in weak style analysis the factor exposures do not necessarily sum to one nor they are positive. Although the intercept in the strong style regression indicates whether the fund under or outperforms the mimicking portfolio on a relative basis, it may only be interpreted as the Jensen measure for a very specific group of investors.

When style analysis is used to construct efficient portfolios from mutual funds that have fixed exposures for certain asset classes, the portfolio and positivity constraints lead to a portfolio that does not have the desired exposure. The fact that in semi-strong and strong style analysis the correlation between the error term and the benchmark assets is not necessarily equal to zero, leads to inconsistency in the style estimates which subsequently lead to portfolios with undesired factor exposure. This can be avoided by simply applying weak style analysis.

Sometimes style analysis is suggested as a method to estimate portfolio holdings of fund managers. However, weak, semi-strong and strong style analysis will not consistently estimate the actual portfolio holdings over the sample period when fund managers invest in a subset of assets underlying an index only, and give the assets different weights than the index. Even in the case that the fund manager holds securities that on average have a beta of one relative to their own assets will in general lead to style estimates that deviate seriously from the actual holdings. The fact that the indices used in style analysis usually are not mutually exclusive can lead to style residuals that may be correlated with the style indices.

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