Econometric Institute Reports EI 2002 - 17

A scenario aggregation based approach for determining a robust airline fleet composition

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Abstract

Strategic airline fleet planning is one of the major issues addressed through newly initiated decision support systems, designed to assist airlines and aircraft manufacturers in assessing the benefits of the emerging concept of dynamic capacity allocation. We present background research connected with such a system, which aims to explicitly account for the stochastic nature of passenger demand in supporting decisions related to the fleet composition problem. We address this problem through a scenario aggregation based approach and present results on representative case studies based on realistic data. Our investigations establish clear benefits of a stochastic approach as compared with deterministic formulations, as well as its implementation feasibility using state-of-the-art optimization software.

Keywords: airline fleet composition, fleet assignment, dynamic capacity allocation, stochastic programming, scenario aggregation

1 Introduction

Airlines around the world are facing nowadays steadily declining passenger yields. As competition intensified, owing to liberalisation and deregulation, airlines were forced to cut costs and uphold revenues, while their marginal profits came under tremendous pressure. One of the major factors contributing to the problems in airlines operations is the stochastic nature of passenger demand. While seasonal variations in demands are usually taken into account, there are also typical random demand fluctuations all over airline network, which generally lead to (relatively) low average load factors and a significant number of not accepted passengers (spill). A newly envisioned concept to deal with this high variability is the dynamic allocation of airline fleet capacity. Essentially, this emerging new operating philosophy aims to use the most recent estimates of customers demands for accordingly updating the assignments of aircrafts to the flight schedule, shortly before the actual operations, in order to better match the available capacity to the demands and boost the total operating profit over the entire network (see for instance the discussion on Demand Driven Dispatch in Berge and Hopperstad (1993)).

Presently some airlines manually swap aircraft assignments at various stages in response to demand variation. However, a systematic application of the dynamic capacity

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allocation concept on a structural basis would imply major reorganization changes for the airlines. In order to provide insight into the concept benefits and the necessary changes it would trigger, prospective studies and appropriate decision support systems started to emerge. ORTEC Consultants B.V., The Netherlands, initiated the Dynamic Capacity Management (DCM) system, designed to assist airlines and aircraft manufacturers throughout this process. In this system a strategic and an operational level are distinguished. The strategic tool addresses the airline fleet planning and its impact on the network dynamics. Despite advanced mathematical optimization techniques, the approaches to date are deterministic in nature.

The purpose of this paper is to present background research connected with the DCM system, aiming to come up with an approach which accounts for the random demand fluctuations when deciding upon airline fleet composition. Our guiding idea was to search for a fleet composition which appropriately supports dynamic allocation, depending on the flight schedule under consideration and the associated stochastic demands on its flight legs. Owing to this strong dependency, such an approach should support strategic fleet planning in a model based way and should be applicable to various networks and schedule scenarios. We believe that not only the fleet operation, but also the decision process of fleet composition planning should already be placed into the perspective of applying a dynamic allocation of its capacity on a structural basis. From this perspective, the design of a strategic support tool should incorporate explicit means for determining a fleet composition flexible enough for the successful implementation of the dynamic allocation concept.

The remainder of the paper is organized as follows. Section 2 describes the fleet problem under consideration as well as our various assumptions and research questions. The mathematical modelling of the described problem is further elaborated in Section 3. In Section 4 we discuss our implementation of the involved algorithms in a suite of applications, called FleetComp. Case study results on representative networks are presented in Section 5. Finally, Section 6 formulates our documented conclusions.

The findings of our investigations show promise in providing conceptually more robust solutions than deterministic formulations. Moreover, they clearly assess the potential benefits of using a stochastic approach and the feasibility of its implementation within a practice-oriented decision support system.

2 The fleet composition problem

Given a flight schedule and a set of aircraft types, the fleet composition problem - concisely formulated - is to determine the number of aircrafts of each type the fleet should consist of in order to be the most profitable when assigned to the schedule. The various factors and assumptions considered for this problem are subsequently specified.

Weekly flight schedules given as a list of flight legs are considered here, where the corresponding index week is supposed to offer a representative worth of flights for the airline's operations. For each flight leg in the schedule the following data are given: the origin airport, the destination airport, the departure time, the arrival time, the (expected) demand for each fare class (economy, business) and the flight distance. Demands for seats are assumed to follow independent normal distributions (truncated at zero), with the variability specified as the K-factor (the ratio of the standard deviation to the mean).

Each aircraft type is defined by its fixed costs (per week), its operational costs, its capacity for each fare class, its range capability and its family indicator. An aircraft type may be assigned to a flight leg only if its range is greater than the flight distance, so that certain type/leg combinations are prohibited. Types allowed to perform a leg are assumed to have the same flying time and this to be exactly the one required by the flight specifications. Moreover, we assume that all the aircraft types considered have identical turn-around-time over the whole network (this assumption can be easily relaxed, leading to type-dependent flight connections). When demand for a fare class exceeds the corresponding capacity of a type, the excess demand is spilled and the spilled passengers not recaptured afterwards. For a given schedule there exists a constant representing the minimum total number of airplanes (independent of type) needed to carry out the whole schedule, which can be easily computed (see the modeling section). Only fleet compositions with this minimum total number of airplanes are taken into consideration.

The main measure of fleet performance is expressed in terms of the profit it can generate by operating the schedule from which the fixed costs of its aircrafts have to be subtracted. This profitability measure, which results from the dynamic interaction between the actual demand values and the aircrafts' characteristics, especially their capacities, is driving both the search process for an appropriate fleet composition as well as the evaluation of any established fleet configuration. More precisely, once a fleet composition has been specified, the quantification of its performance can be achieved by means of demand simulation, fleet reassignment and calculation of average scores. Such an evaluation follows essentially the same macro-flow structure as the Demand Driven Dispatch method proposed by Berge and Hopperstad (1993). This structure forms the conceptual basis for implementing operational support systems for dynamic aircraft assignment. However, the manner in which the stochastic nature of demands could be taken into account for actually determining a suitable fleet composition remains to be investigated and it's dealt with in this paper. Our investigations were driven by the need to answer the following questions:

1. By which mathematical optimization techniques could the stochastic nature of passenger demands be taken into account in the fleet composition problem ?
2. To what extent would the solution given by such an approach be more robust as compared with a deterministic solution ?
3. Could such an approach determine an appropriate composition for an interchangeable fleet (which allows swapping assignments of planes within an aircraft family) ?

Answers to these questions are provided based on the models and their solution methodology presented in the next section.

3 Modeling

The fleet composition problem can be formulated as a multicommodity flow problem based on the construction of a space-time network, customarily used for the fleet assignment (see Berge and Hopperstad (1993), Hane et al. (1995)). The stream of arrivals and departures in the schedule is translated into activity time lines, one such line for each airport. Each leg adds its departure time to the time line of its departure airport and its arrival time to the time line of its arrival airport. At this point the arrival times incorporate also the turn-around-times resulting in actual ready-to-takeoff times, such that proper connections are established. A node in the network represents an airport during a block of time, comprising consecutive arrivals followed by consecutive departures. An arc in the network is either a flight arc between two nodes from two airports or a ground arc between two consecutive nodes from the same airport. We augment the network with one source and one sink for each time line and consider the following additional ground arcs: one ground arc from a source to the first actual node of the corresponding line, representing the initial number of aircraft at that airport (before the schedule is carried out), and one ground arc from the last actual node of a time line to the corresponding sink, representing the final number of aircraft at that airport (after the schedule is carried out). This allows for model formulations with or without restrictions on the number of aircrafts at airports at the beginning and/or at the end of the planning period. With these conventions the activity time line at an airport can be represented as in Figure 1.

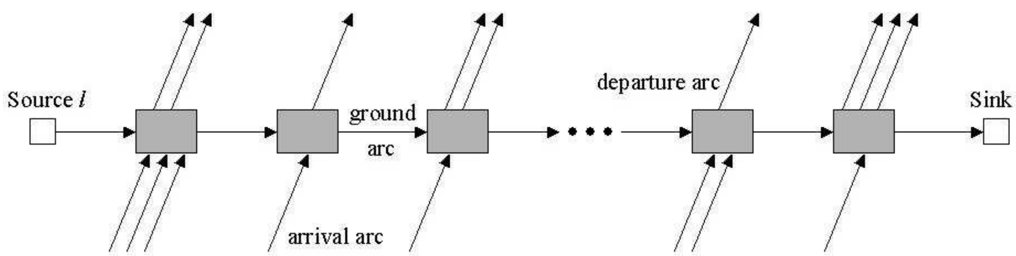

Figure 1: Time line of activity at airport l

The constructed network enables an efficient formulation of the underlying mathematical model, which is discussed in subsection 3.1. For any pre-specified set of demand values, with the associated profits for the potential assignments, the model searches for the corresponding optimal fleet composition. For instance, the expected values of demand may be used in this sense. Clearly, such an approach does not account for the impact of demand variability on the assignments profit. A reasonable alternative is to compute beforehand for any allowed assignment type/leg the expected profit based on demand distribution and to run the deterministic model with these expected profits as parameters. We will refer to this second variant as the deterministic approach to our problem. While apparently this approach uses more information from demand distributions, the profit of any potential assignment type/leg remains however a priori fixed. Thus, using only the deterministic model in either variant has the drawback that it looks for a fleet composition without reflecting the profits variability as a major determinant factor. Drawing on a single set of parameters, the deterministic approach rather corresponds to a static allocation of airline's capacity. The need arises for an approach to explicitly account for the information offered by demand distribution while searching for a fleet which is robust with respect to the variability of the actual profits. A way to achieve this is to generate a set of representative demand scenarios consistent with the known distributions and to address the composition of the fleet through a stochastic programming approach which accounts for these scenarios. Since the scenarios may be as well interpreted as multiple demand realisations over a number of (consecutive) weeks, such a stochastic approach better reflects the decision process which pursues maximum fleet flexibility for dynamic capacity allocation. The modeling of this point of view is explained in subsection 3.2, after the introduction of the deterministic model in the next subsection. Subsequently, subsection 3.3 presents a method for tackling the proposed stochastic model. The envisioned scenario generation method is described later in subsection 3.4. The last subsection of this modeling section discusses the evaluation of fleet performance and the terms of comparison between solutions.

3.1 The underlying deterministic model

The set of flight legs in the schedule is denoted by N and the set of potential aircraft types by K. For each flight $i \in N$ we denote by K_i the set of aircraft types which may perform flight i and similarly, for each type $k \in K$ we denote by N_k the set of flights which may be performed by type k. The set of airports serviced by the schedule is denoted by L. Also, we denote by V the set of all the nodes (except sources and sinks) and by G the set of all the ground arcs in the space-time network. We make a small abuse of notation for simplicity and use N to denote also the flight arcs in the network. Consequently $\mathrm{arr}(v)$ and $\mathrm{dep}(v)$ denote the flights arriving at and respectively departing from node v. In the same vein we use $l \in L$ to denote also the first actual node of the time line of airport l, such that $g|\mathrm{in}\ l$ represents the first ground arc at airport l.

The model parameters are f_k, the fixed cost of a plane type k and p_i^k, the profit of the assignment of aircraft type k to the flight leg i. The computation of revenues, costs and profit functions for the allowed assignments is discussed in the Appendix.

When type k is allowed to perform flight i, the decision variable x_i^k has value 1 if aircraft type k flies the flight leg i and 0 otherwise. For each type k, the variables y_g^k count the number of aircraft of this type on every ground arc $g \in G$. They may be defined as continuous variables, because in any solution with integral assignments x, the y variables are forced by the model formulation to be integral as well. The variables z_k represent the total number of planes type k in the fleet (also defined as continuous).

Using this notation the underlying deterministic model for the fleet composition problem states as

$$(P) \quad \max \quad \sum_{k\in K}(-f_k)z_k + \sum_{k\in K}\sum_{i\in N_k} p_i^k x_i^k$$

$$\text{s.t.} \quad \sum_{k\in K_i} x_i^k = 1 \quad \forall\, i \in N \tag{1}$$

$$y^k_{g|\mathrm{in}\ v} - y^k_{g|\mathrm{out}\ v} + \sum_{i\in\ \mathrm{arr}(v)\cap N_k} x_i^k - \sum_{i\in\ \mathrm{dep}(v)\cap N_k} x_i^k = 0 \quad \forall\, k \in K,\ \forall\, v \in V \tag{2}$$

$$z^k - \sum_{l\in L} y^k_{g|\mathrm{in}\ l} = 0 \quad \forall\, k \in K \tag{3}$$

$$z_k \geq 0 \quad \forall\, k \in K \tag{4}$$

$$y_g^k \geq 0 \quad \forall\, k \in K,\ \forall\, g \in G \tag{5}$$

$$x_i^k \in \{0,1\} \quad \forall\, k \in K,\ \forall\, i \in N_k \tag{6}$$

This formulation corresponds to a mixed integer multicommodity flow problem on the constructed space-time network, where the commodities correspond to the aircraft types. Constraints (1), called cover rows, force each flight leg to be performed by exactly one aircraft type. The balance constraints (2) assure the conservation of flow of each aircraft type at each node. Constraints (3) determine the fleet composition by counting the number of aircraft of each type on the ground before the actual schedule is carried out. They are added to the model for the clarity of the formulation and for easing some integer programming extensions. For example if the number of aircraft of a type k must be within certain limits, upper and/or lower bounds can be imposed on z_k. When all the variables z_k are fixed, this results in the fleet assignment problem for that particular fleet composition. The model may be extended to include a fixed start location of the aircrafts in a given fleet, by fixing the first ground arcs variables $y^k_{g|\text{in } l}$ for the time line of each airport l and for every aircraft type k. Moreover, if the start location and the end location of the planes must be the same, constraints equaling the first ground arc variable to the last ground arc variable for each time line and each aircraft type may be added. When at least 3 aircraft types are considered, this problem is proven to be NP-hard (see Gu et al. (1994)).

The minimum number of planes (independent of type) necessary to fly the whole schedule can be easily determined by running formally the above model with $f_k = 1$ for every k and $p^k_i = 0$ for every k and every i.

While this deterministic model and its extensions capture the basic features of the problem, it has obvious limitations when it comes to cope with fluctuating customer demands in deciding upon the fleet composition. For this purpose we present a more advanced model in the next subsection.

3.2 A robust fleet composition

As suggested previously, deciding upon a robust fleet composition can be achieved by accounting for a number of demand scenarios, which may be generated as explained in subsection 3.4. Once the uncertainty of demand is modeled by, say, S representative scenarios, one may find a solution (z_s, y_s, x_s) to the individual scenario s problem, concisely written as:

$$(P_s) \quad \max \quad f(z, y, x, s)$$
$$\text{s.t.} \quad (z, y, x) \in C_s$$

We remark that in the model formulation considered here only the objective function f depends on the scenario, the feasible set C_s is actually the same $C_s = C$ for every scenario s and it is convex and closed. A solution to (P_s) would generate a fleet composition z_s appropriate for scenario s.

When all the scenarios are considered and a probability p_s is assigned to each scenario s, we are interested in a solution of the form $(z,\ (y_s, x_s)$ for every $s)$ to the stochastic programming problem:

$$(SP) \quad \max \quad \sum_s p_s\, f(z, y_s, x_s)$$
$$\text{s.t.} \quad (z, y_s, x_s) \in C \quad \forall\, s$$

with the fleet composition z as first stage decision (it must not depend on s) and the assignments (y, x) as second stage decisions (depending on s). That is, we want to find now just one fleet composition which maximizes the expected profit over a number of possible future situations with respect to the uncertain demand. Such a solution represents a possible decision and therefore it is called an implementable solution. The difficulty is of course that problem (SP) is in general much larger than individual scenario problems (P_s) and therefore much harder to solve. Since (P_s) is already a hard problem, it's clear that (SP) can't be tackled directly, except the case when the deterministic version has a particularly limited size.

Preliminary tests in solving small stochastic fleet composition problems to integrality showed that a large number of assignments become integer in the solution of the linear relaxation of (SP), denoted by (LSP). This is in agreement with previous reports concerning the deterministic fleet assignment problem (see Subramanian et al. (1994), Hane et al. (1995), Rushmeier and Kontogiorgis (1997)), where fixing a significant part of the (integer) variables after solving the linear relaxation is an essential step in the solution methodology. Moreover, when we solve the stochastic problem our interest is mainly focused on the fleet composition, that is on the first stage decisions, which are by the nature of this problem very few (the number of potential aircraft types is limited). Yet they depend on the large number of second stage variables (the potential assignments) in such a way that expectedly the contribution of few (fractional) assignments to the determining of the whole fleet composition is quite minor. Therefore a solution to (LSP) will already give good insight into the candidate integer configurations for a robust fleet. This encouraged us to pursue the strategy of finding first a solution to the linear relaxation (LSP) of the stochastic problem and then to use a simple rounding procedure to generate integer fleet compositions. As practically also (LSP) can't be tackled directly due to its overwhelming dimensionality and computer memory requirements, we resorted to the scenario aggregation technique described in the next subsection.

3.3 The scenario aggregation based approach

Scenario aggregation was developed as a decomposition-type of method for multi stage stochastic programming problems (see Rockafellar and Wets (1991), Wets (1989)), which is not directly related to the well-known Dantzig-Wolf decomposition principle. The idea is to aggregate successive solutions of perturbed scenario problems in successive overall solutions that are implementable and converge to the solution of the stochastic problem. This technique gives a reliable mathematical basis for solutions derived from individual scenarios and can be applied to linear problems in order to improve on pure scenario analysis. We apply it here in order to find (a good estimate for) the solution to (LSP), as the first step suggested above.

3.3.1 The scenario aggregation algorithm

First we describe the algorithmic part of the method and then we formulate the comments which are in place. The principal set-up of the scenario aggregation algorithm for the (linear relaxation of the) stochastic fleet composition problem states as follows:

Step 0. Set $\hat{z}^0 = 0$ and $\hat{y}_s^0 = 0,\ \hat{x}_s^0 = 0$ for every s.
Set $w_s^0 = 0$ for every s.
Choose $\rho > 0$ and set $\nu = 1$.

Step 1. For each scenario s, solve the perturbed scenario problem

$\max f^\nu(z, y, x, s)$ subject to $x \in C$

where

$$f^\nu(z, y, x, s) = f(z, y, x, s) - w_s^{\nu-1} z - \tfrac{1}{2}\rho \,||(z, y, x) - (\hat{z}^{\nu-1}, \hat{y}_s^{\nu-1}, \hat{x}_s^{\nu-1})||^2$$

Let $(z_s^\nu, y_s^\nu, x_s^\nu)$ denote the solution vector.

Step 2. Calculate $\hat{z}^\nu = \sum_s p_s z_s^\nu$ and set $\hat{y}_s^\nu = y_s^\nu\ ,\ \hat{x}_s^\nu = x_s^\nu$.
For every s, update the perturbation term

$$w_s^\nu = w_s^{\nu-1} + \rho(z_s^\nu - \hat{z}^\nu)$$

Return to Step 1 with $\nu = \nu + 1$.

At each iteration $\nu = 1, 2, ...$ one generates an admissible decision $(z_s^\nu, y_s^\nu, x_s^\nu)$ for each scenario s, as a solution to the perturbed problem for scenario s having as objective the augmented Lagrangian $f^\nu(z, y, x, s)$. These solutions are blended into an implementable solution $(\hat{z}^\nu, (\hat{y}_s^\nu, \hat{x}_s^\nu)$ for every s), which is not necessarily admissible, in the sense that the "assignments" $\hat{y}_s^\nu$ and $\hat{x}_s^\nu$ are not necessarily feasible for the "fleet" $\hat{z}^\nu$ in scenario s. Besides the multipliers $w_s^{\nu-1}$ and the fixed parameter ρ corresponding to the quadratic term, the augmented Lagrangian $f^\nu(z, y, x, s)$ also involves the implementable solution $\hat{z}^{\nu-1}$ and $\hat{y}_s^{\nu-1}, \hat{x}_s^{\nu-1}$ resulted from the previous iteration. On the basis of scenario solutions and the aggregated solution, the multipliers w are updated for the next iteration. These multipliers are interpreted as information prices that can be associated with the implicit constraints that the feasible solutions must be implementable, that is the individual scenario solutions must generate afterwards the same fleet composition. What typically happens is a "fight" between the scenario solutions z_s^ν and the aggregated solution $\hat{z}^\nu$, the individual solutions trying to pull away from the implementable one. This tendency is "corrected" by updating w multipliers and when they become properly adjusted the scenario solutions will agree with the implementable solution. The stopping criteria must reflect a measure of this agreement. We use in this sense the conditional variance of the error with respect to z variables, $\theta_\nu := \sum_s p_s ||z_s^\nu - \hat{z}^\nu||^2$. According to the convergence results in Rockafellar and Wets (1991), θ_ν converges to 0, so the algorithm may stop when $\theta_\nu \le \epsilon$ for a given tolerance $\epsilon > 0$. In the fleet composition problem, such a tolerance may be expressed as a given small percentage of the minimum total number of planes.

So the scenario aggregation algorithm generates a sequence $\{\hat{z}^\nu\ ,\ \nu = 1, 2, ...\}$ of estimates of the optimal first stage decision $z^\star$ of the relaxed stochastic problem (LSP), by insisting progressively that the scenario solutions must be implementable, that is they produce the same fleet composition. A great advantage of this approach in our case is that we can capture already at an early stage the direction in which the sequence $\{\hat{z}^\nu, \nu = 1, 2, ...\}$ moves. Moreover, since we are actually interested in an integer solution to (SP), it becomes clear that we don't need at all to pursue the search for an optimal solution of the relaxation to the end, but rather to stop with a reliable estimate, which we have always at hand in the last $\hat{z}^\nu$ generated.

3.3.2 A rounding procedure

Although the best available estimated first stage solution to (LSP) consists of fractional values, through the progressive hedging effect of the scenario aggregation algorithm it provides already good insight into the candidate integer fleet configurations to be considered. Suppose that $z = (z_1, z_2, ..., z_m)$ is a fractional first stage solution with $\sum_{k=1}^{m} z_k = M$, where M denotes the (constant) total number of planes. For any real number u we denote by $[u]$ the integer part of u, that is the largest integer smaller then or equal to u and by $\{u\}$ the fractional part of u, that is $\{u\} = u - [u]$. Suppose c is a constant between 0 and 0.5. For each $k = 1, ..., m$, z_k can be rounded to an integer a_k as follows: if $\{z_k\} < c$ then $a_k = [z_k]$; if $\{z_k\} > 1 - c$ then $a_k = [z_k] + 1$; if $c \leq \{z_k\} \leq 1 - c$ then $a_k = [z_k]$ or $a_k = [z_k] + 1$. The constant c can be defined as a value deemed relevant for the structure of z (for instance 0.2 or 0.25 appear to work well in most cases). The higher is c, the fewer rounding possibilities will result and vice versa. We consider all the integer vectors $a = (a_1, a_2, ..., a_m)$ which result as possible combinations of these individual a_k, $k = 1, ..., m$, such that $\sum_{k=1}^{m} a_k = M$. Clearly, some of the vectors a represent a rounding of z which are intuitively more justified than others. Therefore we order these integer vectors a in increasing order of the euclidian distance to z. Our typical experience when evaluating these potential integer fleet compositions over the scenarios is that only a limited number of configurations from the beginning of the list give a significant improvement of the total expected profit. Moreover, as we go further in the list the total expected profit decreases considerably. So the number of fleet compositions from the list to be checked can be decided (or alternatively pre-specified) in each case at hand, based on its characteristics and practical considerations. From the evaluated configurations we retain that fleet composition which generates the maximum expected profit over the scenarios and we refer to it as the solution of the scenario aggregation based approach.

3.4 Scenario generation

The random demand parameters are originally assumed to follow continuous (independent) distributions. However, the modeling needs to reflect the dynamic interaction between actual demand values and the aircrafts capacities. In order to achieve this through a reduced but yet representative set of scenarios, we select the demand realisations and their mutual combinations by using the descriptive sampling method (Saliby (1990), Jönsson and Silver (1996)). Descriptive sampling is based on a purposive selection of the sample values - aiming to achieve the closest fit with the represented distribution - and the random permutations of these values. It is therefore relevant for problems where the sample sequence plays a major role, such as in our situation.

Suppose for simplicity that only one payload class is available in each aircraft type. The demand for seats of each flight leg $i = 1, 2, ..., N$ is assumed to follow a normal distribution $d_i \sim N(\mu_i, \sigma_i)$ with probability distribution function F_i (the demands are assumed to be independent). We specify in advance the number of scenarios we wish to generate, say S. Then exactly S values $d_i[1]$, $d_i[2]$,..., $d_i[S]$ are sampled from distribution i and they are set at equally spaced quantiles of the distribution, that is

$$d_i[j] = F_i^{-1}(\frac{j - 0.5}{S}), \qquad j = 1, 2, ..., S$$

The idea is that in this way we generate more sample values from a range where the distribution has higher density and less values from low density regions. Since the inverse of the distribution function F_i is not available analytically, we use accurate numerical approximations generated with the Newton-Raphson method. Subsequently, a random permutation of the values $d_i[j],\ j = 1, 2, ..., S$, is generated for each $i = 1, 2, ..., N$. Then each vector $(d_1[j], d_2[j], ..., d_N[j]),\ j = 1, 2, ...S$ represents a scenario which is assigned probability $\frac{1}{S}$. So we maintain the sample variability by a random combining of the S values of each distribution with each other. This is of particular interest in the fleet composition problem since the dynamic allocation concept tries to improve marginal profits based on adjusting the available capacity to the demand fluctuations on connecting flights.

When two payload classes (economy, business) are considered for each aircraft type, values can be generated by descriptive sampling for each class, either assuming that the two classes are independent or assuming some dependency between them (for instance, positive correlation).

3.5 Fleet performance evaluation

The evaluation of fleet performance can be achieved by simulating demands with random sampling from the given distributions, assigning the fleet to the schedule in the optimal way for each drawn set of values and finally calculating its average scores. Namely, we record the average estimates for the following performance indicators: the load factor, the spill percentage, the total revenues, the total operational costs and the total profit (which also accounts for the fixed costs of the component aircrafts). In order to reflect the fleet capabilities for dynamic use, we place the conceptual evaluation flow above in two specific settings. The first variant aims to assess the generic fleet flexibility with respect to demand fluctuations as resulting from its capacity distribution only, irrespective to the family affiliation of its aircrafts. The second setting focuses more specifically on the fleet interchangeability within families in order to adjust its capacity to the actual demands. We describe subsequently either variant in turn.

The generic fleet flexibility is evaluated as follows. We make a number of random draws for demand values and at every draw the fleet is completely reassigned to the schedule in the best possible way. By complete reassignment we mean that there are no constraints related to the start location or the family of any plane and the assignment at each draw is made independently. The average indicators recorded with such a scheme can give reasonable insight into the appropriateness of the fleet capacity distribution among types for the typical demand variations in the schedule.

The fleet interchangeability within families has to be assessed relatively to an existing fleet assignment. Therefore we make first one random draw of demands and record the optimal fleet allocation based on these drawn values. We refer to this as the fixed assignment of the given fleet. Subsequently we make a number of random draws. At each draw the fleet is again reassigned in the best way to the schedule, but subject to the following extra constraints: 1) the start location of the tails is identical with the one in the fixed assignment; 2) an aircraft type k is allowed to perform a flight leg i only if leg i is flown in the fixed assignment by a type $k_0(i)$ belonging to the same family as type k. Given these extra constraints as well as the original flow conservation constraints, the reassignment of the fleet generates in this case actual swaps of its planes within families, relatively to the fixed assignment, in such a way that the overall profit is maximized for each drawn set of demand values. The undertaken steps admit the following interpretation. The fixed assignment corresponds to an initial capacity allocation for the index week, based on the forecasted demands (cast by the first draw) at a relevant planning point in time, preceding the week's operations. As this initial capacity assignment also determines the scheduling of the crews, whose dynamic assignment would be both difficult and expensive, it is required that the actual operation of each flight to be done by an airplane belonging the same family the assigned crew is certified to fly. As the time comes closer, more accurate information about the actual demands is accumulated and the initially assigned capacity is to be adjusted. Each subsequent draw captures a possible state of the world shortly before the index week operation. Where possible, the planes are swapped in order to better match their capacities to the actual demands, increase the passenger loads, decrease the spill and improve the operating profits.

In either case, the fleet composition given by the deterministic approach based on expected profits (EP) and the fleet composition given by the scenario aggregation based approach (SA) can be compared on the basis of the average performance indicators achieved.

4 Implementation issues

The numerical analysis of the case studies was performed on a Windows NT-based 933MHz Pentium III PC with 256MB RAM using our own FleetComp suite of C applications with the CPLEX Callable Library version 7.1 (see ILOG (2000)). The components of the FleetComp suite are schematically illustrated in Figure 2.

Based on the data red in the *Input* module, the *DynNetGen* module generates the dynamic space-time network of flights as described in the Modelling section, which further serves for all models formulations within *FleetNet, FleetSA* and *FleetSim* modules. The *fleetnet* application can address the deterministic model either based on profit parameters corresponding to particular demand values or based on the expected profits for each allowed type/leg combination. It can be run with the fleet composition to be determined as well as with a pre-specified fleet configuration. The *fleetstoch* application solves the stochastic model in extensive form; it is only useful for small cases in order to validate the scenario aggregation based method.

The main scenario aggregation algorithm is implemented in the *fleetsa* application. In this context, two issues need to be clarified. The first is the choice of the ρ perturbation parameter. Following the argument in Rockafellar and Wets (1991) and after appropriate numerical experiments, we decided to use low values between 50 and 100 for ρ in order to encourage progress in the primal sequence $\{\hat{z}^\nu\}$ (instead of the dual sequence $\{w^\nu\}$). The second issue concerns the value of the ϵ tolerance for the θ_ν convergence measure. We

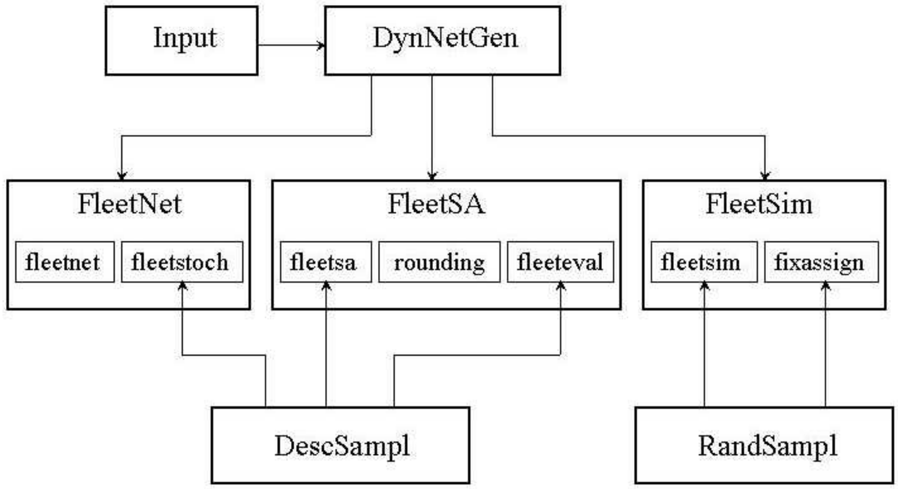

Figure 2: FleetComp suite components

set this tolerance to a small percentage (3%) of the minimum total number of planes in each particular case. This means for instance that if a total of 100 planes were required, the scenario aggregation procedure would stop when the sum of the deviations of all (first stage) scenario solutions from the implementable solution (weighted by scenario probabilities) is no more than 3. Although not our main concern here, the scenario aggregation algorithm greatly facilitates parallel computation, such that its execution can potentially be spread out to utilize all available computational power, leading to substantial running time reduction. The *rounding* application implements the rounding procedure with an adjustable c rounding constant. The candidate fleets from the resulted ordered list are passed further for evaluation over scenarios to the *fleeteval* application. The advantage of using *fleeteval* is that it evaluates a given configuration over the descriptive sampling based scenarios, which are more limited, and it avoids therefore the application of the computationally much more expensive simulation too many times. This way, the *fleetsim* application can be finally used to assess the actual performance of few fleets with typical characteristics. It can address the complete reassignment studies as well as the plane swapping studies starting from a fixed assignment generated by the *fixassign* application.

The perturbed scenario problems within the scenario aggregation procedure take the form of concave quadratic programming problems and are solved using CPLEX Barrier Optimizer. The other model based applications make use of the branch-and-cut algorithm exploited by the CPLEX Mixed Integer Optimizer with several tuning options, which are specified in turn below.

In order to avoid overly tight optimality criteria and speed up the computation process, we used relative mip gap tolerances between 0.02% and 0.04%. The presolver and aggregator were set on at all the times. The setting we found most robust uses the CPLEX Barrier LP solver for the relaxation at the root of the branch-and-bound tree followed by dual crossover for obtaining an optimal basis, before entering the branching phase. Often CPLEX automatically performed objective costs perturbation in order to avoid dual degeneracy. Moreover, in our experiments CPLEX fixed a significant number of integer variables after solving the root relaxation and before performing the crossover. Furthermore, the heuristic supported by CPLEX Mixed Integer Optimizer seemed to be very effective, especially when invoked at the root, after an optimal basis was found. For the LP relaxations at nodes the experiments led us to finally choose the dual simplex solver with the steepest edge pricing strategy.

An option to use the assignment constraints as prioritized Type I Special Ordered Sets (SOS) was implemented. This option could complementarily improve performance on many fleet assignment instances, especially where the default branching rules required more time. For its implementation the aircraft types were sorted in increasing order of their total capacity and an initial priority was computed for each assignment constraint i as $\text{prior}(i) = \sqrt{\sum_{k \in K_i} (p_i^{k_a} - p_i^k)^2}$, where k_a is the type preceding type k allowed to fly leg i and the summation starts with the second type in K_i. That is, prior(i) gives a measure of variability in the objective coefficients corresponding to leg i. The interval between the minimum and the maximum initial priority was then divided in a number of equal intervals and legs belonging to the same interval were assigned the same (final) priority. The number of priority classes is easily adjustable through the program. The node selection strategy we chose emphasized feasibility and preferred more recently created nodes until an integer feasible solution was found. An upper limit (500) was set on the number of nodes in the tree, but usually the tree was pruned without exhausting it.

Of great effectiveness during the branching phase appeared to be the *Gomory fractional cuts*. Therefore they were often encouraged at the nodes of the tree. Actually, on the type of mixed integer programming models involved, they made a great difference in the CPU-time between CPLEX software release 7.1 we finally used and our trials with the previous versions. Finally, we would remark that especially the fleet assignment model exploited in *fleeteval* and *fleetsim* showed a great variety of instances resulting from various combinations of a given fleet with particular demand realizations, such that encountering also some problematic instances was inevitable. In spite of this, the tuning options chosen provided good trade-offs for most instances, resulting in reasonable overall running times.

5 Case study results

The benefits of the presented method were established through application to several case studies based on realistic data, set up in agreement with ORTEC airline consultant. We summarize these benefits by discussing two representative cases: a small case in which we validated the method and a large case which better shows the extent to which our method improves on the deterministic approach.

For simplicity, we assume in both cases the same K-factor for all flight legs, specific only to each fare class, namely 0.5 for economy and 0.6 for business. The yield multiplier and the yield exponent (see appendix) equal 1.7 and 0.35 for the business class, respectively 1.5 and 0.40 for the economy class. Up to 9 aircraft types from 3 families A, B, C, denoted by A1, A2, A3, A4, B1, B2, C1, C2, C3, were considered. They range in capacity from 70 to 175 seats with 40% business seats and 60% economy seats. A minimum turn around time of 25 minutes was considered for all aircraft types at all airports.

Initially, it was thought that in principle increasing the number of scenarios would generally produce significantly better results. Therefore, we experimented with stochastic models based on 50, 80 and respectively 100 scenarios in the case presented in the next subsection. Such experience reveals that a reasonable number of scenarios generated by descriptive sampling suffice for capturing demand variations which actually impact the fleet composition and no significant gain results from excessively increasing the number of scenarios relative to the corresponding computational effort.

5.1 A small case and method validation

The low sized hub-spoke system considered in this case provided early feedback to validate the solution method. The network consists of 342 flight legs per week serving 18 airports with a fleet of 15 airplanes. The stochastic models discussed in this case are based on a number of 50 scenarios. In both studies presented below the scenario aggregation based approach generated a fleet composition which turned out to be the optimal (first stage) solution of the stochastic model, as verified by solving the deterministic equivalent to optimality. Moreover, most of the alternative fleets from the top of the list constructed by our method generated profits close to the optimal when evaluated over the scenarios.

5.1.1 Generic fleet flexibility study

For this study all the 9 aircraft types were considered. This setting translates into a deterministic model with 5,068 variables, 2,430 constraints and 12,783 non-zero's, whose solving required 2 seconds. The corresponding stochastic model with 50 scenarios has 252,959 variables, 121,500 constraints and 639,150 non-zero's. Solving its extensive form to optimality required almost 2 hours of computation. By comparison, the scenario aggregation procedure stopped after 12 minutes by satisfying the stopping criterion and the rounding procedure generated 10 candidate fleet compositions, each of them requiring approximately 1 minute for evaluation over scenarios. The third fleet from the list turned out to be the optimal one. The expected profits generated by the first 5 candidate configurations from the top of the list were significantly better than those generated by the last 3 fleets in the list. The fleet composition given by the deterministic approach (EP) and the fleet composition generated by the scenario aggregation based approach (SA) are given in Table 1.

	Aircraft types								
Fleet	A1	A2	A3	A4	B1	B2	C1	C2	C3
EP	1	0	5	0	6	0	2	1	0
SA	2	2	1	2	4	1	1	1	1

Table 1: Fleet composition with 9 aircraft types (small case)

The performance of each of these configurations was established through a simulation run with 200 draws and complete reassignment, requiring 3 minutes (EP), respectively 5 minutes (SA). Their average performance indicators based on *weekly* figures are presented in Table 2.

In the EP fleet some aircraft types (such as A3 and B1) are preferred, because their capacities render themselves more profitable when related to the (fixed) expected profits of the flight legs. However, when actual varying profits are cast in scenarios and the

	EP fleet	SA fleet	SA – EP (% of EP)
Load factor (%)	67.34	68.97	1.63
Spill (%)	6.04	3.64	−2.40
Revenues($)	2,543,799	2,584,269	40,470 (1.59%)
Operating costs($)	1,487,056	1,498,223	11,167 (0.75%)
Fleet cost($)	915,500	928,500	13,000 (1.42%)
Profit($)	141,243	157,546	16,303 (11.54%)

Table 2: Fleet performance with 9 aircraft types and complete reassignment (small case)

overall expected profit over these scenarios is aimed to be maximized, these types are partly replaced in the SA fleet by several other types with various capacities. This change results in 1.4% increase in the fixed costs of the planes and likewise, a relatively small increase in operating costs. However, the SA fleet generates a higher average load factor with an impressive simultaneous decrease in the average spill, accounting for a much more significant increase in revenues. This increase not only covers the extra investment and operational costs, but moreover, it makes a substantial bottom line contribution in such a way that the SA fleet achieves overall 11.54% improvement in the average total profit. Translating this improvement to a yearly basis would result in about $56,500 added per airplane per year.

5.1.2 Fleet interchangeability within families

Since aircraft types from different families can hardly be swapped without directly impacting crews rosters, it is assumed that when fleet interchangeability is aimed for, aircraft types from fewer families would consequently be acquired. Therefore only the 6 aircraft types from A and B families were considered in this study and planes were allowed to be exchanged only within family as explained in the Modeling section. The deterministic model with 3,340 variables, 1,734 constraints and 8,406 non-zero's required 1 second for solving in this case. The stochastic model with 50 scenarios has 166,706 variables, 86,700 constraints and 420,300 non-zero's and was solved to optimality (in its extensive form) in 21 minutes of computation. The scenario aggregation procedure required 8 minutes. Subsequently, 9 candidate fleet configurations were generated by the rounding procedure, each fleet requiring about 30 seconds in order to be evaluated over scenarios. The second fleet in the list turned out to be the optimal one. The first 4 fleets from the top of the list generated significantly better total expected profits than the last 3 fleets in the list. The EP fleet and the SA fleet compositions are presented in Table 3.

The performance of each of these fleet compositions was established through a simulation run with 200 draws and plane swapping relative to a fixed assignment, a priori generated by one draw. The simulation runs required 1 minute for the EP fleet, respectively 1 1/2 minutes for the SA fleet. The average performance indicators recorded are given in Table 4 (*weekly* figures).

Fleet	Aircraft types					
	A1	A2	A3	A4	B1	B2
EP	3	1	5	0	6	0
SA	3	2	2	2	4	2

Table 3: Fleet composition with 6 aircraft types (small case)

	EP fleet	SA fleet	SA – EP (% of EP)
Load factor (%)	65.76	67.10	1.34
Spill (%)	6.87	4.93	−1.94
Revenues($)	2,496,191	2,529,469	33,278 (1.33%)
Operating costs($)	1,481,805	1,493,187	11,382 (0.77%)
Fleet cost($)	913,000	924,000	11,000 (1.20%)
Profit($)	101,386	112,282	10,896 (10.75%)

Table 4: Fleet performance with 6 aircraft types and plane swapping (small case)

The EP fleet with 6 types rather little differs from the EP fleet with 9 types and it is still based on aircraft types tailored on the (fixed) expected profits of the flight legs. The 3 planes of C family from the SA fleet with 9 types are replaced in the SA fleet with 6 types by 2 different planes of A family and 1 plane of B family. The increase in operational costs is roughly the same as in the previous case, but the difference in fixed costs between the SA fleet and the EP fleet is somewhat smaller in this case. However, while there are only limited potential swapping possibilities within the planes of A family in the EP fleet, the SA fleet composition offers clearly more potential swapping opportunities within both A and B families. These differences are directly reflected in the average performance indicators achieved: the higher load factor and the lower spill of the SA fleet translate into a significant revenues increase, which covers the extra costs and contributes further to the bottom line for an overall 10.75% improvement in the average total profit. On a yearly basis this improvement would add about $37,800 per airplane per year.

5.2 A case study on a large network

The large network with multiple hubs addressed in this case allows a better assessment of the benefits of our method as compared with the deterministic approach. The system operates 1978 flight legs per week, serving 50 airports with a total of 68 planes. The stochastic models applied in this case are based on 25 scenarios.

5.2.1 Generic fleet flexibility study

For this study we considered again all the 9 aircraft types. The deterministic model contains in this case 27,078 variables, 11,806 constraints and 70,497 non-zeros. Its solving required 2 minutes. The stochastic model with 25 scenarios would consist of 676,734 variables, 295,150 constraints and 1,762,425 non-zeros. Clearly, such a large-scale model can not be tackled directly, but the scenario aggregation algorithm generated a (fractional) first-stage estimated solution within the prescribed accuracy in 4 1/2 hours of computation. The rounding procedure generated 12 integer fleets, whose evaluation over scenarios required on average 10 minutes per fleet. The first fleet from the list produced the highest profit over scenarios and was retained as the scenario aggregation based solution. However, we have to remark that in this case, owing to more flexibility conferred by the larger total number of planes, as many as the first 7 candidate configurations from the top of the list generated comparable expected profits over scenarios (in a range from 0.05% to 0.1% less than the best of them). The EP fleet composition and the SA fleet composition resulted in this study are given in Table 5.

	Aircraft types								
Fleet	A1	A2	A3	A4	B1	B2	C1	C2	C3
EP	6	13	7	0	15	0	20	7	0
SA	10	6	8	6	16	5	8	6	3

Table 5: Fleet composition with 9 aircraft types (large case)

The two fleet compositions were compared by means of a simulation run with 75 draws, with complete reassignment at each draw. The simulation run required 3 hours 40 minutes for the EP fleet, respectively 12 hours 20 minutes for the SA fleet. The corresponding average performance parameters (*weekly* figures) are presented in Table 6.

	EP fleet	**SA fleet**	**SA – EP (% of EP)**
Load factor (%)	68.22	70.81	2.59
Spill (%)	6.78	3.46	−3.32
Revenues($)	13,960,397	14,268,352	307,955 (2.21%)
Operating costs($)	8,749,105	8,878,502	129,397 (1.47%)
Fleet cost($)	4,186,500	4,216,000	29,500 (0.70%)
Profit($)	1,024,792	1,173,850	149,058 (14.55%)

Table 6: Fleet performance with 9 aircraft types (large case)

The larger scale network from this case offers more opportunities to exploit the advantages of dynamic allocation. Although these network opportunities would potentially favour both fleets, the SA fleet clearly proves itself more appropriate for dynamic use, as reflected by the almost 15% increase in the average total profit when compared with the EP fleet. Here again the improvement is achieved with an expanded fleet, but which incurs in this case a smaller relative extra investment and a somehow larger relative increase in operating costs. In exchange, the SA fleet capacity is distributed over all aircraft types, including significant number of planes from types which are totaly absent from the EP fleet.

Through this typical adjustment, the SA fleet more effectively matches its capacity to the various demands. Therefore it considerably increases the overall load factor and reduces the average spill in an even more impressive manner. This way it accounts for a revenues increase which contributes with almost $150,000 added to the bottom line average profit per week. On a yearly basis this would add $114,000 per airplane per year.

5.2.2 Fleet interchangeability within families

For this plane swapping study we restrict again the aircraft types to the A and B families. The deterministic model contains in this case 17,816 variables, 8,530 constraints and 46,290 non-zeros and was solved in 30 seconds. The stochastic model based on 25 scenarios would consist of 445,256 variables, 213,250 constraints and 1,157,250 non-zeros. The estimated first-stage solution to this model was generated by the scenario aggregation procedure within 2 hours of computation. The 8 candidate configurations subsequently given by the rounding routine required for evaluation over scenarios in average 5 minutes per fleet. The second fleet from the top of the list generated the highest expected profit over scenarios and was retained as the SA fleet. Table 7 illustrates this fleet composition as well as the EP fleet resulted in this case.

	Aircraft types					
Fleet	A1	A2	A3	A4	B1	B2
EP	22	24	7	0	15	0
SA	15	13	11	7	16	6

Table 7: Fleet composition with 6 aircraft types (large case)

The fleets were evaluated through a simulation run with 75 draws, where the plane swapping setting was applied. The simulation run required 7 minutes for the EP fleet, respectively 20 minutes for the SA fleet. In Table 8 the resulted average performance indicators are given (*weekly* figures).

	EP fleet	**SA fleet**	**SA – EP (% of EP)**
Load factor (%)	66.58	68.26	1.68
Spill (%)	7.42	5.08	−2.34
Revenues($)	13,497,134	13,774,628	277,494 (2.06%)
Operating costs($)	8,698,664	8,849,680	151,016 (1.73%)
Fleet cost($)	4,181,000	4,232,000	51,000 (1.21%)
Profit($)	617,470	692,948	75,478 (12.22%)

Table 8: Fleet performance with 6 aircraft types and plane swapping (large case)

The multiple hubs system addressed in this case involves situations where more planes belonging to the same family are simultaneously on the ground at a hub airport and the plane swapping is more prevalent. While swapping opportunities are restricted to 3 types from A family in the EP fleet, the capacity distribution of the SA fleet enables it to more effectively profit from swapping combinations within both A and B families. Besides the expectable capital investment increase, this change also incurs a higher percentage increase in operational costs in this case. However, these extra costs are by far compensated by the over 2% increase in the average revenues, mainly based on spill reduction, but also on significant load factor increase. Moreover, the revenues increase accounts further for over 12% increase in the bottom line profit. Translating this extra profit to a yearly basis would result in $57,700 added per airplane per year.

6 Conclusions

The investigations presented in this paper emphasize that the stochastic nature of passenger demands should be explicitly taken into account in the airline fleet composition problem, approached from the perspective of applying a dynamic allocation of its capacity to the flight schedule. Acquiring suitably distributed aircraft capacity, depending on the involved network and its dynamics, is crucial for the successful implementation of the dynamic allocation concept. From this point of view a stochastic approach as ours can generate significantly more robust solutions than deterministic formulations. Given its balanced search between representative demand scenarios, this approach is able to detect situations where it is more profitable to expand the fleet as well as cases where the fleet based on deterministic estimates should be actually downsized in order to increase profitability in a dynamic environment. Therefore the scenario aggregation based approach properly quantifies the effects of fluctuating passenger flows on the fleet planning process, generating flexible fleet configurations which better support dynamic assignments. Such robust compositions showed in our case studies a potential increase of the load factors up to 2.6%, with a simultaneous potential spill decrease up to 3.3%. Moreover, our approach can find an appropriate fleet composition in order to facilitate interchanging of planes within families. A significant pay-off would be achieved with such a fleet if the plane swapping concept was applied. In such settings our results show up to 1.7% higher load factors and up to 2.3% less turned away passengers. Given the typically low operating profit margins from the total operating revenues, such improvements can lead to a substantial increase in the bottom line profits (between 10% and 15% in the presented cases).

Besides the clear utility of the scenario aggregation based approach, the feasibility of its implementing has also been proven using realistic data. Although the primary objective of our implementation was to build a tool in a proof-of-approach sense, the solution procedures performed very well for models involving up to 2000 flights and 9 aircraft types and there is clear indication of their applicability to even larger instances. In such cases, we are confident that further improvement in the efficiency of the various routines can be achieved through motivated future research. Moreover, this methodology offers great opportunity for parallel computations, which could dramatically impact the overall running times, bringing it even closer to a point of potential integration into a practice-oriented decision support system.

Acknowledgement

We would like to thank the experts from ORTEC Consultants B.V., The Netherlands, for providing us data and insight into the fleet composition problem.

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Appendix

The revenues, costs and profit functions

The profit parameters p_i^k used in the description of the underlying model depend on the capacity of the aircraft type k and on the customers demand for seats for the flight leg i, as well as on the operational costs incurred by the assignment of type k to flight i. We use the following functions:

$$p_i^k = \sum_j r_i^{kj} - c_i^k$$

$$r_i^{kj} = rp_i^j \times \min(dem_i^j, cap_k^j)$$
$$rp_i^j = m_j \times (d_i)^{1-e_j}$$
$$c_i^k = cc_k + cg_k \times d_i$$

where

p_i^k	=	profit of assigning aircraft type k to flight leg i
r_i^{kj}	=	revenue of flight i from payload class j when carried out by type k
c_i^k	=	operational costs of performing flight i by type k
rp_i^j	=	revenue per passenger in payload class j of flight i
dem_i^j	=	demand for seats class j for flight i
cap_k^j	=	capacity for class j of aircraft type k
m_j	=	yield multiplier of class j
e_j	=	yield exponent of class j
d_i	=	distance of flight leg i
cc_k	=	constant costs of using aircraft type k on one flight leg
cg_k	=	variable costs of using aircraft type k per unit distance
j	=	payload class index (economy, business)

When the demands for seats dem_i^j follow normal distributions $N(\mu_i^j, \sigma_i^j)$ (truncated at zero) the expected profit of assigning aircraft type k to flight leg i is given by

$$\mathbb{E}[p_i^k] = \sum_j \mathbb{E}[r_i^{kj}] - c_i^k$$

$$\mathbb{E}[r_i^{kj}] = rp_i^j \times (\, \mathbb{E}[dem_i^j \mid 0 \le dem_i^j \le cap_k^j] \; + \; cap_k^j \times \mathbb{P}(cap_k^j < dem_i^j) \,)$$