Inflation Rates: Long-memory, level shifts, or both?

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Abstract

We examine if US inflation rates series can be characterized by a long-memory model, by a model with occasional level shifts or by a new model, which jointly captures the two features. Through simulations we show that this new model can be usefully applied in practice. For 23 inflation rate series we find that generally the long-memory model is best, both in terms of in-sample fit and out-of-sample forecasts.

Key words: Long memory, level shifts, inflation

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1. Introduction and motivation

Applied time series econometrics usually involves the analysis of models for economic time series, which can capture their salient features. Preferably, these models can be reliably used for out-of-sample forecasting or for subsequent multivariate modeling. A typical concern for time series econometrics is that there are usually many possible models for these features, and indeed, to make a selection across these models is an important issue. For example, there are several ways to describe a trend in economic data and the choice between models for trends (including the unit root models) amounts to an important practical decision.

It sometimes happens that time series features seem to require the use of models from different classes. A well-known example, which has attracted much recent interest, concerns the long-memory feature in various economic time series. Loosely speaking, long memory entails that shocks or innovations to a time series do not have a persistent nor a short-run transitory effect, but that they last for a long while. It appears from studies like Granger and Hyung (1999), Bos, Franses and Ooms (1999), and Diebold and Inoue (2001) that apparent long memory can also be caused by neglected occasional level shifts. Indeed, one might intuitively understand that an occasional level shift mimics the effect of a long-lasting shock, and hence one might easily be inclined to think that the data have long memory. Of course, it does matter for out-of-sample forecasting whether one opts for one or the other model, as the forecast generating equations are completely different. Hence, it seems of practical relevance to examine which model, that is, a long-memory model or a model with occasional level shifts, is more appropriate for a given time series at hand.

Several recent studies acknowledge the possibility that occasional level shifts can be confused for long memory. The inclusion of dummy variables can accommodate this, where the locations of the shifts are determined from the outset, see for example Bos, Franses and Ooms (1999). Of course, one might prefer that these level shift locations are determined at the same time as that one estimates the long-memory model parameters. Hence, there seems to be a need to have a joint model (at least to allow for a more systematic comparison of models), which incorporates both long memory and occasional level shifts. In this paper we put forward such a model, which we will label as the FI-BREAK model. This model extends the intriguing new model proposed in Engle and Smith (1999). Once we have examined its properties, we will use this model to see if various US inflation rate series have long memory, have occasional level shifts or have both.

The outline of the paper is as follows. In Section 2, we discuss the representation of the FI-BREAK model. We discuss an estimation method, and through simulations we show that the estimation method is reliable. In Section 3, we consider the model for 23 monthly US inflation rate series, and we compare its fit with nested models for long memory or occasional shifts only. We find that in many cases there is no need to consider the joint model, as one of its nested versions yields a better fit. In fact, we find that the long-memory model generally outperforms in terms of fit. In an out-of-sample forecasting exercise, we find that these in-sample findings carry through. In Section 4, we conclude with some remarks.

2. The FI-BREAK model

We exploit the possibility that the occasional structural break model (or simply BREAK model) and I(d) model can be summarized into one single model. One motivation for this joint model is that both individual models can capture a long-memory component to some extent, and hence that a joint model would be able to capture all long memory components.

2.1. Preliminaries

To construct such a joint model for a time series y_t, one can think of, for example,

$$\begin{aligned} y_t &= m_t + u_t, \\ m_t &= m_{t-1} + q_t h_t, \\ (1-L)^d u_t &= e_t \end{aligned} \qquad (1)$$

where q_t follows an i.i.d. binominal distribution, that is,

$$q_t = \begin{cases} 1, & with\ probability\ p \\ 0, & with\ probability\ 1-p \end{cases} .$$

For simplicity, we assume e_t ~ i.i.d. (0, $s_e{}^2$), and h_t ~ i.i.d. (0, $s_h{}^2$). Here, we may first identify multiple breaks, remove these, and then the residual series contains I(d) components. In other words, the series y_t is decomposed into a break component m_t and a long memory component u_t ~ I(d). One may rewrite this model as

$$(1-L)^d(y_t - m_t) = e_t$$

Clearly, the unconditional mean of y_t is constant up to moments that breaks appear as discrete steps. As the m_t series is a random walk, the series y_t is explosive in the long run unless the number of breaks is finite.

An alternative model may be

$$\begin{aligned} y_t &= m_t + e_t \\ (1-L)^d m_t &= q_t h_t \end{aligned} \tag{2}$$

with the same assumptions for q_t, e_t and h_t as before. If d = 1, occasional events in $q_t h_t$ would establish permanent breaks. When d < 1 but close to 1, m_t ~ I(d) is a non-stationary but mean-reverting process. In this representation, events would have a long-memory effect, not a permanent effect. The time series can then be decomposed into a long-memory and a structural break component (m_t) and a transitory or short memory component (e_t).

Finally, one may consider

$$\begin{aligned} (1-L)^d y_t &= m_t + e_t, \\ m_t &= m_{t-1} + q_t h_t \end{aligned} \tag{3}$$

with again the same assumptions for q_t, e_t and h_t as before. Equation (3) can be rewritten as

$$\begin{aligned} y_t &= \tilde{m}_t + u_t \\ (1-L)^d \tilde{m}_t &= m_t = m_{t-1} + q_t h_t \\ (1-L)^d u_t &= e_t, \end{aligned} \tag{4}$$

The time series y_t can be decomposed into a long-memory break component $\tilde{m}_t$ and a long memory component u_t. This model generalizes the model in van Dijk, Franses and Paap (2002), which is

$$\begin{aligned} (1-L)^d y_t &= x_t \\ x_t &= f_1(L)x_{t-1}(1-G(s_t;g,c)) + f_2(L)x_{t-1}G(s_t;g,c) + e_t, \end{aligned} \qquad (5)$$

where the transition function $G(\cdot)$ is assumed to be logistic function.

$$G(s_t;g,c) = [1+\exp\{-g(s_t - c)/\ s_{st}\}]^{-1},$$

where $g > 0$, s_t is the transition variable and s_{st} is the standard deviation of s_t. This fractionally integrated smooth transition autoregressive model allows only two different regimes corresponding with $G(\cdot) = 0$ and $G(\cdot) = 1$. One may extend this model to have more regimes, but then one should know the number of regimes in advance. And, another related model is the STOPBREAK model of Engle and Smith (1999), that is,

$$\begin{aligned} y_t &= m_t + e_t, \\ m_t &= m_{t-1} + q_{t-1}e_{t-1} \end{aligned} \qquad (6)$$

where the function q_t is specified as

$$q_t = \frac{e_t^2}{g + e_t^2}$$

or

$$q_t = \frac{(e_t + \ldots + e_{t-s+1})^2}{g + (e_t + \ldots + e_{t-s+1})^2},$$

for some value s > 0, see Smith (2000) for further details. This model includes an endogenous smooth transition function to indicate structural breaks, and it can be seen as a contender to the discrete break model.

2.2. Representation

Based on the discussion above, we consider the following representation of a FI-BREAK model, that is,

$$\begin{aligned} a(L)(1-L)^d y_t &= m_t + e_t, \\ m_t &= m_{t-1} + q_{t-1} e_{t-1} \\ q_t &= \frac{(e_t + \ldots + e_{t-s+1})^2}{g + (e_t + \ldots + e_{t-s+1})^2} \end{aligned} \qquad (7)$$

where $a(L) = (1 - a_1 L - \ldots - a_p L^p)$. This general model can be seen to nest several related models by imposing certain parameter restrictions.

I. When d = 0 and $g \to \infty$, the model becomes an AR(p) model. Indeed, as $g \to \infty$, $q_t = 0$ for all t, which implies that $m_t = m_0$ for all t. Furthermore, if additionally d = 0, one has $a(L)y_t = m_0 + e_t$.

II. When 0 < d < 1 and $g \to \infty$, the model becomes an ARFI(p,d) model, that is, $a(L)(1-L)^d y_t = m_0 + e_t$.

III. When d = 0 and $0 < g < \infty$, the model is the familiar STOPBREAK model.[1] If $g < \infty$, this process contains an endogenous smooth break, see Engle and Smith (1999).

IV. When 0 < d < 1 and $0 < g < \infty$, the FI-BREAK model combines an I(d) model and a break model.

We summarize the various results in Table 1. In this paper, we consider only AR models, for estimation convenience. Also, there are other parameter combinations, such as $g = 0$ and d = 1, but we choose to consider only the above models I, II, III and IV. Indeed, if we would know the degree of integration of a series (1 or 2), we can take proper differences, and return to one of the models above.

[1] The original specification of STOPBREAK model of Engle and Smith (1999) is slightly different from ours as they consider $a(L)(y_t - m_t) = e_t$, $m_t = m_{t-1} + q_{t-1} e_{t-1}$.

2.3. Estimation

We can rewrite the FI-BREAK model (7) as

$$\sum_{j=0}^{\infty} p_j \Delta y_{t-j} = e_t - q_{t-1} e_{t-1}$$

where $q_{t-1} = 1 - q_{t-1}$ and $\sum_{j=0}^{\infty} p_j L^j = (1 - a_1 L - \ldots - a_p L^p)(1-L)^d$. Using Theorem 1 in Engle and Smith (1999), we can show that this nonlinear moving average process is invertible with probability 1 if $prob(q_t>0) > 0$ and if

$$q_t \left(1 + \frac{e_t}{q_t} \frac{\partial q_t}{\partial e_t}\bigg|_{e_t} \right) < 2$$

with probability 1. Hence, we can estimate the model parameters using the AML method of Beran (1995), see also van Dijk, Franses and Paap (2002). Similarly, we can claim that the AML estimator for the FI-BREAK model is consistent and asymptotically normal.

We now examine the empirical performance of the estimation procedure for the FI-BREAK model. We simulate three different types of long-memory processes with occasional break of the equation (3). The numbers of e_t and h_t are generated from the standard Gaussian distribution and $s_e^2 = 1$ and $p = 0.01$.

DGP A: $d = 0.1$ and $s_h^2 = 0.5$

This DGP gives data with clear (visual) breaks but weak long memory

DGP B: $d = 0.4$ and $s_h^2 = 0.5$

This DGP gives data with clear breaks and evident long memory

DGP C: $d = 0.4$ and $s_h^2 = 0.1$

This DGP gives data with evident long memory, but with weak evidence of breaks

For each DGP, we set the number of Monte Carlo replications to 250 with sample length T = 300, 600, and 1500. The sample size 300 is similar to the ones used in the empirical analysis below. For DGP B, we also simulate series with length T = 3000 in order to investigate the properties of FI-BREAK estimators in large samples. The DGPs do not contain AR parameters, and when we estimate the models we also impose the AR order to be zero. We estimate the FI-BREAK model parameters while imposing that $0 < g \leq \infty$ and $d \geq 0$. For comparison purposes, we also estimate d in an ARFIMA(0,d,0) model by Beran's (1995) AML method, which corresponds with a FI-BREAK model with $g \to \infty$.

The simulation results are summarized in Figure 1 to Figure 3 and in Tables 2-1 and 2-2. The kernel densities in the graphs are computed using the Epanechnikov function. The first column in figures concerns the estimated values of d in a FI-BREAK model, and the second column shows the densities of the estimated d in an ARFI model. We can observe a clear upward bias in the estimation of d in the ARFI model, which is of course due to neglected level shifts. As the sample size increases, this bias in the ARFI model is getting worse, that is, the distribution moves further to the right. The FI-BREAK model appears to be quite successful in filtering out the break components when estimating d. Table 2-1 shows that for DGPs A and C, the increase of the sample size from 300 to 1500 provides substantial evidence of improvement of estimating d in the FI-BREAK model. For DGP C, that is the DGP with strong long memory but with a weak break component, the estimation of d in the FI-BREAK model is improving quickly as sample size increases. On the other hand, the convergence speed is slow in DGP B, as shown in Table 2-1. We therefore also consider this DGP with a sample size of 3000. For 51 series out of 250 replications (see Table 2-2) we obtain an estimated value of d close to zero with $g \to 0$. From Table 1, we can see that this is the case of $g \to 0$ and d = 0. In this case, $q_t = 1$ for all t, which means $m_t = m_{t-1} + e_{t-1}$. After simple algebra, we get $a(L)\Delta y_t = e_t$, that is, we have an ARI(p,1) process. As DGP B has clear long memory and clear break components together, this process has properties very similar to a unit root process, even in large samples.

Overall, we conclude from the simulation results that the AML estimation method for the FI-BREAK model is reliable. This holds particularly true for data with weak break components but with clear long memory components.

3. US inflation

We now turn to the question in the title of the paper. To answer this question, we will evaluate the empirical merits of the model I to IV, discussed above.

We consider 23 monthly US Consumer Price Index series. These series were randomly chosen from the U.S. City Average data set. The sample period covers 1967:01 - 2000:08 and the base years are 1982 - 1984. All series are seasonally adjusted. Inflation rates are constructed from the price indices by taking 100 times the first differences of the logarithmic transformed series. The series concern All items, Durables, Commodities, Energy Commodities, Commodities less Food, Commodities less Food and Energy, Commodities less Food, Energy and Used Cars and Trucks, Services, Medical care services, Transportation services, Transportation, Housing, Electricity, Fuels, New vehicles, Men's and Boys' apparel, Footwear, Alcoholic Beverages, Eggs, Beef and veal, Fish and seafood, Fruits and vegetables and Potatoes.

First we estimate the FI-BREAK model for all 404 observations with various restrictions. We fix the AR order p for each model at 1, 6 and 12, in order to avoid the effect of an AR order selection procedure. Secondly, we split the sample into two parts. For the in-sample period we estimate the parameters with AR order p = 3 and use the estimated values for all one-step-ahead out-of-sample forecasts. We choose 1967:01-1990:12 as the in-sample period, which has 288 observations, and hence the out-of-sample period is 1991:01-2000:08 with 116 observations.

3.1. In-sample fit

For the comparison of different types of models, we impose restrictions on the parameters of FI-BREAK model, that is, we consider

I. The AR model: $d = 0$ and $g \to \infty$,

II. The ARFI model: $d > 0$ and $g \to \infty$,

III. The STOPBREAK model: $d = 0$ and $0 < g < \infty$,

IV. The new FI-BREAK model: $d \geq 0$ and $0 < g \leq \infty$.

Evidently, the ARFI model is close to (or "nests") the AR model, the STOPBREAK model nests the AR model, but the ARFI model and the STOPBREAK model are not nested. The four models are to be compared by using AIC, BIC, and the log likelihood for three AR orders. The results are summarized in Tables 3, 4 and 5 for the AR lag length of 1, 6 and 12.

For most cases we find that the ARFI model is the best using any of the three model selection criteria. There are only a few cases where the FI-BREAK model is estimated to be different from the STOPBREAK model or the ARFI model. In most cases, the estimated FI-BREAK models are equivalent with the AR, the ARFI or the STOPBREAK model. When p = 1 or p = 6, the FI-BREAK model sometimes ends up to equal the STOPBREAK model. Moreover, when the AR order increases, a linear AR specification apparently captures nonlinear features of inflation. Indeed, when p = 12, there is more evidence in favor of a linear AR model, that is, 12 out of 23 series can be fitted well by an AR model in terms of the log likelihood.

Overall, we find that the ARFI model is the best model for the US inflation series. There is mild evidence of neglected break components, for example for series 2, 6, and 7. For those series, the STOPBREAK model is the best model in terms of in-sample fit. As the ARFI and STOPBREAK models are not nested, one may maintain a FI-BREAK model for these three cases.

3.2. Out-of-sample forecasting

We now turn to an evaluation of the out-of-sample forecasting performance of the four models, where we set the forecast horizon at 1, 3, 12 and 24. Table 6 presents the results of cumulative forecasts. The models are estimated only once using the in-sample period 1967:1-1990:12. We compute the root mean squared forecast errors (RMSFE). For comparison, we report the ratios of the RMSFEs for the AR model, ARFI model, and STOPBREAK model over the RMSFE of FI-BREAK models.

Generally, we find that the ARFI model or the FI-BREAK model performs better than the STOPBREAK model for almost all inflation series. This can be noticed from the fact that the ratios in the columns of ARFI model are almost always 1, and that these are less than the ratios of the STOPBREAK model. For the series with evidence of breaks (that is, series 2 and 6), we see that the STOPBREAK model can deliver better forecasts than the ARFI model. For the other 21 series, the ARFI model is not only the best model in terms of in-sample fit, it is also the best model in terms of out-of-sample forecasting for 19 inflation series. For series 7 and 21, the AML estimation results of the FI-BREAK model point towards the ARFI model, but its forecasts are not better than the forecasts of the STOPBREAK model. We also estimate linear AR models, but their forecasts are far from accurate.

4. Conclusion

In this paper we examined the usefulness of a new and general FI-BREAK model for the purpose of determining whether inflation rates have long memory, levels shifts or both. We used a range of estimation results and forecast evaluations to investigate the relative performance of the FI-BREAK model to a BREAK or an I(d) model.

Our simulation results indicate that the long-memory parameter in the FI-BREAK model can be consistently estimated, in the case that there is an unknown break component. This contribution of our paper may be relevant for other economic data as well.

The results of our analysis of 23 US inflation series broadly indicate the usefulness of the FI-BREAK model. Overall, however, we find that the ARFI model best captures the features of US inflation rates, and hence we are inclined to state that the dominant feature in inflation is long memory and that level shifts are less important.

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Table 1. Parameter Space and Models

	d = 0	0 < d < 1	d = 1
$\gamma \to 0$	ARI(p,1)	ARFI(p,1+d)	ARI(p,2)
$0 < \gamma < \infty$	STOPBREAK	FI-BREAK	Integrated BREAK
$\gamma \to \infty$	AR(p)	ARFI(p,d)	ARI(p,1)

Table 2-1. Estimated values of memory parameter d

		300	600	1500	3000
DGP A (d = 0.1)	ARFI	0.253 (0.109)	0.311 (0.089)	0.373 (0.066)	-
	FI-BREAK	0.126 (0.064)	0.135 (0.044)	0.141 (0.028)	-
DGP B (d = 0.4)	ARFI	0.697 (0.131)	0.720 (0.116)	0.767 (0.091)	0.788 (0.074)
	FI-BREAK	0.175 (0.202)	0.201 (0.199)	0.183 (0.188)	0.225 (0.180)
DGP C (d = 0.4)	ARFI	0.564 (0.116)	0.589 (0.102)	0.634 (0.085)	-
	FI-BREAK	0.298 (0.172)	0.313 (0.161)	0.332 (0.124)	-

Note: The entries are values averaged over 250 replications. The values in parentheses are the standard deviations for the simulated data.

Table 2-2. Estimated models by class of parameters

	Sample Size	300	600	1500	3000
DGP A	$d \approx 0$	3	2	0	-
	$\gamma \approx 0$	0	0	0	-
	$d \approx 0, \gamma \approx 0$	0	0	0	-
DGP B	$d \approx 0$	109	87	83	51
	$\gamma \approx 0$	142	126	132	96
	$d \approx 0, \gamma \approx 0$	109	87	83	51
DGP C	$d \approx 0$	38	30	13	-
	$\gamma \approx 0$	59	46	25	-
	$d \approx 0, \gamma \approx 0$	37	30	13	-

Note: The entries are the numbers out of 250 replications. We consider $d \approx 0$ case when $d < 0.001$, and $g \approx 0$ if $\Sigma q_t >$ (0.25 times the sample size).

Table 3. Estimation results in case p = 1

	AIC				BIC				Log likelihood			
	I	II	III	IV	I	II	III	IV	I	II	III	IV
1	2.041	1.894	1.929	II	2.061	1.924	1.958	II	979	948	955	II
2	2.376	2.305	2.277	III	2.396	2.335	2.307	III	1046	1031	1025	III
3	2.853	2.763	2.816	II	2.873	2.793	2.846	II	1142	1123	1133	II
4	6.587	6.592	I	II	6.607	6.622	I	II	1892	1892	I	II
5	3.084	3.051	3.086	II	3.104	3.081	3.116	II	1188	1181	1188	II
6	1.960	1.821	1.782	III	1.979	1.850	1.812	III	962	933	926	III
7	1.988	1.792	1.771	III	2.008	1.822	1.801	III	968	928	923	III
8	2.232	2.046	2.173	II	2.252	2.075	2.203	II	1017	979	1004	II
9	2.164	1.982	2.058	II	2.184	2.012	2.088	II	1003	966	981	II
10	3.283	3.199	3.262	II	3.303	3.229	3.292	II	1228	1210	1223	II
11	3.917	3.907	I	II	3.937	3.937	I	II	1356	1353	I	II
12	2.346	2.201	2.242	II	2.366	2.131	2.272	II	1040	1010	1018	II
13	4.373	4.278	4.299	II	4.393	4.308	4.329	II	1447	1427	1432	II
14	4.351	4.248	4.286	II	4.371	4.278	4.316	II	1443	1421	1429	II
15	3.429	3.426	I	II	3.449	3.455	I	II	1258	1256	I	II
16	3.388	3.365	3.358	3.360	3.408	3.395	3.388	3.400	1249	1244	1242	1242
17	3.647	3.611	3.598	3.602	3.667	3.641	3.628	3.642	1301	1293	1291	1290
18	2.987	2.973	I	II	3.007	3.003	I	II	1169	1165	I	II
19	8.032	8.036	I	II	8.052	8.066	I	II	2183	2183	I	II
20	5.596	I	I	I	5.616	I	I	I	1693	I	I	I
21	4.538	4.456	4.454	4.457	4.558	4.486	4.484	4.497	1481	1463	1463	1462
22	6.212	I	I	I	6.237	I	I	I	1818	I	I	I
23	8.070	I	I	I	8.090	I	I	I	2190	I	I	I

Note: Sample period, 1967:01 - 2000:08 with 404 observations. I, II, III, and IV denote the AR model, the ARFI model, the STOPBREAK model and the FI-BREAK model, respectively. When an entry contains an I, II or III, it means that the estimation results are the same as for that model.

Table 4. Estimation results in case p = 6

	AIC				BIC				Log likelihood			
	I	II	III	IV	I	II	III	IV	I	II	III	IV
1	1.909	1.895	1.909	II	1.979	1.976	1.989	II	935	932	934	II
2	2.349	2.315	2.295	III	2.419	2.396	2.376	III	1023	1015	1010	III
3	2.785	2.771	I	II	2.855	2.852	I	II	1109	1105	I	II
4	6.591	6.596	I	II	6.661	6.676	I	II	1865	1865	I	II
5	3.064	3.055	I	II	3.135	3.135	I	II	1165	1162	I	II
6	1.858	1.844	1.806	1.811	1.928	1.924	1.886	1.901	925	921	914	914
7	1.788	1.792	1.776	III	1.858	1.872	1.856	III	911	911	908	III
8	2.033	2.027	I	II	2.103	2.107	I	II	960	958	I	II
9	1.981	1.985	I	II	2.051	2.065	I	II	950	949	I	II
10	3.218	3.215	I	II	3.288	3.295	I	II	1195	1193	I	II
11	3.923	3.910	I	II	3.993	3.990	I	II	1335	1331	I	II
12	2.221	2.216	I	II	2.292	2.296	I	II	997	995	I	II
13	4.330	4.303	4.323	II	4.400	4.383	4.403	II	1416	1409	1413	II
14	4.286	4.273	4.290	II	4.356	4.353	4.371	II	1407	1403	1407	II
15	3.444	3.441	I	II	3.515	3.521	I	II	1240	1238	I	II
16	3.376	3.368	3.367	II	3.446	3.448	3.448	II	1226	1224	1224	II
17	3.592	3.561	3.585	II	3.663	3.641	3.665	II	1269	1262	1267	II
18	2.992	I	I	I	3.062	I	I	I	1150	I	I	I
19	8.043	I	I	I	8.114	I	I	I	2153	I	I	I
20	5.605	5.609	I	II	5.676	5.689	I	II	1669	1669	I	II
21	4.476	4.433	4.461	II	4.546	4.513	4.542	II	1445	1435	1441	II
22	6.185	I	I	I	6.256	I	I	I	1784	I	I	I
23	8.085	I	I	I	8.156	I	I	I	2161	I	I	I

Note: See Table 3

Table 5. Estimation results in case p = 12

	AIC				BIC				Log likelihood			
	I	II	III	IV	I	II	III	IV	I	II	III	IV
1	1.870	1.873	I	II	2.002	2.015	I	II	907	907	I	II
2	2.319	2.323	2.289	III	2.451	2.465	2.431	I	995	995	988	I
3	2.760	2.765	I	II	2.892	2.907	I	II	1082	1081	I	II
4	6.579	I	I	I	6.711	I	I	I	1828	I	I	I
5	3.054	I	I	I	3.186	I	I	I	1139	I	I	I
6	1.869	1.871	1.831	III	2.001	2.013	1.974	III	907	907	899	III
7	1.804	1.809	1.785	III	1.936	1.951	1.927	III	895	895	890	III
8	2.028	I	I	I	2.160	I	I	I	938	I	I	I
9	2.007	I	I	I	2.139	I	I	I	934	I	I	I
10	3.210	I	I	I	3.342	I	I	I	1169	I	I	I
11	3.891	I	I	I	4.023	I	I	I	1303	I	I	I
12	2.224	2.229	I	I	2.356	2.371	I	I	977	977	I	I
13	4.135	4.320	I	II	4.447	4.462	I	II	1385	1385	I	II
14	4.289	I	I	I	4.421	I	I	I	1380	I	I	I
15	3.446	3.422	3.412	III	3.578	3.564	3.554	III	1215	1210	1208	III
16	3.371	3.367	3.365	II	3.503	3.509	3.507	II	1201	1199	1199	II
17	3.577	3.577	I	II	3.709	3.719	I	II	1241	1240	I	II
18	3.026	I	I	I	3.158	I	I	I	1133	I	I	I
19	8.010	I	I	I	8.142	I	I	I	2108	I	I	I
20	5.614	I	I	I	5.746	I	I	I	1639	I	I	I
21	4.461	4.461	4.463	II	4.593	4.603	4.605	II	1414	1413	1413	II
22	6.201	6.206	I	II	6.333	6.348	I	II	1754	1754	I	II
23	7.995	I	I	I	8.127	I	I	I	2105	I	I	I

Note: See Table 3.

Table 6. Root Mean Squared Cumulative Forecast Errors, relative to the FI-BREAK model

	AR model				ARFI model				STOPBREAK model			
	1	3	12	24	1	3	12	24	1	3	12	24
1	1.069	1.325	2.052	2.119	1.000	1.000	1.000	1.000	1.003	1.037	1.081	0.962
2	1.158	1.504	2.349	2.216	1.038	1.109	1.289	1.326	1.000	1.000	1.000	1.000
3	1.037	1.236	1.695	1.785	1.000	1.000	1.000	1.000	1.037	1.236	1.695	1.785
4	0.999	1.002	0.997	1.006	1.000	1.000	1.000	1.000	0.999	1.002	0.997	1.006
5	1.001	1.125	1.456	1.628	1.000	1.000	1.000	1.000	1.001	1.125	1.456	1.628
6	1.135	1.545	2.777	2.862	1.068	1.248	1.598	1.699	1.000	1.000	1.000	1.000
7	1.091	1.332	1.985	1.968	1.000	1.000	1.000	1.000	0.940	0.818	0.683	0.718
8	1.159	1.442	1.972	1.887	1.000	1.000	1.000	1.000	1.159	1.442	1.972	1.887
9	1.418	1.843	2.175	1.850	1.000	1.000	1.000	1.000	1.261	1.550	1.744	1.398
10	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
11	0.993	1.075	1.244	1.348	1.000	1.000	1.000	1.000	0.993	1.075	1.244	1.348
12	1.122	1.440	2.350	2.355	1.000	1.000	1.000	1.000	1.010	1.034	1.052	0.960
13	1.045	1.188	1.475	1.444	1.000	1.000	1.000	1.000	1.045	1.188	1.475	1.444
14	1.019	1.052	1.217	1.414	1.000	1.000	1.000	1.000	1.019	1.052	1.217	1.414
15	1.074	1.152	1.155	1.104	1.000	1.000	1.000	1.000	1.074	1.152	1.155	1.104
16	1.020	1.109	1.292	1.257	1.000	1.000	1.000	1.000	1.020	1.109	1.292	1.257
17	1.049	1.172	1.515	1.482	1.000	1.000	1.000	1.000	1.000	1.025	1.114	1.086
18	0.984	0.969	1.034	1.010	1.000	1.000	1.000	1.000	0.984	0.969	1.034	1.010
19	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
20	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
21	1.040	1.169	1.637	1.687	1.000	1.000	1.000	1.000	0.983	0.944	0.781	0.660
22	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
23	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Note: The in-sample period is 1967:01 - 1990:12 with 288 observations and the out-of-sample period is 1991:1 - 2000:8 with 116 observations. The AR order is fixed at 3.

Figure 1. Kernel Density of Estimated Memory Parameter (DGP A)

FI-BREAK model

ARFI model

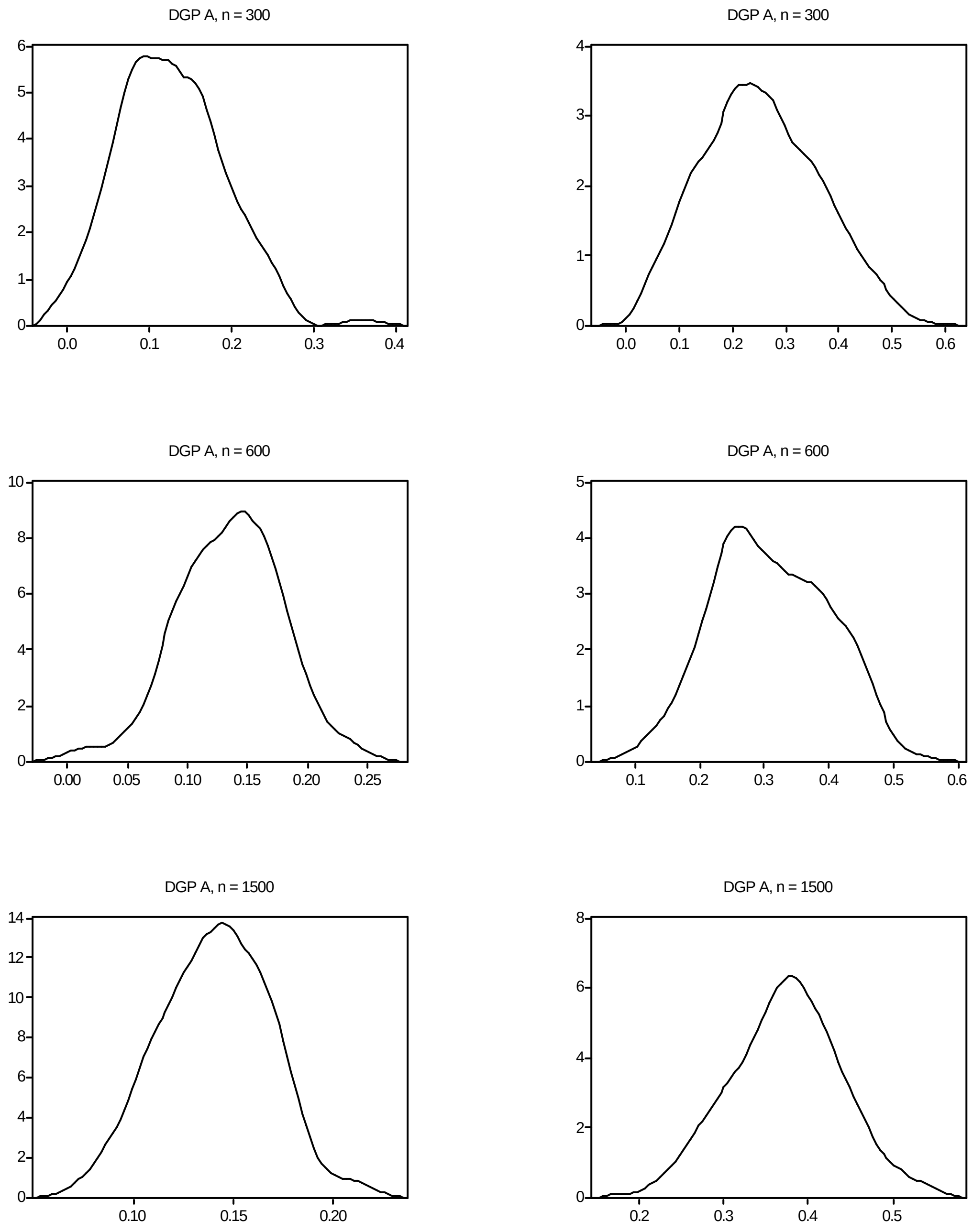

Figure 2. Kernel Density of Estimated Memory Parameter (DGP B)

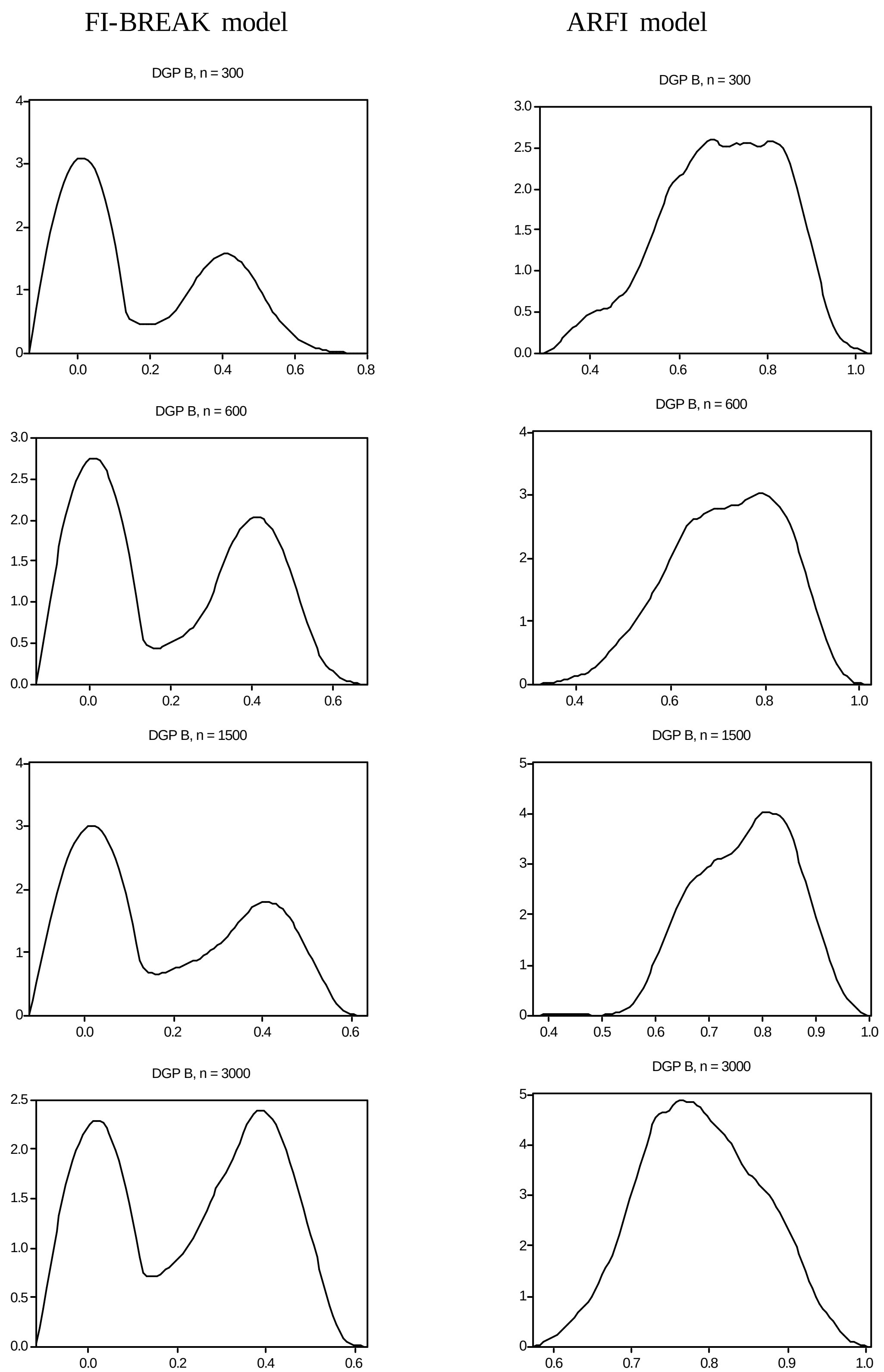

Figure 3. Kernel Density of Estimated Memory Parameter (DGP C)

FI-BREAK model ARFI model