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Abstract

The Net Present Value (NPV) approach is considered to be the right approach to study inventory and production systems. But, approximate average cost (AC) approach is widely used in both practice and theory. However, the opportunity cost interpretation of AC framework is not that straightforward in systems with joint manufacturing and remanufacturing . In such systems the end-product stock contains both manufactured and remanufactured products. Remanufacturing can be used to convert the returns stock into different products. Due to this complex structure, the valuation of inventories at both stocking points is ambiguous. In this paper we analyze a two-product system with manufacturing and remanufacturing in a deterministic setting. By considering two different models under an NPV approach and an AC approach, we determine holding cost rates such that the two approaches are approximately equivalent. Then we demonstrate the negative effect of traditional valuation methodology on the remanufacturing operation dynamics by using these theoretical results.

Keywords: reverse logistics, remanufacturing, net present value, holding cost,inventory control.

Introduction

The management and control of inventory systems with joint manufacturing and remanufacturing has received considerable attention in recent literature. Research has focused on optimal policy structures (e.g. Inderfurth 1997, Fleischmann et al. 2002), heuristic policy structures (Van der Laan et al. 1999b, Toktay et al. 2000, Inderfurth and van der Laan 2001, Kiesmüller 2003, Mahadevan et al. 2003, Teunter et al. 2005), and heuristics to calculate near optimal parameter values (Kiesmüller and Minner, 2003, van der Laan and Teunter, 2005).

In most of these models, the stocks considered are returned items that are not yet remanufactured and serviceable stock that consists of both manufactured and remanufactured items. The setting of holding cost rates of these stocks is an important determinant for the performance of inventory policies in a reverse logistics environment as was shown by (Teunter et al. 2000) in a simulation study. It appeared that an intuitive choice of the holding cost rates easily leads to very poor performance.

The problem with respect to holding cost rates in an average cost (AC) framework arises because the average cost approach does not explicitly take into account the time value of money. The opportunity cost of inventory investment is usually included in the holding cost parameters. The assumption behind this is that the opportunity cost is (approximately) linear in the capital tied up in inventory and the opportunity cost rate. This assumption was validated, using a net present value (NPV) framework, for the EOQ model (Hadley 1964, Trippi and Lewin 1974, Thompson 1975, Hofmann 1998, Klein Haneveld and Teunter 1998), but the conclusion is less clear for multi-echelon systems (Grubbström and Thorstenson 1986) and remanufacturing systems (Teunter et al 2000, Van der Laan 2003). The net present value, or discounted cash flow approach is generally considered to be the right approach in financial decision making, since it focuses directly on cash flows rather than derivative costs and profits. However, due to a simpler structure, the average cost approach is more frequently employed both in practice and academia.

In this paper we analyze a two-product system with joint manufacturing and remanufacturing. For such a system, complications in finding the correct holding cost parameters arise because of two reasons. Firstly, the convergent structure of multiple sources (manufacturing and remanufacturing) means that serviceable inventory contains items that are physically and qualitatively the same, but are produced against different costs. Routinely used valuation methods such as Activity Based Costing (ABC), tell us to differentiate between the two items and set separate holding cost rates since the capital tied up in inventory differs. Secondly, the divergent structure of using returned products for two different end-items means that recoverable inventory contains products that may be qualitatively different, but exactly the same in terms of inventory investment. Traditional valuation methodology tells us to *not* differentiate between these items as the capital tied up in inventory is the same. Our analysis shows that, in this setting, the above methodology is fundamentally wrong on both accounts.

The remainder of the paper is organized as follows. After introducing the system in the next section, we analyze two models under NPV and AC approaches and try to find holding cost rates such that the AC approach is equivalent to an NPV analysis. Then in the following section, we demonstrate the effects of traditional valuation methodology on the remanufacturing operation dynamics by comparing that approach to the theoretical results of our analysis. Finally, we discuss the main results and point to some managerial insights.

Model development and analysis

We consider a two-product, joint manufacturing and remanufacturing environment as depicted in Figure 1. Customer demand for end-products A and B can be satisfied by newly manufactured products and by remanufacturing of used products. The returned products, denoted remanufacturables, are collected in a common stocking point. These products can be either processed by remanufacturing process A, which will turn them into type A products, or by remanufacturing process B, which will turn them into type B products. The type of conversion is either dictated by quality of remanufacturables (model 1) or a decision (model 2). All demand and return rates (number of products per time unit) are constant and deterministic. We assume that for each product type the product recovery rate (the long-run number of products recovered per time unit) is smaller than the demand rate, so to satisfy demand we also need the manufacturing process to replenish the stock of end-products (denoted 'serviceables'). All remanufacturables are eventually used for either product A or B, that is, there is no disposal or yield loss.

In order to clearly point out the difference between the NPV and AC approach we study two variations of the abovementioned remanufacturing system.

In **Model 1** we assume that product returns come in two different quality types, quality type A and quality type B. Remanufacturing process differs according to quality and is commenced

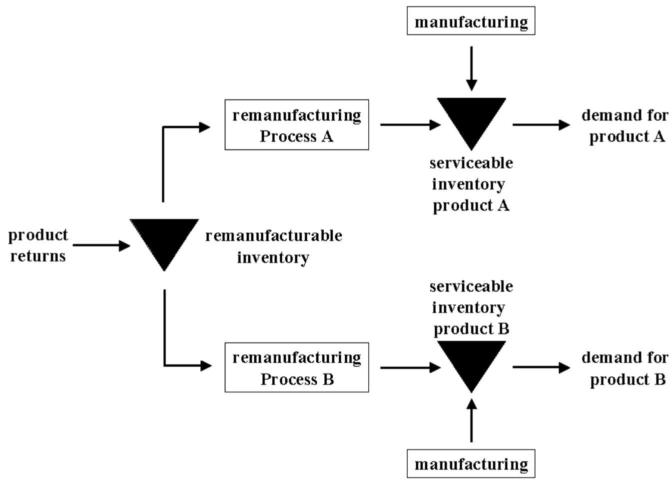

Figure 1: A system with joint manufacturing and remanufacturing operations.

at predetermined intervals for each type.

In **Model 2** we assume returned products can be remanufactured as A or B by incurring different costs. Moreover, we assume there is a limitation on how remanufacturing option is employed. Due to this the remanufacturing facility alternates between remanufacturing processes.

For simplicity of exposition we assume zero acquisition costs for remanufacturables. The implications of non-zero acquisition costs is discussed in the next section. The NPV and AC methodologies do not differ with respect to out-of-pocket holding costs therefore they are excluded from the models. The fundamental difference between the two approaches lies in the treatment of inventory investment, so that is the focus of the analysis.

The notation in the remainder of the paper is as follows.

λ_x : demand rate for product type X; $x \in \{a, b\}$

γ : overall return rate

γ_x : return rate for products used in remanufacturing process X

Q_m^x : manufacturing batch size of type X products

Q_r^x : remanufacturing batch size of type X products

r : discount rate

p_x : sales price for (re)manufactured products of type X

c_m^x : unit manufacturing cost per product of type X

c_r^x : unit remanufacturing cost per product of type X

K_m^x : fixed manufacturing cost per order of type X

K_r^x : fixed remanufacturing cost per order of type X

h_r^x : holding cost rate for remanufacturables that are turned into product type X

$h_s^{m,x}$: holding cost rate for manufactured products of type X

$h_s^{r,x}$: holding cost rate for remanufactured products of type X

Model 1: Quality Differentiation

First, we consider a situation where depending on their quality, returned products are used in remanufacturing process A or B and then added to the respective serviceable inventory. Demand for serviceables of A and B occur at rates λ_a and λ_b respectively. With fixed probability π, a returned product is of quality type A, so $\gamma_a = \pi\gamma$ and $\gamma_b = (1-\pi)\gamma$. We assume $\gamma_a < \lambda_a$ and $\gamma_b < \lambda_b$.

Depending on the nature of the product returns, it may or may not be possible, or cost efficient, to determine the quality of the items upon arrival. First, we consider the case where returned item quality is known at arrival. Then, a remanufacturing batch of size Q_r^a occurs every $T_r^a = Q_r^a/\gamma_a$ time units starting at time T_r^a and similarly a remanufacturing batch of size Q_r^b occurs every $T_r^b = Q_r^b/\gamma_b$. To fully satisfy demand, each $T_m^x = Q_m^x/(\lambda_x - \gamma_x)$ time units a manufacturing batch of size Q_m^x is initiated. Figure 2 presents a visualization of the inventory processes.

Since it takes T_r^x time units to accumulate the first remanufacturing batch, both subsystems need to be initiated by manufacturing batches of sizes equal to $Q_m^x + Q_r^x$.

The annuity stream (r times the net present value; see Grubbstrom, 1980) of the whole system

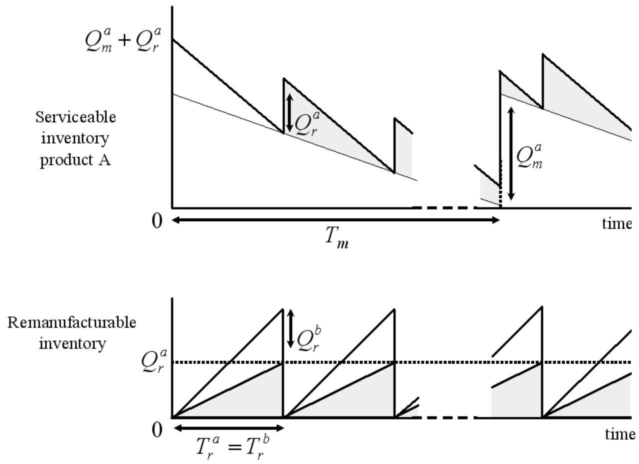

Figure 2: Inventory process of system A for $T_r^a = T_r^b$.

is given by

$$AS = p_a\lambda_a + p_b\lambda_b - r\left(c_m^a Q_r^a + c_m^b Q_r^b + \sum_{n=0}^{\infty}(K_m^a + c_m^a Q_m^a)e^{-rnT_m^a}\right.$$
$$\left. + \sum_{n=0}^{\infty}(K_m^b + c_m^b Q_m^b)e^{-rnT_m^b} + \sum_{n=1}^{\infty}(K_r^a + c_r^a Q_r^a)e^{-rnT_r^a} + \sum_{n=1}^{\infty}(K_r^b + c_r^b Q_r^b)e^{-rnT_r^b}\right) \quad (1)$$

which is equivalent to

$$AS = p_a\lambda_a + p_b\lambda_b - r\left(c_m^a Q_r^a + c_m^b Q_r^b + \frac{(K_m^a + c_m^a Q_m^a)}{1 - e^{-rT_m^a}} + \frac{(K_m^b + c_m^b Q_m^b)}{1 - e^{-rT_m^b}}\right.$$
$$\left. + \frac{(K_r^a + c_r^a Q_r^a)e^{-rT_r^a}}{1 - e^{-rT_r^a}} + \frac{(K_r^b + c_r^b Q_r^b)e^{-rT_r^b}}{1 - e^{-rT_r^b}}\right) \quad (2)$$

A linearization in r, using $\frac{r}{1-e^{-rT}} = \frac{1}{T} + \frac{r}{2} + \mathrm{o}(r^2)$ and $\frac{re^{-rT}}{1-e^{-rT}} = \frac{1}{T} - \frac{r}{2} + \mathrm{o}(r^2)$ gives,

$$\begin{aligned}
\overline{AS} &= \lambda_a p_a + \lambda_b p_b - r c_m^a Q_r^a - r c_m^b Q_r^b - (K_m^a + c_m^a Q_m^a)\left(\frac{1}{T_m^a} + \frac{r}{2}\right) \\
&\quad - \left(K_m^b + c_m^b Q_m^b\right)\left(\frac{1}{T_m^b} + \frac{r}{2}\right) - (K_r^a + c_r^a Q_r^a)\left(\frac{1}{T_r^a} - \frac{r}{2}\right) - \left(K_r^b + c_r^b Q_r^b\right)\left(\frac{1}{T_r^b} - \frac{r}{2}\right) \\
&= \lambda_a p_a + \lambda_b p_b - (\lambda_a - \gamma_a)c_m^a - (\lambda_b - \gamma_b)c_m^b - \gamma_a c_r^a - \gamma_b c_r^b \\
&\quad - \frac{r(K_m^a + K_m^b - K_r^a - K_r^b)}{2} - \frac{(\lambda_a - \gamma_a)K_m^a}{Q_m^a} - \frac{(\lambda_b - \gamma_b)K_m^b}{Q_m^b} - \frac{\gamma_a K_r^a}{Q_r^a} - \frac{\gamma_b K_r^b}{Q_r^b} \\
&\quad - r\left(c_m^a \frac{Q_m^a}{2} + c_m^b \frac{Q_m^b}{2} + (2c_m^a - c_r^a)\frac{Q_r^a}{2} + (2c_m^b - c_r^b)\frac{Q_r^b}{2}\right) \qquad (3)
\end{aligned}$$

To keep the analysis tractable for the AC approach we need to make an assumption about how holding costs are determined for manufactured products and remanufactured products in the serviceable inventory. Since all products of the same type are sold for the same price, it should not matter whether demand is satisfied by a manufactured product or a remanufactured product. In practice one would assign products to demand randomly. In a deterministic setting this is equivalent to assigning remanufactured and manufactured products according to their production rates γ_x and $\lambda_x - \gamma_x$ respectively. This is also visualized in Figure 2. Then, the average inventory of manufactured products is $Q_m^x/2$ and the average inventory of remanufactured products is $Q_r^x/2$. Given the above, the AC approach computes the average profit function as

$$\begin{aligned} AP &= \lambda_a p_a + \lambda_b p_b - (\lambda_a - \gamma_a)c_m^a - (\lambda_b - \gamma_b)c_m^b - \gamma_a c_r^a - \gamma_b c_r^b \\ &- \frac{(\lambda_a - \gamma_a)K_m^a}{Q_m^a} - \frac{(\lambda_b - \gamma_b)K_m^b}{Q_m^b} - \frac{\gamma_a K_r^a}{Q_r^a} - \frac{\gamma_b K_r^b}{Q_r^b} \\ &- h_s^{m,a}\frac{Q_m^a}{2} - h_s^{m,b}\frac{Q_m^b}{2} - (h_s^{r,a} + h_r^a)\frac{Q_r^a}{2} - (h_s^{r,b} + h_r^b)\frac{Q_r^b}{2} \end{aligned} \tag{4}$$

If it is not possible or not operationally viable to determine the quality upon arrival of the product returns, we assume that sorting occurs just prior or even during remanufacturing. Operationally this means that every $T_r = Q_r/\gamma$ time units remanufacturing operation is initiated. On average π percent of returns is of type A, the rest being type B. Therefore, each cycle a batch is pushed to each subsystem with batch sizes $Q_r^a = \pi Q_r$ and $Q_r^b = (1-\pi)Q_r$.

Note that (3) and (4) are equal up to a constant if we choose the holding cost parameters such that the last line of (3) equals the last line of (4). At the same time it is desirable to have holding cost parameters that depend on unit costs but not on system parameters such as γ_x and π. This is the case if the holding cost parameters satisfy the following set of equations.

$$h_s^{m,a} = rc_m^a \tag{5}$$

$$h_s^{m,b} = rc_m^b \tag{6}$$

$$h_s^{r,a} + h_r^a = r(2c_m^a - c_r^a) \tag{7}$$

$$h_s^{r,b} + h_r^b = r(2c_m^b - c_r^b) \tag{8}$$

The parameters $h_s^{m,a}$ and $h_s^{m,b}$ are uniquely defined by (5) and (6), but (7) and (8) each represent an equation with two unknowns, thus presenting an unlimited number of options for setting holding cost rates for remanufacturables and remanufactured items in serviceable stock. However, we can try and use traditional valuation methodology to pick solutions that have a valid economic interpretation and therefore appeal to our intuition.

For example, an option may be using $h_r^a = h_r^b = 0$ as the acquisition cost of remanufacturables is zero, so there is no capital tied up in inventory. Then a unique solution to (5–8) is

$$h_r^a = h_r^b = 0 \Rightarrow \begin{cases} h_s^{m,a} &= rc_m^a \\ h_s^{m,b} &= rc_m^b \\ h_s^{r,a} &= r(2c_m^a - c_r^a) \\ h_s^{r,b} &= r(2c_m^b - c_r^b) \end{cases} \tag{9}$$

According to traditional valuation logic, $h_s^{r,x}$ should represent the added value of the remanufacturing operation (that is $h_s^{r,x} = rc_r^x$), but it does not. Although it includes c_r^x, it has the opposite sign, i.e. when added value increases holding cost rate decreases. Moreover, this set of holding cost rates suggest that remanufactured items should be assigned to demands before manufactured items since they incur cost at a greater rate. As we argued before, this does not make sense from a financial perspective as selling a remanufactured product generates the same cash flow as selling a manufactured product.

Actually, the rationale that h_r^x should represent the acquisition cost (zero in our model) is fundamentally flawed. In this model the return stream is an autonomous process that is independent of the decision variables. Therefore, the NPV of the total acquisition cost is a constant and the unit acquisition cost does not influence any optimization of the decision variables. Including the unit acquisition cost in the holding cost rates would influence optimization, which should not happen.

Using the traditional logic we can set $h_s^{r,x} = rc_r^x$, which results in

$$\left.\begin{array}{rcl} h_s^{r,a} &=& rc_r^a \\ h_s^{r,b} &=& rc_r^b \end{array}\right\} \Rightarrow \begin{cases} h_s^{m,a} &= rc_m^a \\ h_s^{m,b} &= rc_m^b \\ h_r^a &= 2r(c_m^a - c_r^a) \\ h_r^b &= 2r(c_m^b - c_r^b) \end{cases} \tag{10}$$

but also lacks economic interpretation for the rates corresponding to remanufacturable inventory. One important characteristic that follows implicitly from traditional valuation methodology is that the difference between the holding cost rates of consecutive stocking points should represent the added value of the operation that moves a product from one stocking point to the next. That is, $h_s^{r,x} - h_r^x = rc_r^x$. Under this condition we have the following unique solution.

$$\left.\begin{array}{rcl} h_s^{r,a} - h_r^a & = & rc_r^a \\ h_s^{r,b} - h_r^b & = & rc_r^b \end{array}\right\} \Rightarrow \begin{cases} h_s^{m,a} = h_s^{r,a} = rc_m^a \\ h_s^{m,b} = h_s^{r,b} = rc_m^b \\ h_r^a = r(c_m^a - c_r^a) \\ h_r^b = r(c_m^b - c_r^b) \end{cases} \tag{11}$$

This can be seen as a direct extension of the results presented in (Van der Laan 2003) which were for a similar setting, but with just one product type. Note that these holding cost rates have the appealing characteristic that there is no discrimination between manufactured and remanufactured products.

In practice the above holding cost rates can only be applied if the quality of a returned product can be identified prior to remanufacturing. Otherwise it is practically impossible to assign different holding costs for the recoverable products (i.e. $h_r^a = h_r^b = h_r$). Then we have to consider (7) and (8) together, but still there are 3 unknowns. Among the infinitely many options, one could rely on (9), but this has limited or no economic interpretation. Moreover, it is easy to see that, the setting of h_r has effect on both serviceable inventories. In other words cost parameter(s) of one subsystem effect the other if included in h_r. But, if the holding cost rate for remanufacturables is expressed in terms of the cost parameters of just one of the subsystems, this effect is limited to only one serviceable stock. By using the rationale that lead to (11) as a starting point, the following set of solutions follow.

$$h_r^a = h_r^b = r(c_m^a - c_r^a) \Rightarrow \begin{cases} h_s^{m,a} & = & h_s^{r,a} = rc_m^a \\ h_s^{m,b} & = & rc_m^b \\ h_s^{r,b} & = & r(c_m^b + (c_m^b - c_r^b) - (c_m^a - c_r^a)) \end{cases} \tag{12}$$

The interpretation of this set of holding cost rates are as follows. Suppose that initially all the recoverables are sent to subsystem A. Then (12) is equivalent to (11). Now assume that one recoverable item per cycle is diverted to subsystem B. What is the net cost/benefit of that decision? First, one unit of serviceable inventory is shifted from A to B with opportunity cost $r(c_m^a - c_m^b)$. Next, for the remanufactured units in serviceable inventory of subsystem B we have the opportunity cost of $r[(c_m^b - c_r^b) - (c_m^a - c_r^a)]$, which is the net cost of recovering for B instead of A. In total we have the opportunity cost of $r[(c_m^a - c_m^b) + (c_m^b - c_r^b) - (c_m^a - c_r^a)]$, which is exactly the difference $h_s^{r,b} - h_s^{r,a}$ as expected. Note that the system is symmetric in quality types

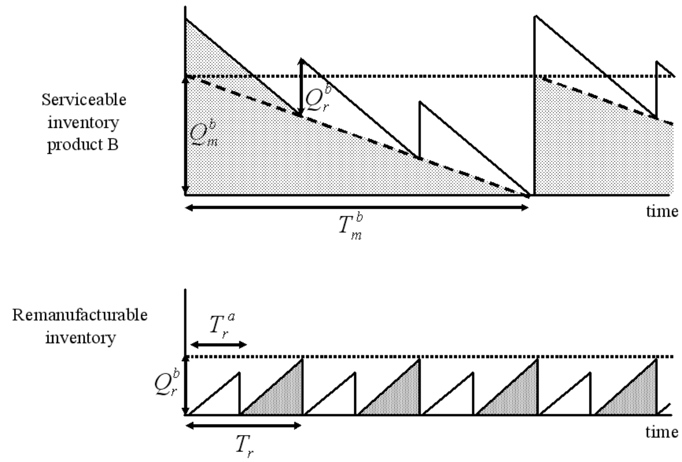

Figure 3: Inventory process for system B.

A and B, so switching superscripts a and b in (12) also results in a valid set of holding cost parameters.

Model 2: Sequential batches

As a second model, we consider a case where a remanufacturing operation is employed such that two systems are supplied with one batch in a sequential manner. The returned products can be converted into either A or B by incurring different costs. Without any additional restrictions, all the returned products will be sent to the channel that is most profitable. However, legislative constraints, capacity constraints, or marketing constraints may dictate that remanufacturing is employed for one of the products at a minimum level. Let π represent the portion of remanufacturing capacity dedicated to meet this minimum for subsystem A. In terms of batch sizes, π plays a similar role as in the previous model; so $Q_r^a = \frac{\pi}{1-\pi} Q_r^b$, $\gamma_a = \pi\gamma$, and $\gamma_b = (1-\pi)\gamma$. The cycle time for remanufacturing is $T_r = \frac{Q_r^a + Q_r^b}{\gamma}$ (see Figure 3).

Without losing generality we assume that the first batch out of remanufacturing process is of type A. Since it takes $T_r^a = Q_r^a/\gamma$ time units for this batch to accumulate, initially we have to cover $(T_r^a/T_r)Q_r^a = \pi Q_r^a$ by manufacturing. Thus, to start up subsystem A, a manufacturing batch of size $Q_m^a + \pi Q_r^a$ is used. To start up subsystem B, similar to model 1, a batch of size $Q_m^b + Q_r^b$ is used. Then the annuity stream is given by,

$$
\begin{aligned}
AS &= p_a\lambda_a + p_b\lambda_b - r\left(c_m^a \pi Q_r^a + c_m^b Q_r^b + \sum_{n=0}^{\infty}(K_m^a + c_m^a Q_m^a)e^{-rnT_m^a}\right. \\
&\quad + \sum_{n=0}^{\infty}(K_m^b + c_m^b Q_m^b)e^{-rnT_m^b} + \sum_{n=0}^{\infty}(K_r^a + c_r^a Q_r^a)e^{-r(T_a+nT_r)} + \left.\sum_{n=1}^{\infty}(K_r^b + c_r^b Q_r^b)e^{-rnT_r}\right) \\
&= p_a\lambda_a + p_b\lambda_b - r\left(c_m^a \pi Q_r^a + c_m^b Q_r^b + \frac{(K_m^a + c_m^a Q_m^a)}{1-e^{-rT_m^a}} + \frac{(K_m^b + c_m^b Q_m^b)}{1-e^{-rT_m^b}}\right. \\
&\quad + \left.\frac{(K_r^a + c_r^a Q_r^a)e^{-rT_a}}{1-e^{-rT_r}} + \frac{(K_r^b + c_r^b Q_r^b)e^{-rT_r}}{1-e^{-rT_r}}\right) \qquad (13)
\end{aligned}
$$

Using $\frac{re^{-rT_r^a}}{1-e^{-rT_r}} = \frac{1}{T_r} + \frac{r}{2} - \frac{rT_r^a}{T_r} + \mathrm{o}(r^2)$, a linearization in r gives,

$$
\begin{aligned}
\overline{AS} &= \lambda_a p_a + \lambda_b p_b - r c_m^a \pi Q_r^a - r c_m^b Q_r^b \\
&\quad - (K_m^a + c_m^a Q_m^a)\left(\frac{1}{T_m^a} + \frac{r}{2}\right) - (K_m^b + c_m^b Q_m^b)\left(\frac{1}{T_m^b} + \frac{r}{2}\right) \\
&\quad - (K_r^a + c_r^a Q_r^a)\left(\frac{1}{T_r} + \frac{r}{2} - \frac{rT_a}{T_r}\right) - (K_r^b + c_r^b Q_r^b)\left(\frac{1}{T_r} - \frac{r}{2}\right) \\
&= \lambda_a p_a + \lambda_b p_b - (\lambda_a - \gamma_a)c_m^a - (\lambda_b - \gamma_b)c_m^b - \gamma_a c_r^a - \gamma_b c_r^b \\
&\quad - \frac{(\lambda_a - \gamma_a)K_m^a}{Q_m^a} - \frac{(\lambda_b - \gamma_b)K_m^b}{Q_m^b} - \frac{\gamma K_r^a}{Q_r^a + Q_r^b} - \frac{\gamma K_r^b}{Q_r^a + Q_r^b} \\
&\quad - r\left(\frac{K_m^a + K_m^b + K_r^a - K_r^b}{2} - \pi K_r^a\right) \\
&\quad - r\left(c_m^a \frac{Q_m^a}{2} + c_m^b \frac{Q_m^b}{2} + (c_m^a - c_r^a)\pi Q_r^a + c_r^a \frac{Q_r^a}{2} + (2c_m^b - c_r^b)\frac{Q_r^b}{2}\right) \qquad (14)
\end{aligned}
$$

The average inventory of remanufacturables can be expressed as $\frac{\pi Q_r^a + (1-\pi)Q_r^b}{2} = \frac{Q_r^b}{2} - \frac{Q_r^a}{2} + \pi Q_r^a$. Then, the AC approach computes the average profit function as,

$$
\begin{aligned}
AP &= \lambda_a p_a + \lambda_b p_b - (\lambda_a - \gamma_a)c_m^a - (\lambda_b - \gamma_b)c_m^b - \gamma_a c_r^a - \gamma_b c_r^b \\
&\quad - \frac{(\lambda_a - \gamma_a)K_m^a}{Q_m^a} - \frac{(\lambda_b - \gamma_b)K_m^b}{Q_m^b} - \frac{\gamma K_r^a}{Q_r^a + Q_r^b} - \frac{\gamma K_r^b}{Q_r^a + Q_r^b} \\
&\quad - h_s^{m,a}\frac{Q_m^a}{2} - h_s^{m,b}\frac{Q_m^b}{2} - h_s^{r,a}\frac{Q_r^a}{2} - h_s^{r,b}\frac{Q_r^b}{2} - h_r\left(\frac{Q_r^b}{2} - \frac{Q_r^a}{2} + \pi Q_r^a\right) \qquad (15)
\end{aligned}
$$

The two approaches are equal (up to a constant value) if the last line of (14) equals the last line of (15). That gives us a system of 2 equations with 3 unknowns where π appears as a coefficient. In this setting, π can be seen as representing a decision, therefore for the sake of robustness it is desirable to pick solutions that do not depend on π. There is only one solution that does not depend on π:

$$\begin{aligned} h_s^{m,a} &= h_s^{r,a} = rc_m^a \\ h_s^{m,b} &= rc_m^b \\ h_s^{r,b} &= r(c_m^b + (c_m^b - c_r^b) - (c_m^a - c_r^a)) \\ h_r &= r(c_m^a - c_r^a) \end{aligned} \tag{16}$$

The interpretation of these set of holding cost rates is similar to (12). However, since the subsystems are not symmetric in this case swapping the superscripts does not lead to a solution. It is worth noting that, for this policy if there is no restriction on how the remanufacturing option is employed in the optimal solution either Q_r^a or Q_r^b would be zero. That is all the returns will be used in the more beneficial option. In the case $c_m^a - c_r^a > c_m^b - c_r^b$, without any restriction on the remanufacturing process, all returns will be used for subsystem A. In that case the rates defined in (16) reduce to the rates suggested in (Van der Laan 2003). The case $c_m^b - c_r^b > c_m^a - c_r^a$ without any restriction, leads to $Q_r^a = 0$, meaning that remanufacturing operation and subsystem A are completely detached. However, the remanufacturable items are still valued against the rate $h_r = r(c_m^a - c_r^a)$. This still has an economic interpretation representing a minimal opportunity, but from an accounting point of view it is not intuitive to use such a measure.

The impact of using the wrong intuition

The traditional intuition regarding holding cost rates is that they should reflect added value, as in the popular Activity Based Costing (ABC) methodology. For our models this suggests that $h_r = 0$, $h_s^{m,x} = rc_m^x$ and $h_s^{r,x} = rc_r^x$. Next we investigate the impact of using those values as compared to the theoretical holding cost rates that were developed in in this study.

For the first model, we assume sorting of items occur at the time of remanufacturing, therefore the holding cost rates shown in (12) apply. In this setting, each subsystem receives a batch every T_r time units. Using in (4) that $Q_r^a = \pi Q_r$ and $Q_r^b = (1-\pi)Q_r$, we differentiate (4) with respect to Q_r and equate to zero. This results in a EOQ-type formula for the total remanufacturing quantity.

$$Q_r^* = \sqrt{\frac{2\gamma(K_r^a + K_r^b)}{\pi(h_s^{r,a} + h_r^a) + (1-\pi)(h_s^{r,b} + h_r^b)}} \quad , \qquad \pi \in (0,1)$$

Table 1 presents a comparison between the optimal values of Q_r found by using our theoretical results and the values computed through the ABC approach.

c_r^a	Q_r (optimal)	Q_r (ABC)	π	Q_r (optimal)	Q_r (ABC)
0	29.8	89.4	0.0^+	36.5	44.7
2	31.1	67.6	0.2	34.8	48.5
4	32.7	56.6	0.4	33.3	53.5
6	34.4	49.6	0.6	32.0	60.3
8	36.5	44.7	0.8	30.9	70.7
10	39.0	41.0	1.0^-	29.8	89.4

Table 1: Comparison model 1; $r = 0.1$, $\lambda = 1$, $\gamma = 0.8$, $c_m^a = c_m^b = 10$, $c_r^a = 2$, $c_r^b = 8$, $K_r = 1000$ $\pi = 0.75$, unless specified differently.

As Table 1 shows, increasing the remanufacturing cost of product A increases the total remanufacturing batch size. Intuitively, from a cash flow perspective as remanufacturing becomes more expensive, it is better to move remanufacturing further into the future to delay costs. This delay naturally brings about a larger batch size. By using (12), it is easily seen that the net effect of increasing the remanufacturing cost of product A leads to a decrease in the denominator. Thus, the true dynamics is reflected using the 'correct' holding costs.

However, the ABC approach fails to capture this behavior. Apart from seriously overestimating the batch size for low and moderate values of remanufacturing cost, it shows a decreasing pattern rather than an increasing one. This is due to the fact that in ABC methodology increasing c_r^a means increasing the inventory investment, thus using smaller batches to offset the increasing holding cost.

Since for the base case $c_r^a < c_r^b$, from a cash flow perspective increasing π makes remanufacturing more beneficial, consequently T_r and Q_r decrease. Again, the ABC approach changes the batch size in the wrong direction since decreasing average remanufacturing cost means decreasing holding cost rate in this approach.

For the second model, using (15) and $Q_r^a = \pi Q_r$, $Q_r^b = (1 - \pi)Q_r$ a similar EOQ type formula is obtained.

$$Q_r^* = \sqrt{\frac{2\gamma(K_r^a + K_r^b)}{\pi h_s^{r,a} + (1-\pi)h_s^{r,b} + (\pi^2 + (1-\pi)^2)h_r}} \quad , \qquad \pi \in (0,1)$$

Table 2 presents a comparison between the theoretical optimal values of Q_r and the values that result from using the added value framework.

c_r^a	Q_r (optimal)	Q_r (ABC)	π	Q_r (optimal)	Q_r (ABC)
0	33.5	89.4	0.0^+	36.5	44.7
2	34.4	67.6	0.2	38.8	48.5
4	35.4	56.6	0.4	38.9	53.5
6	36.5	49.6	0.6	36.9	60.3
8	37.7	44.7	0.8	33.5	70.7
10	39.0	41.0	1.0^-	29.8	89.4

Table 2: Comparison model 2; $r = 0.1$, $\lambda = 1$, $\gamma = 0.8$, $c_m^a = c_m^b = 10$, $c_r^a = 2$, $c_r^b = 8$, $K_r^a + K_r^b = 1000$ $\pi = 0.75$, unless specified differently.

Varying c_r^a in model 2 (see Table 2) we observe a similar monotonous increase of the optimal remanufacturing quantity as compared to model 1. Note that these optimal remanufacturing quantities are higher than the values reported in Table 1. The reason for this is the lower holding costs that are due to splitting batches. Other than this difference, the dynamics and intuition are the same as model 1. Varying π in model 2 shows a concave relation rather than a monotonously decreasing one.

The intuition behind this can be demonstrated as follows. If we consider a fixed Q_r, unlike model 1 changing π has an effect on the timing of remanufacturing batches as well as the total number of products manufactured at time zero. For fixed Q_r, in model 1 this total does not change whereas it is convex in π in the relevant range in model 2. Therefore, considering the cash flows when π moves towards boundaries Q_r goes down to reduce the cash outflow at time zero (i.e. earlier remanufacturing).

Using the theoretical holding cost rates, the effect of split batches is captured by the quadratic term. In contrast ABC approach completely misses the dynamics brought by batch splitting because $h_r = 0$ in this case.

It is worth to point to the fact that the ABC approach fails to differentiate between models 1 and 2 as it computes exactly the same values for the remanufacturing quantities. The explanation for this is easily shown by quantitatively comparing the two models for the NPV approach and the AC approach. The difference in terms of the annuity stream, ΔAS, is (3) minus (14):

$$\Delta \text{AS} = r(c_m^a - c_r^a)(1-\pi)Q_r^a + \text{constant}$$

Assuming that $h_r^a = h_r^b = h_r$, the difference in terms of the average profits, ΔAP, is (4) minus (15):

$$\Delta \text{AP} = h_r(1-\pi)Q_r^a + \text{constant}$$

Thus, the only way to reflect the difference between the two models is through h_r, which apparently should depend on $(c_m^a - c_r^a)$. But ABC logic sets h_r proportional to the acquisition cost which is zero. Consequently, the average cost (AC) framework combined with ABC logic is not able to account for the different dynamics of models 1 and 2.

Summary and Discussion

In this paper, we considered a two source, two product remanufacturing environment in a deterministic setting. The purpose was to develop a general intuition for setting holding cost rates in a multi product manufacturing/remanufacturing environment by comparing an exact NPV analysis and an AC analysis of different policies. Our analysis shows that it is far from trivial to set the holding cost rates such that the average cost approach gives approximately the same results as the net present value approach.

For a single item manufacturing/remanufacturing system, it is already known that (see Van der Laan 2003) the 'correct' holding cost rate of remanufacturables depend on per unit benefit of remanufacturing. In the first model when the two subsystems are not detached (i.e. quality is not known beforehand), the valuation of remanufacturable inventory is an issue because per unit benefit of manufacturing depends on type. In the second model, returns can be converted to either of the products by incurring different costs thus again represent two different opportunities.

Therefore, the two models considered have different dynamics with respect to remanufacturable inventory and thus, timing of cash flows. Although the 'correct' set of holding cost rates determined in this study capture these different dynamics, this is at the expense of differentiating serviceables of one product according to the source. On the other hand, using the traditional logic to set the holding cost rates based on inventory investment leads to completely missing out the dynamics of remanufacturable inventory.

The NPV framework does not know any of the above difficulties. It analyzes the cash flows as they follow from the operational decisions that are taken and gives the corresponding financial consequences.

Summarizing, we have the following managerial implications.

- The classical approach regarding holding costs claims that holding costs can be specified in terms of added value. This appears not to be true in general for systems with manufacturing and remanufacturing. From an application point of view, this has an effect on what type of policy to choose as well as optimizing policy parameters for a given policy.

- The 'correct' holding cost indicate that the remanufacturable items should be valued against an opportunity cost. This is in line with previous findings. The findings of this paper indicates this opportunity cost interpretation extends to the serviceable inventories due to multi product setting.

- Perhaps more important than calibrating frequently used average cost approach, the appropriate holding cost rates derived in this study underline the fact that returned items represent a potential value which is more than the acquisition cost. This has profound effects ranging from daily stocking operations to (reverse) chain design. Specifically, when there is a disposal option, recognizing this inherent value will lead to exercising that option less frequently, thus resulting in a positive effect on profits and the environment.

As evident from this two-product setting, when models become more elaborate to capture a variety of structures and dynamics, using intuition for setting holding cost rates becomes harder. Nevertheless, given the important effect of using the correct holding cost rates, further research will focus on complex system with nonzero leadtimes and multiple components.

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