Bilevel Modelling of Energy Pricing Problem

Sezin Afsar, Luce Brotcorne, Patrice Marcotte, Gilles Savard

Bilevel Modelling of Energy Pricing Problem

Sezin Afşar, Luce Brotcorne, Patrice Marcotte, Gilles Savard

INOCS Team, INRIA Lille-Nord Europe, France

INFORMS Annual Meeting 2015 Philadelphia

1-4 November 2015

Outline

Demand Side Management

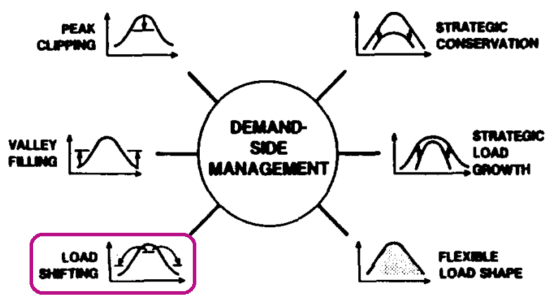
Motivation

- Demand for energy is largely uncontrollable and varies with time of day and season.
  - In UK, given the average demand across the year, the average utilization of the generation capacity is $\leq$ 55%.
  - Minimum demand in summer nights $\doteqdot$ 30% of the winter peak
- Energy is difficult to store in large quantities.
- Supply-demand balance failure $\rightarrow$ system instability
- Total capacity of installed generation must be $\geq$ max demand to ensure the security of supply.

Demand Side Management

Why Necessary?

- DSM: **control** and **manipulate** the demand to meet capacity constraints.
- DSM's role: To improve the efficiency of operation and investment in the system.



[4]C.W. Gellings. The concept of demand side management for electric utilities. *Proceedings of the IEEE*, 73(10):14681470, 1985.

Demand Side Management

Major DSM Techniques

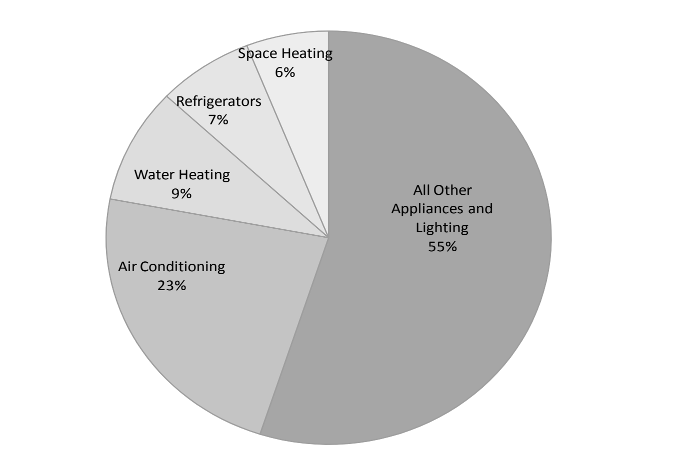
- Direct load control, load limiters, load switching
- Commercial/industrial programmes
- Demand bidding
- Time-of-use pricing
- Smart metering and appliances

New Challenge

"Commitment to market based operation and deregulation of the electricity industry places consumers of electricity in the center of the decision-making process regarding the operation and future development of the system" -G. Strbac, 2008

Demand Side Management

Residential Electricity Use



Bilevel Programming

Classical Mathematical Program

$$\max_{x} \; f(x)$$

s.t.

$$x \in X$$

Bilevel Program

$$\max_{x} \; f(x, y)$$

s.t.

$$(x, y) \in X$$

$$y \text{ solves } \min_{y} \; g(x, y)$$

s.t.

$$(x, y) \in Y$$

Bilevel Programming

Classical Mathematical Program

$$\max_{x} \; f(x)$$

s.t.

$$x \in X$$

Bilevel Program

$$\max_{x} \; f(x, y)$$

s.t.

$$(x, y) \in X$$

$$y \text{ solves } \min_{y} \; g(x, y)$$

s.t.

$$(x, y) \in Y$$

Consumer Behavior
Demand Response

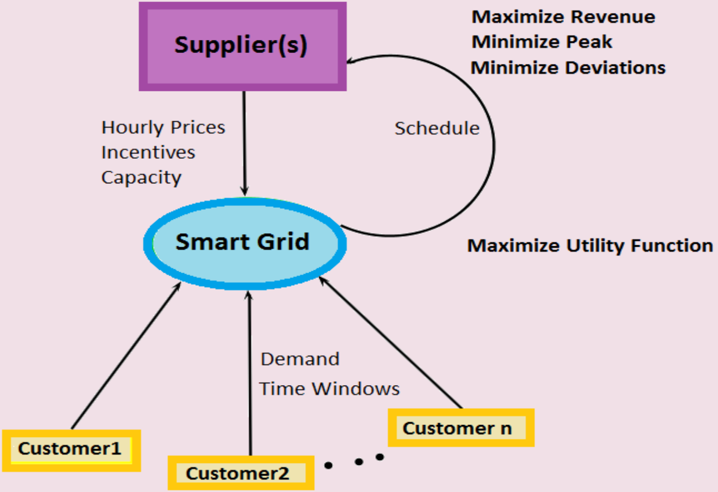
Energy Pricing Problem

Smart Grid Technology

- Every customer is equipped with a device that can receive, process and transfer data: smart meter
- Smart meters communicate with each other $\rightarrow$ smart grid
- Meters are programmable according to the needs of the customer
- Smart grid receives prices from the supplier(s), demand from the customers and schedules the consumption

Energy Pricing Problem

Bilevel Approach



Bilevel Model

Properties

- Stackelberg game / Bilevel programming
- The supplier (upper level) and a group of customers connected to a smart grid (lower level).
- Prices from supplier and demand with time windows from customers are received by the smart grid.
- Time-of-use price control to minimize peak demand and maximize revenue.
- Demand-response to hourly changing prices to minimize cost and waiting time.

Bilevel Model

Objectives

- Leader maximizes (revenue - peak cost) by deciding on prices
- Follower minimizes (billing cost + waiting cost) by deciding on the schedule of consumption.

Assumptions

- A fixed upper bound for prices.
- Demand is fixed.
- All operations are preemptive.
- Every customer has a set of appliances and certain time windows.
- All appliances have power consumption limits.
- One cycle has 24 time slots(hours) in total.

Bilevel Model - Preemptive

$$\max_{p,\Gamma} \sum_{n\in N} \sum_{a\in A_n} \sum_{h\in H_{n,a}} p^h x^h_{n,a} - \kappa\Gamma$$ **Revenue - Peak Cost**

s.t.

$$\Gamma \geq \sum_{\substack{n\in N \\ a\in A_n}} x^h_{n,a} \qquad \forall h \in H$$ **Peak Definition**

$$p^h \leq p^h_{max} \qquad \forall h \in H$$ **Price Limit**

$$\min_{x} \sum_{n\in N} \sum_{a\in A_n} \sum_{h\in H_{n,a}} p^h x^h_{n,a} + \sum_{n\in N} \sum_{a\in A_n} \sum_{h\in H_{n,a}} C^h_{n,a} x^h_{n,a}$$ **Billing Cost + Waiting Cost**

s.t.

$$x^h_{n,a} \leq \gamma^{max}_{n,a} \qquad \forall n \in N, \forall a \in A_n, \forall h \in H_{n,a}$$ **Device Limit**

$$\sum_{h\in H_{n,a}} x^h_{n,a} = E_{n,a} \qquad \forall n \in N, \forall a \in A_n$$ **Demand Satisfaction**

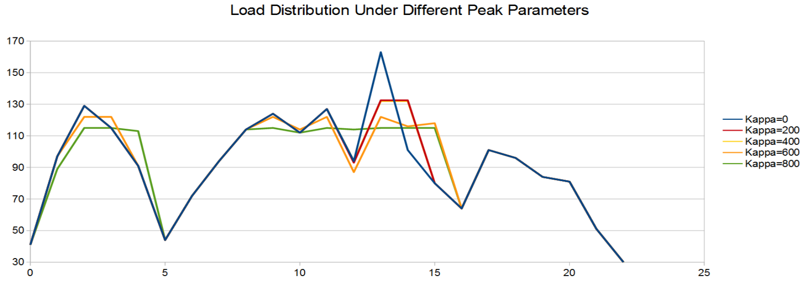
$$x^h_{n,a} \geq 0 \qquad \forall n \in N, \forall a \in A_n, \forall h \in H_{n,a}$$

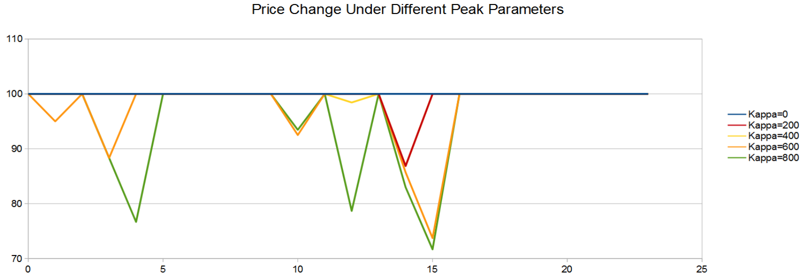
Bilevel Model

Exact Solution Method

- Optimality conditions (primal,dual and complementarity constraints) of the follower are added to the upper level
- Complementary slackness constraints $\rightarrow$ linearized with binary variables
- Objective function is linearized using the follower's dual objective function.
- Single level MIP is solved with CPLEX

Bilevel Model





Price Heuristic

Idea

- Modify the price vector
- Observe the corresponding schedule and peak
- Compute the optimal prices corresponding to this schedule with Inverse Optimization (IO)

Price Heuristic

Initialization

- Set all prices to maximum
- Find the peak, assume it happens at 13:00
- Keep the prices before 13:00 at maximum and randomly generate the rest
- Solve the lower level problem with this price vector
- Solve IO problem to find the optimal corresponding prices
- Compute the net revenue
- Repeat this procedure 50 times, pick the solution pair with best net revenue

Price Heuristic

Iteration

- Find the peak
- Decrease the prices of 4 time slots that come after peak
- Solve the lower level problem with this price vector
- Solve IO problem to find the optimal corresponding prices
- Compute the net revenue
- Repeat this procedure until there is no improvement

Peak Search Heuristic

Idea

- Based on a binary search on the peak value
- Fix a peak value
- Find a feasible schedule with respect to the peak value
- Compute the corresponding prices using Inverse Optimization
- Compute the objective function value

Peak Search Heuristic

Initialization

- UB on peak: assign all jobs to the first preferred hour
- LB on peak: minimize peak under scheduling constraints
- Combing: divide the [LB, UB] interval into 20 subintervals, and follow the previous procedure
- Pick the two best objective values and update LB and UB with corresponding peak values

Peak Search Heuristic

Iteration

- Define (UB-LB)/2 as the new peak limit
- Compute the objective function
- Take its left (L) and right (R) neighbors
- If one of them is better, pick that side, else, STOP

Stopping Criteria

- $|UB - LB| < \epsilon$
- It cannot improve the incumbent solution

Experiments

Details

- For the experiments, CPLEX version 12.3 is used on a computer with 2.66 GHz Intel 283 Xeon CPU and 4 GB RAM, running under the Windows 7 operating system.
- There are 10 randomly generated instances which consist of 10 customers, each owns 3 preemptive appliances. Since it is a system optimal model, we can say that there are 30 jobs.
- Peak weight κ takes 5 values: 200, 400, 600, 800 and 1000.

Comparisons

Experiments

Table: Comparison of Optimality Gap of Heuristics

Kappa	% Gap of PSH	% Gap of PrH
200	0,06	0,00
400	0,00	0,00
600	0,12	0,00
800	0,00	0,13
1000	1,38	0,12
Avg	0,16	0,03

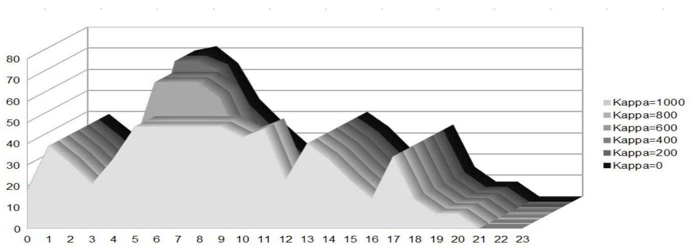
Comparisons

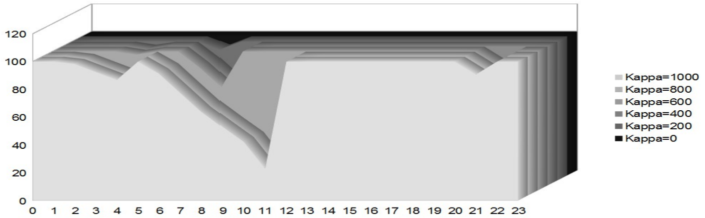
Experiments

Table: Computation Time of Classical Exact Method and Heuristics(sec)

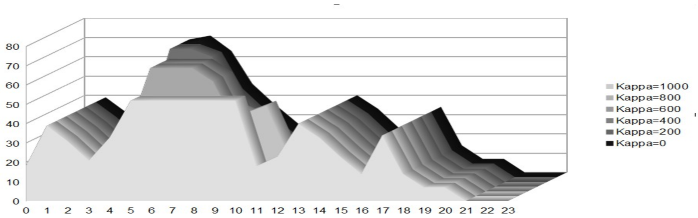
(κ)	CEM	PSH	PrH
200	31.90	21.70	13.50
400	410.40	102.90	74.70
600	2022.70	141.90	122.10
800	4244.20	154.00	152.20
1000	8717.70	153.60	154.00

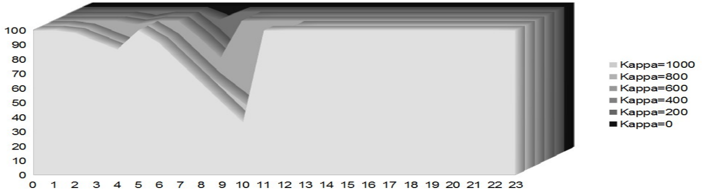
Load and Price Curves of Exact Solution



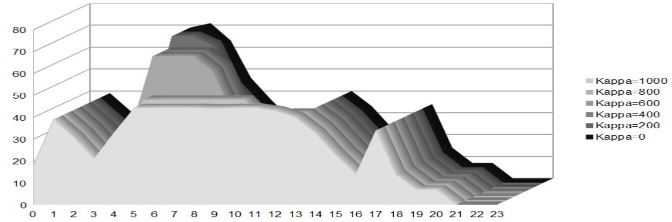


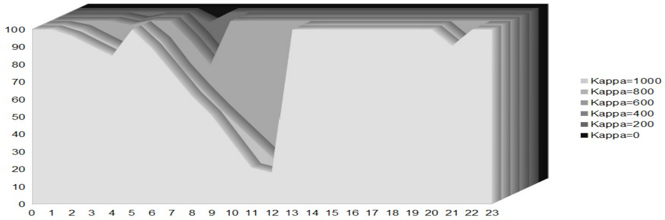
Load and Price Curves of Price Heu Solution



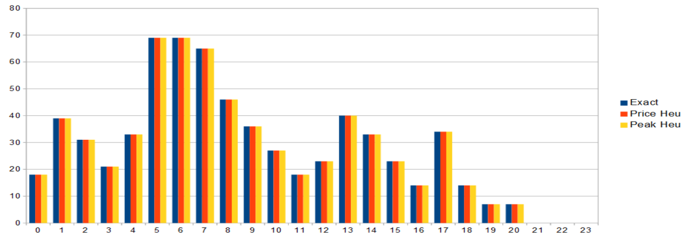


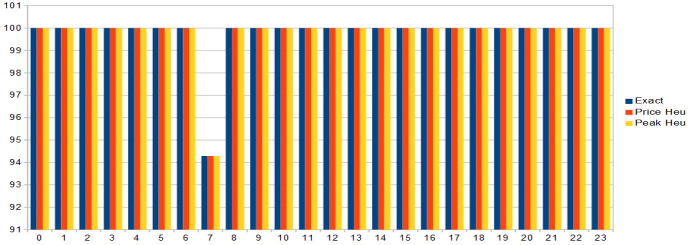
Load and Price Curves of Peak Heu Solution



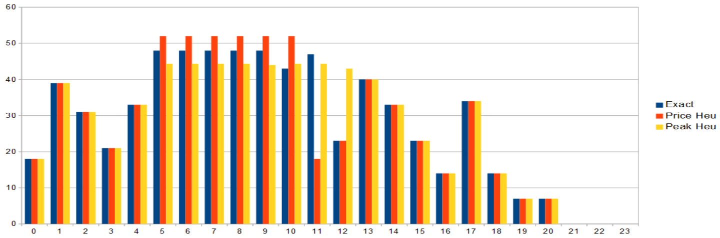


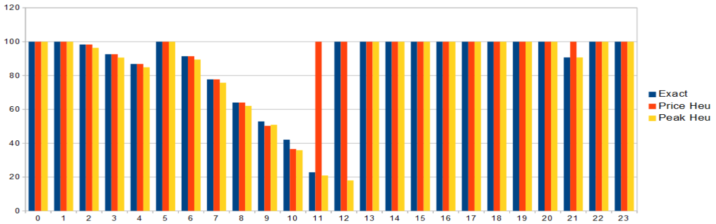
Load and Price Comparison for Low Peak Weight



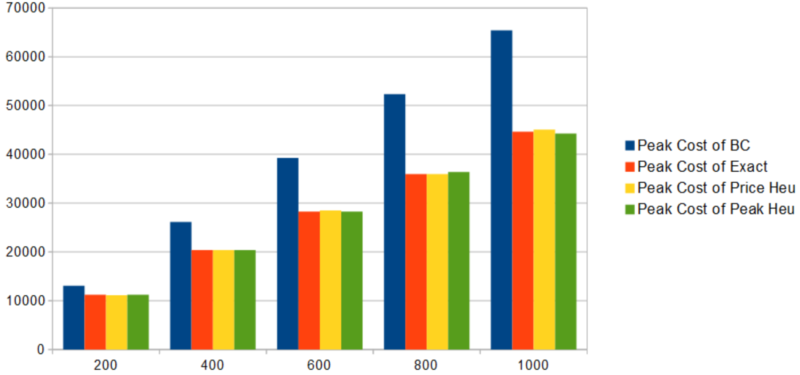


Load and Price Comparison for High Peak Weight

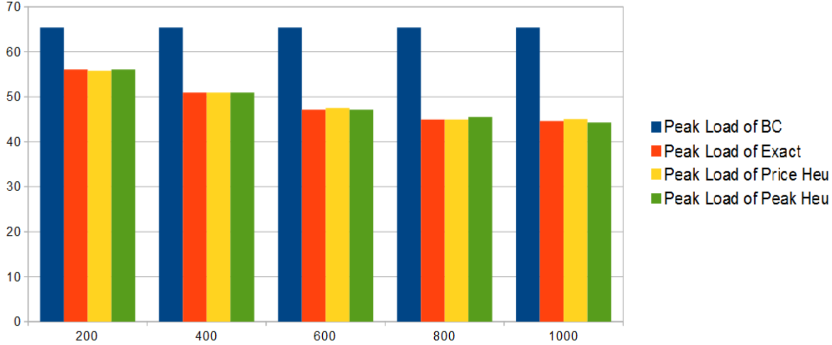




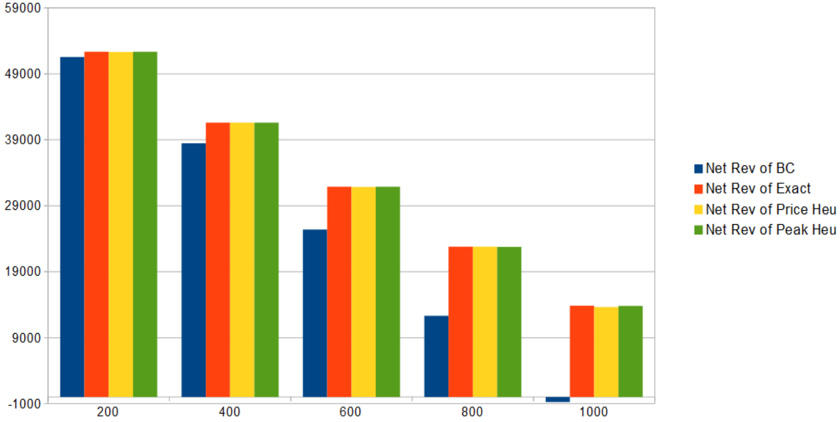
Peak Cost Comparison of All Methods



Peak Load Comparison of All Methods



Net Revenue Comparison of All Methods



Conclusion

Our Work

- Innovative approach for DSM
- Explicitly integrated customer response into the optimization process of the supplier
- Efficient heuristic approach to provide good solutions

What else is Possible?

- Stochastic approach (prices, demand)
- More efficient heuristics
- Robust modelling

THANK YOU FOR LISTENING
Q & A

Bibliography

[1] J.F. Bard and J.T. Moore. A branch and bound algorithm for the bilevel linear programming model. *SIAM Journal on Scientific and Statistical Computing*, 11(2):281–292, 1990.

[2] L. Brotcorne, M. Labbé, P. Marcotte, and G. Savard. A bilevel model and solution algorithm for a freight tariff-setting problem. *Transportation Science*, 34(3):289–302, 2000.

[3] L. Brotcorne, M. Labbé, P. Marcotte, and G. Savard. Joint design and pricing on a network. *Operations Research*, 56:1104–1115, 2008.

[4] C. W. Gellings. The concept of demand-side management for electric utilities. *Proceedings of the IEEE*, 73(10):1468–1470, 1985.

[5] P.B. Hansen and G. Savard. New branch-and-bound rules for linear bilevel programming. *SIAM Journal on Scientific and Statistical Computing*, 13:1194–1217, 1992.

[6] B. Hobbs and S. Nelson. A nonlinear bilevel model for analysis of electric utility deland-side planning issues. *Annals of Operations Research*, 34:255–274, 1992.

[7] I.J. Jùdice and A.M. Faustino. The solution of the linear bilevel programming problem by using the linear complementary problem. *Investigaçâo Oper*, 8:77–95, 1988.

[8] P. Marcotte. Network design problem with congestion effects: A case of bilevel programming. *Mathematical Programming*, 74:141–157, 1986.

[9] G.M. Masters. *Renewable and Efficient Electric Power Systems*. Wiley, Hoboken, NJ, 2004.

[10] A.-H. Mohsenian-Rad and A. Leon-Garcia. Optimal residential load control with price prediction in real-time electricity pricing environments. *IEEE Transactions on Smart Grid*, 1(2):120–133, September 2010.

[11] A.-H. Mohsenian-Rad, V.W.S. Wong, J. Jatskevich, R. Schober, and A. Leon-Garcia. Autonomous demand-side management based on game-theoretic energy consumption scheduling for the future smart grid. *IEEE Transactions on Smart Grid*, 1(3):320–331, 2010.

[12] G. Strbac. Demand side management: Benefits and challenges. *Energy Policy*, 36(12):4419–4426, 2008.