A Comparison of Inventory Control Policies for a Joint Manufacturing / Remanufacturing Environment with Remanufacturing Yield Loss

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Abstract

We consider a joint manufacturing / remanufacturing environment with remanufacturing yield loss. Demand and return follow independent stationary Poisson processes. Returns can be disposed off upon arrival to the system. Manufacturing and remanufacturing operations performed in the same facility at exponential rates. Yield information becomes available after remanufacturing. Demands that are not directly satisfied are lost. We investigate what inventories to consider when making production and disposal decisions, with the objective of maximizing the long-run average expected profit. Four different policies are compared that base disposal decisions on either the local (returns) inventory or the global inventory, and production decisions on either the local (serviceable) inventory or the global inventory. By modelling the system as a Markov process, expressions for the profit associated with each policy are derived. An extensive numerical study shows that it is always optimal to base disposal decisions on the local inventory and production decisions on the global inventory within the parameter sets considered. A sensitivity analysis reveals further insights.

Keywords: inventory control, product returns, remanufacturing, yield loss

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1 Introduction

One of the main characteristics of joint manufacturing / remanufacturing environment is the high degree of uncertainty in the timing, quantity, and quality of returns. Although quality uncertainty has been mentioned as an important complicating factor for material planning activities in a number of studies (Thierry et. al. 1995, Ayres et. al. 1997, Ferrer 1997, Guide and Srivastava 1997, Guide et. al. 2000), this type of uncertainty is ill-researched. Most of the production planning and inventory control literature on reverse logistics only takes uncertainty in timing and quality into account, and assumes that all returns are of the same quality and are successfully remanufactured. See Dekker et al. (2004) for an extensive and recent review.

There have been some results on yield uncertainty. Ferrer (2003) examines a number of distinct situations about the information on the failure of recovery under a deterministic demand, single period setting where the supply of returns is infinite. He includes a number of serial recovery operations and identifies the value of having the yield information available in early stages of the recovery. In addition, the importance of having responsive suppliers in situations with imperfect recovery is shown by numerical experimentation. Ferrer and Ketzenberg (2004) extend this study to the infinite horizon setting. Bayındır et. al. (2005) study the desired level of recovery when both the success and unit cost of recovery increases as the recovery effort increases that is measured by expected time spent for the operation. The analysis is restricted to the case where all items completing their usage time returns to the system. Various inventory control policies that differ in inventory position definition and the time epoch that the regular purchasing decisions are proposed.

In this paper, we investigate the influence of remanufacturing yield uncertainty on the effectiveness of inventory policies. In situations with perfect remanufacturing, it is intuitive that production decisions should be based on the serviceables inventory as well as the returns inventory. However, how to include returns in production decisions is complicated especially when the manufacturing and the remanufacturing operations have non-identical lead times under the practical inventory control policies. A detailed discussion is provided by van der Laan et al. (2004).

The situation is more complicated when the remanufacturing option is not perfect simply due to the fact that not all but a portion of the returns can be used to satisfy the customer demand. Therefore, we will compare policies that base manufacturing decisions on the serviceable inventory (local) to those that consider the returns inventory as well (global). Furthermore, we compare policies that base disposal decisions on the returns inventory (local) to those that consider the serviceable inventory as well (global). So, in total, four different types of policies are compared. All are characterized by a dispose-down-to level for returns and an order-up-to level for production.

The production system is modelled as follows. Manufacturing and remanufacturing operations are performed in the same facility, but by dedicated resources. When the item under consideration is produced, the manufacturing line is always operating (so there is no shortage of input materials for manufacturing) and the remanufacturing line is operating as long as returns are available. The manufacturing and remanufacturing rates are both exponential. Due to quality variations in the returns, there is remanufacturing yield loss. Whether or not a remanufactured item is serviceable becomes known after remanufacturing has ended, as in the case of tire remanufacturing (Ferrer(1997)).

Demand and return follow independent Poisson processes. We note that the independency is a common assumption in reverse logistics, which is justified if the time lag between a sale and a return is considerable. See Fleischmann (2000) for a discussion. Disposal of returns is allowed upon arrival. Demands that can not be immediately satisfied from serviceables on hand are lost. The objective is to maximize the long-run average profit.

By modelling the system as a Markov process, expressions are derived for the profit associated with each of the four above mentioned policies types. In an extensive numerical study, we determine the optimal policy parameters for each type using complete search (with certain bounds on the two policy parameters) for a large number of examples. The main result is that it is always optimal to base disposal decisions on the local (returns) inventory and production decisions on the global (returns and serviceables) inventory. A sensitivity analysis reveals further insights.

The remainder of the paper is organized as follows. In Section 2, the assumed environment is explained in detail, and the policy types are introduced. In Section 3, a generic average long-run profit function is derived. In Section 4, results of the computational study are discussed. Finally, in Section 5, conclusions are presented and further research directions are indicated.

2 Environment and control rules

The notation is summarized in Table 1.

INSERT TABLE 1

In this study we consider a single item, joint manufacturing / remanufacturing environment, which is illustrated in Figure 1.

INSERT FIGURE 1

The demand for the item is Poisson with rate λ, and customers are indifferent between manufactured and remanufactured products. Unsatisfied demand is lost. We do not include an explicit lost sales cost, since it is difficult to estimate it in practice. Instead we consider a profit maximization objective, thereby including the effect of losing profit margin whenever a stock out occurs. We restrict ourselves to lost sale case due to the fact that the assumption makes the system easily analyzed analytically.

The system is subject to stochastic returns following a Poisson process with rate $r\lambda$. We restrict our attention to the case where the demand rate is greater than the return rate on the average, so $0 < r < 1$. Returns are either remanufactured or disposed off. The decision whether or not a return is remanufactured results from the inventory policy, for which we consider several variants that will be discussed in detail later. Remanufacturing is successful with probability ω, $0 < \omega \leq 1$. That is, the remanufacturing yield is $100\omega\%$. Returns that are successfully remanufactured enter the stock of serviceable items, and those that do not satisfy the required standards after remanufacturing are disposed off.

The production facility consists of a remanufacturing line as well as a manufacturing line. The latter is obviously needed, since there are more demands than returns. Both lines have exponential production rates that are denoted by μ_r and μ_m, respectively. Both manufacturing and remanufacturing lines process items on one by one basis. Of course, the remanufacturing line can only be utilized as long as returns are available. Therefore, the total production rate of the facility is $\mu_r + \mu_m$ if returns are available, and μ_m otherwise.

The objective is to maximize the expected (long run) steady state profit per time unit. There is a revenue p for each satisfied demand. Costs included are: (i.) unit costs; c_m for manufacturing, c_r for remanufacturing, and c_d for disposal, (ii.) per unit time unit holding costs; h_r for returns and h_s for serviceables. We remark that the unit disposal cost c_d is only incurred for units that are not successfully remanufactured, but disposed on arrival. For ease of presentation in later sections, without loss of generality the expected disposal cost per remanufactured unit, which is $(1-\omega)c_d^{'}$ if the cost for disposing of a remanufactured unit is $c_d^{'}$, is included in c_r. We remark that the disposal cost can be negative if there are recycling revenues or if not (successfully) remanufactured returns can be sold on a secondary market.

Consistently with ignoring fixed costs, we restrict our attention to base stock inventory policies. A policy is characterized by a pair of base stock levels: a dispose-down-to level D and an order-up-to level S. It is defined as follows. If the *serviceable inventory position (SIP)* drops to or below S, then the production facility is opened until the SIP again reaches S. If the *disposable inventory position (DIP)* is D or more, then returns are disposed off.

Both the SIP and the DIP can be defined in two ways: local and global. Indeed, the main aim of this study is to investigate what definitions maximize profit under the restriction that each piece of information is either fully utilized, or completely ignored. The local SIP is defined as the serviceable inventory on hand, and is denoted by I_s. The local DIP is defined as the returns on hand plus in process, and is denoted by I_r. The global SI/DI is defined as the sum of I_s and I_r, and is denoted by I_g. Since both SI and DI can be defined in two ways, there are four possible policies, which are listed and numbered in Table 2.

INSERT TABLE 2

We remark under Policy II and IV (global SI), the policy parameters are restricted to $D < S$, since otherwise the system cannot leave a state where there are "many" returns and no serviceable inventory and consequently incurs infinitely many lost sales.

The optimal control policy structure for the considered system can be sought by modelling it as a continuous time Markov decision process. Of course, other inventory definitions and policy structures are possible. Indeed, we do not claim that the overall optimal policy is of one of the four proposed base-stock policy types. That overall optimal policy may have a (much) more complex structure and utilize complicated inventory definitions, and may be state-dependent. However, for reasons of practicality, we restrict our attention to the base-stock policy structure and to four proposed types where each information piece on inventory levels is either fully utilized or completely ignored.

Based on intuition, we expect that policies II and IV (global SI) perform better relative to policies I and III (local SI) if the yield ω is higher and if the remanufacturing rate μ_r is larger compared to the manufacturing rate μ_m, since remanufacturing is more reliable and faster under those conditions. It is difficult to predict the influence of the other model parameters on the comparative performance of local versus global DI policies. To get insights into the effect of model parameters on the relative performance of the four policies, and to determine (rough) guidelines into what policy performs best under what conditions, we will perform an extensive numerical study in Section 4. In that section, we compare the optimal policies of types I-IV, which can be determined using the analysis in the next section.

3 Determining the optimal policy parameters

Under all policy types proposed, evolution of both return and serviceable inventories are triggered by exponential events. Therefore, the system under each policy can be modelled as a Markov Process on the state space $\mathcal{M} = \{(I_s, I_r) \mid I_s \geq 0, I_r \geq 0\}$, which for some policies is further reduced by policy type specific restrictions. Ignoring these specific conditions for the moment, there are five possible events that lead the system to enter a state $(i, j) \in \mathcal{M}$. These are:

I.1 a return arrival from state $(i, j-1)$ with rate $r\lambda$, (if $j > 0$)

I.2 a manufactured item completion from state $(i-1,j)$ with rate μ_m, (if $i>0$)

I.3a a successful remanufactured item completion from $(i-1,j+1)$ with rate $\omega\mu_r$, (if $i>0$)

I.3b an unsuccessful remanufactured item completion from state $(i,j+1)$ with rate $(1-\omega)\mu_r$ (if $j+1>0$),

I.4 a demand occurrence from state $(i+1,j)$ with rate λ.

Similarly, the outflows from such a state can be caused by five possible events;

O.1 a return arrival to state $(i,j+1)$ with rate $r\lambda$,

O.2 a manufactured item completion to state $(i+1,j)$ with rate μ_m,

O.3a a successful remanufactured item completion to $(i+1,j-1)$ with rate $\omega\mu_r$, (if $j>0$)

O.3b an unsuccessful remanufactured item completion to state $(i,j-1)$ with rate $(1-\omega)\mu_r$ (if $j>0$),

O.4 a demand occurrence to state $(i-1,j)$ with rate λ, (if $i>0$).

As remarked above, there are policy type specific restrictions on the state space. These are discussed in the Appendix for each policy type separately. So, the state space under policy $k \in \{I, II, III, IV\}$ is a subset of $\mathcal{M}$, which we will denote by $\mathcal{M}_k$. For each policy type k, the set $\mathcal{M}_k$ as well as the generator matrix of the Markov process based on the above listed inflow and flow events, are given in the Appendix. Moreover, using that matrix, the steady state probabilities $\pi_{(i,j)}, (i,j) \in \mathcal{M}_k$ that the system is in state (i,j) are derived in the Appendix. Note that for ease of presentation, the policy type index is omitted from $\pi_{(i,j)}$.

The steady-state probabilities $\pi_{(i,j)}, (i,j) \in \mathcal{M}_k$ can be used to derive the long-run average expected revenue and costs per time unit as a function of the policy parameters S and D. We start by deriving the revenue function. Since demands are satisfied if there are serviceables on hand and lost otherwise, it easily follows that average revenue in the long-run can be written for Policy k is as follows:

$$R_k(S,D) \quad = \quad p\lambda\left(1-\sum_j \pi_{(0,j)}\right) \quad \text{for all } k \in \{I,\ldots,IV\}.$$

Similarly, we get average holding cost

$$H_k(S,D) \quad = \quad \sum_i \sum_j \left\{(ih_s + jh_r)\pi_{(i,j)}\right\} \quad \text{for all } k \in \{I,\ldots,IV\}.$$

What remains is to determine the cost functions for production (manufacturing and remanufacturing) and for disposal. These are not the same for all policy types, since production and disposal decisions are based on different inventory definitions.

Under Policies I and III (local SI), the production facility is open when $I_s < S$. Both the manufacturing line and the remanufacturing line are open if $I_r > 0$ and only the manufacturing line is open otherwise. Hence, the long-run average production cost is

$$CP_k(S,D) = c_m\mu_m\left(1-\sum_j \pi_{(S,j)}\right) + c_r\mu_r\left(1-\sum_j \pi_{(S,j)} - \sum_{i,\ i\neq S}\pi_{(i,0)}\right) \quad \text{for all } k\in\{I,\ III\}.$$

The only difference under Policies II and IV (global SI) is that the production facility is open when $I_g = I_s + I_r < S$. Hence, the long-run average production cost function is

$$CP_k(S,D) = c_m\mu_m\left(1-\sum_{i+j=S} \pi_{(i,j)}\right) + c_r\mu_r\left(1-\sum_{i+j=S} \pi_{(i,j)} - \sum_{i,\ i\neq S}\pi_{(i,0)}\right) \quad \text{for all } k\in\{II,\ IV\}.$$

Under Policies I and II, returns are disposed if $I_r = D$. Hence, the long-run average disposal cost function is (recall from Section 2 that disposal costs for unsuccessfully remanufactured units are included in the remanufacturing cost)

$$CD_k(S,D) = c_d r\lambda\sum_i \pi_{(i,D)} \quad \text{for all } k\in\{I,\ II\}.$$

Under Policies III and IV, returns are disposed if $I_g = I_s + I_r \geq= D$. Hence, the disposal cost function is

$$CD_k(S,D) = c_d r\lambda\sum_{i+j\geq D} \pi_{(i,j)} \quad \text{for all } k\in\{III,\ IV\}.$$

To find the optimal (S,D) pair maximizing the expected average profit in the steady state under each policy $k\in\{I,\ldots,IV\}$, the following problem should be solved.

$$\begin{array}{ll} Max & P_k(S,D) = R_k(S,D) - H_k(S,D) - CP_k(S,D) - CD_k(S,D) \\ s.t & S\in\{0,1,\ldots\} \\ & D\in\{0,1,\ldots\} \\ & D<S \text{ (only for Policies II and IV)} \end{array}$$

Note that we do not have closed for expressions for the steady state probabilities and consequently the objective function under all policy types considered. Therefore, we cannot proof any structural property of the objective function with respect to decision variables analytically. Nevertheless our numerical experience reveals that the function is unimodalar under the parameter ranges considered.

4 Computational study

The main aim of the computational study is to assess the relative performance of the proposed policies under different system conditions. For this reason, an extensive experimental setting is considered. Note that for ease of presentation, we will refer to a 'policy' rather than the 'policy with optimal policy parameters'.

In the experimental setting, the average demand rate λ, the price of the item p, the unit manufacturing cost c_m, and the unit serviceables holding cost h_s are kept constant at $\lambda = 1$, $p = 2$, $c_m = 1$, and $h_s = 0.25$. A full factor experiment is designed on the remaining factors. The levels of the factors are given in Table 3.

INSERT TABLE 3

In these settings, we cover the cases where (1) total system capacity in terms of average production rate is significantly less than, almost equal to, and significantly larger than the average demand rate, (2) the fraction of total production capacity allocated to remanufacturing channel is very low, about half, and very high, (3) the average return ratio, i.e., the ratio of average return rate to average demand rate, is very low, moderately high, and very high, (4) the unit remanufacturing cost is lower than, equal to, and higher than the unit manufacturing cost, (5) the unit disposal cost is zero, significantly lower than unit remanufacturing cost, or slightly lower than that cost. Moreover, for the key yield loss parameter, we consider the wide range [0.1,1] with step-size of 0.1.

We use MATLAB to determine the optimal policy levels S and D for each problem instance under all four policies. It appears from the numerical results that the profit function is unimodular. However, since we do not have no formal proof, a full search procedure is applied (within reasonable bounds). In the remainder of this section, we report the main observations. For interested readers, all raw results are available upon request.

Our main observations can be summarized as follows:

Observation 1: There is a certain minimum yield level required to differentiate the performance of the policies for a given set of all other problem parameters.

Observation 2: Policy II under which production decisions are based on system-wide inventory and disposal decisions are based on returns inventory outperforms the others whenever the performance of the policies can be differentiated.

In the following subsections we discuss these two main observations in detail.

4.1 Threshold Yield Level

The most important observation is that policy II performs at least as good as the others for all problem settings considered. Recall that Policy II basis disposal decisions on the returns inventory and production decisions on the global inventory. The dominance of policy II is rather surprising. We expected (see Section 1) that for small yield levels, it would be better to place production decisions on the serviceable inventory only.

As it turns out, however, there is no significant difference in performance between the four policies for small yield levels. This is illustrated in Figure 2 for specific model parameter settings.

INSERT FIGURE 2

A look at the policy parameters reveals, for this example but also in general, that there is a certain, model parameter dependent, threshold yield level below which remanufacturing is simply not profitable, and all policies set the dispose-down-to level to either zero or one, thereby disposing off (almost) all returns upon arrival. With such small dispose-down-to levels and hence small stocks of returns, the different policies make (almost) identical inventory control decisions. Apparently, as soon as yield becomes sufficiently large for remanufacturing to be profitable, it is better to include the returns in the serviceable inventory positions, even for still relatively low yield levels such as 0.4 in Figure 2.

In Table 4 average threshold yield levels are reported for combinations of values for the return rate and another parameter. Averages are calculated over all cases with those values.

INSERT TABLE 4

It appears from Table 4 the threshold yield level increases as,

- unit disposal cost, c_d, decreases,
- unit remanufacturing cost, c_r, increases,
- unit holding cost of returns, h_r, decreases
- return ratio, r, increases,
- proportion of total system capacity allocated to remanufacturing, $\frac{\mu_r}{\mu_r+\mu_m}$, increases,
- total system capacity, $\mu_r + \mu_m$, decreases.

These results are intuitive. If the remanufacturing cost is relatively large, a higher yield level is required for remanufacturing to be profitable. The threshold level is also high when more returns are available for remanufacturing, the remanufacturing capacity is relatively large, and total capacity is small. Under these conditions, the system performance becomes more dependent on remanufacturing and a larger yield level is required to remain reliable.

4.2 Performance Comparison of the Policies

As mentioned above, Policy II outperforms the other policies for all cases (with yield above the threshold level). The effect of yield level on the relative performance of the policies is typical in Figure 2; the profit improvement by Policy II increases as the yield level increases. Table 5 shows the average increase (over the cases with a difference in profit) in profit compared to Policies I, III, and IV. Note that in Table 5 we report the absolute profit improvements rather than percentages since in some cases Policy II makes an improvement over no profit situation; and consequently percentage improvements are infinity.

INSERT TABLE 5

It appears from Table 5 that the performance of Policy I and III are almost the same on the average. On the other hand, Policy IV is the worst behaving policy.

The explanation for the bad performance of Policy I and III is trivial; they ignore the work-in-process, i.e., returns, while making production decisions. Due to the fact that at higher values of ω most of the items in returns inventory are successfully remanufactured, it is beneficial to count them while making production decisions.

The very poor performance of Policy IV is more difficult to explain. Recall that the only difference with the best behaving Policy II is that Policy IV includes serviceables in the disposable inventory position. It turns out this leads to less than desirable utilization of the remanufacturing option. Policy III suffers from the same disadvantage, but to a lesser extent due to the additional restriction that $D < S$ for Policy IV. (The same restriction does not lower the performance of Policy II since under Policy II, D is defined in terms of the local, i.e. returns, inventory position.) As a result, the relative performance of Policy IV is especially poor when (i.) taking back the returns is most profitable i.e, when both unit remanufacturing and disposal costs and holding cost of returns are low, and the return ratio is high, and (ii.) the serviceable inventory accumulates quickly even without using the remanufacturing line, i.e., when the total system capacity is high and most of the capacity is allocated to manufacturing option. The effect is amplified by the discrete character of D and S.

5 Conclusion

We investigate the performance of a number of inventory control policies for a joint manufacturing-remanufacturing system with remanufacturing yield loss. The policies differ in the type of inventory information, local or global, used for disposal (of returns upon arrival) and production decisions.

The main results are as follows. There is a case-dependent threshold yield level below which remanufacturing is not so profitable and all policies achieve the same performance. In general, the threshold yield level increases in the unit remanufacturing cost and in the proportion of capacity allocated to remanufacturing, and decreases in unit disposal cost. When yield is above the threshold level, the policy that basis disposal decisions on the local (returns) inventory and production decisions on the global (returns + serviceables) inventory outperforms all others. A counter intuitive result is that the policy which basis all decisions on the global inventory performs worst. The numerical results show that this "full information" policy is unable to utilize the remanufacturing option as much as desired.

Our work can be extended into several other environments.

- **Manufacturing and remanufacturing in distinct facilities:** Two facilities are operated independently, (possibly) using different production order-up-to levels and basing decisions on different pieces of inventory information. The increased flexibility complicates the analysis, and optimization by simulation may be required.

- **Partial utilization of information pieces.** Especially for production decisions, it seems natural to only add the expected yield of the available returns to the inventory position. This again complicates the analysis and may also reduce the practical applicability, but those disadvantages may be outweighed by the profit increase.

- **Modeling the remanufacturing operation:** In real life, remanufacturing often consists of a series of testing and repair operations with a portion of the yield information becomes available after each step. Ferrer (2003) shows the importance of having yield information in early stages of the remanufacturing under single period setting. An extension of our model could show whether or not these finds extend to a multi-period setting.

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A Derivation of steady state probabilities

The inflows (I.1, etc.) and outflows referred to in this section are those given in Section 2.

Policy I

Under Policy I, the system can be modelled as a two-dimensional Markov process on the state space $\mathcal{M_I} = \{(I_s, I_r) \mid I_s \in \{0, 1, \ldots, S\}, I_r \in \{0, 1, \ldots, D\}\}$ for given (S, D). The following constraints on inflow and outflows to each state (i, j) are imposed by Policy I.

D.I When $I_r = D$, returns are disposed off. Hence, it is not possible to leave a state $(i, D) \in \mathcal{M}_I$ by outflow O.1.

P.I When $I_s = S$, production stops. Hence, outflows O.2, O.3a, and O.3.b are not possible for $(S, j) \in \mathcal{M}_I$. Similarly, state $(S, j) \in \mathcal{M}_I$ cannot be reached by an I.3b. type inflow.

The Generator Matrix of the Markov process under Policy I, obtained by ordering states lexicographically as (0,0),(0,1), …, $(0, D)$, (1,0), …, $(1, D)$,…, $(S, 0)$, …, (S, D), is as follows:

$$Q = \begin{bmatrix} A_0 & B & 0 & 0 & 0 & \ldots & 0 & 0 & 0 \\ C & A & B & 0 & 0 & \ldots & 0 & 0 & 0 \\ 0 & C & A & B & 0 & \ldots & 0 & 0 & 0 \\ 0 & 0 & C & A & B & \ldots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ldots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \ldots & C & A & B \\ 0 & 0 & 0 & 0 & 0 & \ldots & 0 & C & A_S \end{bmatrix},$$

where, A_0, A, A_S, B and C are $(D + 1) \times (D + 1)$ matrices and given by,

$$A_0 = \begin{bmatrix} -(\mu_m + r\lambda) & r\lambda & 0 & \ldots & 0 & 0 & 0 & 0 \\ (1-\omega)\mu_r & -(\mu_m + r\lambda) & r\lambda & \ldots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \ldots & 0 & (1-\omega)\mu_r & -(\mu_m + r\lambda) & r\lambda \\ 0 & 0 & 0 & \ldots & 0 & 0 & (1-\omega)\mu_r & -\mu_m \end{bmatrix}, \tag{1}$$

$$A = \begin{bmatrix} -(\mu_m + \bar{\lambda}) & r\lambda & 0 & \ldots & 0 & 0 & 0 & 0 \\ (1-\omega)\mu_r & -(\mu_m + \bar{\lambda}) & r\lambda & \ldots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \ldots & 0 & (1-\omega)\mu_r & -(\mu_m + \bar{\lambda}) & r\lambda \\ 0 & 0 & 0 & \ldots & 0 & 0 & (1-\omega)\mu_r & -(\mu + \lambda) \end{bmatrix}, \tag{2}$$

$$A_S = \begin{bmatrix} -\bar{\lambda} & r\lambda & 0 & \dots & 0 & 0 & 0 & 0 \\ 0 & -\bar{\lambda} & r\lambda & \dots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 0 & 0 & -\bar{\lambda} & r\lambda \\ 0 & 0 & 0 & \dots & 0 & 0 & 0 & -\lambda \end{bmatrix}, \tag{3}$$

$$B = \begin{bmatrix} \mu_m & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\ \omega\mu_r & \mu_m & 0 & \dots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 0 & \omega\mu_r & \mu_m & 0 \\ 0 & 0 & 0 & \dots & 0 & 0 & \omega\mu_r & \mu_m \end{bmatrix}, \tag{4}$$

$$C = \begin{bmatrix} \lambda & 0 & \dots & 0 \\ 0 & \lambda & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \lambda \end{bmatrix}, \tag{5}$$

where $\bar{\lambda} = (1+r)\lambda$, $\mu = \mu_r + \mu_m$.

Using the generator matrix, balance and normalization equations can be expressed as

$$\begin{aligned} \Pi_0 A_0 + \Pi_1 C &= 0 && (6) \\ \Pi_{i-1} B + \Pi_i A + \Pi_{i+1} C &= 0 \quad \text{for all } \; i = 1, \dots, S-1 && (7) \\ \Pi_{S-1} B + \Pi_S A_S &= 0 && (8) \\ \sum_{i=0}^{S} \Pi_i \vec{1} = 1 & && (9) \end{aligned}$$

where $\Pi_i = (\pi_{(i,0)}, \pi_{(i,1)}, \dots, \pi_{(i,D)})$ for all $\; i \in \{0, 1, \dots, S\}$.

Using (6) and (7), it can be shown that

$$\Pi_i = \Pi_0 X_i \quad \text{for all } \; i = 1, \dots, S, \tag{10}$$

where

$$\begin{aligned} X_1 &= -A_0 C^{-1}, && (11) \\ X_2 &= A_0 C^{-1} A - B, && (12) \\ X_i &= -(X_{i-2} B + X_{i-1} A) C^{-1} \quad \text{for all } \; i = 3, \dots, S && (13) \end{aligned}$$

Finding X_{S-1} and X_S using (11), (12), and (13) recursively, and substituting them into (8) yields

$$\Pi_0 \bar{X} = 0,$$

where

$$\bar{X} = X_{S-1}B + X_S A_S. \tag{14}$$

Note that $rank(\bar{X}) = S$.

On the other hand, substituting (10) into (9) yields,

$$\Pi_0 \left(\vec{1} + \sum_{i=1}^{S} X_i \vec{1} \right) = 1.$$

Therefore, Π_0 can be found by,

$$\Pi_0 = e_{D+1}^T \bar{X'}^{-1},$$

where $\bar{X'}$ is obtained by replacing $D + 1^{st}$ column of $\bar{X}$ with $\left(\vec{1} + \sum_{i=1}^{S_r} X_i \vec{1} \right)$, and e_{D+1} is the $D + 1^{st}$ elementary vector.

Matrix Π_i, for all $i \in \{1, \ldots, D\}$ can be determined using (10).

Policy II

Under this policy, the system can be modelled as a two-dimensional Markov process on the state space, $\mathcal{M}_{\mathcal{II}} = \{(I_s, I_r) \mid I_s \in \{0, 1, \ldots, S\}, I_r \in \{0, 1, \ldots, D\}\}$ for given (S, D). The following constraints on inflow and outflows to each state (i, j) are imposed by Policy II.

D.II. Returns are accepted according to the same criterion as under Policy I. Therefore, it is not possible to leave a state $(i, D) \in \mathcal{M}_I$ by outflow O.1.

P.II. When $I_g = S$, production is stopped. Hence, outflows O.2, O.3a and O.3.b are not possible for state $(i, j) \in \mathcal{M}_{II}$ such that $i + j \geq S$. Moreover, outflows O.1 is only possible if it is valid for D.II. Similarly, it is not possible to reach such a state by any type of item completion, i.e., I.2, I.3a, and I.3b are not possible. (When $i + j = S$ only I.3b is not possible.)

The Generator Matrix of the Markov process under Policy I, obtained by ordering states lexicographically as (0,0),(0,1), …, $(0, D)$, (1,0), …, $(1, D)$,…, $(S, 0)$, …, (S, D), is as follows:

$$Q = \begin{bmatrix} A_0 & B_0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ C & A_1 & B_1 & 0 & 0 & \dots & 0 & 0 & 0 \\ & C & A_3 & B_3 & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & C & A_4 & B_4 & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \dots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \dots & C & A_{S-1} & B_{S-1} \\ 0 & 0 & 0 & 0 & 0 & \dots & 0 & C & A_S \end{bmatrix},$$

where, A_i, B_i for all $i \in \{0, \dots, S\}$ and C are $(D+1) \times (D+1)$ matrices that can be written similar to the ones under Policy I:

- Matrix A_0, A_S and C are exactly as defined in (1), (3), and (5), respectively.
- Matrix A_i is equal to A given in (2) if $D + i < S$. For $i \geq S - D$ the following modifications are made:
  - $A_i(l, l-1) = 0$ for all $l \in \{2, \dots, D+1\}$ if $l + i - 1 \geq S$,
  - $A_i(l, l) = -(1+r)\lambda$ for all $l \in \{2, \dots, D\}$ if $l + i - 1 \geq S$,
  - $A_i(D+1, D+1) = -\lambda$.
- Matrix B_i is equal to B given in (4) if $D + i < S$. For $i \geq S - D$ the following modifications are made:
  - $B_i(l, l-1) = 0$ for all $l \in \{2, \dots, D+1\}$ if $l + i - 1 \geq S$,
  - $B_i(l, l) = 0$ for all $l \in \{2, \dots, D+1\} <$ if $l + i - 1 \geq S$.

The procedure utilized for finding the steady-state balance equations under Policy I can easily be adapted for this case. In this case the equations (11), (12) and (13) will be replaced by the following expressions, respectively.

$$\begin{aligned} X_1 &= -A_0 C^{-1}, \\ X_2 &= A_0 C^{-1} A_1 - B_0, \qquad (15) \\ X_i &= -(X_{i-2} B_{i-2} + X_{i-1} A_{i-1}) C^{-1} \text{ for } \quad \text{for all } \; i = 3, \dots, S. \qquad (16) \end{aligned}$$

Besides, the matrix $\bar{X}$ (given in equation (14) under Policy I) becomes

$$\bar{X} = X_{S-1} B_{S-1} + X_S A_S \qquad (17)$$

in that case.

Policy III

Under Policy III, the system can be modelled as a two-dimensional Markov process on the state space, $\mathcal{M}_{III} = \{(I_s, I_r) \mid I_s \in \{0, 1, \ldots, S\}, I_r \in \{0, 1, \ldots, D\}\}$ for given (S, D). The following constraints on inflow and outflows to each state (i, j) are imposed by Policy I:

D.III When $I_g = D$, returns arriving to the system are disposed of. Hence, O.1 cannot occur if $i+j \geq D$. Similarly it is not possible to reach a state (i, j) such that $i + j > S$ by I.1.

P.III P.I provided for Policy I is valid under Policy III too.

The Generator Matrix of the Markov process under Policy III, obtained by ordering states lexicographically as (0,0),(0,1), …, $(0, D)$, (1,0), …, $(1, D)$,…, $(S, 0)$, …, (S, D), is as follows.

$$Q = \begin{bmatrix} A_0 & B & 0 & 0 & 0 & \ldots & 0 & 0 & 0 \\ C & A_1 & B & 0 & 0 & \ldots & 0 & 0 & 0 \\ 0 & C & A_3 & B & 0 & \ldots & 0 & 0 & 0 \\ 0 & 0 & C & A_4 & B & \ldots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ldots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \ldots & C & A_{S-1} & B \\ 0 & 0 & 0 & 0 & 0 & \ldots & 0 & C & A_S \end{bmatrix},$$

where, A_i for all $i \in \{0, \ldots, S\}$, B and C are $(D + 1) \times (D + 1)$ matrices that can be written similar to the ones under Policy I:

- Matrix A_0, B and C are exactly as defined in (1), (4), and (5), respectively.
- The following modifications are made on A given in (2) to get matrix A_i for all $i \in \{0, \ldots, S - 1\}$:
  - $A_i(1, 1) = -(\lambda_m + \mu_m)$ if $i \geq D$,
  - $A_i(l, l) = -(\lambda + \mu)$ for all $l \in \{2, \ldots, D + 1\}$ if $l + i - 1 \geq D$,
  - $A_i(l, l + 1) = 0$ for all $l \in \{2, \ldots, D\}$ if $l + i - 1 \geq D$.
- $A_S = -C$.

The procedure utilized for finding the steady-state balance equations under Policy II can easily be adapted for this case; all B_i terms in equations (15), (16) and (17) are replaced by B.

Policy IV

Under this policy, the system can be modelled as a two-dimensional Markov process on the state space, $\mathcal{M}_{\mathcal{IV}} = \{(I_s, I_r) \mid I_s \in \{0, 1, \ldots, S\}, I_r \in \{0, 1, \ldots, S - I_s\}\}$ for given (S, D). The following constraints on inflow and outflows to each state (i, j) are imposed by Policy IV:

D.IV. Due to the fact that the return acceptance is made as in the same way as Policy III, the restrictions given in D.III are also valid under Policy IV.

P.IV. Due to the fact that the production decisions are based on the same information piece as in Policy II, the restrictions given in P.II. are valid under Policy IV, except there is no state (i, j) such that $i + j > S$ in this case.

Unfortunately, the underlying Markov process under Policy IV does not show any specific structure. Therefore, stead-state probabilities are found by directly solving systems of linear equations representing balance equations and normalization.

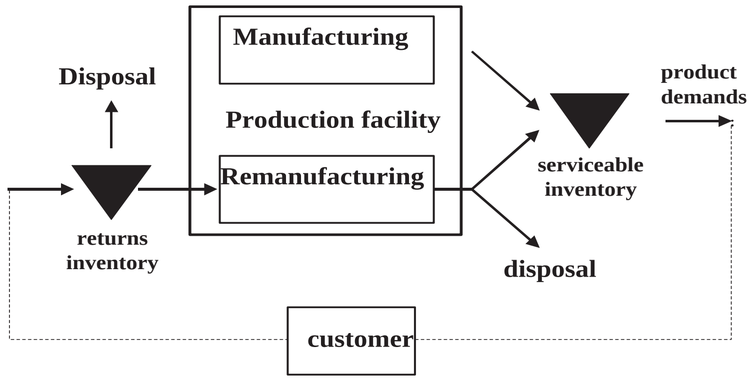

Figure 1: A simple sketch of the system considered

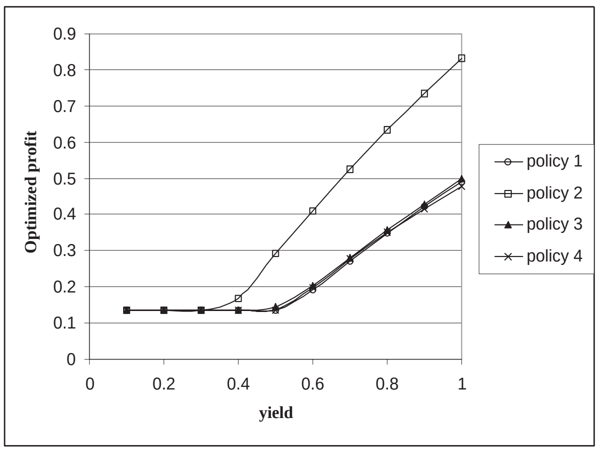

Figure 2: An example case; $\mu_r + \mu_m = 2$, $\mu_r/(\mu_r + \mu_m) = .9$, $c_d = 0$, $h_r = .125$, $r = .95$

Table 1: Notation used

λ:	mean demand rate per unit time
$r\lambda$:	mean return rate per unit time
μ_r:	mean remanufacturing rate
μ_m:	mean manufacturing rate
ω:	probability that a return is successfully remanufactured
I_r:	inventory level of returns
I_s:	inventory level of serviceables
I_g:	system-wide inventory level $I_g = I_r + I_s$
p:	unit price
c_r:	unit remanufacturing cost
c_m:	unit manufacturing cost
c_d:	unit disposal cost
h_r:	unit holding cost of returns per unit time
h_s:	unit holding cost of serviceables per unit time

Table 2: Base information under each policy

Policy	Production	Disposal
I	I_s	I_r
II	I_g	I_r
III	I_s	I_g
IV	I_g	I_g

Table 3: Values of varied parameters in the experiment

Factor	Levels
$\mu_r + \mu_m$	.5, .9, 1.1, 2
$\frac{\mu_r}{\mu_m+\mu_r}$	.1, .45, .9
h_r	0, .1
c_r	.75, 1, 1.25
$\frac{c_d}{c_r}$	0, .25, .5
r	.25, .75, 0.95
ω	.1, .2, .3, .4, .5, .6, .7, .8, .9, 1

Table 4: Yield values at which the performances of policies start to differentiate

		r		
		0.25	0.75	0.95
c_r	0.75	0.23	0.26	0.26
	1	0.26	0.30	0.31
	1.25	0.32	0.36	0.36
c_d/c_r	0	0.31	0.31	0.31
	1/4	0.26	0.28	0.30
	1/2	0.24	0.31	0.33
h_r	0	0.24	0.27	0.27
	0.125	0.31	0.36	0.36
$\mu_r + \mu_m$	0.5	0.32	0.35	0.37
	0.9	0.29	0.34	0.35
	1.1	0.26	0.29	0.28
	2	0.21	0.24	0.25
$\frac{\mu_r}{\mu_r+\mu_m}$	0.1	0.11	0.12	0.11
	0.45	0.21	0.22	0.21
	0.9	0.60	0.60	0.60

Table 5: Average profit improvement by Policy II over the others

		Policy I			Policy III			Policy IV		
		r			r			r		
		0.25	0.75	0.95	0.25	0.75	0.95	0.25	0.75	0.95
c_r	0.75	0.166	0.206	0.219	0.166	0.207	0.220	0.178	0.227	0.239
	1	0.173	0.220	0.221	0.173	0.219	0.221	0.181	0.234	0.233
	1.25	0.180	0.220	0.224	0.180	0.220	0.224	0.186	0.228	0.229
c_d/c_r	0	0.183	0.225	0.235	0.183	0.225	0.235	0.188	0.233	0.243
	1/4	0.171	0.216	0.227	0.171	0.216	0.227	0.179	0.231	0.240
	1/2	0.164	0.202	0.195	0.165	0.203	0.196	0.178	0.225	0.214
h_r	0	0.211	0.255	0.262	0.211	0.256	0.262	0.225	0.279	0.281
	0.125	0.129	0.166	0.170	0.129	0.166	0.169	0.132	0.170	0.174
$\mu_r + \mu_m$	0.5	0.092	0.125	0.129	0.092	0.126	0.129	0.097	0.128	0.132
	0.9	0.154	0.194	0.202	0.154	0.194	0.202	0.163	0.206	0.211
	1.1	0.180	0.223	0.225	0.180	0.223	0.225	0.189	0.239	0.238
	2	0.247	0.288	0.291	0.247	0.287	0.290	0.258	0.312	0.312
$\frac{\mu_r}{\mu_r+\mu_m}$	0.1	0.259	0.272	0.270	0.259	0.272	0.270	0.263	0.275	0.272
	0.45	0.145	0.200	0.206	0.146	0.200	0.206	0.162	0.216	0.219
	0.9	0.064	0.127	0.151	0.064	0.127	0.151	0.070	0.163	0.182