

**AN NPV AND AC ANALYSIS OF A STOCHASTIC INVENTORY SYSTEM  
WITH JOINT MANUFACTURING AND REMANUFACTURING**

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# An NPV and AC analysis of a stochastic inventory system with joint manufacturing and remanufacturing

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## Abstract

While the net present value (NPV) approach is widely accepted as the right framework for studying production and inventory control systems, average cost (AC) models are more widely used. For the well known EOQ model it can be verified that (under certain conditions) the AC approach gives near optimal results. This paper investigates whether the same holds for two-source systems with joint manufacturing and remanufacturing. It appears that the performance of the AC approach stands or falls with the right choice of the holding cost parameters. Through the analysis of a deterministic model a theoretical basis is provided for choosing the parameters. Then, given this set of holding cost parameters, the performance of the AC approach is tested in a stochastic model.

**Keywords:** Net present value, average costs, inventory control, manufacturing, remanufacturing, holding costs.

## 1 Introduction

Several authors (e.g. Hadley, 1964; Trippi and Lewin, 1974; Thompson, 1975; Hofmann, 1998; Klein Haneveld and Teunter, 1998) have argued that *for the EOQ model* the average cost (AC) framework as an approximation to the superior net present value (NPV) framework leads to near optimal results under the following conditions:

- Products are not moving too slow,
- Interest rates are not too high,
- The customer payment structure does not depend on the inventory policy.

The first two conditions have to guarantee that compounded interest does not effect the results. That the latter condition is crucial was first put forward by Beranek (1966), who's concern was confirmed later by Grubbström (1980) and Kim et al. (1984). Grubbström and Thorstenson

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(1996) report that the NPV approach can differ significantly from the AC approach for a multi-level inventory system.

The main objections against the average cost approach, as it is usually applied, as an approximation to the net present value approach is threefold:

- The time value of money is not explicitly taken into account,
- There is no distinction between out-of-pocket holding costs and opportunity costs due to inventory investment, while other sources of opportunity costs/yields (fixed ordering costs, product sales) are not taken into account at all.
- Initial conditions are not taken into account

Yet, the net present value approach is often rather complicated so an approximation may still be preferred.

Several authors have tried to deal with the above problems by showing that a certain transformation of the holding cost parameters in EOQ-type models results in a very good approximation to the NPV equivalent. Unfortunately however, finding the right holding cost transformation requires to compare an NPV analysis with an AC analysis and choose the holding cost parameters in such a way that the results of both approaches (approximately) coincide. Derived solutions can be very counter-intuitive (see e.g. Beranek, 1966) and differ case by case.

A particular type of inventory models that is receiving an increasing amount of attention lately is that of an inventory system with joint manufacturing and remanufacturing (see e.g. Fleischmann et al., 1997; Inderfurth, 1997; Richter, 1999; Van der Laan et al., 1999a, 1999b). In these models product demand can be satisfied both from remanufacturing of old products that have been returned by the customer or from manufacturing new products. The complication that arises here is that the serviceable inventory contains new products as well as remanufactured products that have been produced against different costs. In this situation it is not immediately clear how to set the holding cost parameters, although recently a number of holding cost settings have been proposed for this situation. Teunter et al. (1999) compare the available alternatives through a simulation study, but none of these alternatives are founded on a sound mathematical analysis. This paper does present a mathematical argument for the choice of holding cost parameters. Furthermore, the theoretical results are tested in a stochastic model through an *exact* comparison procedure rather than simulation.

This paper is further organized as follows: In the next section we shortly discuss the fundamental differences between the NPV and AC approach. In Section 3 we introduce the complications that arise with two-source models through an exact analysis of a stochastic model with joint manufacturing and instantaneous remanufacturing, using both the NPV and AC criterion. In Section 4 we extend the model with remanufacturing batches and derive the appropriate holding cost parameters in a deterministic setting. Section 5 then evaluates the performance of the AC criterion with the derived holding cost parameters in the original stochastic model. Finally, in Section 6 we discuss the main results and propose some topics for future research.

## 2 The NPV principle versus the Average Cost principle.

We define the *Net Present Value (NPV)* as the total discounted cash-flow over an infinite horizon. For instance, consider a cyclic cash-flow  $C$  that occurs at stochastic times  $T_1, T_2, \dots$  starting at time  $T_1 = 0$ . The NPV of this series of cash-flows, discounted at rate  $r$ , equals

$$NPV = E \left\{ \sum_{n=1}^{\infty} C e^{-rT_n} \right\}. \quad (1)$$

In the special case in which all inter-occurrence times  $T_i - T_{i-1}$  are independent and have the same probability density function  $f(t)$ , provided maybe the first occurrence time, which has density function  $f_1(t)$ , expression (1) is given by

$$\begin{aligned} NPV &= C \sum_{n=1}^{\infty} \int_0^{\infty} f_1 * f_2 * \dots * f_n(t) e^{-rt} dt = C \sum_{n=1}^{\infty} \int_0^{\infty} f_1 * f^{*(n-1)}(t) e^{-rt} dt \\ &= C \tilde{f}_1(r) \sum_{n=1}^{\infty} \tilde{f}(r)^{(n-1)} = \frac{C \tilde{f}_1(r)}{1 - \tilde{f}(r)}, \end{aligned}$$

where the asterisk denotes convolution and the tilde denotes the Laplace transform.

In Addition to the *NPV* we define the *Annuity Stream (AS)* as

$$AS = r\{NPV\}.$$

The annuity stream is the transformation of a set of cash flows to one continuous stream of cash-flows, such that the latter has the same net present value as the original set of cash-flows. The notion of an annuity stream is useful, since it can be directly compared with average costs. The relation between NPV, AS, and AC is illustrated with the following simple example. Consider the standard EOQ model with Poisson demand with rate  $\lambda$ , zero lead-times and no out-of-pocket holding costs. A product is manufactured against cost variable cost  $c_m$  and sold for a price  $p$ . As soon as the inventory drops below zero a replenishment of size  $Q_m$  follows against fixed cost  $K_m$  per batch. Starting with a replenishment of size  $Q_m$  we have the following sum of expected discounted cash-flows:

$$\begin{aligned} NPV &= p \sum_{n=1}^{\infty} \tilde{f}_D(r)^n - (K_m + c_m Q_m) \sum_{n=0}^{\infty} \tilde{f}_M(r)^n = \frac{p \tilde{f}_D(r)}{1 - \tilde{f}_D(r)} - \frac{K_m + c_m Q_m}{1 - \tilde{f}_M(r)} \\ &= \frac{p\lambda}{r} - \frac{K_m + c_m Q_m}{1 - \left(\frac{\lambda}{\lambda+r}\right)^{Q_m}}, \end{aligned}$$

where we have used that the inter-occurrence times of demands are negative exponentially distributed with Laplace transform  $\tilde{f}_D(r) = \frac{\lambda}{\lambda+r}$  and, consequently, the inter-occurrence times of manufacturing batches are Erlang- $Q_m$  distributed with Laplace transform  $\tilde{f}_M(r) = \left(\frac{\lambda}{\lambda+r}\right)^{Q_m}$ . Thus, the annuity stream is given by

$$AS = rNPV = p\lambda - \frac{r(K_m + c_m Q_m)}{1 - \left(\frac{\lambda}{\lambda+r}\right)^{Q_m}}$$

In order to compare the annuity stream with the average cost approach we compute the first order MacLaurin expansion as follows.

$$\overline{AS} = \lim_{r \downarrow 0} AS + r \lim_{r \downarrow 0} \frac{dAS}{dr} = \lambda(p - c_m) - (\lambda K_m + \alpha)/Q_m - r(K_m + c_m(Q_m + 1))/2, \quad (2)$$

where  $\alpha = r/2$  accounts for the interest component of fixed setup costs.

The traditional average costs approach calculates the average profit ( $AP$ ) function as the sum of average variable costs, fixed costs, and holding costs per time unit,

$$AP = \lambda(p - c_m) - \lambda K_m/Q_m - h(Q_m - 1)/2,$$

where  $h$  is a holding cost parameter that has to take into account the out-of-pocket holding costs (which are zero in this example) *and* the interest component in (2) the so-called ‘opportunity costs’. Provided that  $\alpha$  is small compared to  $\lambda$ , parameter  $h$  can be chosen such that both the NPV and AC approach (approximately) result in the same optimal value of the order size  $Q_m$ : for  $h = rc_m$  we have that  $\overline{AS}$  equals  $AC$  up to a constant. The classical interpretation of this particular value of the holding cost parameter is that the opportunity cost of holding inventories is the interest rate times the (average) inventory investment per product. Although this intuitive interpretation works for the EOQ model we will see in the remainder of this paper that intuition is treacherous when it comes to two-source models.

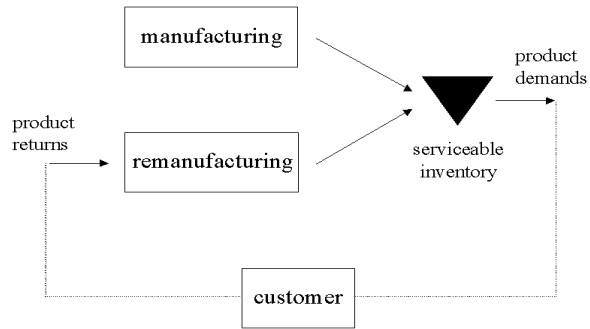
### 3 A stochastic inventory model with manufacturing and instantaneous remanufacturing

#### 3.1 Notation and model development

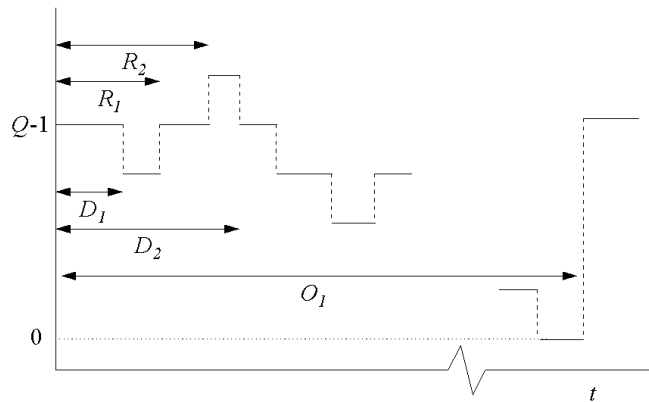
We consider a very basic model with joint manufacturing and remanufacturing (Figure 1). Customer demand can be satisfied by newly manufactured products and remanufacturing of products that have been returned by customers after use. To keep the analysis tractable we assume that the demand process and the return process are *independent* Poisson processes with rates  $\lambda$  and  $\gamma (< \lambda)$  respectively. We assume that remanufacturing occurs instantaneously, so that there is no stocking of remanufacturables. Manufacturing occurs in batches of size  $Q_m$ , following a continuous review ordering policy: as soon as the inventory drops below zero, a remanufacturing batch of size  $Q_m$  is ordered. Note that in case of instantaneous remanufacturing this is an optimal policy (see Fleischmann and Kuik, 1998). After (re)manufacturing products enter the serviceable inventory immediately. Furthermore, all lead-times are zero and backorders are not allowed. To keep the analysis transparent we do not consider out-of-pocket holding costs, so costs related to inventory investment are assumed to be so-called ‘opportunity costs’ only.

The cost structure is as follows:

- $p$  : sales price for (re)manufactured products (serviceables)
- $c_m$  : manufacturing cost per product manufactured
- $c_r$  : remanufacturing cost per product remanufactured
- $K_m$  : fixed ordering cost for manufacturing per order



**Figure 1** A system with joint manufacturing and remanufacturing operations.



**Figure 2** Inventory mutations due to demands ( $D_n$ ), remanufacturing orders ( $R_n$ ) and manufacturing orders ( $M_n$ ).

In the remainder of this paper we assume that product returns are obtained for free. Note that this is a simplification rather than a limitation of the analysis.

Under a net present value criterion all cash in- and out-flows are discounted with opportunity cost rate  $r$ . Define  $\{D_n|n \geq 1\}$ ,  $\{R_n|n \geq 1\}$ , and  $\{M_n|n \geq 1\}$  as the occurrence times of demands, remanufacturing orders, and manufacturing orders respectively (see Figure 2). Note that the timing of remanufacturing orders coincides with the timing of product returns, since we have instantaneous remanufacturing. Then the annuity stream as a function of order size  $Q_m$  reads

$$\begin{aligned} AS(Q_m) &= r \sum_{n=1}^{\infty} E \left( p e^{-rD_n} - c_r e^{-rR_n} - (K_m + c_m Q_m) e^{-rM_n} \right) \\ &= p\lambda - c_r\gamma - r(K_m + c_m Q_m) \sum_{n=1}^{\infty} E \left( e^{-rM_n} \right), \end{aligned} \quad (3)$$

where we have used that  $\sum_{n=1}^{\infty} E \left( e^{-rD_n} \right) = \lambda/r$  and  $\sum_{n=1}^{\infty} E \left( e^{-rR_n} \right) = \gamma/r$ . The annuity stream of manufacturing costs,  $r(K_m + c_m Q_m) \sum_{n=1}^{\infty} E \left( e^{-rM_n} \right)$ , is more complicated to develop and will be left to the next paragraph.

### 3.2 The annuity stream of manufacturing costs

Given that at time 0 the process starts at inventory level  $I_0$  we need to calculate the expected discounted number of manufacturing orders. Since the distribution of their inter-occurrence times cannot be calculated directly we derive a set of recurrence relations. We further note that we are interested in the *Laplace transform* of the distribution, rather than the distribution itself.

Suppose that at time 0 the inventory level is  $I_0 = i (i \geq 0)$ . Either the next occurrence is a demand at time  $t$ , with probability  $g(t) = \lambda e^{-(\lambda+\gamma)t}$ , which moves the inventory down to  $i - 1$ , or the next occurrence is a return, with probability  $h(t) = \gamma e^{-(\lambda+\gamma)t}$ , which moves the inventory level up to  $i + 1$ . If  $f_i(t)$  denotes the distribution of the first occurrence time of a manufacturing order, given that the process starts at inventory level  $i$ , then we have

$$f_i(t) = \begin{cases} g(t) + h * f_1(t), & i = 0, \\ g * f_{i-1}(t) + h * f_{i+1}(t), & i > 0, \end{cases} \quad (4)$$

Taking Laplace transforms and evaluating at  $r$ , (4) becomes

$$\tilde{f}_i(r) = \begin{cases} \tilde{g}(r) + \tilde{h}(r)\tilde{f}_1(r), & i = 0, \\ \tilde{g}(r)\tilde{f}_{i-1}(r) + \tilde{h}(r)\tilde{f}_{i+1}(r), & i > 0, \end{cases} \quad (5)$$

where  $\tilde{g}(r) = \frac{\lambda}{\lambda+\gamma+r}$  and  $\tilde{h}(r) = \frac{\gamma}{\lambda+\gamma+r}$  are the Laplace transforms of  $g(t)$  and  $h(t)$  respectively, and  $\tilde{f}_i(r)$  denotes the discounted first occurrence time of a manufacturing order given that the inventory level starts at state  $i$ . Solving equations (5) for  $\tilde{f}_i(r)$  gives

$$\tilde{f}_i(r) = \left( \frac{1 - \sqrt{1 - 4\tilde{g}(r)\tilde{h}(r)}}{2\tilde{h}(r)} \right)^{i+1}, \quad i \geq 0$$



Then under our control policy we have for the annuity stream of manufacturing cash-flows

$$\begin{aligned}
AS_m(Q_m) &= -r(K_m + c_m Q_m)E\left(\sum_{n=1}^{\infty} e^{-rM_n}\right) \\
&= -r(K_m + c_m Q_m) \sum_{n=0}^{\infty} \tilde{f}_{I_0}(r) (\tilde{f}_{Q_m-1}(r))^n \\
&= -(K_m + c_m Q_m) \frac{r \tilde{f}_{I_0}(r)}{1 - \tilde{f}_{Q_m-1}(r)}, \tag{6}
\end{aligned}$$

### 3.3 Setting the holding cost parameter

Combining (3) and (6) the objective function under the NPV principle reads

$$AS(Q_m) = p\lambda - c_r\gamma - (K_m + c_m Q_m) \left( \frac{r \tilde{f}_{I_0}(r)}{1 - \tilde{f}_{Q_m-1}(r)} \right),$$

with its linearisation in  $r$  (see Appendix A)

$$\begin{aligned}
\overline{AS}(Q_m) = \\
p\lambda - c_r\gamma - \left[ c_m(\lambda - \gamma) + K_m \left( \frac{\lambda - \gamma}{Q_m} \right) + r \left( \frac{1}{2} + \frac{\lambda + \gamma}{2(\lambda - \gamma)Q_m} \right) (K_m + c_m Q_m) \right] \tilde{f}_{I_0}(r). \tag{7}
\end{aligned}$$

The traditional average cost approach computes the average profit function as the marginal profits minus the fixed ordering costs and the opportunity costs for holding inventory:

$$AP(Q_m) = p\lambda - c_r\gamma - c_m(\lambda - \gamma) - K_m \left( \frac{\lambda - \gamma}{Q_m} \right) - h_s \left( \frac{Q_m - 1}{2} + \frac{\gamma}{\lambda - \gamma} \right). \tag{8}$$

Here, the last term denotes the average serviceable inventory (see e.g. Muckstadt and Isaac, 1981) times the serviceable holding cost parameter  $h_s$ . From the traditional average cost point of view it is not immediately clear what the value of  $h_s$  should be. The interpretation that opportunity costs of holding inventories are proportional to the average inventory investment, suggests that  $h_s$  should depend on both  $c_m$  and  $c_r$ , since the inventory of serviceable products is a mixture of manufactured and remanufactured products with *different* marginal costs  $c_m$  and  $c_r$ .

Using the annuity stream there is no reason for confusion. Expression (7) can be rewritten as

$$\begin{aligned}
\overline{AS}(Q_m; I_0) = \\
p\lambda - c_r\gamma - \left[ c_m(\lambda - \gamma) + K_m \left( \frac{\lambda - \gamma + \alpha}{Q_m} \right) + r c_m Q_m / 2 + r K_m / 2 + \alpha c_m \right] \tilde{f}_{I_0}(r), \tag{9}
\end{aligned}$$

with  $\alpha = -r \left( \frac{\lambda + \gamma}{2(\lambda - \gamma)} \right)$ . We can interpret  $\alpha$  as the (relevant) opportunity cost of fixed ordering costs per manufactured product.

Putting the derivative of (9) with respect to  $Q_m$  equal to zero and solving for  $Q_m$  results in the EOQ-type formula

$$Q_m^{\overline{AS}} = \sqrt{\frac{2K_m(\lambda - \gamma + \alpha)}{rc_m}}, \quad (10)$$

Putting the derivative of (8) with respect to  $Q_m$  equal to zero and solving for  $Q_m$  results in

$$Q_m^{AP} = \sqrt{\frac{2K_m(\lambda - \gamma)}{h_s}}, \quad (11)$$

Note that for moderate values of  $\gamma$ , the influence of  $\alpha$  is rather limited. In that case choosing  $h_s = rc_m$  both approaches result in similar optimal ordering quantities.

### 3.4 Preliminary conclusions

The following things can be learned from the above analysis:

- At first glance the NPV analysis looks rather complicated. The reason why we are able to find analytical expressions for the annuity stream at all in this example is because we have instantaneous remanufacturing. This enables us to formulate the model as a one-dimensional Markov-chain with a relatively simple structure. In most other cases however we end up with a two-dimensional system with a complicated interaction between the remanufacturable inventory and the serviceable inventory. In those cases we have to rely on numerical procedures instead. The AC approach, however, suffers from exactly the same problem to calculate the average inventories (see van der Laan, 1997), so in this respect there is not a clear advantage of the AC approach over the NPV approach.
- It seems strange at first that in the above example manufactured as well as remanufactured products have to be valued against manufacturing cost  $c_m$ . This however is only false appearance. A more sophisticated approach would be to use *two* holding cost parameters for the serviceable inventory:  $h_s^m$  for manufactured products and  $h_s^r$  for remanufactured products. The complication then is to calculate the average inventories of manufactured and remanufactured products. In this case it is quite simple. The inventory process can be split up into two parts (see Fleischmann and Kuik, 1998) : A component that resembles a classical inventory system where the inventory goes down because of demand and goes up only because of manufacturing batches. This is the inventory of manufactured products and has expected value  $(Q_m - 1)/2$ . The residual inventory component is due to remanufactured products only with expected value  $\gamma/(\lambda - \gamma)$ . Then it is clear that  $h_s^m = rc_m$ . Since the expected inventory of remanufactured products is just a constant, the value of  $h_s^r$  does not influence the results. In our example we chose the ‘wrong’ value  $h_s^r = h_s^m = rc_m$  with no consequence. Disadvantages of having two holding cost rates for the serviceable inventory are: 1. the complexity of the model increases, since an extra state variable is needed to keep track of both manufactured products and remanufactured products, 2. since an extra degree of freedom is introduced there is no longer a unique set of holding cost parameters such that AC and NPV are equivalent, and 3. splitting up the

serviceable inventory in a ‘cheap’ inventory (remanufactured products) and a more expensive inventory (manufactured products) implies that in selling products priority should be given to manufactured products over remanufactured products. This however does not make sense from an NPV perspective, since selling any product will generate the same cash-flow  $p$ .

- One of the disadvantages of the AC approach is that it does not accurately reflect the ‘true’ profits/costs of the system. First of all it does not take into account the initial conditions of the system, reflected by expression  $\tilde{f}_{I_0}(r)$  in (9). Although this does not influence the optimization, it does affect the net profits. Also, the AC approach does not take the opportunity costs of fixed ordering cost into account, represented by  $\alpha$  in (9). Fortunately, the latter shortcoming influences the optimization only for extreme values of the return rate.

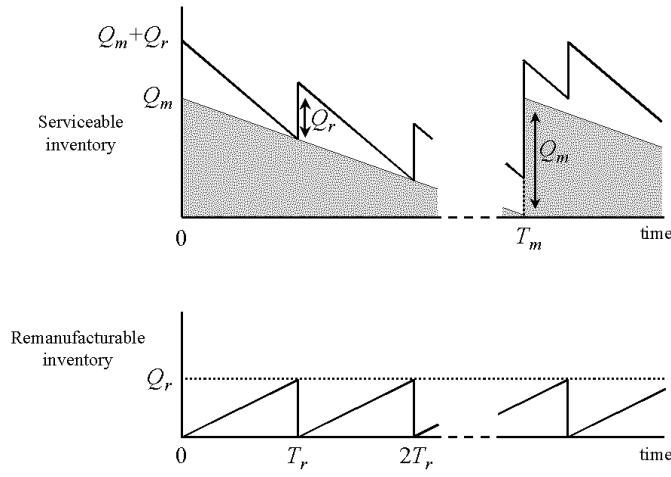
In the next section we investigate the complications that arise when our model is extended with remanufacturing batches.

## 4 Remanufacturing batches

In the model of the previous section remanufacturing occurred as soon as a returned product arrived at the remanufacturing facility. Now we will extend this model with remanufacturing batches, i.e., as soon as  $Q_r$  products have accumulated in the remanufacturable inventory, these products are remanufactured against fixed cost  $K_r$  per batch and enter the serviceable inventory. Note that in the AC framework we now also have a holding cost parameter for remanufacturable products, say  $h_r$ . The purpose of this section is to show that for this type of models it is not trivial to find holding cost parameters such that the AC approach results in similar optimal order sizes as the NPV approach. Even for this simple model it is very hard to find analytic expressions for the annuity stream and average profit function. This means that in general one should use an educated guess which holding cost parameters to use. In order to make this educated guess we look at a *deterministic* system with control operations that resemble the stochastic case.

Consider the following deterministic system with a continuous demand and return flow with rates rate  $\lambda$  and  $\gamma$  respectively. A remanufacturing batch of size  $Q_r$  occurs every  $T_r = Q_r/\gamma$  time units starting at time  $T_r$ . Remanufacturing can only start at time  $T_r$ , since it takes exactly this time for remanufacturables to accumulate to batch size  $Q_r$ . The timing of manufacturing batches in the stochastic system depends on the relation between the average inter-occurrence time of manufacturing batches,  $Q_m/(\lambda-\gamma)$ , and remanufacturing batches,  $Q_r/\gamma$ . If  $Q_m/(\lambda-\gamma) \gg Q_r/\gamma$  infrequent manufacturing batches and during one manufacturing cycle we have frequent remanufacturing orders. If  $Q_m/(\lambda-\gamma) \ll Q_r/\gamma$  we have infrequent remanufacturing orders and frequent manufacturing orders. However, since manufacturing orders are only placed if the serviceable inventory drops below zero, manufacturing batches only occur after a remanufacturing batch is completely depleted. Therefore, for the deterministic system we distinguish between Case A and Case B:

**Case A:**  $Q_m/(\lambda - \gamma) \gg Q_r/\gamma$



**Figure 3** Inventory process of a deterministic system with joint manufacturing and remanufacturing.  
Case A:  $Q_m/(\lambda - \gamma) \gg Q_r/\gamma$

When the average inter-occurrence time of manufacturing batches is much *bigger* than that of remanufacturing batches we have the situation depicted in Figure 3. A manufacturing batch of size  $Q_m$  occurs every  $T_m = Q_m/(\lambda - \gamma)$  time units starting at time  $T_m$ , while remanufacturing batches are ‘pushed’ through the system every  $T_r = Q_r/\gamma$  time units. Because  $T_r \ll T_m$  there will be a lot of remanufacturing batches during one manufacturing cycle. Since it takes  $T_r$  time units to accumulate the first remanufacturing batch a manufacturing batch of size  $Q_m + Q_r$  is initiated at time 0 to start-up the system.

The annuity stream of the deterministic system is given by

$$\begin{aligned} AS(Q_m, Q_r) &= p\lambda - r(c_m Q_r + \sum_{n=0}^{\infty} (K_m + c_m Q_m)e^{-rnT_m} + \sum_{n=1}^{\infty} (K_r + c_r Q_r)e^{-rnT_r}) \\ &= p\lambda - rc_m Q_r - \frac{r(K_m + c_m Q_m)}{1 - e^{-rT_m}} - \frac{r(K_r + c_r Q_r)e^{T_r}}{1 - e^{-rT_r}}. \end{aligned}$$

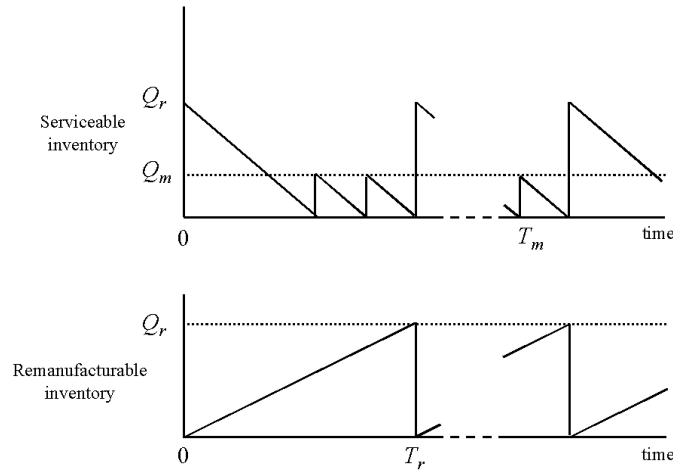
A linearisation in  $r$  gives

$$\begin{aligned} \overline{AS} &= \lambda p - (\lambda - \gamma)c_m - \gamma c_r - rc_m Q_r - (K_m + c_m Q_m) \left( \frac{1}{T_m} + \frac{r}{2} \right) - (K_r + c_r Q_r) \left( \frac{1}{T_r} - \frac{r}{2} \right) \\ &= \lambda p - (\lambda - \gamma)c_m - \gamma c_r - \frac{(\lambda - \gamma)K_m}{Q_m} - \frac{\gamma K_r}{Q_r} \\ &\quad - rc_m \frac{Q_m}{2} - r(2c_m - c_r) \frac{Q_r}{2} - \frac{r(K_m - K_r)}{2}, \end{aligned} \tag{12}$$

The AC approach computes the average profit function as

$$AP = \lambda p - (\lambda - \gamma)c_m - \gamma c_r - \frac{(\lambda - \gamma)K_m}{Q_m} - \frac{\gamma K_r}{Q_r} - h_s^m \frac{Q_m}{2} - h_s^r \frac{Q_r}{2} - h_r \frac{Q_r}{2} \tag{13}$$

Note that there is no unique set of holding costs parameters such that (12) is equal to (13), up to a constant. Therefore we will consider two alternatives that seem natural. First we choose



**Figure 4** Inventory process of a deterministic system with joint manufacturing and remanufacturing;  
Case B:  $Q_m/(\lambda - \gamma) \ll Q_r/\gamma$ .

$h_s^m = h_s^r$  (see the discussion in Section 3.4). This leads to the result

$$h_s^m = h_s^r = h_s \Rightarrow \begin{cases} h_s &= r c_m \\ h_r &= r(c_m - c_r) \end{cases} \quad (14)$$

Since returned products are obtained for free, another option is to put  $h_r = 0$ :

$$h_r = 0 \Rightarrow \begin{cases} h_s^m &= r c_m \\ h_s^r &= r(2c_m - c_r) \end{cases} \quad (15)$$

**Case B:**  $Q_m/(\lambda - \gamma) \ll Q_r/\gamma$

When the average inter-occurrence time of manufacturing batches is much *smaller* than that of remanufacturing batches we have the situation depicted in Figure 4. A remanufacturing batch of size  $Q_r$  occurs every  $T_r$  time units starting at time  $T_r$ , while manufacturing batches only occur *in between* remanufacturing batches. Since it takes  $T_r$  time units to accumulate the first remanufacturing batch, a manufacturing batch of size  $Q_r$  is initiated at time 0 to start-up the system. To keep the analysis tractable we assume that the number of manufacturing batches in between two remanufacturing batches is an integer,  $N$  say. This implies that the following relation holds:

$$\frac{NQ_m + Q_r}{\lambda} = \frac{Q_r}{\gamma}.$$

The above relation states that the time to accumulate one remanufacturing batch equals the time it takes for demand to deplete one remanufacturing batch plus  $n$  manufacturing batches. In this way the inventory process is a recurrent process, since every  $T_r$  time units the system starts all over again.

The annuity stream of the deterministic system is given by

$$\begin{aligned}
& AS(Q_m, Q_r) \\
&= p\lambda - r \left( c_m Q_r + \sum_{n=1}^{\infty} \sum_{i=0}^{N-1} (K_m + c_m Q_m) e^{-rn(T_r + iQ_m/\lambda)} + \sum_{n=1}^{\infty} (K_r + c_r Q_r) e^{-rnT_r} \right) \\
&= p\lambda - rc_m Q_r - \sum_{i=0}^{N-1} \frac{r(K_m + c_m Q_m) e^{-r(T_r + iQ_m/\lambda)}}{1 - e^{Tr}} - \frac{r(K_r + c_r Q_r) e^{Tr}}{1 - e^{Tr}}.
\end{aligned}$$

A linearisation in  $r$  gives

$$\begin{aligned}
\overline{AS} &= \lambda p - rc_m Q_r - (K_r + c_r Q_r) \left( \frac{1}{T_r} - \frac{r}{2} \right) \\
&\quad - (K_m + c_m Q_m) \sum_{i=0}^{N-1} \left( \frac{1}{T_r} + r \left( \frac{1}{2} - \frac{\gamma}{\lambda} \left( 1 - \frac{iQ_m}{Q_r} \right) \right) \right) \\
&= \lambda p - (\lambda - \gamma)c_m - \gamma c_r - \frac{(\lambda - \gamma)K_m}{Q_m} - \frac{\gamma K_r}{Q_r} \\
&\quad - rc_m Q_r - rc_m \left( 1 - (N+1) \left( \frac{\gamma}{\lambda} \right) \right) \frac{Q_m}{2} + rc_r \frac{Q_r}{2} - \frac{r(K_m(1 - (N+1)(\frac{\gamma}{\lambda})) - K_r)}{2}
\end{aligned}$$

which can be further rewritten as

$$\begin{aligned}
\overline{AS} &= \lambda p - (\lambda - \gamma)c_m - \gamma c_r - \frac{(\lambda - \gamma + \alpha)K_m}{Q_m} - \frac{\gamma K_r}{Q_r} \\
&\quad - rc_m \left( 1 - \frac{\gamma}{\lambda} \right) \frac{Q_m}{2} - r \left( \left( 1 + \frac{\gamma}{\lambda} \right) c_m - c_r \right) \frac{Q_r}{2} - \frac{r(K_m(1 - (\frac{\gamma}{\lambda})) - K_r)}{2}, \tag{16}
\end{aligned}$$

where  $\alpha = -r(1 - \frac{\gamma}{\lambda})Q_r$ . Note that the influence of  $\alpha$  will be very limited if  $Q_r$  is small compared to  $\lambda$ .

The AC approach computes the average profit function as

$$AP = \lambda p - (\lambda - \gamma)c_m - \gamma c_r - \frac{(\lambda - \gamma)K_m}{Q_m} - \frac{\gamma K_r}{Q_r} - h_s^m \left( 1 - \frac{\gamma}{\lambda} \right) \frac{Q_m}{2} - h_s^m \left( \frac{\gamma}{\lambda} \right) \frac{Q_r}{2} - h_r \frac{Q_r}{2} \tag{17}$$

Provided that we can discard  $\alpha$ , it follows that expression (16) is equal to (17), up to a constant, if we choose  $h_s^m$ ,  $h_s^r$ , and  $h_r$  as

$$h_s^m = h_s^r = h_s \Rightarrow \begin{cases} h_s &= rc_m \\ h_r &= r(c_m - c_r) \end{cases} \tag{18}$$

or

$$h_r = 0 \Rightarrow \begin{cases} h_s^m &= rc_m \\ h_s^r &= r \left( \left( 1 + \frac{\lambda}{\gamma} \right) c_m - \left( \frac{\lambda}{\gamma} \right) c_r \right) \end{cases} \tag{19}$$

Note that (14) is equal to (18), but (15) is not equal to (19).

Comparing case A and case B it follows that only if we choose  $h_s^m = h_s^r$  we have the same set of holding cost parameters for both cases, i.e.,  $h_s = rc_m$  and  $h_r = r(c_m - c_r)$ . Apart from the advantages already mentioned regarding only one holding cost parameter for serviceable

products, we have the additional advantage that we do not have to differentiate between Case A and B during an optimization procedure. Therefore we prefer (14) over (15) and (19).

In the introduction we already mentioned that the traditional interpretation of holding cost parameters (as an estimate of the opportunity cost of stock-keeping) is the interest rate times the average inventory investment per product. Following this intuition we should choose for the holding cost parameters  $h_s^m = rc_m$ ,  $h_s^r = rc_r$  and  $h_r = 0$ . For this set of holding cost parameters, an increase in  $c_r$  leads to an *increase* in inventory costs, while the ‘correct’ holding costs (14) (15) and (19) lead to the opposite. This also means that the optimal value of  $Q_r$  moves in the opposite direction with an increase in  $c_r$ . For the extreme case  $c_r = 0$  the difference in outcome is quite considerable. For instance, Teunter *et al.* (1999) tested this set of holding cost parameters with simulation for the same system with a slightly different control policy. It appeared that the error in relevant costs could climb up to 30% for low values of  $c_r$ . The above example shows how careful one should be with the traditional intuition in choosing holding cost parameters in systems with joint manufacturing and remanufacturing.

## 5 Numerical evaluation

In this Section we study the *stochastic* system with joint manufacturing and remanufacturing, defined in the beginning of Section 3, under an NPV criterion and an AC criterion in order to evaluate the performance of the average costs approach. In this we will use the holding cost parameters that were developed in the previous section. Although we cannot derive analytic expressions for the annuity stream and average profit function, we can evaluate both functions numerically, given a set of control parameters (See Appendix B). The optimal set of parameters for both criteria can then be found through enumeration.

In order to test the performance of the average cost criterion with the holding cost parameters that were developed in the previous section, i.e.,  $h_s = c_m$  and  $h_r = c_m - c_r$ , we carried out a small numerical study. Note that independent of the used criteria and policy, a cost of  $\delta = (p\lambda - c_m(\lambda - \gamma) - c_r\gamma)$  is incurred. Therefore, for the comparison we will use the relevant annuity stream  $\tilde{A}S = AS - \delta$ , and the relative relevant average profit  $\tilde{A}P = AP - \delta$ . Further, define the absolute relative relevant difference  $R$  as

$$R = \left| \frac{\tilde{A}S(Q_m^{AS}, Q_r^{AS}) - \tilde{A}S(Q_m^{AC}, Q_r^{AC})}{\tilde{A}S(Q_m^{AS}, Q_r^{AS})} \right| \times 100\% .$$

For the numerical comparison of the AC and NPV criterion we use the base case scenario in Table 1 as a starting point. In the numerical study only one parameter is varied at a time, while all other parameters are fixed. The system starts-up with a manufacturing batch of size  $Q_m$ .

The results of Table 2 clearly show that the performance of the AC criterion is outstanding, except for extreme values of the return rate and the initial inventory  $I_0$ . This behaviour follows the results of Section 3.3, where it was shown that large values of  $\alpha$  worsens the performance of the AC criterion.

$\lambda$	=	100.0	$p$	=	20.0
$\gamma$	=	50.0	$c_m$	=	10.0
$K_m$	=	0.5	$c_r$	=	5.0
$K_r$	=	0.5	$r$	=	0.2

**Table 1** Base case scenario

scenario	$Q_m^{AS}$	$Q_r^{AS}$	$Q_m^{AC}$	$Q_r^{AC}$	$R$
base case	5	5	5	5	0.0 %
$\gamma = 20.0$	6	4	6	4	0.0 %
$\gamma = 80.0$	3	6	3	6	0.0 %
$\gamma = 90.0$	3	6	2	6	0.1 %
$\gamma = 95.0$	2	6	1	6	1.3 %
$\gamma = 97.5$	2	7	1	6	4.8 %
$K_m = 0.0$	1	5	1	5	0.0 %
$K_m = 1.0$	7	5	7	5	0.0 %
$K_r = 0.0$	5	1	5	1	0.0 %
$K_r = 1.0$	5	7	5	7	0.0 %
$c_r = 0.0$	5	4	5	4	0.0 %
$c_r = 10.0$	5	7	5	7	0.0 %

**Table 2** Numerical evaluation of the performance of the average cost criterion.

## 6 Summary and topics for further research

This paper presents an exact NPV analysis of an inventory system with joint manufacturing and remanufacturing. For the special case of instantaneous remanufacturing analytic expressions are derived for the average profit function, the annuity stream function, and its linearization. For the case of remanufacturing batches numerical procedures are derived. Purpose was to evaluate the performance of the average costs criterion compared to the preferred net present value criterion. It was shown that the performance of the AC criterion heavily depends on the choice of holding cost parameters. The ‘correct’ holding costs parameters were derived through an exact analysis of two deterministic systems. It was shown that using these holding cost parameters results in an excellent performance of the AC criterion. However, using parameters that are based on the intuition that opportunity costs of inventory investment are equal to the interest rate times the product investment times the average inventory can lead to very poor performance. Additionally, one should note that even if the AC criterion results in reasonable values for the decision variables, the underlying evaluation does not necessarily give an accurate representation of ‘true’ costs and profits.

For the simple systems considered in this paper we were able to derive the appropriate holding cost parameters, but this might be very difficult in more realistic, and thus more complex, systems. In this light, future research should be directed at systems with multiple products, multiple components, and (dis)assembly operations.



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## Appendix A

Define  $\tilde{f}'_i(r) = \frac{d}{dr}\tilde{f}_i(r)$ ,  $\tilde{f}''_i(r) = \frac{d^2}{dr^2}\tilde{f}_i(r)$ , and  $\tilde{f}'''_i(r) = \frac{d^3}{dr^3}\tilde{f}_i(r)$ . It is easily verified that

$$\lim_{r \downarrow 0} \tilde{f}_i(r) = 1, \quad \lim_{r \downarrow 0} \tilde{f}'_i(r) = -\frac{i+1}{\lambda-\gamma}, \quad \lim_{r \downarrow 0} \tilde{f}''_i(r) = \frac{(i+1)^2(\lambda-\gamma) + (i+1)(\lambda+\gamma)}{(\lambda-\gamma)^3},$$

$$\lim_{r \downarrow 0} \frac{r}{1 - \tilde{f}_{Q_m-1}(r)} = \lim_{r \downarrow 0} \frac{1}{-\tilde{f}'_{Q_m-1}(r)} = \frac{\lambda-\gamma}{Q_m},$$

and

$$\begin{aligned} \lim_{r \downarrow 0} \frac{d}{dr} \left( \frac{r}{1 - \tilde{f}_{Q_m-1}(r)} \right) &= \lim_{r \downarrow 0} \frac{1 - \tilde{f}_{Q_m-1}(r) + r\tilde{f}'_{Q_m-1}(r)}{(1 - \tilde{f}_{Q_m-1}(r))^2} = \\ \lim_{r \downarrow 0} \frac{r\tilde{f}''_{Q_m-1}(r)}{-2\tilde{f}'_{Q_m-1}(r)(1 - \tilde{f}_{Q_m-1}(r))} &= \lim_{r \downarrow 0} \frac{\tilde{f}''_{Q_m-1}(r) + r\tilde{f}'''_{Q_m-1}(r)}{-2\tilde{f}'_{Q_m-1}(r)(1 - \tilde{f}_{Q_m-1}(r)) + 2(\tilde{f}'_{Q_m-1}(r))^2} = \\ \frac{\tilde{f}''_{Q_m-1}(0)}{2(\tilde{f}'_{Q_m-1}(0))^2} &= \frac{1}{2} + \frac{\lambda+\gamma}{2(\lambda-\gamma)Q_m}, \end{aligned}$$

so that

$$\begin{aligned} (K_m + c_m Q_m) \frac{r\tilde{f}_{I_0}(r)}{1 - \tilde{f}_{Q_m-1}(r)} &= \\ \left[ c_m(\lambda-\gamma) + K_m \left( \frac{\lambda-\gamma}{Q_m} \right) + r \left( \frac{1}{2} + \frac{\lambda+\gamma}{2(\lambda-\gamma)Q_m} \right) (K_m + c_m Q_m) + O(r^2) \right] &\tilde{f}_{I_0}(r). \end{aligned}$$

## Appendix B

Let  $I_s$  and  $I_r$  be two random variables denoting the serviceable inventory and remanufacturable inventory respectively. Then the system can be formulated as a two-dimensional Markov-chain  $\{(I_s(t), I_r(t)) | t \geq 0\}$  on the state space  $\{0, 1, \dots, \infty\} \times \{0, 1, \dots, Q_r - 1\}$  (see Van der Laan et al., 1998). The transition rates from state  $(i, j)$  to state  $(k, \ell)$  are

$$\nu_{(i,j),(k,\ell)} = \begin{cases} \gamma, & \text{if } \{j < Q_r - 1 \text{ and } \ell = j + 1\} \\ & \text{or } \{j = Q_r - 1 \text{ and } k = i + Q_r \text{ and } \ell = 0\} \\ \lambda, & \text{if } \{i > 0 \text{ and } k = i - 1\} \text{ or } \{i = 0 \text{ and } k = Q_m - 1\} \\ 0, & \text{elsewhere} \end{cases}$$

Solving the balance equations associated with these transition rates numerically result in the long-run state probabilities  $p_{(i,j)}$ , from which we can deduct the average serviceable inventory,  $E(I_s)$ , and the average remanufacturable inventory,  $E(I_r)$ :

$$E(I_s) = \sum_{i=0}^{\infty} \sum_{j=0}^{Q_r-1} i p_{(i,j)} , \quad E(I_r) = \sum_{i=0}^{\infty} \sum_{j=0}^{Q_r-1} j p_{(i,j)} .$$

For the NPV criterion, let  $X$  be the matrix with elements  $x_{(i,j),(k,\ell)}$  defined as

$$x_{(i,j),(k,\ell)} = \begin{cases} \frac{\gamma}{\lambda+\gamma+r}, & \text{if } \{j < Q_r - 1 \text{ and } \ell = j + 1\} \\ & \text{or } \{j = Q_r - 1 \text{ and } k = i + Q_r \text{ and } \ell = 0\} \\ \frac{\lambda}{\lambda+\gamma+r}, & \text{if } \{i > 0 \text{ and } k = i - 1\} \text{ or } \{i = 0 \text{ and } k = Q_m - 1\} \\ 0, & \text{elsewhere} \end{cases}$$

i.e.,  $X$  contains the expected discounted transition times from state  $(i, j)$  to state  $(k, \ell)$ .<sup>1</sup> Define the “manufacturing trigger set”,  $\mathcal{B}_m$ , as

$$\mathcal{B}_m = \{(i, j) | i = 0\} ,$$

i.e., when the system is in state  $(i, j) \in \mathcal{B}_m$ , a demand will trigger a manufacturing batch. Similarly, define the “remanufacturing trigger set”,  $\mathcal{B}_r$ , as

$$\mathcal{B}_r = \{(i, j) | j = Q_r - 1\} ,$$

i.e., when the system is in state  $(i, j) \in \mathcal{B}_r$ , a return will trigger a remanufacturing batch. Further define matrix  $B$  with elements  $b_{(i,j),k}$ ,  $k \in \{0, 1\}$  as

$$b_{(i,j),k} = \begin{cases} \frac{\lambda}{\lambda+\gamma+r}, & \text{if } k = 0 \text{ and } (i, j) \in \mathcal{B}_m \\ \frac{\gamma}{\lambda+\gamma+r}, & \text{if } k = 1 \text{ and } (i, j) \in \mathcal{B}_r \end{cases}$$

If  $(i_0, j_0)$  is the initial state of the system, we define the initial state vector  $\mathbf{a}$  with elements  $a_{(i,j)}$  as

$$a_{(i,j)} = \begin{cases} 1, & \text{if } (i, j) = (i_0, j_0) \\ 0, & \text{elsewhere} \end{cases}$$

Now, we can express the total annuity stream as

$$AS = p\lambda - r \left( C_0 + \sum_{n=0}^{\infty} \mathbf{a}^T \prod_{i=0}^n X^i B \begin{pmatrix} K_m + c_m Q_m \\ K_r + c_r Q_r \end{pmatrix} \right) ,$$

where  $C_0$  are the cash-flows incurred at time 0.

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<sup>1</sup>One can think of  $X$  as a two-dimensional matrix in which the system states  $(i, j)$  of each dimension are put in lexicographic order.

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