Risk-based stock decisions for projects

Willem van Jaarsveld and Rommert Dekker*

Econometric institute, Erasmus University Rotterdam,
P.O. Box 1738, 3000 DR Rotterdam, The Netherlands

Report Econometric Institute EI 2009-02

December 2008

Abstract

In this report we discuss a model that can be used to determine stocking levels using the data that comes forward from a Shell RCM analysis and the data available in E-SPIR. The model is appropriate to determine stock quantities for parts that are used in redundancy situations, and for parts that are used in different pieces of equipment with different downtime costs. Estimating the annual production loss using the model consists of a number of steps. First, we need to determine which spares are used for the repairs of which failure modes. In the second step, we estimate the average waiting time for spares as a function of the number of spares stocked. In the third step, the annual downtime costs are determined. We combine the downtime costs with the holding costs to determine the optimal number of parts to stock.

* Corresponding author: E-mail: rdekker@few.eur.nl

Contents

1 Introduction

In this report we will discuss how the information from E-SPIR 2000 can be combined with the information from a Shell Reliability Centered Maintenance (S-RCM) study to give recommendations for minimum stock level provisioning for spare parts of equipment.

E-SPIR 2000 (Electronic Spare Parts and Interchangeability Record) is a software package for obtaining spare parts information from equipment suppliers in a standard format. Information about the price, the leadtime, and the equipment for which the part is needed is contained in the E-SPIR database. This information streamlines the initial spare parts provisioning for newly built facilities.

Initial spare parts provisioning is important, because spares are needed for efficient operation of the plant. When equipment breaks down, the downtime can be significantly reduced if all spares needed for repair are available. If on the other hand spares are not available, the waiting time for the spares can cause costly production losses. Because the costs of keeping spare parts can be high[1], it is not obvious whether we should keep stock - either how many - to avoid downtime, or whether we should refrain from keeping stock to avoid holding costs. There is a formula available in E-SPIR that helps to answer this question (Trimp, Sinnema, Dekker and Teunter 2004). The main ingredients of this formula are price, leadtime, usage and downtime costs. While price and leadtime are available in the E-SPIR, the usage and in particular the downtime costs are not always available to a spare parts reviewer. Moreover, the spare part may be used in multiple equipment in the plant, and the downtime often depends on the precise piece of equipment that breaks down. We conclude that we need a way to determine downtime costs sufficiently accurate.

Shell RCM is a structured approach to ensure that all available data and knowledge is used to arrive at an optimum maintenance regime (Vago, Macki, Festen, Lorenz, Vonk, Slangen, Wiegerinck, Woltman and Wisman 2004). While Shell RCM is mainly focused on optimizing the frequency of maintenance activities, the information that is gathered during the study can be very useful for determining stock quantities as well. In particular, a Shell RCM study gives production loss equations for the different failure modes of equipment in the plant. These production loss equations are closely related to the downtime costs of the equipment.

Shell RCM enables us to use downtime costs that are based on a thorough analysis of people that are actually working with the equipment. We however also find that the downtime costs in Shell RCM consist of more than a single number. Every equipment in which the spare part is used is a potential source of downtime costs. There is another complication, which is redundancy. When there are two pieces of equipment, of which only one is needed to keep the plant running, there is redundancy in the system. The current model used in E-SPIR is unappropriate when multiple equipment have different downtime costs or when there is redundancy in the system.

In this report we will describe a model that can be used to give recommendations for stock quantities using the information from E-SPIR and Shell RCM. In section 2 we discuss the collection of the data that is needed to apply the model to determine minimum spare parts stock levels. In section 3 the model will be described, and we give formulas that can be used to calculate estimated downtime costs and holding costs for different stock levels. In

[1] At Shell, a holding cost fraction of 25% is often used.

section 4 we show how to calculate optimal spare parts stock levels. In section 5 we will give a recommendation for order quantities. We conclude in section 6.

2 Data collection

2.1 Spare parts packages

Spare parts are used when repairing equipment. A specific piece of equipment in the plant is called a **tag** in this document. When optimizing the number of spare parts that is to be kept on stock, it is relevant to look at the tags in which the particular spare part is used. This is possible, because in E-SPIR this information is given. Then we need to determine in what **failure mode** that particular spare part is used. A failure mode is a particular way in which a piece of equipment can break down. It is possible that the part is used for the repair of multiple failure modes, but we will focus first on situation where the part is only used for just one failure mode. Later on in this section, we will discuss the situation where the part is used for multiple failure modes.

Because it is reasonable to assume that repair of equipment can only be completed if all parts needed for the repair are available, we need to determine which other parts are needed for repair of the failure mode. Together with the original part, these parts form a group of spare parts, that we will call a **spare parts package** in this document. A spare parts package is thus a group of spare parts that can be used to repair a piece of equipment when it fails according to a particular failure mode. For the spare parts in the package, **price** and **leadtime** (based on procurement data) are known in E-SPIR. From these, we can calculate the price and the leadtime of the package as a whole. The price of the package is the sum of the prices of the individual parts, while the leadtime of the package is the maximum of the individual leadtimes.

Example: *We consider the failure mode seal leakage of sample pump* 522.101*. The parts needed for repair of this failure mode are*

Description	*ID*	*price*	*procurement time*
seal	38.10.33.20	$8k\$$	22 *weeks*
sleave	522.364.2	$1k\$$	10 *weeks*
soft parts set	522.364.9	$0.30k\$$	1 *week*

The spare parts package "seal repair of sample pump 522.101*" thus has a leadtime based on procurement of* 22 *weeks and a price of* $9.3k\$$*.*

Some parts can be **refurbished**. This means that instead of ordering them new at the supplier, the defect parts are repaired themselves. To avoid notational ambiguities related to the repair of equipment, we will refer to the repair of the spare parts as refurbishment. If refurbishment is an option, it can often be done in a shorter time interval than the leadtime, based on procurement, of the part. We can thus also define the leadtime of the spare parts package based on refurbishing. The leadtime based on procurement is given in E-SPIR. The leadtime based on refurbishment was added to the model to enable practitioners to assess the effect of refurbishment on the optimal stock level, in case the optimal level based on procurement is unsatisfactory. Refurbishment costs are not applied in the model, just "new" costs are used.

Example: *In the above example the seal has a refurbishment time of* 2 *weeks, the sleave a refurbishment time of* 1 *week, and the soft parts set has a refurbishment time of* 1 *week. This means that the leadtime of the package based on refurbishing is* 2 *weeks.*

Refurbishment assessment related to the minimum stock level determination is relevant to get proper contracts for refurbishment in place with suitable contractors.

Practical considerations

By giving recommendations for entire packages of spare parts, we are able to reduce the workload when determining initial provisioning. We do need a leadtime for the package, however. We have chosen to use the maximum of the leadtimes within a spare parts package as the leadtime of the package. In some cases, this might have an undesirable side effect. In particular if a spare parts package consists of a part with a long leadtime and a low price, and a part with a short leadtime and a high price. These situations need to be verified and it is recommended to provide a report that shows the groups of spares where the longest lead time is not within the group of most costly spare parts.

In some cases a part is used in the repair of multiple failure modes. This means that the part will belong to multiple spare parts packages. In this case, we will use the model to give a recommendation for all these packages. The number of parts stocked can then be chosen to be equal to the sum of the number of packages stocked for the two failure modes, but for generic items this might lead to overstocking.

Example: *If at each impeller failure also the bearings are renewed, the spare bearings need to be linked to both the failure mode "bearing failure" and the failure mode "impeller failure".*

The method of applying spare parts packages focuses on the most expensive parts in inventory. When determining stock levels for generic parts and parts with a short leadtime, secondary logic might be used by the user to improve on the outcomes of the model. In E-SPIR, the user is automatically warned when giving a recommendation for a part that is also installed in other equipment.

No part - failure mode combinations

If a part is not used in any known failure mode that was assessed in the Shell RCM study, it is unclear how many parts we should stock. In principle, it is rational to refrain from stocking the part, as in most RCM studies all critical equipment is assessed with its dominant failure modes. If a part is not in a spare parts package, it is thus not considered to be installed in critical equipment. Especially for expensive spare parts this needs to be verified and it is recommended to provide a report showing all expensive spare parts that are not part of a spare parts package.

2.2 Failure mode information

In the Shell RCM study, for each of the tags involved, for each failure mode that can occur, an estimate is made how often the failure occurs, and how large the production loss is in case of downtime. In the RCM study, redundancy situations are also identified. The tags involved in redundancy of a process function are assessed as a functional group. In this section, we will describe how the data obtained during the RCM study will be interpreted.

Downtime costs

When a tag in a functional group goes down, downtime costs are incurred. These downtime costs of course depend on the length of the interval in which the functional group is down. Because we will calculate the average production loss based on the average waiting time, we apply a *linearized production loss equation* because using a nonlinear production loss equation will increase the errors when using the average waiting time.

This means that we will disregard any nonlinearities in the downtime costs as function of the length of the downtime, and we always work with linear downtime costs. This is a simplification of the S-RCM functionality. The simplification reduces the error that results from ignoring the variations in the waiting time. Furthermore, the simplification adds value because of the increased usability of the model as the data needed to apply the model is significantly decreased. In most cases, the simplification will have a negligible impact: in case of downtime as a result of spares shortages this downtime will in general be much larger than the time interval in which the production loss is nonlinear in the downtime.

To linearize the model, we use the downtime costs when the equipment is down for one week ($\mathrm{PLE}_{\mathrm{week}}$). We will then use $\mathrm{PLE}_{\mathrm{week}}/7$ as the linearized daily downtime costs.

Example: *When the tag* P-205 *goes down, the first* 24 *hours there is no production loss because of pipeline storage capacity. After these* 24 *hours, the production loss is* $4.333k\$$ *per day. To linearize the model, we use the downtime costs when the system is down for* 7 *days. These downtime costs are* $6 * 4.333 = 28k\$$. *We will thus calculate with a production loss of* $28/7 = 4k\$$ *per day starting from the moment the equipment goes down. When the pump is down for 4 days, we calculate with a production loss of* $4 \cdot 4k\$ = 16k\$$, *while in reality we have a production loss which is* $3 \cdot 4.333 = 13k\$$.

Health, Safety & Environment effects

In S-RCM the consequences of a equipment failure are translated into Economical, Health and Safety, and Environmental effects. The HSE effects are calculated back into monetary values and added to the Economical consequences. In the minimum stock calculations the HSE aspects are not taken into account, because often the HSE effects occur at the equipment failure and not during the waiting time for repair. In case the HSE effects occur during the waiting time for repair - like flaring caused by an equipment not being available - this shall be taken into account.

Redundancy

Among the tags in which the failure mode can occur, some tags perform a backup function for other tags. These tags can all perform the desired process function, and the production loss depends on the number of tags that are down among the tags in the group. We will refer to a number of tags performing the same function involving redundancy as a **functional group**. In the previous example, tag P-205 did not have a redundancy relation with any other tags. We will assign tag P-205 to functional group 2; this group then only contains this particular tag. We now give an example a functional group that consists of multiple tags.

Example: *Functional group* 1 *is formed by the tags* P-201A *an* P-201B*. If one of these tags is in repair, the other can take over, and the downtime costs are* 0*. If however the other pump fails while the first is still in repair, then both pumps are down and downtime costs of* $30k\$$ *per day are incurred. In RCM, this is denoted by giving* P-201A *downtime costs of* 0*, and by giving* P-201B *downtime costs of* $30k\$$ *per day. Note that this can give rise to confusion, as the downtime costs for* P-201B *are only incurred if* P-201A *is down as well.*

MRTBF

For each tag a **mean time between failure** (MTBF) is given. This is an estimate of the total number of years between failures of the tag, irrespective if the equipment is running or not. In general, the MTBF can be different for different tags within one functional group. Apparently, some tags make more running hours on average than other tags. To be able to calculate downtime probabilities later on, we need the **mean running time between failure** (MRTBF). Because the two tags are equal except for their running fraction, it is reasonable to assume that the mean running time between failure is the same for the two pumps. This MRTBF can then be viewed as a property of the functional group: namely the mean time in which the system is running until a failure occurs.

Example: *In our example the MTBF of the P-201A is* $MTBF_a = 3$ *years, while the MTBF of the P-201B is* $MTBF_b = 5$ *years. The MTBF differs between these two tags, but it is reasonable to assume that the MRTBF for the two pumps is equal. If we denote the fraction of time that pump P-201A is running by* α*, then we have* [2]

$$\begin{aligned} \alpha MTBF_a &= MRTBF, \\ (1-\alpha) MTBF_b &= MRTBF. \end{aligned}$$

From these, we can determine:

$$\begin{aligned} \alpha &= \frac{MTBF_b}{MTBF_a + MTBF_b} = \frac{5}{8} = 0.625 = 62.5\%, \\ MRTBF &= \frac{1}{1/MTBF_a + 1/MTBF_b} = \frac{1}{1/3\ yrs + 1/5\ yrs} = \frac{15}{8} = 1.875\, yrs. \end{aligned}$$

So pump P-201A is running 62.5% *of the time and pump P-201B is running* 37.5% *of the time and the process function experiences a tag failure once every* 1.875 *years on average.*

Using a similar argument as given in the above example, we can show that when a functional group consists of 3 tags A, B and C, then the MRTBF of the functional group is given by

$$\mathrm{MRTBF} = \frac{1}{1/\mathrm{MTBF_a} + 1/\mathrm{MTBF_b} + 1/\mathrm{MTBF_c}}.$$

[2] In addition, we assume that either pump P-201A or pump P-201B is running. In reality, the system will be down for a small fraction of time. We can safely ignore this because the system will be down for only a small fraction of time.

The number of active equipment

In a 1ooN system, we always have one piece of equipment working, unless the system is down. The other pieces of equipment are not working. It therefore is reasonable to assume that when one piece of equipment is still working, the expected time until the next failure is equal to the MRTBF of the functional group, as calculated with the appropriate formula derived above.

In a 2ooN (2oo2 or 2oo3) system, the discussion is somewhat more complex. In principle we could let the expected time until the next failure depend on the number of equipment that are running (1 or 2). If only 1 piece of equipment is running, this expected time might increase with respect to the situation where two pieces of equipment are running as there is only one pump that can break down. On the other hand, as the amount of work that has to be performed by the remaining equipment might increase, the MRTBF can also decrease.

For simplicity, we therefore assume that the MRTBF does not depend on the number of active pieces of equipment. This simplifies the model, as the above considerations do not have to be taken into account. Furthermore, assuming that the expected time until the next failure does not depend on the number of running equipment enables us to use the same calculations for the 1ooN and the 2ooN systems.

Multiple pumps failing in a single incident

In 2oo2 and 2oo3 situations, sometimes both of the running equipments can break down as a result of a single event. The statistics used in the model do not take this possibility into account. The model is therefore not suitable for use in situations where the probability of simultaneous breakdown caused by a single event is significant. Simultaneous breakdown is considered to be rare, and therefore it is left out of consideration.

Example: *When the common seal oil system in a 2oo3 redundant pump system breaks down, both seals run dry. This results in the breakdown of both running pumps. This is considered a rare event.*

Multiple failure modes in a single piece of equipment

If some equipment type can fail according to multiple failure modes, the model can give a biased estimate of the total downtime in situations when the equipment is used in redundant combinations. In general, one failure mode will occur much more often than other failure modes. In this case, the overestimation will often be small. When multiple failure modes all occur regularly, this overestimation can be significant. It is important to be aware of this.

Example: *Consider pump* P-201A *and* P-201B*, which are 1oo2 redundant. In previous examples, we considered the failure mode "seal leakage". Suppose that these pump can also fail as a result of the failure mode "bearing failure". Then, it is possible that pump* P-201A *will fail as a result of seal leakage. Before we are able to repair* P-201A*,* P-201B *can fail as a result of bearing failure, resulting in loss of production capacity.*

We see from this example that there can be interaction between different failure modes. In the model, we ignore this interaction, which may result in underestimation of the downtime.

2.3 Exclusions

We already discussed a number of aspects that can be relevant for stocking decisions, but that are excluded from the model to increase usability. In this section we will make a few remarks on some other things that are excluded.

Scheduled repairs

Some spare parts are used in scheduled repairs, and ordered in a scheduled sequence. As these parts can be ordered separately so that they arrive just before they are needed, this usage can be left out of the stock level assessment.

Cannibalism

When a pump with high downtime costs goes down, parts can be cannibalized from a low priority pump to repair the high criticality pump as fast as possible. Solving problems in this way can not be considered standard operation, and we feel that the model should be based on standard operation. The model thus excludes this. Furthermore, including cannibalization would increase the complexity of the model and decrease its useability.

Prioritizing

As we have seen above, the same spare part may be used in equipment of different criticality. Hence, if only one part is left over, one may choose to keep it in stock for high critical equipment. This is not included in the present model. Models with demand priority are for instance discussed in Dekker, Hill and Kleijn (2002).

Speeding up replenishment

When a particular spare parts is needed fast, replenishment can often be speeded up by paying extra. Again, this will not be considered normal operation, and including this in the model would mean that this way of solving problems would become part of normal operation. Also, obtaining the data needed to include this in the model would decrease the useability of the model significantly. We therefore do not include the speeding up of orders in the model.

3 The model

In this section, we will describe a model that can be used to determine the total expected downtime costs per year and the total stocking costs per year for a given number of spare parts packages to stock. We will use this data to determine the optimal number of spare parts packages to be stocked. Note that in this assessment the optimum re-order quantity is not taken into account.

3.1 Notation

Leadtime, Cost, Packages Stocked

We assume that a spare part package with a cost C ($k\$$) and a leadtime L (wks) is given. The leadtime can either be based on procurement of a new part, or it can be based on refurbishment. The spare parts package is used for repair of a failure mode of a particular equipment type. We will denote the number of spare parts packages we keep stocked by S (packages). If we would for instance keep S packages stocked, we make sure that we have S packages to start with. Immediately after a package is used, a new package is ordered (in case of new parts procurement) / we start refurbishing the old package (in case of refurbishing).

Example: *In the example we have that $L = 22$ weeks (based on procurement), and $C = 9.3k\$$. The package we consider is used for repair of the failure mode seal leakage of the sample pump* 522.101*.*

Downtime costs

We assume that the tags in which the considered failure mode can occur are known, as well as the functional groups. We will denote each functional group by $f \in \{1, \ldots, N\}$. For each of these functional groups f it is known of how many tags the group consists (denoted by R_f). We also assume that the downtime costs are known: the daily downtime costs when $i \in \{1, \ldots, R_f\}$ pumps are down in functional group f are denoted by c_{fi} ($k\$$/day). We assume the MRTBF is the same for all tags within a functional group, we will denote the MRTBF of functional group f by MRTBF_f(yrs).

Example: *We have $N = 3$ functional groups. We discussed functional group 1 and 2 in previous examples. The last functional group is functional group 3 that consists of $R_3 = 3$ tags (*P-1108A, P-1108B *and* P-1108C*). When one pump is down, there are no production loss costs ($c_{31} = 0k\$/day$). When two pumps are down, the downtime costs are $c_{32} = 20k\$/day$, and when all three pumps are down, the downtime costs are $c_{33} = 100k\$/day$. Pumps* P-1108A, P-1108B *and* P-1108C *have a MTBF of 2,3 and 5 years, respectively. We thus know that the MRTBF of the third functional group can be calculated to be $MRTBF_3 = 1/(1/2 + 1/3 + 1/5) \approx 0.97\text{yrs}$. We now present all relevant functional group data in a table:*

Functional group	*Number of tags*	*Downtime costs* ($k\$$/day) *1 down*	*2 down*	*3 down*	*Mean running time between failures*
$f = 1$	$R_1 = 2$	$c_{11} = 0$	$c_{12} = 30$	-	$MRTBF_1 = 1.875\text{yrs}$
$f = 2$	$R_2 = 1$	$c_{21} = 4$	-	-	$MRTBF_2 = 2\text{yrs}$
$f = 3$	$R_3 = 3$	$c_{31} = 0$	$c_{32} = 20$	$c_{33} = 100$	$MRTBF_3 = 0.97\text{yrs}$

Holding costs

Keeping spare parts packages on stock gives rise to annual holding costs, which we calculate as a fixed percentage of the procurement price of the package (regardless of whether we decide to procure or refurbish the parts, as the parts on stock must be obtained at the procurement price in either case). The holding costs come forward from interest of capital involved, warehouse and handling costs.

Example: *The cost of the spare parts package in the example is* $9.3k\$$*. We use an annual holding cost percentage of* $25\%/yr$*. So the annual holding costs are* $2.325k\$/yr$ *per package.*

Using a fixed percentage of the procurement cost of the part as holding cost is a simplification. In general, for items with a higher value the holding cost *fraction* may be lower than for lower value parts. We will ignore these subtleties to increase the usability of the model.

Number of repairs

Keeping spares in stock enables the repairs of defect equipment to be done faster, as stock reduces the waiting time for spares. We will denote the total number of expected repairs per year by λ (1/yr). It can be calculated using the MRTBF of the individual functional groups. We have

$$\lambda = \sum_{i=1}^{N} 1/\mathrm{MRTBF}_i \text{ per year.} \tag{1}$$

Example: *In our example, we can easily calculate that* $\lambda = 1/1.875 + 1/2 + 1/0.97 = 2.07$*, which means that we expect on average about 2 repairs every year, which is the demand rate on the spare parts package involved.*

Repair times

If equipment breaks down, we assume that we first have to wait until a spare parts package is completely available. We denote the time that we need to wait by t_w(wks). We will explore the waiting time in more detail in section 3.2. When all the spare parts that are needed for the repair are available, we assume that there is a remaining time needed to complete the repair. We will refer to this time as the **repair time when all spare parts are available** ($t_\triangledown$). The **total repair time** is denoted by t. We have:

$$t = t_w + t_\triangledown$$

Example: *The waiting time depends on the specific repair. The repair time when all spare parts are available of the failure mode "Seal leakage" of sample pump in our example is* 2 *weeks. We thus have that* $t_\triangledown = 2$ *weeks. The total repair time is thus* $t = t_w + 2$.

3.2 The average repair time

The waiting time varies

The waiting time depends on the specific repair that we consider. When the package is on stock at the moment it is needed, the waiting time will be zero. If the package is neither on stock, nor on order, the waiting time is L. But whether the package is available from stock depends on the amount stocked, and on the time interval between the repair, and earlier repairs demanding the package.

Example: *Say we keep one package on stock ($S = 1$). At a certain moment in time, the pump P-1108A breaks down. It is repaired using the package on stock ($t_w = 0$), and a new package is ordered at the supplier. The total repair time is thus $t = t_\bigtriangledown = 2$ weeks. 12 weeks later, the pump* P-201A *breaks down and needs the package. The package that we ordered will only arrive in another $22 - 12 = 10$ weeks, so we have a waiting time of 10 weeks until the ordered package arrived ($t_w = 10$ weeks). We immediately do order a new package, however, which means that we have 2 packages on order. 10 weeks later, the package that was ordered upon the failure of pump* P-1108A *arrives, and in another* 2 *weeks* P-1108A *is repaired. The total repair time of* P-1108A *is thus* 12 *weeks.* 10 *weeks later, the part ordered upon the failure of* P-1108A *arrives. Another* 19 *weeks later,* 41 *weeks after the breakdown of pump* P-1108A, *pump $P - 105$ breaks down. As the package is already on stock, the waiting time is zero. A new package is ordered at the supplier. 4 weeks later, pump P-201A breaks down (again). We have to wait for another $t_w = 18$ weeks for the package to arrive, and the total repair time t for this pump is 20 weeks. The events in this example are shown again in the following table.*

time (wks)	*event*	*parts on stock*	*remarks*
0	*Breakdown of P-1108A*	*1*	
0	*Withdrawal for repair P-1108A*	*0*	$t_w = 0$
0	*Ordering of new stock*	*0*	1 *package on order*
2	*Completion of repair P-1108A*	*0*	$t = t_w + t_\bigtriangledown = 2$ *wks*
12	*Breakdown of P-201A*	*0*	*no spare parts available*
12	*Ordering of new stock*	*0*	2 *packages on order*
22	*Arrival of new stock*	*1*	*Ordered upon failure of P-1108A*
22	*Withdrawal for repair P-201A*	*0*	$t_w = 10$
24	*Completion of repair P-201A*	*0*	$t = t_w + t_\bigtriangledown = 12$ *wks*
34	*Arrival of new stock*	*1*	*Ordered upon failure of P-201A*
53	*Breakdown of P-105*	*1*	
53	*Withdrawal for repair P-105*	*0*	$t_w = 0$
53	*Ordering of new stock*	*0*	1 *package on order*
55	*Completion of repair P-105*	*0*	$t = t_w + t_\bigtriangledown = 2$ *wks*
57	*Breakdown of P-201A*	*0*	*no spare parts available*
57	*Ordering of new stock*	*0*	2 *package on order*
75	*Arrival of new stock*	*1*	*Ordered upon failure of P-105*
75	*Withdrawal for repair P-201A*	*0*	$t_w = 18$
77	*Completion of repair P-201A*	*0*	$t = t_w + t_\bigtriangledown = 20$ *wks*

In the 4 *repairs considered in this example the waiting times were* 0,10,0 *and* 18 *weeks. The average waiting time over these* 4 *repairs is thus* 7 *weeks.*

Note that we expect on average 1 repair every 0.48 yrs. In the above example, the breakdowns do not occur in equally spaced time intervals, even though the average inter-arrival time is about 0.48 yrs. In practice, repair times are not equally spaced in time, and the above scenario is quite realistic. Also, from the above discussion we see that long waiting times occur when breakdowns occur shortly after each other. We thus need to assess the risk of breakdowns occurring shortly after each other.

The average waiting time

From the above example, it is clear that the waiting time for spare parts is different for various repairs. To be able to do a risk based assessment and simplify the discussion we will use a METRIC like approach (Sherbrooke 1968): we work with the **average waiting time for spares** ($\bar{t}_w$) when determining downtime costs. This waiting time depends on the leadtime L of the spare parts package, the total expected demand per year λ, and the number of packages kept on stock S. In the above example we saw how the average waiting time should be interpreted. In practice, the moment of arrival of the different repairs is not known. It is quite reasonable however to assume that the time interval between successive repairs is exponentially distributed in each functional group. We then know that the number of defects in a time interval is Poisson distributed. We can then use queueing theory to determine the average waiting time. However, the results from queuing theory do not give much insight or understanding. We therefore first derive some approximate results for our example.

Example: *Assume we stock $S = 1$ part. When a tag breaks down, the spare is on stock if no tag broke down in the last 22 weeks. If a tag did break down in this period, we have to wait. We expect on average $\lambda = 2.07$ repairs per year. The probability that a tag has broken down in this period is approximately $22 \cdot 7/365.5 \cdot 2.07 = 87\%$. If we have to wait, the waiting time until the spare arrives is on average about 11 weeks. So we expect the average waiting time when we stock 1 spare to be about $0.87 \cdot 11 = 9.5$ weeks. Now, suppose we stock $S = 2$ parts. The probability that we have to wait is now equal to the probability that we have 2 demands in the 22 preceding weeks. This probability can be approximated to be $(22 \cdot 7/365.5 \cdot 2.07)^2/2 = 38\%$. If we have to wait, we can expect that we have to wait on average $22/3 = 7.33$ weeks. We thus have to wait on average $0.38 \cdot 7.33 = 2.77$ weeks. Note that the formulas above to estimate the probability of no demand / less than 2 demand in the 22 week period are only approximate.*

We now give the exact results, which are less insightful. We use a result from queueing theory to calculate the expected waiting time for spares as a function of the yearly demand λ, the leadtime L, and the number of packages kept on stock S. In particular, we use Little's formula to calculate the average waiting time $\bar{t}_w$ (see Sherbrooke (1968))

$$\bar{t}_w(S) = L - \frac{S}{\lambda} + \frac{1}{\lambda}\sum_{i=0}^{S-1} \frac{(S-i)(\lambda L)^i}{i!} e^{-\lambda L}. \tag{2}$$

Example: *We have $\lambda = 2.07$ demand per year. Furthermore, we have $L = 22 \cdot 7/365.5$ years. We can thus easily tabulate $\bar{t}_w(S)$ for different values of S using equation 2.*

S	$\bar{t}_w(S)$	$\bar{t}_w(S)$
0	0.421 *yrs*	22.00 *weeks*
1	0.140 *yrs*	7.31 *weeks*
2	0.035 *yrs*	1.83 *weeks*
3	0.007 *yrs*	0.36 *weeks*
4	0.001 *yrs*	0.06 *weeks*
5	0.000 *yrs*	0.01 *weeks*

The value for the average waiting time when we have no stock (S=0) corresponds to the lead-time of the package as expected. When we keep one part on stock, the average waiting time *decreases significantly, to* 7 *weeks. Note that our approximation gave* 9.5 *weeks. Stocking* 2 *parts gives an average waiting time of* 1.83 *weeks, deviating from the approximate result* 2.8 *weeks. As the amount stocked increases, the average waiting time steadily decreases to zero.*

Average repair time

Previously, we saw how to calculate the average waiting time for spares $\bar{t}_w$ as a function of S, λ and L. We can use this average waiting time to compute the **average repair time** $\bar{t} = \bar{t}_s + t_{\bigtriangledown}$.

Example: *We assume that we choose to stock one part* ($S = 1$)*. We then have an average waiting time* $\bar{t}_w = 7.31$ *weeks. The average repair time is then* $\bar{t} = 9.31$ *weeks or* 0.178 *year.*

3.3 Downtime costs

Given the repair time $\bar{t}$ calculated in the last section we want to calculate the downtime costs. Let us consider an arbitrary functional group f. Recall from section 3.1 that we denote MRTBF of this group by MRTBF_f. The group consists of N_f tags. When i tags are down, costs equal to c_{fi} per day are incurred.

Under the assumption of exponential time intervals between successive defects in the functional group, it is clear that the long term fraction of time in which there are n defective tags in the group is given by

$$\begin{aligned} F_n &= P(\text{Defect tags} = n) \qquad\qquad n \in \{0, \dots, N_f\} \\ &= \frac{(\bar{t}/\mathrm{MRTBF}_f)^n/n!}{\sum_{i=0}^{N_f} (\bar{t}/\mathrm{MRTBF}_f)^i/i!} \\ &= \frac{\bar{t}^n/n!}{(\bar{t} + \mathrm{MRTBF}_f) \cdot \mathrm{MRTBF}_f^{n-1}} (1 + \mathcal{O}(\bar{t}/\mathrm{MRTBF}_f) \\ &\approx \frac{\bar{t}^n/n!}{(\bar{t} + \mathrm{MRTBF}_f) \cdot \mathrm{MRTBF}_f^{n-1}} \end{aligned}$$

where the approximation is valid under the assumption that the expected number of failures during the repair time is sufficiently small ($\bar{t}/\mathrm{MRTBF}_f \ll 1$). If we specialize this expression for the fraction of time in which there are 1, 2, and 3 defects, we get:

$$F_1 = \text{Fraction of time with 1 defect} = \frac{\bar{t}}{\bar{t} + \mathrm{MRTBF}_f} \tag{3}$$

$$F_2 = \text{Fraction of time with 2 defects} = \frac{\bar{t}^2/2}{(\bar{t} + \mathrm{MRTBF}_f) \cdot \mathrm{MRTBF}_f} \tag{4}$$

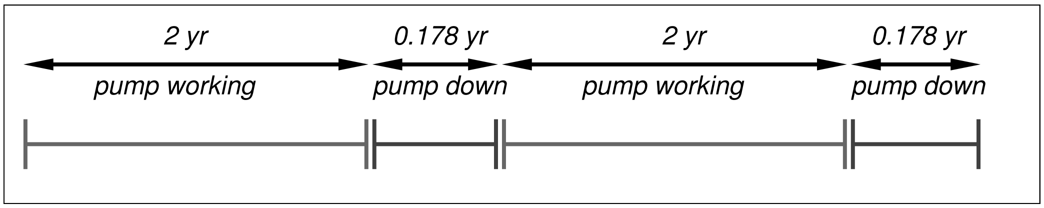
$$F_3 = \text{Fraction of time with 3 defects} = \frac{\bar{t}^3/6}{(\bar{t} + \mathrm{MRTBF}_f) \cdot \mathrm{MRTBF}_f^2} \tag{5}$$

The annual downtime costs for functional group f can thus be calculated to be

$$\mathrm{costs}_f = \sum_{i=1}^{N_f} F_i \cdot c_{fi} \cdot 365.5. \tag{6}$$

We will now calculate the downtime costs for the different functional groups in the example. Although we will use equations 3,4 and 5, we will try to illustrate the formulas with a graphical representation.

Example: *We assume that we stock* 1 *spare parts package. Then, we know that the average repair time is* $\bar{t} = 0.178$ *yr. Consider first functional group* 2*, for which the discussion is simplest because it consists of only one tag. The MRTBF for this group is* $MRTBF_2 = 2$ *years. The situation for this tag can be illustrated as follows:*



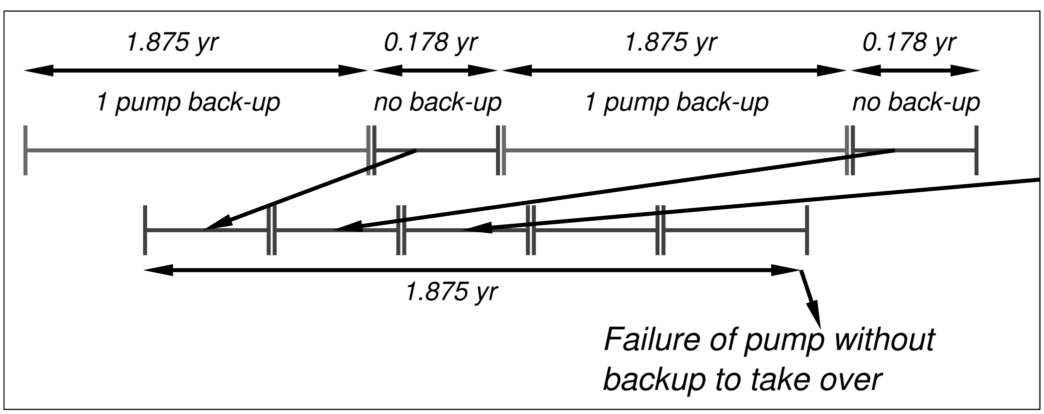
From this figure (or from equation 3) it is clear that the fraction of time that the pump is down is given by

$$F_1 = \frac{\bar{t}}{\bar{t} + MRTBF} = \frac{0.178}{2.178} = 0.089$$

and the downtime costs for the pump are thus:

$$costs_1 = 0.089 \cdot 4 \cdot 365.5 = 119.69k\$/yr$$

We now take a look at functional group 1*. We know that* $MRTBF_1 = 1.875$*. Because there are no costs when only one pump is down, we are mainly interested in the fraction of time in which two pumps are down. The situation for this functional group can be illustrated as follows:*



We have thus every

$$\frac{\bar{t}}{(\bar{t} + MRTBF_1)MRTBF_1}$$

years a failure of a pump without a backup to take over. When a pump fails without a backup to take over, the expected length of the downtime is on average $\bar{t}/2$*. We thus have* *that the long term fraction of time with two pumps down is given by*

$$F_2 = \frac{\bar{t}^2/2}{(\bar{t} + MRTBF_1)MRTBF_1} = 0.0041$$

We could of course also have used equation 4 to calculate this fraction. The downtime costs of functional group 1 *are thus given by*

$$costs_1 = F_1 \cdot c_{12} \cdot 365.5 = 45.28k\$/yr$$

Finally, we consider functional group 3. We will use formulas 4 and 5 to calculate the fraction of time in which there are 2 and 3 pumps down, respectively.

$$F_2 = \frac{\bar{t}^2/2}{(\bar{t} + MRTBF_3) \cdot MRTBF_3^1} = 0,014$$
$$F_3 = \frac{\bar{t}^3/6}{(\bar{t} + MRTBF_3) \cdot MRTBF_3^2} = 0,0009$$

The annual downtime costs for functional group 3 *can thus be calculated to be*

$$\text{costs}_3 = F_2 \cdot c_{32} \cdot 365.5 + F_3 \cdot c_{33} \cdot 365.5 = 137k\$/\text{yr}$$

The total downtime costs can be easily calculated as the sum of the downtime costs of the individual packages

$$\text{downtime costs} = \sum_{f=1}^{N} \text{costs}_f \tag{7}$$

Example: *From this, we conclude that the expected total annual downtime costs when we stock one package is given by*

$$\text{downtimecosts}(S = 1) = \text{costs}_1 + \text{costs}_2 + \text{costs}_3 = 302k\$/\text{year}$$

4 The optimal stock level

We are able to calculate the annual downtime costs for a given number of packages to be stocked (S). In short, such a calculation consists of the following steps:

1. Calculate the MRTBF for each functional group. Calculate the total repair rate λ.
2. Calculate the expected waiting time for spares $\bar{t}_w$ using equation 2. Calculate the expected repair time using $\bar{t} = \bar{t}_w + t_\bigtriangledown$.
3. Calculate the downtime costs per functional group using equation 6.
4. Calculate the total downtime costs using equation 7.

Now that we know how to calculate the annual holding costs for a given stock level S, we are able to find an optimal stock level S. The optimal stock level S is given as the stock level for which the total annual costs are minimized. The total annual costs are given as the sum of the annual downtime costs and the annual holding costs, which are given by

$$S \cdot C \cdot h \tag{8}$$

where h is the annual holding costs fraction [3].

Example: *For different values of S, we can perform the calculation that is described above. Note that if S changes we get another value for $\bar{t}_w$ which means that almost all other values change as well. The results are:*

spare parts packages on $\bigtriangledown$ (S)	*average repair time $\bar{t} = \bar{t}_w + t_{\bigtriangledown}$*	*annual downtime costs*	*annual holding costs*	*total annual costs*
0	*24 weeks*	*1539.37 k\$*	*0 k\$*	*1539.37 k\$*
1	*9.31 weeks*	*301.98 k\$*	*2.33 k\$*	*304.30 k\$*
2	*3.83 weeks*	*81.87 k\$*	*4.65 k\$*	*86.52 k\$*
3	*2.36 weeks*	*43.73 k\$*	*6.98 k\$*	*50.71 k\$*
4	*2.06 weeks*	*36.89 k\$*	*9.30 k\$*	*46.19 k\$*
5	*2.01 weeks*	*35.77 k\$*	*11.63 k\$*	*47.39 k\$*

We find that the optimal number of packages to stock is 4. *One other thing that is immediately clear from the number in the fifth column is that the total costs are not so sensitive to a small difference in the stock level.*

In the example, we see that around the minimum, the total costs vary only slightly. This will in fact often be the case. As a lot of the data in the model are approximations, and as data is often only approximate, the optimal policy has some inaccuracy bandwidth.

5 Order quantities

When determining order quantities, we recommend to look at parts on item level, not on spare parts package level. This means that after we have determined the minimum stock level for the different spare parts packages, we translate these minimum stock levels to levels for the individual parts. The order quantity for these parts can then be determined using the EOQ. In chapter 5 of Trimp et al. (2004), this EOQ is discussed. Here, we only give a short discussion, as the results in Trimp et al. (2004) are still applicable.

The EOQ formula is given by

$$\text{EOQ} = \sqrt{\frac{2 \cdot \text{Annual usage} \cdot \text{Order costs}}{\text{Annual holding cost per unit}}} \tag{9}$$

The annual holding costs can be estimated using the price of the item and the price of the item. If the part is used in the repair of only one failure mode, the annual usage corresponds to the average annual number of repairs λ (as given by equation 1) for that failure mode.

[3] At shell, a fraction of 0.25 is often used. For more information on holding costs see section 3.1.

The order costs must be determined for every particular plant, and depend on the way the replenishment are organized. The EOQ has to be rounded to a positive integer, for more information on this see Trimp et al. (2004). If the part is used in multiple failure modes, the total usage of the part is equal to the sum of the usage of the packages that contain the part.

In general, the EOQ is mostly interesting for cheap items with relatively high usage, most of the time generic parts. For expensive parts with low usage the EOQ is typically 1.

Example: *For non-generic parts, an average usage of once every two years is not uncommon. Assuming ordering costs of* \$200 *and fractional annual holding costs of* 25%*, with this usage even parts with a price as low as* \$500 *would receive an EOQ that will be rounded down to* 1*, which would mean that a base stock policy is best for the item.*

For generic items, using the EOQ to control order costs can be beneficial.

Example: *Generic parts such as gaskets can easily be used* 10 *times a year. Assuming again ordering costs of* \$200 *and fractional annual holding costs of* 25%*, the EOQ for a gasket with this usage and a price of* \$500 *would be* 6*. The annual advantage of ordering* 6 *instead of* 1 *would be* \$1300 *annually for this part.*

6 Conclusion

In this report we have developed a model that can be used to calculate optimal stocking levels for groups of spare parts that are used when equipment in a plant fails following a given failure mode. The model uses data that is obtained when doing an Shell RCM study, and data that is available from E-SPIR. The model assumes the failure modes can occur in multiple tags in the plant, which need not have the same downtime costs. The model also can cope with tags that are redundant. This general downtime structure increases the usability of the model, we feel that the most relevant aspects of stock control in a plant are incorporated in the model. At the same time, we have tried to exclude aspects from the model for which the added value of including them does not outweigh the decreases usability of the model when including them.

The model has been implemented as an Excel Spreadsheet. In the spreadsheet, the data from Shell RCM and E-SPIR can be entered. Using this data, we then calculate optimal stock levels using the methods described in this report.

References

Dekker, R., Hill, R. and Kleijn, M.: 2002, On the (s-1,s) lost sales inventory model with priority demand classes, *Naval Research Logistics* **49**, 593–610.

Sherbrooke, C. C.: 1968, METRIC: A Multi-Echelon Technique for Recoverable Item Control, *Operations Research* **16**, 122–141.

Trimp, M., Sinnema, S., Dekker, R. and Teunter, R.: 2004, Optimise initial spare parts inventories: an analysis and improvement of an electronic decision tool, *Technical Report EI 2004-52*, Econometric institute, Erasmus University Rotterdam.

Vago, A., Macki, S., Festen, M., Lorenz, M., Vonk, J., Slangen, P., Wiegerinck, J., Woltman, A. and Wisman, R.: 2004, *Risk & Reliability Management (RRM), version 2*.