# Modeling the Volatility in Global Fertilizer Prices*

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# Abstract

The main purpose of this paper is to estimate the volatility in global fertilizer prices. The endogenous structural breakpoint unit root test and alternative volatility models, including the generalized autoregressive conditional heteroskedasticity (GARCH) model, Exponential GARCH (EGARCH) model, and GJR model are estimated for six global fertilizer prices and the crude oil price. Weekly data for 2003-2008 for the seven price series are analysed. The empirical results suggest that the volatility of global fertilizer prices and crude oil price from March to December 2008 are higher than in other periods, and that the peak crude oil price caused greater volatility in the crude oil price and global fertilizer prices.

**Keywords:** Volatility, Global fertilizer price, Crude oil price, Non-renewable fertilizers, Structural breakpoint unit root test.

## I. Introduction

The world population in 2000 was more than 6 billion, and is expected to reach 8 billion in 2025, based on projections by United Nation Population Division. The increase in global population, combined with economic development, will place increasing demand on agricultural food products, especially grains, rice, soybeans, and sugarcane. The derived demand for energy crops has been increased significantly due to the development of bio-fuel. Such development can lead to food shortages and increasing international food prices, which will encourage farmers to expand planted acreage. This predicament has increased the derived demand for global fertilizers and increased fertilizer prices.

Fertilizers are combinations of nutrients that enable plants to grow. The essential elements of fertilizers are nitrogen, phosphorus, and potassium. Urea fertilizer is the major fertilizer that provides the element of nitrogen, and is produced through converting atmospheric nitrogen using natural gas. Ammonia and phosphoric acid are also produced using energy. Thus, prices for urea, ammonia, and acid will be affected by energy prices. Monoammonium phosphate (hereafter MAP) and muriate of potash (hereafter MOP) are two other important fertilizers that are sources of phosphorus and potassium. As most of the world's phosphate for fertilizer is mined, and hence is non-renewable, over the last decade the prices of phosphate and potash fertilizers have risen more steeply than the price of nitrogen-based urea.

Figure 1 shows the trends in six fertilizer prices and Dubai crude oil price during the period 2003-2008. It is clear that most of these prices changed dramatically in 2007 and 2008. Figure 2 shows the trends in the prices of the main fertilizers, including MAP, MOP and urea, and Dubai crude oil weekly prices, from 2003-2008. This figure shows that fertilizers and Dubai crude oil price exhibit positive trends. Moreover, MAP and MOP prices had upsurge in early 2008. These figures show there is a clear positive

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relationship between global fertilizer prices and crude oil price. The main purpose of this paper is to estimate the volatility in global fertilizer prices and in the crude oil price. Such empirical results should provide useful information regarding the risk associated with variations in global fertilizer prices, with significant implications for global agricultural production.

The remainder of the paper is organized as follows. Section 2 introduces the data, the empirical models are discussed in Section 3, and the empirical results are analysed in Section 4. Some concluding remarks related to the policy implications of the volatility of global fertilizer prices are given in the final section.

## II. Data

The source of the data is divided into two parts. The weekly global fertilizer supply prices are obtained from the Fertilizer Market Bulletin (hereafter FMB) weekly fertilizer report, while the weekly Dubai crude oil prices are obtained from the database in the Bureau of Energy during the period 2003-2008. Table 1 gives the descriptive statistics of six fertilizer prices, including monoammonium phosphate, urea, ammonia, phosphoric acid, phosphate rock, and potassium chloride, and Dubai crude oil prices. The monoammonium phosphate prices show a steady upward trend, but have a sharp price spike in February 2008, as shown in Figure 1. The prices of urea and ammonia vary considerably, with steady increases over time. The phosphoric acid, phosphate rock, and potassium chloride supply prices do not fluctuate significantly, but generally have upward trends. The trend in crude oil prices is relatively stable.

## III. Model Specifications

The generalized autoregressive conditional heteroskedasticity (GARCH) model will be used to model the volatility in global fertilizer and crude oil prices. Before

estimating the GARCH models, the Lee and Strazicich (2003) approach will be used to capture the structural breakpoint in fertilizer prices, which should enable identification of alternative time periods for the volatility in fertilizer prices.

**3.1 Minimum LM unit root test with two endogenous breaks**

Most traditional empirical studies use regression methods to estimate relationships among variables under the assumption of stationarity. However, spurious regression results may arise when some or all of the variables are non-stationary. The Dickey-Fuller (1979, 1981) test, Augmented Dickey-Fuller (ADF) test (Said and Dickey, 1984), and Phillips-Perron (1988) test are widely-used unit root tests, but they are based on data generation processes with no structural breaks. Ignoring possible structural breaks can lead to non-rejection of the null hypothesis of non-stationarity, so that the effects of structural breaks may be attributed to the existence of a unit root. Nelson and Plosser (1982) used the Dickey-Fuller unit root test to examine U.S. macroeconomic time series, and found that widespread non-stationarity.

In order to tackle the problem of structural breaks, Perron (1989) proposed a unit root test with a structural breakpoint, which used an exogenous structural break to re-examine Nelson and Plosser's (1982) data. The empirical results showed that most macroeconomic time series do not have unit roots, and the data features displayed by variables with a structural change are similar to those displayed by variables with unit roots. Thus, it is important to test for structural change, otherwise an incorrect outcome of the unit root test is likely.

Banerjee et al. (1992) and Zivot and Andrews (1992) modified the unit root test with a known breakpoint to a unit root test with an unknown breakpoint. Lumsdaine and Papell (1997) and Lee and Strazicich (2003) transformed the unit root test with an unknown breakpoint into a unit root test with two unknown breakpoints. However, Lee

and Strazicich (2003) establish minimum LM unit root test with two unknown structural change points to compensate for the shortcomings of the test. Both the null and alternative hypotheses are specified for series with two endogenous structural breakpoints.

**3.2 Conditional Mean and Conditional Volatility Models**

Engle (1982) captured time-varying volatility through the autoregressive conditional heteroskedasticity (ARCH) model. Subsequent extensions, such as the generalized ARCH (GARCH) model of Bollerslev (1986), have been used to capture dynamic volatility for univariate and multivariate processes. The GARCH model is most widely used for symmetric shocks. In the presence of asymmetric shocks, whereby positive and negative shocks of equal magnitude have different impacts on volatility, the GJR model of Glosten et al. (1992) and the EGARCH model of Nelson (1991) are very useful. Further theoretical developments in specification, estimation and asymptotic theory have been suggested in Ling and Li (1997), Ling and McAleer (2002a, 2002b, 2003a, 2003b), and McAleer (2005).

The following model is based on McAleer (2005) and McAleer et al. (2007). The methods have been extended detect the volatility in patent growth (Chan, Marinova and McAleer, 2005a), in analyzing the volatility of USA ecological patents (Marinova and McAleer, 2003; Chan, Marinova and McAleer, 2005b), in modelling the volatility of environment risk (Hoti, McAleer and Pauwels, 2005), and the volatility of atmospheric carbon dioxide concentrations (McAleer and Chan, 2006). However, there does not yet seem to have been any empirical analysis of such volatility models on global fertilizer prices.

In this paper, we consider the stationary AR(1)-GARCH(1,1), or ARMA(p,q)-GARCH(1,1), model for the global fertilizer price series data, namely $y_t$:

$$y_t = \phi_1 + \phi_2 y_{t-1} + \varepsilon_t, \qquad \text{for } t = 1, \ldots, n, \qquad\qquad (1)$$

$$y_t = ARMA(p,q) + \varepsilon_t$$

where $\varepsilon_t$ is the unconditional shock (or movement in global fertilizer prices), and is given by:

$$
\begin{aligned}
&\varepsilon_t = \eta_t \sqrt{h_t}, \; \eta_t \sim iid\,(0,1), \\
&h_t = \omega + \alpha \varepsilon_{t-1}^2 + \beta h_{t-1},
\end{aligned}
\qquad\qquad (2)
$$

and $\omega \geq 0, \alpha \geq 0, \beta \geq 0$ are sufficient conditions to ensure that the conditional variance $h_t \geq 0$. Ling and McAleer (2003b) indicated equation (2) in the AR(1) process could be modified to incorporate a non-stationary ARMA(p,q) conditional mean and a stationary GARCH(r,s) conditional variance. In (2), the $\alpha$ (or ARCH) effect indicates the short run persistence of shocks, while the $\beta$ (or GARCH) effect indicates the contribution of shocks to long run persistence (namely, $\alpha + \beta$).

The parameters in equations (1) and (2) are typically estimated by the maximum likelihood method. Ling and McAleer (2003b) investigate the properties of adaptive estimators for univariate non-stationary ARMA models with GARCH(r,s) errors. The conditional log-likelihood function is given as follows:

$$\sum_{t=1}^{n} l_t = -\frac{1}{2} \sum_{t=1}^{n} (\log h_t + \frac{\varepsilon_t^2}{h_t}).$$

As the GARCH process in equation (2) is a function of the unconditional shocks, the moments of $\varepsilon_t$ need to be investigated. Ling and Li (1997) showed that the ARCH(p,q)

model is strictly stationary and ergodic if the second moment is finite, that is, $(\alpha + \beta)^2 + 2\alpha^2 < 1$. Ling and McAleer (2003a) showed that the Quasi MLE (QMLE) for GARCH(p,q) is consistent if the second moment is finite. Ling and Li (1997) demonstrated that the local QMLE is asymptotically normal if the fourth moment is finite, that is, $E(\varepsilon_t^4 < \infty)$, while Ling and McAleer (2003a) proved that the global QMLE is asymptotically normal if the sixth moment is finite, that is, $E(\varepsilon_t^6 < \infty)$. Using results from Ling and Li (1997) and Ling and McAleer (2002a, 2002b) (see also Bollerslev (1986) and Nelson (1990)), the necessary and sufficient condition for the existence of the second moment of $\varepsilon_t$ for GARCH(1,1) is $\alpha + \beta < 1$ and, under normality, the necessary and sufficient condition for the existence of the fourth moment is $(\alpha + \beta)^2 + 2\alpha^2 < 1$.

For the univariate GARCH(p,q) model, Bougerol and Picard (1992) derived the necessary and sufficient condition, namely the log-moment condition or the negativity of a Lyapunov exponent, for strict stationarity and ergodicity (see also Nelson (1990)). Using the log-moment condition, Elie and Jeantheau (1995) and Jeantheau (1998) established it was sufficient for consistency of the QMLE of GARCH(p,q) (see Lee and Hansen (1994) for the proof in the case of GARCH(1,1)), and Boussama (2000) showed that it was sufficient for asymptotic normality. Based on these theoretical developments, a sufficient condition for the QMLE of GARCH(1,1) to be consistent and asymptotically normal is given by the log-moment condition, namely

$$E(\log(\alpha\eta_t^2 + \beta)) < 0. \tag{3}$$

However, this condition is not straightforward to check in practice, even for the GARCH(1,1) model, as it involves the expectation of a function of a random variable

and unknown parameters. The extension of the log-moment condition to multivariate GARCH(p,q) models has not yet been shown to exist, although Jeantheau (1998) showed that the ultivariate log-moment condition could be verified under the additional assumption that the determinant of the unconditional variance of $\varepsilon_t$ in (1) is finite. Jeantheau (1998) assumed a multivariate log-moment condition to prove consistency of the QMLE of the multivariate GARCH(p,q) model. An extension of Boussama's (2000) log-moment condition to prove the asymptotic normality of the QMLE of the multivariate GARCH(p,q) process is not yet available.

The effects of positive shocks on the conditional variance, $h_t$, are assumed to be the same as the negative shocks in the symmetric GARCH model. In order to accommodate asymmetric behavior, Glosten et al. (1992) proposed the GJR model, for which GJR(1,1) is defined as follows:

$$h_t = \omega + (\alpha + \gamma I(\eta_{t-1}))\varepsilon_{t-1}^2 + \beta h_{t-1}, \tag{4}$$

where $\omega > 0, \alpha \geq 0, \alpha + \gamma \geq 0, \beta \geq 0$ are sufficient conditions for $h_t > 0$ and $I(\eta_t)$ is an indicator variable defined by

$$I(\eta_t) = \begin{cases} 1 & \varepsilon_t < 0. \\ 0 & \varepsilon_t \geq 0, \end{cases}$$

as $\eta_t$ has the same sign as $\varepsilon_t$. The indicator variable differentiates between positive and negative shocks, so that asymmetric effects in the data are captured by the coefficient $\gamma$, with $\gamma \geq 0$. The asymmetric effect, $\gamma$, measures the contribution of shocks to both short run persistence, $\alpha + \gamma/2$, and to long run persistence,

$\alpha + \beta + \gamma / 2$.

Ling and McAleer (2002b) derived the unique strictly stationary and ergodic solution of a family of GARCH processes, which includes GJR(1,1) as a special case, a simple sufficient condition for the existence of the solution, and the necessary and sufficient condition for the existence of the moments. For the special case of GJR(1,1), Ling and McAleer (2002b) showed that the regularity condition for the existence of the second moment under symmetry of $\eta_t$ is

$$\alpha + \beta + \frac{1}{2} < 1, \tag{5}$$

and the condition for the existence of the fourth moment under normality of $\eta_t$ is

$$\beta^2 + 2\alpha\beta + 3\alpha + \beta\gamma + 3\alpha\beta + \frac{3}{2}\gamma^2 < 1, \tag{6}$$

while McAleer et al. (2007) showed that the weaker log-moment condition for GJR(1,1) was given by

$$E(\ln[(\alpha + \gamma I(\eta_t))\eta_t^2 + \beta]) < 0, \tag{7}$$

which involves the expectation of a function of a random variable and unknown parameters.

An alternative model to capture asymmetric behavior in the conditional variance is the Exponential GARCH (EGARCH(1,1)) model of Nelson (1991), namely:

$$\log h_t = \omega + \alpha \left| \eta_{t-1} \right| + \gamma \eta_{t-1} + \beta \log h_{t-1}, \quad \left| \beta \right| < 1 \tag{8}$$

where the parameters $\alpha$, $\beta$ and $\gamma$ have different interpretations from those in the GARCH(1,1) and GJR(1,1) models.

As noted in McAleer et al. (2007), there are some important differences between EGARCH and the previous two models, as follows: (i) EGARCH is a model of the logarithm of the conditional variance, which implies that no restrictions on the parameters are required to ensure $h_t > 0$; (ii) Nelson (1991) showed that $\left| \beta \right| < 1$ ensures stationarity and ergodicity for EGARCH(1,1); (iii) Shephard (1996) observed that $\left| \beta \right| < 1$ is likely to be a sufficient condition for consistency of QMLE for EGARCH(1,1); (iv) as the conditional (or standardized) shocks appear in equation (4), $\left| \beta \right| < 1$ would seem to be a sufficient condition for the existence of moments; and (v) in addition to being a sufficient condition for consistency, $\left| \beta \right| < 1$ is also likely to be sufficient for asymptotic normality of the QMLE of EGARCH(1,1).

Furthermore, EGARCH captures asymmetries differently from GJR. The parameters $\alpha$ and $\gamma$ in EGARCH(1,1) represent the magnitude (or size) and sign effects of the conditional (or standardized) shocks, respectively, on the conditional variance, whereas $\alpha$ and $\alpha + \gamma$ represent the effects of positive and negative shocks, respectively, on the conditional variance in GJR(1,1).


**IV. Empirical Results**

**4.1 Minimum LM unit root test with one and two breaks**

The empirical results for the unit root tests are shown in Table 2, and they generally indicate that the ADF test does not reject the null hypothesis of a unit root. However,

MAP, Urea, and Rock reject the null hypothesis at the 1□ significance level, which

shows that there is no unit root for these prices, as shown in Table 3, when the minimum

LM unit root test with two breaks by Lee and Strazicich (2003) is used. However, the

price series for ammonia are tested using the minimum LM test unit root with one break

as two breakpoints were not detected.

## 4.2 ARMA(p,q) Processes

In order to investigate global fertilizer price volatility, an appropriate time series

model needs to be determined that satisfies the appropriate regularity conditions. The

first task is to determine the processes for the mean equation. We choose the ARMA

processes with the smallest Schwarz Bayesian Information Criterion (BIC) value for the

seven series in each period. The p-values of the Ljung-Box Q statistics of the residuals

from the fitted models indicate that there is no autocorrelation at the 5% significance

level. Therefore, the specifications of the conditional mean and variance equations for

the seven series are given in Table 4.

## 4.3 Alternative Volatility Models for Crude Oil and Six Global Fertilizer Prices

The appropriate volatility models for each of the six fertilizer prices and crude oil

price are chosen on the basis of BIC and the regularity conditions for the moments to

exist, and hence for consistency and asymptotic normality of the QMLE to hold. The

QMLE will be consistent and asymptotically normal when the weak log-moment

condition is satisfied.

The empirical estimates for the alternative volatility models for the seven price

series are given in Tables 5-11 for the three different time periods (that is, with. one or

two structural breakpoints). Suitable models for crude oil price (given as Poil) are GJR

(1,1) for the first two periods, and GARCH(1,1) for the third period, as shown in Table 5. Periods 1 and 2 have asymmetric effects (with $\square\square 0$ in the GJR(1,1) model). The

short run persistence of shocks in periods 1, 2, and 3 are 0.079, 0.311 and 0.282, respectively, while the long run persistence of shocks in period 3 is 0.768, which is higher than in periods 1 and 2 of 0.314, and 0.519, respectively. These empirical outcomes indicate that a higher peak in the crude oil price is associated with greater volatility.

For the MAP price series, a suitable model in the first period is GJR(1,1), while a suitable model in periods 2 and 3 is GARCH(1,1), as shown in Table 6. The estimated coefficients satisfy the sufficient conditions for the conditional variance to be positive ($h_t \geq 0$). Time period 1 has an asymmetric effect (with $\square\square 0$ in the GJR(1,1) model).

The short run persistence of shocks for MAP in periods 1, 2 and 3 are 0.239, 0.286 and 0.356, respectively, while long run persistence is 0.374, 0.588 and 0.877, respectively, which indicates that MAP has the largest long run persistence of shocks in the third period. As compared with both the short and long run persistence of MAP and crude oil price, we find that both price series have same volatility effects in these three periods. In other words, both the level and volatility of MAP prices may be highly correlated with crude oil prices.

Table 7 shows that the GARCH(1,1) model is the appropriate model for the three periods for the Urea series. The estimates show that the weak log-moment condition is satisfied, so that the QMLE in the three periods for Urea are consistent and asymptotically normal. The short run persistence of shocks for Urea in periods 1, 2 and 3 are 0.080, 0.185 and 0.318, respectively, and the long run persistence of shocks in periods 1, 2 and 3 are 0.373, 0.595 and 0.960, respectively. The long run persistence of shocks in period 3 is more substantial than in the other two periods, which is similar to

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the case of the crude oil and MAP prices.

The appropriate model for the Ammonia series in the first and second periods is GJR(1,1), as shown in Table 8. Both of these two periods have asymmetric effects (with ☐☐0 in the GJR(1,1) model). The short run persistence of shocks in periods 1 and 2 are

0.269 and 0.522, respectively, while the long run persistence of shocks in periods 1 and 2 are 0.494 and 0.908, respectively. The long run persistence of shocks in the second period is greater than its counterpart in period 1.

Appropriate volatility models for Rock, Acid, and MOP prices for three different time periods are shown in Tables 9-11. For the Rock price series, the suitable models in the three time periods are GARCH(1,1), GARCH(1,1), and GJR(1,1), respectively, as shown in Table 9. For the Acid price series, as shown in Table 10, the best model in the first period is GJR(1,1), while GARCH(1,1) is best in the second and third periods. For the MOP price series, as shown in Table 11, the best model for all three time periods is GJR(1,1).

The empirical results show that the long run persistence of shocks in periods 1, 2 and 3 are 0.436, 0.605 and 0.817, respectively, for Rock prices, so that the Rock price in period 3 has the largest long run persistence of shocks. For Acid prices, the long run persistence of shocks in periods 1, 2 and 3 are 0.378, 0.627 and 0.733, respectively, so that the long run persistence in period 3 is the largest. With regard to MOP prices, the long run persistence of shocks in the three periods are 0.391, 0.589 and 0.916, respectively, so that the third period again has the largest long run persistence of shocks. Moreover, these three price series behave in a similar manner to crude oil prices.

## 4. Concluding Remarks

The main purpose of the paper was to evaluate empirically the volatility of global

fertilizer prices, and to link them to the volatility in crude oil prices. An empirically adequate model of volatility of the six global fertilizer prices was determined by checking the regularity conditions of the estimated models, and then detecting whether structural breaks existed in the six fertilizer price series. First, three time periods with two structural breakpoints were determined endogenously for six global fertilizer prices and the crude oil price using the Lee and Strazicich (2003) approach. Second, symmetric and asymmetric univariate conditional volatility models, including the widely used GARCH, GJR and EGARCH models, were estimated and selected on the basis of the BIC criterion and the regularity conditions for the QMLE to be consistent and asymptotically normal.

The contribution of shocks to the long run persistence of crude oil prices during the third period was found to be greater than during the first and second periods. This would suggest that the volatility in crude oil prices has recently increased in both strength and frequency. Therefore, the strength and frequency of global fertilizer prices has increased gradually over time. As the volatility in global fertilizer prices has increased, global agricultural production is likely to be affected significantly, which may lead to future instability in agricultural food prices.

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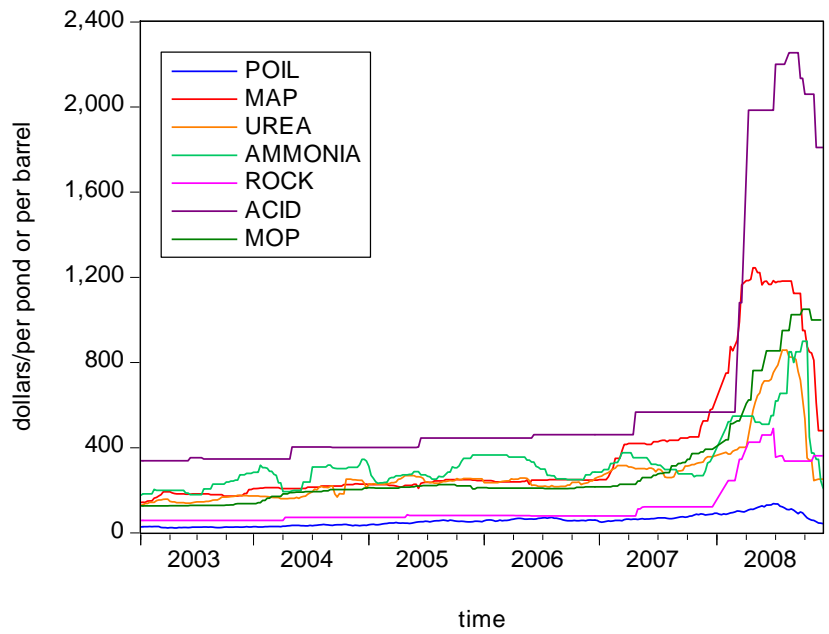
Figure 1.   Price Trends for Global Fertilizers and Crude Oil, 2003-2008
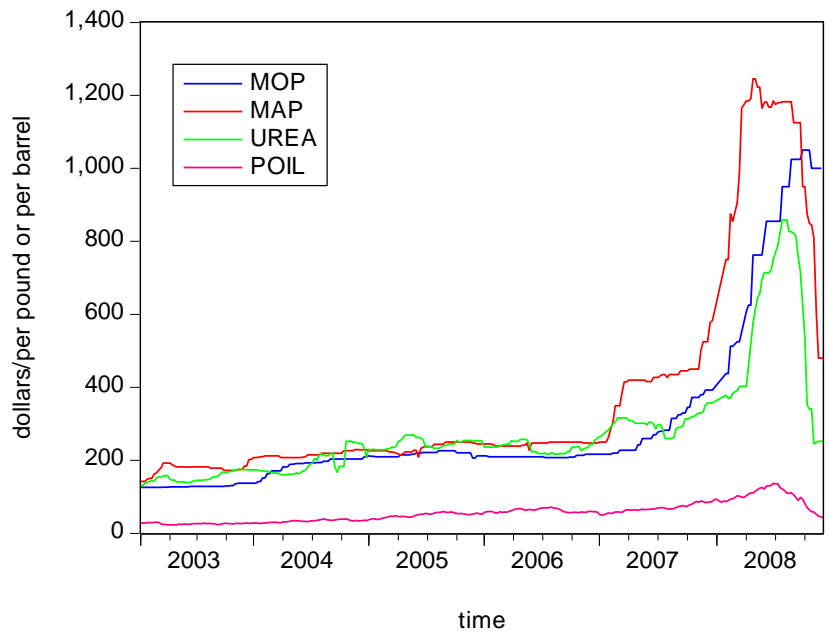


Figure 2.   Higher Energy Use Fertilizer Prices and Crude Oil Price, 2003-2008

**Table 1. Descriptive Statistics of Seven Price Series**

| Statistics | MAP (US$ /metric ton) | Urea (US$ /metric ton) | Ammonia (US$ /metric ton) | Acid (US$ /metric ton) | Rock (US$ /metric ton) | MOP (US$ /metric ton) | Poil (Price of Oil. US$/Bale) |
|---|---|---|---|---|---|---|---|
| Sample | 254 | 254 | 254 | 254 | 254 | 254 | 254 |
| Mean | 258.07 | 225.80 | 280.72 | 428.30 | 78.46 | 206.18 | 48.29 |
| Medium | 237 | 234.50 | 278.25 | 445.00 | 79.50 | 210.00 | 51.56 |
| Maximum | 582.5 | 357.5 | 357.5 | 566.25 | 121.5 | 392.5 | 88.32 |
| Minimum | 142.5 | 50.5 | 176 | 338.5 | 58 | 126 | 22.97 |
| Std. Dev. | 89.39 | 55.51 | 53.89 | 70.01 | 18.97 | 57.95 | 17.23 |

**Table 2. Augmented Dickey-Fuller (ADF) Unit Root Tests**

| Series | ADF tests | | Critical values | |
| | With constant | With constant and trend | With trend | With constant and trend |
| --- | --- | --- | --- | --- |
| **Poil** | -1.326(1) | -0.493(1) | | |
| **MAP** | -2.154(9) | -2.248(9) | -3.457 (1□) | -3.995 (1□) |
| **Urea** | -2.439(3) | -3.125(3) | | |
| **Ammonia** | -1.089(9) | -2.301(9) | -2.873 (5□) | -3.428 (5□) |
| **Rock** | -2.372(0) | -2.681(0) | -2.573 (10□) | -3.137 (10□) |
| **Acid** | -2.179(0) | -1.926(0) | | |
| **MOP** | 3.280(0) | 1.327(0) | | |

Note: BIC is used to select the optimal lag length. The values in parentheses denote the number of lags.

**Table 3. LM Unit Root Tests with Two Breaks**

| Series | LM$_\tau$ | k | TB1 | TB2 |
|---|---|---|---|---|
| **Poil** | -6.0177*** | 8 | 20071129 | 20080327 |
| **MAP** | -8.2394*** | 8 | 20071108 | 20080327 |
| **Urea** | -8.2641*** | 8 | 20071220 | 20080424 |
| **Ammonia** | -5.7755** | 7 | | 20080320 |
| **Rock** | -7.9262*** | 8 | 20070412 | 20080313 |
| **Acid** | -15.9207*** | 0 | 20071220 | 20080410 |
| **MOP** | -9.5491*** | 8 | 20071213 | 20080424 |

Notes: The 1□, 5□ and 10□ critical values are -5.823, -5.286, and -4.989, respectively

(see Lee and Strazicich, 2003). *, ** and *** denote significance at the 10□, 5□

and 1□ levels, respectively.

**Table 4. Optimal ARMA Processes for Seven Prices**

|  | Period | | |
|---|---|---|---|
|  | Period 1 | Period 2 | Period 3 |
| **Poil** | 2003/01/09-2007/11/22<br>ARMA(3,2) | 2007/11/29-2008/03/20<br>ARMA(2,1) | 2008/03/27-2008/12/04<br>ARMA(3,3) |
| **MAP** | 2003/01/09-2007/11/01<br>ARMA(2,1) | 2007/11/08-2008/03/20<br>ARMA(1,1) | 2008/03/27-2008/12/04<br>ARMA(3,1) |
| **Urea** | 2003/01/09-2007/12/13<br>ARMA(3,2) | 2007/12/20-2007/04/17<br>ARMA(2,1) | 2008/04/24-2008/12/04<br>ARMA(2,1) |
| **Ammonia** | 2003/01/09-2008/03/13<br>ARMA(3,2) | 2008/03/20-2008/12/04<br>ARMA(2,1) | |
| **Rock** | 2003/01/09-2007/04/05<br>ARMA(2,1) | 2007/04/12-2008/03/06<br>ARMA(1,1) | 2008/03/13-2008/12/04<br>ARMA(3,2) |
| **Acid** | 2003/01/09-2007/12/10<br>ARMA(3.2) | 2007/12/17-2008/03/31<br>ARMA(2,1) | 2008/04/07-2008/12/04<br>ARMA(3,2) |
| **MOP** | 2003/01/09-2007/12/06<br>ARMA(3,1) | 2007/12/13-2008/04/17<br>ARMA(2,1) | 2008/04/24-2008/12/04<br>ARMA(3,2) |

**Table 5. Volatility in Crude Oil Prices**

| Period | 2003/01/09-2007/11/22 | 2007/11/29-2008/03/20 | 2008/03/27-2008/12/04 |
|---|---|---|---|
| **Series** | ARMA(3,2) | ARMA(2,1) | ARMA(3,3) |
| **(Poil)** | GJR(1,1) | GJR(1,1) | GARCH(1,1) |
| **Mean Equation** | | | |
| **AR(1)** | 0.519 | 0.393 | 0.617 |
| | (0.062) | (0.016) | (0.030) |
| **AR(2)** | 0.154 | 0.280 | 0.199 |
| | (0.007) | (0.002) | (0.010) |
| **AR(3)** | -0.181 | | 0.032 |
| | (0.061) | | (0.087) |
| **MA(1)** | 0.473 | -0.268 | 0.323 |
| | (0.064) | (0.065) | (0.011) |
| **MA(2)** | -0.753 | | -0.293 |
| | (0.050) | | (0.013) |
| **MA(3)** | | | 0.012 |
| | | | (0.077) |
| **Variance Equation** | | | |
| $\omega$ | 0.527 | 0.372 | 0.007 |
| | (0.178) | (0.164) | (0.014) |
| $\alpha$ | 0.133 | 0.238 | 0.282 |
| | (0.034) | (0.085) | (0.031) |
| $\beta$ | 0.235 | 0.207 | 0.485 |
| | (0.108) | (0.199) | (0.079) |
| $\gamma$ | -0.108 | 0.147 | |
| | (0.075) | (0.096) | |
| **Log moment** | -0.859 | -0.761 | -0.202 |
| **Second moment** | 0.421 | 0.519 | 0.768 |
| **Short run persistence** | 0.079 | 0.311 | 0.282 |
| **Long run persistence** | 0.314 | 0.519 | 0.768 |
| **BIC** | 2.491 | 3.814 | 4.601 |

Note: Values in parentheses denote standard errors.

## Table 6. Volatility in MAP Prices

| Period | 2003/01/09-2007/11/01 | 2007/11/08-2008/03/20 | 2008/03/27-2008/12/04 |
|---|---|---|---|
| **Series** | ARMA(2,1) | ARMA(1,1) | ARMA(3,1) |
| **(MAP)** | GJR(1,1) | GARCH(1,1) | GARCH(1,1) |
| **Mean Equation** | | | |
| **AR(1)** | 0.665 | 0.817 | 0.633 |
| | (0.008) | (0.043) | (0.010) |
| **AR(2)** | -0.279 | | 0.158 |
| | (0.089) | | (0.013) |
| **AR(3)** | | | -0.150 |
| | | | (0.070) |
| **MA(1)** | -0.138 | -0.127 | -0.457 |
| | (0.081) | (0.074) | (0.083) |
| **Variance Equation** | | | |
| $\omega$ | 0.409 | 0.221 | 0.006 |
| | (0.226) | (0.246) | (0.087) |
| $\alpha$ | 0.209 | 0.286 | 0.356 |
| | (0.047) | (0.049) | (0.042) |
| $\beta$ | 0.135 | 0.302 | 0.521 |
| | (0.104) | (0.169) | (0.179) |
| $\gamma$ | 0.061 | | |
| | (0.095) | | |
| **Log moment** | -0.614 | -0.015 | -0.199 |
| **Second moment** | 0.374 | 0.588 | 0.877 |
| **Short run persistence** | 0.239 | 0.286 | 0.356 |
| **Long run persistence** | 0.374 | 0.588 | 0.877 |
| **BIC** | 5.354 | 7.474 | 8.268 |

Note: Values in parentheses denote standard errors.

**Table 7. Volatility in Urea Prices**

| Period | 2003/01/09-2007/12/13 | 2007/12/20-2008/04/17 | 2008/04/24-2008/12/04 |
|---|---|---|---|
| **Series** | ARMA(3,2) | ARMA(2,1) | ARMA(2,1) |
| **(Urea)** | GARCH(1,1) | GARCH(1,1) | GARCH(1,1) |
| **Mean Equation** | | | |
| **AR(1)** | 0.763 | 0.314 | 0.897 (0.032) |
| | (0.009) | (0.042) | |
| **AR(2)** | -0.274 | 0.123 | -0.461 |
| | (0.027) | (0.023) | (0.013) |
| **AR(3)** | -0.026 | | |
| | (0.011) | | |
| **MA(1)** | -0.057 | -0.397 | -0.432 |
| | (0.048) | (0.103) | (0.029) |
| **MA(2)** | 0.243 | | |
| | (0.023) | | |
| **Variance Equation** | | | |
| $\omega$ | 0.053 | 0.291 | 0.051 |
| | (0.083) | (0.163) | (0.061) |
| $\alpha$ | 0.080 | 0.185 | 0.318 |
| | (0.036) | (0.023) | (0.005) |
| $\beta$ | 0.293 | 0.410 | 0.704 |
| | (0.159) | (0.192) | (0.078) |
| $\gamma$ | | | |
| **Log moment** | -0.085 | -0.276 | -0.356 |
| **Second moment** | 0.373 | 0.595 | 0.960 |
| **Short run persistence** | 0.080 | 0.185 | 0.318 |
| **Long run persistence** | 0.373 | 0.595 | 0.960 |
| **BIC** | 5.222 | 6.845 | 6.896 |

Note: Values in parentheses denote standard errors.

**Table 8. Volatility in Ammonia Prices**

| Period | 2003/01/09-2008/03/13 | 2008/03/20-2008/12/04 |
|---|---|---|
| Series | ARMA(3,2) | ARMA(2,1) |
| (Ammonia) | GJR(1,1) | GJR(1,1) |
| **Mean Equation** | | |
| AR(1) | 0.758 | 0.724 |
| | (0.031) | (0.056) |
| AR(2) | 0.449 | -0.309 |
| | (0.008) | (0.004) |
| AR(3) | -0.403 | |
| | (0.043) | |
| MA(1) | -0.066 | 0.263 |
| | (0.028) | (0.099) |
| MA(2) | -0.346 | |
| | (0.017) | |
| **Variance Equation** | | |
| $\omega$ | 0.067 | 0.304 |
| | (0.034) | (0.096) |
| $\alpha$ | 0.220 | 0.241 |
| | (0.056) | (0.136) |
| $\beta$ | 0.150 | 0.459 |
| | (0.041) | (0.122) |
| $\gamma$ | 0.099 | 0.160 |
| | (0.168) | (0.014) |
| **Log moment** | -0.601 | -0.764 |
| **Second moment** | 0.494 | 0.908 |
| **Short run persistence** | 0.269 | 0.522 |
| **Long run persistence** | 0.494 | 0.908 |
| **BIC** | 5.368 | 7.439 |

Note: Values in parentheses denote standard errors.

## Table 9. Volatility in Rock Prices

| Period | 2003/01/09-2007/04/05 | 2007/04/12-2008/03/06 | 2008/03/13-2008/12/04 |
|---|---|---|---|
| **Series** | ARMA(2,1) | ARMA(1,1) | ARMA(3,2) |
| **(Rock)** | GARCH(1,1) | GARCH(1,1) | GJR(1,1) |
| **Mean Equation** | | | |
| **AR(1)** | 0.334 | 0.963 | 0.178 |
| | (0.061) | (0.054) | (0.074) |
| **AR(2)** | 0.248 | | 0.223 |
| | (0.009) | | (0.009) |
| **AR(3)** | | | -0.203 |
| | | | (0.056) |
| **MA(1)** | 0.371 | -0.223 | 0.472 |
| | (0.061) | (0.027) | (0.017) |
| **MA(2)** | | | -0.153 |
| | | | (0.061) |
| **Variance Equation** | | | |
| ω | 0.005 | 0.121 | 0.086 |
| | (0.004) | (0.164) | (0.040) |
| α | 0.109 | 0.226 | 0.401 |
| | (0.022) | (0.084) | (0.045) |
| β | 0.327 | 0.339 | 0.324 |
| | (0.196) | (0.105) | (0.038) |
| γ | | | 0.183 |
| | | | (0.051) |
| **Log moment** | -0.081 | -0.184 | -0.759 |
| **Second moment** | 0.436 | 0.605 | 0.817 |
| **Short run persistence** | 0.109 | 0.266 | 0.493 |
| **Long run persistence** | 0.436 | 0.605 | 0.817 |
| **BIC** | 1.751 | 2.611 | 2.315 |

Note: Values in parentheses denote standard errors.

## Table 10. Volatility in Acid Prices

| Period | 2003/01/09-2007/12/10 | 2007/12/17-2008/03/31 | 2008/04/07-2008/12/04 |
|---|---|---|---|
| **Series** | ARMA(3.2) | ARMA(2,1) | ARMA(3,2) |
| **(Acid)** | GJR(1,1) | GARCH(1,1) | GARCH(1,1) |
| **Mean Equation** | | | |
| **AR(1)** | 0.525 | 0.501 | 0.624 |
| | (0.026) | (0.093) | (0.041) |
| **AR(2)** | 0.244 | 0.294 | 0.453 |
| | (0.010) | (0.067) | (0.010) |
| **AR(3)** | 0.048 | | -0.238 |
| | (0.003) | | (0.077) |
| **MA(1)** | 0.464 | -0.207 | 0.610 |
| | (0.072) | (0.089) | (0.033) |
| **MA(2)** | -0.261 | | -0.387 |
| | (0.015) | | (0.059) |
| **Variance Equation** | | | |
| $\omega$ | 0.113 | 0.007 | 0.007 |
| | (0.106) | (0.049) | (0.042) |
| $\alpha$ | 0.179 | 0.151 | 0.032 |
| | (0.005) | (0.082) | (0.003) |
| $\beta$ | 0.250 | 0.476 | 0.706 |
| | (0.104) | (0.139) | (0.017) |
| $\gamma$ | -0.102 | | |
| | (0.058) | | |
| **Log moment** | -0.652 | -0.087 | -0.214 |
| **Second moment** | 0.378 | 0.627 | 0.733 |
| **Short run persistence** | 0.128 | 0.151 | 0.032 |
| **Long run persistence** | 0.378 | 0.627 | 0.733 |
| **BIC** | 6.285 | 7.307 | 7.074 |

Note: Values in parentheses denote standard errors.

**Table 11. Volatility in MOP Prices**

| Period | 2003/01/09-2007/12/06 | 2007/12/13-2008/04/17 | 2008/04/24-2008/12/04 |
|---|---|---|---|
| **Series** | ARMA(3,1) | ARMA(2,1) | ARMA(3,2) |
| **(MOP)** | GJR(1,1) | GJR(1,1) | GJR(1,1) |
| **Mean Equation** | | | |
| **AR(1)** | 0.875 | 0.704 | 0.856 |
| | (0.085) | (0.051) | (0.030) |
| **AR(2)** | -0.335 | -0.178 | -0.187 |
| | (0.021) | (0.019) | (0.005) |
| **AR(3)** | -0.343 | | 0.095 |
| | (0.084) | | (0.038) |
| **MA(1)** | 0.243 | -0.178 | 0.148 |
| | (0.052) | (0.062) | (0.051) |
| **MA(2)** | | | -0.139 |
| | | | (0.029) |
| **Variance Equation** | | | |
| ω | 0.184 | 0.122 | 0.029 |
| | (0.139) | (0.074) | (0.034) |
| α | 0.131 | 0.277 | 0.319 |
| | (0.033) | (0.056) | (0.152) |
| β | 0.218 | 0.233 | 0.507 |
| | (0.106) | (0.102) | (0.097) |
| γ | 0.084 | 0.158 | 0.182 |
| | (0.029) | (0.072) | (0.104) |
| **Log moment** | -0.774 | -0.744 | -0.849 |
| **Second moment** | 0.391 | 0.589 | 0.916 |
| **Short run persistence** | 0.173 | 0.356 | 0.410 |
| **Long run persistence** | 0.391 | 0.589 | 0.916 |
| **BIC** | 3.582 | 4.515 | 3.023 |

Note: Values in parentheses denote standard errors.