## Developing New Methods for Efficient Container Stacking Operations

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# Developing New Methods for Efficient Container Stacking Operations 

Ontwikkelen van nieuwe methoden voor efficiënt container in- en uitslaan operaties

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To my parents and sister

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## Chapter 1

## Introduction

An ocean freight container also called container, shown in Figure 1.1, is a reusable steel box used for transporting products (Levinson, 2008). Intermodal freight transport involves the transportation of freight in containers, using multiple modes of transportation such as a ship, truck, train, or barge (see Figure 1.2), without any handling of the freight itself when changing modes (Crainic and Kim, 2007). Bundling the freight in containers reduces cargo handling, and thereby improves security, may reduce damages and loss, and may allow freight to be transported faster (Agerschou et al., 1983). The dimensions of containers have been standardized so that different modalities can transport them all around the globe. The term


Figure 1.1. An ocean freight container (Source: Boxxes, 2012) twenty-feet-equivalent-unit (TEU) is used to refer to one container with a length of twenty feet. A container of 40 feet is expressed by two TEU. Table 1.1 shows the dimensions and other specifications of twenty- and forty-feet-equivalent-units. Based on the international convention for safe containers, every container has a container safety convention (CSC) plate containing all specifications of the container (International Maritme Organization (IMO), 1977, Admiralty and Maritime Law Guide, 1972). Furthermore, every container has an international identification code, the so-called bureau international des containers (BIC) code, which is a unique code in all international transport and customs declaration documents (Bureau International des Containers et du Transport Intermodal (BIC), 2012).

Containerized transportation has become an essential part of intermodal freight transport (Kim and Kim, 1999a, Steenken et al., 2004). More than $90 \%$ of all cargo is now transported by ships (Taggart, 1999, Henwood et al., 2006). Most of this cargo is broken down into standard units that can be handled by


Figure 1.2. Different modalities participating in intermodal freight transport (Source: Europe Container Terminals (ECT), 2012)

Table 1.1. Container specifications (Source: Barrington Freight, 2012)

|  |  | 20-feet-equivalent-unit |  | 40-feet-equivalent-unit |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | imperial | metric | imperial | metric |
| External dimensions | Length | $20^{\prime} 00^{\prime \prime}$ | 6.096 m | $40^{\prime} 00^{\prime \prime}$ | 12.192 m |
|  | Width | $8^{\prime} 00^{\prime \prime}$ | 2.438 m | $8^{\prime} 00^{\prime \prime}$ | 2.438 m |
|  | Height | $8^{\prime} 6^{\prime \prime}$ | 2.591 m | $8^{\prime} 6^{\prime \prime}$ | 2.591 m |
| Interior dimensions | Length | $18^{\prime} 10 \frac{5}{16}$ | 5.758 m | $\overline{39} 9^{\prime \prime} \frac{45}{64}$ | $\overline{12} .0 \overline{3} 2 \mathrm{~m}$ |
|  | Width | $7{ }^{1} 8 \frac{19}{}{ }^{\prime \prime}$ | 2.352 m | $7^{\prime} 8 \frac{19}{}{ }^{\prime \prime}$ | 2.352 m |
|  | Height | $7{ }^{\prime} 9 \frac{51}{64}$ | 2.385 m | $7^{\prime} 9 \frac{57^{\prime \prime}}{64}$ | 2.385 m |
| Door aperture | W-īdth |  | 2.343 m |  | 2.343 m |
|  | Height | $7^{\prime} 5 \frac{8}{4}{ }^{\prime \prime}$ | $2.280 \mathrm{~m}$ | $7{ }^{\prime} 5 \frac{3}{4}{ }^{\prime \prime}$ | 2.280 m |
| Volume |  | $1.169 \mathrm{ft}^{3}$ | -33.1 m ${ }^{3}$ | $2.385 \mathrm{ft}^{3}$ | $67.5 \mathrm{~m}^{3}$ |
| Maximum gross mass |  | 66,139 lb | $30,400 \mathrm{~kg}$ | 66,139 lb | $30,400 \mathrm{~kg}$ |
| Empty weight |  | $4,850 \mathrm{lb}$ | $2,200 \mathrm{~kg}$ | $8,380 \mathrm{lb}$ | $3,800 \mathrm{~kg}$ |
| Net load |  | 61,289 lb | 28,200 kg | $57,759 \mathrm{lb}$ | 26,600 kg |

containers. Between 1990 and 2015, the total number of full containers shipped internationally is expected to grow from 28.7 million TEU to 177.6 million TEU (United Nations: ESCAP, 2007). A simple calculation shows that enough containers exist on the planet to build more than two 8 -foot-high wall around the equator
(Taggart, 1999). This trend necessitates all firms involved in the intermodal freight transport supply chain to contribute to a better performance. As Figure 1.3 shows, in intermodal freight transport, different firms such as terminal operators, freight forwarders, information service providers, infrastructure managers, shippers, and receivers play a role. In the chain of international transport, container terminals are of special importance since all containers should pass through at least one of them during their drayage. Container terminals are the nodes where all different sorts of modalities meet to transport containers. They are involved in linking sea terminals with inland terminals, or linking terminals with end points in the chain, such as warehouses. A large terminal handles millions of containers on an annual basis (Drewry, 2011). Table 1.2 shows that the terminals of the Port of Rotterdam handled more than 11 million TEU in 2010 (Port of Rotterdam Authority, 2012). As a result of stacking millions of containers, the land needed for the related supply chain activities has become short. Lack of space has driven container terminal operators to have higher container stacks, saving much footprint. The other trend is that the size of the ships has also grown during the past decades. The largest Post-Panamax ships are able to carry about 15,000 TEU whereas the first-generation ships had a capacity of about 400 TEU. Consequently, large ships experience relative long in-port times compared to smaller ships. For instance, $24 \%$ of overall voyage time for a 8,000 TEU ship is in-port time, compared to $17 \%$ for a 4,000 TEU Panamax ship (Midoro et al., 2005). Handling a large number of containers entering and leaving terminals by different modalities including the new mega-size ships significantly affects the performance of terminals and makes them a bottleneck, if their container handling operations are not well managed.


Figure 1.3. The intermodal freight transport supply chain (source: Veenstra et al., 2012)

Container terminal operators are always looking for new technologies and smart solutions to maintain efficiency. Several instances will be discussed later in this dissertation. An example is a recent innovation started in the Netherlands is to integrate supply chain and transportation through extending the sea terminal gate into the hinterland as the gateway to the European hinterland (Veenstra et al., 2012). It enables them to better connect with shippers and receivers in the network. The main purpose of all these new methods such as the extended gate project is to somewhat leverage the effect of the increasing flow of containers on

Table 1.2. Container and TEU traffics in the Port of Rotterdam (Source: Port of Rotterdam Authority, 2012)

| Year | Number of containers | Number of TEU | Net weight of load $(\mathrm{kg})$ |
| :---: | :---: | :---: | :---: |
| 2005 | $5,636,570$ | $9,288,399$ | $70,998,184$ |
| 2006 | $5,846,433$ | $9,653,232$ | $73,820,345$ |
| 2007 | $6,488,646$ | $10,790,829$ | $81,770,729$ |
| 2008 | $6,485,464$ | $10,783,825$ | $83,012,059$ |
| 2009 | $5,900,114$ | $9,743,290$ | $77,803,233$ |
| 2010 | $6,745,760$ | $11,145,804$ | $85,929,163$ |

terminal operations. In this dissertation, we study different operations at a container terminal and focus on container stacking operations as one of the prime activities of marine container terminals. We develop optimization and heuristic methods that help for better managing the stacking operations. The remaining of this chapter is organized as follows. In Section 1.1, we briefly explain the main logistic operations of a container terminal. Section 1.2 introduces the focus of the dissertation. Then, in Section 1.3, we present a literature review of the main operations of a container terminal. Finally, in Section 1.4, an outline of the dissertation is given.

### 1.1 Container terminal operations

Figure 1.4a depicts a top view of a container terminal and different container handling systems. Material handling equipment includes quay cranes (QCs), yard cranes (YCs), Automated Guided Vehicles (AGVs), and Straddle carriers (SCs). These systems are shown in Figures 1.4b-e respectively, and are used to transship containers from ships to barges, trucks and trains, and vice versa. The containers can be transshipped directly from one mode of transportation to another. Alternatively, containers can be stored for a certain period in a stack, before they are transferred to another mode. Material handling equipment used at a terminal is very expensive, regardless of whether they are automated or manned. The investment in a large modern container terminal can easily amount to €1-5 billion and the payback period varies between 15-30 years (Wiegmans et al., 2002, De Koster et al., 2009, Roy and De Koster, 2012). Extensive overviews of various processes and decision making at terminals are given by Vis and De Koster (2003), Stahlbock and Voß (2008a), and Steenken et al. (2004).

Sea container terminals are divided into a seaside, landside, and stacking area (see Figure 1.5). At a typical automated container terminal, QCs load and unload containers from ships berthed along the quay at the seaside. AGVs transport containers from the seaside to the stacking area where YCs take them over. Finally, SCs transport containers between the YCs and trucks and trains at the landside. In general, three different groups of containers can be distinguished in container terminals: import, export, and transshipment containers (Kim and Park, 2003). Import containers are carried to the terminal by ships, stay for a while

(a) Container terminal

(b) QC

(c) YC

(d) AGV

(e) SC

Figure 1.4. A top view of a container terminal and material handling equipment (Source: Europe Container Terminals (ECT), 2012)
and are transferred outside by trains, trucks or barges. Export containers are delivered to the terminal by trains, trucks or barges, stay for a short time, and are then loaded onto ships for other ports. Typically, these containers start arriving as early as seven days in advance; containers scheduled for earlier ships are likely to arrive sooner than the ones belonging to the later ships (Lee and Chao, 2009). Transshipment containers arrive and leave the terminal by ship.

At a container terminal, containers are stacked in container blocks. Figure 1.6a depicts a typical container terminal layout with several blocks of containers in the stack area; other terminal layouts are studied by Wiese et al. (2010). Each block consists of multiple rows, tiers, and bays as shown in Figure 1.6b. Containers arrive or leave the terminal from the seaside and landside and spend a period of time in these blocks. Input/output (I/O) points are located at each end of a block and a single YC is used to stack and retrieve containers in that block. The YC can simultaneously move along the rows and bays of the block. When a container is to be retrieved, the YC picks it up from its location in the block and drops it at an I/O point at the container's destination side of the block. At the seaside, an AGV is available at the I/O point on which the YC drops


Figure 1.5. Loading and unloading processes of containers at a typical container terminal (adopted from Brinkmann (2010))
off the container to be transported to a QC. At the landside, depending on the type of container terminal, a truck, a chassis, or an SC takes away the container from the I/O point. In case of a storage request, the YC picks up a container from an AGV, SC, chassis, or truck at an I/O point and stores it in a given location in the block. Recently, container terminals have started using two or even three YCs to retrieve and stack containers in every block of containers (Dorndorf and Schneider, 2010, Li et al., 2009, Vis and Carlo, 2010). Depending on the design of the blocks and YCs, the YCs can or cannot pass each other. In all cases during stacking operations, for security reasons, the distance between the two YCs cannot be less than a minimum safety limit.


Figure 1.6. Schematic representation of a container terminal layout

YCs can be classified into two types: rail-mounted gantry (RMG) cranes and rubber-tired gantry (RTG) cranes. The former are automated, but their movements are limited to a block or to a few adjacent blocks of a row. In contrast, RTG cranes are more manual and can move freely from one block to another. RMG cranes can be automated or manual, where automated RMG cranes are sometime called Automated stacking cranes (ASCs), according to Stahlbock and Voß (2008b). A survey by Wiese et al. (2010) using the data of 114 container terminals world wide shows that YCs are the most common equipment for stacking, as in $63.2 \%$ of the terminals use them. This percentage is as high as $75.5 \%$ in Asia.

Containers are mainly stacked in container blocks according to their loading sequence onto the ships. The sequence depends on their ships, ports of destination, and weights. Kang et al. (2006) define two containers to be of the same type if they belong to the same ship, port of destination and weight group. Obviously, containers have to be retrieved from the block in the sequence of the departure of their corresponding ships. Furthermore, containers of the destinations that will be visited by the ship later have to be loaded onto the ship earlier. Finally, containers have to be loaded according to their weight which is mainly divided into three groups: Heavy (H), Medium (M) and Light (L). In order to ensure a ship's stability, heavier containers should be loaded before lighter ones. In other words, containers should be loaded in a sequence of $\mathrm{H}, \mathrm{M}$ and L . Therefore, they have to be stacked in a reverse sequence in a block. Otherwise, reshuffling occurs. A reshuffle is an unwanted movement of a container stacked on top of the one which should be retrieved (Kim et al., 2000, De Castillo and Daganzo, 1993, Caserta and Voß, 2009). Note that in a container terminal, multiple ships simultaneously berth at the quay and five or six QCs usually load and unload containers to and from a large ship. The QCs load and unload the ship bay by bay, and the unloading operations precede the loading operations with a small overlap. Therefore, the retrieval sequence is more flexible than explained above. As an example, a heavy container which should be loaded to the ship by a specific QC can be retrieved from the block earlier than a heavy container that should be loaded onto the ship by another QC. Even in the case of a single QC, the sequence can be more flexible. For example, while loading containers of a bay, the QC can load a light container of a pile earlier than a heavy container of another pile. However, in general the sequence explained above results in the minimum number of reshuffles.

Nowadays, large ships visit about five destinations in every trip. Thus, the area of a ship along its deck is divided into five subareas with several bays corresponding to each destination. Furthermore, particular areas of the ship may be assigned to special containers such as refrigerated containers. A refrigerated container is equipped with an integral refrigeration unit which controls the temperature of the container. This integral refrigeration unit has to be connected to the on-board power supply system after the container has been loaded onto the ship.

### 1.2 Focus of the dissertation

Operators of automated container terminal are confronted with many strategic, tactical, and operational decisions. At the strategic level, decisions include deciding the berthing capacity, layout of the terminal, type of equipment for handling containers at the seaside and landside, and type of vehicles to used for container transport between seaside and the landside. At the tactical and operational levels, decisions include berth allocation, number of ASCs and transport vehicles, and container stacking policies. Murty et al. (2005) describe a variety of inter-related decisions made during daily operations at a container terminal.

Container terminal managers need to know how different operations at the terminal interact and affect the performance of the terminal as a whole. Many authors have tried to evaluate the performance of different operations and their effects on the overall performance of the terminal, using different methods such as data envelop analysis (De Koster et al., 2009, Cullinane et al., 2005, 2006, Wang and Cullinane, 2006). The results show that one of the key performance indicators is the berthing times of ships (Linn and Zhang, 2003, Böse, 2011). The shorter the berthing times are, the better the performance is. The cost of holding a 2000 TEU ship idle is $\$ 20000-\$ 25000$ per day (Agarwal and Ergun, 2008). In order to minimize the berthing time of a ship, QCs loading and unloading containers from the ship should not fall idle. Closely related to the ship turnaround time, another important measure is the average QC rate, which is the quay cranes throughput measure during a period, given by (Murty et al., 2005):

$$
\begin{equation*}
\text { QC rate }=\frac{\text { no. of containers unloaded, loaded }}{\text { total no. of QC hours of all QCs that worked }} \tag{1}
\end{equation*}
$$

Over the years, the speed of QCs has improved substantially. Currently, a modern QC can handle 35 containers per hour. We now see QCs with double hoists able to handle two TEU simultaneously, or even up to four TEU. This means that in order to keep up with the speed of QCs, response times of ASCs should be as short as possible. Typically, the throughput of an ASC, calculated as the number of handled containers per hour, is approximately one-third of a modern QC (Murty, 2007). As a result, in order to fully utilize QCs, (1) containers to be loaded by each QC must be distributed over several blocks (at least three blocks), and (2) operations of each ASC must be properly scheduled to minimize the cycle time so as to prevent QCs becoming idle. Otherwise, the ASCs become a bottleneck and the efficiency of the QCs drop. Dekker et al. (2007) show that while retrieving containers of a jumbo ship, the percentage of time that the workload of the ASC is more than $100 \%$ can be around $10 \%$. In other words, the terminal seaside performance heavily depends on the performance of the stacking operations at the stacking area (Zhang et al., 2002, Kim and Kim, 1999a). An efficient block stacking operation can significantly affect the overall performance of the container terminal (Zhang et al., 2002, Kim and Kim, 1999a). The stacking operation decisions include not only the scheduling of the ASC but also allocating containers in the block based on the retrieval sequence in order to avoid reshuffling which is time-consuming and can significantly increase the response time of the

ASC.
The importance of stacking operation decisions becomes more clear considering the fact that while retrieving containers for a ship, each ASC also should retrieve containers leaving the terminal by truck and train at the landside. In addition, it should stack containers arriving at the I/O points of the block by AGVs from the seaside or by trucks and trains from the landside. Stacking operations of the ASC should be optimized to prevent it from becoming a bottleneck. Otherwise, not only the berthing time of the ship may significantly increase but also the waiting times of trucks, and trains. It is therefore important to minimize the makespan of all requests to be performed by the ASC by optimally sequencing them. In practice, usually simple ASC scheduling rules are deployed, such as nearest neighbor (NN) or first-come-first-served (FCFS). In the NN heuristic, the ASC carries out the nearest container, and in the FCFS heuristic, it carries out the containers based on their arrival sequence. A more proper schedule might considerably reduce the total travel time of the ASC, which consequently reduces the makespan of ships and waiting times of trucks, barges, and trains. In case of a container terminal with multiple rows and a straddle carrier to store and retrieve containers, Vis and Roodbergen (2009) show that the time difference between the optimal and FCFS sequence is at least $30 \%$. Although an ASC operates differently from a straddle carrier, substantial travel time reduction may be gained for the ASC as well.

This dissertation focuses on the operations at the stacking area of a container terminal. More specifically, we consider the problem of scheduling ASCs to stack and retrieve containers. We also study where storage containers have to be stacked not only from the viewpoint of reducing the ASC travel time but also to minimize the number of reshuffles. In the next section, we review the previous theoretical papers on these topics, and then in Section 1.4, the outline of the dissertation is explained.

### 1.3 Literature review

The literature on tactical and operational decision problems at container terminals is dense (see, for example, Günther and Kim, 2005, Steenken et al., 2004, Stahlbock and Voß, 2008a). In some research papers, the interaction between different systems at a container terminal is discussed, whereas in the other papers the focus is on improving the performance of an individual operation. Simulation is the prime tool to analyze the integration and interaction of different systems (Petering and Murty, 2009, Petering et al., 2009, Petering, 2011a, 2010, Liu et al., 2002, 2004, Roy and De Koster, 2012). More than 40 papers on simulation models can be found in the literature which range from strategic to operational decision making (Petering et al., 2009).

Exact and heuristic methods are more common when it comes to analyzing the individual processes at a container terminal. For example, routing and dispatching AGVs are studied by Kim and Bae (2004), Vis et al. (2001), Evers and Koppers (1996), Vis et al. (2005), Briskorn et al. (2006), Lehmann et al. (2006), and Meersmans (2002). Scheduling SCs is studied by Vis and Roodbergen (2009), Kim and Kim (1999c,b),

Kozan and Preston (1999), and Kozan (2000). Scheduling QCs can be found in Daganzo (1989), Kang et al. (2008), Goodchild and Daganzo (2006), Choo et al. (2010), and Zhen et al. (2011). Some papers focus on the berth allocation problem. The problem involves the allocation of ships to berth places in time in order to optimize a certain objective function. In most of the reported studies, the objective is to minimize each ship's turnaround time (Hendriks et al., 2010, Kim and Moon, 2003, Guan and Cheung, 2004, Cordeau et al., 2005, Imai et al., 2005, Monaco and Sammarra, 2007, Moorthy and Teo, 2006). A limited number of studies consider a multi-objective problem, where besides the turnaround times, the weighted deviations from predetermined berth positions are minimized (Wang and Lim, 2007, Hansen et al., 2008). Last but not least, scheduling and routing ships are studied by Agarwal and Ergun (2008), Christiansen et al. (2004), Sherali et al. (1999), Brown et al. (1987), Appelgren (1969, 1971), Ronen (1983) and Ronen (1993).

Since this dissertation focuses on the ASC scheduling and the container reshuffling problem, we particulary pay attention to these two sections of the literature in the following subsections. Surprisingly, in spite of the importance of these issues for daily terminal operations, relatively little research attention has been given to these topics.

### 1.3.1 Yard crane scheduling

ASC, or yard crane (YC) scheduling, has not been studied well. Most of the relevant papers do not specify any special type of YC (i.e. manual or automated RMG or RTG crane) and as such the models and solution methods developed are applicable to all sorts of YC. However, the assumptions considered often show that the models are more suitable for a special type of crane and must be tweaked in different ways in order to be used to schedule another type of crane. All in all, since most papers do not specifically mention the type of crane, we use the general term "yard crane (YC)" to review the literature.

Kim and Kim (1999a) schedule a YC to retrieve containers from several blocks in the stacking area of a terminal. They propose a discrete time network model in which the objective is to minimize the total travel time of the crane to carry out all retrieval requests. Narasimhan and Palekar (2002) also study a model in which a single YC retrieves containers from a single block. Containers are classified into several types and while retrieving a container of a specific type, the ASC selects one of the containers of that type available in different locations of the block. They prove that the problem is $\mathcal{N} \mathcal{P}$-hard and develop a branch-and-bound algorithm. For large size instances, they propose a heuristic with a worst-case performance ratio of 1.5. Ng (2005) schedules several YCs to carry out a set of retrieval requests with different ready times in a yard zone, defined as multiple blocks located behind each other in a row. The YCs cannot pass each other and cannot exit the zone. They propose a discrete time mixed integer model and solve it by means of a heuristic based on dynamic programming. The objective function is to minimize the total completion time. Ng and Mak (2005) have later proposed an exact branch-and-bound algorithm for the same problem, but with a single crane. The objective function is to minimize the total waiting time of all requests.

In more recent papers, retrieval and storage requests are considered simultaneously. Zhang et al. (2002)
propose a discrete time mixed integer linear model for a problem in which several YCs carry out a given workload in multiple blocks. Based on their definition, a workload can consist of storage and retrieval requests. The objective is to minimize the total unfinished workload at the end of each time period. They propose a Lagrangian relaxation model and a heuristic method to solve the problem. Cheung et al. (2002) study a similar problem and prove that it is $\mathcal{N} \mathcal{P}$-hard in the strong sense. In order to solve the problem, they propose a Lagrangian relaxation exact algorithm, as well as an approximation method which formulates the problem as a network flow model with a piecewise-linear objective function.

Since the use of twin cranes limited to a block of containers is a new technology, scheduling models for such configurations can only be found in more recent papers. Li et al. (2009) introduce a discrete time model to schedule two ASCs carrying out the storage and retrieval requests in a single block with an I/O point located at one side of the block along the bays. The ASCs cannot pass each other and must be separated by a safety distance. The requests have different due times and the objective is to minimize a weighted combination of earliness and lateness of all requests, compared to their due times. They introduce a rolling horizon algorithm in which a horizon of a specific length is defined, and all requests falling within this horizon are considered and optimized by CPLEX. The horizon is updated whenever all its requests have been scheduled. Vis and Carlo (2010) also consider a similar setting. However, in their problem the ASCs can pass each other but cannot work on the same bay simultaneously. In their problem, requests do not have any due time and can be scheduled in any sequence. They formulate the problem as a continuous time model and minimize the makespan of the ASCs. They solve it by a simulated annealing algorithm and use the single-row method proposed by Vis (2006) to compute a lower bound.

Note that some papers focus on scheduling SCs (Vis and Roodbergen, 2009, Hartmann, 2004, Steenken et al., 1993, Kim and Kim, 1999c,b). An SC can only operate on a single row of containers, where containers of that row pass a landside or seaside I/O point located at the end of it depending on the side of destination. Since traveling from one row to another one is time-consuming, the SC completes all requests associated with a given row consecutively. Based on this property of the problem, Vis and Roodbergen (2009) first use dynamic programming to route the SC among the rows of containers and then use a single-row-method proposed by Vis (2006) to optimally route the SC in each row in a polynomial time.

Finally, in order to increase the performance of a container terminal, YCs and QCs not only must be scheduled optimally but also must be synchronized with AGVs. In this regard, some authors deal with optimizing the number of AGVs or synchronizing them with other material handling equipment at the terminal (Vis et al., 2001, 2005, Bish et al., 2005, Kim and Bae, 2004, Li and Vairaktarakis, 2004, Roy and De Koster, 2012).

### 1.3.2 Minimizing container reshuffling

Papers dealing with container reshuffling study three main subjects: (1) estimating the number of reshuffles, (2) pre-marshalling, and (3) stacking methods to reduce the number of reshuffles. As it will be discussed
later in this section, due to the complexity of the models concerning these issues, many focus on only a single bay with few piles for stacking or pre-marshalling containers (Froyland et al., 2008). In order to solve a single-bay problem, some authors try to increase the quality and speed of their algorithms by local search methods (Caserta and Voß, 2009, Lee and Chao, 2009). As a result, the need for studying a holistic problem which considers a whole block of containers exists.

In order to estimate the expected number of reshuffles which results from a given number of storage handlings, Kang et al. (2006) use simulated annealing assuming that containers belong to a single ship. Based on this, they introduce a probabilistic formula to estimate the total expected number of reshuffles. Kim (1997) proposes a method based on dynamic programming in combination with two heuristic algorithms to estimate the total number of reshuffles. In a recent paper, Lee and Kim (2010) employ this estimation in a model to optimize the block size taking into consideration the required throughput of YCs. De Castillo and Daganzo (1993) also develop general expressions to calculate the expected number of reshuffles to retrieve a container under segregation and non-segregation stacking strategies. Under the segregation strategy, containers are separated based on their duration of stay. They assume no new container will be stacked.

Some researchers focus on how to reduce the number of reshuffles by pre-marshaling containers in a way that fits the ships' stowage plans. Pre-marshaling is the repositioning of containers of the block prior to the ship arrival so that no or few reshuffles are needed when containers are loaded onto the ships. Lee and Hsu (2007) propose an integer programming model for a container pre-marshaling problem preventing reshuffling. They develop a multi-commodity network flow model for obtaining a plan on how to pre-marshal containers stacked in some piles of containers. They also propose a simple heuristic for large-scale problems. Lee and Chao (2009) develop a neighborhood-based heuristic model to pre-marshal containers of a single bay in order to find a desirable final bay plan. Caserta and Voß (2009) also study a similar container pre-marshaling problem. They propose a dynamic programming model to pre-marshal containers of a single bay. In order to quickly find the solution, they propose a corridor method. In this local search method, when a container is being pre-marshaled it can only be stacked in a corridor which consist of the next few predecessor or successor piles of the bay with a specific limit on the number of empty locations.

Some papers focus on how to avoid reshuffling by proposing methods to properly locate incoming containers in a container block. Dekker et al. (2007) investigate different stacking policies, using simulation based on real data. They allocate the containers to the block based on the containers' expected duration of stay. Kim and Park (2003) also propose a heuristic algorithm based on the containers' duration of stay to locate containers. Kim et al. (2000) propose a stochastic dynamic programming model for determining storage positions of export containers in a single bay of a block. To avoid solving a time-consuming dynamic programming model for each incoming container, they build decision trees, using the optimal solutions of the dynamic programming model. The trees tell where to store an incoming container. The validity of the recursive function of the dynamic programming model is proven by Zhang et al. (2010).

### 1.4 Outline of the dissertation

Motivated by the discussions in section 1.2 and the literature in section 1.3, this dissertation proposes, develops, and tests optimization methods to support the decisions of container terminal operators in the stacking area of a terminal. We focus on operational stacking problems, which are treated in four chapters. The first three chapters focus on sequencing a given set of container storage and retrieval requests in a single block. All three problems are complex and modeled as continuous time integer programming models. The objective is to minimize the makespan to carry out all requests. As discussed in section 1.2, we can improve the performance of the terminal by minimizing the makespan. We try to optimally solve the problems as long as the complexity allows it. Otherwise, heuristic algorithms are developed to obtain near-optimal solutions. The last chapter links indirectly to the first three and will be discussed later. Although some theoretical studies have already addressed parts of the problems considered in this dissertation, our specific models incorporating the constraints that are faced in practice have, to the best of our knowledge, not yet been addressed in literature.

In Chapter 2, we minimize the travel time of a single ASC to carry out all requests. The ASC must move retrieval containers from the block to the I/O points, and must move storage containers from the I/O points to the block. Locations as well as destination sides of retrieval containers and I/O points of storage containers are known. The problem is formulated as an asymmetric traveling salesman problem (ATSP). The objective is to minimize the total travel time of the ASC to perform all requests. Different than the literature discussed above, we optimally solve the ATSP model for instances of all sizes. In most of the previous research papers, discrete time models are proposed and mainly heuristics are used to solve the models.

In Chapter 3, we consider a similar problem with the difference that storage locations are not given. In other words, in case of a storage request, the ASC picks up a container from an I/O point, and drops it off in a location selected from a set of open locations suitable for stacking the container. Since container terminal operators often separate containers based on several criteria such as destination and weight, a set of open suitable locations is in many cases available to stack the container. We formulate the problem as a generalized asymmetric traveling salesman problem (GATSP). Similar to the previous problem, we minimize the total travel time of the ASC to perform all requests. In this problem, a single location may be suitable for stacking different containers, and thus sets of open locations may overlap. Locations in the intersection of multiple sets make the problem complex. Extra constraints are necessary to stack at most one container in such a location. In the literature, no continuous time model for such problem is proposed. We formulate the problem and solve it for small and medium instances. For large instances, we use a heuristic algorithm to obtain near-optimal solutions.

Chapter 4 considers a problem in which two ASCs carry out the requests. The ASCs can never pass each other and must operate sufficiently far from each other. Furthermore, the storage locations are not given and, in addition, containers can have different priorities in order to be stacked or retrieved. The most important reasons for this are as follows:

- The performance of a container terminal is often evaluated based on the berthing times of ships (Böse, 2011). Therefore, seaside containers usually have a higher priority than landside containers.
- To ensure the stability of ships, heavy containers must be loaded before light containers, in lower tiers on the ship and must therefore be retrieved earlier (see, for example, Gharehgozli et al., 2012c, Kim et al., 2000, Sammarra et al., 2007, Dekker et al., 2007).
- Trucks, trains and ships arrive at a container terminal to deliver or pick up containers at different points in time (see, for example, Froyland et al., 2008, Petering, 2011b, Newman and Yano, 2000), which induces precedence constraints on the stacking and retrieving of containers.

Preventing reshuffling also imposes additional precedence constraints on the operations. A precedence constraint can prohibit a container to be stacked in a pile before retrieving a container located in the same pile, or it can force containers stacked on top of a specific container to be retrieved earlier.

We formulate this multi-crane problem as a multiple asymmetric generalized traveling salesman problem with precedence constraints (mAGTSP-PC), which generalizes the single version of the problem without such constraints (see, for example, Laporte et al., 1987, Noon and Bean, 1991). The objective is to minimize the makespan of the two ASCs. The model also contains additional constraints regarding the interactions of the ASCs and the selection of open storage locations which lie in the intersection of multiple sets. Stacking problems with two ASCs have hardly researched. The combination with precedence constraints, and multiple open locations for storage containers is new. The new extra constraints make the problem so complex that we develop an adaptive large neighborhood search heuristic to solve the problem.

In these chapters, the storage locations can be selected from sets of open locations. However, it is assumed that these locations are still determined beforehand at a higher level in the decision making hierarchy. Chapter 5 is an attempt to find proper locations among all available empty locations when containers arrive at the container terminal. The objective function is to minimize the expected number of reshuffles.

## Chapter 2

## Scheduling a Single Yard Crane with Given Storage Locations*

Container terminals play a vital role in the organization of efficient global trade (Taggart, 1999, Henwood et al., 2006). A large terminal handles millions of containers annually, which are transported by deep-sea vessels of increasingly larger sizes (Drewry, 2011). Containers arriving at a terminal are stored in a very large yard until they can be loaded onto proper outbound transport modes, like other deep-sea ships, barges, trains or trucks. At a terminal, containers are stacked in container blocks, each operated by a single automated stacking crane (ASC) (see Chapter 1). The stack decouples flows between different transport modes, like deepsea and shortsea vessels, barges, trains, and trucks, in time. Particulary for large vessels, it is known some time in advance which containers have to be unloaded and loaded, to and from which position in the stack. Modes with smaller drop sizes, such as barges and trucks have to be scheduled in between the large-scale operations of deep-sea vessels. Minimizing the makespan of deep-sea vessels is a prime overall objective at a container terminal. However, the ASc has to handle requests for other modes as well within the given time frame. Since the ASC is often a heavily utilized resource at a container terminal, it is necessary to schedule the stacking operations over a given horizon with throughput time minimization as a prime objective (Böse, 2011). In practice, storage and retrieval requests are often executed in a first-come-first-served (FCFS) order or by the nearest neighbor (NN) search, as provided by commercial software companies (for example, Cosmos NV and Modality Software solutions b.v.). A more proper schedule can considerably reduce the total travel time of the ASC, which consequently reduces the makespan of ships and waiting times of trucks, barges, and trains.

We consider the problem of sequencing a given set of requests to be stacked or retrieved by an ASC in a single block. The objective is to minimize the travel time of the ASC to carry out the requests. We formulate

[^0]this problem with multiple I/O points as a special kind of an asymmetric traveling salesman problem (ATSP), in which to travel between two request locations in the block, the ASC, in most cases, must visit an I/O point. We show that due to the existence of multiple I/O points and the special movements of the ASC, the problem is so complex and in a special case is $\mathcal{N} \mathcal{P}$-hard. For quickly solving the problem, we propose a two-phase solution method. In the first phase of the solution, we develop a new merging algorithm to patch subtours of an optimal solution of the assignment problem (AP) relaxation of the problem without adding extra travel time. In this phase, we first search for two arcs from every two different subtours visiting a common I/O point, and swap the destinations of them to merge the subtours. Next, based on the fact that in some arcs, the ASC has multiple I/O point options with the same travel time to visit, we can create further opportunities to merge more subtours. We show that the first phase runs in a polynomial time, and often finds an optimal solution. Otherwise, a branch-and-bound ( $B \& B$ ) algorithm is used in the second phase to find an optimal solution of the problem. In this phase, the merging algorithm is again used in each node of the $\mathrm{B} \& \mathrm{~B}$ tree to save computation time.

The numerical results show the two-phase solution method is quite efficient. For instances up to 200 requests, an optimal solution can be obtained in less than a second. Furthermore, we show that the travel time reduction between the optimal and FCFS sequence is around $30 \%$, on average. The reduction is around $14 \%$, in case the NN heuristic is used to find the sequence. Finally, in order to evaluate the complexity of the problem and the performance of our algorithm, we compare our results with CPLEX results. The results show that the two-phase solution method can obtain significantly better results than CPLEX truncated after five hours. For instances with 100 requests, CPLEX cannot obtain a feasible solution after five hours.

In general, scheduling a yard crane to carry out storage and retrieval requests can be modeled as an ATSP or one of its special cases such as the rural postman problem, or Chinese postman problem, depending on the properties of the problem (Vis and Roodbergen, 2009). Simply stated, the ATSP is a combinatorial problem in which from a given list of locations with given pairwise travel times, a tour must be determined passing every location exactly once in such a way that the total travel time is minimized (Lawler et al., 1985). In general, ATSP is proved to be $\mathcal{N} \mathcal{P}$-hard (see, for example, Srour and Van de Velde, 2011). Several algorithms have been proposed to solve the ATSP. Among the most efficient ones is the B\&B algorithm based on the subtour elimination approach proposed by Carpaneto and Toth (1980) and later improved by Carpaneto et al. (1995) and Miller and Pekny (1991). The core idea of the B\&B algorithm in all three studies is the AP relaxation. In each node of the $B \& B$ tree, an $A P$ with some extra constraints is solved. Constraints exclude a number of arcs and require some others to appear in the AP solution so that some subtours can be avoided. Carpaneto et al. (1995) and Miller and Pekny (1991) also propose two merging algorithms. In these algorithms, the reduced costs of the arcs are used to merge subtours, and they are thus time consuming. Therefore, Carpaneto et al. (1995) for example use the merging algorithm in every node if the number of zero-reduced cost arcs is more than a threshold. In addition, the objective value may increase after merging subtours. These issues make their algorithms different than the one developed in this chapter, in a sense that
our algorithm uses swappable arcs with common visiting I/O points to quickly patch subtours without extra travel time.

Reviewing the papers on scheduling a single crane to carry out storage and retrieval requests of containers at a container terminal (Vis and Roodbergen, 2009) or of unit loads in a warehouse (Ratliff and Rosenthal, 1983, De Koster and Van der Poort, 1998, Van den Berg and Gademann, 1999) reveals that specific properties of each individual problem, modeled as a form of ATSP, determine whether a polynomial solution method can be developed. These properties are: (1) the number of I/O points, and (2) the number of rows in which containers are stored or retrieved. The literature can therefore be categorized based on these properties as follows.

In case of a single I/O points, polynomial solution methods can be developed (see Burkard et al. (1998) for solvable TSP cases). In this case, all subtours of an optimal solution of an AP relaxation to the ATSP can be merged as a complete Hamiltonian tour, since the crane returns to the same I/O point for every request. Having returned to the I/O point, the crane can select any storage request. It can also select a retrieval request in case in the optimal AP solution that retrieval request is sequenced after another retrieval request. Therefore, all subtours can be merged without adding extra travel time. Nevertheless, by increasing the number of I/O points, the problem becomes more complex. In fact, we show that the general problem with multiple I/O points is $\mathcal{N} \mathcal{P}$-hard.

Van den Berg and Gademann (1999) discuss a problem with both an output point and an input point, which is a special case of a problem with a single I/O point. As a result, although locations are spread in multiple rows, an optimal solution can efficiently be found. They formulate the problem as a transportation problem using a bipartite graph and prove that each feasible solution of the transportation problem corresponds to a retrieval and storage sequence of the main problem. Note that they assume storage requests are executed in a FCFS order. The problem is to find which retrieval requests must be interleaved after each storage.

Vis and Roodbergen (2009) consider a problem with a single block where the block corresponds to multiple rows separated by aisles. Each row has one I/O point at each end which serve that row. A single straddle carrier stores or retrieves all containers in the rows, where each container passes an appropriate I/O point of its row. The straddle carrier must exit the current row at the end in order to travel from one row to the other one, which is time-consuming. Therefore, it usually finishes all requests in that row before traveling to another. Based on this specific characteristic of straddle carriers, they can decompose the problem with multiple rows into several problems with one row and two I/O points and solve them separately. They propose a solution method based on dynamic programming to determine the shortest path for the straddle carrier crossing multiple rows. The dynamic programming allows multiple visits of a single row. It uses an optimal storage and retrieval request sequence in each row. Using an optimal AP solution to sequence the requests in a row with two I/O points results in maximum two subtours. They prove that these subtours can be optimally merged by enumerating and exchanging arcs in $O(N)$ time because in case of a single row
and two I/O points, because the travel time matrix has the properties of a Monge matrix (see Gilmore and Gomory, 1964). Vis and Carlo (2010) also use the same method to find a lower bound for their container sequencing problem with a single block and two ASCs. To obtain the lower bound, they collapse all rows of the block together as a single row and assume that a single ASC carries out all requests. Considering two ASCs to carry out the requests makes the problem complex that they resort to a metaheuristic to solve the problem.

Vis and Roodbergen (2009) extend the models proposed by Ratliff and Rosenthal (1983) and De Koster and Van der Poort (1998) for routing an orderpicker in a warehouse. Ratliff and Rosenthal (1983) consider a warehouse with parallel aisles (comparable to a container terminal with parallel rows), a central I/O point and no storage request, whereas De Koster and Van der Poort (1998) consider the same situation but the warehouse can have an I/O point at the end of each aisle. Dynamic programming is used in both studies to decompose and solve the problems.

Different from the previous line of research, in this paper, a continuous time integer programming model is proposed to stack and retrieve containers in a block with multiple I/O points and rows densely located together. Since the ASC can simultaneously move across the rows and does not return to the row ends to switch rows, the problem is more complex. Compared to the current ASC position, many locations in neighbor rows have identical travel times. In order to obtain the optimal solution, all rows and I/O points must be considered at the same time in the solution method. Decomposing the block into multiple single-row blocks for adopting the methods proposed by Vis and Roodbergen (2009), De Koster and Van der Poort (1998), Ratliff and Rosenthal (1983) is not possible, since returning to an I/O point located at either end of each row is essential in the proposed network models and solution methods. Extra arcs are necessary in the network to model switching rows without returning to the I/O points. Even if the block is divided into multiple single-row blocks, the optimal solution of each row cannot be obtained using the enumerative method. The reason is that containers are not dropped off or picked up at dedicated I/O points at the end of the row. Some requests are connected to other I/O points, which means that by solving an AP, the number of subtours is not necessarily two and can be more.

The rest of this chapter is organized as follows. In Section 2.1, we describe the technical aspects of the problem and present the mathematical model. In Section 2.2, the solution method is developed. Section 2.3 presents numerical results and Section 2.4 contains the conclusions.

### 2.1 Problem description and model

This section describes the research problem, introduces notations, and then formulates the problem.

### 2.1.1 Problem description

We study how to sequence $N$ container storage and retrieval requests in a single block of containers consisting of $X$ rows, $Y$ bays, $Z$ tiers, and $M$ I/O points. Let $I / O_{m}, m=1, \ldots, M_{s}$, be the location of the $m^{t h}$ I/O point at the seaside whereas $I / O_{m}, m=M_{s}+1, \ldots, M$, be the location of the $m^{t h} \mathrm{I} / \mathrm{O}$ point at the landside. A single ASC moves $n$ storage containers from the I/O points to given locations in the block and moves $N-n$ retrieval containers from the block to the I/O points. The objective is to minimize the total travel time of the ASC. From a given starting point, the ASC can carry out the requests in any sequence in order to minimize total travel time. We denote by 0 and $0^{\prime}$, respectively, the starting and ending locations of the ASC. These locations can be anywhere in the block. The ASC can only carry a single container at a time and containers must not be dropped off at temporary locations. Storage and retrieval containers can be of different sizes (i.e. 20 or 40 feet) and leave or arrive at the container terminal by truck, train or ship.

We denote by $V=\left\{1, \ldots, N, 0,0^{\prime}\right\}$ the set of all storage and retrieval locations including the ASC starting and ending locations. Note that every request corresponds to a unique location in the block and a unique container. Thus, for ease of notation, we use their notations interchangeably as follows. Container $i$ of storage request $i$ is currently located at an $\mathrm{I} / \mathrm{O}$ point and is waiting to be stored in location $i$ in the block, $i=1, \ldots, n$. Container $j$ of retrieval request $j$ is already stacked in location $j$ in the block, $j=n+1, \ldots, N$, and is waiting to be delivered to an I/O point either at the seaside or landside, depending on whether they should be transported by truck/train, or by ship, respectively. Due to the abundance of internal transport vehicles (like AGVs and chassis) to pick up containers at the seaside and landside, retrieved containers can be delivered to any I/O point at the landside or the seaside.

Locations as well as destination sides of retrieval containers and I/O points of storage containers are known; in practice, they are determined at a higher hierarchical planning level. This level focuses on minimizing the number of reshufflings (De Castillo and Daganzo, 1993, Kim et al., 2000) and speeding up the loading and unloading process of a ship (Kim and Kim, 1999a, Vis and Roodbergen, 2009). Due to the abundance of internal transport vehicles (like AGVs and chassis) to pick up containers at the seaside and landside, retrieved containers can be delivered to any I/O point at each side. Because of the same reason, retrieval containers can be quickly moved away from all I/O points, and they can therefore be delivered to the same I/O point that a storage container has to be picked up. The other issue is that storage and retrieval locations do not coincide. This is a reasonable assumption as $N$ is usually much smaller than the block size, the block utilization is often fairly low, and container terminal operators usually separate containers based on destination and type. Finally, we assume that containers do not have explicit precedence constraints. This assumption can be explained based on the fact that all containers are categorized into several precedence groups (for example, based on their latest departure times) and we can consider the sequencing problem group by group. In Chapter 4, we present an extension of the problem in which precedence constraints are considered.

Figure 2.1 shows a top view of a small block $(X=7, Y=10$, and $Z=4)$ of containers with three
storages and four retrievals. Locations of storages, retrievals and I/O points are highlighted with different colors. The first storage container must be picked up at $I / O_{1}$, the second at $I / O_{1}$, and the third at $I / O_{7}$. On the other hand, retrieval container 4 must be moved to one of the I/O points at the seaside and containers 5,6 and 7 must be moved to one of the I/O points at the landside.


Figure 2.1. A top view of a block with storage and retrieval locations

In order to mathematically formulate the problem and calculate the total travel time of the ASC, the pairwise travel times between the storage and retrieval locations in the block must be calculated. If the ASC travels directly from location $i,\left(x_{i}, y_{i}, z_{i}\right)$, to location $j,\left(x_{j}, y_{j}, z_{j}\right)$, the travel time of the ASC can be calculated as follows:

$$
\begin{equation*}
t_{i j}=\max \left\{\left|x_{i}-x_{j}\right|,\left|y_{i}-y_{j}\right|\right\}+\left|z_{i}\right|+\left|z_{j}\right| \tag{1}
\end{equation*}
$$

where, $\left(x_{i}, y_{i}, z_{i}\right)$ and $\left(x_{j}, y_{j}, z_{j}\right)$ are the Cartesian coordinations of locations $i$ and $j$, with $0 \leq x_{i}, x_{j} \leq T_{x}$, $0 \leq y_{i}, y_{j} \leq T_{y}$, and $0 \leq z_{i}, z_{j} \leq T_{z}$. Here, $T_{x}, T_{y}$, and $T_{z}$, are the furthest travel times of locations in the block in $X, Y$, and $Z$ directions, respectively. The first term of the right-hand side of Equation (1) emphasizes that the ASC can move in the $X$ and $Y$ directions simultaneously.

In order to calculate the travel times between storage and retrieval locations, we additionally must consider that the ASC needs to travel through I/O points to pick up or deliver containers. For example, to move from storage location $i$ to storage location $j$, the ASC needs to travel first to the I/O point where container $j$ is located and then move container $j$ to location $j$. Table 2.1 summarizes how to calculate the travel times between different locations. The travel time of the ASC satisfies the triangle inequality.

Table 2.1. Computation of pairwise travel times $\left(t_{i j}\right)$

| Arc (i,j) ${ }^{1}$ | Computation of $t_{i j}{ }^{2}$ | I/O |
| :---: | :---: | :---: |
| $\begin{aligned} & \text { (storage location } i \text {, retrieval location } j \text { ) } \\ & \text { (storage location } i \text {, storage location } j \text { ) } \\ & \text { (storage location } \left.i, 0^{\prime}\right)^{4} \\ & \hline \end{aligned}$ | $\begin{aligned} & l_{i j} \\ & t_{i, I / O_{k}}+t_{I / O_{k}, j} \\ & 0 \end{aligned}$ | $\begin{array}{\|l\|} \hline \text { None } \\ I / O_{k}{ }^{3} \\ \text { None } \end{array}$ |
| (retrieval location $i$, retrieval location $j$ ) | ```\(\min _{m=1, \ldots, M_{s}}\left\{t_{i, I / O_{m}}+t_{I / O_{m}, j}\right\}\), if container \(i\) must be delivered to the seaside (or \(m=M_{s}+1, \ldots, M\) ) (or landside).``` | $P^{5}$ |
| (retrieval location $i$, storage location $j$ ) | (1) $\overline{t_{i, I} / O_{k}}+\bar{t} \overline{t_{I}} / O_{k}, j, \overline{i f}$ container $\bar{i}$ must $\overline{\mathrm{b}} \overline{\mathrm{e}}$ delivered to the same side where container $j$ is already located. <br> (2) $\min _{m=1, \ldots, M_{s}}\left\{t_{i, I / O_{m}}+t_{I / O_{m}, I / O_{k}}+t_{I / O_{k}, j}\right\}$, if container $i$ must be de(or $\begin{gathered}\left.m=M_{s}+1, \ldots, M\right)\end{gathered}$ <br> livered to the seaside (or landside) and container $j$ is already located at the opposite side. | $\begin{aligned} & P \quad \text { and } \\ & I / O_{k}{ }^{6} \end{aligned}$ |
| (retrieval location $\left.i, 0^{\prime}\right)^{7}$ |  <br> ```(or \(\left.\begin{array}{c}m=1, \ldots, M_{s} \\ m=M_{s}+1, \ldots, M\end{array}\right)\) landside).``` |  |
| (0, retrieval location $j$ ) | $t_{0, j}$ | None |
| (0, storage location $j$ ) | $t_{0, I / O_{k}}+t_{I / O_{k}, j}$ | $I / O_{k}$ |
| Other cases | $\infty$ | None |

${ }^{1}$ In arc $(i, j)$, the YC moves from location $i$ to location $j$.
${ }^{2} t_{i i}=\infty$.
${ }^{3}$ Let $I / O_{k}$ be the I/O point where storage container $j, j=1, \ldots, n$, is located.
${ }^{4}$ The YC stops at location $i$, if storage request $i$ is the last request.
${ }^{5}$ Let $P$ be the set of I/O points that the YC can deliver retrieval container $i$ to either one and travel to another location with the minimum travel time.
${ }^{6}$ In this case, two I/O points must be visited: $I / O_{k}$ and an I/O point in $P$.
${ }^{7}$ The YC delivers the container $i$ to an I/O point and stops there, if retrieval request $i$ is the last request.

The mathematical formulation of the problem requires a unique travel time between every two locations. However, in an arc in which the ASC travels from a retrieval location to another location, the ASC is allowed to deliver the retrieval container to any of the multiple I/O point options. Each possible I/O point results in a different travel time. Due to the following theorem, the ASC selects the I/O point which gives the minimum travel time between the retrieval location and the immediate next storage or retrieval location.

Theorem 2.1 In order to minimize the travel time, the ASC selects the I/O point which gives the minimum pairwise travel time to deliver a retrieval container and travel to the next storage or retrieval location.

The proof is straightforward and is given in the Appendix. Note that multiple I/O points can result in the minimum travel time, but for now, we randomly select one of them. Later in the solution method section, we explain how multiple I/O point options can help to solve the problem.

### 2.1.2 Mathematical model

The problem of sequencing storage and retrieval requests is mathematically stated as the following integer programming model. $x_{i j}$ is the binary decision variable which equals 1 if and only if location $j$ is visited immediately after location $i, i, j \in V, i \neq j$.

The objective function is to minimize the total travel time of the ASC:

$$
\begin{equation*}
\text { minimize } \quad Z=\sum_{i \in V \backslash\left\{j, 0^{\prime}\right\}} \sum_{j \in V \backslash\{0\}} t_{i j} x_{i j} . \tag{2}
\end{equation*}
$$

The model has the following constraints.

Visiting all locations and carrying out all requests: Each location must be entered and exited exactly once. Of course, the starting point has no ingoing arc and the ending point has no outgoing arc:

$$
\begin{align*}
& \sum_{i \in V \backslash\{j\}} x_{i j}=1, \quad \forall j \in V \backslash\{0\} .  \tag{3}\\
& \sum_{j \in V \backslash\{i\}} x_{i j}=1, \quad \forall i \in V \backslash\left\{0^{\prime}\right\} . \tag{4}
\end{align*}
$$

Subtour elimination constraints: The ASC must avoid traveling subtours. Therefore, for any subset of locations $O \subseteq V \backslash\left\{0,0^{\prime}\right\}, O \neq \emptyset$ where $|O|$ is the size of $O$, we have the following constraint:

$$
\begin{equation*}
\sum_{i, j \in O} x_{i j} \leq|O|-1 \tag{5}
\end{equation*}
$$

All the variables are binary:

$$
\begin{equation*}
x_{i j} \in\{0,1\}, \quad \forall i, j \in V . \tag{6}
\end{equation*}
$$

The problem is defined as an ATSP $\left(t_{i j} \neq t_{j i}\right)$, consisting of assignment constraints plus subtour elimination constraints. In this problem, containers are dispatched on a plane (the block), are picked up or dropped off at different I/O points located at two ends of the plane, and the crane carrying out the containers can simultaneously move along the X and Y directions of the plane. Therefore, the travel times are defined based on the Chebyshev distance (see equation (1)) and the properties of Monge matrix do not hold. As a result, even in the case of a limited number of I/O points, the enumerative methods which merge subtours by exhaustively exchanging arcs among subtours and find the one with the minimum amount of extra travel time are not applicable. In fact, the complexity of the problem increases in the number of I/O points and requests. A special case of the problem, where the number of I/O points is not limited $(N=M)$, is $\mathcal{N} \mathcal{P}$-hard. This can be easily proved by polynomial time reduction of the stacker crane problem with uninterrupted movements on a general graph which has been proven to be $\mathcal{N} \mathcal{P}$-hard (Frederickson et al., 1978).

### 2.2 Solution method

A relaxation of the problem is an AP that can be efficiently solved in $O\left(N^{3}\right)$ steps where $N$ is the number of requests (Kuhn, 1955). An optimal AP solution often contains subtours since the subtour elimination constraint (Equation (5)) may not be satisfied. In order to merge subtours to obtain an optimal solution of our problem, we first develop some unique properties of the problem. Using these properties, an algorithm is developed in subsection 2.2 .1 to merge subtours. If all subtours have been merged into a single tour, an optimal solution is found. Otherwise, some subtours have not been merged, and a B\&B algorithm is developed in subsection 2.2.2 to merge all remaining subtours to obtain an optimal solution of our problem.

### 2.2.1 Merging algorithm

In this phase, we first in subsection 2.2.1 explain how to patch subtours of an optimal AP solution two by two by swapping arcs visiting a common I/O point. Then, in subsection 2.2.1, we introduce arcs in which the ASC can has multiple I/O point options to travel from the origins to the destinations of the arcs. These arcs can be used to create more opportunities to merge subtours. Finally, in subsection 2.2.1, we present the merging algorithm. We show that the objective value does not change after this merging.

## Merging subtours based on an optimal AP solution

In this section, we find some properties of the problem to merge subtours of the AP solution without adding extra travel time.

Definition 1 The visiting I/O point of an arc is the I/O point that the ASC needs to travel through either to pick up or to deliver a container in order to travel the arc connecting two requests.

In order to visualize the visiting I/O point of an arc, we need to distinguish carrying and non-carrying moves. In case of a retrieval request, the ASC starts with a non-carrying move from a storage location (or an I/O point) to the retrieval location, and then continues with a carrying move from the retrieval location to an I/O point. In case of a storage request, the ASC starts with a carrying move from an I/O point to the storage location, and then continues with a non-carrying move from the storage location to an I/O point or retrieval location. This property is implicitly considered in travel time of different arcs connecting requests, based on the calculations summarized in Table 2.1.

Figure 2.2b shows carrying and non-carrying moves of the arcs of an optimal AP solution presented in Figure 2.2a. For clarity, in Figure 2.2b, only the first and second subtours are considered. For example, storage request 1 is connected to $I / O_{1}$ by a carrying move and to retrieval request 4 by a non-carrying move. Furthermore, retrieval request 4 is connected to $I / O_{1}$ by a carrying move and to storage request 1 by a non-carrying move. As a result, $I / O_{1}$ is the visiting I/O point of arc $4 \xrightarrow{x_{41}=1} 1$.

Arcs in two different subtours that have a common visiting I/O point can be swapped between those subtours resulting in a merged subtour without extra travel time. These so-called swappable arcs have the


An optimal AP solution: subtour 1: $0 \xrightarrow{x_{02}=1} 2 \xrightarrow{x_{25}=1} 5 \xrightarrow{x_{50^{\prime}}=1} 0^{\prime}$; subtour 2: $1 \xrightarrow{x_{14}=1} 4 \xrightarrow{x_{41}=1} 1$; subtour 3: $3 \xrightarrow{x_{36}=1}$ $6 \xrightarrow{x_{67}=1} 7 \xrightarrow{x_{73}=1} 3$.

Figure 2.2. (a) An optimal AP solution consisting of subtours 1, 2 and 3, (b) Carrying and non-carrying moves of subtours 1 and 2, (c) Merging subtours 1 and 2 using the swappable arcs $x_{41}$ and $x_{02}$
following characteristics.
Definition 2 Swappable arcs are a pair of arcs in two different subtours that have a common visiting I/O point.

Theorem 2.2 If two subtours contain a pair of swappable arcs, they can be merged without adding extra travel time.

The proof can be found in the Appendix.

Corollary 2.1 If $x_{i j}=x_{k l}=1$ are two arcs of two subtours, the two subtours can be merged by setting $x_{i j}=x_{k l}=0$ and $x_{i l}=x_{k j}=1$. If arcs are swappable, the total travel time remains the same, otherwise it may increase.

The proof is straightforward and follows similar arguments as the proof of Theorem 2.2.

In the optimal AP solution presented in Figure 2.2a, $x_{41}$ and $x_{02}$ define a pair of swappable arcs because both arcs visit $I / O_{1}$ (follow the carrying moves in Figure 2.2b). Therefore, based on Theorem 2.2, replacing them with $x_{42}$ and $x_{01}$ results in merging subtours 1 and 2 without adding extra travel time. Merged subtour $1+2$ is shown in Figure 2.2c: $0 \xrightarrow{x_{01}=1} 1 \xrightarrow{x_{14}=1} 4 \xrightarrow{x_{42}=1} 2 \xrightarrow{x_{25}=1} 5 \xrightarrow{x_{50^{\prime}}=1} 0^{\prime}$.

## Merging subtours using replaceable I/O points

In the previous section, we used the following type of swappable arcs in Theorem 2.2 to patch subtours of an optimal AP solution.

Definition 3 An arc with a single irreplaceable visiting I/O point is an arc in which the ASC has a unique visiting I/O point option with the minimum travel time to visit and travel from the origin to the destination of the arc.

The assumption that all arcs have a single irreplaceable visiting I/O point results in missing some merging opportunitis. As the last column of Table 2.1 shows, in some arcs, the ASC may be able to visit one of multiple I/O points with the same travel time. Arcs without any I/O point such as moving from a storage location to a retrieval location are not a matter of attention here because swappable arcs need to have a common visiting I/O points. Therefore, we can introduce the following type of arcs.

Definition $4 A n$ arc with replaceable visiting I/O points is an arc in which the ASC has multiple visiting I/O point options with the minimum travel time to visit and travel from the origin to the destination of the arc.

Based on the last column of Table 2.1, an arc with replaceable visiting I/O points can be realized if the request corresponding to the location at the origin of an arc is a retrieval request. The retrieved container must be delivered to an I/O point which results in the minimum travel time for the ASC in order to travel from the origin to the destination of the arc (Theorem 2.1). In some cases, multiple I/O points may give the same minimum travel time one of which can be chosen. Arcs with replaceable visiting I/O points create extra opportunities to merge subtours.

During the merging algorithm, we check each arc to find out the number of visiting I/O points. However, in order to speed up the algorithm, we introduce the following theorem which can be used to identify the types of arcs that always have a single irreplaceable visiting I/O point. The other arcs may have replaceable or irreplaceable I/O points.

Theorem 2.3 Arc $i \xrightarrow{x_{i j}=1} j$ has a single irreplaceable visiting I/O point if:

1. $i$ and $j$ are both storage requests, or
2. $i$ is a retrieval request, $j$ is a storage request and retrieval container $i$ must be delivered to the same side where storage container $j$ must be picked up.

The proof is given in the Appendix.

In Theorem 2.3, the single irreplaceable visiting I/O point is the I/O point where storage container $j$ is located. As an example, Arc $x_{41}$ in Figure 2.2-a shows an arc with a single irreplaceable visiting I/O point, as $I / O_{1}$ is the only I/O point that can be visited (see Figure 2.2-b). In Figure 2.3-a where subtours $1+2$ and 3 are shown, arc $x_{50^{\prime}}$ is an arc with four replaceable visiting I/O points, $I / O_{m}, m=5,6,7,8$, that give the same minimum travel time, $t_{50^{\prime}}$. Arc $x_{67}$ is also an arc with replaceable visiting I/O points, $I / O_{m}, m=6,7$. In Figure 2.3-b, carrying and non-carrying moves of $x_{50^{\prime}}$ and $x_{67}$ are shown in black and gray, respectively.


Figure 2.3. (a) Subtour $1+2$ and subtour 3, (b) Carrying and non-carrying moves of arcs $x_{50^{\prime}}$ and $x_{67}$ with replaceable visiting I/O points in subtours $1+2$ and 3, (c) Merging subtours $1+2$ and 3 by replacing $x_{50^{\prime}}=x_{67}=1$ with $x_{57}=x_{60^{\prime}}=1$

The following corollaries derived from Theorem 2.2 explain which I/O point must be selected for merging subtours when swappable arcs with replaceable visiting I/O points are used for this purpose.

Corollary 2.2 If two subtours contain a pair of swappable arcs of which at least one has an irreplaceable visiting I/O point, the subtours can be merged without extra travel time by using that single common visiting I/O point.

The proof follows similar arguments as Theorem 2.2.

Corollary 2.3 discusses a more general case of this theorem when both swappable arcs have replaceable visiting I/O points.

Corollary 2.3 If two subtours contain a pair of swappable arcs, both with replaceable visiting I/O points, using any of the common visiting I/O points, the subtours can be merged without extra travel time.

The proof follows similar arguments as Theorem 2.2.
Corollaries 2.2 and 2.3 state that arcs replacing the swappable arcs should visit one of the common visiting I/O points. Visiting other I/O points will increase the total travel time, thus these I/O points must be omitted. An example of Corollary 2.2 is merging subtours 1 and 2 using swappable arcs $x_{41}$ and $x_{02}$ where the single common visiting I/O point is $I / O_{1}$ (see Figure 2.2). An example of Corollary 2.3 is presented in Figure 2.3. Swappable arcs of subtours $1+2$ and 3 shown in Figure 2.3a are $x_{67}$ and $x_{50^{\prime}}$. The carrying, non-carrying moves and replaceable visiting I/O points of these arcs are shown with different colors in Figure 2.3b. Based on Corollary 2.3, common visiting I/O points, $I / O_{m}, m=6,7$, can be used to merge subtours $1+2$ and 3. A complete tour is shown in Figure 2.3c. In this solution, $I / O_{6}$ is for example selected to deliver the retrieved container 6 .

## Merging algorithm steps

Having considered different properties of the problem, the merging algorithm can be presented. In the algorithm, merging subtours will be carried out by using the following arcs consecutively: (1) swappable arcs with a single irreplaceable visiting I/O point, (2) swappable arcs with replaceable visiting I/O points, and (3) swappable arcs of which one has a single irreplaceable visiting I/O point and the other has replaceable visiting I/O points. The reason behind this sequence is as follows:

In case both swappable arcs have a single irreplaceable visiting I/O point, merging two subtours results in a merged subtour in which the same common visiting I/O point will be visited by the new arcs replacing the swappable arcs (based on corollary 2.2). This is the ideal case because the subtours are merged without omitting any I/O point. In the other two cases, the number of common I/O points that will be kept in the merged subtours determines the sequences. In case swappable arcs have replaceable I/O points, the two related subtours can be merged using one of the common I/O points (based on corollary 2.3). On the other hand, in the third case, arcs replacing the swappable arcs visit only a single irreplaceable common visiting I/O point (based on corollary 2.2). In other words, in the third case only a single common visiting I/O point can be used to join the merged subtour to the other subtours whereas in the second case at least one common visiting I/O point exist. Therefore, the second case has priority in subtour merging over the third case.

In the Algorithm (1), $U_{m}$ and $U_{m}^{\prime}$ are the sets of all arcs in an optimal AP solution with irreplaceable and replaceable visiting $\mathrm{I} / \mathrm{O}$ points respectively, which visit $I / O_{m}, m=1, \ldots, M$.

The merging algorithm complexity is $O(M N)$, where $N$ is the total number of requests and $M$ is the number of I/O points. The complexity of the algorithm comes from identifying each type of arc. We need to

```
Algorithm 1 Merging algorithm
Require: An optimal AP solution;
Ensure: An optimal solution of the problem with the same optimal AP objective value
    and a complete Hamiltonian tour, or an optimal AP solution with the same optimal AP
    objective value and a few subtours;
    procedure MERGE
            classify and assign arcs of the AP optimal solution to the sets of arcs with irreplaceable
    and replaceable visiting I/O points, \(U_{m}\) and \(U_{m}^{\prime}, m=1, \ldots, M\) using Definitions 3 and
    4;
        for \(m=1, \ldots, M\) do
            if \(\left|U_{m}\right|>1\) then
                based on Corollary 2.2, the subtours can be merged using the common I/O
    point, \(I / O_{m}\). Merge subtours two by two using Corollary 2.1;
                update \(U_{m}\) and \(U_{m}^{\prime}, m=1, \ldots, M\);
            end if
        end for
        repeat steps 3-8 for \(U_{m}^{\prime}\). If \(\left|U_{m}^{\prime}\right|>1, m=1, \ldots, M\), based on Corollary 2.3, subtours
    can be merged using the common I/O point, \(I / O_{m}\);
        repeat step 3-8 for \(U_{m} \cup U_{m}^{\prime}\). If \(\left|U_{m} \cup U_{m}^{\prime}\right|>1, m=1, \ldots, M\), based on Corollary
    2.2 , subtours can be merged using the common I/O point, \(I / O_{m}\);
        if the number of subtours is 1 then
            an optimal solution of the problem is obtained; output the solution;
        else
            an optimal solution is not found because it does not satisfy Equation (5). The
    B\&B algorithm presented in subsection 2.2.2 will be called to find an optimal solution;
        end if
    end procedure
```

go through all the arcs and decide whether they have replaceable or irreplaceable I/O points.

### 2.2.2 Branch-and-bound algorithm

If, upon completion of the merging algorithm, the solution still contains at least two subtours, a further step is needed. This section introduces a modified B\&B algorithm to merge all subtours as a complete Hamiltonian tour.

## Basic branch-and-bound algorithm

## Step 1. Initialization

The output of the merging algorithm is an optimal AP solution with $|W|$ subtours, where $W$ is the set of subtours. The output serves as the input of the $B \& B$ algorithm in node 0 . In addition, determine the best solution found so far $X^{*}$, with the objective value $Z^{*}$ (which serves as an upper bound for the $\mathrm{B} \& \mathrm{~B}$ algorithm) by using a heuristic algorithm such as the FCFS or NN algorithm.

## Step 2. First branching

The $\mathrm{B} \& \mathrm{~B}$ algorithm continues by selecting a subtour $w \in W$ containing the minimum number of arcs to branch node 0 (Carpaneto and Toth, 1980). The branches divide the solution space into complementary
solution subspaces such that $w$ can be removed. This is carried out by sequentially excluding a next arc of the (now broken) subtour $w$ and including (keeping) the preceding arcs. The excluding arcs must disappear in the next solution and including arcs must appear. Assume subtour $w$ can in general be represented as $[1] \xrightarrow{x_{[1][2]}=1}[2] \rightarrow \cdots \rightarrow[K] \xrightarrow{x_{[K][1]}=1}[1]$, where $[i]$ is the $i^{\text {th }}$ location, $i=1, \ldots, K$, that will be visited by the ASC in this subtour. Subtour $w$ results in $K$ child nodes, each with the following corresponding sets of excluded and included arcs. Arcs can be excluded and included by assigning 0 and 1 to their corresponding variables, respectively.

$$
\begin{equation*}
E_{j}=\left\{[j] \xrightarrow{x_{[j][j+1]}=0}[j+1]\right\}, \quad j=1, \ldots, K, \tag{7}
\end{equation*}
$$

$$
\begin{equation*}
I_{j}=\left\{[1] \xrightarrow{x_{[1][2]}=1}[2], \ldots,[j-2] \xrightarrow{x_{[j-2][j-1]}=1}[j-1]\right\}, \quad j=2, \ldots, K, \tag{8}
\end{equation*}
$$

where, $I_{1}=\emptyset$, and $[K+1]=1$ in Equation (7).
Figure 2.4 shows the steps of the $B \& B$ algorithm. Note that using the merging algorithm, we can merge all subtours of the AP relaxation of the problem presented in Figure 2.1 as a single tour in Figure 2.3-c. However, assume that the problem has two more requests: retrieval request 9 and storage request 8 . These requests cause an extra subtour which cannot be merged with the other ones because it does not have any common visiting I/O point with the other subtours. Therefore, the outcome of the merging algorithm would be the solution presented at node 0 . The $\mathrm{B} \& \mathrm{~B}$ algorithm chooses subtour 4 for branching because it has only 2 arcs whereas the other one has $8 \operatorname{arcs}$.

## Step 3. Fathoming and bounding

At each child node, $l=1, \ldots, K$, we solve a modified AP, introduced by considering constraints forced by Equations (7) and (8) as well as other general constraints of the AP (Equations (2)-(4) and (6)).

Assume $Z_{\mathrm{AP}}^{l}$, and $X^{l}, l=1, \ldots, K$, are the optimal AP objective value and an optimal AP solution at node $l$. If $Z_{\mathrm{AP}}^{l} \geq Z^{*}$, fathom the node. If $Z_{\mathrm{AP}}^{l}<Z^{*}$ and the optimal solution to the modified AP at node $l$ is a complete Hamiltonian tour, update $Z^{*}=Z_{\mathrm{AP}}^{l}, X^{*}=X^{l}$ and fathom the node. Otherwise, if $Z_{\mathrm{AP}}^{l}<Z^{*}$ and the optimal solution is not a complete tour, add $l$ to the set of live nodes, $L$ ( $L=\emptyset$ at node 0 ). The set of live nodes is the set of nodes that are not fathomed yet and must be considered.

Figure 2.4 shows that solving the modified APs at child nodes 1 and 2 results in solutions each consisting of 2 subtours, namely subtours 6 and 7 for $l=1$, and subtours 8 and 9 for $l=2$. However, $Z_{\mathrm{AP}}^{2}=281.46 \geq$ $280.02=Z^{*}=Z_{\mathrm{NN}}$, so node 2 must be fathomed. Note that $X^{*}$ and $Z^{*}$ are obtained by the NN heuristic. On the other hand, $Z_{\mathrm{AP}}^{1}=277.96<280.02=Z^{*}=Z_{\mathrm{NN}}$. Therefore, node 1 will be added to the set of live nodes.

## Step 4. Further branching

Choose one of the live nodes from $L$ to be evaluated next. In our implementation, this is carried out by


Figure 2.4. The basic B\&B algorithm
using the depth-first-search (DFS) as the overall strategy and the best-first-search (BeFS) when a choice is to be made between nodes at the same level of the tree (see Clausen, 2003). This search strategy is denoted by DFS-BeFS. In case several nodes can be chosen, choose one randomly. Furthermore, if $L$ is empty, $X^{*}$ is an optimal solution and $Z^{*}$ is the optimal value.

Assume node $\hat{l} \in L$ is selected. If $Z_{\mathrm{AP}}^{\hat{l}} \geq Z^{*}$, omit this node from the set and choose another one; otherwise, find out which subtour must be split. We follow Carpaneto and Toth (1980), who propose to select the subtour with the minimum number of not-included arcs to be split. A not-included arc of a subtour is an arc that is not forced by $I_{\hat{l}}$ to appear in the optimal AP solution of node $\hat{l}$. The subtour
with the minimum number of not-included arcs generates the minimum number of child nodes from node $\hat{l}$. Assume $\hat{l}$, has $Q$ subtours where $G_{q}$ is the set of arcs in the $q^{t h}$ subtour. As a result, subtour $\bar{q}$ will be chosen to be split if $M=\left|G_{\bar{q}}\right|-\left|G_{\bar{q}} \cap I_{\hat{l}}\right|=\min _{q=1, \ldots Q}\left\{\left|G_{q}\right|-\left|G_{q} \cap I_{\hat{l}}\right|\right\}$, where $\left|G_{q} \cap I_{\hat{l}}\right|$ is the number of arcs showing up both in the $q^{\text {th }}$ subtour and in the set of included arcs of node $\hat{l}$.

In order to break subtour $\bar{q}$, we use the same branching rule explained in Equations (8) and (7). However, instead of considering all arcs in subtour $\bar{q}$, we only consider the not-included arcs because we cannot include or exclude a previously-included arc. Consider that subtour $\bar{q}$ can be represented as $[1] \xrightarrow{x_{[1][2]}=1}[2] \rightarrow \cdots \rightarrow[K] \xrightarrow{x_{[K][1]}=1}[1]$ of which the set of not-included in total contains $C$ arcs, $\left\{[1 o] \xrightarrow{x_{[10][1 d]}=1}[1 d], \ldots,[C o] \xrightarrow{x_{[C o][C d]}=1}[C d]\right\}$, where [io] and [id] are the origin and destination of the $i^{\text {th }}$ not-included arc, $i=1, \ldots, C$, respectively. The sequence of arcs in the set of not-included arcs can be determined randomly. Node $\hat{l}$ results in $C$ child nodes, each with the corresponding sets of excluded and included arcs as follows:

$$
\begin{equation*}
E_{B+j}=E_{\hat{l}} \cup\left\{[j o] \xrightarrow{x_{[j o[j d]}=0}[j d]\right\}, \quad j=1, \ldots, C, \tag{9}
\end{equation*}
$$

$$
\begin{equation*}
I_{B+j}=I_{\hat{l}} \cup\left\{[1 o] \xrightarrow{x_{[1 o[[1 d]}=1}[1 d], \ldots,[j-1, o] \xrightarrow{x_{[j-1, o][j-1, d]}=1}[j-1, d]\right\}, \quad j=2, \ldots, C, \tag{10}
\end{equation*}
$$

where, $I_{B+1}=I_{\hat{l}} \cup \emptyset$ and $B$ is the total number of nodes in the $\mathrm{B} \& \mathrm{~B}$ tree before generating the new child nodes.

The same fathoming and bounding strategy discussed in step 3 for the child nodes of node 0 applies here. In each child node, an optimal solution of the modified AP and the best solution found so far determine whether (1) the child node must be fathomed because the optimal AP solution is worse than the best solution found, or (2) the child node must be fathomed because a better solution is found, or (3) the child node must be added to $L$. Next, based on the DFS-BeFS, another node will be chosen and the algorithm will repeat until the set of live nodes is empty, $L=\emptyset$.

## Modified branch-and-bound algorithm

In Step 4, after selecting the next live node, if $Z_{\mathrm{AP}}^{\hat{l}}<Z^{*}, \hat{l} \in L$, the $\mathrm{B} \& \mathrm{~B}$ algorithm chooses a subtour to be split and generates the branches. However, in this section, before choosing a subtour and branching, we introduce an extra step (denoted by Step $4^{*}$ ) which tries to patch the subtours by using the merging algorithm. The smaller the number of subtours is, the fewer nodes will be produced afterwards in the B\&B tree. Furthermore, we may also obtain a complete tour which fathoms the node and makes further branching unnecessary. As a result, the number of nodes of the $B \& B$ tree decreases significantly.
Step 4*. Merging algorithm. Merge subtours of the optimal AP solution in the selected live node using the
merging algorithm. If the output of the merging algorithm is a complete Hamiltonian tour update $Z^{*}=Z_{\mathrm{AP}}^{\hat{l}}$, $X^{*}=X_{\hat{l}}$, fathom the node and select another live node. Otherwise, choose a subtour and branch the node.


Figure 2.5. (a) The child nodes of node 1 generated by the basic $\mathrm{B} \& \mathrm{~B}$ algorithm, (b) Fathoming node 1 by the modified B\&B algorithm

Figure 2.5 is used to explain Step $4^{*}$ of the algorithm. Figure 2.5a shows the rest of the B\&B tree, if the basic $B \& B$ algorithm is used. The basic $B \& B$ algorithm must evaluate 4 more nodes ( 7 nodes in total), nodes $3,4,5$, and 6 which are the child nodes of node 1 , to find an optimal solution. On the other hand, Figure 2.5 b shows that no extra branching is necessary if the modified $\mathrm{B} \& \mathrm{~B}$ algorithm is used ( 3 nodes in total). As a result, computation time is reduced. This is due to the fact that, in Step $4^{*}$, using the merging algorithm, all subtours of node 1 are merged and therefore, it must be fathomed. Furthermore, in this specific case, $X^{1}$ is also an optimal solution with the optimal value $Z^{*}=277.96$.

### 2.3 Computational expriments

Multiple numerical experiments have been performed to investigate the effectiveness of the two-phase solution method consisting of the merging algorithm and the modified $B \& B$ algorithm. In the basic scenario, we
consider a single block of containers with 30 bays, 10 rows, four tiers, and 10 I/O points. Furthermore, 100 requests are considered of which the percentage of retrieval requests, $\mathbb{P}$, is $50 \%$. At large container terminals, the inbound and outbound flows of a container block are usually reasonably balanced. The ASC starts its operation from the location of the first I/O point and it either ends at the storage position of a storage request if it is the last request in the sequence or at an I/O point if a retrieval request is the last request in the sequence. Other input data can be found in Table 2.2. The two-phase solution method is used to optimally sequence all the requests.

Table 2.2. Inputs of the simulation study

| Variable | value |
| :--- | :--- |
| Gantry speed of the ASC |  |
| Trolley speed of the ASC | $240 \mathrm{~m} / \mathrm{min}$ |
| Hoisting speed of the ASC | $60 \mathrm{~m} / \mathrm{min}$ |
| Container length ${ }^{2,3}$ | $72 \mathrm{~m} / \mathrm{min}$ |
| Container width | 5.89 m |
| Container hight | 2.33 m |
| 1 No acceleration or deceleration time is considered. | 2.38 m |
| 2 $^{\text {2 }}$ All containers are 20 feet long. |  |
| ${ }^{3}$ No separating space between containers is considered. |  |

In each scenario of the simulation study, 100 realizations of $N$ requests with their storage and retrieval locations and corresponding I/O points are randomly generated by Monte Carlo simulation. The study is performed on a Notebook with 2.40 GHz Intel ${ }^{\circledR}$ Core $^{\mathrm{TM}}$ i5 processor, with 4.00 GB of RAM and the programming language is MATLAB ${ }^{\circledR}$ 2010a.

In Table 2.3, the results of the two-phase solution method are presented. In Table 2.4, our solution method is compared with the FCFS and NN heuristics. The comparisons are carried out in the same way by generating 100 realizations for each scenario and use the two-phase solution method as well as the heuristics to find the total travel time. The number of realizations satisfies $N_{\text {realz. }} \geq \sigma^{2}\left(\frac{(1+\varepsilon) \mathbb{Z}_{1-\alpha / 2}}{\varepsilon \mu}\right)^{2}$ with a $90 \%$ confidence level ( $\alpha=10 \%$ ), where $\sigma^{2}$ and $\mu$ are respectively the variance and mean of the objective values, $\mathbb{Z}_{1-\alpha / 2}$ is the $1-\alpha / 2$ percentile of the normal distribution, and $\varepsilon=5 \%$ is for determining the confidence interval (Law and Kelton, 1999). Based on the results presented in Tables 2.3 and 2.4, the following insights can be obtained:

- The results in Table 2.3 show that the two-phase solution method can find the optimal solution of the problem in less than a second in all scenarios.
- The merging algorithm is quite efficient. This can be seen in the column presenting $\rho$, which is the percentage of the problems solved in the first phase of the solution method. When $\rho=100 \%$, the problem is completely solved in phase 1 and we do not need to enter phase 2 . The same conclusion can

Table 2.3. The results of the solution method

| X | $Y$ | $N$ | $\mathbb{P}$ (\%) | M | Computation time (sec.) |  |  | Number of nodes |  |  | $\frac{Z_{\mathrm{AP}}^{*}}{Z^{*}}$ | $\begin{gathered} Z^{*} \\ \text { (Sc.) } \end{gathered}$ | $\rho(\%)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 10 | 30 | 50 | 50 | 10 | 0.0397 | 0.171 | 0.0256 | 1.32 |  | 1 | 0.999995 | 2325.709 | 0.93 |
| 10 | 30 | 100 | 50 | 10 | 0.1321 | 0.3936 | 0.1072 | 1.04 | 5 | 1 | 1 | 4650.213 | 0.99 |
| 10 | 30 | 150 | 50 | 10 | 0.2973 | 0.3377 | 0.2397 | 1 | 1 | 1 | 1 | 6965.121 | 1 |
| 10 | 30 | 200 | 50 | 10 | 0.5462 | 0.7231 | 0.4062 | 1 | 1 | 1 | 1 | 9190.737 | 1 |
| 5 | $30^{-}$ | -100 | 50 | 10 | -0.1295 | 0.1554 ${ }^{-}$ | 0.1038 |  |  | I |  | 4595.805 |  |
| 6 | 30 | 100 | 50 | 10 | 0.1315 | 0.4104 | 0.0999 | 1.04 | 5 | 1 | 1 | 4605.67 | 0.99 |
| 7 | 30 | 100 | 50 | 10 | 0.1419 | 0.6872 | 0.1034 | 1.16 | 9 | 1 | 1 | 4551.678 | 0.97 |
| 8 | 30 | 100 | 50 | 10 | 0.1353 | 0.413 | 0.1086 | 1.08 | 5 | 1 | 1 | 4609.479 | 0.98 |
| 9 | 30 | 100 | 50 | 10 | 0.1312 | 0.3934 | 0.0968 | 1.04 | 5 | 1 | 1 | 4612.963 | 0.99 |
| 10 | 20 | -100 | 50 | 10 | 0.112 | 0.165 | 0.0851 | -1- |  | I |  | 3788.589 |  |
| 10 | 40 | 100 | 50 | 10 | 0.1215 | 0.3179 | 0.094 | 1.04 | 5 | 1 | 1 | 5410.98 | 0.99 |
| 10 | 50 | 100 | 50 | 10 | 0.1272 | 0.4288 | 0.0931 | 1.08 | 5 | 1 | 1 | 6360.285 | 0.98 |
| 10 | $30^{-}$ | -100 | 50 | 2 | 0.1237 | 0.1593- | 0.0878 | ${ }^{-1}$ | T | I | 1 | 4760.263 |  |
| 10 | 30 | 100 | 50 | 8 | 0.1301 | 0.1524 | 0.1123 | 1 | 1 | 1 | 1 | 4633.471 | 1 |
| 10 | 30 | 100 | 50 | 14 | 0.1415 | 0.4124 | 0.0923 | 1.2 | 5 | 1 | 1 | 4592.33 | 0.95 |
| 10 | 30 | 100 | 50 | 20 | 0.139 | 0.4238 | 0.1054 | 1.2 | 5 | 1 | 0.999998 | 4645.63 | 0.95 |
| 10 | $30^{-}$ | -100 | - | I0 | 0.1147 | $0.3789{ }^{-}$ | 0.0978 ${ }^{-}$ | $\square^{-}$ |  | I | -- $1^{---}$ | $6 \Gamma 30.303$ | $]^{-}$ |
| 10 | 30 | 100 | 20 | 10 | 0.1401 | 0.248 | 0.1113 | 1 | 1 | 1 | 1 | 5330.784 | 1 |
| 10 | 30 | 100 | 40 | 10 | 0.1324 | 0.2342 | 0.1116 | 1 | 1 | 1 | 1 | 4672.246 | , |
| 10 | 30 | 100 | 60 | 10 | 0.1068 | 0.8279 | 0.0822 | 1.12 | 13 | 1 | 1 | 4800.1 | 0.99 |
| 10 | 30 | 100 | 80 | 10 | 0.0952 | 0.137 | 0.0809 | 1 | 1 | 1 | 1 | 5436.168 | 1 |
| 10 | 30 | 100 | 100 | 10 | 0.0942 | 0.1807 | 0.0781 | 1 | 1 | 1 | 1 | 6197.658 | 1 |
| 10 | $30^{-}$ | -100 | 50 | 50 | 0.1976 | I.1485 | 0.093 | -2.44 | 17 | I |  | 4595.972 | 0.75 |
| 10 | 30 | 100 | 50 | 100 | 0.2034 | 1.3404 | 0.0865 | 2.62 | 21 | 1 | 1 | 4624.975 | 0.79 |
| 10 | 30 | 100 | 50 | 150 | 0.3221 | 4.5623 | 0.084 | 4.88 | 83 | 1 | 1 | 4633.177 | 0.68 |
| 10 | 30 | 100 | 50 | 200 | 0.4459 | 3.8641 | 0.0761 | 7.02 | 75 | 1 | 0.999999 | 4620.406 | 0.57 |

NOTE. We denote by Ave, Max, and Min the average, maximum and minimum computation time and number of nodes of the two-phase solution method. The table also presents the ratio of the objective value of the optimal AP solution (obtained from the original travel time matrix) to the objective value of the optimal solution $\left(Z_{\mathrm{AP}}^{*} / Z^{*}\right)$, the total travel time of the $\operatorname{ASC}\left(Z^{*}\right)$, and the percentage of the realizations optimally solved in the first phase of the solution method $(\rho)$. The other notations are explained in the text.
be obtained from the columns showing the average, maximum and minimum number of nodes needed in each scenario. In most of the scenarios, the average number of nodes of the $\mathrm{B} \& \mathrm{~B}$ tree is close to 1 .

The merging algorithm also performs well for large instances. The reason is that as the number of requests $N$ increases, the number of arcs visiting each I/O point increases, and consequently the number of swappable arcs increases. Therefore, most of the subtours can be merged in the first phase and the second phase becomes unnecessary.

- From Table 2.3 it is clear that for a fixed number of requests, the computation time and number of nodes of the two-phase solution method are quite insensitive to changes in problem inputs such as: number of bays, rows, requests and percentage of retrieval requests in different scenarios. The reason is that the performance of the solution method depends on the performance of the merging algorithm which mainly depends on the ratio of the number of requests to the number of I/O points. When the number of requests is 100 and the number of I/O points is less than 10 , the ratio of the requests to I/O points is high enough (even in case of 10 I/O points, on average 10 containers are picked up or delivered at each I/O point) for the merging algorithm in order to be able to patch all subtours in the first phase of the algorithm. The computation time of the first phase of the solution method is about

Table 2.4. Comparing the FCFS and NN heuristics with the optimal solution

| X | $Y$ | $N$ | $\mathbb{P}(\%)$ | M | $Z^{*}$ | $Z_{\text {FCFS }}{ }^{1}$ | $Z_{\mathrm{NN}^{2}}$ | $G_{\text {FCFS }}(\%)^{3}$ | $G_{\text {NN }}(\%)^{4}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 10 | 30 | 20 | 50 | 10 | 976.42 | 1367.03 | 1113.1 | 28.57 | 12.28 |
| 10 | 30 | 50 | 50 | 10 | 2325.7 | 3416.29 | 2730.6 | 31.92 | 14.83 |
| 10 | 30 | 100 | 50 | 10 | 4650.2 | 6841.4 | 5406.32 | 32.03 | 13.99 |
| 10 | 30 | 150 | 50 | 10 | 6965.1 | 10269.3 | 8006.24 | 32.18 | 13.00 |
| 10 | 30 | 200 | 50 | 10 | 9190.7 | 13727 | 10692.1 | 33.05 | 14.04 |
| 5 | $30^{-}$ | 100 | 50 | 10 | 4595.8 | $6750.53{ }^{-}$ | -5238.2 | 31.92 | 12.26 |
| 6 | 30 | 100 | 50 | 10 | 4605.7 | 6762.95 | 5288.25 | 31.90 | 12.91 |
| 7 | 30 | 100 | 50 | 10 | 4551.7 | 6765.15 | 5294.51 | 32.72 | 14.03 |
| 8 | 30 | 100 | 50 | 10 | 4609.5 | 6802.99 | 5264.34 | 32.24 | 12.44 |
| 9 | 30 | 100 | 50 | 10 | 4613 | 6836.42 | 5338.95 | 32.52 | 13.60 |
| 10 | 20 | 100 | 50 | IO | 3788.6 | 5524 | 4423.2- | 31.42 | 14.35 |
| 10 | 40 | 100 | 50 | 10 | 5411 | 8763.42 | 6336.47 | 38.25 | 14.61 |
| 10 | 50 | 100 | 50 | 10 | 6360.3 | 9691.24 | 7421.24 | 34.37 | 14.30 |
| 10 | $30^{-}$ | 100 | 50 | 2 | 4760.3 | $7081.74{ }^{-}$ | 5561.32 | 32.78 | 14.40 |
| 10 | 30 | 100 | 50 | 8 | 4633.5 | 6916.51 | 5386.99 | 33.01 | 13.99 |
| 10 | 30 | 100 | 50 | 14 | 4592.3 | 6867.52 | 5404.58 | 33.13 | 15.03 |
| 10 | 30 | 100 | 50 | 20 | 4645.6 | 6852.88 | 5394.12 | 32.21 | 13.88 |
| 10 | 30 | 700 | $\sigma$ | - IO | 6130.3 | 7308.65- | 6312.60 | 16.12 | 2.89 |
| 10 | 30 | 100 | 20 | 10 | 5330.8 | 7056.59 | 5850.07 | 24.46 | 8.88 |
| 10 | 30 | 100 | 40 | 10 | 4672.2 | 6899.69 | 5368.84 | 32.28 | 12.97 |
| 10 | 30 | 100 | 60 | 10 | 4800.1 | 6864.83 | 5485.70 | 30.08 | 12.50 |
| 10 | 30 | 100 | 80 | 10 | 5436.2 | 6916.22 | 5754.48 | 21.40 | 5.53 |
| 10 | 30 | 100 | 100 | 10 | 6197.7 | 7154.74 | 6219.10 | 13.38 | 0.34 |
| 10 | 30 | 100 | 50 | -50 | 4596 | $7168.31^{-}$ | $5428.3-$ | 35.88 | 15.33 |
| 10 | 30 | 100 | 50 | 100 | 4625 | 6857.15 | 5363.7 | 32.55 | 13.77 |
| 10 | 30 | 100 | 50 | 150 | 4633.2 | 6868.57 | 5415 | 32.55 | 14.44 |
| 10 | 30 | 100 | 50 | 200 | 4620.4 | 6846.41 | 5380.8 | 32.51 | 14.13 |

${ }^{1} Z_{\text {FCFS }}$ is the average objective value obtained by the FCFS heuristic over 100 runs.
${ }^{2} Z_{\text {NN }}$ is the average objective value obtained by the NN heuristic over 100 runs.
${ }^{3} G_{\mathrm{FCFS}}(\%)=\left(Z_{\mathrm{FCFS}}-Z^{*}\right) /\left(Z_{\mathrm{FCFS}}\right) \times 100$.
${ }^{4} G_{\mathrm{NN}}(\%)=\left(Z_{\mathrm{NN}}-Z^{*}\right) /\left(Z_{\mathrm{NN}}\right) \times 100$.
the same for all scenarios with a fixed number of requests.

- The last four rows of Table 2.3 investigate theoretical cases with 50 or more I/O points, in order to find out the effect of the number of I/O points on the computation time complexity. By increasing the number of I/O points the probability of merging subtours in the first phase decreases. As a result, $\rho$, the computation time and the number of nodes increases. However, in this problem, when the number of I/O points is 50 , each I/O point is on average shared by 4 requests $(N=100)$. Therefore, it is still probable that the subtours can be merged in the first phase using swappable arcs with common visiting I/O points.
- Based on the results presented in Table 2.4, when there is a balance between the number of storage and retrieval requests $(\mathbb{P}=50 \%)$, the average objective value gap is more than $30 \%$ in case of the FCFS heuristic, and $14 \%$ in case of the NN heuristic. On the other hand, in case of an imbalance between the number of storage and retrieval requests (moving toward $\mathbb{P}=0$ or $100 \%$ ), both FCFS and NN heuristics perform better. The reason is that in the optimal solution, more pure storage and retrieval requests must be individually executed by the ASC, and there is less opportunity to sequence retrieval requests after storage requests for minimizing the empty travel time of the ASC.

We also compare the results of four different instances obtained by the two-phase solution method with those results obtained by CPLEX 12.2 coded in C++ using the Concert Technology framework and executed by a g++ compiler on a 2.40 GHz AMD Opteron ${ }^{\text {TM }}$ Processor 250 , with 8.00 GB of RAM under the Linux operating system. The results presented in Table 2.5 confirm the complexity of the problem, as CPLEX cannot solve the problems in a reasonable time. The computation in CPLEX was truncated after 18000 seconds. Table 2.5 shows that there is a large gap between the feasible objective value obtained by CPLEX in 18000 seconds and the optimal objective value (obtained in less than a second). By increasing the number of requests, the gap between the two values increases.

Table 2.5. Comparing the two-phase solution method and truncated CPLEX

| Inst. | $X$ | $Y$ | $N$ | $\mathbb{P}(\%)$ | $M$ | CPLEX |  | Our method |  | $G_{\text {CPLEX }}(\%)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | CPU | $Z$ | CPU | $Z^{*}$ |  |  |  |  |
| 1 | 10 | 40 | $\mathbf{1 0}$ | 50 | 10 | 4019 | 699.10 | 0.00 | 699.10 | 0 |
| 2 | 10 | 40 | $\mathbf{2 0}$ | 50 | 10 | 18000 | 1100.10 | 0.01 | 1100.10 | 0 |
| 3 | 10 | 40 | $\mathbf{5 0}$ | 50 | 10 | 18000 | 2994.09 | 0.03 | 2751.34 | 8.11 |
| 4 | 10 | 40 | $\mathbf{1 0 0}$ | 50 | 10 | 18000 | NA | 0.13 | 5122.75 | NA |

Notes. Let $Z$ and $Z^{*}$ be the objective value obtained by truncated CPLEX and the optimal objective value obtained by the two-phase solution method, respectively. Column CPU is the computation time in seconds. If the computation time of CPLEX is less than 18000 seconds, the optimal value is obtained.
The gap is calculated as: $G_{\mathrm{CPLEX}}(\%)=\left(\left(Z-Z^{*}\right) / Z\right) \times 100$.

### 2.4 Conclusion

We model and solve a difficult operational problem in which a set of container storage and retrieval requests must be sequenced. We minimize the total travel time of a single ASC carrying out the requests in a single block of containers. An efficient ASC scheduling operation can significantly affect the overall performance of the container terminal. We formulate the problem as an integer model, and show that it is $\mathcal{N} \mathcal{P}$-hard in a special case. We benefit from properties of the problem to develop a two-phase solution method which can efficiently solve this problem. The merging algorithm proposed in the first phase to patch subtours of an optimal AP solution works efficiently specially for large-scale problems. As the number of requests increases, the average number of requests per I/O point increases and therefore, the merging algorithm has more opportunity to patch subtours. As a result, in most of the realizations of the scenarios an optimal solution can be obtained in the first phase by optimally solving the AP and using the merging algorithm. If an optimal solution cannot be obtained in the first phase, a $B \& B$ algorithm in the second phase rapidly finds an optimal solution. The computation time of the solution method is low, i.e. it is less than a second when the number of requests is 200 . The gaps between the optimal value and the objective values of the FCFS and NN heuristics are on average more than $30 \%$ and $14 \%$, respectively. Furthermore, the two-phase solution method significantly outperforms CPLEX for small-size instances. For instances with 100 requests,

CPLEX cannot obtain a feasible solution after five hours.
The model developed in this paper can be extended in several directions. We assume that containers do not have any precedence constraints. However, in reality, many issues such as the priority of seaside operations to landside operations, the arrival time of different modes of transport, and ensuring the stability of the ship impose precedence constraints to the model. Furthermore, we consider that all storage containers are available at the I/O points and a retrieval container can be dropped off at the same I/O point from where another container has to be stacked in the block. But safety regulations and space availability cause some limitations. Extra constraints are required to model reality. Finally, one may study an online version of the model where the list of requests is updated in some time periods. Some of these issues are tackled in the next chapters.

## Appendix

## 2.A Proof of Theorem 2.1

Choosing the I/O point which minimizes the travel time doesn't affect the optimal sequence of requests. Let $t_{i j}$ be the travel time between locations $i$ and $j$ in an optimal solution. If $t_{i j}$, is larger than $t_{i j}^{\prime}$, then the optimal solution can be improved by replacing $t_{i j}$ with $t_{i j}^{\prime}$. As a result, in order to have the minimum objective value, the minimum travel time must be always selected for every arc.

## 2.B Proof of Theorem 2.2

Consider two subtours resulting from an optimal AP solution with the swappable arcs $x_{k l}=1$ and $x_{r s}=1$ and the common visiting I/O point, $I / O_{m}$. By setting arcs $x_{k l}=x_{r s}=0$ and $x_{k s}=x_{r l}=1$, we can show that subtours are merged without extra travel time.

The objective function of the problem can be shown as follows:

$$
\begin{equation*}
\min Z=\sum_{i \in V \backslash\left\{j, 0^{\prime}, r, k\right\}} \sum_{j \in V \backslash\{0, l, s\}} t_{i j} x_{i j}+t_{k l} x_{k l}+t_{r s} x_{r s}+t_{k s} x_{k s}+t_{r l} x_{r l}, \tag{11}
\end{equation*}
$$

Before merging, $x_{k l}=x_{r s}=1$ and $x_{k s}=x_{r l}=0$, so $Z_{\text {Before }}=\sum_{i \in V \backslash\left\{j, 0^{\prime}, r, k\right\}} \sum_{j \in V \backslash\{0, l, s\}} t_{i j} x_{i j}+t_{k l}+$ $t_{r s}$. On the other hand, from Table 2.1, $t_{k l}=t_{k, I / O_{m}}+t_{I / O_{m}, l}$ and $t_{r s}=t_{r, I / O_{m}}+t_{I / O_{m}, s}$. Consequently, $Z_{\text {Before }}=\sum_{i \in V \backslash\left\{j, 0^{\prime}, r, k\right\}} \sum_{j \in V \backslash\{0, l, s\}} t_{i j} x_{i j}+t_{k l}+t_{r s}=\sum_{i \in V \backslash\left\{j, 0^{\prime}, r, k\right\}} \sum_{j \in V \backslash\{0, l, s\}} t_{i j} x_{i j}+$ $t_{k, I / O_{m}}+t_{I / O_{m}, l}+t_{r, I / O_{m}}+t_{I / O_{m}, s}$.

After merging, $x_{k l}=x_{r s}=0$ and $x_{k s}=x_{r l}=1$, so $Z_{\text {After }}=\sum_{i \in V \backslash\left\{j, 0^{\prime}, r, k\right\}} \sum_{j \in V \backslash\{0, l, s\}} t_{i j} x_{i j}+$ $t_{k s}+t_{r l}$. From Table 2.1, we have $t_{k s}=t_{k, I / O_{m}}+t_{I / O_{m}, s}$ and $t_{r l}=t_{r, I / O_{m}}+t_{I / O_{m}, l}$, therefore $Z_{\text {After }}=$ $\sum_{i \in V \backslash\left\{j, 0^{\prime}, r, k\right\}} \sum_{j \in V \backslash\{0, l, s\}} t_{i j} x_{i j}+t_{k, I / O_{m}}+t_{I / O_{m}, s}+t_{r, I / O_{m}}+t_{I / O_{m}, l}=Z_{\text {Before }}$.

## 2.C Proof of Theorem 2.3

The proof follows from the calculations of Table 2.1. Each case will be proven separately. Note that this theorem is not bi-conditional, which means that if an arc has a single visiting I/O point, it is not necessarily the following types of arcs.
Case 1: consider storage request $j$ is immediately after storage request $i$. Furthermore, $I / O_{l}$ is the I/O point where container $j$ must be picked up. Obviously, traveling to an I/O point, $I / O_{l^{\prime}}$, doesn't give the minimum travel time because the ASC must travel to $I / O_{l}$, and $t_{I / O_{l}, I / O_{l^{\prime}}}$ will be added to the total travel time of the ASC.

Case 2: consider storage request $j$ is immediately after retrieval request $i$. Furthermore $I / O_{l}$ is the I/O point where container $j$ must be picked up and $I / O_{l^{\prime}}$ is the $\mathrm{I} / \mathrm{O}$ point where container $i$ will be delivered. If container $i$ must be delivered into the same side that the storage container is currently located in an I/O point, the travel time of the ASC is minimum, if $I / O_{l}=I / O_{l^{\prime}}$. In words, the container must be delivered to the same I/O point where the other container must be picked up. If $I / O_{l} \neq I / O_{l^{\prime}}$, then we add $t_{I / O_{l}, I / O_{l^{\prime}}}$ to the total travel time of the ASC.

## Chapter 3

## Scheduling a Single Yard Crane with Flexible Storage Locations*

In Chapter 2, we studied the problem of scheduling an automated stacking crane (ASC) to carry out a set of storage and retrieval requests in a single block of containers. We considered that in the case of a retrieval request, the ASC picks up a container from a location in the block and drops it off in an input/output (I/O) point, whereas in the case of a storage request, the ASC picks up a container from an I/O point and drops it off at a given location. In this chapter, we study an extension where multiple open locations are available to stack the storage container. Since container yard operators often physically separate containers based on criteria such as destination and weight, a set of suitable open locations are available to stack the container. Obviously, a single location may be suitable for stacking different containers, and thus sets of open locations may overlap. Figure 3.1 b is a top view of a block of containers with multiple storage and retrieval locations. The sets of open locations available for different storage containers are highlighted by the contours.

We study where to stack storage containers while sequencing a given number of requests in a single block of containers. The objective is to minimize the travel time of the ASC carrying out the requests. We formulate the problem as a generalized asymmetric traveling salesman problem (GATSP). In this formulation, locations in the overlap of multiple sets make the problem more complex. For such open locations, we need extra constraints to avoid selecting one of them for stacking more than a single storage container. The GATSP is $\mathcal{N} \mathcal{P}$-hard, and the extra constraints increase the complexity even more. We propose a three-phase solution method to optimally solve the problem. The main idea of the algorithm is to simplify the problem to an ATSP and use its optimal solution to obtain an optimal GATSP solution. An optimal ATSP solution can be quickly obtained by the solution method proposed in Chapter 2, which uses the particular structure of the problem, namely the I/O points to obtain a complete tour. In the first phase of our algorithm, we

[^1]

Figure 3.1. Schematic representation of a container yard layout
Note. Locations highlighted by each contour are suitable to stack a given container.
generate an ATSP model by pre-selecting the locations resulting in the minimum pairwise travel times. This can be realized by redefining the original travel time matrix. We show that an optimal solution of the ATSP model generated in this way is identical to an optimal solution of the GATSP in which the extra constraints related to the open locations in the overlap of multiple sets are relaxed. In the second phase, we check for two special cases in which an optimal ATSP solution directly provides an optimal GATSP solution. In the third phase, we develop a branch and bound ( $B \& B$ ) algorithm which guarantees that every open location belonging to multiple sets can be occupied by at most a single container. In every node of the B\&B tree, we allocate an open location in the interval of two or more sets selected for stacking multiple containers to a single set. Then, we generate a new ATSP model of which the solution bounds the node.

The three-phase solution method is efficient for small and medium-scale instances. We propose a heuristic algorithm, for large-scale instances. The heuristic method first finds an optimal solution of the ATSP which is generated by pre-selection of locations. In case the solution is not a feasible GATSP solution, an assignment problem (AP) formulation is used to replace all open locations which store multiple storage containers by other locations, with a minimum increase in the ATSP optimal value. The numerical results show the objective value gap between our heuristic algorithm and the lower bound is less than $1 \%$, on average. Note that an optimal ATSP solution provides a lower bound for the GATSP. Furthermore, the heuristic algorithm outperforms the nearest neighbor (NN) and first-come-first-served (FCFS) heuristics by up to $20 \%$ and $37 \%$, respectively. We also compare the results obtained by our solution method and CPLEX. The results show that our solution method can obtain significantly better results than CPLEX truncated after five hours.

Scheduling an ASC to carry out storage and retrieval requests can generally be modeled as an ATSP or one of its special cases such as the rural postman problem, or Chinese postman problem, depending on the properties of the problem (see Chapter 2). These papers merely focus on sequencing storage and retrieval requests and assume that storage locations are already selected by other models (for example, see Kim et al., 2000, Dekker et al., 2007). However, the decisions made in one problem substantially affect the decisions in the other problem. Intuitively, given different storage locations may result in different sequences of performing requests, and vice versa. Scheduling an ASC to carry out storage and retrieval requests in a block with multiple open locations incorporating the storage location problem can be modeled as a GATSP. Contrary to the dense literature of the ATSP and symmetric TSP (see for example Applegate et al. (2007) and Laporte (1992) for an overview on the TSP or Burkard et al. (1998) for an overview on solvable cases), the GATSP problem has been only considered in a few papers (Fischetti et al., 1997, Laporte et al., 1987, Noon and Bean, 1991). In the GATSP, multiple sets of locations exist and the problem calls for a tour passing at least one location of each set in such a way that the total travel time is minimized. If the tour is forced to pass exactly a single location of each set, the resulting problem is sometimes called E-GATSP, where E stands for equality. Note that in this chapter, sets of open locations can overlap. In previous more theoretical papers, sets of open locations either do not overlap, or if they do, all of the overlapping sets are visited simultaneously by selecting one of the locations in the overlap of some sets. However, in our problem, a separate location of each set has to appear in a feasible solution.

Laporte et al. (1987) are among the pioneers to study the GATSP. They propose a B\&B algorithm to solve the problem. The core idea of the $\mathrm{B} \& \mathrm{~B}$ algorithm is solving the AP relaxation of the problem and removing the infeasibility. An optimal AP solution may generate an infeasible GATSP solution because (1) some sets are not visited, and/or (2) multiple subtours exist in the solution. Laporte et al. (1987) force the tour to visit a not-visited set by requiring a location of that set in the solution. A subtour is avoided by forbidding one of its arcs and requiring the others to appear in the solution. As a result, in each node of the B\&B algorithm, an AP requiring and forbidding some nodes and arcs is solved. The algorithm is designed to first include all sets and then remove subtours. In fact, the $B \& B$ algorithm is an extension of the $B \& B$ algorithm previously proposed by Carpaneto and Toth (1980) to solve the ATSP by removing the subtours of the AP relaxation. The latter algorithm was later improved by Miller and Pekny (1991) and Carpaneto et al. (1995) who propose two merging algorithms to patch subtours in each node and decrease the number of branches. Noon and Bean (1991) also propose a similar B\&B algorithm to solve the GATSP. To enhance the performance of the B\&B algorithm, they first use Lagrangian relaxation to improve the ATSP and AP bounds. Second, in order to reduce the density of the distance matrix, some of the locations and arcs are eliminated using the reduced costs. Furthermore, they develop a heuristic algorithm to obtain an upper bound to fathom some of the nodes.

Fischetti et al. (1995) study the facial structure of the GTSP and E-GTSP polytopes where sets of locations do not overlap and pairwise travel times are symmetric. Using the theoretical foundation of the
paper, they then develop a branch-and-cut algorithm to solve the GTSP (Fischetti et al., 1997). The solution method uses a B\&B scheme. In each node, an LP relaxation of the problem is solved while adding some valid inequalities. Valid inequalities tighten the solution space of the problem in order to obtain a feasible integer solution.

The main contribution of this chapter is twofold. We model and solve the simultaneous request scheduling and storage location selection problem, which is relevant in practice to use the highly utilized ASC efficiently. The algorithm proposed to solve the problem is new. In the first phase of the algorithm, we use the I/O points to simplify the problem to an ATSP model by preselecting open locations, which differs from previous methods (for example, see Dimitrijević and Šarić, 1997, Lien et al., 1993). Furthermore, the specific structure of the problem, which requires a distinct open location for each storage container, allows us to define a new $B \& B$ algorithm in the third phase of the solution method. In our $B \& B$ algorithm, branching is carried out in order to assign a location in the overlap of multiple sets selected for storing multiple containers to a single set, so that a separate location of each set can be selected. The method is fast for different sizes of the problem if sets of open locations do not overlap. An accurate heuristic algorithm is proposed to solve large-scale problems. The methods are sufficiently general to be applied to other GATSP problems with a similar structure.

The rest of this chapter is organized as follows. In Section 3.1, we describe the technical aspects of the problem and present the mathematical model. In Section 3.2, the solution method is developed. Section 3.3 presents numerical results. Finally, in Section 3.4, we conclude the chapter.

### 3.1 Problem description and model

We seek to determine the sequence to carry out $n$ storage requests and $N-n$ retrieval requests in a single block of containers consisting of $X$ rows, $Y$ bays, $Z$ tiers and $M$ I/O points. $I / O_{m}$ is the location of the $m^{t h}$ I/O point at the seaside, $m=1, \ldots, M_{s}$, or at the landside, $m=M_{s}+1, \ldots, M$. Let $\mathcal{R}$ be the set of all storage and retrieval requests. The objective is to minimize the total travel time of a single ASC performing these requests. The ASC can only carry a single container at a time. From a given starting point, the ASC can perform the requests in any sequence in order to minimize total travel time. We denote by 0 and $0^{\prime}$, respectively, the starting and ending locations of the ASC which can be anywhere in the block. Containers can be of different sizes (i.e. 20 or 40 feet) and leave or arrive at the container terminal by truck, train or ship. In addition, containers should not be dropped off in temporary locations within the block.

Every storage or retrieval request $r \in \mathcal{R}=\{1, \ldots, N\}$ corresponds to a unique container. For storage request $r$, a ASC moves a container from the I/O point where it is initially located to a location $i$ in the block selected from a given set $\mathcal{L}_{r}$ of available open locations. For retrieval request $r$, a ASC moves a container from location $j$ in the block to an I/O point at its destination side. For ease of notation, we write that location $j$ of retrieval request $r$ belongs to $\mathcal{L}_{r}$. Thus, the set $\mathcal{V}$ of all locations can be defined as:
$\mathcal{V}=\{\underbrace{\left\{1, \ldots,\left|\mathcal{L}_{1}\right|\right.}_{\mathcal{L}_{1}}, \underbrace{\left|\mathcal{L}_{1}\right|+1, \ldots,\left|\mathcal{L}_{1}\right|+\left|\mathcal{L}_{2}\right|}_{\mathcal{L}_{2}}, \ldots, \underbrace{\sum_{i=1}^{n-1}\left|\mathcal{L}_{i}\right|+1, \ldots, \sum_{i=1}^{n}\left|\mathcal{L}_{i}\right|}_{\mathcal{L}_{n}}, \underbrace{\sum_{i=1}^{n}\left|\mathcal{L}_{i}\right|+1}_{\mathcal{L}_{n+1}}, \ldots, \underbrace{\sum_{i=1}^{n}\left|\mathcal{L}_{i}\right|+N-n,}_{\mathcal{L}_{N}}$ $\underbrace{0}_{\mathcal{L}_{0}}, \underbrace{0^{\prime}}_{\mathcal{L}_{0^{\prime}}}\}$.
in which $\mathcal{L}_{r}$ consists of $\left|\mathcal{L}_{r}\right| \geq 1$ locations for $r=1, \ldots, n$, but only one location for $r=n+1, \ldots, N$. For each storage location in $\mathcal{V}$ lying in the overlap of multiple sets, a copy is created for each set to which it belongs. The copy locations will be used in the next section to model the constraints associated with locations in the overlap areas.

Storage and retrieval locations as well as destination sides of retrieval containers and I/O points of storage containers are known; in practice, they are determined at a higher hierarchical planning level. This level focuses on minimizing the number of reshufflings (De Castillo and Daganzo, 1993, Kim et al., 2000) and speeding up the loading and unloading process of a ship (Kim and Kim, 1999a, Vis and Roodbergen, 2009). Due to the abundance of internal transport vehicles (like AGVs and chassis) to pick up containers at the seaside and landside, retrieved containers can be delivered to any I/O point at each side. Because of the same reason, retrieval containers can be quickly moved away from all I/O points, and they can therefore be delivered to the same I/O point that a storage container has to be picked up. The other issue is that storage and retrieval locations do not coincide. This is reasonable since $N$ is usually much smaller than the block size, the block utilization is often fairly low, and container terminal operators usually separate containers based on destination and type. Finally, we assume that containers do not have explicit precedence constraints. This assumption is based on the fact that all containers are categorized into several precedence groups (for example, based on their latest departure times) and we can consider the sequencing problem group by group. In Chapter 4, we present an extension of the problem in which precedence constraints are considered.

We use figure 3.2 to explain some of the notations. This figure shows a top view of a block ( $X=7$, $Y=10$, and $Z=4$ ) with 3 storages, and 2 retrievals. For storage requests 1,2 , and 3 , the sets of open storage locations are $\mathcal{L}_{1}=\{1,2\}, \mathcal{L}_{2}=\{3,4,5,6\}$, and $\mathcal{L}_{3}=\{7,8,9\}$, respectively. Location 7 is a copy of location 4 and location 8 is a copy of location 6 . For retrieval requests 4 and $5, \mathcal{L}_{4}=\{10\}$, and $\mathcal{L}_{5}=\{11\}$, respectively.

In order to mathematically formulate the problem and calculate the total travel time of the ASC, we must first calculate the pairwise travel times between the storage and retrieval locations in the block. If the ASC travels directly from location $i,\left(x_{i}, y_{i}, z_{i}\right)$, to location $j,\left(x_{j}, y_{j}, z_{j}\right)$, the travel time of the ASC can be calculated as follows:

$$
\begin{equation*}
t_{i j}=\max \left\{\left|x_{i}-x_{j}\right|,\left|y_{i}-y_{j}\right|\right\}+\left|z_{i}\right|+\left|z_{j}\right| \tag{1}
\end{equation*}
$$

where, $\left(x_{i}, y_{i}, z_{i}\right)$ and $\left(x_{j}, y_{j}, z_{j}\right)$ are the Cartesian coordinates of locations $i$ and $j$, with $0 \leq x_{i}, x_{j} \leq T_{x}$, $0 \leq y_{i}, y_{j} \leq T_{y}$, and $0 \leq z_{i}, z_{j} \leq T_{z}$. Here, $T_{x}, T_{y}$, and $T_{z}$, are the farthest travel times of the ASC in the block in $X, Y$, and $Z$ directions, respectively. The maximum in the first term of the right-hand side


Figure 3.2. A top view of a block with storage, retrieval and I/O locations
of Equation (1) emphasizes that the ASC can move in $X$ and $Y$ directions simultaneously in Chebyshev distance. Term $\left|z_{i}\right|+\left|z_{j}\right|$ shows the time needed for the hoist to roll up or down to pick up or drop off a container. The necessary hoisting actions are also included in these terms.

In order to calculate the travel time between storage and retrieval locations, we should also consider that the ASC needs to travel through I/O points to pick up or deliver containers. For example, to move from storage location $i$ to storage location $j$, the ASC first needs to travel from storage location $i$ to an I/O point where the container that should be stacked in storage location $j$ is initially located, and then move that container to storage location $j$. Table 3.1 summarizes how to calculate pair-wise travel times. The travel time of the ASC satisfies the triangle inequality.

The problem of sequencing storage and retrieval requests, incorporating the problem of finding storage locations of storage containers, can now be stated mathematically as the following integer programming model. Let $x_{i j}$ be the binary decision variable which equals 1 if and only if location $j \in \mathcal{V} \backslash\{0\}$ is visited immediately after location $i \in \mathcal{V} \backslash\left\{0^{\prime}\right\}$.

Objective function: The objective function below minimizes the total travel time of the ASC.

$$
\begin{equation*}
\text { minimize } \quad Z=\sum_{i \in \mathcal{V} \backslash\left\{0^{\prime}\right\}} \sum_{\substack{j \in \mathcal{V} \backslash\{0\} \\ j \neq i}} t_{i j} x_{i j} . \tag{2}
\end{equation*}
$$

Table 3.1. Calculation of pairwise travel times of the travel time matrix, $T$

${ }^{1} t_{i i}=\infty$.
${ }^{2}$ Let $I / O_{r}$ be the I/O point where container of storage request $r$ that must be stacked in storage location $j \in \mathcal{L}_{r}$ is located. The Cartesian coordinates of $I / O_{r}$ are $\left(x_{r}, y_{r}, z_{r}\right)$.
${ }^{3}$ The ASC stops at storage location $i$ if the associated storage request is the last request.
${ }^{4}$ The Cartesian coordinates of $I / O_{m}$ are $\left(x_{m}, y_{m}, z_{m}\right), m=1, \ldots, M$.
${ }^{5}$ When the ASC travels from a retrieval location to another location, it obviously selects the I/O point which gives the minimum pairwise travel time to deliver the retrieval container. Note that multiple I/O points may result in the same minimum travel time. Any of these I/O points can be selected to deliver the retrieved container. Let $\mathcal{P}$ be the set of I/O points where the ASC can deliver the retrieved container with the minimum travel time.
${ }^{6}$ In this case, two I/O points must be visited: $I / O_{r}$ and an I/O point in $\mathcal{P}$.
${ }^{7}$ The ASC delivers the container from location $i$ to an I/O point and stops if the associated retrieval request is the last request.

Carrying out all requests: Constraints (3)-(4) below guarantee that each set must be entered and exited exactly once. The ASC cannot enter 0 from any other location and exit $0^{\prime}$ to any other location. In these constraints, $\overline{\mathcal{L}}_{r}=\underset{\substack{q \in \mathcal{R} \cup\left\{0,0^{\prime}\right\} \\ q \neq r}}{ } \mathcal{L}_{q}, \forall r \in \mathcal{R} \cup\left\{0,0^{\prime}\right\}$; in other words, $\overline{\mathcal{L}}_{r}$ is the set of all locations which do not specifically belong to $\mathcal{L}_{r}$.

$$
\begin{align*}
& \sum_{i \in \overline{\mathcal{L}}_{r}} \sum_{\substack{j \in \mathcal{L}_{r} \\
j \neq i}} x_{i j}=1, \quad \forall r \in \mathcal{R} \cup\left\{0^{\prime}\right\}  \tag{3}\\
& \sum_{i \in \mathcal{L}_{r}} \sum_{\substack{ \\
j \in \overline{\mathcal{L}}_{r} \\
j \neq i}} x_{i j}=1, \quad \forall r \in \mathcal{R} \cup\{0\} \tag{4}
\end{align*}
$$

Network flow conservation: Constraints (5) are the network flow conservation constraints. Constraints (5)-(6) ensure that in an optimal solution, each location can receive at most one container, and each container is stacked somewhere. In constraint (6), $\mathcal{I}$ is the set of storage locations in the overlap of multiple sets, and
$\mathcal{Q}_{i}$ is the set of all copies of storage location $i \in \mathcal{I}$. These constraints are of special importance when locations in the overlap of multiple sets are considered. Without these, multiple containers may be stacked in a single location in the overlap of these sets:

$$
\begin{align*}
& \sum_{\substack{i \in \mathcal{V} \backslash\left\{0^{\prime}\right\} \\
i \neq j}} x_{i j}=\sum_{\substack{k \in \mathcal{V} \backslash\{0\} \\
k \neq j}} x_{j k} \leq 1, \quad \forall j \in \bigcup_{r=1}^{n} \mathcal{L}_{r},  \tag{5}\\
& \sum_{i \in \mathcal{V} \backslash\left\{0^{\prime}\right\}} \sum_{\substack{j \in \mathcal{Q}_{k} \\
j \neq i}} x_{i j} \leq 1, \quad \forall k \in \mathcal{I} . \tag{6}
\end{align*}
$$

Subtour elimination constraints: Constraints (7) forces the ASC to avoid traveling subtours. In this constraint, $\mathcal{O}, 2 \leq|\mathcal{O}| \leq N-1$, is any subset of $\mathcal{R}$ (see Noon and Bean, 1991 for a complete discussion):

$$
\begin{equation*}
\sum_{r \in \mathcal{O}} \sum_{i \in \mathcal{L}_{r}} \sum_{q \notin \mathcal{O}} \sum_{j \in \mathcal{L}_{q}} x_{i j} \geq 1, \quad \forall \mathcal{O} . \tag{7}
\end{equation*}
$$

Finally, constraints (8) are the integrality constraints:

$$
\begin{equation*}
x_{i j} \in\{0,1\}, \quad \forall i, j \in \mathcal{V} \tag{8}
\end{equation*}
$$

Constraints (5) and (6) are the extra constraints specific to this problem. These constraints increase the complexity of the problem even more than the original GATSP.

### 3.2 Solution method

Decreasing the number of open locations in each set of open locations to a single location simplifies the problem to an ATSP. In Section 3.2.1, for this purpose, we first develop some properties of the problem to redefine the travel time matrix. The redefined travel time matrix allows us to choose one open location for every storage container and identify the ATSP. We show that an optimal solution of the ATSP is identical to an optimal solution of the GATSP where constraints (6) are relaxed. In other words, an optimal ATSP solution may not provide an optimal solution of the original GATSP model and sometimes results in an infeasible GATSP solution. The reason is that a single location in the overlap of multiple sets may be selected for stacking containers of more than one of the sets. However, the infeasible solution can be modified later to obtain an optimal GATSP solution. Section 3.2 .2 presents two special cases in which the optimal ATSP solution directly provides an optimal GATSP solution. Section 3.2.3 explains how to use a B\&B algorithm to modify the optimal ATSP solution to obtain an optimal GATSP solution. Section 3.2.4 introduces a heuristic algorithm to obtain a near-optimal solution for large-scale instances.

### 3.2.1 Simplifying the problem to an ATSP

In this section, we identify the ATSP, and solve it. The optimal ATSP solution provides a lower bound for the GATSP, since it may violate constraints (6). In order to find the ATSP, we develop some properties of the problem that enable us to redefine $T$ and to select a single open location to stack the associated container of every storage request.
Redefining the travel time matrix and identifying the ATSP: In order to redefine the travel time matrix, we calculate the travel times from or until the I/O points instead of location to location. The redefined travel time matrix allows us to select a single location for every storage container and generate the ATSP. We use carrying and non-carrying arcs to redefine $T$, presented in Table 3.1, as $T^{\prime}$, given in Table 3.2. Performing each storage or retrieval request requires a non-carrying and a carrying move. A carrying (non-carrying) move is a move of the ASC while it carries (does not carry) a container. Figure 3.3b shows carrying and non-carrying moves of four sample arcs presented in Figure 3.3a.

Based on Table 3.2, the main difference between $T$ and $T^{\prime}$ occurs in the calculation of travel times between storage locations and other locations. In $T$, the travel time of the ASC from the location of a request to a storage location includes the travel time to that storage location. In other words, the carrying move of the storage request (from the I/O point of the storage request until the storage location in the block) is included in that travel time. However, in $T^{\prime}$, the travel time of the ASC is only calculated until the I/O point of the storage container. Hence, the carrying move of the storage request is excluded from the travel time, but will be included in the travel time of the ASC from the storage location to another location in the block. Note that the same carrying and non-carrying moves are used to calculate the travel times in $T$ and $T^{\prime}$, but they may be used in the calculation of travel times of different arcs.

We can now use $T^{\prime}$ to pre-select a single location $i \in \mathcal{L}_{r}$, resulting in the minimum travel time, for every storage request $r, r=1, \ldots, n$, and identify the ATSP. In general, the ASC can carry out four types of movements between requests: storage request $\rightarrow$ retrieval request, storage request $\rightarrow$ storage request, retrieval request $\rightarrow$ storage request, and retrieval request $\rightarrow$ retrieval request. Based on the calculations in Table 3.2, moving from retrieval request $r$ to retrieval request $q$ results in a unique travel time, because $\mathcal{L}_{r}$ and $\mathcal{L}_{q}$, each contain only a single location and no selection is necessary. Moving from retrieval request $r$ to storage request $q$ also results in a unique travel time which includes the travel to the I/O point where the container of request $q$ is located. Now, consider that the ASC moves from storage request $r$ to retrieval request $q$. In $T^{\prime}$, several travel times exist which are calculated by traveling from the I/O point of storage request $r$ to storage location $i \in \mathcal{L}_{r}$ and then to the retrieval location. In order to generate the ATSP, we select a location that gives the minimum travel time because it provides a lower bound. Similarly, when the ASC travels from storage request $r$ to storage request $q$, a location that gives the minimum travel time can be chosen to locate the container of storage request $r$. Note that this pairwise travel time is calculated until the I/O point of storage request $q$. So, our selection does not affect the travel times from storage request $q$ to other requests.

Table 3.2. Calculation of pairwise travel times of the redefined travel time matrix, $T^{\prime}$

${ }^{1} I / O_{r}$ is the I/O point where container of storage request $r$ that must be stacked in storage location $j \in \mathcal{L}_{r}$ is located, whereas $I / O_{q}$ is the I/O point where container of storage request $q$ that must be stacked in storage location $i \in \mathcal{L}_{q}$ is located.

The four sample arcs and their carrying and non-carrying moves shown in Figures 3.3a and 3.3b can be used as an example of how pre-selection can be performed. In order to move from request 4 to request 1 (retrieval request $\rightarrow$ storage request), based on the calculations in Table 3.2, $t_{10,1}=t_{10,2}=t_{10, I / O_{2}}$. As a result, stacking the container of storage request 1 in location 1 or 2 does not have any effect on the travel time between requests 1 and 4, and either open location can be selected to stack the container. On the other hand, moving from request 1 to request 4 (storage location $\rightarrow$ retrieval location) results in two different travel times: $t_{1,10}=t_{I / O_{2}, 1}+t_{1,10}$, and $t_{2,10}=t_{I / O_{2}, 2}+t_{2,10}$. In this case, the location that gives the minimum travel time is selected to stack the container without any effect on the travel times to request 1 or from request 4.
Obtaining an optimal ATSP solution: The ATSP model obtained above can be solved by the two-phase solution method proposed in Chapter 2. In the first phase, the subtour elimination constraint of the ATSP model is removed to form an AP model. The AP model can be efficiently solved in $O\left(N^{3}\right)$ steps where $N$ is the number of requests (Kuhn, 1955). The AP solution often contains subtours since the subtour elimination constraint is not satisfied. To obtain an optimal ATSP solution, an algorithm is developed to merge all subtours by exchanging arcs that have a common I/O point. In the second phase, the authors use a B\&B algorithm to merge all the remaining subtours to obtain an optimal solution to the ATSP. The next theorem states that the ATSP solution is a good starting point for the GATSP solution.

Theorem 3.1 An optimal solution of the ATSP generated by preselecting a single location from each set of open locations provides an optimal GATSP solution to our problem where constraints (6) are relaxed.

Proof. See the proof in Appendix 3.A.


Note. Storage container 1 is at $I / O_{2}$ and retrieval container 4 has a seaside destination.
Figure 3.3. (a) A top view of a block of containers with four exemplary arcs, (b) The carrying and non-carrying moves, (c) An infeasible solution of the GATSP.

Corollary 3.1 An optimal solution of the ATSP generated by preselecting a single location from each set of open locations provides a lower bound for the GATSP.

The proof is intuitive and follows from the fact that an ATSP solution is not constrained by constraints (6). Violation of constraints (6) results in an infeasible GATSP solution due to "multi-selected" locations:

Definition 1 A multi-selected location is an open storage location contained in the overlap of multiple sets of open storage locations, which is selected in an infeasible GATSP solution to stack multiple containers.

Figure 3.3c presents an optimal ATSP solution, which is infeasible for the GATSP. In this solution, container 2 is stacked in location 4 and container 3 is stacked in location 7 which is a copy of location 4 . This location gives the minimum travel time to travel from storage requests 3 and 2 to retrieval requests 5 and 4, respectively. Therefore, location 4 is the multi-selected location.

In the next section, we will discuss two special cases in which the optimal ATSP solution provides a feasible GATSP solution. Since an optimal ATSP solution provides a lower bound for the GATSP, it gives an optimal GATSP solution if it is feasible for the GATSP. If it is not feasible, we remove the infeasibility through a B\&B algorithm (Section 3.2.3).

### 3.2.2 Optimal GATSP solution in two special cases

In the following two cases, the optimal ATSP solution provides an optimal GATSP solution.
Case 1: no overlap among every two sets of open locations: Assume sets of open locations do not overlap. In this case, the optimal ATSP solution cannot contain multi-selected locations, since a multiselected location only appears in the overlap of two or more sets. The following theorem states that in this case an optimal ATSP solution directly provides an optimal GATSP solution:

Theorem 3.2 If sets of open locations of storage requests do not overlap, an optimal ATSP solution is an optimal GATSP solution.

The proof is intuitive. When sets of open locations do not overlap, no multi-selected location can appear in the solution. In other words, the optimal ATSP solution satisfies constraints (6), and is thus a feasible GATSP solution. According to Corollary (3.1), the optimal ATSP solution is an optimal GATSP solution.
Case 2: multi-selected locations can all be replaced by alternative open locations: If sets of open locations overlap, an optimal ATSP solution may consist of multi-selected locations. In this case, we may still be able to fix the optimal ATSP solution to obtain an optimal GATSP solution by replacing all multi-selected locations with other open locations. Because of simultaneous movements of the ASC over the bays and rows of the block as well as properties of Chebyshev distance, the travel time of the ASC in the horizontal direction calculated in equation (1) is the maximum travel time in the $X$ and $Y$ directions. As a result, when the ASC moves from a storage request to another request, multiple open locations may result in the minimum travel time. We call these locations "alternative open" locations and define them as follows:

Definition 2 Alternative open locations are two or more open locations within a set of open locations which result in the minimum ASC travel time when used for stacking the storage container followed by traveling to a next location.

In order to use alternative open locations and remove multi-selected locations, we develop an AP model. The AP assures that every storage container will be located in a different alternative open location. In this way, we avoid generating new multi-selected locations while we remove the other ones. The cost matrix of the AP model is established by using the travel times of the alternative open locations. If a location is an alternative open location for a storage request, then the cost equals the travel time, otherwise infinity. Note that the cost matrix includes all storage requests with sets of open locations that overlap with other sets. These storage requests are either already involved in a multi-selected locations or may result in a multiselected location if not included in the AP model. Obviously, storage requests with sets of open locations which do not overlap with other sets do not cause multi-selected locations. Therefore, they do not need to be included in the model.

If the AP has an optimal solution less than infinity, a separate alternative open location can be found for each storage container. Therefore, we can remove all multi-selected locations and obtain an optimal GATSP solution. Otherwise, the number of alternative open locations is smaller than the number of storage requests,
and thus an optimal solution of the GATSP cannot be obtained using the optimal ATSP solution. We then use the $\mathrm{B} \& \mathrm{~B}$ algorithm discussed in Section 3.2.3 to remove the multi-selected locations.

### 3.2.3 Optimal GATSP solution in the other cases

The input of this section is the optimal ATSP solution which is not a feasible GATSP solution even after using the alternative open locations algorithm. We develop a B\&B algorithm to remove multi-selected locations and to find a complete tour. The B\&B algorithm forbids a single multi-selected location to appear in the solution, one at a time. This is carried out by keeping and removing locations from sets of open locations in different nodes. Next, we first explain the main steps of the $B \& B$ algorithm, and then present the algorithm.
Branching: We design a branching rule to remove one multi-selected location at a time. The branches divide the solution space into complementary solution subspaces such that the multi-selected location cannot appear in any of them. The branching works by keeping the multi-selected location, selected to stack containers of several sets in the previous solution, in a single set at a time and removing that location from the other ones. We select the multi-selected location that is in the overlap of more numbers of sets to perform the branching.

Suppose that in a node, the multi-selected location $i$ is in the overlap of $U$ sets of which some have caused the multi-selected location. The branching rule produces $U$ child nodes. In each child node $u \in\{1, \ldots, U\}$, location $i$ appears in only one set and all its copies belonging to $\mathcal{Q}_{i}$ are removed from all the other sets. Sets of open locations that do not contain this specific location remain the same as the parent node.
Bounding and selection of a live node: In section 3.2.2, we solved an ATSP which provides a lower bound for the GATSP and serves as the input for the B\&B algorithm in node 0 , if it contains multi-selected locations. In all the other nodes of the $\mathrm{B} \& \mathrm{~B}$ tree, the travel time matrix of the parent node must be modified first considering the changes in sets of open locations forced by the branching rule, and then an AP is solved. If an optimal AP objective value is more than the incumbent objective value, the node is fathomed. Otherwise, alternative open locations and the two-phase solution method proposed in Chapter 2 are consecutively used to remove multi-selected locations and subtours. This method results in a complete tour but may increase the objective function. Using alternative open locations does not increase the objective function, but multi-selected locations may still exist in the solution (see Section 3.2.2). As a result, we need to check for the objective value and multi-selected locations. If the objective value is more than the incumbent objective value, the node is fathomed; if not, presence of multi-selected locations must be checked. If no multi-selected location exists, a new incumbent solution is obtained, and the node must be fathomed. If at least one multi-selected location exists, the node is added to the set of live nodes, $E$.

Selecting a live node, $e$, from $E$ is carried out by using the depth-first-search (DFS) as the overall strategy and the best-first-search (BeFS) when choice is to be made between nodes at the same level of the tree. We mention this strategy as DFS-BeFS. Clausen (2003) mentions that in the DFS strategy, we select the live node with the highest level. The level of a specific node in the $\mathrm{B} \& \mathrm{~B}$ tree is defined as the number of nodes between that node and the root node. In the BeFS strategy, the next node to be evaluated is the one with
the lowest bound among all the live nodes.

### 3.2.4 Near-optimal heuristic for large-scale problems

The three-phase solution method can find an optimal GATSP solution. However, the computation time can be long if the number of requests is large $(N \geq 100)$. Therefore, instead of solving the problem to optimality, we use the following heuristic algorithm to quickly find a near-optimal solution. This solution can also serve as an upper bound for the $\mathrm{B} \& \mathrm{~B}$ algorithm.

The heuristic algorithm uses the optimal ATSP solution obtained in Section 3.2.1 and tries to find a feasible GATSP solution by stacking containers in locations other than the multi-selected locations. Note that in Section 3.2.2, we have already tried to find alternative open locations and to remove multi-selected ones. In this algorithm, locations that are chosen are not necessarily alternative open locations, and may increase the total travel time.

The heuristic algorithm performs very much like the alternative open locations algorithm presented in Section 3.2.2. It tries to remove multi-selected locations using an AP model. However, in order to establish the cost matrix of the AP model, instead of the travel times of the alternative open locations, all open locations of the sets are included. An AP solution results in removing multi-selected locations and obtaining a feasible GATSP solution with the minimum increase in the total travel time of the ATSP solution. Note that we always have enough locations to stack all containers; otherwise, the GATSP model does not have a feasible solution. The gap between the new feasible GATSP solution and the optimal ATSP solution which is a lower bound to the problem provides a good indicator to evaluate the performance of the heuristic.

### 3.3 Computational experiments

We conducted multiple numerical experiments to evaluate the performance of the three-phase solution method and the heuristic algorithm. In each scenario, we consider a single block of containers with 40 or 50 bays, 10 or 20 rows, four tiers, and two, 10 , or 20 I/O points. Other inputs regarding the size of containers and the speed of the ASC can be found in Table 2.2 in Chapter 2. The general data is provided by one of the largest container terminals in the Port of Rotterdam. The number of storage and retrieval requests are $N=20,50,100,150$ and 200. Their locations and corresponding I/O points are randomly generated by Monte Carlo simulation. Storage requests have multiple open locations, but the number of potential open locations per storage request in each scenario is equal and is set to one, four, seven, or 10. All locations are uniformly distributed over the block, and an equal number of requests have to be picked up or delivered to the landside and the seaside. The ASC starts its operation from the location of $I / O_{1}$, and it either ends at the storage location of a storage request if it is the last request in the sequence, or at an I/O point if a retrieval request is the last request in the sequence.

The study is performed on a Notebook with 2.40 GHz Intel ${ }^{\circledR}$ Core ${ }^{\mathrm{TM}}$ is processor, with 4.00 GB of RAM and the programming language is MATLAB ${ }^{\circledR}$ 2010a.

### 3.3.1 Evaluating the performance of the three-phase solution method

In this section, we evaluate the performance of the three-phase solution method. Table 3.3 shows the results of the three-phase solution method. The results are categorized into small-scale ( 20 requests), medium-scale ( 50 requests), and large-scale ( 100 requests) problems. For small-scale and medium-scale problems, 100 realizations are randomly generated for each scenario. Due to the long computation time for large-scale problems, we randomly generate 20 realizations for each scenario and terminate the computation after 500 seconds. As explained in chapter 2 , the number of realizations satisfies $N_{\text {realz. }} \geq \sigma^{2}\left(\frac{(1+\varepsilon) \mathbb{Z}_{1-\alpha / 2}}{\varepsilon \mu}\right)^{2}$ with a $90 \%$ confidence level ( $\alpha=10 \%$ ), where $\sigma^{2}$ and $\mu$ are respectively the variance and mean of the objective values, $\mathbb{Z}_{1-\alpha / 2}$ is the $1-\alpha / 2$ percentile of the normal distribution, and $\varepsilon=5 \%$ is for determining the confidence interval (Law and Kelton, 1999).

The results in Table 3.3 show that the three-phase solution method can quickly obtain the optimal solution to small-scale $(N \leq 20)$ and medium-scale $(N \leq 50)$ problems. Average computation time is less than 10.45 seconds. Still, by increasing the number of requests, the computation time becomes longer. The solution to large-scale problems with more than 100 requests cannot be found in a reasonable time. If sets of open locations have a single member $\left(\left|\mathcal{L}_{r}\right|=1, \forall r \in R\right)$, we have an ATSP instead of a GATSP. Therefore, the problem can be solved in less than a second by obtaining an optimal ATSP solution. The first four scenarios of each problem size in Table 3.3. In addiction, column 11 shows that the minimum number of nodes of the $B \& B$ tree in some realizations is 1 . These are the special cases where sets of open locations do not overlap (case 1 in Section 3.2.2), or alternative open locations can be found (case 2 in Section 3.2.2). The solution can be obtained rapidly because an optimal ATSP solution directly provides an optimal GATSP solution.

Table 3.4 is used to evaluate how the locations in the set intersections affect the complexity of the problem and the total travel time of the ASC. We consider an instance with 20 requests in which the number of storage and retrieval requests is in balance. The number of open locations is also set to 20 . Let $\mathbb{I}$ be the percentage of locations that are in the intersection of all sets. The other locations do not belong to any intersection and are randomly dispatched among all sets. Thus, $\left|L_{r}\right| \geq 20 \times \mathbb{I}, r=\{1, \ldots, n\}$. The results show that by increasing the number of locations in the set intersections, the computation time significantly grow. On the other hand, by increasing $\mathbb{I}$, the ASC has more flexibility to stack every container which can be used to decrease the total travel time.

We also compare the results of 17 randomly selected instances obtained by the three-phase solution method with those results obtained by CPLEX 12.2. Due to the long computation time of CPLEX to find an optimal solution of the problem, we truncated the computation after 18000 seconds. The results presented in Table 3.5 show a large gap between the feasible objective value obtained by CPLEX in 18000 seconds and the optimal objective value (obtained in less than a second). By increasing the number of requests, the gap

Table 3.3. The results of the three-phase solution method

| X | $Y$ | $N$ | $\left\|\mathcal{L}_{r}\right\|^{1} \quad M$ | M | CPU |  |  | Number of nodes |  |  | $Z^{*}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  | Ave | Max | Min | Ave | Max | Min |  |
| 10 | 40 | 20 | 1 | 2 | 0.01 | 0.02 | 0.01 | 1 | 1 | 1 | 1303.44 |
|  |  |  | 1 | 10 | 0.01 | 0.08 | 0.01 | 1 | 1 | 1 | 1285.89 |
|  |  |  | 1 | 20 | 0.01 | 0.08 | 0.01 | 1 | 1 | 1 | 1255.18 |
|  |  |  | 4 | 2 | 0.01 | 0.08 | 0.01 | 1.3 | 7 | 1 | 1248.87 |
|  |  |  | 4 | 10 | 0.02 | 0.14 | 0.01 | 1.44 | 7 | 1 | 1214.37 |
|  |  |  | 4 | 20 | 0.02 | 0.10 | 0.01 | 1.29 | 8 | 1 | 1237.48 |
|  |  |  | 7 | 2 | 0.02 | 0.16 | 0.01 | 1.42 | 9 | 1 | 1207.81 |
|  |  |  | 7 | 10 | 0.03 | 0.13 | 0.01 | 1.45 | 6 | 1 | 1180.38 |
|  |  |  | 7 | 20 | 0.03 | 0.10 | 0.01 | 1.35 | 4 | 1 | 1186.74 |
|  |  |  | 10 | 2 | 0.03 | 0.13 | 0.01 | 1.45 | 6 | 1 | 1180.38 |
|  |  |  | 10 | 10 | 0.03 | 0.21 | 0.01 | 1.52 | 7 | 1 | 1174.13 |
|  |  |  | 10 | 20 | 0.04 | 0.32 | 0.01 | 1.61 | 8 | 1 | 1156.86 |
| 10 | 40 | 50 | 1 | ${ }^{2}-$ | 0.03 | 0.06 | 0.02 | I | 1 | I | 3231.64 |
|  |  |  | 1 | 10 | 0.04 | 0.12 | 0.02 | 1 | 1 | 1 | 3126.32 |
|  |  |  | 1 | 20 | 0.04 | 0.63 | 0.02 | 1 | 1 | 1 | 3179.74 |
|  |  |  | 4 | 2 | 0.30 | 2.26 | 0.03 | 4.79 | 41 | 1 | 3024.50 |
|  |  |  | 4 | 10 | 0.35 | 3.42 | 0.03 | 5.56 | 37 | 1 | 2920.33 |
|  |  |  | 4 | 20 | 0.41 | 10.28 | 0.03 | 6.01 | 139 | 1 | 2971.37 |
|  |  |  | 7 | 2 | 1.84 | 64.96 | 0.03 | 8.46 | 195 | 1 | 2978.87 |
|  |  |  | 7 | 10 | 2.42 | 87.44 | 0.03 | 8.62 | 239 | 1 | 2948.98 |
|  |  |  | 7 | 20 | 2.97 | 117.80 | 0.03 | 10.8 | 334 | 1 | 2930.63 |
|  |  |  | 10 | 2 | 10.45 | 198.26 | 0.03 | 20.9 | 293 | 1 | 2920.68 |
|  |  |  | 10 | 10 | 3.10 | 48.34 | 0.03 | 6.58 | 50 | 1 | 2875.11 |
|  |  |  | 10 | 20 | 5.54 | 55.94 | 0.04 | 9.37 | 79 | 1 | 2859.76 |
| 10 | 40 | 100 | 1 | $2^{-}$ | - 0.13 | 0.18 | 0.09 | I | $1^{-}$ | I | 6526.89 |
|  |  |  | 1 | 10 | 0.12 | 0.17 | 0.08 | 1 | 1 | 1 | 6313.35 |
|  |  |  | 1 | 20 | 0.15 | 0.30 | 0.09 | 1 | 1 | 1 | 6266.78 |
|  |  |  | 4 | 2 | $>108.60$ | 500.00 | 0.11 | $>133$ | 803 | 1 | 6271.07 |
|  |  |  | 4 | 10 | $>97.59$ | 500.00 | 0.11 | $>127$ | 740 | 1 | 5736.32 |
|  |  |  | 4 | 20 | >209.69 | 500.00 | 0.12 | $>243$ | 621 | 1 | 6114.36 |
|  |  |  | 7 | 2 | $>186.41$ | 500.00 | 0.12 | $>96.4$ | 307 | 1 | 5971.92 |
|  |  |  | 7 | 10 | >169.14 | 500.00 | 0.11 | $>82.4$ | 320 | 1 | 5790.94 |
|  |  |  | 7 | 20 | $>126.58$ | 500.00 | 0.13 | >71 | 228 | 1 | 6007.79 |
|  |  |  | 10 | 2 | $>157.83$ | 500.00 | 0.13 | $>62.5$ | 204 | 1 | 6088.78 |
|  |  |  | 10 | 10 | $>241.55$ | 500.00 | 0.19 | $>87.6$ | 330 | 1 | 5522.12 |
|  |  |  | 10 | 20 | $>223.97$ | 500.00 | 0.19 | $>73.7$ | 204 | 1 | 6120.98 |

NOTE. We denote by Ave, Max, and Min the average, maximum and minimum computation (CPU) time and number of nodes
of the solution method over 100 realizations. Computation time is calculated in seconds. Furthermore, let $\left|\mathcal{L}_{r}\right|$ be the size of each set of open locations, $r=1,2, \ldots, n$. Column $Z^{*}$ shows the average optimal value for the instances with 20 and 50 requests and it shows the average feasible value for the instances with 100 requests.

Table 3.4. The effect of the set intersections on the performance of the algorithm

| X | $Y$ | $N$ | M | II | CPU |  |  | Number of nodes |  |  | $Z^{*}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  | Ave | Max | Min | Ave | Max | Min |  |
| 10 | 40 | 20 | 10 | 0 | 0.02 | 0.25 | 0.01 | 1 | 1 | 1 | 1045.15 |
|  |  |  |  | 25 | 4.89 | 114.70 | 0.01 | 758 | 16563 | 1 | 980.70 |
|  |  |  |  | 50 | 6.85 | 272.05 | 0.01 | 691 | 24833 | 1 | 945.9345 |
|  |  |  |  | 75 | 26.38 | 500.00 | 0.07 | 1969 | 33474 | 4 | 944.7448 |
|  |  |  |  | 100 | 151.65 | 500.00 | 0.20 | 6762 | 24413 | 5 | 924.63 |

Notes. Column CPU is the computation time in seconds. The computation is truncated after 500 seconds. Each instance has the same number of retrieval and storage requests. For the last two scenarios, colume $Z$ shows the Each instance has the same number of retrieval and storage requests.
between the two values increases. Note that in the first and second instances where the computation time of CPLEX is less than 18000 seconds the optimal value is obtained. We also try to improve the performance of CPLEX by using set the solution of our heuristic algorithm and the CPLEX feasible solution obtained
in the first 18000 seconds as start points for solving the model. The results show that CPLEX is not still efficient. Furthermore, although our heuristic algorithm provides the optimal solution in some cases, it takes a considerable amount of time to prove the optimality.

Table 3.5. Comparing the three-phase solution method and truncated CPLEX

| Inst. | X | $Y$ | $N$ | M | $\left\|\mathcal{L}_{r}\right\|$ | CPLEX |  | Our method |  | $G_{Z}^{Z^{*}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 |  |  |  |  |  | 4036.14 | 617.97 | 0.00 | 617.97 | 0 |
| 2 |  |  |  |  | 1 | 3680.59 | 455.60 | 0.00 | 455.6 | 0 |
| 3 | 10 | 40 | 10 | 10 | 4 | 18000 | 556.24 | 0.02 | 555.47 | 0.14 |
| 4 |  |  |  |  |  | 18000 | 517.57 | 0.03 | 517.57 | 0 |
| 5 |  |  |  |  | 7 | 18000 18000 | 582.10 610.77 | 0.01 0.01 | 576.43 603.63 | 0.97 |
| 7 |  |  |  |  |  | 18000 | -1233.75 | 0.02 | 1169.92 | 5.17 |
| 8 |  |  |  |  | 1 | 18000 | 981.83 | 0.01 | 981.83 | 0 |
| 9 | 10 | 40 | 20 | 10 | 4 | 18000 | 1048.88 | 0.01 | 1012.42 | 3.48 |
| 10 | 10 | 40 | 20 | 10 | 4 | 18000 | 1098.79 | 0.04 | 991.58 | 9.76 |
| 11 |  |  |  |  | 7 | 18000 | 1251.04 | 0.02 | 1057.12 | 15.50 |
| 12 |  |  |  |  | 7 | 18000 | 1263.27 | 0.02 | 1097.54 | 13.12 |
| 13 |  |  |  |  |  | 18000 | 3688.04 | 0.04 | 2669.07 | 27.63 |
| 14 |  |  |  |  | 1 | 18000 | 3004.97 | 0.05 | 2593.2 | 13.70 |
| 15 | 10 | 40 | 50 | 10 |  | 18000 | 2611.34 | 0.14 | 2459.02 | 5.83 |
| 16 | 10 | 40 | 50 | 10 | 4 | 18000 | 3426.06 | 0.09 | 2472.02 | 27.85 |
| 17 |  |  |  |  | 7 | 18000 | 2840.63 | 0.06 | 2496.02 | 12.13 |
| 18 |  |  |  |  | 7 | 18000 | 2831.90 | 0.19 | 2430.94 | 14.16 |

Notes. Let $Z$ and $Z^{*}$ be the objective value obtained by truncated CPLEX and the optimal objective value obtained by the three-phase solution method, respectively. The gap is calculated as: $G_{Z}^{Z}=\left(\left(Z-Z^{*}\right) / Z\right) \times 100$ Column CPU is the computation time in seconds. Each instance has the same number of retrieval and storage requests.

### 3.3.2 Evaluating the performance of the heuristic algorithm

We use our heuristic algorithm to quickly find the solution to large-scale problems. Table 3.6 shows the results. We randomly generated 100 realizations for each scenario. Furthermore, we considered a larger block with 50 bays, 20 rows, and four tiers to be able to randomly generate separate locations for large-scale problems. The average objective value provided by the heuristic algorithm is compared with the lower bound which is obtained by optimally solving the ATSP based on Corollary 3.1. The results in Table 3.6 show that the gap between the lower bound and the heuristic algorithm is less than $1 \%$, on average. The results show that the computation time of our heuristic becomes longer by increasing the number of requests and number of open locations in each set of open locations. The computation time of our heuristic is mainly determined by selecting a location which gives the minimum travel time and simplifying the problem to an ATSP. The scenarios with a single location for each storage request show that the optimal ATSP solution can be obtained in less than a second. Using the AP model to remove multi-selected locations can also be performed in less than a second.

Finally, we compare our heuristic with the NN and FCFS ASC-scheduling heuristics, commonly used at container terminals, as provided by commercial software companies (for example, Cosmos NV and Modality Software solutions b.v.). The results are presented in Table 3.7. Apparently, when the number of storage and

Table 3.6. Performance of our heuristic algorithm for largescale problems

| X | $Y$ | $N$ | $\left\|\mathcal{L}_{r}\right\|$ | M | $Z^{\text {LB }}$ | $Z^{\text {he }}$ | $G_{h e}^{L B}$ | CPU |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 20 | 50 | 100 | 1 | 2 | 8067.88 | 8067.88 | 0.00 | 0.12 |
|  |  |  | 1 | 10 | 7126.70 | 7126.70 | 0.00 | 0.12 |
|  |  |  | 1 | 20 | 7357.73 | 7357.73 | 0.00 | 0.13 |
|  |  |  | 4 | 2 | 7521.19 | 7528.91 | 0.10 | 0.46 |
|  |  |  | 4 | 10 | 6969.84 | 6978.14 | 0.12 | 0.52 |
|  |  |  | 4 | 20 | 6867.85 | 6876.85 | 0.13 | 0.47 |
|  |  |  | 7 | 2 | 7544.59 | 7552.01 | 0.10 | 1.16 |
|  |  |  | 7 | 10 | 6761.50 | 6767.23 | 0.08 | 0.96 |
|  |  |  | 7 | 20 | 6934.53 | 6941.27 | 0.10 | 1.07 |
|  |  |  | 10 | 2 | 7506.82 | 7513.7 | 0.09 | 2.17 |
|  |  |  | 10 | 10 | 6833.78 | 6841.45 | 0.11 | 2.49 |
|  |  |  | 10 | 20 | 6836.95 | 6842.83 | 0.09 | 2.13 |
| 20 | 50 | 150 | 1 | 2 | -11944.07 | 11944.07 | -0.00 | 0.28 |
|  |  |  | 1 | 10 | 10916.10 | 10916.10 | 0.00 | 0.28 |
|  |  |  | 1 | 20 | 10731.47 | 10731.47 | 0.00 | 0.27 |
|  |  |  | 4 | 2 | 11379.60 | 11397.40 | 0.16 | 1.80 |
|  |  |  | 4 | 10 | 10409.75 | 10430.39 | 0.20 | 1.87 |
|  |  |  | 4 | 20 | 10557.51 | 10576.15 | 0.18 | 1.68 |
|  |  |  | 7 | 2 | 11225.87 | 11242.31 | 0.15 | 3.38 |
|  |  |  | 7 | 10 | 10167.16 | 10184.39 | 0.17 | 4.52 |
|  |  |  | 7 | 20 | 10423.00 | 10439.80 | 0.16 | 4.26 |
|  |  |  | 10 | 2 | 11152.43 | 11168.22 | 0.14 | 6.94 |
|  |  |  | 10 | 10 | 10439.52 | 10454.83 | 0.15 | 8.09 |
|  |  |  | 10 | 20 | 10219.01 | 10238.87 | 0.19 | 7.95 |
| 20 | 50 | 200 | 1 | 2 | 15995.47 | 15995.47 | -0.00 | 0.48 |
|  |  |  | 1 | 10 | 14347.18 | 14347.18 | 0.00 | 0.46 |
|  |  |  | 1 | 20 | 14191.38 | 14191.38 | 0.00 | 0.46 |
|  |  |  | 4 | 2 | 15227.22 | 15255.70 | 0.19 | 4.08 |
|  |  |  | 4 | 10 | 13707.66 | 13742.79 | 0.26 | 4.83 |
|  |  |  | 4 | 20 | 13950.1 | 13984.4 | 0.25 | 4.79 |
|  |  |  | 7 | 2 | 14804.83 | 14835.89 | 0.21 | 10.32 |
|  |  |  | 7 | 10 | 13626.49 | 13657.35 | 0.23 | 11.11 |
|  |  |  | 7 | 20 | 13612.46 | 13644.67 | 0.24 | 11.60 |
|  |  |  | 10 | 2 | 15059.98 | 15094.22 | 0.23 | 20.10 |
|  |  |  | 10 | 10 | 13671.52 | 13705.30 | 0.25 | 23.04 |
|  |  |  | 10 | 20 | 13375.98 | 13407.71 | 0.24 | 21.56 |

retrieval requests are balanced $(\mathbb{P}=50 \%)$, the average objective value gap is up to $37 \%$ for the FCFS heuristic, and $20 \%$ for the NN heuristic. On the other hand, when the number of storage and retrieval requests are not balanced (moving toward $\mathbb{P}=0$ or $100 \%$ ), the gaps with both FCFS and NN heuristics become smaller. The reason is that in the solution, more pure storage and retrieval requests must be individually executed by the ASC, and there is less opportunity to sequence retrieval requests after storage requests for minimizing the ASC's empty travel time.

The large performance gaps between our heuristic and NN can be understood by studying the extreme cases $\mathbb{P}=100 \%$ or $0 \%$, with relatively small gaps. When $\mathbb{P}=100 \%$, all requests are retrieval requests. In this case, we solve an ATSP and do not have any multi-selected location. The solution is relatively similar to the solution obtained by the NN heuristic. In fact, in case of a single I/O point, the solutions of our heuristic and NN are identical. However, when $\mathbb{P}=0 \%$, all requests are storage requests. In this case, due to the pre-selection of empty locations, the ATSP provides a good lower bound for the problem. As a result, the
heuristic solution obtained by using the AP model for replacing the multi-selected locations with the nearest empty locations is near optimal and far better than the NN solution.

Table 3.7. Comparing our heuristic with the FCFS and NN heuristics

| $X$ | $Y$ | $N$ | $\left\|\mathcal{L}_{r}\right\|$ | $M$ | $\mathbb{P}$ | $Z^{h e}$ | $Z^{N N}$ | $Z^{F C F S}$ | $G_{N N}^{h e}$ | $G_{F C F S}^{h e}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 20 | 50 | 100 | $\mathbf{1}$ | 10 | 50 | 6371.02 | 7548.02 | 9915.28 | 15.59 | 35.75 |
| 20 | 50 | 100 | $\mathbf{4}$ | 10 | 50 | 6145.97 | 7519.09 | 9621.08 | 18.26 | 36.12 |
| 20 | 50 | 100 | $\mathbf{7}$ | 10 | 50 | 5980.09 | 7420.03 | 9463.83 | 19.41 | 36.81 |
| 20 | 50 | 100 | $\mathbf{1 0}$ | 10 | 50 | 6027.92 | 7526.66 | 9493.05 | 19.91 | 36.50 |
| 20 | 50 | 100 | 7 | 10 | $\mathbf{0}$ | 7673.75 | 8432.70 | 9783.38 | 9.00 | 21.56 |
| 20 | 50 | 100 | 7 | 10 | $\mathbf{2 0}$ | 6769.90 | 7979.79 | 9541.50 | 15.16 | 29.05 |
| 20 | 50 | 100 | 7 | 10 | $\mathbf{4 0}$ | 6050.70 | 7489.51 | 9528.86 | 19.21 | 36.50 |
| 20 | 50 | 100 | 7 | 10 | $\mathbf{6 0}$ | 6335.97 | 7603.10 | 9498.38 | 16.67 | 33.29 |
| 20 | 50 | 100 | 7 | 10 | $\mathbf{8 0}$ | 7322.12 | 8059.38 | 9744.66 | 9.15 | 24.86 |
| 20 | 50 | 100 | 7 | 10 | $\mathbf{1 0 0}$ | 8452.75 | 8471.89 | 10104.69 | 0.23 | 16.35 |
| Notes. Let $Z^{N N}$ and $Z^{F C F S}$ be the average objective values of the NN and FCFS heuristics. The objective values |  |  |  |  |  |  |  |  |  |  |
| are averaged over 100 |  |  |  |  |  |  |  |  |  |  |
| realizations. Furthermore, $\mathbb{P}$ indicates the percentage of retrievals. The gaps are calculated |  |  |  |  |  |  |  |  |  |  |
| as: $G_{a}^{b}(\%)=((a-b) / a) \times 100$. |  |  |  |  |  |  |  |  |  |  |

### 3.4 Conclusion

This chapter studies an operational problem of sequencing container storage and retrieval requests in a single block of containers with multiple open locations for each storage container. The objective is to minimize the travel time of a ASC carrying out the requests. We show that the problem can be modeled as a GATSP which is $\mathcal{N} \mathcal{P}$-hard. The model also includes some extra constraints regarding the single selection of locations in the overlap of multiple sets. We use the I/O points to redefine the travel time matrix and simplify the problem to an ATSP. If sets of open locations for storage containers do not overlap, an optimal ATSP solution results in an optimal solution of the GATSP. However, if they overlap, an optimal ATSP solution may contain multiselected locations which is an infeasible GATSP solution. In this case, if multi-selected locations cannot be removed using alternative open locations, the optimal ATSP solution should be embedded in a B\&B algorithm to find an optimal GATSP solution. The numerical experiments show that if sets of open locations do not overlap or alternative open locations for multi-selected locations exist, an optimal solution can be obtained in less than a second. In general, the optimal solution of small and medium-scale instances can be quickly obtained. In addition, we can obtain significantly better results than CPLEX truncated after five hours. For large-scale instances with sets of open locations that overlap, we develop a heuristic to quickly find a near-optimal solution. The results show that the gap between the solution of our heuristic and the lower bound of the problem is less than $1 \%$, on average. Performance gaps with heuristics applied in practice, such as NN and FCFS, are up to $20 \%$ and $37 \%$, respectively. In real container terminals, the benefits might be smaller though, due to terminal-specific constraints. The methods proposed in this chapter can be applied to any GATSP problem with a similar structure to obtain high quality solutions. The Model can also be extended to include scheduling priorities, online requests, container rehandling, and synchronization with

## Appendix

## 3.A Proof of Theorem 3.1

Suppose that in an optimal GATSP solution, an open location is chosen that does not give the minimum time in order to travel from the associated storage request to a next following request. Then, the total travel time between these two requests can be reduced by replacing this open location with another open location which gives a shorter travel time. However, replacing all such travel times in the optimal GATSP solution with minimum travel times results in an optimal solution to the ATSP model in which open locations that result in the minimum travel time are pre-selected. Of course, this may not be a feasible GATSP solution, since constraints (6) may be violated and a location in the overlap of multiple sets may be chosen to locate multiple containers.

## Chapter 4

## Scheduling Two Non-Passing Yard

## Cranes*

In this chapter, we study an extension of chapter 2 and 3 where we minimize the makespan of the two non-passing automated stacking cranes (ASCs) carrying out a set of storage and retrieval requests in a block of containers. In addition to determining the order in which to carry out the requests, we determine a location for each storage request under the following operational constraints. When a container is to be retrieved to the seaside, the seaside ASC picks it up from its location in the block and drops it at the seaside input/output (I/O) point. When a storage request is to be stacked from the seaside, the seaside ASC picks it up and stores it in a location in the block, selected from a set of available open locations specified for that request. Each open storage location can be occupied by at most one container. Landside operations can be carried out similarly. The ASCs can carry out single cycle or double cycle movements to carry out requests. In a single cycle movement, the ASC carries out a single storage or retrieval request and returns to the I/O point to carry out another request, whereas in a double cycle, the ASC combines a storage request with a retrieval request in order to reduce the empty travel time (Goodchild and Daganzo, 2006, 2007). The ASCs can never pass each other and must operate sufficiently far from each other. Since the ASCs cannot pass each other, seaside requests are carried out by the seaside ASC and landside requests are carried out by the landside ASC. Containers should not be dropped off in any location other than their final locations since this requires additional ASC movements which are time-consuming. We consider different priorities in stacking or retrieving containers. The most important reasons for this are as follows:

- The performance of a container terminal is often evaluated based on the berthing times of ships (Böse, 2011). Therefore, seaside containers usually have a higher priority than landside containers.
- To ensure the stability of ships, heavy containers must be loaded before light containers, in lower tiers

[^2]on the ship and must therefore be retrieved earlier (see, for example, Chapter 5, Kim et al., 2000, Sammarra et al., 2007, Dekker et al., 2007).

- Trucks, trains and ships arrive at a container yard to deliver or pick up containers in different time windows (see, for example, Froyland et al., 2008, Petering, 2011b, Newman and Yano, 2000), which induces precedence constraints on the stacking and retrieving of containers.

Preventing reshuffles also imposes additional precedence constraints on the operations. A reshuffle is an unwanted movement of a container stacked on top of another one that has to be retrieved (Kim et al., 2000, De Castillo and Daganzo, 1993, Caserta and Voß, 2009). For example, a precedence constraint can prohibit a container to be stacked in a pile before retrieving a container located in the same pile, or it can force containers stacked on top of a specific container to be retrieved earlier.

We formulate this multi-crane problem as a multiple asymmetric generalized traveling salesman problem with precedence constraints (mAGTSP-PC), which generalizes the single version of the problem without such constraints (see, for example, Laporte et al., 1987, Noon and Bean, 1991). The model also contains additional constraints regarding the interactions of the ASCs and the selection of open storage locations which lie in the intersection of multiple sets. We will elaborate on these constraints in the problem description section. We develop an adaptive large neighborhood search heuristic for this problem.

Our scientific contribution is to develop and solve a continuous time model to schedule two interacting ASCs working in a single block of containers. Our model incorporates precedence constraints, ASC interaction constraints, and constraints that assign each container to a storage location selected from a given set. Considering these constraints is valuable not only from a theoretical standpoint but also from a practical point of view. In practice, containers have different priorities, and a set of suitable open locations is available for each container that has to be stacked. The mAGTSP and its variants have been shown to be $\mathcal{N} \mathcal{P}$-hard (see, for example, Cheung et al., 2002, Narasimhan and Palekar, 2002) and the new features introduced in this study make the problem even more difficult to solve.

The remainder of this chapter is organized as follows. In Section 4.1, we describe the technical aspects of the problem and present our mathematical model. Section 4.2 describes the heuristic. Section 4.3 presents numerical results, and Section 4.4 contains the conclusions.

### 4.1 Problem description and model

This section describes the research problem, introduces notations, and then formulates the problem as an mAGTSP-PC.

### 4.1.1 Problem description

We seek to determine the sequence to stack containers of $n$ storage requests and retrieve containers of $N-n$ retrieval requests in a single block of containers consisting of $X$ rows, $Y$ bays, $Z$ tiers and two I/O points, one at each end of the block. We denote by $S$ and $L$ the seaside and landside I/O points, respectively. Let $\mathcal{R}$ be the set of all requests, whereas $\mathcal{R}_{s}$ and $\mathcal{R}_{l}$ be the sets of requests that must be picked up or dropped off at $S$ and $L$, respectively.

The objective is to minimize the makespan of the two ASCs carrying out these requests. We denote by $A S C_{s}$ the seaside ASC assigned to $\mathcal{R}_{s}$, and by $A S C_{l}$ the landside ASC assigned to $\mathcal{R}_{l}$. Each ASC can carry only one container at a time. From a given starting point, each ASC can execute its requests in any sequence in order to minimize its total travel time. We denote by $0_{s}$ and $0_{s}^{\prime}$, respectively, the starting and ending locations of $A S C_{s}$, whereas $0_{l}$ and $0_{l}^{\prime}$ are the corresponding locations for $A S C_{l}$. These locations can be anywhere in the block. The ASCs cannot pass each other and have to be be separated by a distance taking $\tau$ time units to travel. The storage and retrieval containers can be of different sizes ( 20 or 40 feet) and leave or arrive at the container terminal by truck, train or ship. Containers should not be dropped off at temporary locations. Finally, the speed of an empty or full ASC is the same.

Every storage or retrieval request $r \in \mathcal{R}=\{1, \ldots, N\}$ corresponds to a unique container. For storage request $r$, an ASC moves a container from the I/O point where it is located to a location $i$ in the block selected from a given set $\mathcal{L}_{r}$ of available open locations. For retrieval request $r$, an ASC moves a container from location $j$ in the block to a specified I/O point. For ease of notation, we write that the location $j$ of retrieval request $r$ belongs to $\mathcal{L}_{r}$. Thus, the set $\mathcal{V}$ of all locations can be defined as:
$\mathcal{V}=\{\underbrace{\left\{1,2, \ldots,\left|\mathcal{L}_{1}\right|\right.}_{\mathcal{L}_{1}}, \underbrace{\left|\mathcal{L}_{1}\right|+1,\left|\mathcal{L}_{1}\right|+2, \ldots,\left|\mathcal{L}_{1}\right|+\left|\mathcal{L}_{2}\right|}_{\mathcal{L}_{2}}, \ldots, \underbrace{\sum_{i=1}^{n-1}\left|\mathcal{L}_{i}\right|+1, \sum_{i=1}^{n-1}\left|\mathcal{L}_{i}\right|+2, \ldots, \sum_{i=1}^{n}\left|\mathcal{L}_{i}\right|}_{\mathcal{L}_{n}}$,
$\underbrace{\sum_{i=1}^{n}\left|\mathcal{L}_{i}\right|+1}_{\mathcal{L}_{n+1}}, \ldots, \underbrace{\sum_{i=1}^{n}\left|\mathcal{L}_{i}\right|+N-n}_{\mathcal{L}_{N}}, \underbrace{0_{s}}_{\mathcal{L}_{0_{s}}}, \underbrace{0_{l}}_{\mathcal{L}_{0_{l}}}, \underbrace{0_{s}^{\prime}}_{\mathcal{L}_{0_{s}^{\prime}}}, \underbrace{0_{l}^{\prime}}_{\mathcal{L}_{0_{l}^{\prime}}}\}$.
For each storage location in $\mathcal{V}$ lying in the intersection of multiple sets, a copy is created for each set to which it belongs. The copy locations will be used in the model to avoid stacking multiple containers in the same location. The same as the previous two chapters, we assume that storage and retrieval locations as well as destination sides of retrieval containers and I/O points of storage containers are known; in practice, they are determined at a higher hierarchical planning level. This level focuses on minimizing the number of reshufflings (De Castillo and Daganzo, 1993, Kim et al., 2000) and speeding up the loading and unloading process of a ship (Kim and Kim, 1999a, Vis and Roodbergen, 2009). Furthermore, due to the abundance of internal transport vehicles (like AGVs and chassis) to pick up containers at the seaside and landside, retrieved containers can be delivered to any I/O point at each side. Because of the same reason, retrieval containers can be quickly moved away from all I/O points, and they can therefore be delivered to the same I/O point that a storage container has to be picked up. The other issue is that storage and retrieval locations do not
coincide. This is reasonable since $N$ is usually much smaller than the block size, the block utilization is often fairly low, and container terminal operators usually separate containers based on destination and type. Finally, we do not consider the collaboration between the ASCs. In other words, if a container comes from the seaside and has to be stacked at the landside, the seaside ASC stacks it at its location, while the landside ASC has moved sufficiently far away in order to satisfy the safety distance requirements.

In order to present the model, we also have to define the pairwise travel times between the storage and retrieval locations of requests in the block. If the ASC travels directly from location $i$ to location $j$ in the block, the travel time of the ASC can be calculated as follows:

$$
\begin{equation*}
t_{i j}=\max \left\{\left|x_{i}-x_{j}\right|,\left|y_{i}-y_{j}\right|\right\}+\left|z_{i}\right|+\left|z_{j}\right|, \tag{1}
\end{equation*}
$$

where, $\left(x_{i}, y_{i}, z_{i}\right)$ and $\left(x_{j}, y_{j}, z_{j}\right)$ are the Cartesian coordinations of locations $i$ and $j$, with $0 \leq x_{i}, x_{j} \leq T_{x}$, $0 \leq y_{i}, y_{j} \leq T_{y}$, and $0 \leq z_{i}, z_{j} \leq T_{z}$. Here, $T_{x}, T_{y}$, and $T_{z}$, are the furthest travel times of locations in the block in $X, Y$, and $Z$ directions, respectively. The first term of the right-hand side of Equation (1) emphasizes that the ASC can move in the $X$ and $Y$ directions simultaneously. If the ASC travels from location $i$ to an I/O point to pick up or deliver a container and then travels to location $j$ to stack or retrieve another container, then the travel time to and from the I/O point must be considered. Table 4.1 summarizes how to calculate pairwise travel times of $A S C_{s}$. Travel times of $A S C_{l}$ can be calculated similarly. These times satisfy the triangle inequality.

Table 4.1. Computation of pairwise travel times for $A S C_{s}$

| Case ${ }^{1}$ | Computation of $t_{i j}$ |
| :---: | :---: |
| ```(storage location i, retrieval location j) (storage location i, storage location j) (storage location i, 0 (storage location i, (0s)``` | $\begin{aligned} & \max \left\{\left\|x_{i}-x_{j}\right\|,\left\|y_{i}-y_{j}\right\|\right\}+\left\|z_{i}\right\|+\left\|z_{j}\right\|^{2} \\ & \max \left\{\left\|x_{i}-x_{s}\right\|,\left\|y_{i}-y_{s}\right\|\right\}+\left\|z_{i}\right\|+\left\|z_{s}\right\|+\max \left\{\left\|x_{s}-x_{j}\right\|,\left\|y_{s}-y_{j}\right\|\right\}+\left\|z_{s}\right\|+\left\|z_{j}\right\|^{3} \\ & 0 \\ & \infty \end{aligned}$ |
| ```(retrieval location i, retrieval location j) (retrieval location i, storage location j)```  ```(retrieval location i, 0s)``` | $\begin{aligned} & \max \left\{\left\|x_{i}-x_{s},,\left\|y_{i}-y_{s}\right\|\right\}+\left\|z_{i}\right\|+\left\|z_{s}\right\|+\max \left\{\left\|x_{s}-x_{j}\right\|,\left\|y_{s}-y_{j}\right\|\right\}+\left\|z_{s}\right\|+\left\|z_{j}\right\|\right. \\ & \max \left\{\left\|x_{i}-x_{s}\right\|,\left\|y_{i}-y_{s}\right\|\right\}+\left\|z_{i}\right\|+\left\|z_{s}\right\|+\max \left\{\left\|x_{s}-x_{j}\right\|,\left\|y_{s}-y_{j}\right\|\right\}+\left\|z_{s}\right\|+\left\|z_{j}\right\| \\ & \max \left\{\left\|x_{i}-x_{s}\right\|,\left\|y_{i}-y_{s}\right\|\right\}+\left\|z_{i}\right\|+\left\|z_{s}\right\| \\ & \infty \end{aligned}$ |
| $\begin{aligned} & \left(0_{s}^{\prime}, \text { retrieval location } j\right) \\ & \left(0_{s}^{\prime}, \text { storage location } j\right) \\ & \left(0_{s}^{\prime}, 0_{s}^{\prime}\right) \\ & \left(0_{s}^{\prime}, 0_{s}\right) \\ & \hline \end{aligned}$ | $\begin{aligned} & \infty \\ & \infty \\ & \infty \\ & \infty \end{aligned}$ |
| $\begin{aligned} & \left(0_{s}, \text { retrieval location } j\right) \\ & \left(0_{s}, \text { storage location } j\right) \\ & \left(0_{s}, 0_{s}^{\prime}\right) \\ & \left(0_{s}, 0_{s}\right) \\ & \hline \end{aligned}$ | $\begin{aligned} & \max \left\{\left\|x_{0_{s}}-x_{j}\right\|,\left\|y_{0_{s}}-y_{j}\right\|\right\}+\left\|z_{0_{s}}\right\|+\left\|z_{j}\right\| \\ & \max \left\{\left\|x_{0_{s}}-x_{s}\right\|,\left\|y_{0_{s}}-y_{s}\right\|\right\}+\left\|z_{0_{s}}\right\|+\left\|z_{s}\right\|+\max \left\{\left\|x_{s}-x_{j}\right\|,\left\|y_{s}-y_{j}\right\|\right\}+\left\|z_{s}\right\|+\left\|z_{j}\right\| \\ & \infty \\ & \infty \end{aligned}$ |

${ }^{1} t_{i j}=\infty$ if $i=j$.
${ }^{2}$ Pick up and drop off times are included in times necessary for vertical movements.
${ }^{3}\left(x_{s}, y_{s}, z_{s}\right)$ are the Cartesian coordinates of $S$, with $0 \leq x_{s} \leq T_{x}, 0 \leq y_{s} \leq T_{y}$, and $0 \leq z_{s} \leq T_{z}$.
${ }_{5}^{4} A S C_{s}$ stops at location $i$, if the associated storage request is the last request.
${ }^{5} A S C_{s}$ delivers the container from location $i$ to $S$ and stops there, if the associated retrieval request is the last request.

### 4.1.2 Mathematical Model

The problem of sequencing storage and retrieval requests to be carried out by two ASCs, incorporating the storage location decision problem, can now be stated mathematically as the following integer linear program. Let $x_{i j}^{k}$ be a binary decision variable equal to 1 if and only if $A S C_{k}, k \in \mathcal{K}=\{l, s\}$ visits location $j \in V$ immediately after location $i \in \mathcal{V}$. We already know that the requests that have to be picked up or dropped off at $S$ or $L$ have to be carried out by the associated ASC. Denote by $\mathcal{V}_{k}$ the set of locations of requests in $\mathcal{R}_{k}$, which includes the starting and ending locations of $A S C_{k}$. As a result, we simplify $x_{i j}^{k}$ as the binary decision variable $x_{i j}$ which equals 1 if and only if location $j \in \mathcal{V}_{k}$ is visited immediately after location $i \in \mathcal{V}_{k}, \forall k \in \mathcal{K}$.

Objective function: The objective function minimizes $C$, the makespan of the two ASCs:

$$
\begin{equation*}
\text { minimize } C \tag{2}
\end{equation*}
$$

The model has the following constraints.

Carrying out all requests: Constraints (3)-(4) below mean that each set must be entered and exited exactly once. Of course, the starting point has no ingoing arc and the ending point has no outgoing arc. In these constraints, $\mathcal{L}_{r}^{\prime}=\bigcup_{\substack{e \in \mathcal{R}_{k} \\ e \neq r}} \mathcal{L}_{e}, \forall r \in \mathcal{R}_{k}, \forall k \in \mathcal{K}$; in other words, $\mathcal{L}_{r}^{\prime}$ is the set of all locations which do not specifically belong to $\mathcal{L}_{r}$ :

$$
\begin{align*}
& \sum_{i \in \mathcal{L}_{r}^{\prime}} \sum_{\substack{j \in \mathcal{L}_{r} \\
j \neq i}} x_{i j}=1, \quad \forall r \in \mathcal{R}_{k} \backslash\left\{0_{k}\right\}, \forall k \in \mathcal{K},  \tag{3}\\
& \sum_{i \in \mathcal{L}_{r}} \sum_{\substack{j \in \mathcal{L}_{r}^{\prime} \\
j \neq i}} x_{i j}=1, \quad \forall r \in \mathcal{R}_{k} \backslash\left\{0_{k}^{\prime}\right\}, \forall k \in \mathcal{K} . \tag{4}
\end{align*}
$$

Network flow conservation: Constraints (5) are the network flow conservation constraints. Constraints (5)-(6) ensure that in an optimal solution, each location can receive at most one container, and each container is stacked somewhere. In constraint (6), $\mathcal{I}$ is the set of storage locations in the intersection of multiple sets, and $\mathcal{Q}_{i}$ is the set of all copies of storage location $i \in \mathcal{I}$. These constraints are of special importance when locations in the intersection of multiple sets are considered. Without these, multiple containers may be stacked in a single location in the intersection of these sets:

$$
\begin{equation*}
\sum_{\substack{i \in \mathcal{V}_{k} \\ i \neq j}} x_{i j}=\sum_{\substack{l \in \mathcal{V}_{k} \\ l \neq j}} x_{j l} \leq 1, \quad \forall j \in \mathcal{V}_{k}, \forall k \in \mathcal{K}, \tag{5}
\end{equation*}
$$

$$
\begin{equation*}
\sum_{l \in \mathcal{V} \backslash\left\{0^{\prime}\right\}} \sum_{\substack{j \in \mathcal{Q}_{i} \\ j \neq l}} x_{l j} \leq 1, \quad \forall i \in \mathcal{I} . \tag{6}
\end{equation*}
$$

Makespan: Constraints (7) define $C_{i}$, the arrival time of an ASC at location $i, \forall i \in \mathcal{V}$. In these constraints, $M$ is a large constant. Constraints (8) are used to define the makespans of the ASCs based on their arrival times at their finishing points, $0_{s}^{\prime}, 0_{l}^{\prime}$ :

$$
\begin{equation*}
C_{i}+t_{i j}-C_{j} \leq M\left(1-x_{i j}\right), \quad \forall i, j \in \mathcal{V}_{k}, \forall k \in \mathcal{K}, \tag{7}
\end{equation*}
$$

$$
\begin{equation*}
C \geq C_{j}, \forall j \in\left\{0_{s}^{\prime}, 0_{l}^{\prime}\right\} \tag{8}
\end{equation*}
$$

Precedence constraints: Let $\mathcal{P}_{i}^{-}$be the set of storage or retrieval locations that have to be visited before location $i \in \mathcal{V}$ if they appear in a feasible solution. The precedence sets of storage or retrieval locations are defined on the basis of the precedence sets of requests corresponding to these locations. Constraints (9) ensure that location $i$ is visited after all locations $j, \forall j \in \mathcal{P}_{i}^{-}$. There exists a feasible solution if the precedence constraints do not induce any subtours:

$$
\begin{equation*}
C_{i} \geq C_{j} \quad \forall i \in \mathcal{V}, \forall j \in \mathcal{P}_{i}^{-} \tag{9}
\end{equation*}
$$

Interaction of the ASCs: ASCs should not pass each other and should be separated by at least $\tau$ time units. Let $\mathcal{B}_{i}$ be the set of all locations behind location $i$ that cannot be visited by the other crane because of the no-passing constraint. Furthermore, $\mathcal{A}_{i}$ is the set of all locations in front of location $i$ that can only be visited by the other crane at least $\tau$ time units after the moment that location $i$ has been visited. Note that I/O points help to distinguish the front and behind sides of locations. The I/O point where the corresponding container of a location must be picked up or dropped off shows the behind side of that location. Figure 4.1 depicts a top view of a block of containers with multiple storage and retrieval locations. The sets of open locations available for different storage containers are highlighted by the contours. For example, in Figure 4.1, $A S C_{l}$ must retrieve the container in location 10 to $L$, thus $\mathcal{A}_{10}=\{4,5,11\}$ and $\mathcal{B}_{10}=\{1,2,3,8,9\}$. Having defined $\mathcal{A}_{i}$ and $\mathcal{B}_{i}$, then the interaction between the ASCs can be modeled by constraints (10)-(12). In these constraints $z_{i j}$ indicates whether location $i \in \mathcal{V}_{k}, \forall k \in \mathcal{K}$ has to be visited before location $j \in \mathcal{V}_{k^{\prime}}, \forall k^{\prime} \in \mathcal{K}$. These variables are different from $x_{i j}$. Indeed, the latter variables indicate whether location $i$ is visited immediately before location $j$, whereas the former only indicate that $i$ is visited before $j$. Note that in these constraints, we do not keep track of the positions of the ASCs, but the time that each ASC arrives at the location of every corresponding request. We delay the arrival time of an ASC to a location if the safety
distance is violated. In other words, the constraints guarantee that at the moment that an ASC arrives at the location of each request that has to be carried out by that ASC, the other one has enough time to move to a position where the safety distance requirements can be satisfied. These constraints including with constraints (7)-(8) determine the arrival time of the corresponding ASC at each location.


Figure 4.1. Bird's eye view of a block of containers with storage and retrieval locations

$$
\begin{equation*}
z_{i j}+z_{j i}=1, \forall i, j \in \mathcal{V}, i \neq j \tag{10}
\end{equation*}
$$

$$
C_{i}+\left(\tau-\left|y_{i}-y_{j}\right|\right) \leq M\left(1-z_{i j}\right)+C_{j}, \quad \forall i \in \mathcal{V}, \forall j \in \mathcal{A}_{i}
$$

$$
\begin{equation*}
C_{i}+\left(\tau+\left|y_{i}-y_{j}\right|\right) \leq M\left(1-z_{i j}\right)+C_{j}, \quad \forall i \in \mathcal{V}, \forall j \in \mathcal{B}_{i} \tag{12}
\end{equation*}
$$

Finally, constraints (13)-(14) are the integrality and non-negativity constraints:

$$
\begin{equation*}
C_{i} \geq 0 \quad \forall i \in \mathcal{V} \backslash\left\{0_{s}, 0_{l}\right\}, \quad C_{i}=0 \quad \forall i \in\left\{0_{s}, 0_{l}\right\} \tag{13}
\end{equation*}
$$

$$
\begin{equation*}
x_{i j}, z_{i j} \in\{0,1\}, \quad \forall i, j \in \mathcal{V} \tag{14}
\end{equation*}
$$

### 4.2 Adaptive large neighborhood search heuristic

The complexity of the problem means that only relatively small instances can be solved to optimality within a reasonable time. To solve real-size problems, we have developed an adaptive large neighborhood search (ALNS) heuristic. This type of heuristic was initially proposed by Ropke and Pisinger (2006a), and is based on the LNS search scheme previously developed by Shaw (1997) and Bent and Van Hentenryck (2004) for different problems. The ALNS is a powerful heuristic framework applicable to the solution of combinatorial problems. It has been applied by several authors in a variety of contexts including crew scheduling (Bartodziej et al., 2009), fixed charge network flow routing (Hewitt et al., 2010), capacitated arc routing (Laporte et al., 2010), and vehicle routing (i.e., Ropke and Pisinger, 2006a, Pisinger and Ropke, 2007, Ropke and Pisinger, 2006b, Pepin et al., 2009). To be effective, the implementation of this heuristic has to be tailored to the problem at hand.

```
Algorithm 2 ALNS metaheuristic
Require: A feasible solution \(X, q \in \mathbb{N}\);
Ensure: A near-optimal feasible solution, \(X_{\text {best }}\);
    procedure \(\operatorname{ALNS}(X, q)\)
        \(X_{\text {best }} \leftarrow X\);
        while stop criterion is not met do
            \(X^{\prime} \leftarrow X ;\)
            apply one operator to remove \(q\) requests from \(X^{\prime}\)
            apply one operator to reinsert removed requests into \(X^{\prime}\);
            if \(Z\left(X^{\prime}\right)<Z\left(X_{\text {best }}\right)\) then
                \(X_{\text {best }} \leftarrow X^{\prime} ;\)
            end if
            if \(X^{\prime}\) is selected as the incumbent solution then
                \(X \leftarrow X^{\prime} ;\)
            end if
        end while
        return \(X_{\text {best }}\);
    end procedure
```

The basic outline of the ALNS metaheuristic is presented in Algorithm (2). The search starts from a feasible solution generated by a simple construction heuristic (see Section 4.3.2). The ALNS attempts to improve this solution by sequentially applying several elementary operators to remove a subset of requests from the solution and reinsert them. The main difference between ALNS and LNS is that the ALNS applies multiple removal and insertion operators with adaptive weights, whereas the LNS uses only one removal operator and one insertion operator. We now explain the removal and insertion operators developed, and
then in Section 4.2.3, we describe how to update and use the weights to select a removal and an insertion operator in each iteration.

### 4.2.1 Removal operators

We have developed several removal operators, all of which are applied to a feasible solution. Note that in this problem, one of the ASCs needs to visit a unique location for each request. In our operators, we refer to corresponding requests of locations.

Relatedness removal (Shaw removal): The Shaw operator, initially introduced by Shaw (1997), is one of the most widely used removal operators. It removes the requests which are somewhat related to each other and can be inserted interchangeably in different positions of the solution. In our implementation, the relatedness of requests $r$ and $e$ is defined as:

$$
\begin{equation*}
R(r, e)=\varphi \mid \text { type }_{r}-\text { type }_{e}|+\chi| \text { side }_{r}-\text { side }_{e}\left|+\psi \|\left|\mathcal{P}_{r}\right|-\left|\mathcal{P}_{e}\right|\right|+\omega\left|v_{r}-v_{e}\right|, \tag{15}
\end{equation*}
$$

where, type $e_{r}$ equals 1 if request $r$ is a storage request, and 0 otherwise; side equals 1 if request $r$ has to be picked up or dropped of at $S$, and 0 otherwise; $\mathcal{P}_{r}$ is the set of requests that must appear either before or after request $r$; and $v_{r}$ is the location associated with request $r$. In order to apply equation (15), all variables are scaled to take values between 0 and 1 . The smaller $R(r, e)$ is, the more related $r$ and $e$ are. The complexity of this operator is $O(N)$.

Random removal: In this type of removal, $q$ requests are randomly removed from the solution. The complexity of this operator is $O(1)$ and it is repeated $q$ times.

Worst removal: Assume that $\Delta(r, X)$ is the travel time of request $r$ to its predecessor and successor requests in the solution. The worst removal operator removes $q$ requests with the largest $\Delta(r, X)$. Note that we do not define $\Delta(r, X)$ as the effect of removing request $r$ in the objective function. Indeed, because of the interactions between the ASCs, and because of the precedence constraints, fully evaluating the objective function for each partial solution is rather time consuming. Our simplified approach avoids this computation. The complexity of this operator stems from the sorting process involved in the operator which is $O\left(N^{2}\right)$.

Precedence constraint removal: This operator sorts the requests of the solution in non-decreasing order of $\left|\mathcal{P}_{r}\right|$, and then selects the first $q$ requests. The rationale is that since these requests have smaller number of predecessors and successors, their reinsertion in the solution is likely to be easier. The complexity of this operator is $O\left(N^{2}\right)$.

Group removal: In the group removal operator, either $q$ storage requests or $q$ retrieval requests are removed from the solution. The idea is that storage or retrieval requests can be used to generate double cycles when
they are reinserted in the solution. In our implementation, we choose the first $q$ storage or retrieval requests in the solution. However, one may remove those with the largest $\Delta(r, X)$ or $\left|\mathcal{P}_{r}\right|$. The complexity of this operator is $O(N)$ and it is repeated $q$ times.

Single cycle removal: This operator removes $q$ single cycles from the current solution. It is expected that better double cycles than the current ones in terms of the objective function can be generated by reinserting their requests in the solution. In this operator, as in the group removal operator, we remove the first $q$ single cycles, but one can alternatively remove those with the largest $\Delta(r, X)$ or $\left|\mathcal{P}_{r}\right|$. If there is no single cycle location, the random removal operator removes requests until a new single cycle can be found. The complexity of this operator is $O(N)$ and it is repeated $q$ times.

Location removal: One of the features of our problem is the presence of sets of open locations to stack storage containers. In the heuristic used to generate a feasible initial solution, the location for each storage container is randomly selected from its set of available open locations. If no operator is applied to assign new locations, the selected locations do not change, which results in lost opportunities to improve the solution. The location removal operator, randomly selects $\max \{n, q\}$ storage requests and removes the locations assigned to them. A location insertion operator then selects other locations for these requests (see the next section). The complexity of this operator is $O(N)$ and it is repeated $\max \{n, q\}$ times.

### 4.2.2 Insertion operators

We have developed four operators to reinsert the $q$ requests that have been removed by one of the above operators. The insertion operators regain solution feasibility, by sequencing all requests, while satisfying the precedence and interaction constraints.

Greedy reinsertion: Let $\Delta f_{r}$ be the travel time of request $r$ to its predecessor and successor request by inserting it in its best position. The greedy operator inserts the request minimizing $\Delta f_{r}$, over all removed requests. After inserting a request, the operator updates the $\Delta f_{r}$ values and is reapplied. The complexity of this operator is $O(N)$ and it is repeated $q$ times.

Random reinsertion: This operator randomly reinserts the requests into the partial solution. The complexity of this operator is $O(1)$ and it is repeated $q$ times.

Interaction insertion: The main idea of this operator is to insert seaside and landside requests in the partial solution so as to minimize the interaction of the corresponding ASCs. When a request is inserted in a position, the operator counts the immediate requests after and before the request that have to be delivered to the same side. It then inserts the request in a position yielding the largest number. The complexity of this operator is $O(N)$ and it is repeated $q$ times.

Location insertion: The location removal operator described in Section 4.2.1 can only be followed by the location insertion operator. For each storage request with a removed location, this operator randomly selects a new location from its set of available open locations. The complexity of this operator is $O(1)$ and it is repeated $\max \{n, q\}$ times.

### 4.2.3 Choosing a removal operator or a insertion operator

We run the ALNS algorithm for a preset number of iterations. At each iteration, the selection of a removal or insertion operator is governed by a roulette-wheel mechanism which selects operator $o$ from the set of removal or insertion operators with probability $w_{o} / \sum_{o^{\prime}=1}^{O} w_{o^{\prime}}$, where $w_{o}$ is the weight of operator $o$, and $O$ is the number of operators of the set.

In the ALNS, an initial weight is assigned to each operator and the weights are then updated after every $\delta$ iterations of the algorithm. The updates are based on the score that each operator has gained during the past $\delta$ iterations. At each iteration, the score of each operator is updated based on its performance: the weight of each removal or insertion operator used in the current iteration is incremented by $\sigma_{1}$ if their application result in a new best solution; by $\sigma_{2}$ if result in a better incumbent solution; and by $\sigma_{3}$ if result in a solution which is not better than the best solution found and incumbent solution but is accepted, where $\sigma_{1} \geq \sigma_{2} \geq \sigma_{3}$. Let $\pi_{o}$ and $\theta_{o}$ be the total score of operator $o$ and the number of times it has been selected after $\delta$ iterations, respectively. Furthermore, let $\zeta_{o} \geq 1$ be a normalization factor which reflects the computational effort that operator o requires (see Ropke and Pisinger, 2006b). The weight of operator $o$ is then updated as $w_{o} \leftarrow w_{o}(1-\rho)+\rho \pi_{o} / \zeta_{o} \theta_{o}$, where $\rho \in[0,1]$ is the reaction factor which controls how quickly the weight adjustment reacts to changes in the operator performance. Note that $w_{o}$ does not change if $\theta_{o}=0$.

### 4.2.4 Acceptance of the new solution and stop criterion

In order to produce sufficient diversification in the search process, the ALNS metaheuristic is executed within a simulated annealing (SA) framework (Kirkpatrick et al., 1983), which has proven to work well for the solution of several combinatorial problems (e.g., Vis and Carlo, 2010, Bozer and Carlo, 2008, Wilhelm and Ward, 1987). In the SA framework, if the solution found at the current iteration is better than the current solution, it is accepted. Otherwise, it is accepted with probability $e^{-\left(Z\left(S^{\prime}\right)-Z\left(S^{*}\right)\right) / T}$, where $T$ is the temperature, $Z\left(S^{\prime}\right)$ and $Z\left(S^{*}\right)$ are the objective values of the solution at the current iteration and best solution, respectively. The temperature starts at $T_{\text {start }}$ and is updated at each iteration as $T \leftarrow T \phi$, where $0<\phi<1$ is a cooling rate.

### 4.3 Computational experiments

Extensive numerical experiments were performed to assess the effectiveness of our ALNS metaheuristic. In Section 4.3.1, we discuss how its parameters were tuned. Section 4.3.2 compares the results of the ALNS with those results of the LNS, truncated CPLEX, and some other simple heuristics.

The ALNS and the other heuristics were coded in C++ using Microsoft Visual Studio 2008 and were executed on a Notebook with 2.40 GHz Intel ${ }^{\circledR}$ Core ${ }^{\mathrm{TM}}$ i5 processor, with 4.00 GB of RAM. Exact results were obtained by CPLEX 12.2 coded in C++ using the Concert Technology framework and executed by a g++ compiler on a 2.40 GHz AMD Opteron ${ }^{\text {TM }}$ Processor 250 , with 8.00 GB of RAM under the Linux operating system.

### 4.3.1 Tuning and initial solution

In our experiments, we consider a single block of containers with 40 bays, 10 rows, and four tiers. The number $N$ of requests is set to 10,25 and 45 , which is approximately the workload of a single ASC without any interaction in a common 40 to 180 minute planning horizon (Vis and Carlo, 2010). We assume an equal number of open locations $\left|\mathcal{L}_{r}\right|$ for storage requests $r=1, \ldots, n$, and we set $\left|\mathcal{L}_{r}\right|$ equal to 1,3 , or 5 . As a result we perform nine basic experiments, numbered as one to nine, on five randomly generated instances in each case. Each instance has the same number of retrieval and storage requests. All locations are uniformly distributed over the block, and an equal number of requests have to be picked up or delivered to the landside and the seaside. The landside ASC starts its operations from $L$ and ends either at the storage position of a storage request if this is the last request in the sequence, or at $L$ if a retrieval request is the last request in the sequence. Similarly, the seaside ASC starts from $S$ and either finishes in a storage position or at $S$.

We assume that the containers are categorized in five priority levels of decreasing importance. In addition, we consider a sixth priority level dedicated to containers that can be moved at any time. Since the seaside process time of a terminal has a relatively higher importance, the seaside containers have to be moved before the other ones, which gives them the priority levels 1,2 , and 3 . Eighty percent of remaining containers are categorized with the priority levels 4 to 6 and $20 \%$ with the priority levels 1 to 3 . Other inputs regarding the size of containers and the speed of the ASC can be found in Table 2.2 in Chapter 2.

We have used the first instance of Experiment 5 with 25 requests and three open locations for each storage requests to tune the parameters of the ALNS. As Ropke and Pisinger (2006a) suggest, we have applied an ad hoc trial and error strategy to set the parameters. To this end, we have fixed all parameters and changed only one parameter at a time. The ALNS was applied five times and the parameter value resulting in the best objective was selected. The parameters used for all scenarios are as follows: $\left(\varphi, \chi, \psi, \omega, \sigma_{1}, \sigma_{2}, \sigma_{3}, \rho, \phi, \delta\right)=$ $(4,2,3,6,33,13,9,0.2,0.94,200)$. In our implementation, we have used $\zeta_{o}=1$ for all removal and insertion operators except for the single cycle and worst removal where $\zeta_{o}=5$. We set the number of ALNS iterations to 25000. The insertion or removal operators have equal weights at the first iteration. The starting temperature
of the SA was set in such a way that a solution $30 \%$ worse than the initial solution should be accepted with probability 0.5 . In the first third of all scenarios, the number of $q$ requests to be removed and inserted was randomly drawn from the interval $[1,0.5 N]$, using a semi-triangular distributions with a negative slope. The interval was narrowed down to $[1,0.25 N]$ and $[1,0.1 N]$ for the second and third tiers of the iterations, respectively.

### 4.3.2 Computational results

The results obtained by the ALNS are now compared with those results obtained by truncated CPLEX and with other heuristics.

## Comparing ALNS and truncated CPLEX

Tables $4.2,4.3$, and 4.4 compare the makespan and computation time of the ALNS and truncated CPLEX over different instances. For each instance, we apply the ALNS 20 times each with a different random initial solution and calculate the minimum, average, and maximum makespan, in addition to the average computation time. The CPLEX algorithm is applied to each instance only once in order to find a solution for the mAGTSP-PC and mAGTSP which is a relaxed problem without the precedence constraints and the ASC interaction constraints. The computation time of CPLEX for finding an optimal or even a feasible solution for the mAGTSP-PC is very high and we therefore truncate the computation after 14400 seconds. On the other hand, the mAGTSP can be quickly solved. The reason is that when $\left|\mathcal{L}_{r}\right|=1, r=1, \ldots, n$, by relaxing the precedence and ASC constraints we can easily find an optimal solution of the mAGTSP by solving an assignment problem (AP). Since there is only a single I/O point for each ASC, by using it all subtours obtained by the AP can be easily merged. When $\left|\mathcal{L}_{r}\right|>1, r=1, \ldots, n$, the situation is more complicated since we have to choose a location for each storage container. However, since the number of requests is relatively small in our instances, we can quickly find the optimal solution with CPLEX. The optimal mAGTSP solution value provides a lower bound for the original problem.

Table 4.2 compares the optimal results obtained by CPLEX with the ALNS results for small instances. Column $G_{\text {Ave }}^{Z}$ shows that the ALNS objective values are on average within $0.34 \%$ of the optimal values. On the other hand, as column $G_{Z}^{L B}$ shows, even on small-size instances, the gap between the lower bound and the optimal mAGTSP-PC solution is large. The gap stems from the fact that the precedence constraints and interaction constraints significantly affect the sequence of requests carried out by the ASCs and increase the objective function. Omitting these constraints results in a relaxed problem whose optimal solution provides a very poor lower bound for the original problem. This suggests that although the gap between the average ALNS solution value and the lower bound may be large in some cases, this does not necessarily reflect on the quality of the ALNS algorithm. Therefore, column $G_{L B}^{A v e}$ which shows the gap between the lower bound and average ALNS solution is omitted from the table. A fair assessment is to compare the average result of the

ALNS with the best known objective value, as Ropke and Pisinger (2006a) recommend. This is carried out in column $G_{A v e}^{m i n}$ which shows that the average results are within $0.34 \%$ of the best know objective value.

Tables 4.3 and 4.4 compare the ALNS results for medium and large instances and those results of the truncated CPLEX. In all instances, columns $G_{\text {Ave }}^{Z}$ and $G_{\text {min }}^{Z}$ correspondingly show that the gaps between the average and minimum ALNS objective values and the truncated CPLEX objective values are on average negative or very small. Furthermore, the gaps between the minimum and average ALNS results shown in column $G_{\text {Ave }}^{m i n}$ are on average less than $2.85 \%$ which is quite promising.

Table 4.2. Results of the ALNS and CPLEX for Experiments 1,2 , and 3

| Inst. | CPLEX results |  |  |  |  | ALNS results (20 runs for each instance) |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | LB | CPU | Z | CPU | $G_{Z}^{L B}$ | Min | Max | Ave | $G_{\text {Ave }}^{\text {min }}$ | $G_{\text {Min }}^{Z}$ | $G_{\text {Ave }}^{Z}$ | CPU |
| Experiment $1\left(N=10,\left\|\mathcal{L}_{r}\right\|=1, r=1, \ldots, 5\right)$ |  |  |  |  |  |  |  |  |  |  |  |  |
| 1 | 348.83 | 0.00 | 387.59 | 0.10 | 10.00 | 387.59 | 387.59 | 387.59 | 0.00 | 0.00 | 0.00 | 0.75 |
| 2 | 288.29 | 0.01 | 386.53 | 0.40 | 25.42 | 386.53 | 386.53 | 386.53 | 0.00 | 0.00 | 0.00 | 0.73 |
| 3 | 343.05 | 0.00 | 426.17 | 0.02 | 19.50 | 426.17 | 426.17 | 426.17 | 0.00 | 0.00 | 0.00 | 0.80 |
| 4 | 337.50 | 0.01 | 398.74 | 0.12 | 15.36 | 398.74 | 398.74 | 398.74 | 0.00 | 0.00 | 0.00 | 0.75 |
| 5 | 234.95 | 0.01 | 289.52 | 0.06 | 18.85 | 289.52 | 289.52 | 289.52 | 0.00 | 0.00 | 0.00 | 0.79 |
| Ave- | -310.52 | $0.01{ }^{-}$ | 377.71 | 0.14 | 17.85 | 377.71 ${ }^{-}$ | $377.71^{-}$ | $377.71{ }^{-}$ | 0.00 | 0.00 | 0.00 | 0.76 |
| Experiment $2\left(N=10,\left\|\mathcal{L}_{r}\right\|=3, r=1, \ldots, 5\right)$ |  |  |  |  |  |  |  |  |  |  |  |  |
|  | 288.29 | 0.01 | 356.32 | 3.54 | 19.09 | 356.32 | 356.32 | 356.32 | 0.00 | 0.00 | 0.00 | 0.81 |
| 2 | 319.38 | 0.01 | 419.15 | 2234.84 | 23.80 | 419.15 | 419.15 | 419.15 | 0.00 | 0.00 | 0.00 | 0.93 |
| 3 | 313.97 | 0.02 | 346.08 | 1699.35 | 9.28 | 346.08 | 350.05 | 346.88 | 0.23 | 0.00 | 0.23 | 0.89 |
| 4 | 275.06 | 0.02 | 291.56 | 13.80 | 5.66 | 291.56 | 291.56 | 291.56 | 0.00 | 0.00 | 0.00 | 0.84 |
| 5 | 336.81 | 0.01 | 392.46 | 6.55 | 14.18 | 392.46 | 419.05 | 397.53 | 1.27 | 0.00 | 1.27 | 0.86 |
| Àve | 306.70 | 0.01 | 361.11 | 791.62 | 14.40 | 361.11 | 367.23 | $362.29{ }^{-}$ | 0.30 | 0.00 | $0.30^{-}$ | O 8 T |
| Experiment $3\left(N=10,\left\|\mathcal{L}_{r}\right\|=5, r=1, \ldots, 5\right)$ |  |  |  |  |  |  |  |  |  |  |  |  |
| 1 | 370.14 | 0.02 | 469.90 | 14317.17 | 21.23 | 469.90 | 489.23 | 475.63 | 1.21 | 0.00 | 1.21 | 1.03 |
| 2 | 282.59 | 0.02 | 355.49 | 462.38 | 20.51 | 355.49 | 355.49 | 355.49 | 0.00 | 0.00 | 0.00 | 1.04 |
| 3 | 425.48 | 0.02 | 459.36 | 94.06 | 7.38 | 459.36 | 459.36 | 459.36 | 0.00 | 0.00 | 0.00 | 1.00 |
| 4 | 311.74 | 0.02 | 322.69 | 18.73 | 3.39 | 322.69 | 322.69 | 322.69 | 0.00 | 0.00 | 0.00 | 0.93 |
| 5 | 300.00 | 0.03 | 367.29 | 1782.91 | 18.32 | 367.29 | 379.47 | 369.12 | 0.49 | 0.00 | 0.49 | 0.89 |
| A- ${ }^{\text {eve }}$ | -337.99 | 0.02 | 394.95 | -3335.05 | 14.17 | 394.95 | 401.25 | 396.46 | 0.34 | 0.00 | 0.34 | 0.98 |

Note. LB is the optimal solution of the AGTSP. $Z$ is the feasible or optimal solution of mAGTSP-PC. Columns CPU show the computation time in seconds. For CPLEX, when CPU $<14400$ seconds, the optimum is attained. The computation time of the ALNS is an average over 20 runs. The gaps are calculated as: $G_{a}^{b}(\%)=((a-b) / a) \times 100$.

Although we have shown the superior performance of our heuristic by the gaps discussed in Tables 4.24.4, readers may be still curious about the gaps between the optimal results and those results obtained by the ALNS for large instances. We can do this by omitting the interaction and precedence constraints, and obtain the exact and ALNS results for the mAGTP and the AGTP. The mAGTSP is a relaxation of our problem without the precedence and interaction constraints. Its solution provides a lower bound for our problem, as shown in Section 4.3.2. The AGTSP is the problem in which a single ASC carries out all the requests, and no precedence or interaction constraints are imposed. Table 4.5 shows that the ALNS can quickly find near-optimal results.

Table 4.3. Results of the ALNS and CPLEX for Experiments 4,5 , and 6

| Inst. | CPLEX results |  |  |  |  | ALNS results (20 runs for each instance) |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | LB | CPU | Z | CPU | $G_{Z}^{L B}$ | Min | Max | Ave | $G_{\text {Ave }}^{\text {min }}$ | $G_{\text {Min }}^{Z}$ | $G_{\text {Ave }}^{Z}$ | CPU |
| Experiment $4\left(N=25,\left\|\mathcal{L}_{r}\right\|=1, r=1, \ldots, 13\right)$ |  |  |  |  |  |  |  |  |  |  |  |  |
| 1 | 660.26 | 0.02 | 728.36 | 14400 | 9.35 | 728.36 | 756.00 | 739.19 | 1.47 | 0.00 | 0.11 | 3.50 |
| 2 | 608.06 | 0.02 | 747.62 | 11730.83 | 18.67 | 747.62 | 789.15 | 761.44 | 1.81 | 0.00 | 0.14 | 3.26 |
| 3 | 733.75 | 0.02 | 749.54 | 14400 | 2.11 | 749.54 | 776.57 | 758.57 | 1.19 | 0.00 | 0.09 | 3.35 |
| 4 | 675.55 | 0.01 | 805.03 | 14400 | 16.08 | 805.03 | 812.33 | 806.77 | 0.22 | 0.00 | 0.02 | 2.97 |
| 5 | 809.92 | 0.02 | 827.04 | 14400 | 2.07 | 826.78 | 854.89 | 834.26 | 0.90 | -0.03 | 0.07 | 3.12 |
| A-Ave | -697.51 | 0.02 | 771.52 | 13866.17 | 9.66 | 771.47 | 797.79 | 780.05 | 1.12 | -0.01- | 0.09 | 3.24 |
| Experiment $5\left(N=25,\left\|\mathcal{L}_{r}\right\|=3, r=1, \ldots, 13\right)$ |  |  |  |  |  |  |  |  |  |  |  |  |
| 1 | 653.96 | 0.07 | 952.04 | 14400 | 31.31 | 952.04 | 952.04 | 952.04 | 0.00 | 0.00 | 0.00 | 4.41 |
| 2 | 715.18 | 0.06 | 846.43 | 14400 | 15.51 | 842.57 | 849.37 | 845.80 | 0.38 | -0.46 | -0.01 | 4.42 |
| 3 | 624.06 | 0.06 | 869.14 | 14400 | 28.20 | 869.14 | 906.20 | 885.17 | 1.81 | 0.00 | 0.16 | 4.09 |
| 4 | 738.29 | 0.06 | 776.28 | 14400 | 4.89 | 767.41 | 824.80 | 809.12 | 5.15 | -1.16 | 0.33 | 4.96 |
| 5 | 659.84 | 0.06 | 860.45 | 14400 | 23.31 | 860.45 | 870.99 | 863.61 | 0.37 | 0.00 | 0.03 | 4.66 |
| Ave- | - $6 \overline{7} \overline{8} . \overline{2} \overline{7}$ | $\overline{0} .0 \overline{6}$ | $\overline{8} \overline{60} .87$ | $\overline{144} \overline{0} 0$ | ${ }^{2} 0 . \overline{64}{ }^{-}$ | $\overline{8} 58 . \overline{32}$ | $8 \overline{8} 0 . \overline{6} 8$ | $87 \overline{1} .15$ | 1.54 | $-0 . \overline{3} 2$ | $\overline{0} .1 \overline{0}$ | ¢ 51 |
| Experiment $6\left(N=25,\left\|\mathcal{L}_{r}\right\|=5, r=1, \ldots, 13\right)$ |  |  |  |  |  |  |  |  |  |  |  |  |
| 1 | 618.33 | 0.16 | 763.93 | 14400 | 19.06 | 713.34 | 773.40 | 733.86 | 2.80 | -7.09 | -0.30 | 5.06 |
| 2 | 641.96 | 0.15 | 1071.37 | 14400 | 40.08 | 1071.37 | 1071.37 | 1071.37 | 0.00 | 0.00 | 0.00 | 5.72 |
| 3 | 706.86 | 0.16 | 886.09 | 14400 | 20.23 | 886.09 | 920.08 | 893.48 | 0.83 | 0.00 | 0.07 | 6.19 |
| 4 | 688.31 | 0.16 | 715.19 | 14400 | 3.76 | 705.01 | 725.63 | 708.63 | 0.51 | -1.44 | -0.07 | 4.77 |
| 5 | 667.79 | 0.15 | 721.13 | 14400 | 7.40 | 693.15 | 722.71 | 703.08 | 1.41 | -4.04 | -0.18 | 5.10 |
| A-Ave | -664.65 | 0.16 | - 831.54 | 14400 | 18.10 | 813.79 | 842.64 | 822.08 | 1.11 | - $2.511^{-}$ | -0.09 | 5.37 |

Table 4.4. Results of the ALNS and CPLEX for Experiments 7,8 , and 9

| Inst. | CPLEX results |  |  |  |  | ALNS results (20 runs for each instance) |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | LB | CPU | $Z$ | CPU | $G_{Z}^{L B}$ | Min | Max | Ave | $G_{\text {Ave }}^{\text {min }}$ | $G_{\text {Min }}^{Z}$ | $G_{\text {Ave }}^{Z}$ | CPU |
| Experiment 7 $\left(N=45,\left\|\mathcal{L}_{r}\right\|=1, r=1, \ldots, 23\right)$ |  |  |  |  |  |  |  |  |  |  |  |  |
| 1 | 1240.20 | 0.03 | 1326.44 | 14400 | 6.50 | 1321.95 | 1367.07 | 1339.21 | 1.29 | -0.34 | 0.95 | 12.64 |
| 2 | 1341.59 | 0.03 | NA | 14400 | NA | 1406.13 | 1503.34 | 1456.24 | 3.44 | NA | NA | 11.44 |
| 3 | 1451.60 | 0.03 | 1775.20 | 14400 | 18.23 | 1778.04 | 1832.83 | 1791.75 | 0.77 | 0.16 | 0.92 | 11.98 |
| 4 | 1187.91 | 0.03 | 1418.68 | 14400 | 16.27 | 1401.92 | 1447.53 | 1425.19 | 1.63 | -1.20 | 0.46 | 13.22 |
| 5 | 1368.85 | 0.03 | 1461.50 | 14400 | 6.34 | 1422.84 | 1542.50 | 1469.79 | 3.19 | -2.72 | 0.56 | 13.00 |
| -Ave | -1318.03 | 0.03 | 1495.46 | -14400- | 11.83 | ${ }^{-1466.18}$ | 1538.65 | 1496.44 | $2.06{ }^{-}$ | -1.02- | 0.72 | 12.46 |
| Experiment $8\left(N=45,\left\|\mathcal{L}_{r}\right\|=3, r=1, \ldots, 23\right)$ |  |  |  |  |  |  |  |  |  |  |  |  |
| 1 | 1313.27 | 0.31 | 1441.71 | 14400 | 8.91 | 1392.42 | 1477.36 | 1436.01 | 3.04 | -3.54 | -0.40 | 19.07 |
| 2 | 1250.54 | 0.31 | NA | 14400 | NA | 1516.23 | 1610.82 | 1556.11 | 2.56 | NA | NA | 20.30 |
| 3 | 1260.58 | 0.30 | 1597.99 | 14400 | 21.11 | 1464.64 | 1583.35 | 1511.18 | 3.08 | -9.10 | -5.74 | 20.44 |
| 4 | 1132.84 | 0.30 | 1392.69 | 14400 | 18.66 | 1284.52 | 1401.77 | 1335.53 | 3.82 | -8.42 | -4.28 | 14.30 |
| 5 | 1178.74 | 0.30 | 1515.75 | 14400 | 22.23 | 1439.96 | 1513.48 | 1465.41 | 1.74 | -5.26 | -3.44 | 17.64 |
| A-vē | -1227.19 ${ }^{-}$ | -0.30 | 1487.04 | -14400- | -17.73 | -1419.55 | 1517.36 | 1460.85 | $2.85{ }^{-}$ | -6.58 | -3.46 | 18.35 |
| Experiment $9\left(N=45,\left\|\mathcal{L}_{r}\right\|=5, r=1, \ldots, 23\right)$ |  |  |  |  |  |  |  |  |  |  |  |  |
| 1 | 1188.81 | 1.26 | 1468.33 | 14400 | 19.04 | 1350.82 | 1424.86 | 1395.43 | 3.20 | -8.70 | -5.22 | 27.31 |
| 2 | 1203.28 | 1.28 | 1486.10 | 14400 | 19.03 | 1414.11 | 1515.43 | 1444.65 | 2.11 | -5.09 | -2.87 | 28.56 |
| 3 | 1162.50 | 1.27 | 1519.36 | 14400 | 23.49 | 1406.12 | 1524.84 | 1443.78 | 2.61 | -8.05 | -5.23 | 27.08 |
| 4 | 1120.03 | 1.27 | 1433.51 | 14400 | 21.87 | 1262.19 | 1333.39 | 1285.71 | 1.83 | -13.57 | -11.50 | 22.30 |
| 5 | 1370.13 | 1.28 | 1788.55 | 14400 | 23.39 | 1612.51 | 1686.91 | 1640.71 | 1.72 | -10.92 | -9.01 | 28.37 |
| A- ${ }^{-}$ | -1208.95 | 1.27 | 1539.17 | -14400- | - 21.36 | ${ }^{-1409.15}$ | 1497.09 | 1442.06 | $2.29{ }^{-}$ | -9.27 | $-6.77$ | 26.72 |

## Comparing the ALNS with the LNS and other heuristics

In this section, we investigate the effect of each removal or insertion operator on the quality of the ALNS solution. We also perform several numerical experiments to identify the advantages of the adaptive strategy

Table 4.5. Solution gaps between the ALNS and exact solution values for the mAGTSP and AGTSP

|  | mAGTSP |  |  |  | AGTSP |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Inst. | ALNS | CPU | $Z$ | $G_{\text {ALNS }}^{Z}$ | ALNS | CPU | $Z$ | $G_{\text {ALNS }}^{Z}$ |
| Expr. 5 Inst. 1 | 656.44 | 1.63 | 653.96 | 0.38 | 1322.09 | 1.52 | 1307.68 | 1.09 |
| Expr. 6 Inst. 1 | 622.24 | 1.31 | 618.33 | 0.63 | 1258.88 | 1.19 | 1239.41 | 1.55 |
| Expr. 8 Inst. 1 | 1331.97 | 4.23 | 1313.27 | 1.40 | 2353.34 | 4.00 | 2286.96 | 2.82 |
| Expr. 9 Inst. 1 | 119.97 | 4.38 | 1188.81 | 0.60 | 2188.19 | 3.28 | 2122.98 | 2.98 |

Note. Gaps are averaged over 20 ALNS runs. columns $Z$ show the exact results.
of the ALNS compared with different LNS heuristics in which only a single removal operator and a single insertion operator are applied. Finally, we compare the results of the ALNS with those of some other heuristics, and with the optimal solution values of two relaxed problems.

Table 4.6 shows the effect on the performance of the ALNS when one of the removal or insertion operators is removed. Since we cannot insert new open locations for storage requests without removing them first, these two operators have to be removed together. Our results show that all operators have a positive impact on improving the objective value, especially for larger instances. In particular, location removal and insertion operators have a major effect on the quality of the solution. Without these operators, the locations chosen for stacking storage containers in the initial solution do not change during the algorithm. As a result, we lose the opportunity to improve the solution by choosing the other locations for storage containers. Since most of the operators are simple, the computation time does not change significantly by removing them.

Table 4.6. Effect of removing removal and insertion operators on the performance of the ALNS

| Inst. | Removal operators |  |  |  |  |  |  | Insertion operators |  |  | Location removal and insertion |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  | Group removal |  |  |  |  |  |
|  | Shaw | Random | Worst | Precedence | Single cycle | Retrievals | Storages | Random | Greedy | Interaction |  |
| Expr. 2 Inst. 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 3.90 | 0.00 | 0.00 | 3.58 |
| Expr. 5 Inst. 1 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.00 | 0.19 | 0.00 | 0.00 | 2.76 |
| Expr. 8 Inst. 1 | 4.46 | 4.98 | 4.51 | 5.12 | 4.54 | 5.26 | 5.29 | 7.64 | 5.43 | 5.15 | 8.06 |
| Ave ${ }^{-}$ | $1.49{ }^{-}$ | 1.66 | 1.50 | 1.71 | 1.51 | 1.76 | 1.76 | $3.91{ }^{-}$ | 1.81 | 1.72 | 4.80 |

Note. The numbers show the gaps which are the percentage of difference between the average objective value over 20 runs and the best objective value found by the ALNS.

Table 4.7 shows the effect of different insertion and removal operators in an LNS setting. This experiment was carried out on the first instance of Experiment 5 with $N=25$ and $\left|\mathcal{L}_{r}\right|=3, r=1, \ldots, 13$. We report the results of the LNS for 20 runs in each setting. The gap shows the difference between the average objective value of the 20 runs and the best objective value found during the full experiment. For example, in the first combination we use the Shaw removal and random insertion. The result shows that the average objective value is $14.65 \%$ more than the best known value. Table 4.7 shows that the ALNS can find better results than the LNS. We also test whether the poor performance of the LNS is due to the absence of the location removal and insertion operators in this algorithm. In Table 4.6, the location operators are shown to have a positive effect. The results of Table 4.7 show that although the performance of the LNS improves by including these
operators, it is not still as good as that of the ALNS.
Table 4.7. Comparing the ALNS with the LNS

|  | Comb. | Removal operators |  |  |  |  |  |  |  | Insertion operators |  |  |  | $G$ | $\underset{G}{\substack{\text { LNS }+ \\ \text { loc rem } \\ \hline \mathrm{ins}}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Shaw | Random | Worst | Precedence | Single cycle | ${ }_{\text {Retrievals }}^{\text {group re }}$ | group removal | Location | Random | Greedy | Interaction | Location |  |  |
|  | 1 | $\checkmark$ |  |  |  |  |  |  |  | $\checkmark$ |  |  |  | 14.65 | 9.97 |
|  | 2 | $\checkmark$ |  |  |  |  |  |  |  |  | $\checkmark$ |  |  | 21.18 | 16.96 |
|  | 3 | $\checkmark$ |  |  |  |  |  |  |  |  |  | $\checkmark$ |  | 18.86 | 14.49 |
|  | 4 |  | $\checkmark$ |  |  |  |  |  |  | $\checkmark$ |  |  |  | 3.43 | 0.01 |
|  | 5 |  | $\checkmark$ |  |  |  |  |  |  |  | $\checkmark$ |  |  | 8.09 | 2.95 |
|  | 6 |  | $\checkmark$ |  |  |  |  |  |  |  |  | $\checkmark$ |  | 2.92 | 0.49 |
| LNS | 7 |  |  | $\checkmark$ |  |  |  |  |  | $\checkmark$ |  |  |  | 6.13 | 4.63 |
|  | 8 |  |  | $\checkmark$ |  |  |  |  |  |  | $\checkmark$ |  |  | 15.11 | 10.24 |
|  | 9 |  |  | $\checkmark$ |  |  |  |  |  |  |  | $\checkmark$ |  | 11.32 | 9.19 |
|  | 10 |  |  |  |  |  |  |  |  | $\checkmark$ |  |  |  | 11.55 | 11.21 |
|  | 11 |  |  |  | $\checkmark$ |  |  |  |  |  | $\checkmark$ |  |  | 23.29 | 22.58 |
|  | 12 |  |  |  | $\checkmark$ |  |  |  |  |  |  | $\checkmark$ |  | 19.42 | 16.11 |
|  | 13 |  |  |  |  |  |  |  |  | $\checkmark$ |  |  |  | 4.21 | 1.56 |
|  | 14 |  |  |  |  | $\checkmark$ |  |  |  |  | $\checkmark$ |  |  | 13.28 | 10.96 |
|  | 15 |  |  |  |  | $\checkmark$ |  |  |  |  |  | $\checkmark$ |  | 8.72 11.02 | 5.95 782 |
|  | 16 |  |  |  |  |  | $\checkmark$ |  |  | $\checkmark$ |  |  |  | 11.02 | 7.82 |
|  | 17 |  |  |  |  |  | $\checkmark$ |  |  |  | $\checkmark$ |  |  | 18.85 | 16.48 |
|  | 18 |  |  |  |  |  | $\checkmark$ |  |  |  |  | $\checkmark$ |  | 20.70 | 18.67 |
|  | 19 |  |  |  |  |  |  | $\checkmark$ |  | $\checkmark$ |  |  |  | 7.99 | 6.24 |
|  | 20 |  |  |  |  |  |  | $\checkmark$ |  |  | $\checkmark$ |  |  | 19.19 | 14.42 |
|  | 21 |  |  |  |  |  |  | $\checkmark$ |  |  |  | $\checkmark$ |  | 14.63 | 13.38 |
| Avē | 22 |  |  |  |  |  |  |  | $\checkmark$ |  |  |  | $\checkmark$ | 22.07 |  |
| ALNS | 23 | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | 0.00 |  |

Note. The numbers in the last two columns show the gaps. The first gap is the percentage of difference between the average LNS objective value over 20 runs and the best objective value found. For obtaining the second gap shown in column LNS + loc rem/ins, we add the location removal and insertion operators to the pure LNS heuristic and obtain the average objective value over 20 runs. The gap is the percentage of difference between this average objective value and the best objective value found.

Finally, we compare the ALNS with some simple constructive heuristics such as nearest neighbor (NN), nearest neighbor in combination with double cycles (NNDC), farthest neighborhood (FN), and random heuristics. In the NN heuristic, each ASC travels to the nearest request until all requests are carried out. Due to the precedence constraints, sequencing the requests of each ASC regardless of the other one may result in a poor solution. For example, a seaside request which should precede some landside requests may be carried out last by the seaside ASC, which may delay the execution of some landside requests. As a result, we use a modified NN in which we choose a seaside or landside ASC with $50 \%$ probability and travel to the nearest requests of the previously carried out requests of that ASC. Obviously, we have to satisfy the interaction and precedence constraints. The NNDC heuristic is similar to NN, with the difference that the ASC not only travels to the nearest request but also tries to generate double cycles. The FN is the same as the NN with the difference that we always move to the farthest request. Each of these heuristics can be used in the first phase of the ALNS to generate an initial solution. In our implementation, we use the random heuristic to generate the initial solution. Table 4.8 shows that the ALNS outperforms all these heuristics in terms of the objective value. The average gap is more than $24 \%$ over all instances.

### 4.4 Conclusion

We have modeled and solved a difficult operational problem arising in a container terminal, consisting of scheduling two ASCs to execute a set of storage and retrieval requests in a block of containers. Several practical and theoretical constraints were considered: (1) the ASCs cannot pass each other and, for security

Table 4.8. Solution gaps between the ALNS and heuristic solution values for several constructive heuristics

| Inst. | ALNS | NN | $G_{N N}^{A L N S}$ | NNDC | $G_{N N D C}^{A L N S}$ | FN | $G_{F N}^{A L N S}$ | rand | $G_{\text {rand }}^{A L N S}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Expr. 2 Inst. 1 | 356.32 | 473.25 | 24.71 | 478.21 | 25.49 | 532.68 | 33.11 | 490.70 | 27.39 |
| Expr. 5 Inst. 1 | 952.04 | 1293.09 | 26.37 | 1339.93 | 28.95 | 1376.21 | 30.82 | 1348.89 | 29.42 |
| Expr. 8 Inst. 1 | 1436.01 | 2337.18 | 38.56 | 2254.73 | 36.31 | 2459.81 | 41.62 | 2388.56 | 39.88 |

Note. CPU times are omitted here since they are all small. Gaps are averaged over 20 runs.
reasons, the ASCs must be separated by a safety distance; (2) each storage container must be stacked in a location selected from a set of available open locations; and (3) because of waiting times and of the presence of several container transport modes, containers have different storage and retrieval priorities. We have formulated the problem as a multiple AGTSP with precedence constraints and ASC interaction constraints. Due to the complexity of the problem, it can be only solved exactly for small size instances. We have therefore developed an ALNS heuristic capable of solving instances of realistic sizes. Our experiments demonstrate the impact of the new constraints and the efficiency of our heuristic. For small instances which can be optimally solved by CPLEX, the gaps between the ALNS and optimal solution values are on average less than $0.34 \%$. For large instances, compared to CPLEX truncated after four hours, the ALNS can quickly obtain up to $6.77 \%$ better results, on average. It also yields results that are $24 \%$ better compared with alternative less sophisticated heuristics. Furthermore, for two relaxed problems, the ALNS obtain results that are on average within $3 \%$ of the optimal solution values. The Model can be extended to include collaboration of the ASCs and online update of the list of requests.

## Chapter 5

## Minimizing the Expected Number of Reshuffles *

As a result of the exponential container handling growth in the last decades, containers should be stacked multi-high at container terminals, using handling equipment like straddle carriers, rubber-tired gantry (RTG) cranes, or rail-mounted gantry (RMG) cranes. Stacking containers multi-high often leads to reshuffling. A reshuffle is the removal of a container stacked on top of a desired container. Containers are commonly stacked in sequence of their arrivals which can conflict with the retrieval sequence required by the ships' stowage plans. This results in a number of reshuffles. Containers of a ship are usually classified into different classes based on their weights and ports of destination. In order to ensure the ship's stability, heavier containers should be stacked in lower tiers of a ship. In addition, containers of ports of destination that will be visited by the ship later should be loaded onto the ship earlier. It is therefore beneficial if containers are stacked in a reverse sequence in every pile of the block. Moreover, if containers of multiple ships and ports of destination are mixed in a pile, a container leaving earlier should be stacked at a higher tier in order to avoid reshuffling.

It is rather difficult, or even impossible, to control the arrival sequence of containers at the terminal, as many containers arrive one by one by individual trucks or by internal transport trucks from rail or barge terminals. Therefore, they should be stacked onto piles as they arrive, which makes later reshuffling inevitable. Since reshuffling is time consuming and increases berthing times of ships, reducing the number of reshuffles, while retrieving containers, is a top concern for stacking containers (Kim, 1997, Zhao and Goodchild, 2010).

We study how to stack incoming containers in a container block with the objective of minimizing the expected number of reshuffles. We develop a heuristic algorithm to quickly find near-optimal solutions for large-scale problems with a realistic block size (expressed in number of piles). Our heuristic algorithm uses the results of a stochastic dynamic programming (DP) model built on work of Kim et al. (2000). We show that

[^3]the total number of states of the DP model increases exponentially in the dimensions of the block and types of containers. It follows that optimal solution of only small-scale problems can be obtained in a reasonable time. Therefore, we develop the heuristic algorithm in a way that the solution of large-scale problems can be quickly obtained. The algorithm maps all the states of the DP model for small-scale problems onto decision trees which help to recognize the correlation between the decisions made by the DP model for different states of the block. We are then able to simplify and generalize the trees into generalized decision trees using these similarities. The simplification and generalization significantly reduce the complexity of the trees compared to the original ones. As a result, the generalized trees can quickly locate incoming containers in a block with a real number of piles. We can always improve the decisions made by the generalized trees as long as DP results of larger-scale problems are available. The numerical experiments show that the decisions can be made in less than a second. The decisions are optimal for small-scale and near-optimal for large scale problems. The results obtained by the proposed heuristic are significantly better than the results obtained by stacking policies commonly used in practice. Using these trees, we also study the effect of a shared-stacking (SS) policy on reducing the expected number of reshuffles. An SS policy allows containers of multiple ships to be stacked on top of each other in a pile. This differs from a dedicated-stacking (DS) policy, where containers of a single ship cannot be mixed in a pile. Finally, the decision trees can be used to determine the storage locations in the models developed in the previous three chapters.

The decision tree (DT) heuristic is novel and differs from the one proposed by Kim et al. (2000) in several aspects. (1) Our method uses the DP results differently as our method maps all the states of DP solutions in the decision trees as nodes and branches and then simplifies and generalizes them, while Kim et al. (2000) use "machine learning logic" which produces the branches of decision trees one by one based on the amount of information each branch will provide for making decision on where to locate an incoming container. (2) Our heuristic can optimally solve small-scale problems. Furthermore, it can quickly find the solution of large-scale problems with a realistic number of piles that cannot be solved by the DP while Kim et al. (2000) solve problems up to six piles only. The quality of their solution even in case of six piles depends on the $C V P$. (3) Our decision trees are open for further improvements; when a larger problem is solved by the DP, our decision trees can be extended to include the new information provided by the DP and more accurately solve larger problems. Kim et al. (2000) need to rerun the algorithm to find new decision trees as their trees are not open to extension.

The rest of this chapter is organized as follows. In Section 5.1, we formulate a stochastic DP model to locate incoming containers. In Section 5.2, a heuristic solution algorithm is developed. Section 5.3 presents numerical results. Finally, in Section 5.4, we conclude the chapter and present possible future research topics.

### 5.1 Problem description and model

This section describes the research problem, introduces notations, and then formulates the problem as a stochastic DP model.

### 5.1.1 Problem description and notations

We study how to stack containers in a single container block by minimizing the expected number of reshuffles. We use the following notations and assumptions.

- Containers are of the same size (i.e. 20 feet) and are to be loaded onto $S$ ships. Let $Q_{s}$ be the total number of containers belonging to ship $s=1,2, \ldots, S$, and $Q=\sum_{s=1}^{S} Q_{s}$.
- In Section 1, we explained that to maintain the weight stability of a ship, heavier containers of that ship should be loaded onto lower tiers of the ship. Furthermore, ports of destination of containers should be taken into consideration when they are loaded onto the ship. Finally, we study a problem where containers of multiple ships can be stacked on top of each other in each pile of the block. Obviously, containers should be loaded onto the ships in the sequence of the departure of their corresponding ships.

In order to satisfy the constraints mentioned above, we define $C$ as the set of all types of incoming containers. Containers of type $c$ must be retrieved earlier than containers of type $c+1, \forall c \in C$. Containers of the same type can be retrieved in any order. The size of the set of all container types depends on the number of ships, number of weight groups and the number of ports of destination. Assume that $w_{s}$ and $d_{s}$ are the numbers of weight groups and ports of destination of ship $s, s=$ $1,2, \ldots, S$. Then, $|C|=\sum_{s=1}^{S}\left(w_{s} \times d_{s}\right)$ is the cardinality of $C$.

- Containers arrive at the container block individually and randomly and they are stored in the block based on a first-come-first-served (FCFS) sequence. The probability $\mathbb{P}(c)$ of receiving a container of type $c \in C$ is a function of its weight group, port of destination and the size of the corresponding ship. We assume uniform probabilities of receiving a container of type $c$ of ship $s$, or $\mathbb{P}(c)=\frac{Q_{s}}{\sum_{j=1}^{S}\left(w_{j} \times d_{j}\right) \times \sum_{i=1}^{S} Q_{i}}$. Obviously, $\sum_{c \in C} \mathbb{P}(c)=1$. The probabilities do not change by receiving containers.
- Each pile can be shared by containers of different ships up to $T$ tiers. We distinguish the type of a pile, $P_{c t}$, according to the type of its first-leaving container (not necessarily the topmost container), and the number of open slots available above the upmost container, $t, 1 \leq t \leq T-1$. Pile types with $t=0$ and $T$ are denoted as $P_{F}$ (full pile) and $P_{E}$ (empty pile), respectively.
- The block has $B$ bays, $R$ rows and $T$ tiers. The total number of piles, $V=B \times R$. Let $V_{c t}$, $V_{F}$ and $V_{E}$ denote the numbers of pile types $P_{c t}, P_{F}$ and $P_{E}$, respectively. Then, we have $V=$ $\sum_{c \in C} \sum_{t=1}^{T-1} V_{c t}+V_{F}+V_{E}$.
- Selecting the pile where an incoming container should be stacked is our decision variable. In order to avoid reshuffling, an incoming container should be assigned to a pile type whose first-leaving container will be retrieved later than the incoming container. This implies containers in each pile should be stacked according to the categories in $C$.


### 5.1.2 Dynamic programming (DP) model

In this section, the basic concepts of the stochastic dynamic programming (DP) are explained. To ease the understanding of different steps of the DP model in this section and the heuristic algorithm in Section 5.2, we use an example (denoted as Example 1) throughout this chapter. Example 1 is defined by $S=2, V=$ $2, T=4, Q_{1}=Q_{2}=4, w_{1}=w_{2}=3, d_{1}=d_{2}=1, C=\{1,2, \ldots, 6\}$, and $\mathbb{P}(c)=\frac{1}{6}, \forall c \in C$.
$X_{n}$ is the input state of the $n^{\text {th }}$ stage, $n=1, \ldots, N$. Let $N$ be the total number of empty slots in the current state of the block, where $N=V \times T$ is the maximum number of stages. The state of the DP model can be represented by a vector whose members show the number of different piles per pile type at stage $n$. Without loss of generality, the elements are sequenced in an increasing order of the container types and the number of empty slots, then $V_{F}$ and $V_{E}:\left(V_{11}, V_{12}, \ldots, V_{1, T-1}, V_{21}, V_{22}, \ldots, V_{2, T-1}, \ldots, V_{|C| 1}, V_{|C| 2}, \ldots, V_{|C|, T-1}, V_{F}, V_{E}\right)$. Table 5.1 illustrates state $X_{3}$ for a given block state of Example 1 where $N=2 \times 4=8$. For simplicity, in the rest of the chapter, we represent a state with its nonzero entries only.

Table 5.1. A schematic illustration of a state of a block


| $X_{3}$ |  |
| :---: | :---: |
| Full representation |  |
| $\left(V_{11}, V_{12}, V_{13}, V_{21}, V_{22}, V_{23}, V_{31}, V_{32}, V_{33}\right.$, |  |
| $\left.V_{41}, V_{42}, V_{43}, V_{51}, V_{52}, V_{53}, V_{61}, V_{62}, V_{63}, V_{F}, V_{E}\right)=$ |  |
| (0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0) |  |
| Simplified representation: |  |
| $X_{3}=\left(V_{12}, V_{21}\right)=(1$ |  |
| Note. The types of the to ers in $P_{12}$ are $c=3$ and $c$ | ost and the first leaving contain1 , respectively. |

$D_{n}$ is the decision variable at stage $n$. It tells where (onto which type of pile) to stack an incoming container, $c_{n}$, at stage $n$. This pile can be any non-full pile in the studied block, $D_{n} \in\left\{P_{c t} \mid V_{c t} \geq 1, c \in\right.$ $C, t=1, \ldots, T-1\} \cup\left\{P_{E} \mid V_{E} \geq 1\right\}$. For example, in case of Table 5.1, $D_{3} \in\left\{P_{12}, P_{21}\right\}$.
$S_{n}\left(X_{n}, D_{n}, c_{n}\right)$ is the state transfer function. Starting from state $X_{n}$, an incoming container $c_{n}$ is located in pile $D_{n}$ leading to state $X_{n-1}$. In Table 5.1, assume $c_{3}=1$, then $S_{3}\left(X_{3}, D_{3}, 1\right)$ maps $X_{3}$ to $X_{2}=\left(V_{11}, V_{21}\right)=(1,1)$, or $X_{2}=\left(V_{12}, V_{F}\right)=(1,1)$ by locating container $c_{3}=1$ in piles $P_{12}$ or $P_{21}$, respectively.
$g_{n}\left(X_{n}, D_{n}, c_{n}\right)$ is the marginal number of reshuffles added at stage $n$ due to stacking incoming container $c_{n}$ in pile $D_{n}$ with block state $X_{n} \cdot g_{n}\left(X_{n}, D_{n}, c_{n}\right)$ is either 0 or 1 . If stacking the incoming container in a pile of the current state of the block causes reshuffling, $g_{n}\left(X_{n}, D_{n}, c_{n}\right)=1$, otherwise $g_{n}\left(X_{n}, D_{n}, c_{n}\right)=0$.

In the previous example, $g_{3}\left(X_{3}, P_{12}, 1\right)=g_{3}\left(X_{3}, P_{21}, 1\right)=0$, whereas if $c_{3}=2$, then $g_{3}\left(X_{3}, P_{12}, 2\right)=1$ and $g_{3}\left(X_{3}, P_{21}, 2\right)=0$.

By repeating this calculation for every incoming container, we obtain the total (final cumulative) number of reshuffles of the block. For example, in a problem with only a single pile with 3 tiers, if the pile is stacked in a sequence of $c_{3}=1$ at the bottom, $c_{2}=3$ in the middle, and $c_{1}=3$ on the top, the total number of containers to be reshuffled is 2 . Stacking the first container does not generate any reshuffling, $g_{3}\left(X_{3}, P_{E}, 1\right)=0$. The arrival of the second container which is of type 3 generates one reshuffle and $g_{2}\left(X_{2}, P_{12}, 3\right)=1$. When the last container which is the second container of type 3 arrives, because the first-leaving container is of type 1 , we have another reshuffle which means that $g_{1}\left(X_{1}, P_{11}, 3\right)=1$. In total, the number of reshuffles of the pile becomes 2. Note that this is not the expected number of reshuffles, since the problem is deterministic in this case.
$f_{n}\left(X_{n}\right)$ is the total expected number of reshuffles up to stage $n$. The total expected number of reshuffles to completely fill up an empty block with the capacity of total number of $N$ containers can be formulated as follows:

$$
\begin{equation*}
f_{N}\left(X_{N}\right)=\sum_{c_{N} \in C} \min _{D_{N}} \cdots \sum_{c_{1} \in C} \min _{D_{1}}\left[\prod_{m=1}^{N} \mathbb{P}\left(c_{m}\right) \sum_{n=1}^{N} g_{n}\left(X_{n}, D_{n}, c_{n}\right)\right] \tag{1}
\end{equation*}
$$

where, $f_{0}^{*}\left(X_{0}\right)=0$. It can be concluded that in order to minimize the total expected number of reshuffles up to stage $n$, starting from state $X_{n}$ and filling up the entire block with incoming containers, the following recursive function of the DP model can be obtained from Equation (1) (see Zhang et al., 2010).

$$
\begin{equation*}
f_{n}^{*}\left(X_{n}\right)=\sum_{c_{n} \in C} \mathbb{P}\left(c_{n}\right) \min _{D_{n}}\left[g_{n}\left(X_{n}, D_{n}, c_{n}\right)+f_{n-1}^{*}\left(X_{n-1}\right)\right] \tag{2}
\end{equation*}
$$

where, $X_{n-1}=S_{n}\left(X_{n}, D_{n}, c_{n}\right), f_{0}^{*}\left(X_{0}\right)=0$, and the decision variable in stage $n$ is $D_{n} \in\left\{P_{c t} \mid V_{c t} \geq 1, c \in\right.$ $C, t=1, \ldots, T-1\} \cup\left\{P_{E} \mid V_{E} \geq 1\right\}$. The DP model can be solved by successively solving $f_{1}^{*}\left(X_{1}\right), f_{2}^{*}\left(X_{2}\right)$, $\ldots, f_{N}^{*}\left(X_{N}\right)$. The optimal location, $D_{n}^{*}$ can be obtained for each $c_{n}, n=1, \ldots, N$. In case of a tie in a state, the optimal location is randomly selected. $f_{N}^{*}\left(X_{N}\right)$ is the minimum expected number of reshuffles for stacking $N$ containers, which implicitly assumes $N=Q$. For $Q \leq N$, we only need to solve a part of the model until stage $Q, f_{1}^{*}\left(X_{1}\right), \ldots, f_{Q}^{*}\left(X_{Q}\right)$.

Table 5.2 presents all the required states in stages $n=1$ and 2 to show how the DP model calculates the expected number of reshuffles in state $X_{3}=\left(P_{12}, P_{21}\right)=(1,1)$ of Example 1. Consider that the incoming container is $c_{3}=1$. Given $X_{3}$, by locating $c_{3}=1$ in $P_{21}$, we obtain $g_{3}\left(X_{3}, P_{21}, 1\right)=0$ and $X_{2}=\left(V_{12}, V_{F}\right)=$ $(1,1)$ which yields $g_{3}\left(X_{3}, P_{21}, 1\right)+f_{2}^{*}\left(X_{2}\right)=0+\frac{60}{36}$. However, by locating $c_{3}=1$ in $P_{12}$, we obtain $g_{3}\left(X_{3}, P_{1,2}, 1\right)=0$ and $X_{2}=\left(V_{11}, V_{21}\right)=(1,1)$, which results in $g_{3}\left(X_{3}, P_{12}, 1\right)+f_{2}^{*}\left(X_{2}\right)=0+\frac{49}{36}<\frac{60}{36}$. Therefore, $P_{12}$ is selected to locate $c_{3}=1$, because it results in a smaller expected number of reshuffles. The
optimal solution of the other states can be obtained by the DP model in a similar fashion.
Table 5.2. Optimal objective values of some states of the DP model

| State | Incoming container $\left(c_{n}\right)$ |  |  |  |  |  | $f_{n}^{*}\left(X_{n}\right)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | 4 | 5 | 6 |  |
| $X_{1}=\left(P_{11}, P_{F}\right)=(1,1)$ | $0+0$ | $1+0$ | $1+0$ | $1+0$ | $1+0$ | $1+0$ | $\frac{5}{6}$ |
| $X_{1}=\left(P_{21}, P_{F}\right)=(1,1)$ | $0+0$ | $P_{11}$ | $P_{11}$ | $P_{11}$ | $P_{11}$ | $P_{11}$ | $1+0$ |
|  | $P_{21}$ | $P_{21}$ | $P_{21}$ | $1+0$ | $P_{21}$ | $P_{21}$ | $1+0$ |
| $X_{21}$ | $\frac{4}{6}$ |  |  |  |  |  |  |
| $X_{2}=\left(P_{12}, P_{F}\right)=(1,1)$ | $0+\frac{5}{6}$ | $1+\frac{5}{6}$ | $1+\frac{5}{6}$ | $1+\frac{5}{6}$ | $1+\frac{5}{6}$ | $1+\frac{5}{6}$ | $\frac{60}{36}$ |
|  | $P_{12}$ | $P_{12}$ | $P_{12}$ | $P_{12}$ | $P_{12}$ | $P_{12}$ | 4 |
|  | $P_{11}$ | $0+\frac{5}{6}$ | $1+\frac{4}{6}$ | $1+\frac{4}{6}$ | $1+\frac{4}{6}$ | $1+\frac{4}{6}$ | $\frac{49}{36}$ |
| $X_{3}=\left(P_{12}, P_{21}\right)=(1,1)$ | $0+\frac{49}{36}$ | $0+\frac{60}{36}$ | $1+\frac{49}{36}$ | $1+\frac{49}{36}$ | $1+\frac{49}{36}$ | $1+\frac{49}{36}$ | $\frac{449}{216}$ |

Theorem 5.1 The total number of states in the DP model is $\binom{J+V-1}{V}-1$, where $V$ is the total number of piles in the block, $J=|C| \times(T-1)+2$ is the total number of pile types, $C$ is the set of all types of incoming containers and $T$ is the number of tiers.

Proof. See the proof in 5.A.
Our DP model is adapted from the one proposed by Kim et al. (2000) for the case of the DS policy. Intuitively, the SS policy outperforms the DS policy in terms of the expected number of reshuffles since the SS policy is a strict relaxation of the DS model in which containers of different ships must be stacked separately in different piles. Unfortunately, solving the DP model is very time consuming because the total number of states grows exponentially with the size of the problem. Kim et al. (2000) have explained that even in case of $S=1$ ship (the DS policy), the DP model requires too long computation time. They therefore develop a heuristic. Solving the DP model with multiple ships ( $S \geq 2$ ) is even more complex. Based on Theorem (1), $\binom{11+4-1}{4}-1=1000$ states must be enumerated for stacking containers of 1 ship, divided into 3 types (i.e., heavy, medium, light) with a single port of destination, in a container block with only 4 piles, each 4 tiers, whereas in case the same block is shared between 2 or 3 ships, the number of states increases to 8854 and 35959, respectively. Due to this significant increase of the number of states, we have to resort to a heuristic algorithm, especially in case a real size block with more than 100 piles of which the number of states grows to more than $25 \times 10^{21}$ states in case of 2 ships and 4 -tier piles.

### 5.2 Decision-tree heuristic

In this section, we deploy the optimal solutions of the DP model obtained for problems with a small number of piles to generate decision trees in a decision-tree (DT) heuristic which can be applied to solve large-scale
problems with an a realistic number of piles. A tree classifies a database (i.e., the DP results) and helps to make fast and comprehensible decisions (Quinlan, 1986). It consists of nodes where a logical decision must be made and connecting branches that are chosen according to the result of this decision. The nodes and branches that are followed constitute a sequential path through a decision tree that reaches a final decision in the end.

The intuition for using the DT approach is as follows. Every state of a block can be represented by a path, where the decision where to locate the next incoming container in that state is given by the last node in that path. Collecting all these paths together constructs a decision tree. By increasing the size of the problem in terms of the number of piles, the number of paths grows exponentially fast. The reason is that we need to represent every state of the block with an individual path and according to Theorem (5.1), the number of states in the DP model increases exponentially. So, it is difficult to find a specific path in the tree to make a quick decision. We therefore simplify the tree by merging some of its paths. We then generalize the merged paths so that they can represent the states of a larger scale problem. These trees however, might lead to rather poor solutions as not all paths are represented, so we improve them with better paths, if the optimal DP results are available. The three types of trees that result from this approach are called Basic, Simplified and Generalized trees, and are more formally defined in the following three steps:

1. Step 1. Representing the optimal results of the DP as basic decision trees: In this phase, we represent all states of the DP model for small-scale problems as basic decision trees. In these trees, each state and its corresponding decision is illustrated by a path where nodes represent the pile types and weighted links represent the number of piles per pile type in the current state. Decisions can be found in the last node of each path. The basic decision trees are quite helpful for solving small-scale problems. However, the number of paths of the trees grows exponentially and solving the problem becomes time consuming as the size of the problem increases.
2. Step 2. Simplifying and generalizing the basic decision trees: This phase simplifies the basic decision trees, generated in the previous step so that the solution can be obtained more quickly. Furthermore, we generalize them to blocks with a larger number of piles. The simplification can be carried out by clustering multiple paths of the decision trees with the same decision as a single path. Simplifying the decision trees significantly decreases the number of paths. These trees can be used for quickly solving small-scale problems which are already solved by the DP model. We need to generalize them so that they can solve real-scale problems. Generalizing is realized by using the decisions of simplified decision trees for problems with larger number of piles. The generalized decision trees can quickly make decisions for large-scale problems since they have few paths, whereas the DP model needs to evaluate an enormous number of states.
3. Step 3. Improving the generalized decision trees: If the DP results for larger size problems are available, the generalized decision trees generated in step 2 can be improved to more precisely solve larger
size problems. This is realized by comparing the decisions made by the current generalized decision trees and the DP model of a larger problem for a specific state of the block and removing possible inconsistencies in the decisions. Removing an inconsistency from a generalized decision tree adds new nodes and branches.

Steps 1, 2, and 3 of the algorithm are explained in detail in Sections 5.2.1, 5.2.2, and 5.2.3, respectively.

### 5.2.1 Step 1. Representing the optimal results of the DP by basic decision trees

As long as the number of piles $V$ is small, for every incoming container $c$, the DP can make an optimal decision $D_{n}^{*}$ according to the state of the block $X_{n}$. For a fixed number of piles $V$, we construct a basic decision tree for every type of incoming container $c \in C$, denoted by $B_{c V}$. Each state $X_{n}$ is represented as a path in this tree basically counting the number of piles of each type in $X_{n}$. A path consists of the root node, inner nodes, weighted links, and defines the optimal decision $D_{n}^{*}$ in a so-called leaf node. The elements of a tree are defined as follows:

1. Paths. A path is a sequence of nodes and links from the root node to a decision node. In basic decision trees, all nodes in a path together with the weighted links uniquely represent a single block state, $X_{n}$. Note that each path represents the full representation of a state with all the zero entries (see Table 5.1).
2. Nodes. Every node in each path that has at least one child node represents a pile type of the corresponding state $\left(P_{n t}, n \in C, 1 \leq t \leq T-1, P_{F}\right.$ or $\left.P_{E}\right)$. Node B is a child node of node A if it is located immediately below node A and linked to node A. We call the topmost node of a tree the root node. It is the reference point to follow and find a path that represents a DP state.
3. Weighted links. A link corresponds to a condition to branch a node into child nodes. The weight of the link from pile type $P_{n t}$ (a parent node) to $P_{m l}$ (a child node) represents the number of piles of the type of the parent node, $V_{n t}$, in the current state that the path represents. If no pile of type $P_{n t}$ exist in the path, in other words if $V_{n t}=0$, then the weight of the link is 0 . In basic decision trees, the summation of the weights of all weighted links in a path always equals $V$.
4. Leaf or decision nodes. A node for which the sum of the weights in the path leading to it from the root node equals $V$ and which does not have any child node is called a leaf or decision node. In every leaf node, a decision, $D_{n}$, is made to locate an incoming container $c$ in one of the piles of state $X_{n}$.

Figure 5.1 shows a part of $B_{12}$. In this figure, the path $P_{13} \xrightarrow{0} P_{12} \xrightarrow{1} P_{11} \xrightarrow{0} P_{23} \xrightarrow{0} P_{22} \xrightarrow{0} P_{21} \xrightarrow{1} P_{12}$ highlighted by the gray dotted arcs represents state $X_{3}=\left(V_{12}, V_{21}\right)=(1,1)$ with the decision $D_{3}^{*}=P_{12}$ to stack $c_{3}=1$. This is one of the states of Example 1 discussed in Table 5.1 of Section 5.1.2. Furthermore,

Table 5.2 in Section 5.1 .2 shows why $D_{3}^{*}=P_{12}$ is the optimal location. Using the elements of a tree, basic decision trees can be built in 4 sub-steps as follows:


Figure 5.1. A part of $B_{12}$

Note. The gray dotted arcs represent the path $P_{13} \xrightarrow{0} P_{12} \xrightarrow{1} P_{11} \xrightarrow{0} P_{23} \xrightarrow{0} P_{22} \xrightarrow{0} P_{21} \xrightarrow{1} P_{12}$, which corresponds to $X_{3}=\left(V_{12}, V_{21}\right)=(1,1)$ with $D_{3}^{*}=P_{12}$. The paths in the contour specify the states in which $V_{13}>1$. In all these sates, the optimal decision is $P_{13}$.

Step 1.1. Solve the DP model and obtain the optimal results for a problem with $V$ piles.
Step 1.2. Choose the type of incoming container for which a basic decision trees will be built: a basic decision tree $B_{c V}$ will be built for every type of incoming container $c \in C$.
Step 1.3. Map all the states of the DP model on $B_{C V}$ : in a basic decision tree, every state of the DP model and the corresponding decision made is represented by an individual path from the root node to a leaf node. In other words, the total number of paths of each basic decision tree equals the total number of states of the DP model. In each path, the pile types, number of piles per type, and decision made for a state of a block appear on the nodes, weighted links and leaf node, respectively.

Step 1.4. Order the weighted links and nodes on $B_{c V}$ : without a good representation of $X_{n}$, the decision trees may become large. In order to speed up searching the trees to find a specific state, weighted links connected to each node, are sorted from left to right in decreasing order of their weights. Furthermore, we sort the pile types, which appear in the nodes of different levels of $B_{c V}$, in decreasing order of the total number of times that each pile type is chosen in every possible state of the block to locate the incoming container.

### 5.2.2 Step 2. Building generalized decision trees

The basic decision trees, $B_{c V}, \forall c \in C$, contain optimal decisions for problems solved by the DP model for $V$ or smaller number of piles. In order to solve larger problems, we aim to generalize such trees to obtain generalized decision trees, $G_{c V}, \forall c \in C$. The generalized trees still give optimal decisions for the problems solved by the DP model. In order to obtain the generalized trees, we first simplify $B_{c V}, \forall c \in C$. A closer look at a basic decision tree, for a given $c \in C$, reveals that the optimal decision to locate an incoming container is the same in many paths. These paths yielding the same decision can be replaced by a single path. We then generalize the decision made in each path to problems with real number of pules.

Followings, it is explained how to simplify and generalize $B_{c 2}$ in order to obtain $G_{c 2}, \forall c \in C$. Generalizing basic decision trees of larger instances with $V>2$ is more complicated, and also unnecessary since those generalized decision trees can more easily be obtained by comparing $G_{c 2}$ with basic decision trees of such large instances and then improving $G_{c 2}, \forall c \in C$. Improving generalized decision trees is explained in step 3 in Section 3.3.

## Step 2.1. Simplifying basic decision trees

The basic decision trees have many nodes and branches. As a result, searching through them is slow implying they can only be used to solve small-scale problems. We simplify the basic decision trees by clustering all paths with a common decision as a single path. Clustering the paths of these basic decision trees results in significantly decreasing the density of the trees. Therefore, they can quickly solve small-scale problems. Furthermore, in the next step, the clusters and their corresponding decisions made are used to generalize the trees so they can solve large-scale problems. The simplification is carried out as follows.

We start from the root node of $B_{c 2}$. Let the pile type of the root node be $P_{q t}$. If for every path in which $V_{q t}>0$, the optimal decision is $D_{n}^{*}=P_{q t}$, we can simplify $B_{c 2}$ by replacing all those paths connecting the root node to a leaf $D_{n}^{*}=P_{q t}$ by a single path. The path connects the root node to the leaf node with a link whose weight is an integer contained in the interval 1 to $2\left(P_{q t} \xrightarrow{0<V_{q t} \leq 2} P_{q t}\right)$, indicating that if there is at least one pile of type $P_{q t}$, then the decision is always to locate the incoming container $c$ in that pile. One path can therefore represent multiple states, but one state is only represented by one path. Now, suppose in one of the paths with $V_{q t}>0, D_{n}^{*}=P_{q^{\prime} t^{\prime}} \neq P_{q t}$. In this case, in order to represent all optimal decisions made in different paths in $B_{c 2}$ with the minimum number of paths in simplified $B_{c 2}$, we then replace all paths with 2 paths: $P_{q t} \xrightarrow{0<V_{q t} \leq 2} P_{q^{\prime} t^{\prime}} \xrightarrow{1 \leq V_{q}^{\prime} t^{\prime}} P_{q^{\prime} t^{\prime}}$ and $P_{q t} \xrightarrow{0<V_{q t} \leq 2} P_{q^{\prime} t^{\prime}} \xrightarrow{V_{q^{\prime} t^{\prime}}=0} P_{q t}$. The second path indicates that if only a single pile $P_{q t}$ exists in the block and the second pile is not $P_{q^{\prime} t^{\prime}}$, then the incoming container will be located in $P_{q t}$ regardless of the type of the second pile.

Having simplified all paths of $B_{c 2}$ with $V_{q t}>0$, we next select the child node of the root node connected with the weighted link $V_{q t}=0$. We look for all the paths from the child node for which at least 1 pile of that type exists in the block. These paths are simplified in the same way as we did for the root node. Note that only that part of the path connected to this child node will be simplified. This is repeated by moving to a node connected to the child node with a link whose weight is zero until all paths in the basic decision tree
are simplified.
The following property quantifies the complexity (in terms of the number of paths) of the simplified decision trees compared to the basic decision trees. The simplified decision trees are generated based on $B_{c 2}, \forall c \in C$. The number of paths of $B_{c 2}$ for a given $c \in C$ can be calculated based on Theorem (5.1). On the other hand, each simplified decision tree has in total $J-1$, with $J=|C| \times(T-1)+2$, paths if starting from the root node all decisions are consistent and we do not need to create extra paths to remove some conflicts as mentioned above. However, in some instances, a few more paths may be needed to cover all decisions made by the basis decision trees. The new paths do not significantly increase the complexity of the simplified decision trees. So, we have the following property:

Proposition 5.1 The percentage of reduction of paths in the simplified $B_{c 2}$ compared with $B_{c 2}$, for a given $c \in C$, is less than or equal to $\left(1-\frac{J-1}{\binom{J+2-1}{2}-1}\right) \%=\frac{J}{J+2} \%$.

In Example 1, 209 paths of $B_{12}$ are replaced with the 19 paths of the simplified $B_{12}$ which means that more than $90 \%$ of the paths are omitted. Figure 5.2 a shows a part of the simplified $B_{12}$. The single path contained in the contour represents all the paths contained in the contour in $B_{12}$ shown in Figure 5.1. $P_{13} \xrightarrow{0<V_{q t} \leq 2} P_{13}$ means that if at least 1 pile of type $P_{13}$ exists, $c_{n}=1$ will be located in $P_{13}$ regardless of the type of the second pile. In $B_{12}$ shown in Figure 5.1, it can be observed that the decision is independent of the type of the second pile as long as $V_{13} \geq 1$.


Figure 5.2. A part of (a) simplified tree $B_{12}$, and (b) generalized tree $G_{12}$

Step 2.2. Generalizing simplified basic decision trees
Simplified decision trees cannot be used to solve small-scale problems. The reason is that all weighted links are limited to at most two piles and the summation of the weights of all weighted links in a path always equals two piles. We aim to generalize decisions made by the simplified decision trees so that we can solve a block with a large number of piles. As a result, one obvious solution to generalize the simplified decision trees is to remove the upper bounds. The generalized decision trees generated in this way can be used to solve larger problems with more than two piles. These trees still give optimal decisions for the problems with two piles.

Figure 5.2 depicts $G_{12}$ of Example 1. This trees is generated by removing the upper bounds from the weighted links of $B_{12}$. In this tree, to make decision for any state of the block with any number of piles, we start from the root node. If $V_{13}>0$ in the block, an incoming container of type 1 will be located in $P_{13}$ regardless of the number of piles in the block $(V>2)$. Otherwise, we check $P_{12}$, if $V_{12}>0$, the incoming container will be located there; otherwise, we will continue further down the tree until we find a pile type of which at least one pile exists in the block.

The following property quantifies the complexity (in terms of the number of paths) of $G_{c 2}$, compared to $B_{c V}$, for a given $c \in C$ and a realistic number of piles. The number of paths of $B_{c V}, \forall c \in C$, can be calculated based on Theorem (5.1), since each of them includes all paths from the DP model. The number of paths in $G_{c 2}$, for a give $c \in C$, is in total $J-1$, if the corresponding simplified $B_{c 2}$ does not have extra branches to cover all decisions of $B_{c 2}$. Extra branches however do not significantly increase the complexity. We can formulate this as follows.

Proposition 5.2 The percentage of reduction of paths in $G_{c 2}$, generated based on the simplified $B_{c 2}$, compared with $B_{c V}$, for a given $c \in C$, is less than or equal to $\left(1-\frac{J-1}{\binom{J+V-1}{V}-1}\right) \%$.

In Example 1, $G_{12}$ shown in Figure 5.2b has 19 paths, whereas $B_{13}$ and $B_{14}$ have 1539 and 8854 paths, respectively. As a result, $G_{12}$ can make decision for problems with three and four piles where the number of paths in $G_{12}$ is $98 \%$ and $99 \%$ less than $B_{13}$ and $B_{14}$, respectively. Similarly, the complexity of larger basic decision trees can be compared with $G_{12}$.

### 5.2.3 Step 3. Improving the generalized decision trees

A generalized decision tree, $G_{c i}, \forall c \in C$, can be improved using a basic decision tree, $B_{c V}, \forall c \in C$, with $V>2$. In general, $G_{c i}, i>2$, is generated by comparing $G_{c, i-1}$ and $B_{c i}$, and removing all inconsistencies. An inconsistency occurs when the decisions made by a generalized decision tree and the optimal decision made by a basic tree differ for the same state. There is no theoretic limit on the number of piles used to improve the generalized decision trees. The improvement can consider all the piles in the block. However, the
number of piles included is limited by the computation time needed. Based on the numerical results reported below, improving the generalized trees based on the basic trees of problems with only a small number of piles suffices to obtain good solutions for much larger problems.

## Step 3.1. Finding inconsistencies

An inconsistency can be found by comparing the decisions made by $G_{c, i-1}$ and $B_{c i}$ for the same state of the block. Therefore, for every state of the block with $V=i$, we must find the paths in $G_{c, i-1}$ and $B_{c i}$, representing that state. If the decisions in the leaf nodes of the two paths are not the same, an inconsistency is detected.

Figure 5.3 shows an example of an inconsistency. Figure 5.3a and Figure 5.3b depict parts of $G_{12}$, and $B_{13}$, respectively. These trees are used to locate $c=1$ in a block with 3 piles. In both trees the gray dotted arcs depict state $X_{5}=\left(V_{21}, V_{22}\right)=(1,2)$. The decision nodes reveal that $P_{21}$ is selected by $G_{12}$ to locate $c_{5}=1$, whereas $B_{13}$ shows that the optimal decision is $P_{22}$. In other words, decisions are not consistent.


Figure 5.3. A part of (a) generalized tree $G_{12}$, (b) basic tree $B_{13}$, and (c) improved generalized tree $G_{13}$

Note. The gray dotted arcs show state $X_{5}=\left(V_{21}, V_{22}\right)=(1,2)$. The decisions made by the basic and generalized trees are shown in the squares when $c_{5}=1$.

## Step 3.2. Removing inconsistencies

In order to remove an inconsistency from a path in $G_{c, i-1}$ representing state $X_{n}$, that path will be split into multiple paths in $G_{c i}$. This is carried out by adding a node for each pile type of which at least one pile exists in $X_{n}$ in $B_{c i}$ to the path representing $X_{n}$ in $G_{c, i-1}$, in case that node does not exist in that path
in $G_{c, i-1}$. Each of these nodes presenting a pile type of $X_{n}$ will have two links in $G_{c i}$ of which the weight of one is larger than or equal to the number of piles of that type in $X_{n}$ in $B_{c i}$, and the other one has a weight smaller than this number. Using only two weighted links allows us to remove the inconsistency with a minimum increase in density of the generalized trees. Finally, to comply with the optimal decision made by $B_{c i}$, the decision in the leaf node of the path representing $X_{n}$ will be the decision made by $B_{c i}$ and the decision of others will be the decision made by $G_{c, i-1}$.

In order to remove the inconsistency from $G_{12}$ shown in Figure 5.3a, we need to add a node for all the pile types in $X_{5}=\left(V_{21}, V_{22}\right)=(1,2)$ to $P_{13} \xrightarrow{0} \ldots \xrightarrow{0} P_{21} \xrightarrow{>0} P_{21}$, which represents $X_{5}$ in $G_{12}$. Therefore, $P_{22}$ must be added to the path since $P_{21}$ already exists in the path. In $G_{13}$, the node representing $P_{21}$ has two links with the weights $\geq 1$ and $0(<1)$ because $V_{21}=1$ in $X_{5}$. Furthermore, the node representing $P_{22}$ has two links with the weights $\geq 2$ and $<2$, as $V_{22}=2$ in $X_{5}$. Finally, the decision in the path $P_{13} \xrightarrow{0} \ldots \xrightarrow{0} P_{21} \xrightarrow{\geq 1} P_{21} \xrightarrow{\geq 2} P_{22}$ changes to $P_{22}$, since $B_{13}$ has the optimal decision for $X_{5}$. A part of $G_{13}$ is shown in Figure 5.3c. In order to speed up the search to find a solution, we try to obtain simpler trees by generating a smaller number of branches. Therefore, we only add and branch a node for the first and last pile types in a state. Note that different pile types in all states are sequenced. For example, if other pile types also exist between $P_{21}$ and $P_{22}$, only these two are again added and branched.

### 5.3 Numerical experiments

Multiple numerical experiments have been performed to evaluate the performance of the DT heuristic. In Section 5.3.1, we compare the results of the DT heuristic with the results of the DP model. Then, through a simulations study, we compare the SS and DS policies. In Section 5.3.2, through a case study, we show that the DT heuristic outperforms heuristics commonly used at large container terminals in terms of the expected number of reshuffles for real-scale problems.

The study is performed on a Notebook with 2.40 GHz Intel ${ }^{\circledR}$ Core $^{\mathrm{TM}}$ i5 processor, with 4.00 GB of RAM and the programming language is MATLAB ${ }^{\circledR}$ 2010a.

### 5.3.1 Algorithm performance evaluation

In this section, we evaluate the performance of generalized decision trees by comparing them with optimal results. We focus on a small-scale problem with seven piles and four tiers where containers of three types should be stacked upon each other.

Table 5.3 presents the results of comparing $G_{c V}, V=2, \ldots, 7$, with $B_{c 7}, c \in\{1,2,3\}$ which contains all the states and optimal decisions for a seven pile problem. As explained in Section 5.2, $G_{c 7}$ also contains all the optimal decisions in $B_{c 7}, c \in\{1,2,3\}$ for every state. Table 5.3 shows that $G_{c 7}, c \in\{1,2,3\}$ leads to the same decisions as $B_{c 7}, c \in\{1,2,3\}$ for stacking incoming containers in different states of the block. Using
$G_{c V}, c \in\{1,2,3\}$, where $V<7$ results in decisions that are not always in line with decisions made by $B_{c 7}$, $c \in\{1,2,3\}$. However, one can observe that the percentage of wrong decisions is low (less then $5.72 \%$, on average) even if $G_{c 2}, c \in\{1,2,3\}$ are used for stacking containers in a seven pile problem. The other point is that the complexity of generalized decision trees is significantly lower compared with the basic decision trees. For example, the largest generalized decision trees, $G_{c 7}, c \in\{1,2,3\}$, on average have less than 547.7 paths, whereas each $B_{c 7}$, for every $c \in\{1,2,3\}$ has 19477 paths. The average number of paths decreases to 10 for $G_{c 2}, c \in\{1,2,3\}$.

We also compare our generalized trees with decision trees generated by the heuristic proposed by Kim et al. (2000). Table 5.4 show when we have the complete DP results of a problem with $V=6$ piles, our heuristic algorithm generates generalized trees which can make optimal decisions in every state of the block whereas the decisions made by the trees generated by the heuristic proposed by Kim et al. (2000) depend on $C V P$ which is a value that specifies the amount of pruning of the trees.

Table 5.3. A comparison of different generalized trees and $B_{c 7}, c \in\{1,2,3\}$

|  | $c \in C=\{1,2,3\}$ |  |  | Average |
| :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 |  |
| Number of paths in $B_{c 7}$ | 19477 | 19477 | 19477 | 19477 |
| Number of paths in $G_{c 7}$ | 336 | 711 | 596 | 547.7 |
| Number of wrong decisions by $G_{c 7}$ | 0 | 0 | 0 | 0 |
| Percentage of wrong decisions | 0 | 0 | 0 | 0 |
| Number of paths in $\overline{\text { in }} \bar{B}_{c 6}^{-}$ | $80 \overline{0} \overline{7}$ | 8007 | $\overline{800} \overline{7}$ | $\overline{80} \overline{0} \overline{7}$ |
| Number of paths in $G_{c 6}$ | 150 | 335 | 245 | 243.3 |
| Number of wrong decisions by $G_{c 6}$ | 123 | 327 | 297 | 249 |
| Percentage of wrong decisions | 0.63 | 1.68 | 1.52 | 1.28 |
| Number of paths in $\overline{\mathrm{i}}^{-} \bar{B}_{c 5}^{-}$ | 3002 | $\overline{30} \overline{2}$ | $\overline{3} 00 \overline{2}$ | 3002 |
| Number of paths in $G_{c 5}$ | 78 | 148 | 88 | 104.7 |
| Number of wrong decisions by $G_{c 5}$ | 245 | 824 | 610 | 559.7 |
| Percentage of wrong decisions | 1.26 | 4.23 | 3.13 | 2.87 |
| Number of paths in $\overline{\text { in }} \bar{B}_{c 4}^{-}$ | 1000 | 1000 | 1000 | 1000 |
| Number of paths in $G_{c 4}$ | 37 | 63 | 46 | 48.7 |
| Number of wrong decisions by $G_{c 4}$ | 411 | 1335 | 798 | 848 |
| Percentage of wrong decisions | 2.11 | 6.85 | 4.10 | 4.35 |
| Number of paths in $\overline{\text { in }}^{-} \bar{B}_{c 3}$ | $\overline{2} \overline{5}$ | $\overline{2} \overline{8} 5$ | $\overline{2} \overline{8} 5$ | $\overline{2} 85$ |
| Number of paths in $G_{c 3}$ | 22 | 23 | 18 | 21 |
| Number of wrong decisions by $G_{c 3}$ | 467 | 1784 | 1154 | 1135 |
| Percentage of wrong decisions | 2.40 | 9.16 | 5.92 | 5.83 |
| $\overline{\text { Number }} \overline{-} \overline{\text { of paths }} \overline{\text { in }} \bar{B}_{c 2}^{----}$ | 65 | -65 | 65 | $6 \overline{5}$ |
| Number of paths in $G_{c 2}$ | 10 | 10 | 10 | 10 |
| Number of wrong decisions by $G_{c 2}$ | 512 | 1573 | 1258 | 1114.3 |
| Percentage of wrong decisions | 2.63 | 8.08 | 6.46 | 5.72 |

Table 5.5 compares the decisions made by $G_{c V}, V=2, \ldots, 7, c \in\{1,2,3\}$ and the optimal decisions made by the DP model. The first column shows the decisions made by the DP model and the first row shows the

Table 5.4. A comparison of $G_{6}^{c}, B_{6}^{c}, c \in\{1,2,3\}$ and decision trees proposed by Kim et al. (2000)

|  | C |  |  | Average |
| :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 |  |
| Number of paths in $B_{6}^{c}$ | 8007 | 8007 | 8007 | 8007 |
| Number of paths in $G_{6}^{\text {c }}$ | 150 | 335 | 245 | 243.3 |
| Number of wrong decisions by $B_{6}^{c}$ | 0 | 0 | 0 | 0 |
| Percentage of wrong decisions | 0 | 0 | 0 | 0 |
| Decision trees proposed by Kim et al. (2000) |  |  |  |  |
| Number of paths ( $C V P=0.05$ ) | 152 | 235 | 162 | 183 |
| Number of wrong decisions | 9 | 143 | 98 | 83.3 |
| Percentage of wrong decisions | 0.11 | 1.79 | 1.22 | 1.04 |
| Number of branches ( $\bar{C} \bar{V} \bar{P}=\overline{0}-4)$ | $\overline{3} 5$ | 10 | 10 | 18.3 |
| Number of wrong decisions | 288 | 645 | 397 | 443.3 |
| Percentage of wrong decisions | 3.60 | 8.06 | 4.96 | 5.5 |

decisions made by $G_{c 7}, c \in\{1,2,3\}$. In the first line of each entry, the three numbers in parentheses indicate the number of times that the decisions made respectively by $G_{17}, G_{27}$, and $G_{37}$ is the pile type shown in the first row of the table, whereas the decision made by the DP model is the pile type shown in the first column of the table, considering all states of the block. In each entry, the other lines correspond to the generalized trees with $V=6, \ldots, 2$, respectively. Table 5.5 shows that decisions made by $G_{c 7}, c \in\{1,2,3\}$ are all in line with decisions made by the DP model which is also supported by Table 5.3. The accuracy of decisions decreases as $V$ decreases.

Next, we carry out a simulation study to compare the results obtained by $G_{c V}, V=2, \ldots, 7, c \in\{1,2,3\}$ and optimal results obtained by the DP model. Both methods are used to stack incoming containers of three types in a bay with seven piles and four tiers. Note that $G_{c 7}, c \in\{1,2,3\}$ already provides optimal results for this configuration. The results are presented in Figure 5.4a and 5.4b. In each scenario, a sample of $M$ sequences of $N=4 \times 7=28$ containers is randomly generated by Monte Carlo simulation to fill up an empty block. The sample size must satisfy the following equation with a $90 \%$ confidence level (Law and Kelton, 1999):

$$
\begin{equation*}
M \geq S^{2}(M)\left(\frac{(1+\varepsilon) \mathbb{Z}_{1-\alpha / 2}}{\varepsilon \bar{f}(M)}\right)^{2} \tag{3}
\end{equation*}
$$

where, $S^{2}(M)$ and $\bar{f}(M)$ are respectively the variance and mean of the number of reshuffles in a preliminary run, $\mathbb{Z}_{1-\alpha / 2}$ is the $1-\alpha / 2$ percentile of the normal distribution and $\alpha=10 \%$ in case of the $90 \%$ confidence level. $\varepsilon=5 \%$ is for determining the confidence interval. Depending on the scenario, $M$ varies between 2000 and 6000 random container sequences.

Based on the results in Figure 5.4a and 5.4b, we can draw the following conclusions:

- The gap between the objective values of the DT heuristic and the optimal solution depends on the size

Table 5.5. Decisions made by the DP model and $G_{c V}, V=$
$2, \ldots, 7, c \in\{1,2,3\}$ in different states of the block


Note. In the first line of each entry, $(a, b, c)$ presents the number of times that the decision made by $\left(G_{17}, G_{27}, G_{37}\right)$ is the pile type shown in the first row of the table, whereas the decision made by the DP model is the pile type shown in the first column of the table, considering all states of the block. The other lines correspond to the generalized decision trees with six, five, four, three, and two piles, respectively. Empty entries show that all three numbers are zero.
of the problem whose optimal solutions obtained by the DP model are used to improve the generalized decision trees.

- For the generalized trees improved based on the optimal solutions of a block with $V$ piles, the DT heuristic produces zero-gap solutions to any problem with no more than $V$ piles. For larger size problems, the gap is a decreasing convex function of the number of piles of problems whose optimal solutions are used to build the generalized trees.

For example, Figure 5.4a shows that for a seven pile problems the relative gap between the optimal and $G_{C V}$-based solutions decreases to zero very fast, as the optimal solutions of more piles are used to improve the generalized trees. Generalized decision trees modified based on a four pile problem already generate near-optimal solutions for the seven pile problem.

A Larger number of piles causes a higher flexibility in stacking decisions which enables generalized trees, modified based on a small number of piles, to provide good solutions for those problems. In
such problems, these trees can systematically differentiate different types of incoming containers, stack them and avoid reshuffling up to a point when only few empty locations remain. At that point, the problem transforms to a small-scale problem, which can efficiently be solved by generalized trees.

- The computation time of the DT heuristic is less than a second for our examples, and is much shorter than that of the DP. Figure 5.4 b shows that already for a block of seven piles, the DP requires 20 minutes for finding an optimal solution, but the DT heuristic only needs a second.


Figure 5.4. (a) Percentage difference in expected number of reshuffles between the optimal and heuristic DT solution based on $V$ piles for a seven-pile problem, (b) Computation time of the DP and DT heuristic solution based on $V$ piles (in seconds)

Finally, We conduct another simulation study to compare the performance of the DS and SS policies. The inputs of the study are $S=2, \ldots, 5, T=4$, and an empty block with $V=20$ piles will be completely filled up by containers of $S$ equally sized ships. We consider that containers of each ship belong to three weight groups and the same port of destination. Therefore, we define $C=\{1,2, \ldots, 3 S\}$ of which the cardinality depends on the number of ships. Figure 5.5 compares the SS and DS policies for different numbers of ships. In the SS policy containers of $S$ ships are piled upon each other, whereas in the DS policy containers of a single ship can be stacked on top of each other in a pile. The results show that by increasing the number of ships the improvement of the SS policy over the DS policy increases. This can be also directly concluded from the DP model. Allowing containers of multiple ships to be stacked on top of each other in multiple piles is a strict relaxation of the problem in which containers of each ship are forced to be stacked separately.

### 5.3.2 Case study

Several stacking policies are used by container terminal operators to stack incoming containers, including random stacking and horizontal stacking. We compare the DT heuristic with vertical stacking. A major container terminal in Antwerp uses vertical stacking which appears to outperform other stacking heuristics


Figure 5.5. Comparing the DS and SS policies
Note. Improvement $(\%)=\frac{f_{D S}-f_{S S}}{f_{D S}} \times 100 \%$ where $f_{D S}$ and $f_{S S}$ are the expected number of reshuffles of DS and SS respectively calculated by using the DT heuristic. The number of realizations satisfies Equation (3).
such as random stacking, nearest-neighborhood stacking, and horizontal stacking, according to the initial simulation study we have carried out (which is omitted here). The vertical stacking policy works as follows: Vertical stacking policy. For an incoming container, the policy assigns the container onto a pile type whose first-leaving container has the same type as the incoming container or will be retrieved later. If such a pile does not exist in the block, the policy assigns the container to an empty pile. Finally, if any empty pile cannot be found, a non-full pile is randomly selected for stacking the incoming container.

Figure 5.6 shows the results of the vertical stacking policy and DT heuristic by varying the number of piles. The results show that the DT heuristic outperforms the vertical stacking policy by more than $70 \%$. The reason is as follows. When the utilization of the block increases, the vertical stacking policy has more difficulty to find a pile for stacking without adding a reshuffle. When a block is filled to $80 \%$, the vertical stacking policy starts performing similar to random stacking. In other words, the performance of the vertical stacking policy is independent of the number of piles, but the DT heuristic performs better and better with an increase in the number of piles. Therefore the performance gap between the DT heuristic and the vertical stacking policy increases with an increase in the number of piles.

### 5.4 Conclusion and future research

In this chapter, we study how to stack incoming containers in a real-size container block. The objective is to minimize the expected number of reshuffles. We use a stochastic dynamic programming (DP) model and develop a decision-tree (DT) heuristic. The heuristic uses the results of the stochastic DP model for smallscale problems with a small number of piles to generate generalized decision trees that can solve large-scale problems. These generalized decision trees can be continuously improved as results of the DP for larger scale problems with more piles become available. For small-scale problems with a small number of piles, we compare the DT heuristic and the DP model results. Our generalized decision trees can obtain the same optimal results but much faster. For large-size problems, we perform a simulation study to compare our


Figure 5.6. The comparison of the performance of the DT and vertical stacking heuristics

Note. Improvement $(\%)=\frac{f_{V}-f_{D T}}{f_{V}} \times 100 \%$, where $f_{D T}$ and $f_{V}$ are the expected number of reshuffles produced by the DT heuristic and the vertical stacking policy, respectively. The number of realizations satisfies Equation (3).
results with those results obtained by heuristics commonly used in practice. Our heuristics performs better than common heuristic in practice. We also use the DT heuristic to show that the shared-stacking (SS) policy significantly outperforms the dedicated-stacking (DS) policy.

Future research derived from this topic can go in different directions. First, although we study the SS policy, we focus on a small number of ships, each with a single port of destination. This is due to the large number of states involved in solving a problem by the DP model. In reality, container terminal operators may deal with containers of a larger number of ships each with multiple ports of destination, leading to hundreds of container types that need to be handled. The insights obtained from this chapter can be used to develop a solution method dealing with more types of containers. Note that the solutions can still be provided to the DT heuristic to make generalized decision trees. Second, in the sequence of loading a ship, it is implicitly assumed that containers are loaded onto the ships with a single quay crane. Loading a ship with multiple quay cranes allows us to deviate from the given sequence. It may be allowed that a light container leaves the block earlier than a heavy container of the same ship which must be loaded by a different quay crane. Considering multiple quay cranes to load ships, new stacking policies can be developed.

## Appendix

## 5.A Proof of Theorem 5.1

In our problem, we have $V$ piles of containers and $J=\left|\left\{P_{c t} \mid c \in C, t=1, \ldots, T-1\right\} \cup\left\{P_{E}, P_{F}\right\}\right|=|C| \times$ $(T-1)+2$ types of piles. In each state of the block, each pile of containers has a specific type. Obviously, multiple piles may have the same type. The number of different states of the DP model can be calculated using the following classic combinatorial problem where a combination of size $V$ must be chosen from $J$ objects and repetition is allowed (Rosen et al., 2000). The total number of combinations is $\binom{J+V-1}{V}$. However, the total number of states of the DP model is one less. The reason is that repetition is allowed and considering $J$ as the total number of pile types means that in one of the states all the piles can be full which is not feasible as no container can be located there.

## Chapter 6

## Conclusions and Further Research

Container terminal operators have to make a large number of decisions every day. Among these, decisions made regarding stacking operations at the stacking area of the terminal are studied in this dissertation. All containers arriving from the seaside and landside must remain in this area for a period of time. Without properly controling the stacking operations in this area, it may become a bottleneck and consequently reduce the performance of the whole container terminal. With this motivation, in this dissertation four models have been developed in order to make (near-) optimal decisions to deal with stacking operations.

### 6.1 Conclusions

In Chapter 2, we studied an automated stacking crane (ASC), stacking and retrieving containers from a single block of containers. The block has multiple input/output (I/O) points at both seaside and landside. The ASC moves storage containers from I/O points to the block, whereas it moves the retrieval containers from the block to the I/O points. We formulate the problem as an asymmetrical traveling salesman problem (ATSP) and propose a two-phase solution method to optimally solve it. In the first phase of the solution, we develop a new merging algorithm to patch subtours of an optimal solution of the assignment problem (AP) relaxation of the problem without adding extra travel time. In this phase, we first search for two arcs from every two different subtours visiting a common I/O point, and swap the destinations of them to merge the subtours. Next, based on the fact that in some arcs the ASC has multiple I/O point options with the same travel time to visit, we can create further opportunities to merge more subtours. We show that the first phase runs in a polynomial time, and often finds an optimal solution. Otherwise, a branch-and-bound $(\mathrm{B} \& \mathrm{~B})$ algorithm is used in the second phase to find an optimal solution of the problem. In this phase, the merging algorithm is again used in each node of the $B \& B$ tree to save computation time. The takeaways of Chapter 2 are as follows:

- Not only the number of requests but also the number of I/O points affects the complexity of the
problem. The problem is $\mathcal{N} \mathcal{P}$-hard in the number of requests and I/O points.
- I/O pints can be used to develop a merging algorithm which results in efficiently solving the problem. Since all containers have to be moved by the ASC to an I/O point or from an I/O point, the merging algorithm can successfully solve many instances of the problem, especially larger instances. Given a limited number of I/O points, by increasing the number of requests, the number of requests connected to each I/O point will increase. Thus, the merging algorithm has more opportunities to patch subtours.
- The numerical experiments show that problems up to 200 requests can be optimally solved in less than a second. Furthermore, the travel time reduction between the optimal and first-come-first-served (FCFS) sequence is around $30 \%$, on average. The reduction is around $14 \%$, in case the nearest neighbor (NN) heuristic is used to find the sequence.

In Chapter 3, we extend the problem by considering sets of open locations for storing containers. In the case of a storage request, the ASC picks up the container from an I/O point, and drops it off in an open storage location in the block. We formulate the problem as a generalized asymmetrical traveling salesman problem (GATSP). However, locations in the intersection of multiple sets make the problem more complex. Extra constraints are necessary to stack at most one container in such a location. The GATSP is $\mathcal{N} \mathcal{P}$-hard, and the new constraints increase the complexity even more. We propose a three-phase solution method to optimally solve the problem. The main idea of the algorithm is to simplify the problem to an ATSP and use its optimal solution to obtain an optimal GATSP solution. An optimal ATSP solution can be quickly obtained using the solution method studied in Chapter 2. In the first phase of the algorithm, we generate a corresponding ATSP model by pre-selecting the locations resulting in the minimum pairwise travel times. This can be realized by redefining the original travel time matrix. In the original travel time matrix, the pairwise travel times between two locations are calculated from the first location until the other one, whereas in the redefined travel time matrix, the pairwise travel times are calculated from or until I/O points in an arc, if available. We show that an optimal result of the ATSP model generated in this way is identical to an optimal solution of the GATSP in which the new extra constraints are relaxed. In other words, an optimal ATSP solution provides a lower bound for the problem, since pre-selection may result in a solution that one location in the intersection of multiple sets is selected to stack multiple containers. In the second phase, we present two special cases in which an optimal ATSP solution directly provides an optimal GATSP solution. In the first case, sets of open locations do not overlap. In the second case, locations in the intersection of multiple sets selected to stack containers of more than one set can be replaced with alternative open locations. Otherwise, in the third phase, we embed the optimal ATSP solution in a B\&B algorithm which tries to avoid stacking containers of more than one set in a location by assigning it to a single set in each node. The takeaways of Chapter 3 are as follows:

- The larger the cardinality of sets of open locations is, the harder it is to solve the problem. In fact, if the cardinality is one for every set of open locations, the GATSP simplifies to the ATSP discussed in

Chapter 2. The ATSP can be solved in less than a second.

- The other important factor for quickly solving problems is the locations in the intersections of sets. If there is no overlap between any two pairs of sets, no matter what the sizes of the sets are, we can simplify the problem to an ATSP. An optimal ATSP solution directly provides an optimal GATSP solution. As a result, it is important not only to limit the number of locations in every set but also the locations in the overlap of the sets.
- The proposed three-phase solution method is efficient for small and medium instances. For large instances with more than 100 requests, we propose a heuristic algorithm. The heuristic method first finds an optimal solution of the ATSP which is generated by pre-selection of locations. In case the solution is not a feasible GATSP solution, an AP will be used to replace all locations which are selected to store more than one storage container by other locations with the minimum increase in the ATSP optimal value. The numerical results show the objective value gap between our heuristic algorithm and the lower bound is less than $1 \%$, on average. Note that an optimal ATSP solution provides a lower bound for the GATSP.
- Compared to the NN and FCFS heuristics used in practice, travel time savings of up to $20 \%$ and $37 \%$, respectively can be achieved.
- The high quality of the heuristic method is due to the fact that the ASC moves over the bays and rows simultaneously. When the ASC moves from a storage request to another request, multiple open locations associated to the storage request result in the minimum travel time or different travel times which are close to each other. Therefore, the ATSP model generated by pre-selecting the empty locations provides a good lower bound for the problem. As a result, the heuristic solution obtained by using the AP model for replacing the multi-selected locations with the nearest empty locations is near optimal.

In Chapter 4, we extend the problem further. In the problem studied there, two ASCs carry out the requests, which have different priorities. We model the problem as a multiple generalized asymmetrical traveling salesman problem (mGATSP-PC) including the precedence constraints and some other extra constraints. Several practical and theoretical constraints were considered: (1) the ASCs cannot pass each other and, for safety reasons, the ASCs must be separated by a safety distance; (2) each storage container must be stacked in a location selected from a set of available open locations; and (3) because of waiting times and of the presence of several container transport modes, containers have different storage and retrieval priorities. Due to the complexity of the problem, it can only be solved exactly for small size instances. We have therefore developed an adaptive large neighborhood search (ALNS) heuristic capable of solving instances of realistic sizes. The takeaways of Chapter 4 are as follows:

- The ALNS is a very powerful heuristic to solve combinatorial problems. Our experiments demonstrate the impact of the new constraints and the efficiency of our heuristic. For small instances which
can be optimally solved by CPLEX (a high-performance mathematical programming solver for linear programming, mixed integer programming, and quadratic programming), the gaps between the ALNS and optimal solution values are on average less than $0.34 \%$. For large instances, compared to CPLEX truncated after four hours, the ALNS can quickly obtain up to $6.77 \%$ better results, on average. It also yields results that are $24 \%$ better compared with alternative less sophisticated heuristics. Furthermore, for two relaxed problems, the ALNS obtains results that are on average within $3 \%$ of the optimal solution values.
- The precedence constraints substantially affect the objective value. There is a big gap between the mGATSP-PC and the mGATSP where the precedence constraints are relaxed.

In Chapter 5 , we study a different problem which is indirectly related to the previous ones. The problem is how to stack incoming containers in a real-size container block shared by containers of multiple ships. The objective is to minimize the expected number of reshuffles. We use a stochastic dynamic programming (DP) model and develop a decision-tree (DT) heuristic. The heuristic uses the results of the stochastic DP model for small-scale problems with a small number of piles to generate generalized decision trees that can solve large-scale problems. These generalized decision trees can be continuously improved as results of the DP for larger scale problems with more piles become available. The takeaways of Chapter 5 are as follows:

- For small-scale problems with a small number of piles, we show that the gap between the optimal and DT heuristic solutions is a decreasing convex function of the number of piles of problems of which optimal solutions are used to improve the generalized trees. For larger scale problems, these trees also provide near-optimal solutions since a larger number of piles results in a higher flexibility to stack containers and to avoid reshuffling. In addition, the DT heuristic is compared with the vertical stacking policy which is one of the good heuristics commonly used in practice. We show that the DT heuristic significantly outperforms vertical stacking.
- Furthermore, we also use the DT heuristic to show that the shared policy significantly outperforms the dedicated policy in which containers of a single ship are stacked on top of each other. The shared stacking policy allows to increase the utilization of the block, whereas in the dedicated policy, containers of only one ship can be stacked in the block.


### 6.2 Future research

Container terminals have received an increasing attention from the academic community due to the opportunities and challenges they offer in research. In this dissertation, some of the operational problems in the stacking area of a container terminal are studied. The results show that major improvements can be achieved, using the exact and heuristic models developed in this dissertation. However, there is still enough room for conducting further research. Container terminals have seen major developments over the last decades. These
developments impact container terminal operations and create new opportunities for research studies. Followings, we discuss some new research directions that are interesting from both theoretical and practical perspectives.

Role of container terminals in networks: handling millions of containers arriving at and leaving from the deep-sea terminals introduces many challenges such as: congestion, delay, and pollution. This has driven container terminals to transform their static supply chains into adaptive business networks to increase their competitiveness and robustness (Vervest and Li, 2009, Heinrich and Betts, 2003). An example is a recent project started in the Netherlands to integrate supply chain and transportation through extending the sea terminal gate into the hinterland as the gateway to the European hinterland Veenstra et al. (2012). These initiatives thus shift container terminals, in addition to all their container handling activities, from being a stock point to a flow operator. The result is a better connection between the shippers and receivers in the network. This change comes with serious and unexplored challenges, but it also provide a tremendous opportunity to develop a sustainable competitive advantage. The seamless flow of goods from seaports to locations far into the hinterland will be a major enabler to prevent negative external effects from the transport, such as congestion in seaports, congestion on motorways due to too much trucking, and enhance the competitiveness of multimodal inland nodes for warehousing and value added activities. On the other hand, network configurations give rise to several strategic and operational problems such as the selection of hinterland terminals to be included in the network, the number of modalities required for transporting containers, the percentage of containers that have to be handseled by each modality, the path that each modality has to travel, and the terminals that each modality has to visit in its path.

Sustainable container terminals: sustainability, next to efficiency, has caught the attention of academic and professional communities during the last decade. The world has become aware of the limited resources of the earth and the necessity to preserve the environment. Therefore, many companies are looking for new methods to manage their operations more sustainably considering the environmental, economic, and social impacts of every decision. Although with a delay, container terminals have also started to move toward more sustainable operations. Container terminal operators are currently involved in redesigning their operations considering that while profitable, their operations should not harm the society and environment. As a result, traditional models in the literature which consider only profitability and efficiency have to be modified in order to include broader considerations of the terminal's internal and external stakeholders and environmental impacts. The result is many new operational problems in which it is tried to reduce the internal and external energy use, make the operations eco-friendly, and design durable facilities and handling systems.

New technologies and automation: nowadays, traditional manual container terminals are transforming into modern automated terminals which are much faster and safer. For example, manual yard cranes are replaced with single, double, or even triple automated yard cranes to stack and retrieve containers, straddle carriers are automated and are able to lift two TEU at the same time, modern quay cranes and yard cranes
are capable of simultaneous lifting of four TEU, traditional truck appointment systems are replaced with gate appointment system, and etc. However, in order to be efficient, the new modern technologies and methodologies need careful planning. New exact or heuristic models which consider all features and constraints introduced by automation and new technologies have to be developed.

Security and safety: security and safety are among the most important and least studied topics in container terminal operations. Many containers daily pass through a container terminal. Handling these containers involve many sorts of security risks such as terroristic attack and smuggling. Although the probability of such incidents is low, but the cost is extremely high. As an example, Abt (2003) estimates that the cost of the detonation of a nuclear device in a port is within $\$ 55-\$ 220$ billion. A promising solution to avoid these risks is inspecting all containers arriving and leaving container terminals. However, the $100 \%$ screening is costly and time consuming, and therefore other more efficient screening policies can be developed (Bakshi et al., 2011). Other methods such as isolating some parts of the terminal which limits access only to certified workers may also reduce the security risk. All in all, considering the cost involved in occupance of these incidents requires the container terminal operators and scholars to consider security at container terminals more thoroughly. Safety on the other hand requires other measures. Container terminal operators have already acquired different methods to increase the safety in their terminals. For example, terminals ask drivers to exit their cabs and stand inside a booth before yard cranes can load their chassis. Furthermore, many terminals feature remotely controlled cranes which minimize exposure to potential safety hazards. However, in order to increase the safety at terminals, many other safety regulations can be still developed.

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## About the author



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## Summary

The first containers were introduced to the world in April 1956 when Malcolm McLean moved fifty-eight 35 -foot containers from Newark to Houston by a refitted oil tanker, "SS Ideal X" (see the figure). Nowadays, containerized transportation has become an essential part of intermodal freight transport. More than $90 \%$ of all cargo is now transported by ships. Most of this cargo is handled in containers. A large terminal handles millions of containers on an annual basis. For example, the terminals in the Port of Rotterdam has handled more than 11 million containers in 2010. Container terminal managers attempt to efficiently manage the logistic process of the terminals to keep up with the increasing number of containers to be handled. The stacking area is particularly critical since most of the containers transiting through a container terminal must be stored for a certain length of time, possibly in different blocks. Efficiently managing block operations can significantly improve the overall performance of the container terminal.


First container transshipment, April 1956

In this dissertation, we first study how to minimize the makespan to stack and retrieve a set of containers in a block of containers. In Chapters 2 and 3, a single automated stacking crane (ASC) handles the requests. The block has multiple input/output ( $\mathrm{I} / \mathrm{O}$ ) points. The ASC must move retrieval containers from the block to the I/O points, and must move storage containers from the I/O points to the block. We formulate the
problems as continuous time integer programming models and propose exact and heuristic solution methods to solve them. The numerical experiments reveal that the optimal makespan results are significantly shorter than results obtained by heuristics commonly used in practice such as first-come-first-served and selecting the nearest request. In Chapter 4, two ASCs carry out the requests. The ASCs can never pass each other and must operate sufficiently far from each other. We formulate this problem as an integer programming model and propose a metaheuristic algorithm to solve it. The results show that the proposed algorithm significantly outperforms other heuristics and truncated CPLEX.

In Chapters 2-4, we consider that locations as well as destination sides of retrieval containers and I/O points of storage containers are known. Note that in Chapter 2, each storage location has a given storage location in the block. In contrast, in Chapters 3 and 4, each storage container can be stacked in a location selected from a set of open locations suitable for stacking that container. Storage locations can be determined using the methodology discussed in Chapter 5, where we use a stochastic dynamic programming model to decide where to stack each incoming container in a block. The objective is to minimize the expected number of reshuffles. The number of states of the dynamic programming model is overwhelming. We propose a decision-tree heuristic algorithm to solve real instances. The heuristic uses the results of the exact model for small-scale problems to generate generalized decision trees. These trees can be used to solve problems with a realistic number of piles. For small-scale problems, the trees can quickly make optimal decisions. For large-scale problems, the decision-tree heuristic significantly outperforms stacking policies commonly used in practice. Those policies perform well as long as the utilization of the block is low. However, since they are myopic, as the utilization increases, they start performing similar to random stacking which results in reshuffles. Using the decision trees, we can compare the performance of a shared-stacking policy, which allows containers of multiple ships to be stacked on top of each other, with a dedicated-stacking policy. Shared-stacking appears to outperform dedicated-stacking.

Finally, in chapter 6 , we conclude the dissertation and summarize the contributions and findings. Together, the exact and heuristic methods developed in this dissertation can be used by container terminal operators to increase the performance at the stacking area, which consequently affects the whole terminal performance.

## Samenvatting (Summary in Dutch)

In april 1956 werden de eerste containers geïntroduceerd door Malcolm McLean; hij vervoerde achtenvijftig 35 -foot containers van Newark naar Houston met een nieuw uitgeruste olietanker, de "SS Ideal X" (zie de figuur). Tegenwoordig maakt containertransport een essentieel deel uit van intermodaal vrachtgoedvervoer. Meer dan $90 \%$ van alle vracht wordt nu getransporteerd met schepen. Het grootste deel van deze lading wordt vervoerd in containers. Een grote terminal verwerkt miljoenen containers op jaarbasis. De terminals van de haven van Rotterdam bijvoorbeeld hebben in 2010 meer dan 11 miljoen containers verwerkt, waarvan een groot aantal opgeslagen of uitgeslagen moet worden. Managers van containerterminals proberen het logistieke proces van de terminal zo efficiënt mogelijk uit te voeren om het groeiende aantal containers te verwerken. Het gebied waar de containers opgestapeld worden is kritiek, omdat de meeste overgeslagen containers tijdelijk opgeslagen moeten worden, mogelijk in verschillende blokken. Het efficiënt managen van de opslag en overslag activiteiten in deze blokken kan de algehele prestaties van de containerterminal significant verbeteren.


Eerste container verscheping, April 1956

In dit proefschrift bestuderen we eerst hoe we de totale doorlooptijd kunnen minimaliseren van het inslaan en uitslaan van een groep containers in een container blok. In hoofdstukken 2 en 3 verwerkt een portaalkraan (automated stacking crane, ASC) een gegeven aantal opdrachten in een blok. Het blok heeft
meerdere in- en output (I/O) punten. De ASC moet de uitslag containers verplaatsen van het blok naar een I/O punt, en moet opslagcontainers verplaatsen van een I/O punt naar locaties binnen het blok. We formuleren de problemen als continue tijd integer programmeringsmodellen en doen een voorstel voor exacte en heuristische oplossingsmethoden om ze op te lossen. De numerieke experimenten tonen dat de optimale doorlooptijd significant korter is dan de resultaten verkregen met in de praktijk gebruikelijke heuristieken zoals "first-come-first-served" en het selecteren can de "nearest neighbor" container. In hoofdstuk 4 wordt het blok bediend door twee ASC's. De ASC's kunnen elkaar niet passeren en moeten met voldoende afstand van elkaar opereren. We formuleren dit probleem als een integer programmeringsmodel en stellen een meta-heuristisch algoritme voor om dit op te lossen. Het resultaat laat zien dat het voorgestelde algoritme significant beter is dan de andere heuristieken en CPLEX dat afgekapt wordt na een bepaalde rekentijd.

In hoofdstuk 2-4 beschouwen we locaties en bestemmingen van uitslag containers en I/O punten van opslagcontainers als bekend. In hoofdstuk 2 had iedere opslagcontainer een gegeven opslag locatie in het blok. In hoofdstuk 3 en 4 kan elke opslagcontainer gestapeld worden op een locatie geselecteerd uit een aantal open locaties passend voor het opstapelen van die container. Opslaglocaties kunnen bepaald worden met de methodologie besproken in hoofdstuk 5 , waar we een stochastische dynamische programmeringsmodel gebruiken om te bepalen waar elke inkomende container in een blok opgestapeld dient te worden. Het doel is het minimaliseren van het verwachte aantal interne verplaatsingen (verkassingen). Het aantal toestanden van het dynamische programmeringsmodel is overweldigend. Als oplossing introduceren we een beslissingsboom met heuristisch algoritme om realistische cases op te lossen. De heuristiek gebruikt de resultaten van het exacte model voor kleinschalige problemen om "generalized" beslissingsbomen te genereren. Deze bomen kunnen gebruikt worden om problemen op te lossen met een realistisch aantal containerstapels. Voor kleinschalige problemen kunnen deze bomen snel optimale beslissingen nemen. Voor grootschalige problemen is de beslisboom-heuristiek significant beter dan de huidige stapelmethoden. De huidige methoden presteren goed zolang de benutting van het blok laag is. Echter, door hun myopische werking gaan ze, zodra de blok benutting toeneemt, steeds meer lijken op een willekeurige stapeling, wat resulteert in extra verkassingen. Gebruik makend van beslisbomen, kunnen we de resultaten vergelijken van een "shared-stacking policy", welke toestaat dat containers van meerdere schepen op elkaar containerstapels kunnen vormen, en een "dedicated-stacking policy". Shared stapeling blijkt minder verkassingen te geven dan dedicated stapeling.

Tenslotte worden in hoofdstuk 6 conclusies getrokken met betrekking tot de bevindingen en bijdragen van dit proefschrift. De exacte en heuristische methoden ontwikkeld in dit proefschrift kunnen gebruikt worden door container terminal operators om de prestaties te verhogen in de container blok, waarmee de prestaties van de hele terminal bïnvloed kunnen worden.

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## DEVELOPING NEW METHODS FOR EFFICIENT CONTAINER STACKING OPERATIONS

Containerized transportation has become an essential part of the intermodal freight transport. Millions of containers pass through container terminals on an annual basis. Handling a large number of containers arriving and leaving terminals by different modalities including the new mega-size ships significantly affects the performance of terminals. Container terminal operators are always looking for new technologies and smart solutions to maintain efficiency. They need to know how different operations at the terminal interact and affect the performance of the terminal as a whole. Among all operations, the stacking area is of special importance since almost every container must be stacked in this area for a period of time. If the stacking operations of the terminal are not well managed, then the response time of the terminal significantly increases and consequently the performance decreases. In this dissertation, we propose, develop, and test optimization methods to support the decisions of container terminal operators in the stacking area. First, we study how to sequence storage and retrieval containers to be carried out by a single or two automated stacking cranes in a block of containers. The objective is to minimize the makespan of the cranes. Finally, we study how to minimize the expected number of reshuffles when incoming containers have to be stacked in a block of containers. A reshuffle is the removal of a container stacked on top of a desired container. Reshuffling containers is one of the daily operations at a container terminal which is time consuming and increases a ship's berthing time.

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[^4]
[^0]:    *This chapter is based on Gharehgozli et al. (2012d).

[^1]:    *This chapter is based on Gharehgozli et al. (2012b)

[^2]:    *This chapter is based on Gharehgozli et al. (2012a).

[^3]:    *This chapter is based on Gharehgozli et al. (2012c).

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