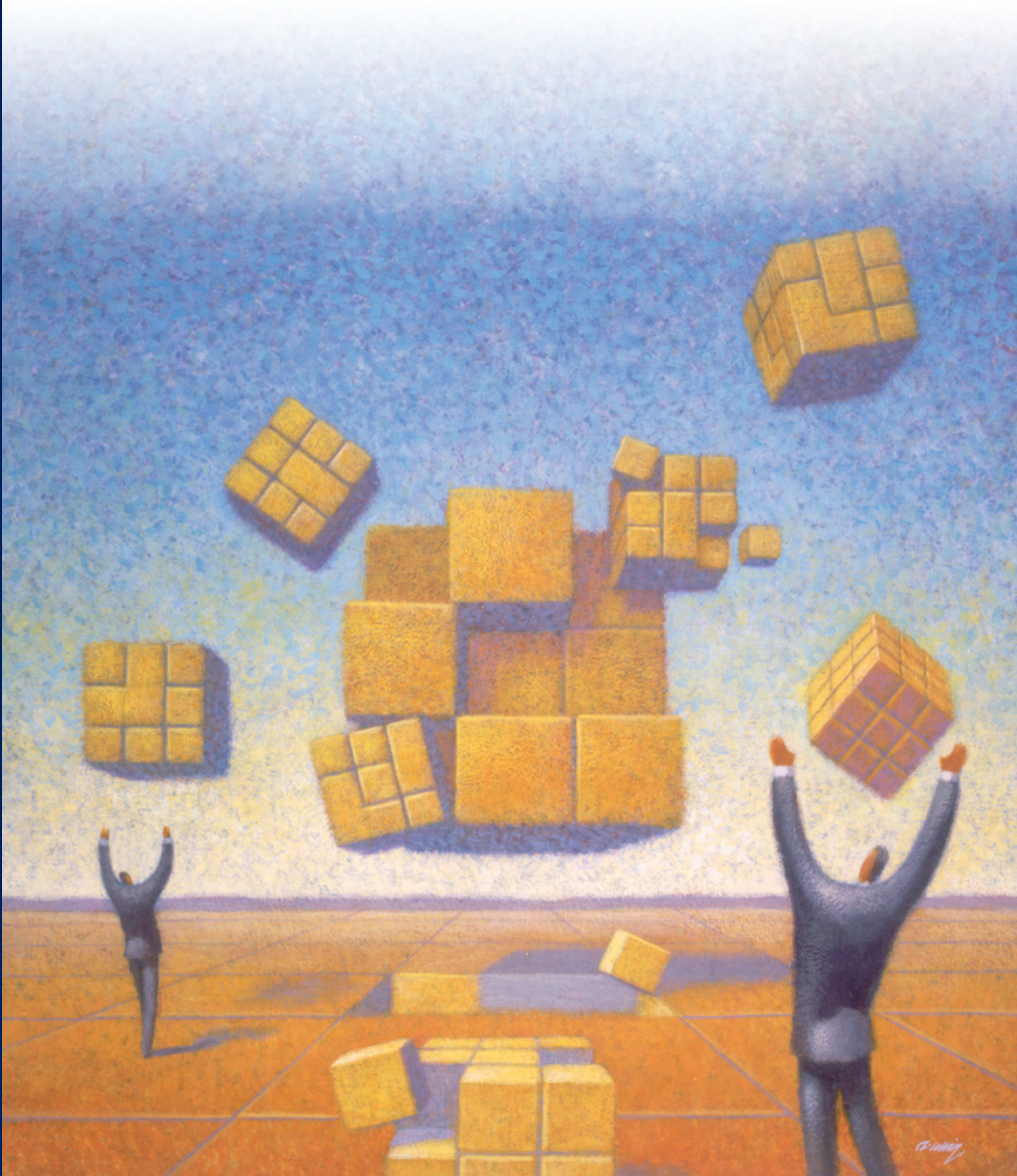
YEMING GONG

Stochastic Modelling and Analysis of Warehouse Operations



Stochastic Modelling and Analysis of Warehouse Operations

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Stochastische modellering en analyse van magazijnoperaties

Thesis

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Born in Tianmen, P.R.China

ERASMUS UNIVERSITEIT ROTTERDAM

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Dedicated to Rona (Guo Zhangrong) and Rachy (Gong Ruiqi)

Preface

The overarching theme of my PhD research is to advance warehouse management by analysing warehouse operations through the theoretical lens of stochastic systems, and by developing novel models aimed at improving operational and financial performance of these systems via real cases. With this goal in mind, I reflect on operations in online retail warehouses, service distributer centers, self-storage warehouses, based on my investigation of over fifty warehouses in Europe, Asia and America. During the process of writing this thesis, I had the honor to obtain much help.

First and foremost, I am full of gratitude to my PhD promotor and daily supervisor René de Koster, who helped me define my research problems and got me on track. My conversations with him shaped many of the ideas in this dissertation. He carefully examined the details of my PhD research and reviewed this thesis word by word for many times. For example, he has revised a chapter on polling models for over thirty times in the last two and half years. I learnt a rigorous academic attitude from him. Moreover, with experiences to advise foreign students, he deeply understands culture differences and international diversities. He also respects academic freedom and encourages independent academic exploration of his PhD students. He deserves the title "the best PhD supervisor" issued by TRAIL, a Dutch academic organization, and I appreciate his patience, insights, and support.

I would like to express my appreciation to my PhD promotor Steef van de Velde, who took his responsibility and did his best to provide me career advice and support. My thesis has benefited a lot from his suggestions and comments.

Donald D. Eisenstein (the University of Chicago) is a professor with brilliant, creative, and novel research ideas, which largely broadened my research vision. He has spent a great deal of time with me, and improved my research skills. He is a highly responsible PhD advisor, and always carefully makes preparation before every meeting with his students. I am extremely appreciative of the help and advice from this leading scholar in my field.

Rommert Dekker taught me a course on "Quantitative Models for Logistics and Transportation", which built a solid foundation for me in the logistics field and gave me a picture of the field. I have benefited a lot from him in my research afterwards.

I have taken a PhD course "Application of Queueing Theory" from Ger Koole (VU Amsterdam). This thesis can be partially regarded as a learning result of this course. I am also grateful he has taken time to answer many of my questions during my PhD study.

I am grateful to Hans Frenk, who has acted as the methodological advisor in my PhD study. Hans is a very popular professor among students. He taught me not only the principles of revenue management but also various stochastic methods. He is a brilliant researcher, and I have learnt much know-how to do academic research from him. I particularly appreciate his help with Chapter 6.

Kees Jan Roodbergen provided valuable comments on Chapter 2. I appreciate the help of Adriana Gabor, who made a contribution to Chapter 6. My other plenary doctoral committee members, J. van Nunen (Erasmus University Rotterdam), I.J.B.F. (Ivo) Adan (Technische Universiteit Eindhoven), and N. Vandaele (Katholieke Universiteit Leuven) gave me valuable feedback on my research.

I also would like to acknowledge the contribution of the co-authors of my five related publications or working papers, René de Koster, Donald D. Eisenstein, Hans Frenk, and Adriana Gabor. During my PhD study, I had other publications or working papers, which are not included in this thesis, with Steef van de Velde, Enver Yücesan, Chien-ming Chen, Xiandong Zhang, Jo van Nuen, Jinlong Zhang,

Erik M.M. Winands and Xiaobin Shu; I learnt a lot from them and appreciate their coauthorship.

Beyond faculty members involved with my PhD study, I appreciate advice and education from my Chinese advisors and teachers. First of all, I am indebted to my advisor Prof. J. Zhang, dean of the management college at Huazhong University of Science and Technology (HUST). Over the last twelve years, Prof. Zhang has been continuously investing a great deal in supporting my career and academic development. I appreciate the advice from Prof. J. Zhou, without which, I would never have thought of going abroad to study and to broaden my academic horizon. I thank Prof. F. Xie. From him, I learnt a scholar should bravely take social responsibilities. Furthermore, I thank the help and career advice from Prof. Y. Ye (Stanford), Prof. C.-P. Teo (NUS), Prof. P. Gao (U.Chicago), Prof. J. Zhuang (SUNY, Buffalo) and Prof. X. Chen (Yale). My uncle Prof. H. Zhou (TAMU) has inspired my academic interests when I was a kid.

During my study in the Netherlands, the Chinese embassy to the Netherlands has provided excellent consulate service. I want to thank the previous ambassador Xue Hanqin and incumbent ambassador Zhang Jun. Particularly, I want to thank education director Luo Ping, who provided high-efficiency service to overseas students. I am pleased that ambassador Zhang Jun, representing the Chinese government, awarded me *the National Award for Outstanding Chinese Overseas Students*, which is a prestigious national honor for any Chinese student.

I am obliged to the help from my previous classmates and alumni. I had a good time with graduate classmates Luo Shangyuan (Lehigh), Shuai Chuanmin (CUG), Shu Xiaobin (RUC), Li Zhonghai (U. Toronto), Wu Yaozhong (NSU), Fang Jinping, Atalay Atasu (Georgia Tech), Zhang Yu (UC-Irene), Xiao Zhixing (CEIBS), Xiang Yi (HKUST), Dai Xianchi (CUHK), Wang Tong (NSU) and my friend Wang Shanshan(U. Chicago). I can never forget my college classmates Gu Hua (UT-Austin), Wan Lin (Amazon), Zhang Tao(HUST). I had a very good time with my friends and classmates in middle school, Xiong Sheng (Temple U.), Wu Bo, Lin Xuncai and Hu Shuang (MIT).

INSEAD provided me an academic foundation, particularly in research methodology. I am thankful to my advisor Enver Yücesan, and two research instructors Steve Chick and Sengupta Kishore.

I appreciate the help from my colleagues in Eramus. Particularly, I am obliged to the help of Carmen and Olga (ERIM PhD office). Without the help from my officemate Bart, my life in the Netherlands would have become much more difficult. I am grateful to my paranimfen Sandra Langeveld and Marlou van der Linden, and thank Henk, José, Morteza, Serge, Irma, Melek, Nima, and Amir. In the Netherlands, I had a good time with my Chinese friends (listed alphabetically), Chen Chien-ming, Chen Hong, Chen Li, Duan Rui, Feng Li, Hu Yamei, Huang Xiaohong, Jiang Hao, Jiang Tao, Jiao Tao, Shi Ying, Wang Zhuangyi, Xu Jingnan, Yang Shengyun, Yang Xu, Yu Yugang, Yuan Taoying, Zhang Xiandong, and Zhang Xueyuan.

Without the support of my family, I could not have finished my PhD study. I am indebted to my parents, my sisters Gong Xiaomei and Gong Yeqing, and thank my brothers Yang Jun, Guo Chuanxiong and Zhang Ziyong.

Yeming (Yale) Gong
Rotterdam, the Netherlands
2009

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Part I

Introduction and Review

1

Introduction to the dissertation

1.1 Introduction to warehouse operations

A warehouse is an important facility to consolidate products to reduce transportation cost, achieve economies of scale in manufacturing or in purchasing (Bartholdi III and Hackman, 2006), provide value-added processes, shorten response time (see, e.g., Gong and De Koster (2008)), or to gain revenue by leasing warehouse space (Gong et al., 2009a).There are various types of warehouses: Ghiani et al. (2004) briefly classify warehouses into production warehouses and distribution centers. Frazelle (2001) classify warehouses by their roles in the supply chain, as raw materials warehouses, work-in-process warehouses, finished good warehouses, distribution warehouses, fulfillment warehouses, local warehouses direct to customer demand, and value-added service warehouses. Ghiani et al. (2004) also classify warehouses by the ownership, and identify three main types: company-owned, public, and leased warehouses. Bartholdi III and Hackman (2006), categorize warehouses, by customer service types, as a retail distribution center, a service parts distribution center, a catalogue fulfillment or e-commence distribution center and a 3PL warehouse.

Admittedly, these heterogeneous warehouses have different operations. However, most of them share some general pattern of material flow, and typical warehouse operations include: receiving, putaway, internal replenishment, order picking, accumulating and sorting, packing, cross docking, and shipping (see Figure-1.1 from Tompkins et al. (2003)). Mainly based on Frazelle (2001) and Tompkins

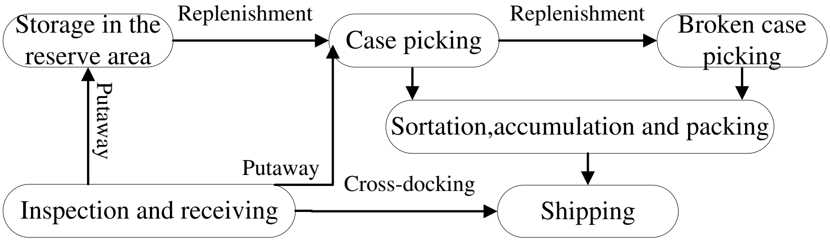


Fig. 1.1. Typical warehouse operations

et al. (2003), these operations may be described as follows. Receiving includes the receipt of all materials into warehouses, assuring the quantity and quality of these materials as ordered and transferring received materials to storage or other places. Receiving typically accounts for about 10% of warehouse operation cost (Drury, 1988). Putaway is to put items in storage, which includes material handling, location verification, and item placement. Putaway typically accounts for about 15% of warehouse operation cost (Drury, 1988). Internal replenishment refers to relocating material from a bulk storage area to an order pick storage area, and documenting this relocation. Order picking is the process of removing items from storage to meet customers' demand; this is rather costly, and typically accounts for about 55% of warehouse operation cost (Drury, 1988). Accumulating and sorting is to group picked items into individual orders. Packing includes checking the completeness of orders and putting items in an appropriate container. Shipping is to transfer the orders to customers, which may involve weighing orders to determine shipping charges, staging and accumulating shipments, and loading trucks.

Warehouse operations must deal with different types of uncertainties, which can be classified by the variance structure as (1) unpredictable events like war, strikes, floods, and hurricanes, which usually are rare events, (2) predictable events like demand seasonality, and (3) internal variabilities like variance of order waiting time for batching, which could be caused by internal randomness. These uncertainty sources can be located at different positions: outside the supply chain, in the supply chain but outside the warehouse, inside the warehouse, or within warehouse control systems. Warehouse uncertainty sources can affect decisions at strategic, tactical and operational levels, classified by the planning horizon. Strategic decisions have a long-term effect, tactical decisions have an effect over the medium term (monthly or quarterly), and operational decisions are made on a daily basis. Decisions on warehouse automation level, layout and systems have a strategic effect. Tactical decisions mainly include a tactical storage plan and a tactical order picking plan. Warehouse operational decisions include daily order picking planning, daily resource planning, and daily warehouse information system management.

To handle these uncertainties, many warehouses have attempted innovative approaches to order receiving, putaway, internal replenishment, order picking, accumulating and sorting, packing, cross docking, and shipping to mitigate risks. This has also led to a bigger complexity of warehouse operations (Frazelle, 2001), which raises our academic curiosity and leads to the research objective and research problems, described in the next section.

1.2 Research objectives and research problems

Warehouses have been going through various challenges: "supply chains are shorter and, hopefully, more integrated, the world is smaller, customers are more demanding, and technology changes occur rapidly" (Tompkins et al. (2003), Page 401). To handle these challenges, warehouses employ techniques like pick-to-light, pick-to-voice, RF communication, and RFID picking, and use new warehouse operations like dynamic storage, real-time processing, dynamic picking, self-storage, and self-organizing order picking to improve warehouse operations. It is unclear whether existing models can be applied to describe these new operations in uncertain settings. Existing models can also be found to improve warehouse performance, like minimizing order throughput time and maximizing warehouse space utilization. However, it is unclear whether similar models can be also applied to uncertain business settings. This research will provide insights into improving both responsiveness and efficiency of warehouse systems in a stochastic setting.

On the other hand, stochastic models and theories have evidently evolved in the last 20 years. But their application to warehouse research is limited. Stochastic models may help understand the impact of different parameters like the batch size in order batch picking (a method to group a set of orders into a number of sub-sets so that they can be retrieved by a single picking tour) and number of zones on the system performance, understand which models are best-suited in new dynamic settings, e.g., in an on line order setting, where the waiting time a customer can tolerate is short, and balance the trade-off between system responsiveness and system efficiency. This thesis attempts to bridge this gap and apply state-of-the-art stochastic models and theories to new-emerging warehouse systems.

To achieve this research objective, we will demonstrate the effective application of stochastic models and analysis to warehouse systems, and identify four research problems. First, we explore optimal batching problems in an e-commence distribution center via stochastic optimization. Second, we model real-time order picking systems by stochastic polling models for the warehouses in online retailers. Third, we research bucket brigades order picking systems with various storage profiles by applied probability models and dynamic systems. Finally, we examine public warehouse design problems and revenue management by queuing models.

1.3 Research methodology

The main methods in this thesis are analytical methods, especially stochastic modelling methods like applied probability modelling, stochastic queue modelling, stochastic network modelling, stochastic analysis methods like infinitesimal perturbation analysis and sample path optimization. Stochastic models are constructed to explore warehouse problems such as real-time picking, batching, storage policies decisions and warehouse revenue management.

I also have conducted empirical research, which is mainly used to formulate models and validate their results. For example, for the warehouse revenue management problem in Chapter 6, I have visited self-storage warehouses in America, Europe and Asia. For the research on dynamic picking systems in Chapter 4, I have visited warehouses of online retailers like “buy.com.cn” to help deepen my understanding of the corresponding warehouse operations.

Another method used in this thesis is simulation, which is mainly used to verify models. My main simulation tools were C^{++} and Matlab.

1.4 Contribution

This PhD research makes contributions by reviewing stochastic modelling and analysis of warehouse operations, computing the approximate optimal order batch sizes in a parallel-aisle warehouse, studying the performance of Bucket Brigades order picking systems with various storage profiles, improving a polling-based dynamic order picking system for online retailers, and redesigning a self-storage warehouse to improve its revenue management.

(1) A review on stochastic modelling and analysis of warehouse operations

Chapter 2 provides an overview of stochastic research in warehouse operations. We identify uncertainty sources of warehousing systems and systematically present typical warehouse operations from a stochastic system viewpoint. Stochastic modelling methods and analysis techniques in existing literature are summarized, along with current research limitations. Through a comparison between potential and existing stochastic warehouse applications, we identify potential new research applications. Furthermore, by comparing potential and existing solution methods, methodological directions relevant to practice and largely unexplored in warehouse literature are identified. This research is based on Gong and De Koster (2009b).

(2) Approximate optimal order batch sizes in a parallel-aisle warehouse

Much of the past warehousing literature dealing with order picking and batching assumes a pre-determined batch size. However, selecting a suitable batch size can significantly enhance system performance. Chapter 3 determines optimal batch sizes in a general parallel-aisle warehouse with stochastic order arrivals. We employ a sample path optimization and perturbation analysis algorithm to search the optimal batch size for a warehousing service provider facing a stochastic demand, and a central finite difference algorithm to search the optimal batch sizes from the perspectives of customers and the total system. We prove the existence of optimal batch sizes, and find that past researches underestimate the optimal batch size, as they focus on warehouse service (lead times) only. This research is based on Gong and De Koster (2009a).

(3) A polling-based dynamic order picking system for online retailers

One of the challenging questions online retailers are facing is how to organize the logistic fulfillment processes during and after transaction. As new information technologies become available to convey picking information in real time and with the ongoing need to create greater responsiveness to customers, dynamic picking can be applied to the warehouses of online retailers. In a DPS (dynamic picking system), a worker picks orders that arrive in real time during the picking operations while the picking information dynamically changes in one picking cycle. In Chapter 4, we build models to describe and analyze such systems via stochastic polling theory and find closed-form expressions for the order line waiting times in a DPS. These analytical results are verified by simulation. By applying polling-based picking to two cases, we show it can generally lead to shorter order throughput times and higher on-time service completion ratios than traditional batch-picking systems using optimal batch sizes. We show how our analysis method can be applied to minimize warehouse cost or to improve service. This research is based on Gong and De Koster (2008). For an extension of this research to consider the sorting and the second moment performance measure, see Gong, Winands, and De Koster (2009b).

(4) Improving order picking productivity via storage profiles and bucket brigades

In Chapter 5, we present closed-form analytic expressions for order pick rates of bucket brigades order picking systems with different storage profiles, and show how to combine storage policies and bucket brigade protocols to improve order picking productivity. We further shed light on reasons why the bucket brigades system can outperform some zone picking systems for a range of storage profiles. This research is based on Eisenstein and Gong (2009).

(5) A novel facility design approach to improve revenue management of public storage warehouses

Public storage is a booming industry. Both private customers and companies can rent temporary space from such facilities. A major question is how to design public storage facilities to fit market segments and accommodate volatile demand in order to maximize revenue. Customers that cannot be accommodated with a space size of their choice can either be rejected or upgraded to a larger space. Based on our survey on 54 warehouses in America, Europe and Asia, we propose models for three different cases: an overflow customer rejection model, and two models with customer upgrade possibilities: one with reservation and another without reservation in Chapter 6. We solve the models for several real warehouse cases, and our results show for all cases the existing public-storage warehouses can be redesigned to bring larger revenues. Finally, we develop the robust design to reduce the loss from the variance of demand to the least. This research is based on Gong, De Koster, Frenk, and Gabor (2009a). A relevant research of mine is Zhang, Gong, De Koster, and Van de Velde (2008).

In my PhD study, I develop a novel framework to evaluate the integral performance of order picking systems with different combinations of storage and order picking policies (see Chen, Gong, De Koster, and Van Nunen (2009)). I also use stochastic optimization to research the multi-location transshipment problem with positive replenishment lead times (see Gong and Yucesan (2009)).

2

A review on stochastic modelling and analysis of warehouse operations

2.1 Introduction

In a dynamic business environment full of uncertainties, today's warehouse operations face challenges like the need of shorter lead times or real-time response, to handle a larger number of orders with greater variety, and to deal with innovative processes with a far greater complexity than before. Some online retailers, for example, face customers who purchase by impulse, then change their minds and cancel orders with impunity. Warehouses of these online retailers face uncertainty from real-time order information updates (Gong and De Koster, 2008). Therefore warehouse managers must consider uncertainties from various sources, both from the outside supply chain and from within the warehouse itself. These uncertainties may come from unpredictable rare events, predictable trends, and internal variability of supply chain processes. Each of the uncertainty sources may cause an unanticipated impact on strategic, tactical, or operational decisions, yet must be met on a daily basis in practice.

Deterministic models and algorithms have successfully been applied to warehouse systems (e.g., Ratliff and Rosenthal (1983), Van den Berg et al. (1998), Karasawa et al. (1980), Lowe et al. (1979), White and Francis (1971)). Even though real-world business problems always have some stochastic factors, deterministic models can provide a good approximation in a stable business setting. However, deterministic models may not always suffice in highly variable environments such as systems with strongly fluctuating order patterns and responsive operations (e.g., an online order setting).

To handle problems with internal variability, a number of stochastic warehouse models have been developed (e.g., Bozer and White (1990), De Koster (1994), Chew and Tang (1999), Bartholdi III et al. (2001)). These pioneering researches provide a valuable start for the exploration of stochastic research methods for warehouse operations. One of our motivations is to provide an overview of existing stochastic research in warehousing, and to identify potential application directions. On the other hand, stochastic models and theory have evidently evolved in the last 20 years. Warehouse practitioners and researchers need suitable methods to research warehouse problems in a stochastic environment. Stochastic models may help understand the impact of stochastic factors on operational processes and system performance. While stochastic models are potentially efficient tools for warehouse research, their application to warehouse research is limited. Therefore, another motivation is to bridge this gap by identifying promising stochastic methods for warehouse research.

Several literature reviews on warehousing research exist. Gu et al. (2007) give a comprehensive overview of warehouse research. Van den Berg and Zijm (1999) present a classification of warehouse management problems. Other researchers focus on one particular aspect of warehouse research. De Koster et al. (2007) review the order picking problem. Cormier and Gunn (1992) have classified the warehouse models into three categories, namely, throughput capacity models, storage capacity models, and warehouse design models. Our research takes a totally different view, and provides insights into method issues in a stochastic setting by identifying the uncertainty sources of warehouse operations, presenting a systematic overview of the stochastic models and analysis of warehouse operations, and further presenting promising research directions.

To identify the relevant academic warehouse literature, we searched via "ScienceDirect", "ISI Web of Knowledge" and "Google Scholar", using key words and their derivatives like "warehouse","distribution center","order picking","storage","order retrieval","order receiving" and "order shipping". We identified 583 articles and 41 books on warehousing from 1948 to May 2008 (for a comprehensive list before 2008, see www.roodbergen.com). Literature on subjects such as Automated Guided Vehicles (AGV), facility layout (other than directly applied to warehousing), facility location and inventory models, has not been included. By carefully reading abstract, introduction and conclusion parts, and checking the remaining parts for research methods used in these 583 articles and 41 books, we identify the research using stochastic methods for warehouse operations. Furthermore, we group these papers by modelling types and methods, by analysis types and methods, and by warehouse processes studied. For each group, we discuss representative papers to illustrate the application of a method. We chose such papers mainly by the degree of fit with the category (e.g. full adoption of a method rather than partial adoption).

2.2 Warehouse operations: a stochastic view

This section identifies uncertainty sources of a warehouse system at strategic, tactical and operational levels, and presents uncertainties of a warehouse system in the warehouse arrival, service, and departure processes, three main processes of a stochastic system. The analysis in this section explains the necessity to research warehouse systems by stochastic methods in uncertain business settings, identifies promising opportunities of warehouse research by stochastic models and analysis, and provides a foundation for the further analysis in subsequent sections.

2.2.1 Uncertainty sources of warehouse systems

Uncertainty sources faced by warehouse systems are quite diverse, both within and external to the warehouse systems (Chopra and Sodhi, 2004). We first present the classification of uncertainty sources, and then study the influence of uncertainty sources on warehouse operations and decisions.

According to the location of uncertainty sources, we classify them as (1) sources outside the supply chain, (2) sources in the supply chain but outside the warehouse, (3) sources inside the warehouse, and (4) sources within warehouse control systems. We present this scope dimension on the horizontal axis of Fig. 2.1. According to the variance structure of uncertainties, we classify uncertainty sources as (1) unpredictable events like war, strikes, floods, and hurricanes,

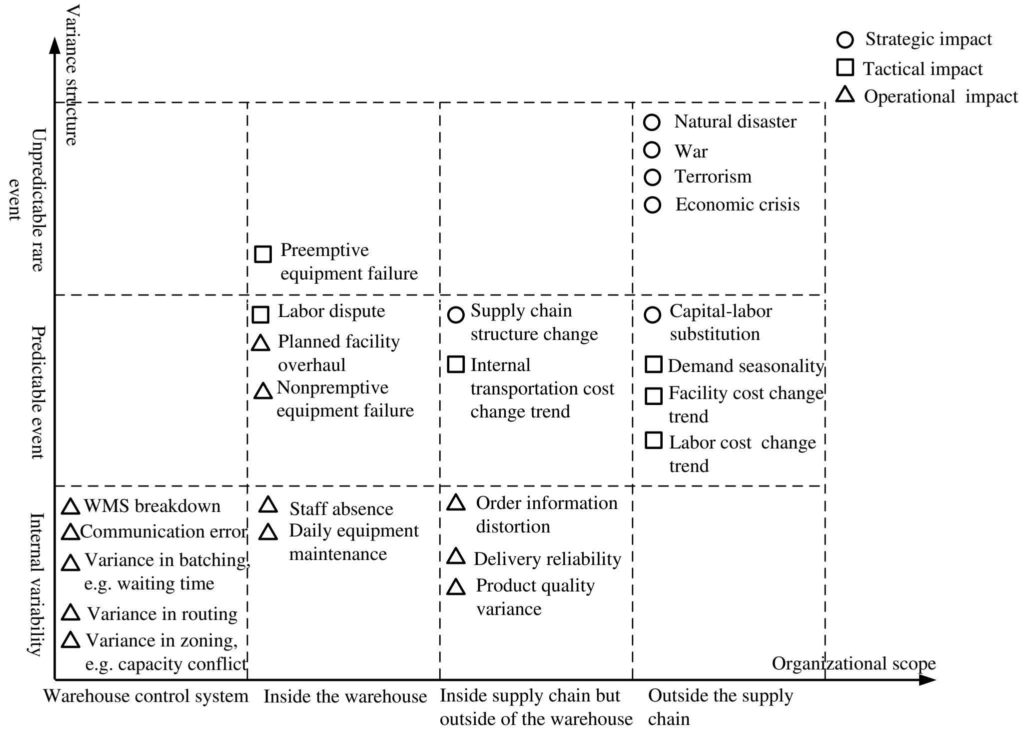


Fig. 2.1. Uncertainty sources of warehouse operations

which usually are rare events, (2) predictable events like demand seasonality, and (3) internal variabilities like variance of order waiting time for batching, which could be caused by internal randomness. We present this classification dimension on the vertical axis of Fig. 2.1. The figure also shows typical examples of uncertainty sources for different types. Examples in Fig. 2.1 primarily distribute along the diagonal of the matrix. External uncertainty sources usually are more unpredictable, and will often bring higher variance to warehouse operations. On the other hand, inside uncertainty sources usually are more predictable and only bring low variance to warehouse operations.

Uncertainty sources can affect decisions at three levels, strategic, tactical and operational, classified by the planning horizon. Strategic decisions have a long-term effect, tactical decisions have an effect over the medium term (monthly or quarterly), and operational decisions have an immediate short-term effect (Ghiani et al., 2004). Decisions on warehouse automation level, layout and systems have a strategic effect. Tactical decisions mainly include storage tactical plan and order picking tactical plan. Warehouse operational decisions include daily order picking planning, daily resource planning, and daily warehouse information system management. We further illustrate the impact of uncertainty sources on these decision levels.

(1) Strategic uncertainty sources

Some system-wide uncertainty sources like natural disaster, war and terrorism can impose a long-term impact on warehouse operations. Other uncertainties, like those in facility price and labor cost, in relation with facility productivity and labor productivity will influence the trade-off between operational capabilities and economic efficiency, and further influence strategic decisions on warehouse automation. Uncertainties in total ownership costs of costly resources (including staff, and key equipment like storage and sorting systems), may affect the financial performance of a warehouse over years.

(2) Tactical uncertainty sources

Tactical uncertainty sources originate from both outside and inside the warehouse's supply chain. Outside sources include economic fluctuation, labor availability, and cost changes of important resources. Preemptive overhaul of key equipments and labor disputes are examples of uncertainty sources inside the warehouse.

(3) Operational uncertainty sources

Uncertainties from human factors have a short-term impact on order picking daily planning, consisting of order batching, routing, and picker task assignment. Among these are manual handling risks and musculoskeletal disorders in distribution centers, as reported by Wright and Haslam (1999). Order information distortion caused by order cancellation can affect daily picking planning. Facility daily planning faces uncertainties of equipment failure and equipment maintenance. Modern warehouses depend heavily on the proper function of information systems. In this respect, they are sensitive to information infrastructure breakdown and errors in the communication with external systems.

We further shed light on the relationship between uncertainty source types and impact levels. Based on the examples distributed along the horizontal axis in Fig. 2.1, we find unpredictable uncertainty sources usually have more strategic or tactical impacts than predictable sources. On the other hand, from examples distributed along the vertical axis in Fig. 2.1, we find outside sources usually have more strategic or tactical impacts, and inside sources usually have more operational impacts. We therefore conclude that, more unpredictable outside uncertainty sources usually have more strategic or tactical impacts, and more predictable inside sources usually have more tactical or operational impacts in warehouse operations.

2.2.2 Uncertainties of warehouse operations

A typical stochastic system can be divided into arrival, service and departure processes. Warehouse operation processes can also be classified likewise. Classifying warehouse processes in these three groups helps us to identify appropriate stochastic models as a clear distinction among arrival, service and departure processes exists. We present the typical warehouse operations in Figure 2.2. This

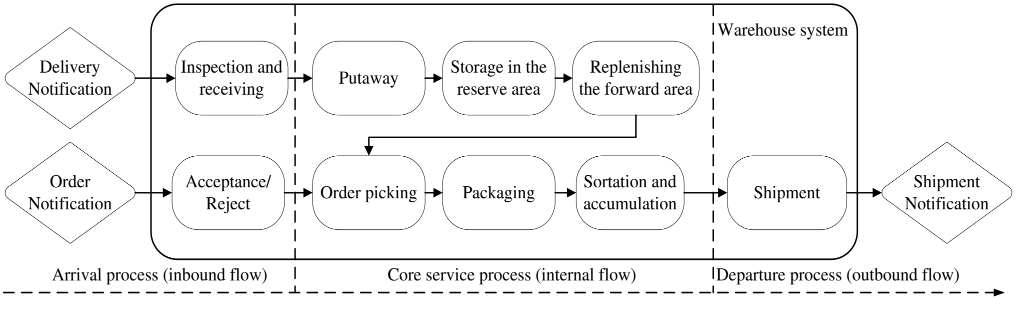


Fig. 2.2. Typical warehouse operations from a stochastic process view

framework is helpful for us to capture the heterogenous stochastic essence and heterogenous uncertainty sources in different operation processes.

In the framework of Figure 2.2, we view warehouse operations associated with inbound flows as arrival processes, which include product arrivals typically followed by an “inspection and receiving” operation, and order arrivals typically followed by an “acceptance and reject” operation. We view warehouse operations, which create or add value and are main processes to support core warehousing functions and mainly deal with internal flows, as core service processes, including putaway, storage in reserve area, replenishing the forward area, order picking, packing, sorting and accumulating. We group warehouse operations associated with outbound flows as departure processes, which mainly includes inspection and shipping. We further describe uncertainty factors in these three processes and summarize them in Table 2.1.

Uncertainties in arrival processes

There are two main arrival processes in a warehouse system. One arrival process is the physical product arrival. The inventory level at suppliers and transportation will influence the arrival rate and uncertainties in the arrival processes. For example, Wilson (2007) investigates the effect of a transportation disruption. Return product arrivals will also increase the arrival variance. After the products have arrived at a warehouse, inappropriate inspection and receiving operations can lead to congestion or delay, and increase the variance of the internal transportation time to the next warehouse operation. Receiving scheduling, prereceiving, receipt preparation have been applied to decrease the uncertainty of the arrival process. Another arrival process is the order arrival process, determined by customer demand, usually a stochastic variable (e.g., seasonality and sales will affect the customer demand; order cancellation will disturb the arrival rate.)

Uncertainties in service processes

Main warehouse operation processes include putaway, storage, order picking, packaging, accumulation and sorting. Not all warehouses will include all these processes.

(1) Putaway. Putaway is a critical operation, since it determines the efficiency, accuracy, and cost of retrieval, and accounts for about 15% of warehouse operational cost (Bartholdi III and Hackman, 2006). Direct putaway eliminates staging and inspection activities. However, without the inspection process, the uncertainties will possibly increase, since potential errors cannot be identified in time. With an efficient WMS, directed putaway can improve efficiency by maximizing location and cube utilization and retrieval productivity.

(2) Storage. Typical storage consists of forward and reserve storage (not all warehouses have their storage system split in forward and reserve). A forward-reserve area storage strategy will improve the efficiencies of order retrieval and picking. In the reserve area, products are stored in pallet racks or block-stacks to achieve a high space utilization. In the forward area with compact size, bin shelving and gravity flow racks are applied to facilitate order picking, and reduce the fluctuation of order picking productivity, compared with the picking in an undivided storage system. The reserve-forward system is a two-echelon inventory system, and imbalance of the inventory level between reserve and forward areas can lead to a greater variance of throughput (e.g., inventory shortage in the forward area will reduce the throughput of order picking).

(3) Order picking. Order picking can be divided in two types of systems: picker-to-parts and parts-to-picker. Parts-to-picker systems include automated storage and retrieval systems, using mostly aisle-bound cranes that retrieve one or more unit loads and bring them to a pick position. Such an automated system may streamline the service process, and can reduce its uncertainty and thereby improve service. In end-of aisle order picking systems, tailored balancing of humans and machines helps to reduce the throughput variance. In picker-to-parts systems, an order picker walks or drives along the aisles to pick items. Two types can be distinguished: low-level picking and high-level picking. In low-level order-picking systems, the order picker picks requested items from storage racks or bins, while traveling along the storage aisles. Pick inaccuracies, i.e., picking a wrong item, can increase the uncertainty of the pick service process. High-level (also called man-aboard) order-picking is used in warehouses with high storage racks. Order pickers travel to pick locations on board of a lifting order-pick truck or crane, which automatically stops in front of the appropriate pick location and waits for

the order picker to perform the pick. If multiple order pickers are used, congestion may occur.

(4) Accumulation, sortation and packaging. Accumulation and sortation of picked orders into individual customer orders is a necessary activity if the orders have been picked in batches. Accumulation and sortation processes usually apply mechanical devices like conveyors, carousel systems and sorters, and man-machine balance will affect the throughput. Mechanical errors like faulty sortations can also cause inaccuracies in accumulation and sortation. Such inaccuracies will increase the uncertainty of the departure process, and may reduce the departure rate. During packing, laborers can check whether customer orders are complete and accurate (Bartholdi III and Hackman, 2006), which can again decrease these uncertainties.

Uncertainties in departure processes

One of the main uncertainties during shipping stems from shipping inaccuracy, i.e., shipping the wrong product to the wrong customer, at the wrong time. Errors in electronic messages can further cause or magnify these uncertainties. Other uncertainties in the departure process arise from departure operations like container loading (e.g., wrong order batch, wrong space calculation for containers), and shipment staging (e.g., human factors cause fluctuations in departure rate). Failure of shipping equipment like trucks, pallet jacks, and counterbalance lift trucks, can also cause uncertainties in this process.

2.3 Stochastic methods in warehouse operations research

Classical deterministic models assume that perfect information is available about the objective function and this information can be used to determine the search direction. However, due to existing uncertainties in warehouse processes (see Table 2.1), such perfect information is usually unavailable. Some researchers therefore resort to stochastic models to analyze warehouse operations. Methods for stochastic models provide a means of coping with inherent system noise and coping with models or systems that are dynamic, stochastic, even unstable, or otherwise inappropriate for classical deterministic methods.

Table 2.1. Warehouse operations with uncertainty factors

Stochastic process	Operation process	Practice	Issues associated with uncertainties
Arrival process	Product arrival	Transportation	Transportation disruption directly affects arrival process and increases the uncertainty.
		Cross-docking	Reduce the variability of throughput time by simplifying operation processes.
		Receiving scheduling	Reduce uncertainty and improve arrival time accuracy by scheduling the receiving resources like dock doors, personnel, equipment, staging space.
		Prereceiving	Reduce uncertainties by capturing information like location assignment and product identification ahead of time.
		Receipt preparation	Decrease arrival uncertainties and improve arrival rate.
	Order arrival	Customer demand	Seasonality and sales will affect the customer demand, order cancellation will disturb the arrival rate.
		Communication	Information system errors between customer and warehouse will increase the uncertainty.
Core service process	Putaway	Direct putaway	Service rate will be improved since direct putaway eliminate staging and inspect activities. Without inspection the uncertainties will possibly increase.
		Directed putaway	Streamline putaway process by maximizing location and cube utilization, and reduce variability of productivity.
		Batch and sequenced putaway	An efficient way to stabilize service rate of putaway and reduce the variability of productivity.
	Storage	Reserve area storage	Achieve the space utilization and reduce the uncertainty of replenishment shortage.
		Forward area Storage	Improve the service rate and reduce the fluctuation of order picking productivity.
	Order picking	Picker-to-parts	Suitable batch and routing polices will improve the service rate. Pick inaccuracy and pick error increase uncertainty.
		Parts-to-picker	The automated system will reduce the uncertainty and improve service rate. Balancing human system and machine systems is helpful to streamline process and reduce uncertainty.
	Packaging, accumulation, sortation	Packaging	Packaging order inaccuracy increase the uncertainty and reduce the departure rate.
		Accumulation and sortation	Sorter mechanical errors can lead to order inaccuracy of accumulation and sortation and increase uncertainty.
Departure process	Shipping	Container loading	Optimization can maximize the cube and utilization of each container and also reduce the uncertainty of utilization.
		Staging activity	The automated operation and direct loading can eliminate staging and its the uncertainty, and improve the departure rate.
		Shipping inaccuracy	Trailer technique can decrease the uncertainty.

Table 2.2. Stochastic models in warehouse operations

Type	method	Research examples	Problem statement
Classical probability models	Urn models	Chew and Tang (1999)	Analyzing the picking systems by urn models.
		Le-Duc and De Koster (2005)	Travel distance estimation in a 2-block class-based storage strategy warehouse.
Classical stochastic models	Renewal process models	Bozer and White (1990)	The basic configuration is modeled as a renewal process in end-of-aisle order picking system.
	Markov chain model	Gue et al. (2006)	Model the circular picking area with two workers as a Markov process.
Queueing models	Single queueing models	Lee (1997)	Analyzing a unit-load AS/RS by a single-server queueing model with two queues and two different service modes.
	Queueing networks models	De Koster (1994)	Performance approximation of pick-to-belt order picking systems.
	Polling models	Gong and De Koster (2008)	A polling-based warehouse dynamic picking system for online retailers.
		Bozer and Park (1999)	Single-device polling-based material handling systems.
Others	Fluid models	Bartholdi III et al. (2001)	Bucket bridges problem when work is stochastic.
	Petri-net models	Hsieh et al. (1998)	Present a Petri-net-based structure to describe and model AS/RS operations.
		Lin and Wang (1995)	Modelling an automated storage and retrieval system using Petri nets.

Various stochastic models have been applied by warehousing researchers (see Table 2.2). First, much order picking work adopts classical probability models, defined by a sample space, events within the sample space, and probabilities of each event, including basic probability models like the binominal, the Bernoulli, the geometric, the hypergeometric models and their derivatives like the urn model. For example, Chew and Tang (1999) analyze order picking operations in a 1-block warehouse and Le-Duc and De Koster (2005) analyze warehousing operations in a 2-block class-based storage strategy warehouse by basic probability models

Table 2.3. Stochastic analysis in warehouse systems

Type	Method	Research examples	Problem statement
Optimization	Stochastic constrained optimization	Azadivar (1986)	To determine the maximum number of storage and retrieval requests in automated warehousing systems.
	Perturbation analysis	Gong and De Koster (2009a)	Approximate optimal order batch sizes in a parallel-aisle warehouse.
	Kuhn-Tucker condition	Jucker et al. (1982)	The simultaneous determination of plant and leased warehouse capacities for a firm facing uncertain demand in several regions.
	Petri-net based technique	Archetti et al. (1991)	Adopted Petri-net models and a stochastic optimization method to study optimal control policies of an AS/RS.
Heuristic	Analytical approximation	Bozer and White (1996)	Present two efficient heuristic algorithms for design and performance analysis for end-of-aisle order-picking system.
Simulation	A tool based on Promodel	Macro and Salmi (2002)	Invented a simulation tool to determine warehouse efficiencies and storage allocations based on Promodel.
	MC simulation	Rosenblatt and Roll (1988)	Analyzing warehouse capacity in a stochastic environment by MC simulation.
	Petri-net based simulation	Hsieh et al. (1998)	Propose a Petri-net based four-layer simulation structure for the AS/RS.
	Enumeration	Stadtler (1996)	Optimize dimensions for automated warehouse systems by a procedure consisting of enumeration simulation.
Others	Determine limiting behavior	Litvak (2006)	Determine a limiting behavior of the shorted rotation time needed to collect large orders in a carousel system
	Matrix geometric analysis	Bastani (1990)	Analyze closed-loop conveyor systems by M/M/s system and an matrix geometric solution.

(specifically, the binominal model and urn models) to determine the locations from which articles must be picked in a pick tour and thereby the tour length. Yu and De Koster (2009) have studied the impact of order batching and picking area zoning on order picking system performance by classical probability models.

Second, classical stochastic models (like renewal process models, Markov models, Martingales models) are also helpful to describe warehousing operation processes. For example, Bozer and White (1990) model order picking operations in an end-of-aisle order picking system as a renewal process, where an event occurs when both pickers and the storage/retrieval (S/R) machine begin service. Gue et al. (2006) model a circular picking area with two workers as a Markov process when they research the effects of pick density on order picking areas with narrow aisles.

Third, various queueing models (including single-server queueing models like $M/M/1$ and $M/G/1$, queueing network models, and their derivatives like the polling model) are frequently used in warehousing research. Lee (1997) has examined a unit-load AS/RS by a single-server (an S/R machine) queueing model with two queues and two different service modes(storage requests and retrieval requests). According to Bozer and Cho (2005), this is the first study using stochastic analysis of a unit-load AS/RS by an analytical method. Queueing networks are also helpful for warehouse modelling. De Koster (1994) has researched performance approximation of zoned order picking systems by a Jackson queueing network. Polling models, a special queueing network type, have also drawn the attention of warehousing researchers. Bozer and Park (1999) have studied single-device, polling-based material handling systems. Gong and De Koster (2008) apply stochastic polling models to a warehouse dynamic order picking system for an online retailer.

Besides the above three main types of methods, several other techniques have been introduced to warehouse research. For example, Bartholdi III et al. (2001) have researched bucket brigades where the work is stochastic by fluid models. Hsieh et al. (1998) and Lin and Wang (1995) model an automated storage and retrieval system using stochastic Petri nets, and their models can be used to evaluate the performance and optimize control policies. These pioneering researches provide new exploration in warehouse research by stochastic methods.

To analyze these stochastic models, researchers have adopted various methods like optimization, heuristics, and simulation. Some typical examples of each of these methods are listed in Table 3. Stochastic optimization refers to the minimization (or maximization) of a function in the presence of randomness in the optimization process, which applies to one or both of the following conditions. (1) There is random noise in the measurement of the objective function; (2) A random (Monte Carlo) choice is made in the search direction as the algorithm iterates towards a solution. By a stochastic constrained optimization algorithm (a simulation optimization algorithm), Azadivar (1986) has determined the maximum number of storage and retrieval requests that can be handled by automated warehousing systems under physical and operational constraints. Jucker et al. (1982) develop an efficient algorithm based on Kuhn-Tucker conditions for simultaneously determining the plant and leased warehouse capacities for a firm facing uncertain demand in several geographical regions. Archetti et al. (1991) have adopted Petri-net models and a stochastic optimization method to study optimal control policies of an AS/RS.

Heuristic methods have also achieved successful application. For example, Bozer and White (1996) present two efficient heuristic algorithms for design and performance analysis of end-of-aisle order-picking operations based on a miniload AS/RS. The algorithm is based on an approximate analytical model developed to estimate the expected picker utilization for a general system configuration.

Simulation has been widely adopted by warehousing researchers. Macro and Salmi (2002) analyze the storage capacity and rack efficiency of a medium volume, low stock-keeping unit (SKU) warehouse and a medium volume, large SKU warehouse by Promodel. The model can be applied to simulate various warehouse configurations like bulk floor storage, push-back, flow-through, drive-in and drive-through racks. Rosenblatt and Roll (1988) have analyzed warehouse capacity in a stochastic environment by Monte Carlo simulation. Stadtler (1996) optimizes dimensions for automated warehouse systems by a procedure consisting of enumeration simulation. Hsieh et al. (1998) propose a Petri-net based four-layer simulation structure as a general tool to model the operations, evaluate the performance and develop control policies of an AS/RS.

Besides these approaches, some other stochastic analytic methods exist. For example, Litvak (2006) determines the limiting behavior of the shortest rotation time needed to collect large orders in a carousel system. Bastani (1990) analyzes closed-loop conveyor systems with breakdown and repair of unloading stations by an M/M/s queueing system and provides an approximation of the steady-state probabilities of the system in different operating states by the matrix geometric technique.

In the last 20 years, we can witness a rapid development of stochastic optimization techniques, including stochastic programming and stochastic approximation. From our review, however, we find while simulation and heuristics are widely applied to warehouse research, stochastic optimization is hardly used.

2.4 Stochastic applications in warehouse operations

This section examines the application of stochastic methods in main warehouse operations, including storage, order picking, packing, sorting, accumulation, and distribution. For order picking, where many stochastic researches exist, we examine applications in three systems: picker-to-parts systems, parts-to-picker systems, and automated picking systems (Van den Berg, 1999).

(1) Storage

Storage is a main function of a warehouse, and a large number of papers research it by deterministic methods. Stochastic research in this area, however, is not abundant (compared with order picking). Noteworthy examples include Van den Berg et al. (1998), who have studied forward-reserve allocation in a warehouse with unit-load replenishments. Roll et al. (1989) present analytical and simulation methods to determine the size of storage containers in a warehouse with an objective to minimize the storage cost. Chang and Wen (1997) research the impact on the rack configuration on the speed profile of storage and retrieval machines and present an analytical procedure to obtain the optimal rack configuration in an AS/RS.

(2) Order picking: picker-to-parts systems

Stochastic research in this area is abundant. For example, Gue et al. (2006) build a stochastic throughput model to explore the effect of pick density on order picking areas with narrow aisles.

(3) Order picking: parts-to-picker systems

Parts-to-picker systems in general have a high automation level, and it is convenient to model such systems by stochastic models. For instance, Bozer and Cho (2005) derive closed-form analytical expressions for throughput performance of an AS/RS under stochastic demand, and also derive an analytical estimate for the expected S/R machine utilization. Park et al. (1999) model an end-of-aisle order picking system as a two-stage cyclic queueing system consisting of

Table 2.4. Stochastic application in warehouse operations

Warehouse operations	Research examples	Problem statement.
Storage	Van den Berg et al. (1998)	Study forward-reserve allocation in a warehouse with unit-load replenishments.
	Roll et al. (1989)	Present an approach to determine the size of a warehouse container.
	Chang and Wen (1997)	Present an analytical procedure to obtain the optimal rack configuration in an AS/RS.
Order picking: Picker to parts	Gue et al. (2006)	Build a stochastic throughput model to explore the effects of pick density on order picking areas with narrow aisles.
	Roodbergen and Vis (2006)	Apply probability models to the layout design in a picker-to-parts warehouse.
Order picking: Parts to picker	Bozer and Cho (2005)	Present an analytical result of throughput performance of AS/RS under stochastic demand.
	Park et al. (1999)	Present queue models for end-of-aisle order picking systems with buffer positions.
Order picking: Automated picking	Azadivar (1986)	Maximize the throughput in a computerized automated warehousing system.
	De Koster et al. (2008)	Consider a newly designed compact three-dimensional AS/RS with automated picking.
Packing, sorting, accumulation	Johnson (1998)	Study the impact of sorting strategies on automated sortation system performance.
	De Jong and Anderson (1995)	Study the setting of shelf heights and the distribution of box sizes in two-dimensional shelf packing.
Distribution	Yu and De Koster (2008)	Research performance approximation and design of a pick-and-pass system.
	Le-Anh (2005)	Study intelligent control of vehicle-based internal transport systems.

one general and one exponential server queue with limited capacity, and present closed-form expressions for system performance measures like throughput.

(4) Order picking: automated picking systems

The number of implementations of automated picking systems is growing. However, only few papers in this area exist. An example is Yu (2008), who studies dynamic picking systems where the pick face is replenished automatically and dynamically from bulk stock. Since automated picking is a rapidly growing area of interest and since order profiles and storage location selection are stochastic, stochastic modelling of these systems could be explored further.

(5) Packing, sorting, accumulation

Although several papers research packing, sorting, accumulation combined with order picking, few papers focus on these processes using stochastic methods. An exception is Johnson (1998), who studies the impact of sorting strategies on automated sortation system performance by a stochastic analytical model.

(6) Distribution

Distribution, including internal transport, inbound and outbound shipping, is critical to improve the overall performance of a warehouse system. Research in internal transport is quite abundant. Many papers research vehicle-based internal transport systems like AGVs. Le-Anh and De Koster (2006) present an overview paper on this topic. However, receiving and shipping processes are not especially studied in current literature.

This section is summarized in Table 2.4. By comparing existing warehouse operations with uncertainties (see Table 2.1 and Figure 2.2) and warehouse operations with stochastic studies (Table 2.4), we can identify interesting academic blanks. These potential research directions will be further presented in Section 6.1.

2.5 Current research limitations: model, parameter, process details

In this section we present limitations of past studies on warehousing, from a stochastic modelling angle. We focus on model inaccuracies referring to limitations from adopting inaccurate mathematical (specially probabilistic or stochastic) models, parameter estimation inaccuracies due to errors in parameter estimation, and process inaccuracies due to oversimplifying warehouse processes or overlooking important processes.

2.5.1 Arrival process

The order arrival process is often modeled as a Poisson process, possibly with a time-varying arrival rate. However, the typical Poisson model cannot always accurately describe arrival processes in practice.

(1) Model inaccuracies. The arrival process might not be well modeled by a Poisson process. One reason is that customer orders can be dependent. For example, students in one business school could order the same book at Amazon. Order arrivals from these students are then correlated. Many researchers model order line arrivals as a Poisson process(e.g., Lee (1997), Axsäer (1995)). However, one order can include several line items, and these line items are dependent. Hence it is inaccurate to model the order line arrival stream as a Poisson process. Some researchers explicitly model correlated products. Frazelle and Sharp (1989) conduct a simulation of a miniload AS/RS where correlated products are stored in the same bins, and report a reduction of 30-40% in the number of retrieval trips compared with that in a setting of random product assignment. A non-homogeneous Poisson process, with a time-dependent rate parameter, may be more suitable for some warehouses. An example of a non-homogeneous Poisson process would be the order arrival rate to the warehouse of an online food retailer, where the arrival rate increases before dinner time and decreases during the remaining parts of the day.

(2) Parameter inaccuracies. Usually we do not know arrival rates or product correlation coefficients and must estimate them. For a time-varying arrival rate, we even need to estimate the arrival rate function. In a warehouse, there are a variety of information sources to use for the estimation. However, existing research usually has not provided a convincing justification for the parameter estimation. A more accurate estimation of the order arrival rate from demand data and the product arrival rate from supplier information is needed. One can assume a parametric form for the arrival-rate function, such as linear or quadratic. Massey et al. (1996) have explored the method to estimate the coefficients of linear arrival rate functions from nonhomogeneous Poisson process data.

(3) Process inaccuracies. Both order and product arrival processes may be inaccurately described. For example, in order arrivals, all research on receiving operations assume a warehouse does not reject orders. But in many cases, a warehouse can reject part of the orders to maximize the revenue. In the case of online retailers, customers can cancel orders, forming a negative arrival process. For product arrivals, existing research hardly considers arrival uncertainty due to

product quality variance, transportation disturbance, and associated rework and product reject flows.

2.5.2 Service process

Warehouse service processes include picking at a storage position, travel between positions, packing, and other processes. However, unsuitable model selection, impractical parameter estimation, process oversimplification can induce inaccuracies.

(1) Model inaccuracies. Many order picking papers assume picking time is constant, but it is not always acceptable to overlook the variance of picking time. Others modeled service time as a sequence of independent and identically distributed random variables, each with an exponential distribution. Bozer and Cho (2005) point out that the coefficient of variation for single command and dual command cycles are known to be considerably less than one in an AS/RS, and "exponentially distributed S/R service times produces results inconsistent with simulation". We can also find other cases where the actual service-time distribution is not exponential. For example, pick time can depend on item types (e.g., the pick time for large items can be longer than for small items) and ergonomic factors (e.g. the pick time of laborers can become longer due to fatigue).

(2) Parameter inaccuracies. Parameters (e.g., pick rate) may be inaccurately estimated. Examples causing parameter inaccuracies include ergonomic factors, which may cause productivity to decrease over time, dependent on, for example, the frequency and length of short breaks, and item heterogeneity which causes the variance of service time. Better parameter estimation can be obtained by analyzing historical data or ergonomic experiments.

(3) Process inaccuracies. The existing literature often pays no attention to several important factors in service processes. First, in several queueing models studying order picking, capacity limitations, including order picker capacity and cart capacity, are overlooked. But this capacity limitation changes the pick process. The second noticeable problem is order correlation, which will affect the order pickers' behavior and picking process, and make an exponential serve time assumption unrealistic. Finally, most literature overlooks the congestion problem,

which has a significant effect on service processes. For example, in the forward-reserve problem (see Van den Berg et al. (1998)), concurrent replenishments may cause congestion in the order picking process. Gue et al. (2006) are among the first to consider the factor of congestion in order picking systems, and describe this process more accurately.

2.5.3 Departure process

The departure process is often modeled as a Poisson process or even overlooked. However,the departure process is directly associated with customer satisfaction. It is important to enhance warehouse performance by improving delivery accuracies and velocities of the departure process.

(1) Model inaccuracies. Departure streams are possibly dependent. For example, departures to the same destination are highly correlated. Furthermore, irregularity uncertainties exist (see Table 2.1). Shipping inaccuracies (e.g., wrong product, wrong destination), which frequently occur in practice,will disturb the departure process (e.g., by changing the destination during the shipping process). In that case, a Poisson process may be unsuitable to model the departure process.

(2) Parameter inaccuracies. Parameter inaccuracies exist also in warehouse shipping. The estimation of departure parameters will benefit by explicitly considering transportation distortion and shipping inaccuracy. It can be done by analyzing historical data.

(3) Process inaccuracies. Existing research often assumes the departure process to be a Poisson process, which may not accurately capture its essence. Batch delivery, a typical departure process in practice, cannot be described by a classical Poisson process. Furthermore, customers may be not satisfied with a shipped product and return it, a process typical for online retailers. Therefore a return flow may exist in the departure process.

2.6 Concluding remarks

In this chapter, we present a literature review of stochastic modelling and analysis of warehouse operations. We identified strategic, tactical, and operational uncertainty sources, and systematically explored uncertainties of arrival, service, and departure processes in a warehouse. These uncertainties explain why researchers might resort to stochastic rather than deterministic models in some uncertain environments.

In the past, deterministic models have achieved successful applications in warehouse research. Researchers may be inclined to think stochastic research is limited in this field. However, we find not only a large number of stochastic applications, but also a great variation in methods. These improve our understanding of warehouse research.

Nevertheless, we find the application of stochastic methods in warehouse research could be explored further. We identify several directions highly relevant to practice and largely unexplored in warehouse literature, including real-time response models, warehouse revenue management, receiving management, and shipping management which can be explored by methods like stochastic programming, stochastic combinatorial modelling, and stochastic networks modelling.

Part II

Stochastic Modelling and Analysis for Online Retailer Warehouses

3

Approximate optimal order batch sizes in a parallel-aisle warehouse

3.1 Introduction

Order picking - the process of retrieving products from storage (or buffer areas) in response to a specific customer request - is the most labor-intensive operation in warehouses with manual systems, and a very capital-intensive operation in warehouses with automated systems (see Goetschalckx and Ashayeri (1989), Tompkins et al. (2003)). Managing order picking systems effectively and efficiently is a challenging process in many warehouses. Order picking efficiency can often be improved by order batching (Gademann and Van de Velde, 2005), which is a method to group a set of orders into a number of sub-sets, each of which can then be retrieved by a single picking tour (De Koster et al., 2007).

Many of the earlier papers dealing with the order batching problem assume the batch size is directly given. In some cases, the batch might be determined by the capacity of the picking cart. When the cart capacity is not restrictive, a natural question is: Are these given batch sizes suitable? Considering setup time and unit service time for order picking, the total service time and batch size are not related linearly: The setup time will take a bigger proportion in the total service time for a small batch, while the unit service time will take a bigger proportion in the total service time for larger batches. Therefore it is an interesting question to explore the optimal batch size when orders arrive according to a stochastic process. A following research question is: How to find an optimal batch size if it exists? Most research involved in optimizing batch sizes, with the objective to minimize total service times assumes the order set is given. Gademann and Van de Velde (2005) have pointed out that the deterministic version of the batching problem with optimal routing is $\mathcal{NP}$- hard when the batch size is larger than 2, in a parallel-aisle layout. It is tough to determine optimal order batching for the stochastic version of the problem. Few papers explore optimum batching, including batch size determination in a stochastic context. In this chapter, we focus on the batch size problem and assume orders arrive online and orders are batched in a FCFS sequence. Chew and Tang (1999) assume orders arrive according to a Poisson process and approximate the travel time in a rectangular warehouse and use this approximate expression to minimize the total throughput time of the first order in a batch. They compare their results with simulation. Le-Duc and De Koster (2007) extend these results by determining the optimal batch size minimizing the throughput time of a random order in a two-block warehouse. All methods use approximation methods and do not directly optimize the batch size, as this is very cumbersome. In this paper we opt for a different approach, by efficient simulation optimization. Simulation optimization can help the search for an improved policy while allowing for complex features that are typically outside the scope of analytical models. We employ SPO (sample path optimization), a simulation optimization technique with the advantage of high efficiency and convenience. However, SPO requires a technique to estimate the gradient of the objective function with respect to the batch size.

A large number of gradient estimation techniques exist, such as Infinitesimal Perturbation Analysis (IPA), Likelihood Ratios, Symmetric Difference, and Simultaneous Perturbation (Fu, 2002). IPA is mainly used to calculate a sample path derivative with respect to an input parameter in a discrete event simulation (Heidelberger, Cao, Zazanis, and Suri, 1988).We will employ this technique since it is an "efficient gradient estimation technique" (Ho, Eyler, and Chien, 1979), which can "expedite the process of performing experiments on discrete event simulation models" (Johnson and Jackman, 1989). The implicit assumption of IPA is that the average of the change which results from the perturbation equals the change in expectation, and it yields an unbiased estimator. Convergence is an important issue for the implementation of IPA. Heidelberger, Cao, Zazanis, and Suri (1988) have studied the convergence properties of IPA sample path derivative, and derived the necessary and sufficient condition for the convergence. Applications of perturbation analysis have been reported in simulations of Markov chains (Glasserman, 1991), inventory models (Fu, 1994), supply chain problems (Gong and Yucesan, 2009), manufacturing systems (Glasserman, 1994), finance (Fu and Hu, 1997), and statistical process control (Fu and Hu, 1999). In some formulations in this chapter, we will face complicated objective functions. In order to obtain optimal batch sizes, we need to compute the gradient of these objective functions. However, either their gradients are not available in explicit forms or they are given by complicated expressions. We therefore resort to a finite difference method, which makes it possible to use arithmetic operations to determine the gradient.

In this chapter, we consider the optimal order batch size problem with stochastic demand in a parallel-aisle warehouse (see Figure 3.1), with cross aisles at the front and back of the aisles. The warehouse faces a demand with a given distribution. An order picker travels at a constant velocity with a S-shape routing policy, one of the most common routing policies in practice. In order to improve picking efficiency, orders are batched.

The research objective in this paper is to minimize the operational costs by optimizing the order batch size, defined as *the set of orders that are picked by one order picker in one route, and batch size q is the number of items in the batch, with constraint $q^{LB} \leq q \leq q^{UB}$, where the upper bound q^{UB} is determined by the capacity of pick devices (pallets or bins) and the lower bound q^{LB} is specified by an additional condition like system stability.* To achieve this research objective, we consider three major research questions and build corresponding models as follows.

First of all, we examine the operational cost from the perspective of a warehousing service provider and build the corresponding Model-1. This model focuses primarily on an internal objective by minimizing the average total service time, which is the sum of setup time and travel time. We exclude retrieval time as this is not influenced by the batching policy. Orders are picked in a FIFO sequence. Model-1 emphasizes the impact of order batching on performance of a warehousing service provider. Secondly,we examine the cost for customers and build a corresponding Model-2, which is taken from Chew and Tang (1999). The contribution in Model-2 is to provide an efficient finite difference algorithm. While using straightforward simulation takes much time to obtain a solution by enumeration, our method takes on average 6 seconds to get one solution. Finally,we consider the total cost for both the warehousing service provider and the customers by combining Model-1 and Model-2 into a new Model-3. The contribution of this research is twofold. First we show SPO and perturbation analysis algorithms are efficient in deriving optimal values. Second we combine the perspectives of both customers and a warehousing provider in one model and show it can also be solved by perturbation analysis and an SPO algorithm.

The remainder of the chapter is organized as follows: in the following section, we search optimal batch sizes for warehousing service providers in a general stochastic parallel-aisle warehouse by sample path optimization and infinitesimal perturbation analysis techniques. Section 3.3.1 is devoted to an efficient finite difference algorithm to search optimal batch sizes for customers. In Section 3.4, we present a model with the objective of minimizing the total cost, and provide an efficient finite difference algorithm to search the optimal batch size. We conclude with final discussion, contribution summary and further research in Section 3.5.

3.2 Optimal order batch sizes to minimize the cost of a warehousing service provider

3.2.1 Model

Model-1's objective is to minimize the total expected operation time of a warehousing service provider $E[T_p(q, D)]$, where q is the decision variable, the subscript p indicates a warehousing service **p**rovider, and D is the demand, the number of order lines per period, generated from a given distribution $f(D)$. $E[T_P(q, D)]$ is the product of the expected number of batches $E[D/q]$ and the expected operational time of one batch. Following Chew and Tang (1999), we do not explicitly consider retrieval time since the batching policy does not influence the total retrieval time for a given demand. But the batch size does influence the total setup time and the total expected travel time. Therefore, in our model the expected operational time of one batch is the sum of a setup time β and an expected travel time $E[L(q)/v]$, where $E[L(q)]$ is the expected travel distance and

v is a constant travel velocity. $E[L(q)]$ depends on the warehouse layout and the routing method. We assume a rectangular, parallel-aisle layout and an S-shape routing method (see De Koster et al., (2007)). For this environment, Chew and Tang (1999) have found a closed form approximate expression for $E[L(q)]$. We have

$$Model-1: \min_{D\sim f(D), 0\leq q\leq E[D]} E[T_P(q,D)]$$
$$s.t. \quad E[T_P(q,D)] = E[L(q)/v+\beta]E[D/q]$$

3.2.2 Algorithm

This section demonstrates how to obtain the optimal batch size quantities in a parallel-aisle warehouse with stochastic demand. Our scheme is to use the simulation optimization algorithm by combining sample path optimization and perturbation analysis to examine optimal order batch sizes.

Algorithm Description

To compute the optimal batch size values, we adopt a sample path optimization technique as main algorithm, where we use IPA (Infinitesimal Perturbation Analysis) to calculate the gradient value. We start with an arbitrary batch size q^1. After randomly generating an instance of the demand, we construct and solve Model-1 in a deterministic fashion. Then, we compute gradient values by the decision tree from the perturbation analysis. The procedure is summarized in a pseudo-code format in the following procedure, where K denotes the total number of steps taken in a search path of the main algorithm, U represents the total number of steps in one inner cycle which is to provide a gradient estimation at one step of the main algorithm, α_k represents the step size at the each iteration k, and q^k represents the batch size at the k^{th} step. The choice of step size is important to guarantee convergence of the batch size. A proper choice will be explained further in Lemma 3.4.

Algorithm 1

(I)*Initialization.*

(I.1) *Initialize* K.

(I.2) *Initialize* U.

(I.3) *Initialize* q^1.

(I.4) *Initialize* $\alpha_1 = \alpha/1$ *for a constant* α.

(II) *Set* $k \leftarrow 1$.

Repeat.

Set $u \leftarrow 1$.

Repeat.

(II.1)A. *Generate the demand* d_u^k *from* $f(D)$.

(II.1)B. *Compute the objective value of Model-1 in a deterministic fashion.*

(II.1)C. *Compute and accumulate gradients* dL_u^k *by perturbation analysis.*

$u \leftarrow u + 1$,

Until u=U.

(II.2) *Compute the desired gradients* $\frac{\partial E[L]}{\partial q}|_{q=q^k} = \frac{1}{U}\sum_{u=1}^{U} dL_u^k$, $E[D]^k = \frac{1}{U}\sum_{u=1}^{U} d_u^k$, *and* $E[L]^k = \frac{1}{U}\sum_{u=1}^{U} L_u^k$ *at the k step.*

(II.3) *Calculate the desired gradients* $dE[T]_q^k = \frac{1}{v}\frac{\partial E[L]}{\partial q}|_{q=q^k}\frac{E[D]^k}{q^k}$ $+(\frac{1}{v}E[L]^k + \beta)(-\frac{E[D]^k}{(q^k)^2})$.

(II.4) *Update the batch size by* $q^{k+1} \leftarrow \lfloor q^k - \alpha_k dE[T]_q^k \rfloor$, *where* $\alpha_k = \alpha/k$.

$k \leftarrow k + 1$,

Until $k = K$.

(III) *Return the* $\{q^k\}_{k=1}^{K}$ *and the objective function value.*

We explain the procedure as follows:

I)Initialization. The algorithm starts with an arbitrary value for the batch size q^1. K and U are given and can be determined by a pilot study to solve the following trade-off: while a small K cannot provide sufficient data, and output will have a big variance, a too large K is inefficient to improve the optimal value.

II) The main loop in step (II) is an outer loop with K steps. Each step includes a U-step inner loop computation in step (II.1), IPA analysis in step (II.2),the desired gradient calculation in step (II.3), and the updating of batch sizes in step (II.4).

We first run an inner loop with U steps. At each step of the inner loop, we generate the demand from distribution $f(D)$, solve the problem of Model-1 in a deterministic fashion once the demand is observed, and calculate the perturbation value dL_u^k. Secondly, we conduct critical computation $\frac{\partial E[L]}{\partial q}|_{q=q^k} = \frac{1}{U}\sum_{u=1}^{U} dL_u^k$, which is just the IPA technique. Thirdly, we compute the gradient of the expected travel time with respect to the batch size by $dE[T]_q^k = \frac{1}{v}\frac{\partial E[L]}{\partial q}|_{q=q^k}\frac{E[D]^k}{q^k} + (\frac{1}{v}E[L]^k + \beta)(-\frac{E[D]^k}{(q^k)^2})$. Finally, we update batch sizes q^{k+1} by $q^{k+1} \leftarrow \lfloor q^k - \alpha_k dE[T]_q^k \rfloor$ at the k^{th} step. Also note that since the algorithm stops at $k = K$, we do not need an extra stopping rule here.

(III) Return the q^k and objective function value at each step. Then we can conduct the output analysis.

Algorithm Justification

If an algorithm converges and the objective function subject to minimization is convex, the algorithm can provide a global optimal value. In order to justify Algorithm 1, we build four lemmas. Lemma 3.1 is to justify the convexity of our objective function. Lemma 3.2, 3.3, and 3.4 will show the convergence of Algorithm 1. In the following,we use the notation below:

M=the number of aisles;

H=the length of aisles. We assume a bin containing one kind of item has one unit length;

ω =the cross distance between two consecutive aisles;

$\overline{D}$= the mean of demand;

N_a=the number of visited **a**isles;

E = the even number set;

Θ= the odd number set;

ξ_i= the aisle position of item i, $\xi_i = 1, ..., M$;

η_i= the location position of item i within an aisle, $\eta_i = 1, ..., H$;

Ω_q=the position set covered by a route when the batch size is q. The position of an item $i \in \{1, .., q\}$ is indicated by (ξ_i, η_i);

$\xi^* = max(\xi_i, i \in \{1, .., q\})$ be the farthest aisle visited;

$\eta^* = max(\eta_i, \forall i, s.t. \xi_i = \xi^*)$ be the farthest position at the farthest aisle visited.

In order to examine the convexity of objective function $E[T_P(q, D)]$. We first establish Lemma 3.1 as follows.

Lemma 3.1. *The objective function* $E[T_P(q, D)] = E[L(q)/v + \beta]E[D/q]$ *is a convex function of* q.

Proof: For item locations with uniform distribution, Chew and Tang (1999) have given an approximate distance estimation for the S-shape routing policy. Chew and Tang (1999) obtained the approximate travelling time by assuming that an additional of half aisle length is traversed regardless whether the number of total aisles visited is odd or even, and the approximate value is within $\frac{H}{2v}$ of the exact expected travelling time. Based on their result, we have

$$E[\frac{L(q)}{v} + \beta] = \frac{M}{v} H[1 - (1 - \frac{1}{M})^q] + 2\frac{\omega}{v}[M - \sum_{j=1}^{M-1} (\frac{j}{m})^q] + \frac{H}{2v} + \beta. \tag{3.1}$$

For a constant θ with $0 < \theta < 1$, $f(q) = -\theta^q$ is a concave function of q. So $E[L(q)/v+\beta]$ here is a concave function. $E[D]/q$ is nonincreasing convex function of q. From Boyd and Vandenberghe(2004), the product $E[L(q)/v + \beta]E[D/q]$ of a concave function $E[L(q)/v + \beta]$ and a nonincreasing convex function $E[D]/q$ is convex. □

In this algorithm, it is critical to find an efficient gradient estimator. We use perturbation analysis to compute this gradient. Perturbation analysis is a powerful technique for the efficient performance analysis of dynamic systems. Its fundamental approach is to keep track of information along a perturbed path. The main principle behind perturbation analysis is that if a decision variable of a system is perturbed by a small amount, the sensitivity of the response of the system to that variable can be estimated by "tracing its pattern of propagation through the system " (Carson and Maria, 1997). This will be a function of "the fraction of the propagations that die before having a significant effect on the response of interest" (Carson and Maria, 1997). The fact that all derivatives can be derived from the same simulation run represents a significant advantage to

IPA in terms of the efficiency. With the support of this technique, we have the Lemma 3.2.

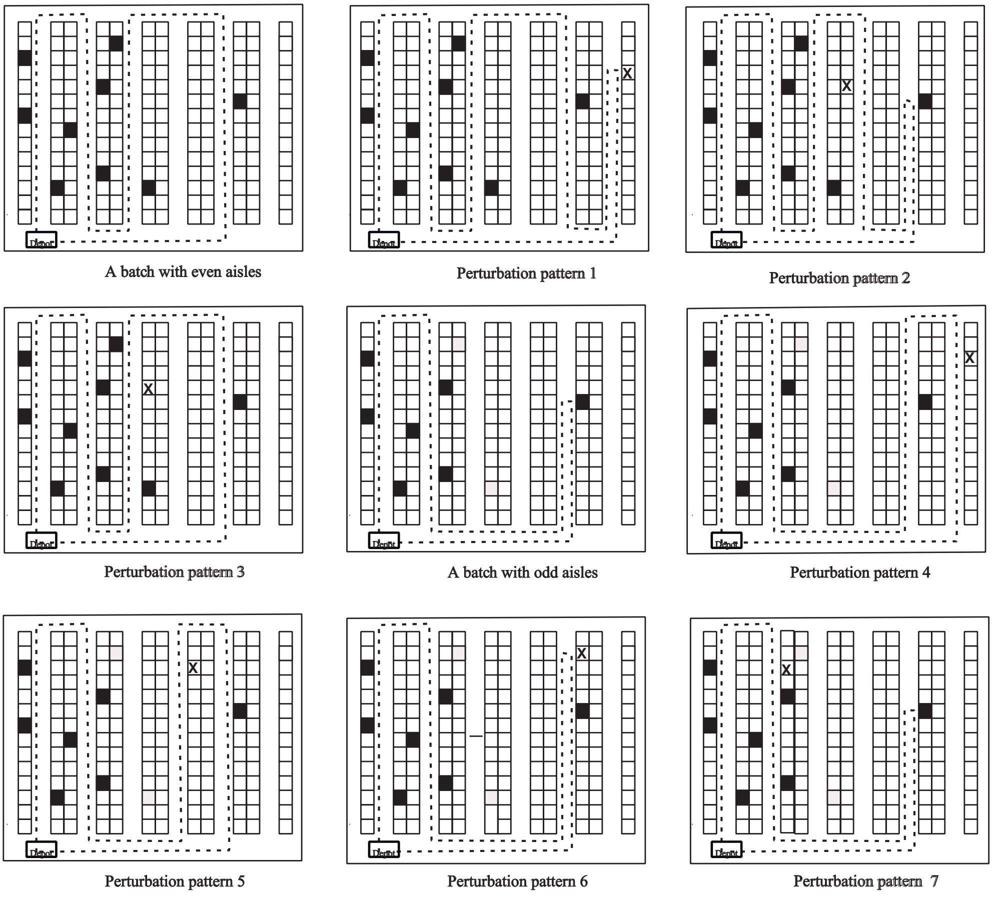


Fig. 3.1. Perturbation analysis for a batch in a parallel-aisle warehouse with S-shape routing policy

Lemma 3.2. *The gradient of expected travel time with respect to batch size can be computed by* $dE[T]_p = \frac{1}{v}\frac{\partial E[L]}{\partial q}\frac{E[D]}{q} + (\frac{E[L]}{v} + \beta)(-\frac{E[D]}{(q^k)^2})$*, where* $\frac{\partial E[L]}{\partial q}$ *can be calculated by the perturbation analysis and decision tree method.*

Proof: From Model-1, we have

$$\left.\frac{\partial E[T_p(q)]}{\partial q}\right|_{q=q^k} = \frac{1}{v}\left.\frac{\partial E[L]}{\partial q}\right|_{q=q^k}\frac{E[D]}{q^k} + (\frac{E[L]}{v} + \beta)(-\frac{E[D]}{(q^k)^2}). \tag{3.2}$$

In this formula, the critical issue is to compute the gradient of the travel distance with respect to batch sizes. Gong and Yucesan (2009) have provided an implementation framework and theoretical justification of SPO and IPA, and they compute the gradient by an analytical duality method. Different from them, we conduct direct perturbation analysis, and then derive a decision tree from perturbation patterns.

We conduct a perturbation analysis for a single batch with S-shape routing policy here. For a batch with batch size q , we give the system a perturbation, i.e., let the batch size increase by 1. In Figure 3.1 the item with a cross “X” is the perturbed item. By comparing the distance before and after perturbation, we can compute the perturbation of distance.When the number of visited aisles is even, there are three perturbation patterns. (see perturbation patterns 1, 2, 3 in Figure 3.1). When the number of visited aisles is odd, there are four perturbation patterns. (see perturbation patterns 4, 5, 6, 7 in Figure 3.1).

By tracing its pattern of propagation through the system, we can build a decision tree for the gradient computation in Figure 3.2. Lemma 3.2 follows from IPA analysis in Figure 3.1 and the decision tree in Figure 3.2. □

Lemma 3.3. *If demand D has a density on* $(0,\infty)$*and* $E[D] < \infty$*, batch size* $q \in \Re^+$ *and* $q < \infty$ *, the gradients obtained by Lemma 3.2 are bounded with probability 1.*

Proof: There are several ways to prove this lemma. We employ the following argument, which is also used to show the computation process in (II.1).C of algorithm 1.

Generate the aisle position ξ_{q+1} and location position η_{q+1} of a perturbed item. For S-shape routing policy, from the decision tree in Figure 3.2, the gradient can be computed as follows:

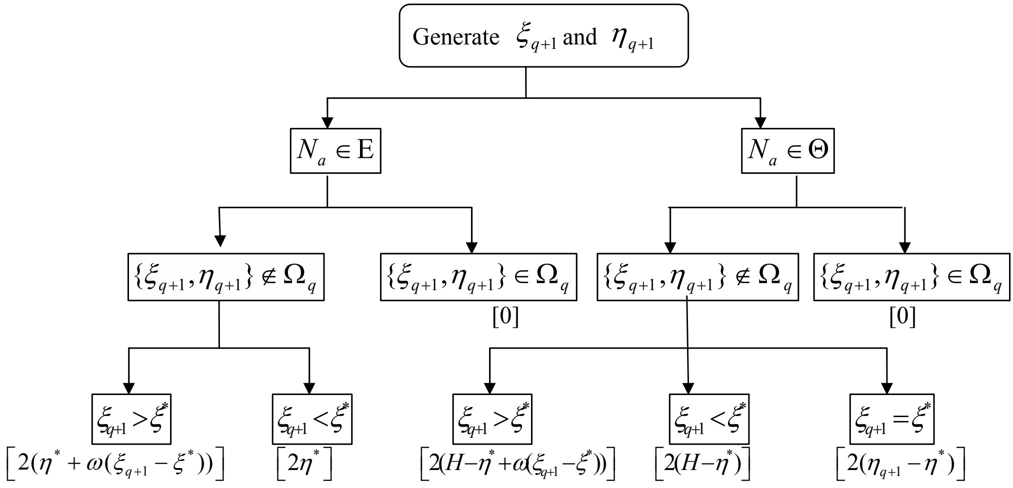


Fig. 3.2. Decision tree from perturbation analysis

$$\frac{\partial L}{\partial q} = \begin{cases} 2[\eta_{q+1} + \omega(\xi_{q+1} - \xi^*)], & \xi_{q+1} > \xi^* \ and\ N_v \in E \\ 2\eta^*, & \xi_{q+1} < \xi^*, (\xi_{q+1}, \eta_{q+1}) \notin \Omega_q \ and\ N_v \in E \\ 0, & \xi_{q+1} = \xi^*, (\xi_{q+1}, \eta_{q+1}) \notin \Omega_q \ and\ N_v \in E \\ 2[H - \eta^* + \omega(\xi_{q+1} - \xi^*)], & \xi_{q+1} > \xi^* \ and\ N_v \in \Theta \\ 2(H - \eta^*), & \xi_{q+1} < \xi^*, (\xi_{q+1}, \eta_{q+1}) \notin \Omega_q \ and\ N_v \in \Theta \\ 2(\eta_{q+1} - \eta^*), & \xi_{q+1} = \xi^*, \eta_{q+1} > \eta^* and\ N_v \in \Theta \\ 0, & (\xi_{q+1}, \eta_{q+1}) \in \Omega_q \ and\ N_v \in \Theta. \end{cases} \tag{3.3}$$

From Equation 3.3, we have

$$\frac{\partial L}{\partial q} \leq max\{2[\eta_{q+1} + \omega(\xi_{q+1} - \xi^*)], 2\eta^*, 2[H - \eta^* + \omega(\xi_{q+1} - \xi^*)],$$

$$2(H - \eta^*), 2(\eta_{q+1} - \eta^*)\} \leq 2H + 2\omega(M - 1). \tag{3.4}$$

The boundedness of gradient follows from Equation 3.4. □

Lemma 3.4. *By the sample path optimization in Algorithm 1 for a proper choice of step size, the batch size* $\{q^k\}_{k=1}^{\infty}$ *converges with probability 1.*

Proof: In order to ensure the convergence, a key issue is the selection of a suitable step size α_k , where we have

Condition (1): A criterion for choosing α_k is to let step size go to zero fast enough so that the algorithm can converge, but not so fast that it will induce a wrong value. One condition to meet that criterion is $\sum_{k=1}^{\infty} \alpha_k = \infty$ and $\sum_{k=1}^{\infty} \alpha_k^2 < \infty$.

For instance, $\alpha_k = \alpha/k$ for some fixed $\alpha > 0$ satisfies Condition (1). The first part of this condition facilitates convergence by ensuring that the steps do not become too small too quickly. However, if the algorithm is to converge, the step sizes must eventually become small, as ensured by the second part of the condition.

For a convex objective function $E[T_p(q)]$, a bounded gradient (see Lemma 3.2 and Lemma 3.3), and a step size α_k which satisfies the condition (1), according to Robbins and Monro (1951), we have a limit point of $\{q^k\}_{k=1}^{\infty}$, which is stationary with probability 1. □

3.2.3 Results

We implement Algorithm 1 in Matlab. Experiments are conducted on a computer with 1.73GHz CPU and 516MB RAM. After acquiring characteristic information like warehouse size and generating the demand by a normal distribution $N(\mu, \sigma^2)$ to specify the problem, the distance computation program can return batch sizes and objective values at each step to the main program. Then by the gradient computing algorithm, the main program can update the batch size until it converges. Here we adopt an initial step size $\alpha_1 = 0.5$ by a pretest experiment. Since the objective function is convex, this convergence will lead to a global optimum.

Without loss of generality, the position of our depot is the first aisle and the first location. The probability to visit an aisle is equal for all aisles and uniformly distributed. We have aisle number $\xi_i \sim U(1, M), \forall i \in \{1, .., q\}$. The probability to visit a location in a visited aisle is also equal, i.e. , location position $\eta_i \sim U(1, H), \forall i \in \{1, .., q\}$. In order to verify the result from the simulation optimization, we compare it with the result of enumeration, where we enumerate all the possible batch size values from q^{LB} to q^{UB}. In combination with Monte Carlo simulation, the enumeration is conducted as follows. First, we generate a very large order number, and then generate the position of each item by given distributions $\xi_i \sim U(1, M)$ and $\eta_i \sim U(1, H), \forall i \in \{1, .., q\}$. Then, for every value of q, we determine the batches of size q in an FCFS sequence. Third we compute the routing length and corresponding warehouse operation time of each batch by the S-shape policy. For every batch size we compute the expected warehousing operation time and hence finally find the optimal batch size.

We present experiments in Table 3.1. We have conducted two groups of experiments: varying the aisle number M(experiments 1 to 5) and varying the aisle length H(experiments 6 to 10). The computation results include the items below:

q^E= the optimal batch size obtained by enumeration;

$\widehat{q}$ = the statistical estimation of batch size by the stochastic simulation algorithms, which includes the mean batch size $\overline{q}$ and half width (HW) of the 95% confidence interval (CI) ;

$R(\overline{q})$= the rounded integer value of the estimated batch size;

$\Delta_1 = |\overline{q} - q^E|/q^E$, the direct bias of statistical estimation;

$\Delta_2 = |R(\overline{q}) - q^E|/q^E$, the indirect bias of rounded statistical estimation;

We compute the average direct bias $\overline{\Delta_1}=1/N \sum_n |\overline{q} - q_n^E|/q_n^E = 0.255\%$ and the average indirect bias $\overline{\Delta_2}=1/N \sum_n |R(\overline{q}) - q_n^E|/q_n^E = 0\%$. The average direct bias of statistical estimation is less than 1%, and the average indirect bias of rounded statistical estimation is negligible.

From the experiment, we observe that

the optimal batch size for Model-1 equals to the upper bound $q^* = q^{UB}$,

which is robust in both groups of simulation experiments. We can understand this result from the limiting system behavior. By increasing the batch size, the travel and setup time per batch will converge to a constant.

Table 3.1. Experiment results for Model-1

$No.$	M	H	ω	q^{UB}	q^{LB}	q^E	$\widehat{q_1} = \overline{q} \pm HW$	$R(\overline{q})$	Δ_1	Δ_2
1	25	20	3	50	1	50	49.8694±0.0876	50	0.26%	0
2	30	20	3	50	1	50	49.7793±0.0773	50	0.44%	0
3	35	20	3	50	1	50	49.8895±0.0271	50	0.22%	0
4	40	20	3	50	1	50	49.9391±0.0763	50	0.12%	0
5	45	20	3	50	1	50	49.8394±0.0745	50	0.32%	0
6	40	25	3	60	1	60	59.7696±0.0876	60	0.38%	0
7	40	27	3	60	1	60	59.9196±0.0773	60	0.13%	0
8	40	28	3	60	1	60	59.8195±0.0272	60	0.30%	0
9	40	30	3	60	1	60	59.9093±0.0765	60	0.15%	0
10	40	32	3	60	1	60	59.8597±0.0743	60	0.23%	0

$$\lim_{q\to\infty} E[\frac{L(q)}{v} + \beta] = \lim_{q\to\infty} \frac{M}{v} H[1 - (1 - \frac{1}{M})^q] + 2\frac{\omega}{v}[M - \sum_{j=1}^{M-1} (\frac{j}{m})^q] + \frac{H}{2v} + \beta$$
$$= \frac{2MH + 4M\omega + H}{2v} + \beta. \tag{3.5}$$

However, with an increasing batch size q, the number of batches $E[D]/q$ will continue to decrease, and therefore the total operation time, which is the product of the two items, will also decrease. That is the reason why the optimal batch size will converge to its upper bound.

3.3 Optimal order batch sizes to improve customer service

3.3.1 Model

Model-1 in Section 2 considers the main operation time from the perspective of a warehousing service provider. It does not measure the waiting time of customer orders and the service level. It is also necessary to examine the time spent by customer orders in a warehousing system. We therefore adopt the throughput time of a consumer order, which is from Chew and Tang (1999), as the objective to build the Model-2 as follows.

$$Model - 2 : Min_{q^{LB} \le q \le q^{UB}} \quad T_{TO}(q)$$

$$s.t. \quad T_{TO}(q) = W_1(q) + W_2(q) + E[S]$$

In Model-2, the objective $T_{TO}(q)$ is the throughput time of a customer order, i.e., the duration an order stays in the system, when the batch size is q with $q^{LB} \leq q \leq q^{UB}$, where q^{LB} is determined by the system equilibrium condition since if the arrival rate λ is too high the system will become unstable and q^{UB} is specified by the capability of picking carts. $T_{TO}(q)$ consists of three parts: expected batch time $W_1(q)$, expected waiting time $W_2(q)$ and expected service time $E[S]$. We assume single-line orders and let the order arrival rate be λ. The expected batch time $W_1(q)$ is given by Chew and Tang (1999) as $W_1(q) = (q-1)/\lambda$. The expected waiting time $W_2(q)$ is approximately computed by a linear combination of expected waiting times of $M/M/1$, $M/D/1$, and $D/M/1$ queueing systems. When the coefficient of variation of both the interarrival time and service time distribution functions are less than or equal to 1, this standard approximation works well for any GI/G/1 queueing systems. The expected service time $E[S]$ consists of travel time, picking and sorting time.

3.3.2 Algorithm

The essential problem in searching for the optimal batch size is the choice of the computation method. From Chew and Tang (1999), our objective function is a complicated function of q, and an analytical gradient computation method is infeasible. Perturbation analysis is also highly complicated in Model-2, especially for the perturbation analysis of W_2. Even if we had obtained the decision tree by perturbation analysis, its computation will not be efficient. Therefore we use the finite differences (FD) as our gradient computation method.

We use the following central finite difference optimization algorithm to solve the order batch problem in a parallel-aisle warehouse, and demonstrate how to obtain the optimal batch size quantities. The procedure is summarized in a pseudo-code format in Algorithm 2, where we start with a batch size q^1, usually $q^1 = q^{UB}$, K denotes the total number of steps taken in a search path, α_k represents the step size at the each iteration k, and q^k represents the batch size at the k^{th} step.

Algorithm 2

(I) *Initialization.*

(I.1) *Initialize* K.

(I.2) *Initialize* q^1 *to* q^{UB}.

(II) *Set* $k \leftarrow 1$.

Repeat.

(II.1) *Compute* $E[T]^k_{TO}(q^k + h)$ *and* $E[T]^k_{TO}(q^k - h)$, *when* k *>1.*

(II.2) *Compute the desired gradients.*

$dE[T]^k_{TO} = \frac{T^k_{TO}(q^k+h)-T^k_{TO}(q^k)}{h}$, *when* k *=1.*

$dE[T]^k_{TO} = \frac{T^k_{TO}(q^k+h)-T^k_{TO}(q^k-h)}{2h}$, *when* k *>1.*

(II.3) *Update the batch size,* $q^{k+1} \leftarrow \lfloor q^k - \alpha_k dT^k_{TO} \rfloor$.

$k \leftarrow k + 1$,

Until $k = K$.

(III) *Return the* $\{q^k\}_{k=1}^K$ *and the objective function value.*

Lemma 3.5. *By the finite difference algorithm 2 and a step size* α_k *which can satisfy Condition (1) , the batch sizes* $\{q^k\}_{k=1}^{\infty}$ *in Model-2 can converge.*

Proof: From Bertsekas (1999), for a finite difference algorithm with a step size which satisfies Condition (1), if the objective function in Model-2 is convex, the batch size converge. Chew and Tang (1999) have showed that $T_{TO}(q) = W_1(q) + W_2(q) + E[S]$ is a convex function of q for $q^{LB} \leq q \leq q^{UB}$. □

3.3.3 Results

Based on Chew and Tang (1999)'s formulation and our optimization algorithm, we implement the optimization procedure in Matlab. One run of simulation takes only 6 seconds on average. Let $q^1 = q^{UB}$, for the number of aisle ranging from 25 to 45, we obtain the search paths indicated in Figure 3.3. All the experiments converge in the last 500 steps. We compute the statistical estimation by the transient deletion technique (see Law and Kelton (2000)).

In order to verify the result from optimization algorithm 2, we compare the result with that by enumeration, where we traverse all possible batch size values

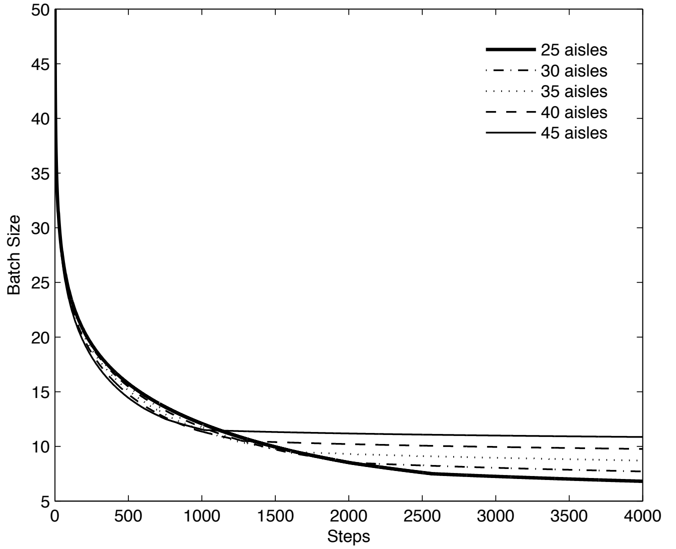


Fig. 3.3. Search path by Algorithm 2

from q^{LB} to q^{UB}. For all possible values of batch size q, we compute the expected objective values from Chew and Tang (1999) and find the optimal batch size. We present the result in Figure 3.4.

We present experiments in Table 3.2. The most left column of Table 3.2 represents the index of the experiments. The middle part of Table 3.2 is the experiment setting: M, H, ω, q^{LB} and q^{UB}. We have conducted two groups of experiments: varying the aisle number M (experiments 1 to 5) and varying the aisle length H (experiments 6 to 10). The right part presents the computation results, which includes the items below:

q^E= the optimal batch size obtained by enumeration;

$\widehat{q}$ = the statistical estimation by Algorithm 2, which includes the mean batch size $\overline{q}$ and HW of the $95\%CI$;

$R(\overline{q})$= the rounded integer value of the estimated batch size;

Δ_1= $|\overline{q} - q^E|/q^E$, the direct bias of statistical estimation;

Δ_2= $|R(\overline{q}) - q^E|/q^E$, the indirect bias of rounded statistical estimation;

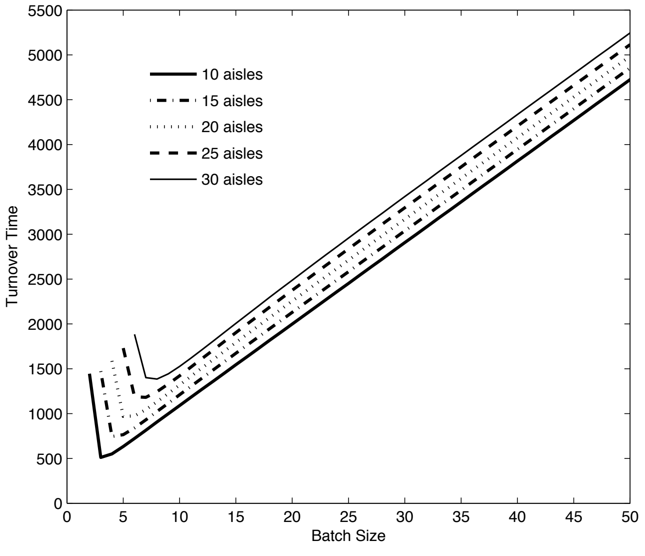


Fig. 3.4. Turnover time versus batch sizes in Model-2

We compute the average direct bias $\overline{\Delta_1}=1/N\sum_n |\overline{q}-q_n^E|/q_n^E = 2.785\%$ and the average indirect bias $\overline{\Delta_2}=1/N\sum_n |R(\overline{q})-q_n^E|/q_n^E = 0.769\%$. The average direct bias of statistical estimation is less than 3%, and the average indirect bias of rounded statistical estimation is less than 1%.

Table 3.2. Experiment results for Model-2

$No.$	M	H	ω	q^{UB}	q^{LB}	q^E	$\widehat{q_1}=\overline{q}\pm HW$	$R(\overline{q})$	Δ_1	Δ_2
1	25	20	3	50	6	7	6.9129±0.0061	7	1.24%	0
2	30	20	3	50	7	8	7.7847±0.0046	8	3.08%	0
3	35	20	3	50	8	9	8.7647±0.0034	9	2.61%	0
4	40	20	3	50	9	10	9.8098±0.0025	10	1.90%	0
5	45	20	3	50	10	11	10.8911±0.0018	11	0.99%	0
6	40	25	3	50	10	12	12.4991±0.00005	12	4.16%	0
7	40	27	3	50	10	13	13.5003±0.00006	14	3.85%	7.69%
8	40	28	3	50	10	14	14.4994±0.00006	14	3.57%	0
9	40	30	3	50	10	15	15.4996±0.00007	15	3.33%	0
10	40	32	3	50	10	16	16.4998±0.00005	16	3.12%	0

From the result we can observe that, the optimal batch size for customers is close to its lower bound and less than its upper bound $q^{LB} < q_2^* < q^{UB}$, which is robust in both groups of simulation experiments.

3.4 Optimal order batch sizes for the total system

The objective functions of both Model-1 and Model-2 focus on one part of warehouse performance. The result from Model-2 is similar to Chew and Tang (1999) and Le-Duc and De Koster (2007), and this result possibly underestimates the positive effect of a batch procedure. The result from Model-1 possibly overestimates the positive effect of batch procedure. While a large batch size brings short-run minimal cost to warehouse service providers, it will also cause long throughput times for the customers, and may therefore harm the long-run interest of warehouse service providers. Considering both sides, we therefore build Model-3 and measure the total system cost.

3.4.1 Model

The objective in Model-2 is the turnover time for a single customer's order while the objective in Model-1 is the total service time for the total customers. So we need to transform the data in Model-1, and compute the time spent by the warehouse on a single customer, that is $\overline{T_P(q)} = \frac{E[L(q)]/v+\beta}{q}$. Without loss of generality, we assume a single customer corresponds to a single order. Let c_1 be the operation cost per unit time for the service provider, and c_2 be the waiting cost per unit time for the customer. Then $c_1\overline{T_P(q)} + c_2 T_{TO}(q)$ is the total system cost $C(q)$ for one customer. We have:

$$Model-3 : Min_{q^{LB} \leq q \leq q^{UB}} \quad C(q)$$

$$
\begin{aligned}
s.t. \quad C(q) &= c_1\overline{T_P(q)} + c_2 T_{TO}(q) \\
\overline{T_P(q)} &= \frac{E[L(q)]/v + \beta}{q} \\
T_{TO}(q) &= W_1(q) + W_2(q) + E[S]
\end{aligned}
$$

The ratio of c_1 and c_2 in Model-3 is used to measure the weight of both sides in the system. We define: the unit cost ratio $\gamma = \frac{c_1}{c_2}$.

3.4.2 Algorithm

We mainly use a central finite difference (FD) method as gradient computation method since the objective function in Model-3 is a complicated function of q. The procedure is summarized in a pseudo-code format in Algorithm 3, where we start with an initial batch size q^1 , for example $q^1 = q^{UB}$, K denotes the total number of steps taken in a searching path, α_k represents the step size at the each iteration k, and q^k represents the batch size at the k^{th} step.

Algorithm 3
(I)*Initialization.*
(I.1) *Initialize* K.
(I.2) *Initialize* q^1 *to* q^{UB}.
(II) *Set* $k \leftarrow 1$.
Repeat.
(II.1) *Compute* $C^k(q^k + h)$ *and* $C^k(q^k - h)$, *when* k >*1.*
(II.2)*Compute the desired gradients.*
$dC^k = \frac{C^k(q^k+h)-C^k(q^k)}{h}$, *when* k =*1.*
$dC^k = \frac{C^k(q^k+h)-C^k(q^k-h)}{2h}$, *when* k >*1.*
(II.3)*Update the batch size,*$q^{k+1} \leftarrow \lfloor q^k - \alpha_k dC^k \rfloor$.
$k \leftarrow k + 1$,
Until $k = K$.
(III) *Return the* $\{q^k\}_{k=1}^K$ *and the objective function value.*

Lemma 3.6. *By the finite difference algorithm 3 and a step size α_k which can satisfy Condition (1), the batch sizes $\{q^k\}_{k=1}^{\infty}$ in Model-3 can converge to a global optimum.*

Proof: From Bertsekas (1999), for the finite difference algorithm with a step size which can satisfy Condition (1), if the objective function in Model-3 is convex, the batch size can converge. In Section 2, we have proven $T_P(q)$ and therefore $c_1\overline{T_P(q)}$ is a convex function. Chew and Tang (1999) have showed that the objective function $T_{TO}(q)$ is a convex function of q. So $C(q) = c_1\overline{T_P(q)} + c_2T_{TO}(q)$ is a convex function of q. The convexity ensures the algorithm will converge to a global optimum. □

3.4.3 Results

Table 3.3. Experiment results for Model-3

No.	M	H	ω	q^{UB}	q^{LB}	γ	q^E	$\widehat{q_1} = \overline{q} \pm HW$	$R(\overline{q})$	Δ_1	Δ_2
1	25	20	3	50	6	20	12	12.2801±3.5136e-004	12	2.33%	0
2	30	20	3	50	7	20	12	12.4999±1.1592e-005	12	4.17%	0
3	35	20	3	50	8	20	12	12.5000±1.3900e-005	13	4.17%	8.33%
4	40	20	3	50	9	20	13	12.7900±9.0225e-004	13	1.62%	0
5	45	20	3	50	10	20	13	13.4995±9.3120e-005	13	3.84%	0
6	40	25	3	50	10	20	14	14.9999±1.8239e-005	14	3.57%	0
7	40	27	3	50	10	20	15	15.4998±1.7737e-005	15	3.33%	0
8	40	28	3	50	10	20	16	15.5001±2.3669e-005	16	3.12%	0
9	40	30	3	50	10	20	16	16.9980±2.4165e-005	16	3.12%	0
10	40	32	3	50	10	20	17	17.5000±2.5194e-006	18	2.94%	5.88%
11	40	20	3	50	10	30	15	15.4992±9.7966e-006	15	3.33%	0
12	40	20	3	50	10	40	18	18.1962±6.0757e-004	18	1.09%	0
13	40	20	3	50	10	50	20	20.4999±6.3132e-006	20	2.50%	0
14	40	20	3	50	10	60	23	22.9373±1.3000e-003	23	0.27%	0
15	40	20	3	50	10	70	25	25.2757±7.0235e-004	25	1.10%	0

Based on the formulation in Model-3, we implement the optimization algorithm 3 in Matlab. The running time ranges from 13 seconds to 19 seconds. For the coefficient γ ranging from 20 to 70, $\lambda = 0.011$ order lines /second.we obtain the search paths in the Figure 3.5. For all the experiments we conducted, we observe the search paths converge in the last 500 steps. We use the transient deletion technique to conduct the statistical estimation.

In order to verify the result from the finite difference optimization, we compare it with the result of enumeration, where we traverse all the batch size values from q^{LB} to q^{UB}. We compute the expected objective values for all possible values of q and find the optimal batch size.

For the aisle numbers ranging from 25 to 45, we respectively compute their "total cost", "part 1 cost" which is the cost of the warehouse operations, and "part 2 cost" which is the cost for the customers. The result is presented in Figure 3.6 and summarized in Table 3.3. We have conducted three groups of experiments: varying the aisle number M (experiments 1 to 5), the aisle length H (experiments 6 to 10) and the cost ratio γ (experiments 11 to 15). The third part of Table 3.3 is the computational results by Algorithm 3 and enumeration, containing q^E, $\widehat{q}$, $R(\overline{q})$, Δ_1, and Δ_2.

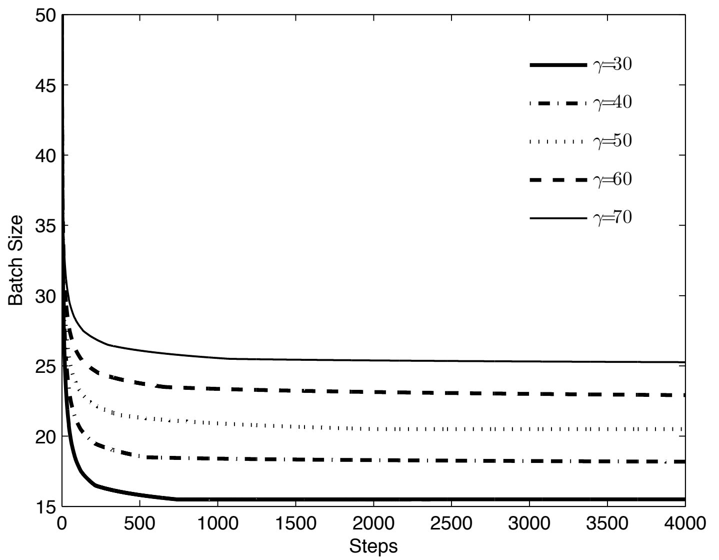


Fig. 3.5. Search path by Algorithm 3

We compute the average direct bias $\overline{\Delta_1}=1/N\sum_n|\overline{q}-q_n^E|/q_n^E = 2.700\%$ and the average indirect bias $\overline{\Delta_2}=1/N\sum_n|R(\overline{q})-q_n^E|/q_n^E = 0.947\%$. The average direct bias of statistical estimation is less than 3%, and the average indirect bias of the rounded statistical estimation is less than 1%.

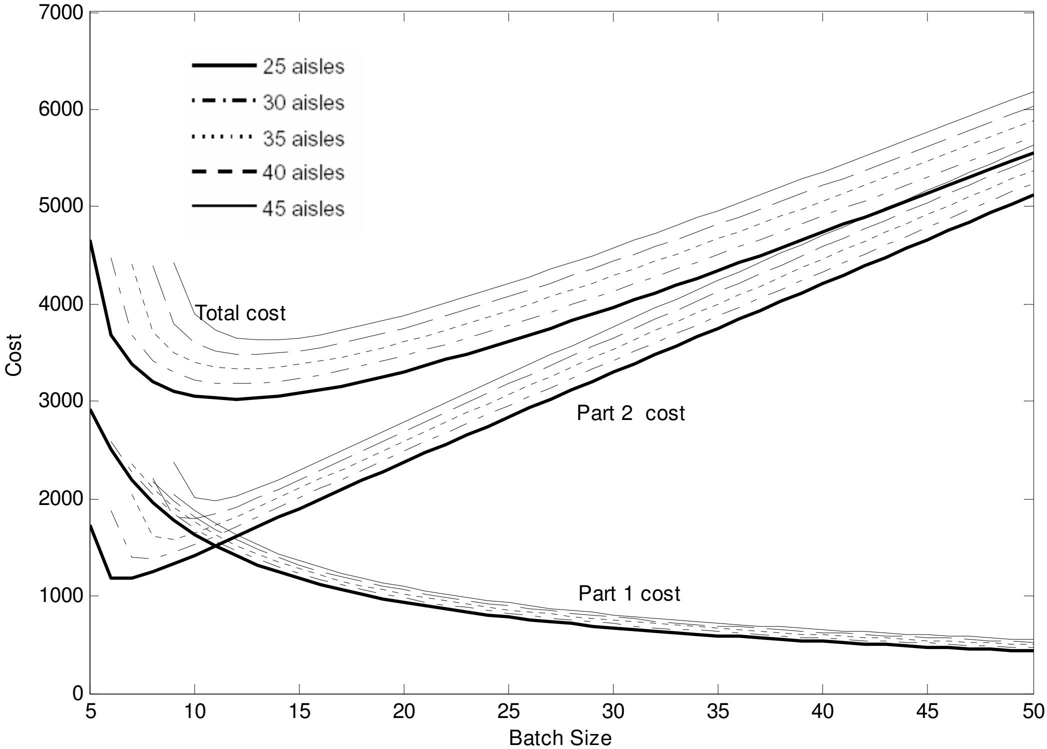


Fig. 3.6. Cost versus batch sizes in Model-3

From the results, we can observe that the optimal batch size for the total system is less than the optimal batch size q_1^* in Model-1 and larger than q_2^* in Model-2. We have $q_2^* \leq q_3^* \leq q_1^*$, which is robust in all the experiments with a different number of aisles, different aisle length, and different cost ratios γ . The result also shows that existing research underestimates the optimal batch size. This underestimation is due to the warehouse internal objective function.

Our problem is a basic economic equilibrium problem with two types of actors: a warehousing service supplier and customers. From the perspectives of different actors, the optimal batch sizes are different.

3.5 Concluding remarks

This chapter studies the optimal order batch size problem in a parallel-aisle warehouse with stochastic order arrivals. The contribution of this chapter is twofold in both application and methodology.

While much existing literature directly assumes the batch size value, this chapter shows that an optimal batch size exists. A too large batch size will harm the throughput time of consumers, and a too small batch size will bring a negative impact to warehousing costs. Existing literature focusing on the customer perspective only claims a suitable batch size will be close to its lower bound. Our research shows an optimal batch size will be larger than its lower bound when the costs of warehousing service providers are considered.

This chapter provides an IPA and SPO stochastic approximate optimal implementation scheme to search the batch sizes for the warehousing service providers in a general parallel-aisle warehouse setting. This chapter also presents an efficient finite difference algorithm to search the optimal batch sizes for customers and the total system. The estimation biases of the proposed algorithms are satisfactory.

A topic for further research could be to investigate the optimal batch size with the different routing policies in a general parallel-aisle warehouse with stochastic order arrivals. This chapter employs an S-shape routing method. It is also possible to research the optimal batch size with other heuristic routing policies like the mid-point routing policy and the largest gap routing policy, or the optimal routing policy.

4

A polling-based dynamic order picking system for online retailers

4.1 Introduction

The number of online retailers has drastically increased since the commercialization of internet in the early 1990s (Ranganathan and Ganapathy, 2002). In this novel business environment, customers, who can order with the ease of a click of a button, expect inexpensive, rapid and accurate delivery (De Koster, 2003). One of the challenging questions that online retailers are facing is how to organize the logistic fulfillment processes during and after order receipt. Compared with traditional retailers, online retailers have a disadvantage on immediacy (Otto and Chung, 2000): When a shopper purchases an item from a physical store, the product can immediately be taken home. But in the case of online retailers, the customer must wait for the shipment to arrive. Besides, many purchases are by impulse and, as customers can change their minds and legally cancel the order within a certain time horizon (about one week in most EU countries), a fast response is critical for online retailers. At the warehouse of Centraal Boekhuis, which distributes the orders of BOL.com, next or same day delivery is necessary to stay competitive. Wehkamp, the largest online retailer in the Netherlands has a 24-hour delivery window. Albert, one of the internet home delivery services of Ahold, is able to deliver in time windows specified by the customer, using different pricing schemes for different windows.

Some online retailers adopt personalized material flows in order to achieve a fast-response competitive advantage. Anckar, Walden, and Jelassi (2002, Page

216) report that, Nettimarket (nettimarket.com), an online retailer in Finland, spends only 45 minutes per order on personal service : “This includes the whole process from obtaining the order to delivering the goods to the customer’s doorstep”. Similar to this Finnish case, “China online shopping” (buy.com.cn), a large-scale online retailer in China, provides a service of “delivery within two hours upon order receipt” in large cities, and promises to home deliver the goods within two hours after order receipt. Obviously, managing order picking systems effectively and efficiently is a challenging process in the warehouses of these online retailers. A prime objective is to shorten throughput times for order picking, and to guarantee the meeting of due times for shipment departures. Many of these online retailers have so-called fluid shipments, which means they ship using a limited number of parcel carriers (like DHL, UPS, TNT, Fedex), who can accept shipments the whole day round, sort them by delivery route in a sorting hub and distribute them to the customers. With a fluid shipment system, the order picking process can therefore start picking for any customer (in the region) at any time.

A traditional way to organize the picking process in case of a large number of daily order lines is to form pick batches and to release these to the shop floor so that items can be picked, sorted by - for example - a sorting machine by customer, and packed. However, batch formation takes time and, as the number of daily orders to be processed increases and as the required lead time becomes shorter, there may be more efficient ways to organize the order picking process. In this chapter we propose a different way to organize it by using dynamic picking. In a Dynamic Picking System (DPS), orders arrive online and are picked in batch, followed by later sorting per customer order (a pick-and-sort system, see De Koster et al. (2007)). More importantly, a picker travels the entire or a part of the warehouse and picks all outstanding order lines in one pick route to a pick cart, including those order lines arriving at the order picker’s current pick position while working there, or arriving further downstream in his or her pick route. We assume that the pick cart has sufficient capacity to accommodate these picks. In the case of Wehkamp for example, the pick cart can contain over a hundred order lines, far more than the average number of lines in a route (about sixty). During a pick cycle, pick information is constantly updated by a pick-by-light, pick-by-

RFID, pick by handheld terminal, or voice-picking system. No paper pick lists are needed. Such systems even work with multiple pickers (pick-by-light, for example, can use different LED colors for different pickers). Compared with static picking, where the pick locations during a pick cycle are given and fixed, dynamic picking can shorten the response time and can thereby improve the customer service.

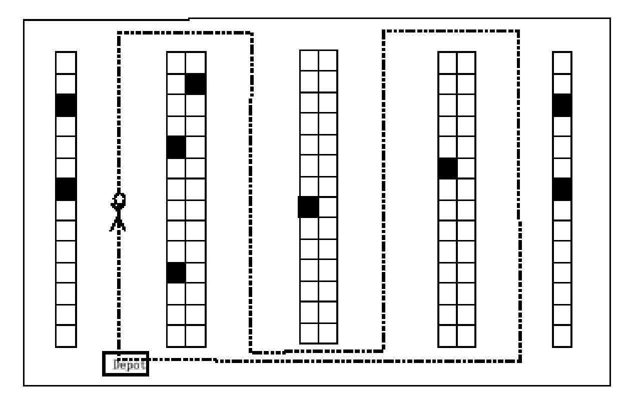
Several companies apply dynamic picking. At "China online shopping", for example, order pickers are instructed through handheld terminals connected to the order processing center, to pick order lines of newly arrived orders and add these to their carts, even during a pick cycle. Currently there is no good way of analyzing the performance of a DPS. A model matching a DPS is a multiple-queue single-server model where the queues represent storage positions, the customers represent the arriving order lines at these positions, and the server corresponds to the order picker who travels from queue to queue in a specified sequence. This system has not been described and analyzed in existing order picking models before.

This chapter therefore has the following research objectives: (1) to identify the stochastic essence of a DPS, and to describe and model it in an online setting, and (2) to provide an analysis method to derive the stability condition of these stochastic models and to analyze the performance of a DPS. To meet these research objectives, the remainder of the chapter is organized as follows. In the following section, we mainly introduce the problem description and stochastic polling models for a DPS in a general parallel-aisle warehouse. Section 4.3 is devoted to the analysis of models, including deriving stability conditions and closed-form expressions for the performance measurement of a DPS with single or multiple order pickers. In Section 4.4, we present our numerical experiments, show the advantage of polling system over traditional batch picking, and show how to apply it in practice. We conclude with final comments in Section 4.5.

4.2 Brief literature review and model

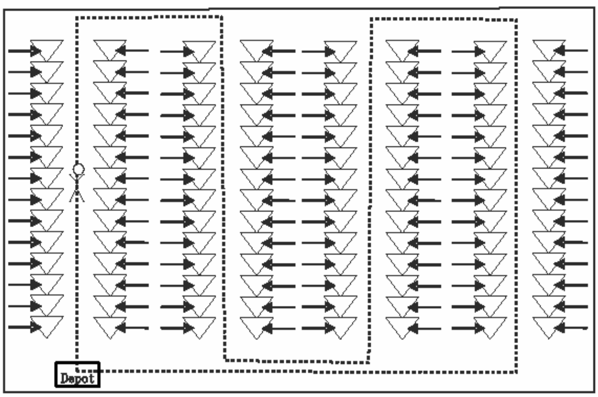
Two major types of order picking systems can be distinguished: parts-to-picker and picker-to-parts systems. The picker-to-parts systems, where the order picker walks or drives along the aisles to pick items, are most common (for a literature review, see De Koster et al. (2007)). Picker-to-parts systems occur in two types: pick by order and pick by article (batch picking). It is also possible to distinguish picker-to-parts systems by the order arrival and release. This can be either deterministic and planned (Gademann and Van de Velde, 2005) or online and stochastic (Le-Duc, 2005). Only few papers study online order arrivals (Chew and Tang, 1999; Van Nieuwenhuyse and De Koster, 2006; Le-Duc and De Koster, 2007). All papers focus on batch picking, where a batch of multiple orders is released to the shop floor and picked in a single pick route. The DPS with online order arrivals and processing can be described by a so-called polling model; a system of multiple queues accessed in cyclic or other specified sequence by a single server, which has been extended to the cases with multiple servers (Srinivasan, 1991). There is considerable literature on polling models, which are used to model an abundant set of systems like computer and telecommunications networks (Takagi, 1988). There are also relevant applications in operations management. For instance, Koenigsberg and Mamer (1982) consider an operator who serves a number of storage locations on a rotating carousel conveyor. Mack et al. (1957) also study an operator who patrols N machines unidirectionally with constant walking and repair times. Xu et al. (2007) consider heavy traffic analysis of a single-vehicle loop in an automated storage and retrieval system. Bozer and Srinivasan (1991) consider tandem configurations for automated guided vehicle systems and analyze single-vehicle loops. Bozer and Park (1999) present single-device, polling-based material handling systems. Although polling systems have been widely researched in these fields, they have not yet received systematic treatment and application in the order picking area.

Warehouse literature studying order picking throughput time usually considers a parallel-aisle layout as sketched in Figure 4.1. Fig.- 4.1(a) represents static picking, where all pick locations are known at the beginning of a pick route and static. Fig. - 4.1(b) represents the DPS, studied in this chapter.



■ : ordered items.

(a) Static picking system.



: The queue of order lines at a storage position.

(b) Dynamic picking system.

Fig. 4.1. Top view of a parallel-aisle warehouse with static picking (a) and dynamic picking (b).

We consider the dynamic order picking system in a general parallel-aisle warehouse with K aisles and L storage positions at each side of one aisle. Without loss of generality, we assume the number of aisles is even. We also assume an order consists of a single order line, which particularly applies to online retailing. An order picker picks the items following a two-sided picking policy, which means he or she picks at both sides in one pass, so called two-sided picking, and therefore we view the storage positions opposite to each other at both sides of an aisle as one position. We assume the picking cart is uncapacitated. In the case of online retailers, this is usually not a real restriction, as the route often finishes before the cart is full. The server here represents the order picker and the queues correspond to corresponding storage positions, each containing a different product. This is a polling system consisting of N queues, attended by a single order picker-while we will consider multiple order pickers later, here we temporarily take a single order picker perspective.

The arrival order line stream at position i (each representing a certain product) is a Poisson process with rate $\lambda_i, 1 \leq i \leq N$. We also assume there is no replenishment required in one picking cycle, and each queue is assumed to have infinite buffer capacity, as usual in the polling literature. At each queue, order lines are served on a First-Come-First-Serve basis. The picking times are assumed to be IID random variables with finite first and second moments. Let $\beta_i^{(h)}$ denote the h^{th} moment of the service time at position $i, i = 1, ...N, h = 1, 2$.

The storage positions are visited according to a strict S-shape routing policy in a cyclic sequence, which means that any aisle is traversed entirely. Since the position and order information is updated in real-time, the order picker cannot choose not to enter some aisles as might be possible in a static picking system. From the last visited aisle, the order picker returns to the pick position 1 via the depot: the depot is on the path from the position to the position 1. At the depot, the order picker drops off the picked items so that other operators can sort and transport them. The picker's depot operating time is independent of the travel time. The models in this chapter do not include sorting operations by other operators.

Let $t_i^{(h)}$ with $i = 1, ..., N-1$ denote the hth moment of the travel times from queue i to queue $i+1$, $t_N^{(h)}$ denotes the hth moment of the travel time from the position N to the position 1, and the depot operating time is denoted with the first and second moments (τ_d, ξ_d). When the order picker travels from one aisle k to the next one $k+1$, the travel time will be larger than that between in-aisle locations. The first and second moments are denoted by (τ_1, ξ_1).When the order picker travels from the last position back to the position 1, the travel time is largest with the first and second moments (τ_2, ξ_2). Within one aisle, the times needed by the order picker to travel from one queue to the next are assumed to be IID random variables with finite first two moments (τ_3, ξ_3). We therefore have:

$$t_i^{(1)} = \tau_1, i = Lk, k = 1, ..., (K-1); \tau_2, i = N; \tau_3, otherwise \tag{4.1}$$

and

$$t_i^{(2)} = \xi_1, i = Lk, k = 1, ..., (K-1); \xi_2, i = N; \xi_3, otherwise \tag{4.2}$$

The mean and second moment of the total travel and depot operation time T during a cycle of picking are respectively specified by,

$$t = \sum_{i=1}^{N} t_i^{(1)} + \tau_d = \tau_1(K-1) + \tau_2 + \tau_3(L-1)K + \tau_d \tag{4.3}$$

and $t^{(2)} = [\tau_1(K-1) + \tau_2 + \tau_3(L-1)K + \tau_d]^2 + (\xi_1 - \tau_1^2)(K-1) +$ $(\xi_2 - \tau_2^2) + (\xi_3 - \tau_3^2)(L-1)K + (\xi_d - \tau_d^2)$

The interarrival times of orders are assumed to be independent of the picking times, travel times and depot operation time of the order picker. The traffic load to queue i is defined by $\rho_i = \lambda_i \beta_i^{(1)}, 1 \leq i \leq N$, and the total traffic load of the system is $\rho = \sum_{i=1}^{N} \rho_i$.

At each queue, the service policy prescribes how the order lines (if any) at each location should be picked. A large number of service policies have been considered in the past polling research. But the main service policies applicable to warehouse operations are:

Definition 1. 1-limited service policy. Each visit at most one order line (the oldest one) is picked at a storage position.

Definition 2. Exhaustive service policy. Each visit the order picker continues to pick all order lines at a storage position until no more order lines are available at the location.

Definition 3. Gated service policy. The order picker picks only those order lines at a storage position which are found there upon his/her visit, and not the order lines that arrive during the course of the current picking operations.

4.3 Analysis

In Section 4.3.1, we analyze the dynamic picking model with a single order picker, including its stability condition and performance analysis. In Section 4.3.2, we present the analysis of a dynamic picking model with multiple order pickers, an extension of the model proposed in Section 4.2.

4.3.1 DPS with one order picker

Before conducting performance analysis, we need to establish stability conditions of the system, which is a fundamental issue for both theoretical analysis and application. Stability conditions also directly provide the guide for warehouse design. Then we calculate the mean order line waiting times.

Stability condition

Kuehn (1979) was the first to establish the stability conditions for a basic polling system. Furthermore, based on the stochastic monotonicity of the state process at the polling instant, Fricker and Jaibi (1994) have derived the overall stability condition for a polling system and furthermore analyzed the stability condition of subsets of queues. Based on these researches, we have the stability condition in a DPS as follows:

Theorem 4.1. *The stability condition is $\rho < 1$ for a DPS with one order picker and exhaustive or gated service policies, and $\rho + max(\lambda_i)t < 1, 1 \leq i \leq N$ for a DPS with one order picker and the 1-limited service policy.*

Proof. Let $E[C]$ be the expected cycle time of the order picker, $\lambda_i E[C]$ be the expected number of arrivals at the storage position in one cycle, and g_i be the average number of order lines at the storage position i which can be picked in one cycle. $E[C]$ can be calculated as $t + \Sigma_{i=1}^{N} \lambda_i E[C] \beta_i^{(1)}$, the sum of expected travel and depot operation time t and expected picking time $\Sigma_{i=1}^{N} \lambda_i E[C] \beta_i^{(1)}$. We have $E[C] = t/(1 - \Sigma_{i=1}^{N} \lambda_i \beta_i^{(1)}) = t/(1 - \rho)$.

For the system with exhaustive or gated service, which does not impose restrictions on g_i, the stability condition is $\rho < 1$ which is established by the finiteness of cycle time. For 1-limited service, we need $g_i = \lambda_i E[C] = t\lambda_i/(1 - \rho) < 1$, and then the stability condition for 1-limited service policy is $\rho + max(\lambda_i)t < 1$. Also notice that the first moment of travel and depot operation time $t = \tau_1(K - 1) + \tau_2 + \tau_3(L - 1)K + \tau_d$.

Performance analysis

In this section it is assumed that the stability condition in Theorem 4.1 is satisfied and that the system is in steady state. The main performance measure concerned in a DPS system is the mean waiting time EW_i of each order line at position $i, i = 1, 2, ..., N$. Since exact analysis is difficult, existing literature adopts the approximation to calculate the mean waiting time $EW_i \approx \omega_i ER[C_i]$ (see Boxma (1989)), where ω_i is a known system parameter mainly determined by the service policy, and $ER[C_i]$ is the expected residual life time of the cycle time at position $i, i = 1, 2, ..., N$. The critical issue is to compute $ER[C_i]$. A natural way is by definition of the expected residual life time $ER[C_i] = E[C_i^2]/2EC_i$, which is mainly used in small-scale systems. In a parallel-aisle warehouse, the number of positions is large, and the calculation of $E[C_i^2]$ is cumbersome. Therefore we resort to a pseudo-conservation law, an accurate expression for a weighted sum of the mean waiting times, which can be used in combination with the approximation of EW_i to approximate the order line waiting times. The earliest applications of a pseudo-conservation law to approximate mean waiting times are due to Everitt (1986) for gated and exhaustive policies and Boxma and Meister (1987) for the 1-limited service. Based on pseudo-conservation laws, Theorem 4.2 is established for a DPS with exhaustive or gated service policies, and Theorem 4.3 is established for a DPS with the 1-limited service policy.

Theorem 4.2. *For a stable DPS with one order picker, the approximate mean order line waiting time* EW_i^E *for the exhaustive service policy and* EW_i^G *for the gated service policy at each position* $i, i = 1, ...N$ *can be respectively calculated by* $EW_i^E \approx \frac{1-\rho_i}{\rho - \Sigma_{i=1}^N \rho_i^2}\{\frac{\rho}{2(1-\rho)}\Sigma_{i=1}^N \lambda_i \beta_i^{(2)} + \frac{\rho t^{(2)}}{2t} + \frac{t}{2(1-\rho)}(\rho^2 - \Sigma_{i=1}^N \rho_i^2)\}$ *and* $EW_i^G \approx \frac{1+\rho_i}{\rho + \Sigma_{i=1}^N \rho_i^2}\{\frac{\rho}{2(1-\rho)}\Sigma_{i=1}^N \lambda_i \beta_i^{(2)} + \frac{\rho t^{(2)}}{2t} + \frac{t}{2(1-\rho)}(\rho^2 + \Sigma_{i=1}^N \rho_i^2)\}$.

Proof. From Watson (1984), the pseudo-conservation laws are respectively established as:

For exhaustive service,

$$\Sigma_{i=1}^N \rho_i EW_i^E = \frac{\rho}{2(1-\rho)}\Sigma_{i=1}^N \lambda_i \beta_i^{(2)} + \frac{\rho t^{(2)}}{2t} + \frac{t}{2(1-\rho)}(\rho^2 - \Sigma_{i=1}^N \rho_i^2). \tag{4.4}$$

For gated service,

$$\Sigma_{i=1}^N \rho_i EW_i^G = \frac{\rho}{2(1-\rho)}\Sigma_{i=1}^N \lambda_i \beta_i^{(2)} + \frac{\rho t^{(2)}}{2t} + \frac{t}{2(1-\rho)}(\rho^2 + \Sigma_{i=1}^N \rho_i^2). \tag{4.5}$$

As the system is stationary, an amount of work ρ_i per time unit is offered to the order picker at position i, so the probability that a picker is working is ρ_i. Based on the assumption of equal mean residual cycle time $ER[C] \equiv ER[C]_i$, we have,

$$EW_i^E \approx (1 - \rho_i)ER[C], 1 \leq i \leq N. \tag{4.6}$$

Based on Boxma (1989), the mean waiting time with gated service consists of two parts: a mean residual cycle time since an order line is never picked in the cycle in which it arrives, and the mean time from the instant the order picker arrives at one position until the service completion. So

$$EW_i^G \approx (1 + \rho_i)ER[C], 1 \leq i \leq N. \tag{4.7}$$

Theorem 4.2 is established by solving for via substituting (4.6) into (4.4) and then applying (4.6) again for exhaustive service, and substituting (4.7) into (4.5) and then applying (4.7) again for gated service. Also notice t is specified by (4.3).

Theorem 4.3. *For a stable DPS with one order picker, the approximate mean order line waiting time EW_i^L for the 1-limited service policy at each position $i, i = 1, ..., N$ can be calculated by* $EW_i^L \approx \frac{1-\rho+\rho_i}{1-\rho-\lambda_i t}\frac{1-\rho}{\rho(1-\rho)+\Sigma_{i=1}^N \rho_i^2}\{\frac{\rho}{2(1-\rho)}\Sigma_{i=1}^N \lambda_i \beta_i^{(2)} + \frac{\rho t^{(2)}}{2t} + \frac{t}{2(1-\rho)}(\rho^2 + \Sigma_{i=1}^N \rho_i^2)\}$.

Proof. From Watson (1984), the pseudo-conservation law is established as,

$$\Sigma_{i=1}^N \rho_i (\frac{1-\rho-\lambda_i t}{1-\rho}) EW_i^L = \frac{\rho}{2(1-\rho)}\Sigma_{i=1}^N \lambda_i \beta_i^{(2)} + \frac{\rho t^{(2)}}{2t} + \frac{t}{2(1-\rho)}(\rho^2 - \Sigma_{i=1}^N \rho_i^2). \tag{4.8}$$

Based on the assumption of equal mean residual cycle time $ER[C] \equiv ER[C]_i$, Boxma and Meister (1987) get,

$$EW_i^L \approx \frac{1-\rho+\rho_i}{1-\rho-\lambda_i t} ER[C], 1 \leq i \leq N. \tag{4.9}$$

Theorem 4.3 is established by solving for via substituting (4.9) into (4.8) and then applying (4.9) again.

To compare the service policies, we have Corollary 4.4.

Corollary 4.4. *For a stable symmetric DPS with one order picker and the same parameters, we have* $EW_i^L > EW_i^G > EW_i^E$.

Proof. The RHS of the pseudo-conservation laws for gated and 1-limited service policies are the same in equations 4.5 and 4.8. So we have,

$$EW_i^L = (\frac{1-\rho-\lambda_i t}{1-\rho})^{-1} EW_i^G > EW_i^G. \tag{4.10}$$

For a symmetric system, we also have, for exhaustive service,

$$EW_i^E = \frac{1}{2(1-\rho)}\Sigma_{i=1}^N \lambda_i \beta_i^{(2)} + \frac{t^{(2)}}{2t} + \frac{t}{2(1-\rho)\rho}(\rho^2 - \Sigma_{i=1}^N \rho_i^2), 1 \leq i \leq N, \tag{4.11}$$

and for gated service,

$$EW_i^G = \frac{1}{2(1-\rho)}\Sigma_{i=1}^N \lambda_i \beta_i^{(2)} + \frac{t^{(2)}}{2t} + \frac{t}{2(1-\rho)\rho}(\rho^2 + \Sigma_{i=1}^N \rho_i^2), 1 \leq i \leq N. \tag{4.12}$$

The corollary is established by comparing (4.10), (4.11) and (4.12).

Besides the efficiency, we also examine the fairness at different positions, and have Corollary 4.5.

Corollary 4.5. *For a stable DPS with one order picker and* $\beta_i = \beta, 1 \leq i \leq N$*,we have* $EW^L_{i_1} > EW^L_{i_2}$*,* $EW^G_{i_1} > EW^G_{i_2}$ *and* $EW^E_{i_1} < EW^E_{i_2}$ *when* $\rho_{i_1} > \rho_{i_2}$ *,* $1 \leq i_1, i_2 \leq N$*.*

Proof. The corollary is established by examining the following equations, $\frac{EW^E_{i_1}}{EW^E_{i_2}} = \frac{1-\rho_{i_1}}{1-\rho_{i_2}}$, $\frac{EW^G_{i_1}}{EW^G_{i_2}} = \frac{1+\rho_{i_1}}{1+\rho_{i_2}}$, and $\frac{EW^L_{i_1}}{EW^L_{i_2}} = \frac{1-\rho+\rho_{i_1}}{1-\rho-\lambda_{i_1}t} / \frac{1-\rho+\rho_{i_2}}{1-\rho-\lambda_{i_2}t}$. For the 1-limited case, just notice that $\rho_i = \lambda_i \beta$.

Corollary 4.5 implies that order lines in light-traffic storage positions will have longer average waiting times than order lines in heavy-traffic storage positions under exhaustive service. The reason is that order lines arriving at heavy-traffic storage positions have a higher probability that their storage positions are currently being picked than those arriving at light-traffic storage positions. The 1-limited service policy is usually considered to be a “fair” policy over different positions and hence different orders since only one order line is picked at each position during one cycle (Boxma, 1991). Exhaustive service is less fair since one heavily loaded position can dominate the system, and will occupy the order picker for a long time. This implies that, although the average waiting time may be smaller for exhaustive service compared to 1-limited service, the maximal waiting time may be larger, particularly in asymmetric systems. If orders cannot be picked before their due time, this will affect the on-time service completion ratio.

4.3.2 DPS with multiple order pickers

For the case with multiple order pickers, the specifications of order arrival, picking, and traveling processes are similar to the model description in Section 4.2. The main difference is the behavior of the multiple order pickers. The storage positions are attended by identical and independent order pickers, with the same picking capacities. They travel from position to position by a strict S-shape policy in a cyclic sequence. From the last visited aisle, the order pickers return to the first position via the depot. pickers can simultaneously visit a storage position. We consider the exhaustive service policy in a warehouse with aisles which are sufficiently wide for overtaking other pickers. A picker arriving at a storage position will pick any order line he or she finds waiting at the storage position, and leaves only when he or she has completed the picking operations and no order lines are waiting. A second picker can overtake a first picker if the first picker is picking a line at a location while the location's queue is empty (Ajmone Marsan et al., 1992).

Stability condition

For this system, we give a stability condition based on the observation that the stability is guaranteed by finiteness of the average cycle times for an exhaustive service policy.

Theorem 4.6. *The stability condition is* $M > \sum_{i=1}^{N} \lambda_i \beta_i$ *for a DPS with multiple order pickers and the exhaustive service policy.*

Proof. Let $E[C]$ be the expected cycle time of an order picker, that is, the time between two consecutive arrivals of this order picker at one position i . The expected cycle time includes two parts: the sum of the expected travel and depot operation time t , and the total picking time t_p . We have $E[C] = t + t_p$, where t is equal to $\tau_1(K-1)+\tau_2+\tau_3(L-1)K+\tau_d$. The critical part is the calculation of t_p. The M order pickers are assumed to be identical, independent, and follow the same visiting order. The expected work of one order picker is therefore $1/M$ of the expected total amount of work arriving during one cycle in a stable system. So we have $t_p = 1/M \sum_{i=1}^{N} \lambda_i E[C] \beta_i^{(1)}$, where $\lambda_i E[C]$ is the expected number of total arrivals at the storage position i in one cycle, and $\beta_i^{(1)}$ is identical for all order pickers at the same position. From $E[C] = t + 1/M \sum_{i=1}^{N} \lambda_i E[C] \beta_i^{(1)}$, we have $E[C] = \frac{[\tau_1(K-1)+\tau_2+\tau_3(L-1)K+\tau_d]M}{M-\sum_{i=1}^{N} \lambda_i \beta_i^{(1)}}$. The polling system is stable if $0 < E[C] < \infty$. Since $[\tau_1(K-1)+\tau_2+\tau_3(L-1)K+\tau_d]M > 0$, the stability condition is established by setting $M > \sum_{i=1}^{N} \lambda_i \beta_i$.

Performance analysis

In this section, we assume the stability condition in Theorem 4.6 is satisfied. The performance analysis of a DPS with multiple order pickers is tougher than that of the single order picker problem. The power series algorithm (PSA) is often used for the numerical analysis of polling systems with multiple servers. However, the required running time of the PSA increases exponentially with the number of queues, and therefore this method is limited only to systems with a small number of queues (Blanc, 1992). Since the number of storage positions and order lines in a parallel-aisle warehouse is usually large, we therefore resort to approximation analyses (Borst and Van der Mei (1998); Brumelle (1971)) by pseudo-conservation laws.

Theorem 4.7. *For a DPS with multiple order pickers and exhaustive service policy, the expected order line waiting time at the position can be calculated as,*

$$EW_i = \frac{\rho_i(1-q_i)}{q_i}\frac{E[V] - 0.5\sum_{i=1}^{N}\lambda_i\beta_i^{(2)}}{\sum_{i=1}^{N}\frac{\rho_i^2(1-q_i)}{q_i}},$$

where $E[V]$ is the mean amount of work in the system and q_i is the steady-state probability that at least one of the order pickers is busy at the storage position i .

Proof. From Brumelle (1971), we have the pseudo-conservation law,

$$\sum_{i=1}^{N}\rho_i EW_i = E[V] - \frac{1}{2}\sum_{i=1}^{N}\lambda_i\beta_i^{(2)}. \tag{4.13}$$

Borst and Van der Mei (1998) calculate the mean amount of work $E[V]$, that is, the sum of the picking time of all order lines in queues and the remaining picking time of all order lines in service (Wolff, 1989), for polling systems with exhaustive service policy as:

$$E[V] = \gamma(\rho)\frac{\sum_{i=1}^{N}\lambda_i\beta_i^{(2)}}{2M[1-\rho/M]} + \Psi\times[\frac{\rho t^{(2)}}{2Mt} + \frac{t}{2(1-\rho/M)}((\frac{\rho}{M})^2 - \sum_i(\frac{\rho_i}{M}))^2], \tag{4.14}$$

where $\Psi = (1-\alpha)[2\frac{\rho}{M}\frac{1+(M-1)t^2/t^{(2)}}{M+1}+\gamma(\rho)(1-\rho/M)]+\alpha M$, $\gamma(\rho)$ is a function of ρ and α is used to measure the degree of clustering. $\alpha \rightarrow 1$ denotes a high degree of clustering, and $\alpha \rightarrow 0$ for a low degree of clustering. Under the exhaustive service policy, with the probability $1-q_i$, no order pickers work and the order lines must wait at the position i, so $EW_i \approx (1-q_i)ER[C_i]$. Based on this relation, Borst and Van der Mei (1998) have specified and established approximation equation:

$$EW_i \approx (1-q_i)\rho_i \Gamma / q_i. \tag{4.15}$$

Γ is a positive constant in 4.15. Theorem 4.7 is established by getting Γ via substituting Equation (4.14) and Equation (4.15) into Equation (4.13) and then applying Equation (4.15) again.

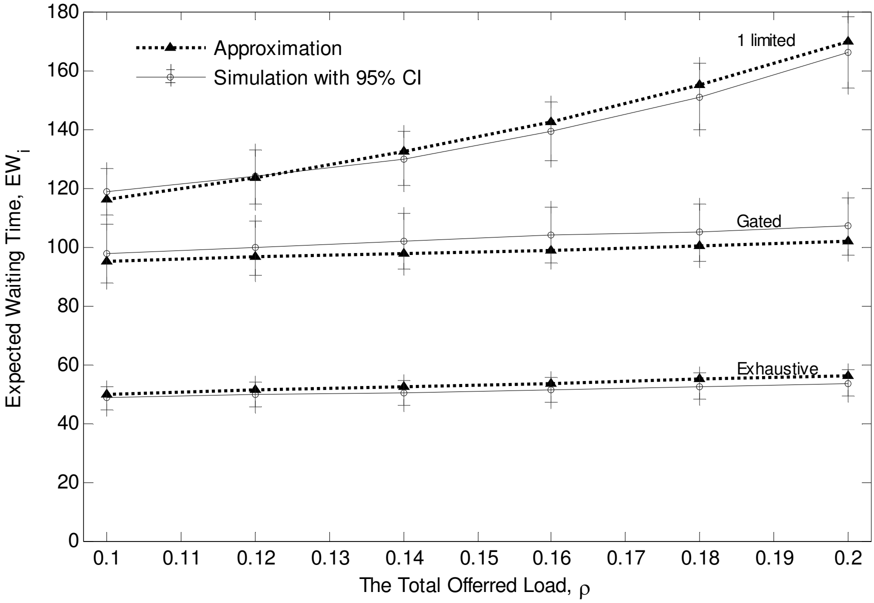
Theorem 4.7 uses approximation methods to derive expected order line waiting times. For the multiple server system, Borst and Van der Mei (1998) have pointed out that the approximations are accurate for symmetric systems, i.e., systems with identical arrival rates and service rates. In the case of asymmetrical arrival rates and service times, the approximation is less accurate but acceptable. Fortunately, even in asymmetrical cases, the approximation accuracy usually increases with the number of queues. Boxma and Meister (1987) explain this phenomenon by the "averaging out" effect which stabilizes systems in case of a large number of queues. A warehouse usually has a large number of storage positions, which positively influences the approximation accuracy.

4.4 Numerical results and application

In this section, we further look into the quality of approximation methods and compare different service policies by numerical experiments in Section 4.4.1. In Section 4.4.2, we compare the polling systems with the traditional batch picking systems with optimal batch sizes, and show the advantage of polling systems. Section 4.4.3 shows how online retailers can further apply polling theory to determine the optimal number of order pickers.

4.4.1 Numerical experiments

In order to establish quality of the approximation methods for the expected waiting times, we compare the results with simulation. Fig. 4.2 presents the results of the approximation method for the order line waiting time and simulated waiting time with various values of ρ. The simulation was conducted with Matlab. Fig. 4.2 also shows half widths of 95% confidence intervals for simulated waiting times.

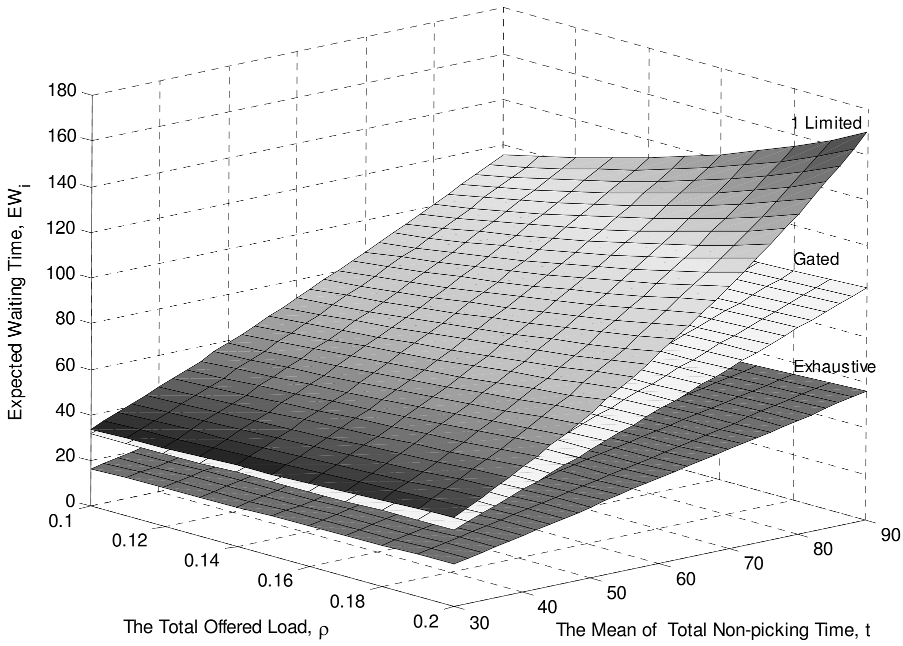


Note: (1)Unit of EW_i:sec.(2)$L = 14, K = 4, N = 56, t = 90\,\text{sec}, \beta_i^{(1)} = 1\,\text{sec}, \beta_i^{(2)} = 1.1\,\text{sec}^2$.

Fig. 4.2. Approximation values versus simulation values.

The bias is defined as $|EW_i^A - EW_i^S|/EW_i^S \times 100\%$, where EW_i^A is the value by analytical method in this chapter and EW_i^S is the value by simulation. The biggest bias is less than 6%, and the average bias is 3.3%. Fig. 4.2 shows all analytical values are within the 95% confidence intervals of simulation values.

Other scholars have found similar results for the pseudo-conservation methods (e.g. Boxma and Meister (1987), Everitt (1986), Fuhrmann and Wang (1988)).

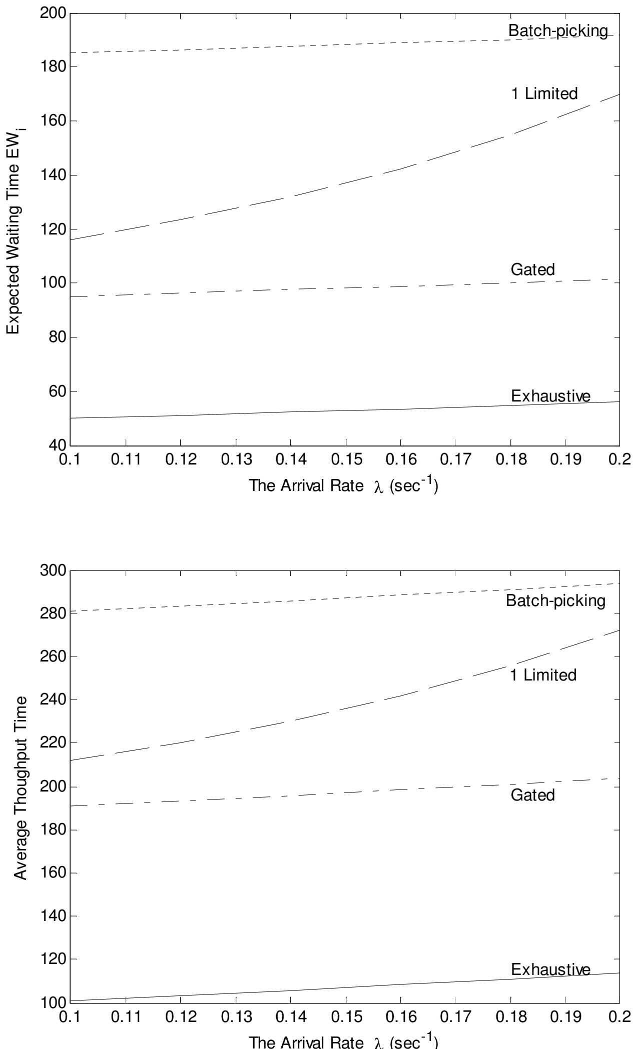


Note: Unit of EW_i and non-picking time:sec.

Fig. 4.3. Performance comparison of different service policies.

Fig. 4.3 shows the values of EW_i with different traffic load ρ and total non-picking time t (travel and depot operation time). Other parameters are identical to those of Fig. 4.2. We see the waiting time with 1-limited service policy is consistently greater than those with exhaustive and gated service policies. The exhaustive service policy leads to the lowest average waiting time. The difference with 1-limited and gated service waiting time increases with a higher offered load ρ or a larger value of t.

4.4.2 Comparison of polling systems and non-polling systems



Note: (1)Unit of EW_i:sec,Thoughput time:sec. (2)Total order lines:20317

(3) $K = 5, L = 20, \beta_i^{(1)} = 1\sec, \beta_i^{(2)} = 1.1\sec^2$.

Fig. 4.4. Performance comparison of a polling system and a batch-picking system in Case 1.

This section presents two examples to compare polling systems and non-polling systems, or traditional batch-picking systems. In the first example, we show a polling system can achieve a shorter throughput time. The second case is based on "China online shopping". This company promises to home deliver the goods to customers within two hours after order receipt. We show polling systems improve the on-time service completion ratio and make this promise feasible.

(1) Case 1

By simulation, we compare the performance measure of a polling system and a traditional batch picking system with optimal batch sizes, for the same arrival rate and service capacity. For the batch picking systems, we use an enumeration method to obtain the optimal batch size (see Le-Duc and De Koster (2007)). The optimal batch size differs for different arrival rates and varies between 4 and 11. For this system, we use an FCFS batching policy, an S-shape routing policy and random storage. We assume single-line orders. The layout is given in Fig. 4.1. In traditional batch picking, the pick locations during a batch-pick cycle are given and fixed, and pick information is not updated during the cycle.

We simulate 20317 randomly generated order lines following a Poisson process with arrival parameters λ varying from 0.1 to 0.2 and compute the average order waiting time and average throughput time per order, consisting of waiting, picking, and travel time. The performance comparison of a polling system and a batch picking system is presented in Fig. 4.4. We can observe the expected order waiting time of a batch picking system is larger than a polling system, and the order throughput time of a polling system is shorter.

(2) Case 2

"China online shopping", an online retailer in consumer electronics in China, provides a "delivery within two hours upon order receipt" service in large cities. This retailer sells over 20000 products in 226 cities. We mainly consider the e-business at their headquarter in Shenzhen. Based on the operations in the second quarter of 2007, the mean transport and delivery time is 96.5 minutes in Shenzhen, the mean time for other operations (order processing / call-center operation, picking, internal travel, sorting, and packing time) is 18.5 minutes. To be able to deliver orders in time, orders should start processing within on average 5 minutes upon receipt. Currently the company has adopted a dynamic order picking system with the aid of an information system based on mobile technology and a call center (order processing center). Major difference between their system and our model is the routing policy. They have not begun to design an optimal or heuristic routing policy yet, and now just adopt a random routing policy.

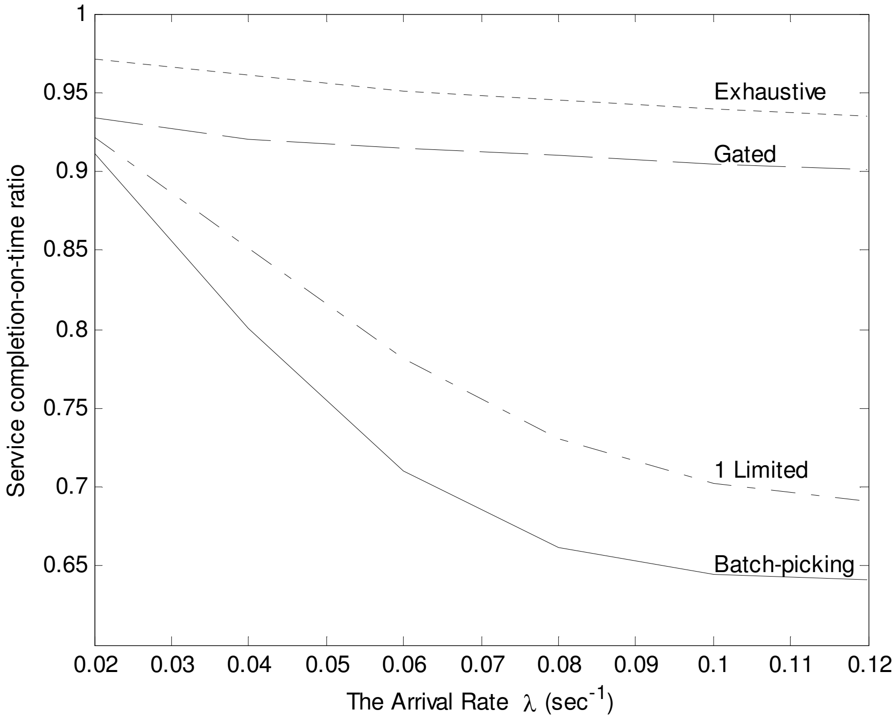


Fig. 4.5. Service completion-on-time ratio versus the arrival rate in China Online Shopping.

We compare the results of our method of polling-based systems with non-polling batch-picking systems using the optimal batch sizes. We simulate one day of work, consisting of 17280 randomly generated order lines following a Poisson process with arrival parameters λ varying from 0.02 to 0.12 and compute on-time service completion ratios for a waiting time upper bound of five minutes beyond the arrival time. In the simulation, we assume single line orders, which in reality form 42.1% of all orders in the second quarter of 2007. Fig. 4.5 shows, for various order arrival rates, the on-time service completion ratios of polling-based picking systems are higher, compared with traditional batch-picking with optimal batch sizes. Note that Fig. 4.5 also shows that differences between polling and batch picking systems are small for small arrival rates. Batch picking with optimal batch size reduces to picking single orders for small arrival rates, and may then outperform polling-based picking due to shorter travel times.

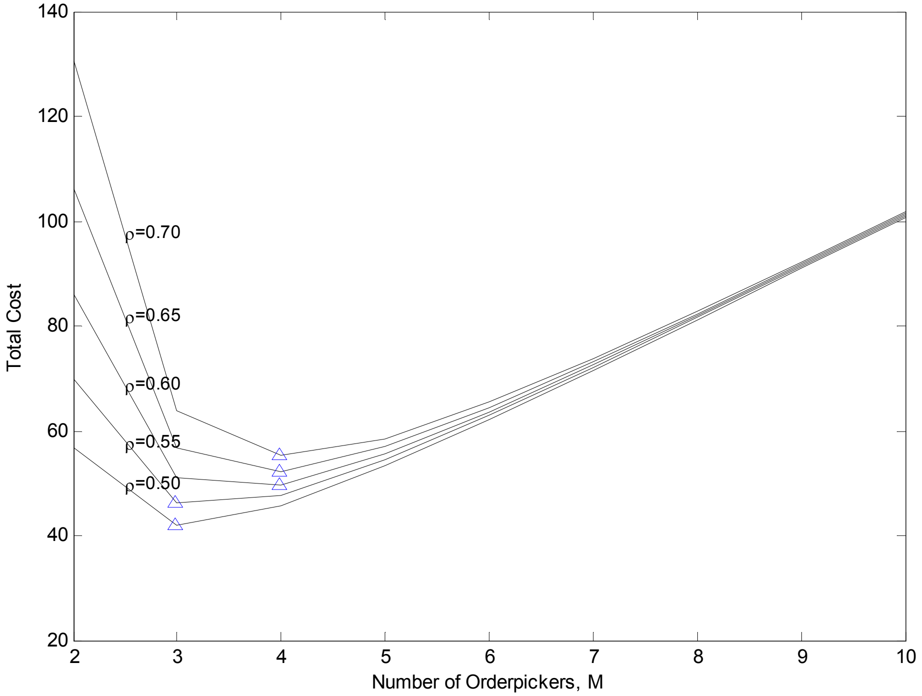
Table 4.1. Parameters of China Online Shopping Shenzhen warehouse

Warehouse	Warehouse area	985 m^2	♯ Aisles	8	♯ Positions per aisle	30
Orders	Ave. order lines/day	17280	Arrival rate	0.01-0.9/sec		
Operations	Travel speed	0.48 meter/sec	Mean pick time/order line	1.51 sec	Depot time	120 sec
	♯ Pickers	20 persons/shift	♯ Shifts	3 shifts/day	♯ Total pickers	60

4.4.3 Applications

This section shows how to apply polling systems in practice, especially to optimize the number of pickers, for the two cases introduced in Section 4.4.2. An important objective for warehouse operations is determining the optimal number of order pickers. This optimum is usually found by trading off the labor cost and service level: while increasing the number of pickers can improve the service level, it will also increase the labor cost. The objective is to minimize the total operational cost, including labor and the customer waiting costs. Waiting costs arise in case of lost sales or when order picking due time cannot be met. We assume it is proportional to the total expected waiting time of all waiting orders (Wolff (1989)).

(1) Case 1



Note: (1)Δ: The optimal number of order pickers. (2)Unit. $\lambda: \sec^{-1}$, cost:EURO.
(3) $L = 20, K = 5, t = 90\sec, \beta_i^{(1)} = 1\sec,$
$\beta_i^{(2)} = 1.1\sec^2, c_L = 10\text{EURO}, c_i = c_W = 10^{-4}\text{EURO}/\sec, T = 3600\sec.$

Fig. 4.6. The total cost versus the number of order pickers in Case 1.

The objective function is the sum of labor cost $c_L M$ and the customer waiting cost $\sum_{i=1}^{N} c_i \lambda_i EW_i T$ during an examined period T, where c_L is the unit labor cost per period per order picker, c_i is the unit time waiting cost at the position i, $\lambda_i T$ is the expected number of arrived order lines in the period T at the position i, $\lambda_i EW_i T$ is the total expected waiting time of these order lines and $c_i \lambda_i EW_i T$ is the total expected waiting cost at the position i. EW_i is specified as a function of $f_i(M)$ by Theorem 4.7. We have the system,

$$min_M \quad c_L M + \sum_{i=1}^{N} c_i \lambda_i EW_i T,$$

subject to

$$EW_i = f_i(M), \forall i.$$

The optimal number of order pickers M^* is determined by the trade-off of the labor cost $c_L M$ which increases linearly with M and the customer waiting cost $\sum_{i=1}^{N} c_i \lambda_i EW_i T$ which decreases nonlinearly with M. Using data of Case 1, the results are presented in Fig. 4.6 for different values of ρ. This example shows how polling theory can be applied in practice to reduce cost. Here we propose a simple model of waiting cost, linearly related to waiting time. For more sophisticated ways of modelling waiting time and related cost, see Hopp et al.(2002).

(2) Case 2

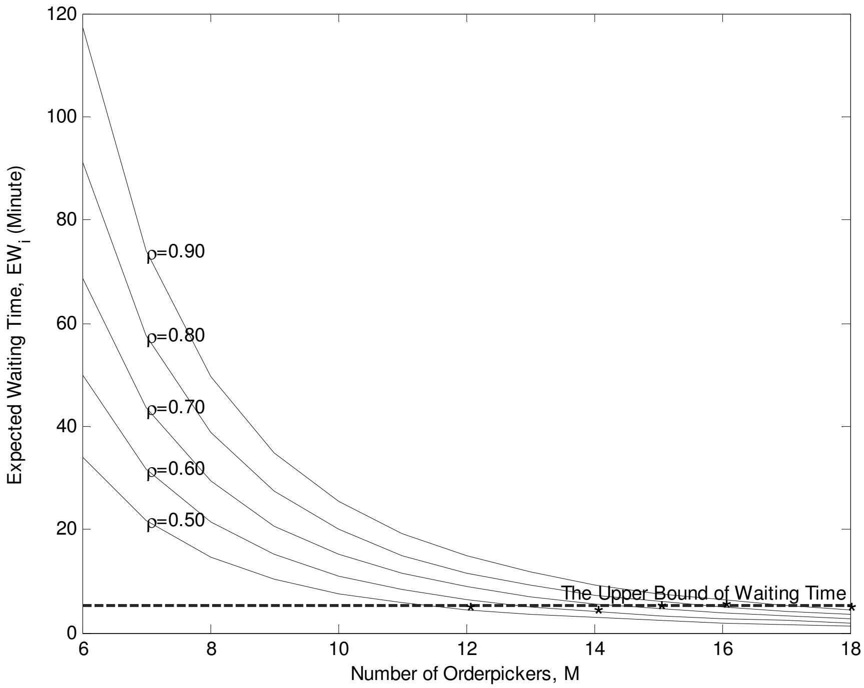
Different from Case 1, "China online shopping" has a given upper bound $\overline{W}$ of five minutes which stems from the promised "delivery within two hours upon order receipt". We are mainly concerned with minimizing the number of order pickers M while keeping the expected waiting time EW_i less than this given upper bound $\overline{W}$. We have the system,

$$min \ M$$

subject to

$$EW_i \leq \overline{W}, \forall i$$

Based on the case data, we present the results in Fig. 4.7. For a given upper bound of waiting time $\overline{W}$ and with different offered loads, we can obtain the optimal number of order pickers. This example shows how polling theory can be applied to determine the number of pickers.



Note: (1)*:The optimal number of order pickers. (2)Unit. $\lambda : \sec^{-1}, EW : \min$.
(3) $L = 30, K = 8, N = 240, t = 960\sec, \beta_i^{(1)} = 1.51\sec, \beta_i^{(2)} = 3.82\sec^2$.

Fig. 4.7. The expected waiting time versus the picker number in China Online Shopping.

4.5 Concluding remarks

In this chapter, we use polling models to describe and identify a dynamic order picking system for online retailers, and provide a stochastic method to analyze such operations in warehouses. We have derived the stability condition of these models, and found approximate closed-form expressions for the order line waiting times in systems with single or multiple order pickers. We show that, particularly for high order arrival rates, polling-based picking systems generally lead to shorter order waiting and throughput time and higher on-time service completion ratios than traditional batch-picking systems using optimal batch sizes. For the case with multiple order pickers, based on the trade-off of labor cost versus service level, we show how to obtain the optimal number of pickers given a maximal waiting time or with an objective to minimize total cost.

This chapter is the first to introduce dynamic order picking systems, and use polling models for their analysis. The models and methods in this chapter are not limited to the settings of a parallel-aisle warehouse, but can be generalized to other storage systems like carousels or paternosters. A carousel system with moving storage positions and a fixed order picker is equivalent to a system with fixed storage positions and a traveling order picker, i.e. a typical polling system with multiple queues and a single server.

Our analysis of dynamic picking lends itself to potential extensions in several directions. For example, the research could be extended to a DPS with multiple-line orders, or to capacitated pick carts. It is also interesting to investigate the optimal routing problem in polling systems. This chapter adopts a cyclic-server routing assumption, which is close to the S-shape routing policy, one of the most common policies used in practice. In an environment of dynamic picking, a policy with local backward routing will probably improve the system performance. It may also be possible to research the combination of DPS with zoning and class-based storage strategies.

Part III

Stochastic Modelling and Analysis for Service Distribution Center

5

Improving order picking productivity via storage profiles and bucket brigades

5.1 Introduction

"Bucket brigades"(BB) are a way to coordinate the efforts of workers along a product line so that the line balances itself. The simplest model (Bartholdi III and Eisenstein, 1996) to capture the essential behavior of bucket brigades is the so-called normative model. The operation of normative model is simple: Each picker carries work forward, from work station to work station, until he either completes an order or it is taken by a downstream colleague; then he walks back to get more work, either from an upstream colleague or from a buffer at the start of the line. If pickers are sequenced from slowest to fastest, the pickers will spontaneously gravitate to the optimal division of work so that the pick rate is maximized. Bucket brigade order picking has been shown to be an effective alternative to zone order picking in many realistic situations (see Bartholdi III et al. (2006) and bucketbrigades.com). The effectiveness of BB is due to its ability to dynamically balance the work load among the pickers. In this research, we examine a new, potential reason that bucket brigades may, in some cases, outperform zone picking systems and show how to improve order picking productivity via storage profiles and bucket brigades.

Picking items from forward storage has two primary cost components: the picking of the items and the walking between picks. The actual cost of picking is typically influenced by the level of technology: pick-to-light, hand-held RF device, or simple printed pick list.

Typical progressive zone picking divides a rack of forward storage (either static or flow rack) into regions or zones. Each worker is assigned a zone. A customer order is passed from one worker to the next, progressing from one zone to another along the length of the pick line. In a pick-to-light implementation, order sequence typically must be maintained, so even if an order were to complete early, it gets passed along the line. The goal of management is to make balancing the zones easy, and so item placement is geared toward smooth uniform demand along the pick line.

The strategic placement of items along the pick line has inherent potential to improve the pick rate of a bucket brigade system. This is because, if popular items are concentrated together, the bucket brigade team can potentially pick many orders within a shrunken or reduced pick line. That is, a bucket brigade is able to expand or reduce the picking range of each worker dynamically, potentially reducing the walking distance required of the team. This potential to save walking distance with a BB team is predicated on a picker not needing to travel to the start of the pick line to begin an order, nor travel to the end of the line to deposit a completed order. So for example, empty totes might be stored along an overhead rail so that a new order can begin at any point, and a completed order can be deposited along an active conveyor running along the pick line. In this research, we do not discuss demand profiles, and how to allocate products to storage positions according to demand profiles. Instead we assume storage profiles are given, and thereby measure and compare the performances of BB in these profiles.

We ignore the congestion issue. One reason is that this is very implementation specific. For example, a pick-to-light implementation may prohibit pickers from picking different orders within a wide bay of flow rack. On the other hand a hand-held or cart-held RF device may not prohibit pickers from picking the same item simultaneously. Other site-specific attributes such as aisle width also affect issues of congestion (see Gue et al. (2006) for example).

We examine various storage policies and model the impact on the picking rate of BB order picking vs. zone order picking. This research obtains closed-form analytic expressions for the main performance measure of order picking bucket brigade systems in different storage profiles, and show how to combine storage policy and bucket brigade protocol to improve order picking productivity. We further show the system outperforms zone picking systems in non-uniform storage profiles.

5.2 Picking protocols

In this section, we consider three picking protocols: uni-directional BB (from Bartholdi III and Eisenstein (1996)), bi-directional BB, and zone picking. This research considers one order picking line with N positions for a unit position length (e.g., 1 foot). At the position x, an order picker needs to pick an item according to a storage density distribution function $f(x)$. O order pickers will work along this line. The o^{th} picker has a travel velocity v_o. We use a configuration from Eisenstein (2008), where there is a conveyor along this line (see Fig. 5.1).

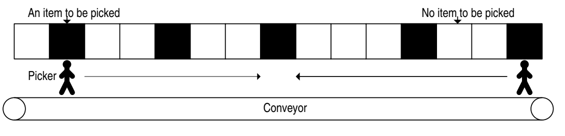


Fig. 5.1. The configuration of bi-directional BB picking systems

In most pick lines, orders are progressed in one, forward, direction — for ease of exposition we will say that picking in the "forward" direction corresponds to orders progressing "left to right".

The uni-directional BB protocol is:

- FORWARD: Pick forward until order taken over or or order completed. Then walk back.
- BACK: Walk back to get more work, either from a predecessor, or at the start of a new order.

In contrast, it may be possible to pick in both directions (see Fig. 5.1). That is when an order is completed moving forward at one end of the pick line, the worker may then walk to the rightmost pick of a new order and begin picking moving backward, or right-to-left. This may be impractical in some settings due to a number of factors; in particular lack of support from the warehouse management system, or simply lack of an active conveyor at both ends to carry completed orders.

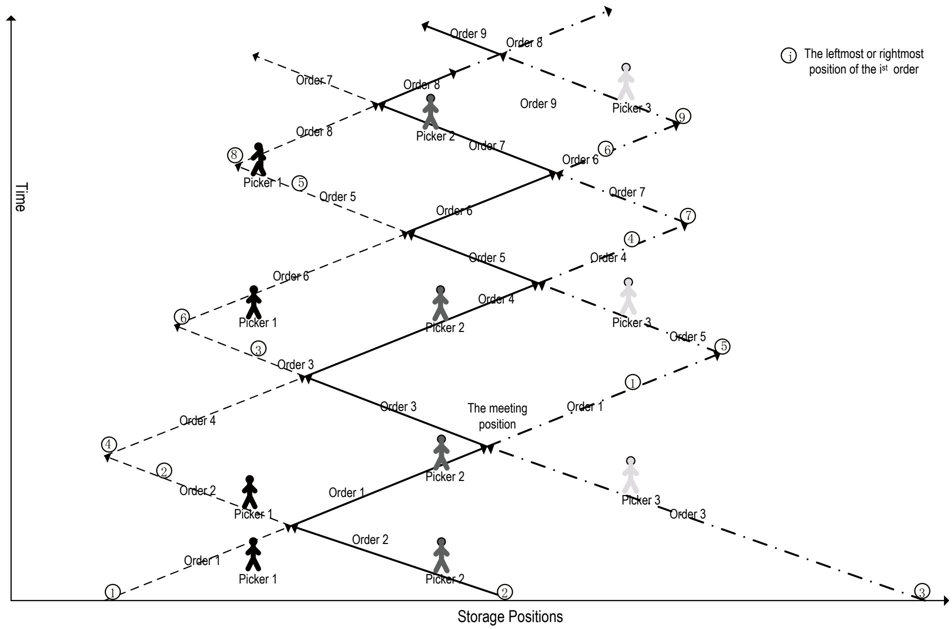


Fig. 5.2. The protocol of bi-directional BB

BB does not build WIP along the pick line, and thus a picker may effectively move in both directions. When BB workers meet, they swap orders. See Fig.-5.2. Since uni-directional BB protocol is explored in Bartholdi III and Eisenstein (1996), we will study here the bi-directional BB protocol.

The bi-directional BB protocol is:

- FORWARD: Pick forward until an order swapped with successor picker or an order completed. Then pick back with an order.

- BACK: Pick back until an order swapped with predecessor, or an order completed. Then pick forward with an order.

Bi-directional picking may be particularly difficult to implement with zone picking, since partially completed orders typically sit on a passive conveyor between zones. Workers would then have difficulty distinguishing forward from backward moving orders and moving orders around those orders moving in the opposite direction. So we assume our zone protocol is typical, by only progressing orders in a forward direction.

The zone protocol defined in this research is:

- Each worker begins an order at the start position (the leftmost or rightmost) of the order in his zone and progresses it to the end position (the rightmost or leftmost) of this order in his zone. A conveyor then takes it to the start of the next zone.
- If no work is available at the start of his zone then he idles, waiting for work.

5.3 Picking rates

In this section, we outline the computation of pick rates ρ, which will be specified combined with the analysis of storage profiles. Throughout the present research, we assume each order consists of m picks. We define the unit of pick rates as orders/ unit time, not picks/ unit time.

5.3.1 BB pick rates

Estimating the pick rate of a bucket brigade system is complicated for a number of reasons.

(1) Walking Distance: The distance walked changes from order to order.
(2) Velocities: Workers walk and pick at different speeds.
(3) Hand-offs: Workers may be slowed due to hand-offs.
(4) Congestion: Workers may be idled due to congestion.

(5) Blocking: Workers may be idled due to blocking, which is more serious than congestion.

Our bucket brigades model will focus on the first two issues: Walking distance and differences in worker velocities. The third issue, hand-off time is considered negligible, and in the case of pick-to-light systems has been observed to be very small. We ignore the fourth issue of congestion, primarily since it is very much a function of a particular implementation — for example, is there room for two workers to pick at neighboring locations? Finally, we do not explicitly model the dynamics of the bucket brigade. We assume that blocking does not occur. Congestion is a short delay which can be removed and blocking is a large delay which may not be removed. Previous work has shown that blocking can be rendered negligible if the workers are sequenced correctly (see, for example, Bartholdi III and Eisenstein (1996), Bartholdi III, Eisenstein, and Lim (2006)).

We assume that the velocity of a worker, both picking and walking, is primarily a function of his agility and motivation. For example, BB has been widely implemented in pick-to-light systems along a flow rack. Since items are picked to light, the velocity of a worker is not a function of product knowledge or learning. So picking, like walking, is more a function of agility and motivation.

We let v_o be the constant rate at which a worker o walks, so that it takes time N/v_o to walk the length of the pick line. Each pick will cost t_p time, independent of the worker. We let L be a random variable representing the leftmost pick in an order, and R the rightmost pick. Then $R - L$ is the pick length the BB team must traverse to pick an order (or equivalently, one can view that the order must travel distance $R - L$ as it is passed from the first to the last BB team member). The BB team has O pickers.

In this research, we mainly examine bi-directional BB protocol since it can reduce efficiently blocking. We now approximate the walking distance, W_{bi}, required of a bi-directional BB team. The last worker will complete an order walking forward, then walk to the rightmost pick of the next order, length W_{RR}, and begin to pick walking backward. The first worker will similarly complete an order walking backward, then walk to the leftmost pick of the next order, W_{LL}, and begin to pick walking forward. The average amount of walking required for an order is $(R-L)+W_{LL}+W_{RR}$. Since this work is completed by O pickers simultaneously, the average travel time per order is $\frac{(R-L)+W_{LL}+W_{RR}}{\sum v_o}$. The average order pick rate of the bi-directional BB line ρ^B is the reciprocal of the sum of average travel time per order and expected picking time per order. A general expression for the pick rate achieved via bi-directional BB systems is

$$\rho^B = 1/\left[\frac{(R-L)+W_{LL}+W_{RR}}{\sum v_o} + mt_p\right]. \tag{5.1}$$

5.3.2 Progressive zoning

In a progressive zone picking system, a bin with items for a single order moves from one zone to another on a conveyor. In each zone, the picker picks the items from their assigned zone for the order. Zone picking systems can have different one-zone configurations (see, e.g. Yu and De Koster (2009)), including a depot at the beginning position, a depot at the middle position, two depots, and a configuration without depots. Eisenstein (2008) has shown that the performance of the configuration without a depot is best, which configuration eliminates the need to walk to and from depot locations. In this research, we therefore assume a picker travels in each zone and picks along one storage line without depot (see Eisenstein (2008)). This implies a conveyor is installed so that when a picker completes an order, he immediately deposits the completed order on the conveyor. We choose this configuration so that it is relatively fair for zone picking in the performance comparison study with BB picking.

We consider a storage line with Z zones, where one order picker is assigned to one zone. Zone picking does not need "hand-off" among pickers. The travel distance includes three parts: $|R-L|_z$, the pick length the picker must traverse to pick items of an order in his zone z; after completing an order walking forward, he will walk to the rightmost pick of the next order, with a length $W_{RR,z}$, and begin to pick walking backward; or after completing an order walking backward, he will walk to the leftmost pick of the next order, $W_{LL,z}$, in his zone and begin to pick walking forward. Different from the bi-directional BB protocol, the sum of average travel distances between two leftmost picks and two rightmost picks per order is $\frac{1}{2}(W_{RR,z} + W_{LL,z})$ since this picker does not need to travel both travel distances between two leftmost picks and two rightmost picks for one order. The average amount of walking required for an order is $|R-L|_z + \frac{1}{2}(W_{RR,z} + W_{LL,z})$ in this zone.

For zone picking systems, it is difficult to perfectly balance zones in terms of both travel time and picking time. Picking rates of zone systems are decided by its bottleneck zone. For a zone, the picker in z^{th} zone has a travel velocity v_z and allocated workload of m_z picks. A general expression for the pick rate achieved via zone picking systems is

$$\rho^Z = 1/max_{z=1,\ldots,Z}\left[\frac{|R-L|_z + \frac{1}{2}(W_{RR,z} + W_{LL,z})}{v_z} + m_z t_p\right]. \qquad (5.2)$$

It is critical to identify the bottleneck zone for zone picking systems. We will specify the pick rate for zone picking in section 6 and 7.What is now left for analysis is first determining the pick rates for various storage profiles. And then to compare the BB pick rate to a more traditional zone picking protocol.

5.4 Storage profiles

In practice, assigning products to storage bins has two fundamental components. First, is how much of each product to store. Items of large physical size and high demand might, for example, benefit from more space in order to minimize overall restocking costs (see Bartholdi III and Hackman (2006) for a model to minimize restocking costs in forward storage). And second is where to locate the item within the forward picking rack. We simplify this discussion by assuming each product receives the same linear facing space, so that the distance to walk past a pick location is the same, regardless of the location of the item.

The decision of where to store an item along a pick line is then a discrete problem of a combinatorial nature — how best to place items into N storage slots in order to maximize the expected pick rate. However, to simplify the analysis we consider the issue in a continuous form. An allocation of the items along the pick line results in a storage density function $f(x)$ which describes the proportion of picking demand at position x, and so $\int_a^b f(x)dx$ is the proportion of picked items slotted between positions a and b. And since $\int_0^N f(x)dx = 1$, then $\int_a^b f(x)dx$ is also the probability an item is slotted between positions a to b along the pick line.

Of course, a warehouse must *react* to the demand for each item, not set it. That is, the warehouse is not able to set $f(x)$ freely. For example, in the unusual circumstance that all items are equally demanded, then regardless of how items are slotted, $f(x)$ is a constant. But more realistically, a large warehouse has items which differ significantly in their demand, and must therefore make allocation decisions as to where to place these items in forward storage (if they are to be placed in forward storage at all, see Bartholdi III and Hackman (2006)). And within this decision a typical warehouse will place much of their items within linear shelving (such as flow racks, each of which are picked independently). This work is then concerned with how to best make this allocation decision — should items of similar demand be placed together or should each flow rack be designed with a wide range of demand? And if the latter, then how best to arrange items with different demands? Storage policies are a set of rules to assign items to storage positions before they can be picked to fulfil customer orders. For a general review on storage assignment, see Section 4 in De Koster et al., (2007). We consider three storage policies to allocate items along the pick line.

(1) Uniform storage system

The uniform storage system, also called random storage, assigns items a location in the storage line that is selected randomly from all eligible empty locations with equal probability $f(x) = \overline{p}$, where $\overline{p} = \frac{1}{N}$ is a constant. The random assignment method results in a high space utilization but often at the expense of increased travel distance.

(2) Volume-based non-uniform storage system (VBS)

VBS assigns items according to their demand. So for example, for a pick line with a depot used to retrieve and deposit orders, a VBS usually assigns items with higher demand to locations near the depot. A VBS is very effective in theory, but in practise can be difficult to manage as item demand changes and items enter and exit the warehouse.

A common VBS policy is the triangle ($f(x) = kx$,k is a constant) storage density distribution (see Figure 5.3-A), which is easier to implement compared

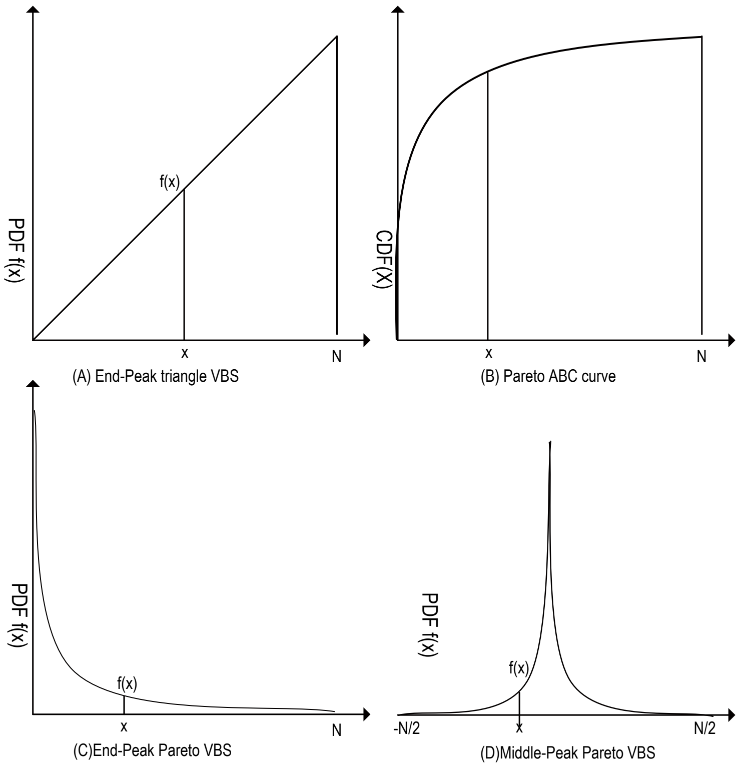


Fig. 5.3. Storage distribution in VBS storage systems

with other VBS policies. From $\int_0^N kxdx = 1$, we have $k = \frac{2}{N^2}$, and $f(x) = \frac{2}{N^2}x, 0 \leq x \leq N$.

One of the most common storage policies is a Pareto-based VBS policy (see Hausman, Schwarz, and Graves (1976)). We consider two cases. The first is an "End-Peak Pareto VBS" as shown in Figure 5.3-C with a peak at the end of the pick line. Here we have $\int_o^x f(x)dx = \varphi x^s$, where s is a constant to determine the shape of storage distribution curve, φ is a constant to grantee storage cumulative distribution function (CDF) $F(x = N) = \varphi N^s = 1$ (see Figure 5.3-B). So we have $f(x) = \frac{s}{N^s}x^{s-1}, 0 \leq x \leq N$. The second case, which we introduce here, we term a "Middle-Peak Pareto VBS" as shown in

Figure 5.3-D. Here we consider a position range $[-\frac{N}{2}, \frac{N}{2}]$. And based on Hausman, Schwarz, and Graves (1976),we have $\int_o^x f(x)dx = \varphi\frac{1}{2}(2x)^s, 0 \leq x \leq \frac{N}{2}$, and $\int_o^{\frac{N}{2}} f(x)dx = 1/2$. Deriving $f(x)$, we have its storage density function $f(x) = \frac{s}{N^s}(2x)^{s-1}, 0 \leq x \leq \frac{N}{2}; f(x) = \frac{s}{N^s}(-2x)^{s-1}, -\frac{N}{2} < x \leq 0$.

(3) Class-based non-uniform storage system (CBS).

A CBS system is a common and more practical restriction to VBS storage. Instead of managing storage down to the individual item level, a CBS partitions items into classes, and then randomly assigns storage within each class (Peterson and Aase, 2004).

A commonly used CBS is the $\alpha - \beta$ two-class storage system $(0 \leq \alpha, \beta \leq 1)$. This system places items in $\alpha \times 100\%$ length of this product line to represent $\beta \times 100\%$ demand in the first storage class, and the remaining $(1-\alpha) \times 100\%$ length of this product line to represent $(1-\beta) \times 100$ % demand in the second class (for three or more classes, we follow the same logic). Within one class, items are evenly distributed. From $\int_0^{\alpha N} f(x)dx = \beta$, we have its storage distribution function $f(x) = \frac{\beta}{\alpha N}, 0 \leq x < N\alpha$, and similarly, $f(x) = \frac{1-\beta}{(1-\alpha)N}, N\alpha \leq x \leq N$.

5.5 Evaluating the pick rate for different storage policies

This section studies the impact of different storage policies on the bi-directional bucket brigade protocol. We use the expected travel distance (per order) and pick rate to measure and compare their performance. We have explored fixed points and stability condition for Bucket Brigade Model with non-uniform storage profiles in Eisenstein and Gong (2009). The stability condition of nonlinear bucket brigade models includes two parts: one is velocities condition to guarantee eigenvalues $(|r(\mathbf{v})|)$ of iteration matrix $|r(\mathbf{v})| < 1$,which is determined by the bucket brigade protocol; another is a storage condition to make Liapunov linearization feasible, which is determined by storage profiles. Throughout this section, we assume the stability condition have been satisfied.

Given the storage density function $f(x)$, we compute the expected pick rates. Let R be the rightmost and L be the leftmost item position in one order, the expected travel time is $\mathbb{E}[R-L]$. Note $\mathbb{E}[R-L] = \mathbb{E}[R] - \mathbb{E}[L]$. Let m be the number of items per order, and their independent positions be $x_1, ..., x_m$, with corresponding CDF's of $F_{X_1}(x_1), ..., F_{X_m}(x_m)$. We have $R = \max\{x_1, ...x_m\}$ and $L = \min\{x_1, ...x_m\}$. The CDF of R is $F_R(r) = F_{X_1}(r)...F_{X_m}(r)$, and the CDF of L is $F_L(l) = 1 - [1 - F_{X_1}(l)][1 - F_{X_2}(l)]...[1 - F_{X_m}(l)]$.

5.5.1 Uniform storage policy

The CDF of the position x of one item is specified as $F_X(x) = \frac{x}{N}$ in uniform storage profiles. The CDF of the rightmost position R is $F_R(r) = \frac{r^m}{N^m}$. The PDF of the position r is $f_R(r) = m\frac{r^{m-1}}{N^m}$. The expected position of the rightmost item is

$$\mathbb{E}[R] = \int_0^N m\frac{r^{m-1}}{N^m}dr = \frac{m}{m+1}N. \tag{5.3}$$

The CDF of the leftmost position L is $F_L(l) = 1 - [1 - \frac{l}{N}]^m$. The PDF of the position l is $f_L(l) = \frac{m}{N}(1 - \frac{l}{N})^{m-1}$. The expected position of the leftmost item is

$$\mathbb{E}[L] = \int_0^N \frac{m}{N}(1 - \frac{l}{N})^{m-1}dl = \frac{1}{m+1}N. \tag{5.4}$$

Besides the expected distance from the leftmost to the rightmost in one order, a picker also needs to travel from the rightmost of one order to the rightmost of the next order, and from the leftmost of one order to the leftmost of the next order. To compute the pick rate, we also need to compute these two expected distances.

First, we compute the expected distance from the rightmost to the rightmost of the next order. Let r_1 be the position of the rightmost item in one order and r_2 be that of the rightmost item in another. We have gotten $f_R(r) = m\frac{r^{m-1}}{N^m}$. The difference $\mathbb{E}[|R_1 - R_2|]$ (without loss of generality, let $R_1 > R_2$) between the rightmost items of two independent orders is

$$\mathbb{E}[|R_1 - R_2|] = 2\int_0^N \int_0^{r_1} (r_1 - r_2)m^2\frac{r_1^{m-1}}{N^m}\frac{r_2^{m-1}}{N^m}dr_2dr_1 = \frac{2m}{(m+1)(2m+1)}N \tag{5.5}$$

Similarly, the difference $\mathbb{E}[|L_1 - L_2|]$ between the leftmost items of two independent orders is

$$\mathbb{E}[|L_1-L_2|] = 2\int_0^N \int_0^{l_1} (l_1-l_2)\frac{m}{N}(1-\frac{l_1}{N})^{m-1}\frac{m}{N}(1-\frac{l_2}{N})^{m-1}dl_2dl_1 = \frac{2m}{(m+1)(2m+1)}N \tag{5.6}$$

We obtain the expected distance spent on one order in a uniform storage distribution is

$$\mathbb{E}[W]_U = \mathbb{E}[R-L] + \mathbb{E}[|R_1-R_2|] + \mathbb{E}[|L_1-L_2|] = \frac{2m^2+3m-1}{(m+1)(2m+1)}N \tag{5.7}$$

Based on the equation 5.7, we have proposition 5.1.

Proposition 5.1. *For a BB order picking system with O order pickers, when using a uniform storage policy and without blocking or congestion, its average pick rate is* $1/[\frac{2m^2+3m-1}{(m+1)(2m+1)}N/\sum_{o=1}^{O} v_o + mt_p]$.

5.5.2 Class-based storage policies

We use a $\alpha-\beta$ two-class "End-Peak CBS" policy as an example to explain its performance (for other CBS policies, see Eisenstein and Gong (2009)). Its storage distribution function is $f(x) = \frac{\beta}{\alpha N}, 0 \le x < N\alpha; \frac{1-\beta}{(1-\alpha)N}, \alpha N \le x \le N$. Here we outline the calculation of pick rates; for an analysis consisting of all details, see Eisenstein and Gong (2009).

(1) The expected distance from the leftmost to the rightmost in one order

The position of the rightmost one $R = max\{x_1, ...x_m\}$, and CDF of the position R is $F_R(r) = (\frac{\beta}{\alpha N}r)^m, 0 \le r < \alpha N; (\beta + \frac{1-\beta}{(1-\alpha)N}(r-\alpha N))^m, \alpha N \le r \le N$. PDF of the position r is $f_R(r) = m(\frac{\beta}{\alpha N})^m(r)^{m-1}, 0 \le r < \alpha N; \frac{(1-\beta)m}{(1-\alpha)N}(\beta + \frac{1-\beta}{(1-\alpha)N}(r-\alpha N))^{m-1}, \alpha N \le r \le N$. The expected position of the rightmost item is

$$\mathbb{E}[R] = (\beta)^m\frac{\alpha m}{m+1}(\frac{\beta}{\alpha})^m N + \Sigma_{i=1}^m \binom{m}{i}(1-\beta)^i(\beta)^{m-i}[\frac{(\alpha-\beta)(-1+\beta^i)+i(1-\beta)(-1+\alpha\beta^i)}{(1+i)(1-\beta)}]N \tag{5.8}$$

The position of the leftmost pick $L = min\{x_1, ...x_m\}$, and CDF of the position l is $F_L(l) = 1 - [1 - \frac{\beta}{\alpha N} l]^m, 0 \leq x < \alpha N; 1 - [1 - \beta - \frac{1-\beta}{(1-\alpha)N}(x - \alpha N)]^m, \alpha N \leq x \leq N$. PDF of the position l is $f_L(l) = \frac{\beta m}{\alpha N}[1 - \frac{\beta}{\alpha N} l]^{m-1}, 0 \leq x < \alpha N; \frac{(1-\beta)m}{(1-\alpha)N}[1 - \beta - \frac{1-\beta}{(1-\alpha)N}(x - \alpha N)]^{m-1}, \alpha N \leq x \leq N$. The expected position of the leftmost item is

$$\mathbb{E}[L] = (1-\beta)^m \frac{(1-\beta)^m(1+m\alpha)}{1+m} N + \Sigma_{i=1}^m \binom{m}{i} (\beta)^i (1-\beta)^{m-i} \frac{1-(1-\beta)^i - i(1-\beta)^i \beta}{(1+i)\beta} N \tag{5.9}$$

(2) Pick rate

First, we compute the expected distance from the rightmost pick of an order to the rightmost of the next order. Let r_1 be the position of the rightmost item in one order and r_2 be that of the rightmost item in another. Without loss of generality, let $R_1 > R_2$. We have gotten $f_R(r) = m(\frac{\beta}{\alpha N})^m (r)^{m-1}, 0 \leq r < \alpha N; \frac{(1-\beta)m}{(1-\alpha)N}(\beta + \frac{1-\beta}{(1-\alpha)N}(r - \alpha N))^{m-1}, \alpha N \leq r \leq N$. The difference $\mathbb{E}(|R_1 - R_2|)$ between the rightmost items of two independent orders is $\frac{2m^2\beta^{2m}}{(\alpha N)^{2m}} \int_0^{\alpha N} \int_0^{r_1} (r_1 - r_2) r_1^{m-1} r_2^{m-1} dr_2 dr_1 + \frac{2(1-\beta)^2 m^2}{(1-\alpha)^{2m} N^{2m}} \int_{\alpha N}^{N} \int_{\alpha N}^{r_1} (r_1 - r_2) \Pi_{i=1}^2 ((1-\beta) r_i + (\alpha - \beta)N)^{m-1} dr_2 dr_1$. We have,

$$\mathbb{E}[|R_1 - R_2|] = \frac{2m^2\beta^{2m}\alpha}{2m+1} N + \frac{2m\Theta_1}{(1-\alpha)^{2m}(1+m)(2m+1)} N, \tag{5.10}$$

where $\Theta_1 = [(1+\alpha-2\beta)^{2m+1} - (2m+1)(-1+\alpha)(1+\alpha-2\beta)^m(-1+\beta)(2\alpha - \beta - \alpha\beta)^m - (2\alpha - \beta - \alpha\beta)^{2m+1}]$.

PDF of the position l is $f_L(l) = \frac{\beta m}{\alpha N}[1 - \frac{\beta}{\alpha N} l]^{m-1}, 0 \leq x < \alpha N; \frac{(1-\beta)m}{(1-\alpha)N}[1 - \beta - \frac{1-\beta}{(1-\alpha)N}(x - \alpha N)]^{m-1}, \alpha N \leq x \leq N$, and the difference $\mathbb{E}[|L_1 - L_2|]$ between leftmost items of two independent orders is $\frac{2m^2\beta^2}{(\alpha N)^2} \int_0^{\alpha N} \int_0^{l_1} (l_1 - l_2)[1 - \frac{\beta}{\alpha N} l_1]^{m-1}[1 - \frac{\beta}{\alpha N} l_2]^{m-1} dl_2 dl_1 + \frac{2(1-\beta)^2 m^2}{(1-\alpha)^2 N^2} \int_{\alpha N}^{N} \int_{\alpha N}^{l_1} (l_1 - l_2) \Pi_{i=1}^2 [1 - \beta - \frac{1-\beta}{(1-\alpha)N}(l_i - \alpha N)]^{m-1} dl_2 dl_1$. We have

$$\mathbb{E}[|L_1 - L_2|] = \frac{2\alpha m[1-(1-\beta)^{2m+1} - (2m+1)(1-\beta)^m \beta]}{\beta(m+1)(2m+1)} N + \frac{2(1-\beta)^{2m} m \Theta_2}{(1-\alpha)^{2m}(m+1)(2m+1)} N, \tag{5.11}$$

where $\Theta_2 = \frac{\Upsilon_1 + \Upsilon_2}{\Gamma(m+1)\Gamma(m+2)}$, and $\Upsilon_1 = -(1-(1-\alpha)^{2m} + (-1-2m+(1-\alpha)^m)(1-\alpha)^m \alpha)\Gamma(m+1)\Gamma(m+2)$, $\Upsilon_2 = (1+2m)\Gamma(m)\{-\Gamma(m+2)((1-\alpha)^m(1+m\alpha) - H[1,-m,1+m,1]) + \Gamma(m+1)((m+1)(1-\alpha)^m - H[2,-m,2+m,1])\}$ and H is a hypergeometric function.

We obtain the expected distance $\mathbb{E}[W]_{CBS}$ spend on one order in a CBS system is

$$\mathbb{E}[W]_{CBS} = \mathbb{E}[R-L] + \mathbb{E}[|R_1 - R_2|] + \mathbb{E}[|L_1 - L_2|] \tag{5.12}$$

We have proposition 5.2.

Proposition 5.2. *For a BB order picking system with O order pickers, when using a* $\alpha - \beta$ *"End-Peak CBS" policy and without blocking and congestion, its average pick rate is* $1/[\mathbb{E}[W]_{CBS}/\sum_{o=1}^{O} v_o + mt_p]$*, and* $\mathbb{E}[W]_{CBS}$ *is specified by equations 5.10,5.11, and 5.12.*

5.5.3 Volume-based storage policies

We use an "End-Peak Pareto VBS" storage policy as an example to explain performance calculation for VBS policies. For the performance analysis of other storage policies, see Eisenstein and Gong (2009). The storage distribution function of "End-Peak Pareto VBS" is $f(x) = \frac{s}{N^s} x^{s-1}, 0 \leq x \leq N$.

(1) The expected distance from the leftmost to the rightmost pick in one order

We firstly compute the expected position of the rightmost item in one order. The CDF of the position x of one item is $F_X(x) = \frac{x^s}{N^s}$. The position of the rightmost one $R = max\{x_1, ...x_m\}$, and CDF of the position R is $F_R(r) = \frac{r^{sm}}{N^{sm}}$. PDF of the position r is $f_R(r) = \frac{smr^{sm-1}}{N^{sm}}$. The expected position of the rightmost item is

$$\mathbb{E}[R] = \int_0^N \frac{smr^{sm-1}}{N^{sm}} r dr = \frac{sm}{sm+1} N. \tag{5.13}$$

We compute the expected position the leftmost item in one order. The position of the leftmost one $L = min\{x_1, ...x_m\}$, and CDF of the position l is $F_L(l) = 1 - [1 - \frac{l^s}{N^s}]^m$. PDF of the position l is $f_L(l) = \frac{sm}{N^s}(1 - \frac{l^s}{N^s})^{m-1} l^{s-1}$. This leads to the following expected position of the leftmost item,

$$\mathbb{E}[L] = \int_0^N \frac{sm}{N^s}(1 - \frac{l^s}{N^s})^{m-1} l^{s-1} l dl = \frac{m\Gamma[m]\Gamma[\frac{1}{s}]}{s\Gamma[1 + m + \frac{1}{s}]}N, \tag{5.14}$$

where $\Gamma[m]$ is the Euler gamma function of m. We obtain the expected distance from the leftmost to the rightmost in one order $\mathbb{E}[R - L] = \mathbb{E}[R] - \mathbb{E}[L] = \frac{sm}{sm+1}N - \frac{m\Gamma[m]\Gamma[\frac{1}{s}]}{s\Gamma[1+m+\frac{1}{s}]}N$.

(2) Pick rate

First, we compute the expected distance from the rightmost pick of an order to the rightmost pick of the next order. Let r_1 be the position of the rightmost item in one order and r_2 be that of the rightmost item in another. Without loss of generality, let $R_1 > R_2$. The PDF of the position r is $\frac{smr^{sm-1}}{N^{sm}}$. The difference $\mathbb{E}[|R_1 - R_2|]$ between rightmost items of two independent orders is $2\int_0^N \int_0^{r_1} (r_1 - r_2)\frac{smr_1^{sm-1}}{N^{sm}} \frac{smr_2^{sm-1}}{N^{sm}} dr_2 dr_1$, and we have $\mathbb{E}[|R_1 - R_2|] = \frac{2sm}{(sm+1)(2sm+1)}N$.

Second, we compute the expected distance from the leftmost pick of an order to the leftmost of the next order. Let l_1 be the position of the leftmost item in one order and l_2 be that of the leftmost item in another. The difference $\mathbb{E}[|L_1 - L_2|]$ between the leftmost items of two independent orders is $2\int_0^N \int_0^{l_1} (l_1 - l_2)\frac{sm}{N^s}(1 - \frac{l_1^s}{N^s})^{m-1} l_1^{s-1} \frac{sm}{N^s}(1 - \frac{l_2^s}{N^s})^{m-1} l_2^{s-1} dl_2 dl_1$ and the value of $\mathbb{E}[|L_1 - L_2|]$ is approximately 0.

We obtain the expected distance $\mathbb{E}[W]_{E,P}$ spent on one order in an "**E**nd-Peak **P**areto VBS" storage policy is,

$$\mathbb{E}[W]_{E,P} = (\frac{sm}{sm + 1} - \frac{m\Gamma[m]\Gamma[\frac{1}{s}]}{s\Gamma[1 + m + \frac{1}{s}]} + \frac{2sm}{(sm + 1)(2sm + 1)})N \tag{5.15}$$

We have proposition 5.3.

Proposition 5.3. *For a bucket brigades order picking system with O order pickers, when using a "End-Peak Pareto VBS" storage policy and without blocking and congestion, its average pick rate is* $1/[(\frac{sm}{sm+1} - \frac{\Gamma[m]\Gamma[\frac{1}{ms}]}{s\Gamma[1+m+\frac{1}{ms}]} + \frac{2sm}{(m+1)(2sm+1)})N/\sum_{o=1}^{O} v_o + mt_p]$.

The pick rate depends on Pareto parameter s. For a bucket brigades order picking system using a Pareto storage policy and without blocking or congestion, the smaller the s is the larger the pick rate is from Proposition 5.3. A smaller s means a more focused storage density, and smaller travel distance. However, s is limited by a lowerbound $\underline{s}$, which is determined by the storage space available per SKU. For example, in a garment pick line with 20 SKUs (the length of each position is a), pickers handle and store on average 108 items per day with 4 storage replenishment operations for the product with the highest turnover; but each position can store at most 15 items. In the position with highest storage density, its stored items number should be smaller than the storage capacity. We therefore have $\int_0^a \frac{s}{(20\times a)^s} x^{s-1} dx \leq \frac{15}{108/4}$, and can determine $s \geq 0.196$. We have Proposition 5.4.

Proposition 5.4. *For a BB order picking system using a Pareto storage policy, the optimal Pareto parameter equals to its lowerbound* $s^* = \underline{s}$, *where* $\underline{s}$ *is determined by the capacity available for the storage of SKU with the highest turnover.*

5.5.4 A summary of performance evaluation of different storage policies

We summarize the analytic results in Table 5.1. For further analysis to support this table, see Eisenstein and Gong (2009). In section 5.7.1, we will verify these analytic results by discrete simulation. By observation, we firstly find two analytical results in two subsequent sections. The remaining insights of Table 5.1 will be uncovered with numerical experiments.

Table 5.1. Comparison different storage profiles

Types	Storage Policies	Expected travel distance per order $\mathbb{E}(R-L)$	Pick rate
Uniform	$f(x) = \frac{1}{N}$	$\frac{m-1}{m+1}N$	$1/[\frac{2m^2+3m-1}{(m+1)(2m+1)}N/\sum_{o=1}^{O} v_o + mt_p]$
CBS	α / β	$[(\beta)^m \frac{\alpha m}{m+1}(\frac{\beta}{\alpha})^m + \Sigma_{i=1}^m C_m^i (1-\beta)^i (\beta)^{m-i}[\frac{(\alpha-\beta)(-1+\beta^i)+i(1-\beta)(-1+\alpha\beta^i)}{(1+i)(1-\beta)}] - (1-\beta)^m \frac{(1-\beta)^m(1+m\alpha)}{1+m} - \Sigma_{i=1}^m C_m^i (\beta)^i (1-\beta)^{m-i} \frac{1-(1-\beta)^i - i(1-\beta)^i \beta}{(1+i)\beta}]N$	$1/(\mathbb{E}[W]_{CBS}/\sum_{o=1}^{O} v_o + mt_p)$
VBS	Pareto	$\frac{sm}{sm+1}N - \frac{m\Gamma[m]\Gamma[\frac{1}{s}]}{s\Gamma[1+m+\frac{1}{s}]}N$	$1/[(\frac{sm}{sm+1} - \frac{\Gamma[m]\Gamma[\frac{1}{ms}]}{s\Gamma[1+m+\frac{1}{ms}]} + \frac{2sm}{(m+1)(2sm+1)})N/\sum_{o=1}^{O} v_o + mt_p]$
	Central Pareto	$\frac{sm}{sm+2}N$	$1/[(\frac{sm}{sm+2} + \frac{4sm}{2^{\frac{sm}{2}}(m+2)(sm+1)})N/\sum_{o=1}^{O} v_o + mt_p]$
	Triangle	$\frac{2m}{2m+1}N - \frac{m\sqrt{\pi}\Gamma[m]}{2\Gamma[1.5+m]}N$	$1/[(\frac{2m}{2m+1} - \frac{m\sqrt{\pi}\Gamma[m]}{2\Gamma[1.5+m]} + \frac{4m}{1+6m+8m^2} + \Theta_1^T)N/\sum_{o=1}^{O} v_o + mt_p]$

Shrink effect

Based on Table 5.1, we find, for some non-uniform storage policies, its expected travel distance per order is shorter than that in a uniform storage profile. We define this phenomenon as follows,

SHRINK EFFECT: For a BB order picking system using some non-uniform storage policies, its expected travel distance per order may be shorter than that in a uniform storage profile, and look like a "shrunken" distance.

The shrink effect can be analytically proven in some storage policies. For example, we compare expected travel distance in " Central Pareto" with that of uniform policy and find $\frac{sm}{sm+2}N/\frac{m-1}{m+1}N < 1$, when $0 < s < 1$. For some expected distances with complicated expressions (e.g.,CBS), we will further verify it with numerical experiments in section 5.7.2. The shrink effect is interesting because BB can efficiently use this effect, via its capacity of self-balancing, combined with different storage profiles to improve order picking productivity, and this effect is more significant in a BB protocol compared with that in other protocols without self-balancing capacity like zone picking, which will be shown in 5.7.3.

Limiting behavior

Based on Table 5.1, we can summarize the limiting behavior of bucket brigades order picking systems in Proposition 5.5.

Proposition 5.5. *For a BB order picking system without blocking or congestion, its limiting behavior is specified by the limiting expected travel distance per order* $\lim_{m\to\infty}\mathbb{E}[R-L]=N$ *and limiting pick rates* $\lim_{m\to\infty}\mathbb{E}[\rho]=\sum_{o=1}^{O}v_o/N$ *for all discussed storage policies if we assume* $t_p=0$.

Order sizes m in most cases are smaller than 50 for a manual order picking storage line according to authors' observation. Based on the results in Table 5.1, we will further discuss the impacts of different storage policies on the performance of bucket brigades models with numerical experiments in section 5.7.2 to derive management insights of storage policies when m is in a practical range.

5.6 Comparison of bucket brigades with zone picking systems

In this section, we compare the BB order picking with sequential zone picking systems since both configurations can adopt multiple pickers to work along one product line. We mainly compare the BB system with sequential zone picking systems where there is no depot but a conveyor along zones (see Section-5.3.2).

We consider two ways to partition zones in a sequential zone picking system along a storage line. One is to divide zones by line length, which has an advantage of easy implementation. Another way is to partition zones by an even workload. It can improve the pick rates; but managers can hardly accurately compute the work loads in time, especially in warehouses with dynamic storage profiles where products are daily updated and replenishment is conducted frequently, e.g., warehouses for online retailers, see Gong and De Koster (2008). We consider their performances in two storage environments, uniform storage and non-uniform storage environments.

Taking the Pareto distribution as an example, we show the BB model outperforms zone picking systems in both zoning methods in a non-uniform storage

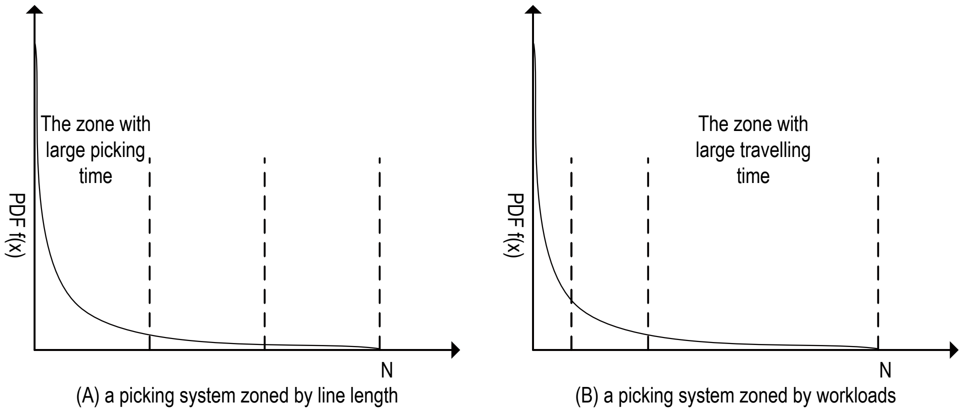


Fig. 5.4. Two zone systems

distribution environment. Picking systems zoned by line lengths have a productivity bottleneck in the head part of the storage distribution line, where the picker must handle more items given equal line length (see Fig.5.4-A). Picking systems zoned by workloads, however, have a productivity bottleneck in the tail part of the storage distribution line, where the picker must travel more time given equal picked items (see Fig.5.4-B).

5.6.1 Comparison with picking systems zoned by line lengths

(1) Uniform storage distribution environment

For a zone picking system with a uniform storage load, we assign each picker one zone with an even line length. The average travel time per order includes three parts, shown in Section 5.3.2. The average pick rate of the zone line ρ^Z is the reciprocal of the sum of average travel time per order and expected picking time per order in a bottleneck zone. The values of O and z are equal in this section since we assume a picker works in one zone.

Let $\underline{v} = min\{v_1, ..., v_O\}$ be the slowest velocity, and $\mathbb{E}[T]_{z,u}$ be the expected **t**ravel time per order of a **z**one system in a **u**niform storage profile. Using a similar logic with equations 5.3 and 5.4, we get order picker team's expected travel distance $|R - L|_{z,u} = \frac{m-z}{m+z}\frac{N}{z}$ for a zone picking system with z zones and z

pickers. Based on equation 5.5, we have $\mathbb{E}[|R_1 - R_2|]_{z,u} = \frac{2m}{(m+z)(2m+z)}N$. Based on equation 5.6, we have $\mathbb{E}[|L_1 - L_2|]_{z,u} = \frac{2m}{(m+z)(2m+z)}N$. From equation 5.2, we obtain the order picker team's expected travel time per order is $\mathbb{E}[T]_{z,u} = \frac{2m^2+mz-z^2}{2m^2+3mz+z^2}\frac{N}{z\underline{v}}$, and expected operation time (travel time plus picking time) per order is $\frac{2m^2+mz-z^2}{2m^2+3mz+z^2}\frac{N}{z\underline{v}} + \frac{m}{z}t_p$.

For bucket brigades, based on Table 5.1, we obtain the expected travel time spend on one order is $\mathbb{E}[T]_{b,u} = (\frac{2m^2+3m-1}{2m^2+3m+1}N)/\sum_{o=1}^{O} v_o$ and expected operation time per order is $(\frac{2m^2+3m-1}{2m^2+3m+1}N)/\sum_{o=1}^{O} v_o + \frac{m}{z}t_p$. By these analyses, we find: (1) Due to self-organization function, we can find variance in pick velocity has less impacts on bucket brigades. For zone picking, the slowest velocity $\underline{v}$ will affect its performance. However, in bucket brigades, the bottleneck effect of $\underline{v}$ will be offset by faster velocities since the performance is dependant on $\sum_{o=1}^{O} v_o$ or the average velocity. (2)In some special cases, zone picking can (but will not always) outperform bucket brigades in terms of the expected travel time spent on one order. We present such an example: When all velocities equal $\overline{v}$, both average picking times are $\frac{m}{z}t_p$, $\mathbb{E}[T]_{z,u} < \frac{2m^2+3mz-z^2}{2m^2+3mz+z^2}\frac{N}{z\underline{v}} = (1 - \frac{2}{2m^2/z^2+3m/z+1})\frac{N}{z\underline{v}}$. The right-hand side of equation is a decreasing function of z, and $(\frac{2m^2+3mz-z^2}{2m^2+3mz+z^2}\frac{N}{z\underline{v}})_{z=1} = \mathbb{E}[T]_{b,u}$. We therefore have $\mathbb{E}[T]_{b,u} > \mathbb{E}[T]_{z,u}$ and $\rho_{b,u} < \rho_{z,u}$, when $z \geq 2$.

Nevertheless, we should notice that we have chosen a protocol favorable to zone picking to guarantee the fairness of comparison. Other typical zone picking configurations include: (1) zone picking systems with one depot and a conveyor, (2) zone picking systems with two depots and a conveyor, (3) a zone picking without a conveyor so that pickers must conduct "hand-off" which means a picker must travel until the end of his zone and leads to more travel distance and an extra delay. These zone picking configurations will lead to a worse performance compared with the zone picking used in this research.

(2) Non-uniform storage distribution environment

In a non-uniform storage distribution environment, we further compare the bucket brigade model with the zone picking system. For a zone picking system with Pareto storage load, its expected travel time is dependent on the first zone $Z_1 = [0, N/z]$ with the heaviest workload. The expected number of items in the first zone is $mF_X(\frac{N}{z}) = m/z^s$. Note the leftmost pick in the first zone is just the leftmost pick of the whole zone, and we have gotten $\mathbb{E}[L] = \frac{m\Gamma[m]\Gamma[\frac{1}{s}]}{s\Gamma[1+m+\frac{1}{s}]}N$ and $\mathbb{E}[L_1 - L_2]$ from the previous analysis in Section 5.5, and need to compute $\mathbb{E}[R]$. Note with probability $P(R \in Z_1) = 1/z^s$, an item will be picked in the first zone and the conditional PDF in the first zone is $f_{R|R\in Z_1}(r) = f_R(r)/P(R \in Z_1) = (\frac{z}{N})^{\frac{sm}{z^s}} \frac{sm}{z^s} r^{\frac{sm}{z^s}-1}$, where $f_R(r)$ is specified in section 5.5.1. We have $\mathbb{E}[R] = \int_0^{N/z} (\frac{z}{N})^{\frac{sm}{z^s}} \frac{sm}{z^s} r^{\frac{sm}{z^s}-1} r dr = \frac{sm}{sm+z^s}\frac{N}{z}$ and $\mathbb{E}[R_1 - R_2] = 2\int_0^{\frac{N}{z}} \int_0^{r_1} (r_1 - r_2)(\frac{z}{N})^{\frac{2sm}{z^s}} \frac{(sm)^2}{z^{2s}} r_1^{\frac{sm}{z^s}-1} r_2^{\frac{sm}{z^s}-1} dr_2 dr_1 = \frac{2sm}{z(sm+z^s)(2sm+z^s)}N$. Based on the analysis above, using $\mathbb{E}[T]^l_{z,p}$ to denote expected travel time per order of a picking system **z**oned by **l**ine length and a **P**areto storage profile, we have proposition 5.6.

Proposition 5.6. *For an order picking system zoned by line length, using an "End-Peak Pareto VBS" storage profile, its pick rate is* $\mathbb{E}[\rho] = 1/(\mathbb{E}[T]^l_{z,p} + \frac{m}{z^s}t_p)$, *where the expected travel time per order* $\mathbb{E}[T]^l_{z,p} = [\frac{sm}{z(sm+z^s)} - \frac{m\Gamma[m]\Gamma[\frac{1}{s}]}{s\Gamma[1+m+\frac{1}{s}]} + \frac{sm}{z(sm+z^s)(2sm+z^s)}]N/v$.

We compare performance of bucket brigades based on Table 5.1 and that of zone picking systems based on proposition 5.6. We find the performance of the bucket brigades picking system is better than zone picking systems in terms of both pick rates and the expected travel time with a non-uniform storage distribution. The advantage of BB is larger in a heavy pick density, i.e., a larger m. The result remains true for all non-uniform storage policies discussed in this research.

For an intuitive explanation, we examine the limiting behavior for systems with z zones, z pickers, and equal velocities v. We obtain the limiting travel time $(\mathbb{E}[T]^l_{z,p})_{m\longrightarrow\infty} = \{[\frac{sm}{z(sm+z^s)} - \frac{m\Gamma[m]\Gamma[\frac{1}{s}]}{s\Gamma[1+m+\frac{1}{s}]} + \frac{sm}{z(sm+z^s)(2sm+z^s)}]N/v\}_{m\longrightarrow\infty} = \frac{N}{zv}$ for zone picking systems and the limiting travel time $(\mathbb{E}[T]_{b,p})_{m\longrightarrow\infty} = \{[(\frac{sm}{sm+1} - \frac{\Gamma[m]\Gamma[\frac{1}{ms}]}{s\Gamma[1+m+\frac{1}{ms}]} + \frac{2sm}{(sm+1)(2sm+1)})N]/zv\}_{m\longrightarrow\infty} = \frac{N}{zv}$ for BB systems are equal. However, considering $0 < s < 1$, the picking time for zone picking is $\frac{m}{z^s}t_p$, larger than that of BB, $\frac{m}{z}t_p$. In a real parameter setting environment, e.g., a 20%/70% policy, the difference in picking time is huge.

5.6.2 Comparison with picking systems zoned by workloads

For order picking systems zoned by workloads in an environment of uniform storage distribution, its performance analysis is the same with that zoned by line length. This section therefore mainly considers the performance comparison in a non-uniform storage distribution environment. We find the performance of zone picking is again worse than that of bucket brigade when the storage line is zoned by workloads.

For a picking system zoned by workloads with Pareto storage profiles, the picking time is equal to $\frac{m}{z}t_p$, and bottleneck is the the expected travel time in the last zone $[hN, N]$ with the longest zone length, where hN is the left boundary of the last zone. From $F(hN) = 1 - \frac{1}{z}$, we obtain the scope of the last zone is $[(1-\frac{1}{z})^{\frac{1}{s}}N, N]$. Note the rightmost pick in the last zone is just the rightmost pick of the whole zone, and we have obtained $\mathbb{E}[R]$ in Section 5.5. To compute the pick rate we need to know $\mathbb{E}[L]$, given $F_L(l) = 1-[1-(\frac{lz}{N})^s]^{\frac{m}{z}}$, which can be calculated as $\int_{hN}^{N} \frac{smz^{s-1}}{N^s}(1-\frac{l^s z^s}{N^s})^{\frac{m}{z}-1} l^{s-1} l dl = \frac{mN}{z^2}[\Gamma(z^s, 1+\frac{1}{s}, \frac{m}{z}) - \Gamma(z^s h, 1+\frac{1}{s}, \frac{m}{z})]$. Based on the analysis above, using $\mathbb{E}[T]_{z,p}^{w}$ to denote expected travel time per order of a picking system **z**oned by **w**orkload and a **P**areto storage profile, we have proposition 5.7.

Proposition 5.7. *For an order picking system zoned by workload, using an "End-Peak Pareto VBS" storage profile, its pick rate is* $\mathbb{E}[\rho] = 1/(\mathbb{E}[T]_{z,p}^{w} + \frac{m}{z}t_p)$, *where the expected travel time per order* $\mathbb{E}[T]_{z,p}^{w} = \{\frac{sm}{sm+1} + \frac{sm}{(sm+1)(2sm+1)} - \frac{m}{z^2}[\Gamma(z^s, 1+\frac{1}{s}, \frac{m}{z}) - \Gamma(z^s h, 1+\frac{1}{s}, \frac{m}{z})]\}N/v$.

We compare performance of bucket brigades based on Table 5.1 and that of zone picking systems based on proposition 5.7 by numerical experiments. We find the performance of the bucket brigades picking system is better than zone picking systems with a non-uniform storage distribution, which will be numerically illustrated in section 5.7. This result remains true for all non-linear storage policies discussed in this research.

5.7 Numerical cases and application

5.7.1 Comparison with discrete simulation experiments

In this section, in order to verify our analytic results, we present numerical cases and compare our analytical results with that of discrete simulation experiments.

For I orders (we set $I = 2000$) and order sizes with a range from 2 to 25, we generate their discrete positions according to storage distribution functions, and then compute the expected distance per order(ft/order), and average pick rate (order/second). We compare the simulation results with the analytic results in Table 5.1. Simulation parameters are $N = 50, v_1 = 0.9ft/sec, v_2 = 0.95ft/sec, v_3 = 1ft/sec, v_4 = 1.05ft/sec$, and SKU unit lengh=1ft. We consider three cases respectively with two pickers (with v_1, v_2), three pickers (with v_1, v_2, v_3), and four pickers (with v_1, v_2, v_3, v_4). This parameter setting is based on an order picking line in a garment warehouse, and within the parameter range based on authors' experiences.

For a uniform storage distribution, we present the comparison in Figure 5.5 (the top panel). For the expected travel distance, the relative error is 0.431%. For the average pick rate, the relative error is 0.435%. We have conducted extensive experiments with different parameter settings; relative errors of our analytic results are less than 0.5 % in most cases.

For a non-uniform storage distribution, we firstly generate the position of an item given a storage density function $f(x)$ using inverse transform method (see Law and Kelton (2000)). We take pareto distribution as an example. Setting $F(X) = \frac{X^s}{N^s} = U$, we generate the positions by the following algorithm: *1.Generate random number U; 2. Set* $X = U^{\frac{1}{s}}N$. We consider two cases with different s value: for a 20%/70% policy, $s = 0.221615$, for a 20%/50% policy, $s = 0.430677$.

We present the comparison in Fig 5.5 (the lower panel). For the expected travel distance, the relative error is 0.405 %; for the average pick rate, the relative error is 0.406%. We conduct extensive experiments in different parameter environments, relative errors of our analytic results are less than 0.6 % in most cases. These experiments show our analytic results are accurate.

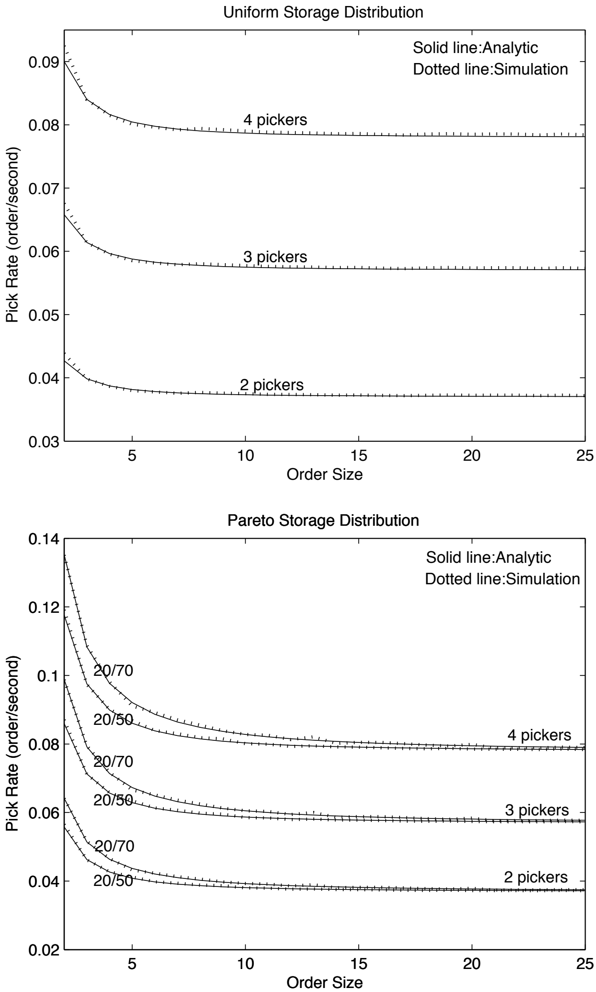


Fig. 5.5. Compare pick rate analytical results with that of discrete simulation

5.7.2 The impact of storage policies

This section examines the impact of different storage policies on bucket brigades models. Based on the analytic expressions in Table 5.1, we first compute the expected travel distance per order in Fig. 5.6 (the top panel), where experiment parameters are $N = 50, s = 0.221615, v_1 = 0.9, v_2 = 0.95, v_3 = 1ft/sec$. To derive more insights, particularly in order to highlight the shrink effect, only in this section we temporarily disregard pick time since it is the same given a specific storage profile and order picking policy. We then compute average pick rates in Fig. 5.6 (the lower panel). By this comparison, we find the following management insights.

(1)We can observe shrink effect in Fig. 5.6 (the top panel). The expected travel distances per order in all 4 non-uniform profiles are "shrunken" compared with that in uniform storage profiles. Correspondingly, we can observe pick rates of all 4 non-uniform profiles are higher than that of uniform storage profile in Fig. 5.6 (the lower panel).

(2)Shrink effects are different in these storage profiles. Volume-based storage policies outperforms class-based storage policies for bucket brigades models. Class-based storage policies outperforms uniform storage distribution. In volume-based storage policies, "middle-peak VBS" outperforms "end-peak VBS" (e.g., see Pareto distribution with a middle peak and that with an end peak). The main reason is that, this can further reduce the expected walk distance of order pickers using the BB protocol.

(3)Parameter values $\frac{\alpha}{\beta}$ impose an influence on the shrink effect. We have calculated the pick rates at different $\frac{\alpha}{\beta}$ values (see Fig.-5.7). When $\frac{\alpha}{\beta}$ is smaller, the performance of bucket brigades models improve. When $\frac{\alpha}{\beta}$ is smaller, shrink effect is larger, that is, items are slotted at a short part of the product line with relatively high density, which can reduce the expected walk distance of order pickers in BB protocol.

(4)When order sizes are small, the effect of different storage policies on performance is significant. Note $\lim_{m\to\infty} \mathbb{E}(R - L) = N$ for all storage policies. When order sizes are large, the expected walk distance per order will approach the line length N, and the effect is slight. But in practice, $m \leq 50$ in most cases. For the high density case with $m > 50$, managers usually will consider automated order picking systems. Therefore, we can conclude that storage profiles usually will impose a significant impact on the performance of bucket brigades in practice.

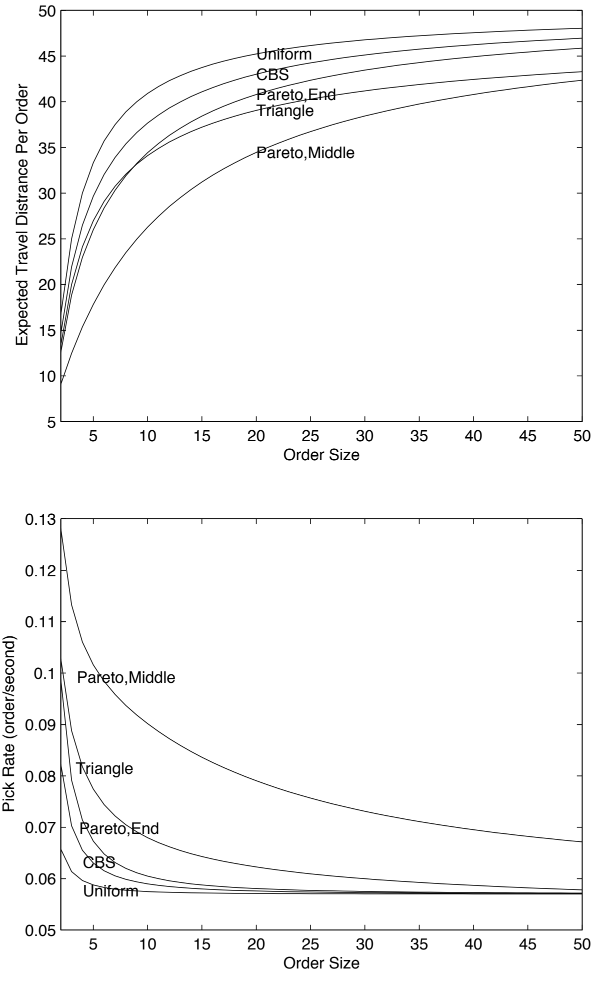


Fig. 5.6. Compare expected travel distances and pick rates in different storage policies

5.7.3 Comparison of bucket brigades model with zone picking systems

In this section, we compare bucket brigades model with the zone picking systems introduced in Section 5.2, including zoned by line length and workload, by simulation. The motivation of this comparison is not just to identify a better protocol among them, but mainly to uncover management insights and show the capacity of BB to use storage profiles and shrink effect to improve pick rates, in non-uniform storage profiles.

We consider a Pareto storage profile with 20%/70% and 20%/50% storage policies, three order pickers with velocities $v = 1$, unit picking time $t_p = 2sec/item$. We examine 400 orders in one picking line, handle them by FCFS sequence, and compare their pick rates.

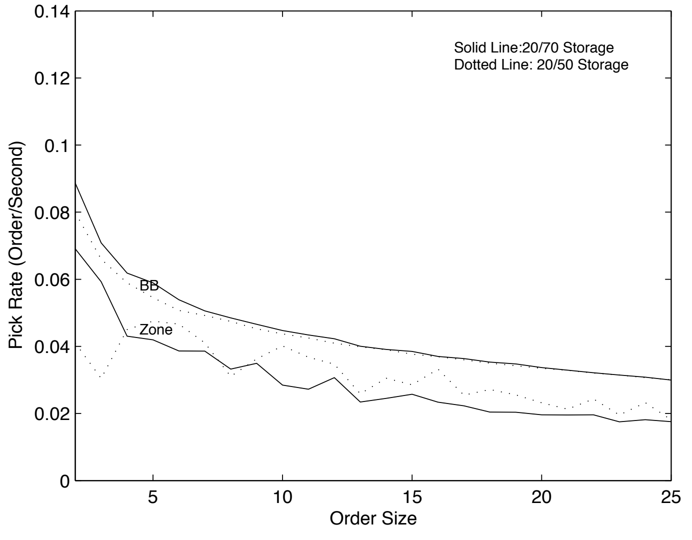


Fig. 5.7. Compare BB and picking systems zoned by line length

First, we compare BB with an order picking system zoned by line length $N = 50$. We present one of our numerical experiments in Figure 5.7. We find, (1)From Figure 5.7, the performance of BB is always better than that of zone picking system. (2) The performance of BB in a 20%/70% policy, with more skewness, is better than that in 20%/50%. However, zone picking systems cannot handle skewed environments very well. Overall, performance of zone picking in a 20%/70% policy is worse than that in 20%/50% since more items will be located in the first zone. This zone then becomes a bottleneck, which cannot be streamlined.

(3) The pick rate curve of BB is more smooth than that of zone picking systems, and BB produces less output variance.

We then compare BB with a picking system zoned by workload. For $N = [50, 200]$ and a 20%/70% policy, we present one of our numerical experiments in Fig 5.8. We find, (1)From Figure 5.8, the performance of BB is consistently better than that of zone picking system at all m and N. (2) When N and m increase, the performance of zone picking becomes worse. (3) The pick rate surface of BB is smoother than that of zone picking systems. We have further compared the pick rates for a 20%/50% policy, and also find BB can handle the storage policies with more skewness, better than zone picking .

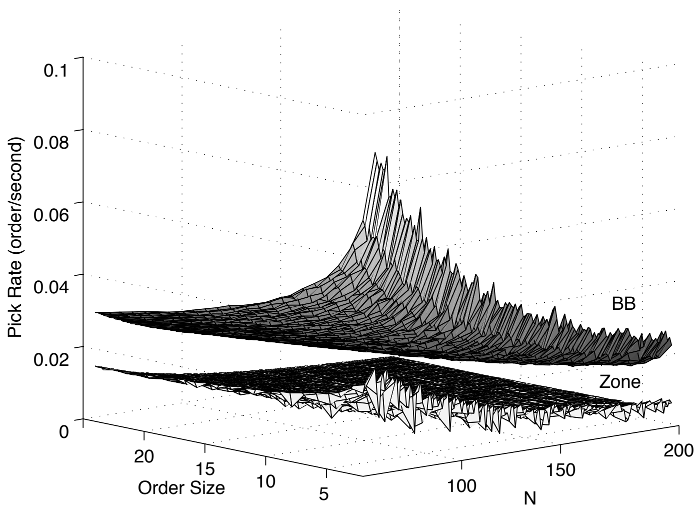


Fig. 5.8. Comparison of BB and picking systems zoned by workload

In sum, while a picking system zoned by line length cannot streamline the unbalance from workload and thereby picking time, a picking system zoned by work load cannot streamline the unbalance from line length and thereby travel time. A BB picking system, however, can balance both sides *simultaneously.* The shrink effect is mainly a positive factor for the BB protocol, but a negative factor for zone picking protocols without self-balancing capacity. Compared with other order picking protocols, the BB protocol is an efficient method with a capacity to capture shrink effect and transfer it to an advantage in productivity. Moreover, the outputs,whatever 2-dimensional or 3-dimensional, from a BB picking system, are smoother, which means a smaller variance in pick rates and less management cost.

5.8 Concluding remarks

We uncover new management insights of applying bucket brigades, particularly bi-directional BB, a new protocol proposed in this research, to warehouse operations in a practical environment than the uniform storage environment. The main contribution is as follows:

(1) We find a shrink effect of BB order picking in non-uniform storage profiles. BB can efficiently use this effect, via its capacity of self-balancing, combined with different storage profiles to improve order picking productivity, and this effect is more significant in a BB protocol compared with that in other protocols without the capacity of self-balancing.

(2) We give closed-form analytic expressions for the order pick rates of bucket brigades model for different storage profiles.

(3) We shed light on reasons why the bucket brigades system outperforms two zone picking systems in non-uniform storage profiles. While the shrink effect of non-uniform storage profiles may be a negative factor for the two zone picking protocols, the BB protocol is an efficient method with a capacity to capture shrink the effect and transfer it to an advantage in productivity.

(4) We show how to use storage profiles and bucket brigade protocol to improve order picking productivity. In general, managers can expect that VBS policies outperform CBS policies for bucket brigades models, and CBS policies outperform uniform storage distribution. In VBS policies, we suggest managers use “middle-peak VBS”, which outperforms “end-peak VBS”. However, we also demonstrate while a larger skewness may improve the productivity of BB order picking, the storage profile design is subject to the available storage space capacity.

Part IV

Stochastic Modelling and Analysis for Public Warehouses

6

A novel facility design approach to improve revenue management of public storage warehouses

6.1 Introduction

Public-storage warehousing is a considerable industry in USA and a booming business in Europe and Asia. In the United States, for example, the Self Storage Association (SSA, the official association representing this industry in the USA) reported, while there were only 6,601 facilities at year-end 1984, the facility number has rocketed to over 51,250 at year-end 2008 (Self Storage Association, 2009). The SSA estimates that the industry had total sales in excess of $20 billion (USD) in 2008 in the United States. In the European marketplace, according to the 2008 industry annual report of the Self Storage Association of UK (the official association representing this industry in the UK), the number of public storage warehouse facilities sharply increased by 117% in Switzerland, 64% in Denmark, 55% in Sweden, 40% in Portugal, 36% in both the Netherlands and Austria, 19% in France and 17% in Belgium from 2007 to 2008, within just a single year (Self Storage Association UK, 2009). In the rest of the world, public-storage is also rapidly developing. For instance, the Self Storage Association of Australasia reports that there are over 1,100 self storage facilities, and " self storage is one of the fastest growing industries in Australasia" (see selfstorage.com.au). Zawya, a leading Middle East business information media headquartered in Dubai, reported that, Big Yellow, a UK self-storage operator, is building the world's largest public-storage warehouse in Dubai, offering a total rentable storage space of 280,000 ft^2 (see zawya.com).

The reasons behind this rapid growth have to do with the public-storage business model, which apparently fills a customer need. Public storage provides both private persons and small businesses a temporary storage opportunity at a centrally located facility. Small-sized rooms in a larger warehouse are rented out during a usually short period of time and can be operated by the renters themselves. One of the continual challenges for managers of such public-storage warehouses is how to design facilities to improve their revenues. Our research has been inspired by conversations with Shurgard-Public Storage (the world's largest public-storage warehouse company) and other public-storage companies. Shurgard is an international corporation providing public-storage warehousing services to both private and business customers. A manager at Shurgard complained he could not get much help from current facility design theories, mainly emphasizing cost control without dealing with revenue aspects. With decades' management experiences in cost control, this industry has been able to control the cost well. According to the SSA at year-end 2008 the large self storage facilities only employed "an average of 3.1 employees per facility" in the USA (Self Storage Association, 2009). For example, a warehouse with about 5000m^2 rentable space only hires two workers per shift to run the facility. In traditional warehouses labor cost is the main cost source (see De Koster et al. (2007)). With the self-storage operation mode, customers handle storage operations themselves, without interference of warehouse personnel (operational details can be found at www.shurgard.eu, for example). The prime objective for these public storage companies is to maximize the expected revenue at a stable cost level. A typical public-storage warehouse contains storage spaces of different sizes and qualities, each with a specific number of storage units. A customer rents a storage unit of an appropriate size for one or multiple months. However, the existing storage sizes or the number of storage units available per size may not fit the needs of the market. The number of available units of some type may be insufficient, while other sizes are abundant. This results in either lost customers and revenue, or inefficient utilization of capacity of one type, which also may bring potential loss in another type. A natural question is therefore whether it is possible to provide a facility design improving the expected revenue, that is with a better fit between storage design (types and numbers) and market demand.

Designing public-storage facilities is an important business in itself. Several storage space design companies like US-based Janus International Corporation (see janusintl.com), UK-based Gliderol self storage solution (see gliderolselfstorage.co.uk), France and UK -based Steel storage (see steelstorage.net), are specialized in developing partitioned storage buildings for the public-storage industry. Public-storage facility design is applied in two situations: the design of new warehouses and the reconfiguration of existing warehouses.

Figure 6.1 shows a typical example of a public storage warehouse (only the second floor is shown), which contains storage rooms of 3, 4.5, 6, 7.5, 9, 12, 15, and 18 m^2, many of which can be merged into larger ones. The height of these storage units is standard. In the USA, the free height is 8 feet. If the height of a facility floor is larger than 8 feet, public storage warehouses will install a roof or a meshwork to cover the storage units in order to standardize the height. In Europe, the height of storage units is also standard in most cases. Throughout this paper, we therefore consider the two-dimensional facility design problem only. In this paper, we focus on the prime managerial issue, i.e., how to determine the appropriate storage types and the number of storage units per type to fit the market segments. We do not consider the remaining engineering problem, i.e., designing a specific storage layout given storage types and the number of storage units per type. For this problem, many heuristics, optimization-based algorithms (see, e.g., chapters 5 and 6 in Tompkins et al. (2003), or Francis et al. (1992)), and commercial software tools (like FactoryCad, FactoryFlow, Flow Planner, and others) are available.

Warehouse facility design is a tactical decision: once a facility has been designed and realized it is difficult to adapt it to a changed environment. However, in public storage, warehouse designs appear to be much more flexible than in other warehouses since public storage warehouses widely apply modular steel-base products like modular corridor, standardized internal wall panels, standardized swing doors and roller doors. Particularly, the internal panel has a special patented "snap together" interlocking seam (see steelstorage.co.uk), rather than fixed jointing, which makes the repartition of warehouse space easier. The Steel Storage Group, a global leader in the design and construction of public storage facilities, can "transform an existing building into a self-storage complex" very

Fig. 6.1. Typical facility layout in a public storage warehouse

rapidly, and claims that their modular units "allow for easily repeatable sizes for rapid installation and easily calculable spaces based on standard metric dimensions" (see steelstorage.co.uk). Most public-storage warehouses have a limited number of storage sizes (in the USA, usually 8 types) and most sizes are an integer multiple of a standard size. In fact, it is usually possible to remove or add non-supporting walls to create for example one 9 m^2 room from a 3 and a 6 m^2 room. Of course there are some constraints: it is impossible to merge or split rooms while they are still occupied and by adding a wall there should still be an access door for both rooms. This space flexibility creates room for public-storage managers to adapt the layout of their facility on a rather short term to changing demand.

To further specify the research problem and understand our problem background, we visited 54 public-storage warehouses, including 33 in USA (Chicago, Philadelphia, Washington DC, New York, Orlando), 14 in Europe (Rotterdam, the Hague, Delft, Lyon, Lille, Brussels, Bonn), 7 in Asia (Hong Kong, Shanghai, Singapore), from December 2007 to July 2009. We collected price, design, and demand information, interviewed managers, customers and laborers. It appears the operation modes are highly dependent on the geographical location. Among the 54 warehouses, 22.2% are located in or close to city centers. These warehouses are inclined to reject customers when capacity is fully occupied as demand appears to be abundant. 64.8% of the warehouses attempt to upgrade customers to larger storage sizes, when the desired storage size is fully occupied, and 13.0% do not (or rarely) reject customers or upgrade customers. The latter group of facilities are located in "wrong" places (e.g., remote outskirts) with low demand, and a substantial amount of previous research addressing various facility location problems (e.g., Ross and Soland (1977), Wesolowsky (1973)) can be applied to solve problems in these kinds of warehouses. This paper thereby primarily focuses on the first two facility types. We build analytical models to provide designs oriented to revenue management according to their demand, with and without upgrade possibilities. Upgrading customers is possible in many cases, if the next larger storage type is still available and price differences are not too large. In practice, managers immediately show customers a larger storage type, when the small one is not available, as customers often have difficulties in expressing their exact space needs. Two situations can then be distinguished: *a priori* reserving some space for upgraded customers, or no *a priori* reservations. For warehouse employees the first policy is more convenient: if a space type is fully occupied and the reserved space of the next larger type is not yet fully booked they can offer it to the customer and ask whether he or she accepts this space. If there is no *a priori* reservation it is not always obvious for an employee whether upgrade space should be offered as this takes away space of the primary customers of the larger space type. However, a policy of no *a priori* reservations might bring a larger expected revenue.

Our contribution in this paper is twofold: (1) Based on practical international public-storage warehouse cases, this paper proposes a facility design approach with the objective to maximize the expected revenue. (2) This paper is one of the earliest to apply revenue management theory in the area of facility design.

The remainder of the paper is organized as follows: In the following section, we review the literature of related application areas and of related methods. Section 6.3 is devoted to a basic design model in an environment with high demand. In Section 6.4, we incorporate the customer upgrade problem by an overflow queue network. We do this both for the case with *a priori* space reservation for upgraded customers and for the case where no *a priori* space has been reserved. Section 6.5 shows results of the models for several warehouse cases. We conclude with final comments and directions for future research in Section 6.6.

6.2 Literature review

Facility planning and design is important for management practice and the economy. The facility plan helps organizations to achieve supply chain excellence in today's competitive global marketplace (Tompkins et al., 2003). Approximately 8% of the US gross national product (GNP) has been spent annually on new facilities in the United States since 1955 (Tompkins et al., 2003). Most facility design methods mainly consider cost control, and focus on minimizing (internal) distance-based cost. Some of the research focuses on optimally sizing the warehouse and renting temporary storage space when demand is stochastic (e.g., Rao and Rao (1998)). Zhang et al. (2008) model the problem of allocating customers to different warehouse spaces given deterministic demand, using a scheduling approach. Few facility design models adopt an objective to maximize the profit. However, without considering the market segment problem, which is viewed as an important common characteristic of revenue management research (see Weatherford and Bodily (1992)), simple profit maximization reduces to cost minimization.

Literature on revenue management (RM) is quite substantial (see Talluri and Van Ryzin (2004)). Several authors have written overview papers on revenue management, like Mcgill and Van Ryzin (1999) and Weatherford and Bodily (1992). These works discuss applications of revenue management to extensive fields (but not warehousing). Solution techniques include mainly heuristics, dynamic programming, and mathematical programming. Airline management is one of main application fields of revenue management techniques (see Talluri and Van Ryzin (2004)). Warehouse revenue management differs from airline revenue management in a number of ways. The major difference lies in the fact that in public-storage warehouses customers may rent a space for multiple periods of time. They also can rent multiple spaces of different sizes, for different periods. Warehouse revenue management is closer to hotel operations management. In the hotel revenue management literature one typically uses (dynamic) Markov decision processes and (static) mathematical programming models to derive optimal or near-optimal strategies for renting hotel rooms of different types to customers with random demands from different market segments under the objective of profit maximization. In case Markov decision processes techniques (dynamic programming) are used, the dynamic arrival process is (in the simplest discrete case) given by a discrete nonhomogeneous Poisson process. For mathematical programming formulations the cumulative distribution function of the demand is assumed to be known in advance. In this case one tries to solve nonlinear optimization problems. In case the models are too complex, heuristics have been developed. The major difference with our paper is that in hotel management one does not focus on design problems, but instead one tries to allocate the existing rooms over the different types of customers given cumulative distribution function (CDF) information about their random demand. An example among many others of a paper using mathematical programming techniques is given by Bitran et al. (1995). Examples of papers using dynamic programming techniques are given by for example Bitran and Gilbert (1996) and Bitran and Mondschein (1995).

We were not able to find papers on facility design focusing on revenue management in the presence of market demand with segments for different space requirements.

6.3 Design model without upgrading

In this section we present a design model without the possibility of upgrading. To introduce this model we assume that there are m different storage type units requested by customers. A storage type i unit, $1 \leq i \leq m$, has an integer storage size area c_i and its rent is given by r_i per unit of time. In the remainder of this paper customers requesting a storage type i unit are called type i customers. Type i customers are arriving according to a Poisson process with arrival rate λ_i and the different Poisson arrival processes are assumed to be independent. The following rental policy is assumed. An available type i storage unit is rented to any arriving type i customer. A type i customer finding upon arrival all storage type i units occupied is lost (no upgrading). We assume that the occupation times of a storage type i unit are given by independent and identically distributed random variables with expected occupation time β_i. The goal is to determine how many storage type i units should be constructed under the objective of revenue maximization.

To model this problem, we introduce the decision variable x_i denoting the number of constructed storage type i units. It is easy to see that the number of occupied storage type i units, $1 \leq i \leq m$, can be modeled as m independent $M/G/x_i/x_i$ queueing loss systems. Hence the total reward is given by $\sum_{i=1}^{m} r_i L_i$ with L_i the long-run average number of customers in such a $M/G/x_i/x_i$ system. To calculate this we will use the following application of Little's law (see page 345 of Tijms (1982)):

In a $G/G/x/x$ loss system with arrival rate λ and x servers, where customers pay a rate r for service, the long-run average revenue is equal to

$$rL = r\lambda(1 - P_{rej})S, \tag{6.1}$$

with L the long run average number of customers in the system, S the expected service time and P_{rej} the rejection probability.

By the PASTA (Poisson Arrivals See Time Averages) property the rejection probability P_{rej} equals (see Gross and Harris (1998) or Cohen (1976)) the Erlang loss formula $B(x_i, \rho_i)$ with $\rho_i := \lambda_i \beta_i$ the load of the system and

$$B(x, \rho) := \frac{\rho^x}{x!} \left(\sum_{j=0}^{x} \frac{\rho^j}{j!} \right)^{-1}. \tag{6.2}$$

Notice that $B(0, \rho) = 1$ for every $\rho > 0$. Hence by the equation (6.1) the long-run average revenue generated by accepted type-i customers equals $r_i \rho_i (1 - B(x_i, \rho_i))$.

Under the restriction that the total area of the warehouse is given by the integer C the problem of maximizing the long-run expected revenue can be formulated as

$$\max \left\{ \sum_{i=1}^{m} r_i v(x_i, \rho_i) : \sum_{i=1}^{m} c_i x_i \leq C, x_i \in \mathbb{Z}_+, 1 \leq i \leq m \right\}, \tag{P}$$

where

$$v(x, \rho) := \rho(1 - B(x, \rho)). \tag{6.3}$$

Problem (P) can be solved by the following dynamic programming algorithm. Introduce for every $1 \leq k \leq m$ and $c \in \mathbb{Z}_+$ the feasible region $\mathcal{F}_k(c) := \{\mathbf{x} \in Z_+^{m+1-k} : \sum_{i=k}^m c_i x_i \leq c\}$ and let

$$J_k(c) := \max\left\{\sum\nolimits_{i=k}^m r_i v(x_i, \rho_i) : \mathbf{x} \in \mathcal{F}_k(c)\right\} \tag{6.4}$$

be the maximal long-run average revenue obtained from storage units of type $k, ..., m$, if the decision maker assigns to these units a total integer capacity c with $c \leq C$. Clearly the optimal solution of problem (P) is given by $J_1(C)$. To compute this value we first observe by the monotonicity of function $x \mapsto v(x, \rho)$ that $J_m(c)$, $c \in \{0, ..., C\}$, is given by

$$J_m(c) = \max\{r_m v(x_m, \rho_m) : x_m \leq \lfloor c c_m^{-1} \rfloor\} = r_m v(\lfloor c c_m^{-1} \rfloor, \rho_m) \tag{6.5}$$

with $\lfloor z \rfloor$ denoting the largest integer smaller than or equal to z. For type $1 \leq k \leq m-1$, the value $J_k(c)$, $c \in \{0, ..., C\}$, can be iteratively calculated by the Bellman equation

$$J_k(c) = \max\nolimits_{x_k \in \{0, ..., \lfloor c c_k^{-1} \rfloor\}} \{r_k v(x_k, \rho_k) + J_{k+1}(c - c_k x_k)\}. \tag{6.6}$$

An easy generalization of problem (P) is to include in this model a service restriction that arriving type i customers, $1 \leq i \leq m$, are rejected with a probability at most equal to σ_i (this parameter is set by the decision maker). In this case we need to solve the problem

$$\max\left\{\sum\nolimits_{i=1}^m r_i v(x_i, \rho_i) : \sum\nolimits_{i=1}^m c_i x_i \leq C, x_i \in \mathbb{Z}_+, B(x_i, \rho_i) \leq \sigma_i, 1 \leq i \leq m\right\}.$$

Observe we always implicitly assume that the feasible region is nonempty. Since the function $x \mapsto B(x, \rho)$ is strictly decreasing, the above optimization problem reduces to

$$\max\left\{\sum\nolimits_{i=1}^m r_i v(x_i, \rho_i) : \sum\nolimits_{i=1}^m c_i x_i \leq C, x_i \geq B_{\rho_i}^{\leftarrow}(\sigma_i), x_i \in \mathbb{Z}_+, 1 \leq i \leq m\right\} \tag{Q}$$

with

$$B_\rho^{\leftarrow}(u) := \min\{x \in \mathbb{Z}_+ : B(x, \rho) \le u\}.$$

For solving optimization problem (Q) we can apply a dynamic programming algorithm similar to the one used for problem (P).

6.4 Design model with upgrading

In this section we consider the following more realistic model: customers who initially are interested in a storage type i unit, may accept with probability p_i to pay a price r_{i+1} for a storage type $i+1$ unit when all storage type i units are occupied. A customer willing to accept this is called an *upgraded* customer. If a storage type $i+1$ unit is available, an upgraded customer will be served, otherwise the customer is lost. As done previously, we will call a customer initially interested in a type i storage unit a type i customer. We again model the arrival process of type i customers as a Poisson process with rate λ_i. However, we now assume that the holding times of a storage type i unit are independent and exponentially distributed with mean β_i . Given the price r_i for each storage type i unit per unit of time the goal is to decide how many units of each type to build (and reserve) such that the long-run average revenue is maximized.

We consider two upgrading models. In the first one, units are reserved *a priori* for upgraded customers, while in the second, upgraded customers of type i may rent upon their arrival any available unit at level $i+1$. For the first model we present an exact method for finding the long-run average revenue. The second model, however, is more complex and as we will see in Section 6.4.2, less tractable. The analysis of this model will be based on a reasonable first moment approximation regarding the overflow process of upgraded customers.

6.4.1 Model with *a priori* reservations for upgraded customers

In this subsection we assume that in the construction phase one reserves units for upgraded customers. The upgrading process can be described as follows. Let x_i, $1 \le i \le m$, be the number of storage type i units built for type i customers at level i and y_i, $1 \le i \le m-1$, the type $i+1$ units reserved for type i customers upgraded to level $i+1$. A type i customer who finds upon arrival all the x_i

storage type i units occupied may choose to be upgraded and use one of the y_i reserved units, if one is available. If all y_i units are occupied, the customer is lost. Customers of type m finding upon arrival all x_m units busy are directly lost. A customer of type $i+1$ is not allowed to occupy one of the y_i units reserved for upgraded type i customers, even if such a unit is available.

To analyze this model we first look at the process followed by each type of customers separately. The long-run average revenue obtained from type i customers can be split in the long-run average revenue obtained from the x_i, respectively y_i units.

Long-run average revenue from the x_i units:

The number of occupied storage type i units among the available x_i can again be modeled by a queueing loss system. Due to our assumption of exponentially distributed occupation times this is an $M/M/x_i/x_i$ loss queue. As before the long-run average revenue obtained from the x_i units is given by $r_i v(x_i, \rho_i)$ with v listed in relation (6.3).

Long-run average revenue from the y_i units:

To calculate the long-run average revenue obtained from the y_i units, we first need to characterize the arrival process of upgraded type i customers to level $i+1$. Clearly this is the same as the overflow process of rejected type i customers willing to upgrade. Since the occupation times of type i units are exponentially distributed, the arrival moments of type i customers finding all x_i storage type i units occupied are regeneration points of the $M/M/x_i/x_i$ loss queue (Wolff (1989)). Hence the overflow process of rejected type i customers is a (delayed) renewal process. By the PASTA property the rate of this process is given by $\lambda_i B(x_i, \rho_i)$. Since each rejected type i customer is willing to upgrade with probability p_i the overflow process of rejected customers willing to upgrade is therefore a delayed renewal process with rate $\eta_{i+1}(x_i)$ given by

$$\eta_{i+1}(x_i) = p_i \lambda_i B(x_i, \rho_i). \tag{6.7}$$

Again by Little's formula, the long-run average revenue generated by the y_i units is equal to

$$r_{i+1}\eta_{i+1}(x_i)\beta_i(1 - P_{rej}(x_i, y_i)) \tag{6.8}$$

with $P_{rej}(x_i, y_i)$ the rejection probability that an upgraded type i customer will find all the reserved y_i units occupied. By Theorem 2, Ch.4, in Takacs (1982), this rejection probability is given by

$$P_{rej}(x_i, y_i) = \frac{1}{\sum_{j=0}^{y_i} \binom{y_i}{j} K_j^{-1}}$$

with $K_0 := 1$ and

$$K_j := \prod_{k=1}^{j} \left(\frac{\theta(x_i, k\beta_i^{-1})}{1 - \theta(x_i, k\beta_i^{-1})} \right). \tag{6.9}$$

for $1 \leq j \leq y_i$. In relation (6.9) the function $\theta(x_i, s)$ represents the Laplace-Stieltjes transform of the inter arrival time at the y_i units between two upgraded type i customers

$$\theta(x_i, s) = \frac{p_i \gamma(x_i, s, \phi)}{1 - (1 - p_i)\gamma(x_i, s, \phi)} \tag{6.10}$$

with

$$\gamma(x_i, s, \phi) = \frac{\sum_{j=0}^{x_i} \binom{x_i}{j} \prod_{k=0}^{j-1} \left(\frac{1-\phi(s+k\mu)}{\phi(s+k\mu)} \right)}{\sum_{j=0}^{x_i+1} \binom{x_i+1}{j} \prod_{k=0}^{j-1} \left(\frac{1-\phi(s+k\mu)}{\phi(s+k\mu)} \right)} \tag{6.11}$$

and $\phi(\alpha) = \frac{\lambda_i}{\alpha + \lambda_i}$ the Laplace-Stieltjes transform of the exponential distribution with parameter λ_i.

The optimization problem of deciding how many units of each type to build/reserve such that the long-run average revenue obtained from all types of customers is maximized can now be written as:

$$\begin{aligned} &\max \textstyle\sum_{i=1}^{m} r_i v(x_i, \rho_i) + \sum_{i=1}^{m-1} r_{i+1} \eta_{i+1}(x_i) \beta_i \left(1 - P_{rej}(x_i, y_i)\right) \\ &\textstyle\sum_{i=1}^{m-1} (c_i x_i + c_{i+1} y_i) + c_m x_m \leq C \\ &x_i, y_i \in \mathbb{Z}_+ \end{aligned} \tag{6.12}$$

The above maximization problem can be solved via dynamic programming. Let again $J_k(c)$ be the maximal long-run average revenue obtained from storage units of type $k, ..., m$ if for those units a total integer capacity c with $c \leq C$ is available. The optimal value $J_1(C)$ of (6.12) can be computed recursively as follows. Since $x \mapsto v(x, \rho)$ is increasing we first obtain for each capacity $c \in \{c_m, ..., C\}$ that $J_m(c)$ is given by

$$J_m(c) = \max_{x_m \in \{0,...,\lfloor c c_m^{-1} \rfloor\}} r_m v(x_m, \rho_m) = r_m v(\lfloor c c_m^{-1} \rfloor, \rho_m) \tag{6.13}$$

Also for $c \in \{0, ..., c_m - 1\}$ it is obvious that $J_m(c) = 0$. Introducing for $1 \leq k \leq m - 1$

$$S_k(c) := \{(x_k, y_k) \in \mathbb{Z}_+^2 : c_k x_k + c_{k+1} y_k \leq c\}$$

we obtain that the Bellman recurrence relations for the functions J_k, $1 \leq k \leq m - 1$ are given by

$$J_k(c) = \max_{(x_k, y_k) \in S_k(c)} \{f(x_k, y_k) + J_{k+1}(c - c_k x_k - c_{k+1} y_k)\},$$

with

$$f(x_k, y_k) = r_k v(x_k, \rho_k) + r_{k+1} \eta_{k+1}(x_k) \beta_k (1 - P_{rej}(x_k, y_k)).$$

Note that in calculating $J_k(c)$, $c \in \{0, ..., C\}$, one does not have to evaluate all the pairs $(x_k, y_k) \in S_k(c)$. Since the function $y \mapsto f(x, y)$ is increasing in y, for a fixed capacity $\tilde{c}_k$ reserved for storage units for type k customers (including the reserved units on level $k + 1$), the maximal revenue obtained from type k customers fixing x_k is obtained reserving $\tilde{y}_k = \lfloor (\tilde{c}_k - c_k x_k) c_{k+1}^{-1} \rfloor$ storage type $k + 1$ units. Hence, one can evaluate $J_k(c)$ by

$$J_k(c) = \max_{\tilde{c}_k \leq c} \max_{\substack{x_k \in \{0,...,\lfloor \tilde{c}_k c_k^{-1} \rfloor\} \\ \tilde{y}_k = \lfloor (\tilde{c}_k - c_k x_k) c_{k+1}^{-1} \rfloor}} \{f(x_k, \tilde{y}_k) + J_{k+1}(c - c_k x_k - c_{k+1} \tilde{y}_k)\}.$$

6.4.2 Model without *a priori* reservations for upgraded customers

In this subsection we assume that one does not reserve in advance capacity for upgraded customers. Instead, a type i customer finding upon arrival all units of type i occupied and choosing for upgrading, may get any available unit of type $i + 1$ (if such an available unit exists). As before, we assume that the arrival of type i customers are independent Poisson processes with rate λ_i and the holding times for all storage units are independent and exponentially distributed with mean β_i for type i customers.

Under this assumption the number of occupied storage type 1 units is a Markovian loss model. Moreover, we have seen in the previous subsection (see also Takacs (1982), Chapter 4) that the overflow process of rejected customers of an $M/M/c/c$ loss queue starting initially empty is a (delayed) renewal process with arrival epochs the overflow epochs. The process at level 2 is however much more complex. The arrival process of type 2 and upgraded customer is the superposition of a delayed renewal process with a given arrival rate and a Poisson process with rate λ_2. Since this loss system at level 2 with two different types of customers is impossible to analyse and at level $i \geq 3$ it becomes even more complicated, we introduce the following approximating (first moment) Poisson assumption for the overflow process of rejected customers. In the sequel H_2 denotes a two-phase hyper exponential distribution.

Assumption *The overflow process of rejected customers in an $M/H_2/x/x$ loss queue with arrival rate λ and mean service time β can be approximated by a Poisson process with rate $\lambda B(x, \rho)$, $\rho = \lambda\beta$ and $B(x, \rho)$ the Erlang loss probability that an arriving customer finds all x servers busy.*

To analyze our model under the above approximation assumption we also need the next result.

Lemma 6.1. *Consider a queueing system with two types of customers and c servers and assume a customer who finds all the servers busy is rejected. The independent arrival processes of the two types of customers are Poisson with rates γ_1, γ_2 and the service times are exponentially distributed with mean ζ_1, ζ_2 respectively. For such a queueing system the long-run probability that a rejected customer is of type i, $i = 1, 2$ is given by $\frac{\gamma_i}{\gamma_1+\gamma_2}$.*

The above lemma also holds for independent and arbitrary distributed service times with mean ζ_1, ζ_2 respectively. We will give a proof of this more general form in the Appendix.

As in the previous sections, we will calculate the long-run average revenue for each type of storage units by using Little's formula. For this, we first need to compute the average occupation time of each type of storage unit and the probability that a customer will find all units of a certain type occupied.

To characterize the rental process of type i storage units we proceed as follows. Let x_i be the number of type i storage units built, η_i the arrival rate of upgraded type $i-1$ customers to units of type i and $B(x_i, \rho_i, \eta_i)$ the probability that a customer interested in a type i unit (an upgraded type $i-1$ customer or a type i customer) finds all type i storage units occupied. Clearly, $\eta_1 = 0$ and $B(x_1, \rho_1, \eta_1) = B(x_1, \rho_1)$, with $B(x_1, \rho_1)$ given by (6.2).

As before the number of occupied storage units of type 1 can be modeled as an $M/M/x_1/x_1$ queue. By the Poisson approximation assumption, the overflow process of rejected customers is a Poisson process with rate $\lambda_1 B(x_1, \rho_1)$. Hence the rate of the arrival process of upgraded type 1 customers at level 2 is a Poisson process with rate $p_1 \lambda_1 B(x_1, \rho_1)$.

Iteratively, for every $i \geq 2$, the arrival process is formed by upgraded customers of type $i-1$ and type i customers. By the Poisson approximation assumption and induction, the arrival process is Poisson with rate $\lambda_i + \eta_i$. If there are storage units available, a type i customer will rent a type i storage unit for an exponential time with mean β_i, while an upgraded type $i-1$ customer will rent such a unit for an exponential time with mean β_{i-1}. Since an arriving customer is of type $i-1$ with probability $\frac{\eta_i}{\lambda_i+\eta_i}$ and of type i with probability $\frac{\lambda_i}{\lambda_i+\eta_i}$, the service time has a two phase hyper exponential distribution with mean

$$\beta_i \frac{\lambda_i}{\lambda_i + \eta_i} + \beta_{i-1} \frac{\eta_i}{\lambda_i + \eta_i}.$$

We can thus conclude that the number of occupied storage units of type i can be modeled by an $M/H_2/x_i/x_i$ loss queue. Also the load of arriving type i and upgraded type $i-1$ customers at level i (arriving at rate $\lambda_i + \eta_i$) is given by

$$\beta_i \lambda_i + \eta_i \beta_{i-1} = \rho_i + \eta_i \beta_{i-1}.$$

Hence by the Poisson approximation assumption the overflow process of rejected customers at this $M/H_2/x_i/x_i$ queue is a Poisson process with rate $(\lambda_i+\eta_i)B(x_i,\rho_i,\eta_i)=(\lambda_i+\eta_i)B(x_i,\rho_i+\eta_i\beta_{i-1})$. This implies using also Lemma 6.1 that the arrival process of upgraded type i customers to storage units of type $i+1$ is therefore a Poisson process with rate η_{i+1} given by

$$\eta_{i+1}=p_i\frac{\lambda_i}{\lambda_i+\eta_i}(\lambda_i+\eta_i)B(x_i,\rho_i+\eta_i\beta_{i-1})=p_i\lambda_iB(x_i,\rho_i+\eta_i\beta_{i-1}). \tag{6.14}$$

Since by the PASTA property

$$\mathbb{P}(\text{arriving customer not blocked at level } i)=1-B(x_i,\rho_i,\eta_i)$$

we also obtain again by Little that the long-run average revenue generated by the x_i type i storage units is given by

$$\begin{aligned} r_i(\rho_i+\eta_i\beta_{i-1})(1-B(x_i,\rho_i,\eta_i)) &= r_i(\rho_i+\eta_i\beta_{i-1})(1-B(x_i,\rho_i+\eta_i\beta_{i-1})) \\ &= r_iv(x_i,\rho_i+\eta_i\beta_{i-1}) \end{aligned} \tag{6.15}$$

with the function v defined in relation (6.3).
The problem of maximizing the long run average revenue for the model without *a priori* reservation can now be formulated as follows:

$$\max\left\{\sum_{i=1}^m r_iv(x_i,\rho_i+\eta_i\beta_{i-1}):\sum_{i=1}^m c_ix_i\le C, x_i\in\mathbb{Z}_+,1\le i\le m\right\} \tag{6.16}$$

First note that for each allocation $\mathbf{x}^\top=(x_1,...,x_m)\in\mathbb{Z}_+^m$ satisfying the feasibility condition $\sum_{i=1}^m c_ix_i\le C$ the (approximated) long-run average revenue can be obtained by iteratively calculating $v(x_i,\rho_i+\eta_i\beta_{i-1})$ using (6.14).

Also observe that the above mathematical program is a nonlinear nonseparable integer programming problem. Solving such a mathematical programming model is extremely hard. Fortunately, by the interpretation of our problem, one can use as follows dynamic programming to find the optimal allocation.

Let $M = \max_{1\leq i\leq m}\{p_i\lambda_i\}$ (with $p_m = 1$). For every $1 \leq k \leq m$ introduce the optimal value function $J_k : \{0, 1, ..., C\} \times (0, M) \to \mathbb{R}, 1 \leq k \leq m$ given by

$$J_k(c,\eta) := \begin{cases} \text{The maximal long-run average revenue obtained from type } i \text{ storage units} \\ i \geq k, \text{if the available capacity for these units is } c \text{ and the} \\ \text{arrival rate of upgraded type } k-1 \text{ customers at level } k \text{ is } \eta. \end{cases}$$

Clearly, when a capacity of c is available for constructing type m storage units, it is most profitable to completely use this capacity and construct $\lfloor cc_m^{-1}\rfloor$ type m storage units, for any arrival rate $0 \leq \eta \leq M$ of upgraded type $m-1$ customers at level m. This implies by relation (6.15) that

$$J_m(c,\eta) = \begin{cases} r_m v(\lfloor cc_m^{-1}\rfloor, \rho_m + \eta\beta_{m-1}) & \text{if } c \in \{c_m, ..., C\} \\ 0 & \text{otherwise} \end{cases} \tag{6.17}$$

To compute the functions $J_k, 1 \leq k \leq m-1$ consider the set $\mathcal{S}$ of real valued functions on the domain $D = \{0, ..., C\}\times (0, M]$ with bounded supnorm

$$\|f\|_\infty := \sup_{(c,\eta)\in D} |f(c,\eta)| \tag{6.18}$$

and introduce the operator $P_k : \mathcal{S} \to \mathcal{S}$ given by

$$P_k f(c,\eta) = \max_{x_k\in\{0,1,...,\lfloor cc_k^{-1}\rfloor\}} \{f(c - c_k x_k, \lambda_k p_k B(x_k, \rho_k + \eta\beta_{k-1})) + r_k v(x_k, \rho_k + \eta\beta_{k-1})\}\,. \tag{6.19}$$

It is easy to verify for every f_1, f_2 belonging to $\mathcal{S}$ that

$$\|P_k f_1 - P_k f_2\|_\infty \leq \|f_1 - f_2\|_\infty. \tag{6.20}$$

By relations (6.14) and (6.15) we obtain for each type $k < m$, that the maximal long run average revenue $J_k(c,\eta)$ obtained from type i storage units, $i \geq k$, given that a total capacity c is assigned to these units and η is the arrival rate of upgraded $k-1$ customers satisfies the (Bellman) equation

$$J_k(c,\eta) = P_k J_{k+1}(c,\eta). \tag{6.21}$$

It is easy to see that the maximum long-run average revenue is given by $J_1(C,0)$ and by backtracking we obtain the optimal allocation. However, the arrival rate η in the recurrent relation (6.19) is continuous. Therefore to solve the equation in relation (6.21) numerically we need to discretize it.

We will next present a discrete dynamic program that gives in each step a lower and an upperbound for $J_k(C,0)$ based on upperbounds and lower bounds of the arrival rates of upgraded customers which are multiples of a step h. We will show for h converging to zero that the value obtained from the discretized dynamic program converges to the optimal value of the continuous one. The algorithm heavily relies on the following two lemma's proven in the Appendix.

Lemma 6.2. *For every fixed $x \in \mathbb{N}$ the function $v : \mathbb{N} \times [0,\infty) \to \mathbb{R}$ given by $v(x,\rho) = \rho(1 - B(x,\rho))$ is increasing in ρ.*

Lemma 6.3. *For each $k \leq m$, and capacity $c \in \{0,...,C\}$, the function $\eta \mapsto J_k(c,\eta)$ is increasing and Lipschitz continuous.*

Before presenting the last lemma to be used, we introduce some notations.

Let $h > 0$ be chosen in such a way that $M = \max_{1\leq i\leq m}\{p_i\lambda_i\}$ is a multiple of h, say $M = nh$ and let $\mathcal{S}_h$ denote the vectors $(u(.,h),...u(.,nh))$ for any u belonging to $\mathcal{S}$. Also introduce the functions $\mathcal{U}_h : \mathbb{R}_+ \to \mathbb{R}$ and $\mathcal{L}_h : \mathbb{R}_+ \to \mathbb{R}$ defined by

$$\mathcal{U}_h(\eta) := \lceil \eta h^{-1} \rceil h \text{ and } \mathcal{L}_h(\eta) := \lfloor \eta h^{-1} \rfloor h$$

with $\lceil z \rceil$ denoting the smallest integer greater than or equal to z. Clearly for every $\eta \geq 0$ we obtain

$$\mathcal{L}_h \eta) \leq \mathcal{U}_h(\eta) \leq \mathcal{L}_h(\eta) + h. \tag{6.22}$$

Also, for any function $f \in \mathcal{S}$ let the functions $f^{(h)}$ and $f_{(h)}$ be defined by

$$f^{(h)}(c,\eta) := f(c,\mathcal{U}_h(\eta))$$

and

$$f_{(h)}(c, \eta) := f(c, \mathcal{L}_h(\eta))$$

For the operator P_k listed in relation (6.19) it is easy to show the following result.

Lemma 6.4. *If the function $\eta \mapsto f(c, \eta)$ is increasing for every $c \in \{0, 1, ..., C\}$ and f belongs to $\mathcal{S}$, then $P_k f_{(h)}, P_k f$ and $P_k f^{(h)}$ are increasing function in η and*

$$P_k f_{(h)} \leq P_k f \leq P_k f^{(h)} \tag{6.23}$$

for every $1 \leq k \leq m - 1$. Moreover, if for every $c \in \{0, 1, ..., C\}$ the function $\eta \to f(c, \eta)$ is Lipschitz continuous with Lipschitz constant $L_{\max}$, then

$$\|P_k f^{(h)} - P_k f_{(h)}\|_\infty \leq L_{\max} h. \tag{6.24}$$

For a given step h, the discretized dynamic program is given below.

Discretization algorithm yielding upperbound and lowerbound for $J_k(C, 0)$.

Step 1. Evaluate for every $q \in \{0, ..., n\}$ and $c \in \{0, ..., C\}$ the values

$$a_m(c, qh) = b_m(c, qh) = J_m(c, qh)$$

with the function J_m listed in relation (6.17).

Step 2. For $k = m - 1$ down to 1 evaluate the vector $a_k = (a_k(c, qh))$ and $b_k = (b_k(c, qh))$ with $c \in \{0, 1, ..., C\}$ and $q \in \{0, 1, ..., n\}$ given by

$$a_k(c, qh) := P_k a_{k+1}(c, \mathcal{U}_h(\eta)) \tag{6.25}$$

and

$$b_k(c, qh) := P_k b_{k+1}(c, \mathcal{L}_h(\eta)) \tag{6.26}$$

Step 3. Output $a_1(C, 0)$ and $b_1(C, 0)$

The correctness of the discretized dynamic program is given by the following theorem.

Theorem 6.5. *The optimal reward, given by $J_1(C,0)$, is bounded from below by $b_1(C,0)$ and from above by $a_1(C,0)$. Also*

$$0 \leq a_1(c,0) - b_1(c,0) \leq Lh$$

for L a Lipschitz constant of the functions $\eta \mapsto J_m(c,\eta)$ for every $c \in \{0,...,C\}$.

Proof: From the description of the algorithm it follows that

$$0 = a_m(c,qh) - b_m(c,qh) \leq Lh$$

for every $c \in \{0,1,...,C\}$ and $q \in \{0,...,n\}$ and that both functions are increasing in q for fixed c. Suppose now by induction that

$$0 \leq a_{k+1}(c,qh) - b_{k+1}(c,qh) \leq Lh$$

for some $k+1 \leq m$ and $q \in \{0,1,..,n\}$ and both functions are increasing in q. Replacing the supnorm by the discrete supnorm over qh, $q \in \{0,1,...,n\}$ in relation (6.20) and using relations (6.25) and (6.26) we obtain by our induction hypothesis

$$a_k(c,qh) - b_k(c,qh) = P_k a_{k+1}(c,\mathcal{U}_h(\eta)) - P_k b_{k+1}(c,\mathcal{L}_h(\eta)) \leq Lh$$

Also by the first part of Lemma 6.4 we obtain by our induction hypothesis that

$$a_k(c,qh) - b_k(c,qh) = P_k a_{k+1}(c,\mathcal{U}_h(\eta)) - P_k b_{k+1}(c,\mathcal{L}_h(\eta)) \geq 0$$

and both functions are increasing in q. This completes the induction and the desired result follows.

For a small value of the discretization step h, the running time of the approximate dynamic algorithm may be high. Hence the model with reservations may be faster to solve.

6.5 Applications

In this section, we numerically investigate our design method for public storage warehouses and explore its management insights. We first apply our method to warehouses with high demand and customer rejections, then we incorporate upgrade operations, followed by sensitivity analysis to test the robustness of the solution approach. We have chosen to demonstrate the application of the method for some real cases exhibiting a particular policy of customer rejection or upgrading, rather than randomly generated data.

6.5.1 Application in warehouses without upgrade operations

Public-storage warehouses in downtown areas primarily follow a customer rejection policy when a particular storage type is not available. Demand per storage type can be modeled by an Erlang-B distribution. For example, at a "Public Storage " warehouse (W. Chicago) near the Hancock center in Chicago, the average demand per month for small storage units is higher than its capacity. The Van Buren warehouse near the University of Illinois Chicago (UIC), has a similar experience: students store their personal belongings in this warehouse during the summer break. The receptionists directly reject customer requests when units of a storage type are fully occupied.

Table 6.1. Design for warehouses without upgrade operations

Warehouse	Items	Class 1	Class 2	Class 3	Class 4	Class 5	Class 6	Class 7	Class 8	Reve. ($)
W. Chicago[a]	Type (ft^2)	5 × 5	5 × 10	7.5 × 10	10 × 10	10 × 15	10 × 20	10 × 25		
	Prices ($)	81	93	170	170	305	348	397		
	Demand	10(2)[c]	15(2)	35(2)	45(2)	20(2)	5(3)	2(3)		
	O. design	21	30	43	120	30	20	15		42109
	N. design	25	34	79	96	45	17	6		48233
Van Buren[b]	Type (ft^2)	5 × 5	5 × 10	7.5 × 10	10 × 10	10 × 15	10 × 20	10 × 25	10 × 30	
	Prices ($)	66	78	102	139	281	348	398	480	
	Demand	19(2)	61(2)	48(2)	80(2)	11(2)	10(3)	8(3)	9(3)	
	O. design	41	63	52	126	43	41	0	0	45649
	N. design	40	103	23	101	19	24	16	19	52693

Notes: [a] For W Chicago, $C = 29500ft^2$. [b]For van Buren, $C = 35325ft^2$. [c] A(B) means the average monthly demand(the average value of single storage duration).

We used prices, design, and demand data (the average monthly demand and the average single storage duration) from these two warehouses of the 2008 summer and applied the basic model with customer rejections. Results can be found in Table 6.1. For the W. Chicago warehouse, its monthly average revenue improves by 14.5% using our new design. This warehouse had just redesigned its layout at the beginning of 2008, and sharply increased the number of units with size 10' × 10' to 120. For the demand of the summer this number apparently is too large; instead smaller size spaces should have been created. For the Van Buren warehouse, its monthly average revenue improves by 14.7% adopting our new design. The Van Buren warehouse has two major customer categories: UIC students and business customers from Chicago Loop (the Chicago central business district). Students in UIC typically rent storage units of size 5' × 10' which are fit for families with "a studio or one-bed room" according to Public Storage's marketing brochure. By our calculation, the number of these small rooms should be increased. In particular the number of 5' × 10' units should be increased to 103. Our optimization computing suggests this warehouse to increase the number of units with sizes 10' × 25' and 10' × 30' to meet the demand from the business customers who need large units.

6.5.2 Design application for warehouses with upgrade operations

Public-storage warehouses with less abundant demand usually try to upgrade customers when they run out of space of a certain type. We apply the upgrade model with *a priori* space reservation to the Spaanse Polder Rotterdam (SP Rotterdam) warehouse and the model with upgrades, but without prior space reservation to the N. Delaware Philadelphia warehouses (ND Philly). SP Rotterdam does not reject customers when the capacity in one class is fully occupied, but instead will attempt to upgrade its customers. The ND Philly warehouse is located near the B. Franklin bridge in Philadelphia, in the downtown area but without convenient transport access, so its demand is not too large. Therefore also the management of this warehouse is not inclined to reject customers but, instead, will try to upgrade them, if space of a storage class runs out.

Based on the data from these two warehouses, we applied the upgrade model, and present the results in Table-6.2. For the SP Rotterdam warehouse, its

Table 6.2. Design for warehouses with upgrade operations

Warehouse	Items	Class 1	Class 2	Class 3	Class 4	Class 5	Class 6	Class 7	Class 8	Revenue
SP Rot. [a]	Type(m^2)	3	6	9	12	15	18	22	27	
	Prices (€)	109	132	177	225	254	372	436	468	
	Demand	31(2)[b]	31(2)	33(2)	13(2)	7 (3)	8(3)	2(4)	2(4)	
	Old design	34	44	58	25	18	27	3	4	38577 €
	New design	15(7)[c]	11(16)	21(14)	53(0)	2(13)	23(6)	9(5)	20(0)	57882€
ND Philly[d]	Type (ft^2)	5 × 5	5× 10	10 × 10	10 × 15	10 × 20	10 × 25	10 × 30	10 × 40	
	Prices($)	65	79	132	227	222	255	326	396	
	Demand	34(2)	90(2)	70(2)	50(2)	40(2)	30(3)	9(3)	2(4)	
	Old design	78	180	144	22	54	22	24	4	67017$
	New design	70	171	129	101	63	6	0	0	71279$

Notes:[a] For SP Rotterdam, $C = 2118m^2$ and $\mathbf{p} = [0.80.90.90.80.90.80.90]$. [b] A(B) means the average monthly demand(the average value of single storage duration). [c] A(B) means the total storage unit number of a storage type(the reserved storage unit number of a storage type).[d] For ND Philly, $C = 53750ft^2$.

monthly average revenue can be significantly improved using our new design with upgrading and reservation operations for the demand data of the summer 2008. Our computations show management should increase the large size units (classes 6, 7, and 8) to admit upgraded orders. For the ND Philly warehouse with demand data of fall 2008, its monthly average revenue can also be improved by using our new design with upgrades. For this warehouse we did not include *a priori* space reservations. It is suggested to substantially increase the number of units of Class 4 to admit upgraded orders of classes 1, 2, and 3; three classes with huge customer numbers. The larger storage units (classes 7 and 8) should no longer be offered in this warehouse.

6.5.3 Sensitivity analysis

So far our results suggest it is possible to create designs that substantially improve the revenue. Although facilities can be flexibly adapted to changing demand to some extent, we here investigate the robustness of the designs by carrying out some sensitivity analyses with respect to demand. We use the SP Rotterdam as an example, since monthly demand data is available for a period of two years. In addition, for the W Chicago warehouse, we use sample-based sensitivity analysis (SBSA) to show the robustness of our approach.

Table 6.3. Sensitivity analysis of the design for SP Rotterdam warehouse

Dem.	Items	Class 1	Class 2	Class 3	Class 4	Class 5	Class 6	Class 7	Class 8	Revenue[a]
	Type(m^2)	3	6	9	12	15	18	22	27	
	Prices[a]	109	132	177	225	254	372	436	468	
	O. design	34	44	58	25	18	27	3	4	
07 Sp.	N. design	70(0)[b]	4(13)	17(13)	46(0)	2(12)	20(6)	10(6)	21(0)	49999(36049)
07 Su.	N. design	61(0)	3(15)	19(11)	45(0)	2(15)	23(6)	10(5)	20(0)	44030(32622)
07 Au.	N. design	15(7)	11(14)	18(16)	47(0)	2(13)	42(0)	4(6)	15(0)	50434(35709)
07 Wi.	N. design	62(0)	4(13)	17(12)	45(0)	2(14)	22(6)	10(6)	21(0)	50201(35724)
08 Sp.	N. design	14(7)	12(13)	18(13)	53(0)	2(14)	23(6)	10(6)	20(0)	63426(39449)
08 Su.	N. design	15(7)	11(16)	21(14)	53(0)	2(13)	23(6)	9(5)	20(0)	57882(38577)
08 Au.	N. design	74(0)	5(12)	18(11)	48(0)	3(9)	40(0)	3(3)	11(0)	53998(37557)
08 Wi.	N. design	12(7)	11(13)	18(12)	40(0)	2(12)	41(0)	3(5)	20(0)	46499(34919)
	Suggestion	-	↓	↓	↑	↓	↓	↑	↑	

Notes:[a] unit:€. [b] A(B)means the total storage unit number of a storage type(the reserved storage unit number of a storage type).

For the SP Rotterdam warehouse the optimal design results have been presented in Table 6.3. Although we use monthly demand data, for space reasons the table only summarizes average quarterly demand data. It appears that in all seasons our design method can improve the monthly average revenue by at least 33.2%. Based on Table 6.3, we can derive some robust design rules. First, units of classes 4, 7, and 8 should be increased, as in 100% of the demand cases a larger number of storage units yield a larger revenue. Apparently, the increased capacity in classes 7 and 8 can serve upgraded orders better. Second, the analysis advises to decrease the number of units of classes 2, 3 and 5, as in 100% of the demand cases a smaller number of storage units yield higher revenue. The number of class 6 should be reduced in 75% of the cases. The analysis gives no clue for redesigning the smallest class 1, since in 50% of the cases a larger number of units gives an increased revenue and in 50% of the cases a smaller number of units. However, this is not serious as class 1 contributes rather little both in terms of revenue and capacity and its demand is volatile. In Section 6.5.4 we will show how to design a robust layout, good for most demand realizations.

For the W Chicago warehouse, we generated 1000 demand samples for every storage type with uniform distribution and given demand upper and lower bounds, which are based on the demand estimation of the last two years. For every demand sample, we calculate the optimal storage design based as expressed

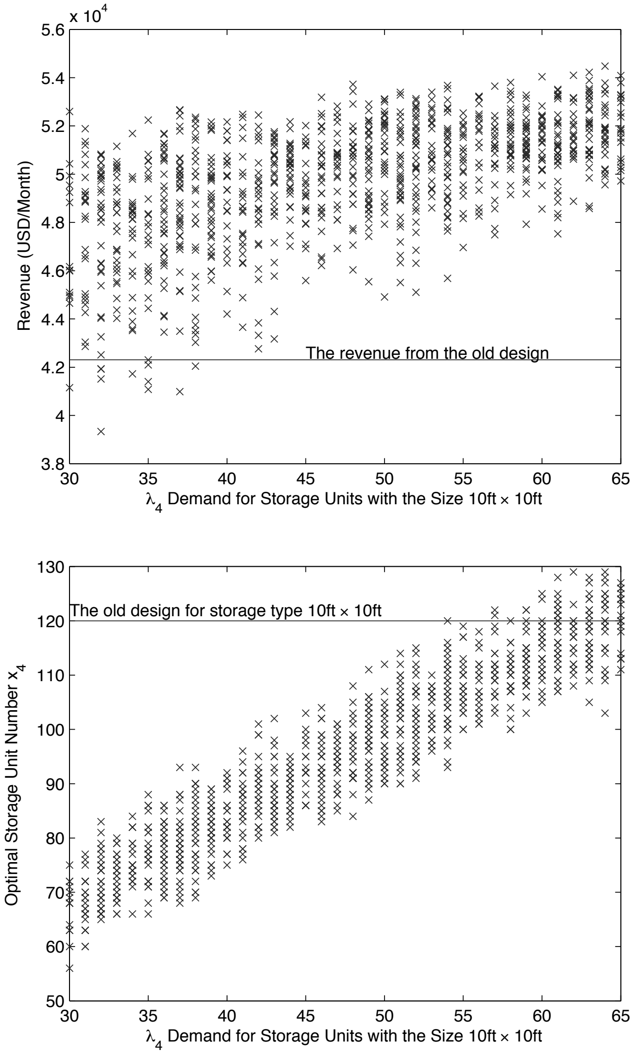


Fig. 6.2. A sample-based sensitivity analysis for W Chicago case

in Table 6.1. The results for storage type 4 are presented in Figure 6.2 by a scatter plot graph. Scatter plots of other storage types show a similar pattern. We find that, with probability 98.9%, our layout can improve the revenue compared with the current revenue of the old design. This shows the robustness of our approach. If the demand increases, the revenue increases. For 1.1% of the samples (see the left lower corner), the revenues are lower than the old revenue. These stem from cases with very low demand realizations. With probability of 95.1%, the old design has too many 10’ × 10’ storage units, showing our design results and suggestion are robust for demand fluctuations.

6.5.4 Robust design

This section will further provide a robust design. Robust optimization plays an important role in revenue management (see Birbil et al., (2009)). We take SP Rotterdam warehouse as an example, to illustrate how to provide a robust design, such that a warehouse can achieve the best revenue performance among the worst revenues from demand data of D quarters. The motivation of robust design is to reduce the loss from the variance of demand patterns to the least. Let $\mathit{\Lambda}$ be the set of D demand data in the last two years (see Table 6.3), we present a robust model as follows,

$$
\begin{aligned}
&\max\{\min_{\lambda_d \in \mathit{\Lambda}, d=1,\dots,D} [\textstyle\sum_{i=1}^{m} r_i v(x_i,\rho_i) + \sum_{i=1}^{m-1} r_{i+1}\eta_{i+1}(x_i)\beta_i \left(1 - P_{rej}(x_i,y_i)\right)]\} \\
&\textstyle\sum_{i=1}^{m-1}(c_i x_i + c_{i+1} y_i) + c_m x_m \le C \\
&x_i, y_i \in \mathbb{Z}_+
\end{aligned}
\tag{R}
$$

We apply a random search robust optimization algorithm to calculate the design. We use optimal results from Table 6.3 to construct the search scope $[\underline{(\mathbf{x},\mathbf{y})}, \overline{(\mathbf{x},\mathbf{y})}]$ for problem (R). For iterations $t = 1, ..., T$, we generate t samples $(\mathbf{x},\mathbf{y})^{\tau}, \tau = 1, ..., t$ from the search scope with considering the capacity constraint. For each of samples $(\mathbf{x},\mathbf{y})^{\tau}$, we compute its revenues in D different demand patterns, and get the worst value $R^{\tau} = min\{R_d((\mathbf{x},\mathbf{y})^{\tau}, \lambda_d), d = 1, ..., D\}$ for this design sample. The optimal revenue corresponding to the robust design is $R(t)^* = max\{R^{\tau}, \tau = 1, ..., t\}$ when the iteration number is t. We can then return the **(x,y)** value corresponding to $R(t)^*$ as the robust design when iteration number is t. The theoretical optimal robust revenue is $R^* = \lim_{t\to\infty} R(t)^*$.

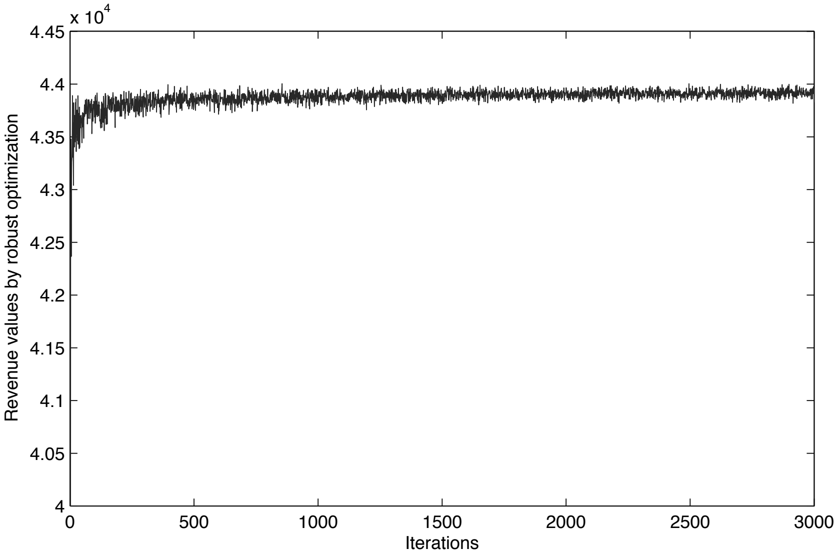


Fig. 6.3. Robust optimization for SP Rotterdam warehouse

We present the result $\{R(t)^*, t = 1, ..., T\}$ in Fig.- 6.3. With setting the criterion of convergence as $\frac{|R(t+1)^* - R(t)^*|}{R(t)^*} \le 0.3\%$, we observe the search converges in the last 500 iterations. We therefore calculate the average value from the last 500 values $\{R(t)^*, t = 2500 : 3000\}$, and obtain the robust optimal revenue value as 43912, which is a significant improvement compared with the current worst revenue value 32622 based on the current design. We use the design value obtained by 3000 iterations $\{50(6), 9(15), 19(16), 46(0), 2(15), 25(5), 9(4), 19(0)\}$ as the robust design, which also confirm the robust design rules from Table 6.3.

6.6 Concluding Remarks

This chapter outlines a design approach to improve revenue management of public-storage warehouses. Based on our visits to 54 of such facilities we can distinguish three groups, based on their demand level, which in turn is for a large part determined by the warehouse location. Facilities with high demand mainly reject customers when space of a storage size runs out. The group with medium demand tries to upgrade customers when space of a storage size runs out. Warehouses with low demand are usually located in the wrong place. We consider models for the first two demand types: models with customer rejection and with customer upgrading. We distinguish two upgrade models: with prior space reservation for upgraded customers and without. Models are solved using dynamic programming. For the case of upgrading without prior reservation we use a first moment approximation of the overflow process of customers from one storage class to the next. Our experiments show the new facility design can significantly improve the expected revenue of public storage warehouses, which is further confirmed by sensitivity analysis.

This chapter is one of the first to apply revenue management, with considering market segments, to facility design. This problem appears to be very relevant for public storage warehouses, but we expect it to be relevant for other facility design situations as well. The design approach can be applied to other fields as well, particularly hotel management. In hotel design, our method may be applied to deciding which room types to build, based on market data. The research may further be applied to parking lot businesses, as parking lots have a similar layout (e.g., different storage types) to that of a self-storage warehouse. In this case the setting is slightly simpler than that in this chapter, as the number of space classes is smaller. Another business where our methods may be applied to is the construction equipment lease business. This is a huge market as most civil construction engineering companies do not purchase all equipments (like bulldozers, shovels, windlasswa, and cranes) but rent it. Equipment lease companies (see,e.g. nakamichi-leasing.co.jp) can determine their equipment types and equipment numbers by our method. There may also be applications in air cargo space design and car rental business (to determine which car types and which numbers to procure).

Appendix

In this Appendix we give a proof of the lemmas mentioned in this chapter. We start with Lemma 6.1.

PROOF OF LEMMA 6.1 Let $\mathbf{t}_n$ denote the arrival time of the nth customer and denote by $\mathbf{L}(t)$ the number of busy servers at time t. Moreover, introduce the random variable $\mathbf{I}_n$ given by

$$\mathbf{I}_n = \begin{cases} 1 \text{ if nth arriving customer is of type 1} \\ 0 \text{ otherwise} \end{cases}$$

Obviously,

$$\mathbb{P}(\text{rejected customer is of type 1}) = \lim_{n\uparrow\infty} \mathbb{P}(\mathbf{I}_n = 1|\mathbf{L}(\mathbf{t}_n^-) = c), \tag{6.27}$$

with $\mathbf{t}_n^-$ denoting the time just prior to the arrival of the nth arriving customer. To compute the conditional probability in relation (6.27) we first observe that

$$\{\mathbf{L}(\mathbf{t}_n^-) = c\} = \{\mathbf{L}(\mathbf{t}_{n-1}^+) = c, \min_{1\leq j\leq c} \mathbf{B}_j^* > \mathbf{t}_n - \mathbf{t}_{n-1}\} \tag{6.28}$$

with $\mathbf{B}_j^*, 1 \leq j \leq c$, the residual service times after time $\mathbf{t}_{n-1}^+$ of the customers in service at time $\mathbf{t}_n^-$. Also by the memoryless property of the Poisson process we obtain

$$\{\mathbf{I}_n = 1, \mathbf{L}(\mathbf{t}_n^-) = c\} = \{\mathbf{X} < \mathbf{Y}, \mathbf{L}(\mathbf{t}_{n-1}^+) = c, \min_{1\leq j\leq c} \mathbf{B}_j^* > \min\{\mathbf{X}, \mathbf{Y}\}\} \tag{6.29}$$

with $\mathbf{X}, \mathbf{Y}$ exponentially distributed with mean $\gamma_1^{-1}, \gamma_2^{-1}$. Applying relation (6.29) this shows

$$\mathbb{P}(\mathbf{I}_n = 1 | \mathbf{L}(\mathbf{t}_n^-) = c) = \frac{\mathbb{P}(\mathbf{I}_n = 1, \mathbf{L}(\mathbf{t}_n^-) = c)}{\mathbb{P}(\mathbf{L}(\mathbf{t}_n^-) = c)} = \frac{\mathbb{P}(\mathbf{X} < \mathbf{Y}, \mathbf{L}(\mathbf{t}_{n-1}^+) = c, \min_{1 \le j \le c} \mathbf{B}_j^* > \min\{\mathbf{X}, \mathbf{Y}\})}{\mathbb{P}(\mathbf{L}(\mathbf{t}_n^-) = c)}. \tag{6.30}$$

To compute the probability in relation (6.30) we introduce the defective distribution $F(z) := \mathbb{P}(\mathbf{X} < \mathbf{Y}, \min\{\mathbf{X}, \mathbf{Y}\} \le z)$. By the conditional probability formula

$$\mathbb{P}(\mathbf{X} < \mathbf{Y}, \mathbf{L}(\mathbf{t}_{n-1}^+) = c, \min_{1 \le j \le c} \mathbf{B}_j^* > \min\{\mathbf{X}, \mathbf{Y}\})$$
$$= \int_0^\infty \mathbb{P}(\mathbf{L}(\mathbf{t}_{n-1}^+) = c, \min_{1 \le j \le c} \mathbf{B}_j^* > \min\{\mathbf{X}, \mathbf{Y}\} | \mathbf{X} < \mathbf{Y}, \min\{\mathbf{X}, \mathbf{Y}) = z) dF(z). \tag{6.31}$$

Since the Poisson arrival process after time $\mathbf{t}_{n-1}^+$ is independent of the number of customers at time $\mathbf{t}_{n-1}^+$ in the system and their residual service times from that time on the random vectors $(\mathbf{1}_{\{\mathbf{L}(\mathbf{t}_{n-1}^+) = c\}}, \min_{1 \le j \le c} \mathbf{B}_j^*)$ and $(\mathbf{1}_{\{\mathbf{X} < \mathbf{Y}\}}, \min\{\mathbf{X}, \mathbf{Y}\})$ are independent. This shows that the conditional density in relation (6.31) reduces to

$$\mathbb{P}(\mathbf{L}(\mathbf{t}_{n-1}^+) = c, \min_{1 \le j \le c} \mathbf{B}_j^* > \min\{\mathbf{X}, \mathbf{Y}\} | \mathbf{X} < \mathbf{Y}, \min\{\mathbf{X}, \mathbf{Y}) = z)$$
$$= \mathbb{P}(\mathbf{L}(\mathbf{t}_{n-1}^+) = c, \min_{1 \le j \le c} \mathbf{B}_j^* > z, | \mathbf{X} < \mathbf{Y}, \min\{\mathbf{X}, \mathbf{Y}\} = z) \tag{6.32}$$
$$= \mathbb{P}(\mathbf{L}(\mathbf{t}_{n-1}^+) = c, \min_{1 \le j \le c} \mathbf{B}_j^* > z).$$

Also conditioning on $\mathbf{X}$ and using again the conditional probability formula we obtain

$$\begin{aligned} F(z) &= \mathbb{P}(\mathbf{X} < \mathbf{Y}, \mathbf{X} < z) \\ &= \gamma_1 \int_0^z \mathbb{P}(x < \mathbf{Y}) \exp(-\gamma_1 z) dz \\ &= \gamma_1 \int_0^z \exp(-(\gamma_1 + \gamma_2) z) dz \\ &= \frac{\gamma_1}{\gamma_1 + \gamma_2} (1 - \exp(-(\gamma_1 + \gamma_2) z). \end{aligned} \tag{6.33}$$

Hence it follows that

$$F(z) = \mathbb{P}(\mathbf{I}_n = 1)\mathbb{P}(\min\{\mathbf{X},\mathbf{Y}\} \leq z). \tag{6.34}$$

and by relations (6.28), (6.31), (6.32) and (6.34) we finally obtain

$$\begin{aligned}
&\mathbb{P}(\mathbf{X} < \mathbf{Y}, \mathbf{L}(\mathbf{t}_{n-1}^{+}) = c, \min\nolimits_{1\leq j\leq c}\mathbf{B}_j^* > \min\{\mathbf{X},\mathbf{Y}\}) \\
&= \mathbb{P}(\mathbf{I}_n = 1)\int_0^\infty \mathbb{P}(\mathbf{L}(\mathbf{t}_{n-1}^{+}) = c, \min\nolimits_{1\leq j\leq c}\mathbf{B}_j^* > z)d\mathbb{P}(\min\{\mathbf{X},\mathbf{Y}\} \leq z) \\
&= \mathbb{P}(\mathbf{I}_n = 1)\mathbb{P}(\mathbf{L}(\mathbf{t}_{n-1}^{+}) = c, \min\nolimits_{1\leq j\leq c}\mathbf{B}_j^* > \min\{\mathbf{X},\mathbf{Y}\}) \\
&= \mathbb{P}(\mathbf{I}_n = 1)\mathbb{P}(\mathbf{L}(\mathbf{t}_n^{-}) = c).
\end{aligned} \tag{6.35}$$

Applying relations (6.30) and (6.35) we have shown that $\mathbb{P}(\mathbf{I}_n = 1|\mathbf{L}(\mathbf{t}_n^{-}) = c) = \mathbb{P}(\mathbf{I}_n = 1)$ and this implies by relations (6.33) and (6.34) the desired result.

Next we give a proof of the lemmas needed for the correctness of the discretization procedure. We start with Lemma 6.2 and continue with Lemma 6.3 and 6.4.

Proof of Lemma 6.2: We know that $v(x,\rho) = \mathbb{E}(\mathbf{L}_\rho(x))$ with $\mathbf{L}_\rho(x)$ denoting the long-run average number of customers in the loss system with load ρ and x servers. By the insensitivity of the Erlang loss formula we may assume without loss of generality that we are dealing with a Markovian loss system with arrival rate ρ and an exponential service time distribution with parameter $\mu = 1$. Since the Markovian loss system is a birth-death process it follows by Proposition 4.2.10 of Stoyan (1983) that $\mathbf{L}_{\rho_1}(x) \geq_d \mathbf{L}_{\rho_2}(x)$ for $\rho_1 \geq \rho_2$ and this shows that $\mathbb{E}(\mathbf{L}_{\rho_1}(x)) \geq \mathbb{E}(\mathbf{L}_{\rho_2}(x))$. Hence the function v is increasing in ρ for fixed x.

Proof of Lemma 6.3: We will prove the claim by induction. First note by Lemma 6.2 and relation (6.3) that for every $k \leq m$ and $c \in \{0,1,...,C\}$, the function $\eta \mapsto r_k v(c, \rho_k + \eta\beta_{k-1})$ is increasing and Lipschitz continuous. Hence by relation (6.17) the function $\eta \mapsto J_m(c,\eta)$ is increasing and Lipschitz continuous for every $c \in \{0,1,...,C\}$. Suppose that $\eta \mapsto J_{k+1}(c,\eta)$ is increasing and Lipschitz continuous for every capacity c. Since the composition of two increasing Lipschitz continuous functions is increasing and Lipschitz continuous this yields by our induction hypothesis that the function $\eta \mapsto J_{k+1}(c - c_k x_k, \lambda_k p_k B(x_k, \rho_k, \eta))$ is increasing and Lipschitz continuous for every $x_k \in \{0, ...\lfloor c c_k^{-1} \rfloor\}$. Again using the last property of Lipschitz continuous functions and the maximum of a finite number of increasing and Lipschitz continuous functions is also increasing and Lipschitz continuous, it follows by relation (6.19) that $\eta \mapsto J_k(c, \eta)$ is increasing and Lipschitz continuous for every c belonging to $\{0, 1, ..., C\}$.

Finally we list a proof of lemma 6.4.

Proof of Lemma 6.4: Since the function $\eta \mapsto B(x_k, \rho, \eta)$ is increasing for every feasible x_k, it follows by our assumption that the function $\eta \mapsto f(c - x_k c_k, \lambda_k p_k B(x_k, \rho_k, \eta))$ is also increasing for every feasible x_k. This shows by (6.19) that

$$P_k f_{(h)} \leq P_k f \leq P_k f^{(h)}.$$

Since also $\eta \mapsto r_k v(x_k, \rho_k + \eta \beta_{k-1})$ is increasing it follows again by (6.19) that $P_k f_{(h)}, P_k f$ and $P_k f^{(h)}$ are increasing functions of η. From (6.20) follows that

$$\|P_k f^{(h)} - P_k f_{(h)}\|_\infty \leq \|f^{(h)} - f_{(h)}\|_\infty. \tag{6.36}$$

Applying now relations (6.22), (6.36) and the function $\eta \to f(c, \eta)$ is Lipschitz continuous for every $c \in \{0, 1, ..., C\}$ the desired inequality in relation (6.24) follows.

Part V

Summary

7

Conclusion and summary of the dissertation

7.1 Summary

This thesis contributes to stochastic modelling and analysis of warehouse operations by the following parts.

(1) A review on stochastic modelling and analysis of warehouse operations

We provide an overview of stochastic research in warehouse operations. We identify uncertainty sources of warehousing systems and systematically present typical warehouse operations from a stochastic system viewpoint. Stochastic modelling methods and analysis techniques in existing literature are summarized, along with current research limitations. Through a comparison between potential and existing stochastic warehouse applications, we identify potential new research applications. Furthermore, by comparing potential and existing solution methods, methodological directions relevant to practice and largely unexplored in warehouse literature are identified.

(2) Stochastic modelling and analysis for warehouses with online order arrivals

Much of the past warehousing literature dealing with order picking and batching assumes batch sizes are given. However, selecting a suitable batch size can significantly enhance the system performance. This research tries to search optimal batch sizes in a general parallel-aisle warehouse with online order arrivals. We employ a sample path optimization and perturbation analysis algorithm to search the optimal batch size for a warehousing service provider facing a stochastic demand, and a central finite difference algorithm to search the optimal batch sizes from the perspectives of customers and total systems. We show the existence of optimal batch sizes, and find past researches underestimate the optimal batch size.

We then research a polling-based dynamic order picking system for online retailers. One of the challenging questions online retailers are facing is how to organize the logistic fulfillment processes during and after the transaction has taken place. As new information technologies become available to convey picking information in real time and with the ongoing need to create greater responsiveness to customers, dynamic picking can be applied in the warehouses of online retailers. In a DPS (dynamic picking system), a worker picks orders arriving in real time during the picking operations while the picking information dynamically changes in one picking cycle. We build models to describe and analyze such systems via stochastic polling theory and find closed-form expressions for the order line waiting times in a DPS. These analytical results are verified by simulation. By applying polling-based picking to two cases, we show it can generally lead to shorter order throughput times and higher on-time service completion ratios than traditional batch-picking systems using optimal batch sizes. We show how our analysis method can be applied to minimize warehouse cost or to improve service.

(3) Stochastic modelling and analysis for service distribution centers

We present closed-form analytic expressions for pick rates of order picking bucket brigades systems in different storage profiles, and show how to combine storage policies and bucket brigade protocols to improve order picking productivity. We further shed light on reasons why the bucket brigades system outperforms zone picking systems in a range of storage profiles. We find a shrink effect of bucket brigades order picking in non-uniform storage profiles. Bucket brigades can efficiently use this effect, via their capacity of self-balancing, combined with different storage profiles to improve order picking productivity, and this effect is more significant in a BB protocol compared with that in other protocols without the capacity of self-balancing.

(4) Stochastic modelling and analysis for public storage

We propose a new facility design approach oriented to improving revenue management. Our experiments show a proper facility design can significantly improve the expected revenue of public storage. This is the first research to apply revenue management theory to a new field, facility design, and identify a new research direction, the interface between revenue management and facility logistics.

7.2 Directions for future works

In this section, we present promising research directions with a potential to be applied to warehouse operations. We focus on recent warehousing phenomena which have received little academic attention, like warehouses with an online front desk, self-storage warehouses, and third-party warehouses, and stochastic research directions which can grasp the inherent decision essence and variability structure in warehouse operations.

7.2.1 Application issues

By comparing Table 2.1, which presents existing warehouse operations with uncertainties, and Table 2.4, which presents warehouse operations with stochastic studies, we can identify warehouse operations with uncertainties but not yet fully explored by stochastic methods.

(1) Warehouse receiving management

We could not find papers explicitly employing stochastic models for receiving processes (see Table 2.4). However, receiving is an important issue for warehouse operations (see Table 2.1) and several interesting research opportunities exist here. The first opportunity is to study storage decisions for returned products.

Many online retailers face this problem. To speed up return processes, it may be helpful to not consolidate them with existing stock, but to store them at separate locations. This will be at the expense of more space needed, which in turn may also increase average storage, retrieval and travel time. The objective is to make the proper decision to take this trade-off into account. Furthermore, warehouse receiving operations (e.g., decentralized receiving, prereceiving) in uncertain environments call for further research by stochastic methods. For example, Yano et al. (1998) conduct a successful research on decentralized receiving operations (receiving occurs not at one or two clusters of receiving docks but at multiple locations) by a mixed integer nonlinear optimization formulation with the objective of minimizing total cost of facilities and labor. Splitting receiving operations over multiple areas can reduce congestion, but usually requires more resources and reduces resource flexibility. It could be interesting to consider this trade-off when product arrival times and order patterns are random.

(2) Warehouse revenue management

Comparing Figure 2.2 and Table 2.4, we find that order acceptance and rejection has been overlooked by past literature. Order acceptance is particularly important in the public storage warehouse, where storage capacity is limited. A manager of such a facility can reject an order to maximize the revenue. For instance, Shurgard (see shurgard.eu in the EU, and publicstorage.com in the USA), an international corporation providing public storage warehouse services, uses stochastic revenue management in allocating storage space to clients. While most warehouse researches focus on cost management, revenue issues in warehouses are unexplored.

(3) Warehouse shipping management

Only few papers deal with outbound material flows (e.g. Yu and Egbelu (2008)) in a deterministic environment. Shipping operations are often overlooked. Nevertheless, many important shipping problems exist. For example, how to allocate shipments to be shipped to different shipping docks. With the increase of innovative warehouse shipping operations like automating pallet loading, automated outbound weight checking, advanced shipping notice preparation, dock assignment optimization (see Frazelle (2001)), it could be an interesting topic to explore shipping operations by stochastic methods.

(4) Real-time response systems

Real-time response constitutes one of most vibrant warehouse research fields. To shorten response time (from order notification to the shipping to customers, see Figure 2.2), new techniques have been introduced, such as online picking (using, for example, pick-by-voice), RFID systems and fluid shipments. In a dynamic environment, decision-makers have insufficient time to collect information, and therefore the negative effect of uncertainties is larger. Deterministic models cannot capture the inherent uncertainty in these systems. Stochastic models might be used to model these systems, to measure the performance of real-time order processing in a stochastic environment, and to optimize these systems.

7.2.2 Methodology issues

By comparing stochastic methods (e.g., see Yao (1994)) with currently used stochastic methods in warehouse operations, and considering current developments in warehousing practice, we can identify promising methodological research directions.

(1) Stochastic networks application

Queueing networks have been applied in warehouse research (e.g., Gong and De Koster (2008), Meng and Heragu (2004)). However, more general stochastic networks, one of the main recent exploration directions in stochastic research (Yao, 1994), have appeared to be promising in the operations and manufacturing areas (Buzacott, 2005), and can be explored further in warehousing. For instance, stochastic fluids models can be used to represent customers in a service facility, or jobs on the work floor(Chen and Mandelbaum, 1994). Stochastic networks are potential tools to handle tough warehouse problems like large order flows in multiple work stations, multi-echelon warehouse, and dynamic scheduling problems.

(2) Stochastic programming application

From our literature review, one of the most obvious blanks of stochastic methodology in warehousing research is stochastic programming (for an introduction, see Birge and Louveaux (1997)). We could not find an application of this important stochastic analytical method in warehouse research (for an introduction to this research stream, see stoprog.net). However, it benefits warehouse optimization problems. Many papers (e.g., Van den Berg et al. (1998), Karasawa et al. (1980)) employ integer programming applications in warehousing since warehouse managers face many integer decision variables like batch sizes and the number of zones. But due to risks and uncertainties in these warehousing decisions (see Table 2.1), stochastic integer models are closer to practice because while deterministic models only consider the first moment of measurements (e.g., the objective) and can cause significant errors, stochastic models can research higher moments of measurements and capture more abundant information. Recently polynomial time algorithms for stochastic integer programming problems have seen increasing research attention (Klein Haneveld and Van der Vlerk (1999)). They might be used for various problems, including product assignment, storage space allocation, the optimal batch size, and optimal zone problems.

(3) Stochastic combinatorial problems

Stochastic combinatorial optimization is a highly promising method in warehouse research, especially the stochastic traveling salesman and stochastic knapsack problems. The application of stochastic traveling salesman models has constituted a main foundation in the logistics field (Bertsimas and Van Ryzin 1991, Bertsimas 1992, Bertsimas and Van Ryzin 1992), and can be applied to the internal picking routing problem in warehouses. Another promising method is the stochastic knapsack model (see Ross and Tsang (1989) and Ross and Yao (1990)), which can be applied to the warehouse storage space allocation problem. Furthermore, Kleywegt and Papastavrou (1998) and Kleywegt and Papastavrou (2001) explore dynamic and stochastic knapsack problems. These methods may be applied to allocate warehouse storage space in static and dynamic environments.

Stochastic researches could also shed light on other questions like, the optimization of dynamic storage and putaway systems in a stochastic environment, the optimal zone problem by using stochastic integer programming, and the optimal batch size problems by infinitesimal perturbation analysis techniques.

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Abstract

This thesis studies stochastic models and analysis of warehouse operations. Based on a review of stochastic research in warehouse operations, where we identify uncertainty sources of warehousing systems and present typical warehouse operations from a viewpoint of stochastic systems, we explore the following topics.

Firstly, we search optimal batch sizes in a parallel-aisle warehouse with online order arrivals. We employ a sample path optimization and perturbation analysis algorithm to search the optimal batch size for a warehousing service provider, and a central finite difference algorithm to search the optimal batch sizes from the perspectives of customers and total systems.

Secondly, we research a polling-based dynamic order picking system for online retailers. We build models to describe and analyze such systems via stochastic polling theory, find closed-form expressions for the order line waiting times, and apply polling-based picking to online retailers.

We then present closed-form analytic expressions for order pick rates of bucket brigades order picking systems with different storage profiles, and show how to combine storage policies and bucket brigade protocols to improve order picking productivity. We further shed light on reasons why the bucket brigades system can outperform some zone picking systems for a range of storage profiles.

Finally, we propose a new warehouse design approach to improve the revenue management of public storage warehouses. Based on our survey of 54 warehouses in America, Europe and Asia, we propose models for three different cases: an overflow customer rejection model and two models with customer upgrade possibilities: one with reservation and another without reservation. We solve the models for several real warehouse cases, and our results show for all cases the existing public-storage warehouses can be redesigned to bring larger revenues. Finally, we develop the robust design to reduce the loss from the variance of demand to the least.

Samenvatting (Summary in Dutch)

Dit proefschrift onderzoekt magazijnoperaties met behulp van stochastische modellen en analyses. Het proefschrift bestaat uit zeven hoofdsukken. Als eerste geven we een overzicht van stochastisch onderzoek in magazijnoperaties in hoofdstuk 2.

In hoofdstuk 3 doen we onderzoek naar een optimale seriegrootte voor een magazijn met parallelle gangen en online orderaankomsten. We maken gebruik van sample-path optimization en perturbation analysis om de optimale seriegrootte voor een magazijn vast te stellen rekening houdend met zowel de order doorlooptijden als de operationele kosten.

In hoofdstuk 4 onderzoeken we een polling-based dynamisch orderverzamelsysteem, wat met name geschikt is voor online retailers. We ontwikkelen modellen om zulke systemen te beschrijven en te analyseren met stochastische polling theorieën. We vinden gesloten uitdrukkingen voor de order wachttijden en passen de resultaten toe op een bedrijfscase.

Daarna presenteren we gesloten analytische uitdrukkingen voor de doorzet van bucket brigade orderverzamelsystemen, waarbij gebruik gemaakt wordt van verschillende opslagprofielen. Deze uitdrukkingen kunnen gebruikt worden om de combinatie van bucket brigades met verschillende opslagprofielen te evalueren.

In hoofdstuk 6 stellen we een nieuwe methode voor om ontwerpen voor self-storage magazijnen te evalueren en te genereren. We tonen aan dat, vergeleken met bestaande magazijnen, de opbrengst aanzienlijk verhoogd kan worden door bij het ontwerp rekening te houden met de te verwachten vraag per type opslag.

About the author

Yeming Gong was born in Hubei, China on 1st January 1976. Before he studies in the Netherlands, he studied production and operations management in INSEAD (European Institute of Business Administration), Fontainebleau, France. His research topic was "Stochastic Optimization for the Multi-location Transshipment Problem with Positive Replenishment Lead times", and he received his MSc degree in 2005.

Since 2006, he joined the PhD program at Rotterdam School of Management, Erasmus University. In the early stage of his PhD study, he had frequently attended the courses and academic activities in LNMB (Dutch Network on the Mathematics of Operations Research) and TRAIL (the Netherlands Research School on Transport, Infrastructure and Logistics), where he laid a solid foundation, particularly in stochastic operations research and logistics.

In Rotterdam School of Management, his main research interests are warehouse operations. His research topics include, approximate optimal order batch sizes in a parallel-aisle warehouse, a flexible performance evaluation framework of order picking systems, storage scheduling decision models for revenue management of self-storage warehouses, a review on stochastic models and analysis of warehouse operations, and a new design method to improve the revenue management of public storage warehouses.

In 2008 summer, he had worked as a visiting PhD student in the PhD program of Booth School of Business, the University of Chicago. His research topic was mainly on bucket brigades order picking systems and storage profiles, and he had also explored the application of nonlinear dynamic models to manufacturing systems.

He presented his papers in international conferences like 2009 EURO Conference in Germany, 2008 INFORMS Annual Meeting in USA, 2008 Annual International Logistics Conference in USA, 2007 Conference on Stochastic Programming in Austria, 2006 International Workshop on Distribution Logistics in Italia. He has published book chapters in publishers like *Springer*, and articles in journals like *IIE Transactions* and *Production and Operations Management*.

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STOCHASTIC MODELLING AND ANALYSIS OF WAREHOUSE OPERATIONS

This thesis studies stochastic models and analysis of warehouse operations. Based on a review of stochastic research in warehouse operations, where we identify uncertainty sources of warehousing systems and present typical warehouse operations from a viewpoint of stochastic systems, we explore three types of warehouses.

Firstly, we study warehouses with online order arrivals. We employ a sample path optimization and perturbation analysis algorithm to search the optimal batch size for a warehouse with online order arrivals, and a finite difference algorithm to search the optimal batch size for its customers. We then build stochastic polling models to describe and analyze a polling-based dynamic order picking system for online retailers, find closed-form expressions for the order waiting times, and apply polling-based picking to online retailers. Subsequently, we study service distribution centers. We present closed-form analytic expressions for pick rates of bucket brigades order picking systems with different storage profiles, and show how to combine storage policies and bucket brigades protocols to improve order picking productivity. Finally, we consider a booming industry, public storage. We propose a novel facility design approach to improve the revenue management of public storage. Our results show existing public-storage warehouses can be redesigned to bring larger revenues. We also develop the robust design to reduce the loss from the variance of demand to the least.

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