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AN OLIGOPOLY MODEL OF FREE BANKING : THEORY AND TESTS*

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1 THE FREE BANKING ERA

While the provision of bank notes is currently a government-operated business, historically there have been several periods of free banking. During the past decade there has been renewed interest in free banking, and the performance of the free banking era has been examined from several different perspectives.[1] Recently, the first of the two alternative proposals by the British treasury for European monetary integration based itself on the idea of currency competition by giving all EC currencies EC-wide legal tender status.

Three major themes are present in the extant literature: (i) sustainability, (ii) uncertainty and information, and (iii) circulation, debasement (inflation) and seigniorage. The first theme arises because the debate about the effects of deregulation of the money supply process is in essence an argument about sustainability; that is, whether a single supplier of money can remain the sole supplier of currency when competitors are free to enter the money supply process. In the Scottish case, for example, White (1984, pp. 25–26) argues that when the Bank of Scotland lost its monopoly rights it was unable to thwart entry. The second theme addresses the argument that the era of free banking in the United States led to more uncertainty about the value of different bank notes in comparison with the pre free-banking era.[2] There is ample evidence that during the free banking era the

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1 One focus has been largely normative in spirit, and coincides with the current interest in deregulation; see the provocative essay by Hayek (1978), and the volume edited by Salin (1984). Another line of research has focused on positive implications of free banking; see the early perceptive analysis by Klein (1974), and the review article by King (1983). In addition, several interesting empirical studies have been conducted. For example, Rockoff (1974) and Rolnick and Weber (1983, 1984) have investigated the free banking era of the United States, while White (1984) has conducted a detailed study of the Scottish experience.

2 See, for example, Rockoff (1974, p. 143) and Friedman (1959).

nonbank public had imperfect information regarding the alternative bank notes and their relative values. Lastly, some have argued that the private provision of money may reduce inflation, since competition would provide a check against overissue; others have argued the converse, see Klein (1974). Currency competition as a means to hold inflation down is the leitmotif of the British treasury proposal.

While the existing literature offers alternative models that address each of these issues separately, a single coherent framework that integrates the three issues is notably absent; see Fischer's (1986) review essay and King (1983, p. 156). The link between theory and empiricism has been especially weak; only the hypothesis of wildcat banking has been investigated empirically. In his study of free banking in the U.S., Rockoff (1974) notes that a number of free bank failures are related to the possibility of wildcat banking. Rolnick and Weber (1983, 1984), however, raise the alternative falling asset prices explanation. This is contested by Rockoff (1989), who argues that asset prices fell precisely because banks had to sell off assets in case of a bank run. While the falling asset hypothesis may explain a portion of bank closures, it would still leave unexplained some 85 percent of the behavior of free banks.

The absence of theoretical models for the free banking era not only obscures discussions about the merits of privatization of the money supply process, but also hampers the empirical analysis as it is unclear what hypotheses are to be tested. The aim of the present paper is to develop an analytical framework that captures themes (i)–(iii) and to test the predictions of the model based on data from the free banking era in the United States.

We begin by noting that the key difference between a government-run central banking system and the free banking system is the difference between a monopoly and an oligopoly model. Hence, we face the difficult task of selecting an appropriate oligopoly model. Toma (1985) has modeled the rivalry between the U.S. Federal Reserve and Treasury in the 1930s and 1940s as a duopoly quantity game. In Toma's model, each institution could reap part of the total seigniorage, by determining their share in the supply of high powered money. The analysis is cast in a Cournot-Nash setting where the two institutions compete for their share in the supply of the U.S. monetary base. Given the institutional background of that time, the respective shares of base money are the appropriate strategic variables.

In contrast, during the free banking era different dollar bills were issued by different banks.[3] These bank notes (inside money) were redeemable in specie, which functioned as the monetary base (outside money). Due to temporal differences in circulation and debasement rates of the different notes, see section 3 below, these notes did not exchange at par in the market place.[4] Different

3 The notes were distinguishable by the imprinted brand names.

4 This may not have been the sole reason for the discounts. For example, the distance between the place of exchange and the location of a bank implicitly determined the transaction costs of arbitrage as the cost of transportation.

debasement rates represented different default risks, see *e.g.* Rolnick and Weber (1987, pp. 4–6) and Rockoff (1989, pp. 39–40). This gave rise to risk premiums in the form of discounts charged by brokers, see Rockoff (1974, p. 144). It follows that when alternative notes coexist, informed agents are not indifferent between the notes if the debasement rates differ. Differences in debasement rates trigger substitution towards holding the notes with lowest rate of debasement (inflation). Therefore, we focus on an oligopoly in the bank note supply process under 'price competition,' rather than 'quantity competition,' where debasement rates are the strategic variables. If the nonbank public were perfectly informed, this would trigger 'price wars' to capture the entire market, see Friedman (1959, p. 7) and Klein (1974, p. 430). Given the costly services of currency brokers and bank note reports, however, it is unlikely that all of the nonbank public was perfectly informed about the rates of debasement of the alternative suppliers of bank notes. Hence, some agents would accept particular notes even if such notes circulated above the lowest rate of inflation. This implies that if the minimum debasement rate is very low, a supplier of notes has an incentive to debase rapidly. This occurs if the lost demand due to agents switching to the notes of another supplier is more than offset by the inflation tax which is levied on the uninformed agents. There are therefore two opposite forces at work: the incentive to debase slowly in order to attract the fraction of note holders who are informed about the debasement rates, and the incentive to debase rapidly to exploit the uninformed public. We show that these two forces preclude the existence of pure strategy equilibrium debasement rates. It is shown, however, that an equilibrium in mixed strategies does exist.

The upshot of our analysis is that the equilibrium strategies are intrinsically stochastic, thus implying that the individual bank note circulation and debasement rates are random over time. However, this source of uncertainty is more a virtue than a vice. While Rolnick and Weber may have refuted the wildcat banking hypothesis, the general belief persists that the era of free banking exhibited more uncertainty. This vice is precisely what is implied by the mixed strategy equilibrium, as it implies uncertainty as an equilibrium phenomenon. But, in contrast to the monopoly provision of bank notes, it has the virtue of producing lower rates of debasement and hence lower inflation. In order to derive these conclusions a model of free banking is developed in the next section.

2 A MODEL OF FREE BANKING

We begin by specifying the demand for money, and describing the technology for issuing bank notes. The money demand functions are based on the quantity equation $MV = PT$.[5] Let k be the money multiplier, which is assumed to be

5 It is not our aim to develop a model which explains why money is used at all, but to model the demand for different monies which are all legal tender. Therefore we adopt one of the most widely

constant. The relationship between bank notes B and money demand M, *i.e.* notes and deposits, thus becomes $M = kB$. We employ Cagan's (1956) specification for velocity, so that $V = exp(\gamma i)$, $\gamma > 0$, where the nominal interest rate i equals the real interest rate r plus the debasement rate π. Furthermore, we assume a fixed relationship between real income and transactions: $T = T(y)$. It follows that the demand for bank notes deflated by the price level P is

$$b = \frac{B}{P} = \frac{T(y)}{k} e^{-\gamma(r+\pi)} .$$

Following Toma (1985), we focus on situations where the real income y and the real interest rate r are exogenous to the model. In this case, the demand for 'real notes' can be written as

$$b = \phi e^{-\gamma\pi} , \tag{1}$$

where $\phi = \phi(y, r) = T(y) e^{-\gamma r}/k$ depends only on y and r, which are assumed to be fixed.

Equation (1) is a reasonable specification of the demand for real notes supplied by one source. During the free banking era, however, several banks issued distinct dollar bills, and one must modify the equation (1) to incorporate interdependencies in the demand for the notes. This is done by allowing for currency substitution, or more appropriately *note substitution*. In particular, we will argue that a separation in information causes some agents to substitute actively, while others remain indifferent. For the period of the antebellum a partition in information, such that some agents were better informed than others, seems natural given the cost involved in gathering information about debasement rates. In equilibrium the number of agents involved in collecting information and engaging in arbitrage is such that the marginal benefits from arbitrage just offset the cost of acquiring the information, see *e.g.* Grossman and Stiglitz (1976), and more specifically Rolnick and Weber (1986). For the period of free banking, the nonbank public can be divided into two classes: those who were informed about the debasement rates of alternative note suppliers and those who were uninformed about the alternative debasement rates. The former class is comprised of brokers of bank notes and the users of bank note reports. For the latter class, comprised of agents primarily involved in local trade, the costs of gathering information exceed the benefits, see *e.g.* Rockoff (1974, p. 145) and Klein (1974). It is assumed that the informed wish to hold the notes of the bank that debases at the lowest rate, see *e.g.* Klein (1974, pp. 432–433), signaling the lowest risk of default, while the uninformed simply

used specifications for money demand, which is also often used in the currency substitution literature, see *e.g.* Kouri and De Macedo (1978). Note that Blanchard and Fischer (1989, chapter 4) employ the same strategy for addressing these two different questions.

hold the 'local' currency. Hence, each bank enjoys some positive demand for its notes from its share of the uninformed. In addition, extra demand materializes if it debases at the lowest rate. In this manner, *direct note substitution* occurred whenever another bank would become the bank with the lowest rate of note creation.

The impact of informed and uninformed agents on the demand for real notes is modeled by means of two multiplicative scale factors, α, β such that $\alpha > \beta$. Suppose there are n banks indexed by $i, j = 1, \ldots, n$. Define the following generic scale factor τ_i:

$$\tau_i = \begin{cases} \alpha & \text{if } \pi_i < \pi_j \, \forall j \neq i \\ \beta & \text{if } \pi_i > \pi_j \text{ for some } j \neq i\,. \end{cases} \tag{2}$$

If T, $T \leq n$, banks simultaneously set the lowest inflation rate, it is assumed that τ_i is a weighted average of α and β.[6] Given this definition of τ_i, the demand for mint i's real notes is given by

$$b_i = \tau_i \phi e^{-\gamma \pi_i}, \quad i = 1, \ldots, n\,. \tag{3}$$

Intuitively, if bank i has the lowest debasement rate, it gets all of the informed and its share of the uninformed, and $\tau_i = \alpha$. If the bank does not have the lowest debasement rate, it loses the informed to the bank with the lowest debasement rate, and τ_i takes on the smaller value of β. Thus the difference $(\alpha - \beta)$ is the note substitution factor by which demand increases when a bank sets the lowest inflation rate. In addition to this *direct note substitution* process, some smooth and more *indirect substitution* process may be present as well.[7] Following Kouri and De Macedo (1978) equation (3) is amended as follows

$$b_i = \tau_i \phi e^{-\gamma \pi_i + \sigma \pi^*}\,. \tag{3'}$$

where π^* is the lowest debasement rate among the other free banks. The semi-elasticity σ signifies the amount of indirect note substitution, while τ_i in (2) gives the amount of direct note substitution. Below we investigate the cases where $\sigma = 0$ and $\sigma > 0$, τ_i constant and τ_i as defined (2).

6 In particular, in this instance we assume $\tau_i = [\alpha + (T-1)\beta]/T$. This assumption is consistent with the following interpretation of α and β: Let I be the number of informed note demanders, and U the number of uninformed note demanders per bank. Then if $\pi_i < \pi_j \forall j \neq i$, bank i gets $\alpha = I + U$ customers, while the other banks get only their share of the uninformed, namely $\beta = U$. When T banks tie for the lowest debasement rate, they share equally the informed note demanders, and $I/T + U$ customers are serviced by the banks that tie for the lowest debasement rate. Given this interpretation of α and β, manipulation reveals that $I/T + U = [\alpha + (T-1)\beta]/T$. Thus α and β reflect the positive scale effects of using the same currency.

7 We are grateful to a referee for raising this issue, and to R. van der Ploeg for discussion on this point.

The supply of bank notes derives from the presumed seigniorage maximization by each individual bank.[8] For simplicity we detach the provision of money business from the other banking activities provided by the banks, *i.e.* we assume that the profit function is separable in the two services. Furthermore, we assume that the cost of maintaining the stock of notes exhibits decreasing average costs due to increasing returns to scale over the relevant range.[9] Several scale factors may be identified: (i) some fixėd costs are incurred in maintaining the stock of notes; (ii) with sufficient cross sectional independence in the agents' stochastic demand for redemption, the specie reserve ratio can decrease with the size of the bank due to Arrow and Lind's diversification argument, see King (1983, p. 133); and (iii), as argued by Friedman (1959, p. 7) the value of the notes must be above marginal cost to 'prevent' the return to a commodity standard. For simplicity, these costs K_i, not necessarily equal for each bank i, are assumed to be constant.

Following Friedman (1953), Bailey (1956) and Toma (1985), and assuming perfect foresight, the revenue from the provision of money equals the inflation or debasement rate π_i times the real base demanded b_i, *i.e.* the 'inflation tax.' Hence, the seigniorage function of the i-th 'mint' is given by

$$S_i = \pi_i b_i - K_i \tag{4}$$

In combination with equation (3), or (3′) the revenues are positive for all inflation rates, and negative in the case of deflation. For K_i sufficiently small and given some π^*, seigniorage is positive over some range as well. The problem of the i-th mint is to maximize S_i by choosing an appropriate π_i.

The purpose of the paper is to investigate several solutions to the maximization problem of the banks. The standard case of a single central bank is represented by equations (3) and (4) with τ_i fixed. Maximizing S_i with respect to π_i yields Bailey's (1956) seminal result $\pi_i = 1/\gamma$. *Inter alia*, note that as π_i increases beyond $1/\gamma$, S_i decreases; this was dubbed the Laffer curve property by Blanchard and Fischer (1989, chapter 10). Turning to multiple note suppliers, it is evident that a cartel of free banks would also implement the monopoly solution $\pi_i = 1/\gamma$. Consider, instead, what happens if there are several competing free banks. Two cases will be investigated: (i) τ_i fixed, and (ii) τ_i as specified in (2). As it turns out, these assumptions about τ_i generate solutions which are qualitatively different from each other. In contrast, whether the other substitution parameter σ is zero or positive only makes a quantitative difference.

8 The actual note supply process during the free banking era was as follows. A free bank deposited an amount of bonds with the state auditor in return for which bank notes were printed and could then be issued. Observe that this process could be repeated by buying new bonds in the market place in return for those notes. This possibility generated the different circulation rates, see *e.g.* Rolnick and Weber (1987, pp. 4–6) and Rockoff (1989, pp. 39–40).

9 The issuance costs like the purchase of printing plates are essentially sunk costs, but the maintenance costs are recurrent.

Start with the assumption that τ_i is fixed, *i.e.* each free bank is assured a fixed segment of the market. Just how many notes each segment wants to hold is determined by the own debasement rate π_i and the own semi-elasticity γ, the opportunity costs π^* and its semi-substitution elasticity $\sigma \geq 0$, *cf.* equation (3) or (3′). Adopting the Nash-Cournot solution, *i.e.* taking the other debasement rates as given, yields again $\pi_i = 1/\gamma$. The similarity between the monopoly and oligopoly solutions derives from the exponential format of the demand equations (3) and (3′).[10]

The solutions derived up to this point rely on the assumption that the demand side is equally uninformed, and hence substitution between notes occurs only indirectly. Consider what might happen if some segment is better informed than the others such that τ_i follows (2) instead of being fixed. Is inflating at $1/\gamma$ still a viable strategy?

Let there be n free banks participating in the money supply process.[11] The informed agents favor the 'good' money over the 'bad' money, *i.e.* prefer low debasement rates, and the low inflation notes experience a discrete demand advantage. Hence, it pays for a particular bank to 'undercut' the other banks if the other banks are debasing at rate $1/\gamma$. By debasing at a slightly lower rate the bank loses some revenue on each note issued, but this loss is more than compensated for by the discrete jump in demand through the informed (higher α). In fact, this type of 'deflationary war' destroys pure strategies for any $\pi_i \in [s, 1/\gamma]$, where $s < 1/\gamma$ is the lower bound debasement rate such that

$$S_i(\pi_i = s \mid \tau_i = \alpha) = S_i(\pi_i = 1/\gamma \mid \tau_i = \beta)\,.$$

If, however, one or more competitors choose $\pi^* = s$, a bank is better off by inflating at $\pi_i = 1/\gamma$. The reason is that, if multiple banks charge the lowest debasement rate, they must share the informed note demanders. With ties at s, the gain from the switch to $1/\gamma$ more than outweighs the loss in demand, *c.f.* equations (2) and (3) or (3′). It follows that no $\pi_i \in [s, 1/\gamma]$ can be a pure strategy Nash equilibrium. The intuition behind this result is as follows. Throughout the interval $[s, 1/\gamma]$, competitive forces exert deflationary pressure on π_i. But at the lower bound s, the incentive to exploit one's monopolistic power over the uninformed becomes dominant. This renders any pure strategy non-viable. What is needed is a strategy such that the two forces - competitive and monopolistic - exactly offset one another for all $\pi_i \in [s, 1/\gamma]$. Such a strategy is the mixed strategy, where the π_i's

10 We are grateful to a referee for inducing us to treat this oligopoly solution, and to Rick van der Ploeg for extensive discussion on this point.

11 While entry was free in principle, the free banking act of Ohio, which is considered in the empirical section, resulted in limited entry only. In effect, the number of free banks can be taken as fixed after the first year of the enactment of the law, see Huntington (1964, p. 210). With free entry n is endogenous and seigniorage is driven to zero, but the qualitative properties of the Nash equilibrium remain the same.

are chosen at random from a particular distribution function. By randomizing the debasement rates, a bank precludes its rivals from being able to systematically undercut its own debasement rate.

The symmetric mixed strategy Nash equilibrium for the above game is derived as follows. Define the profits for the i-th bank, which depend on whether it experiences the scale benefits, as:

$$S_i = \begin{cases} W_i(\pi) & \text{if } \tau_i = \alpha, \\ L_i(\pi) & \text{if } \tau_i = \beta. \end{cases} \tag{5}$$

Recall that $\tau_i = \alpha$ if bank i sets the lowest debasement rate, while $\tau_i = \beta$ if some bank other than i sets the lowest debasement rate, and $\alpha > \beta$. Suppose all other j, $j \neq i$, and $j = 1, \ldots, n-1$ say, banks employ a mixed strategy and set π_j according to a distribution function $F_j(\pi_j) = F(\pi)$, for all j. Hence, with probability: $A(\pi) = (1 - F(\pi))^{n-1}$ the i-th firm debases at the lowest rate and captures the scale benefits α. With probability: $1 - A(\pi)$ it does not inflate at the lowest rate and experiences a scale effect on the demand side of $\beta < \alpha$.[12] Assuming that the i-th firm employs a mixed strategy with distribution function $F_i(\pi)$ in the support $[s, m]$, its expected seigniorage is

$$\mathrm{E}S_i = \int_s^m \{A(\pi)W_i(\pi) + (1 - A(\pi))L_i(\pi)\}\,\mathrm{d}F_i(\pi). \tag{6}$$

The objective of the i-th bank is to maximize $\mathrm{E}S_i$ by choosing $\mathrm{d}F_i(\pi)$ appropriately. We will show that the distribution function (mixed strategy)

$$F_i(\pi) = F(\pi) = 1 - A(\pi)^{1/(n-1)} \quad \text{with support } [s, m],$$

such that

$$A(\pi)W_i(\pi) + (1 - A(\pi))L_i(\pi) = C_i, \tag{7}$$

where C_i is a constant such that $C_i = L_i(m)$, $m = 1/\gamma$, and $W_i(s) = C_i$ implicitly defines s, comprises a Nash equilibrium to the above game.[13] To see intuitively why this comprises a Nash equilibrium, we note that A and $1 - A$ just balance W and L for each π in the support.

12 Note that with an absolutely continuous distribution $F(\pi)$ the case $\pi_i = \pi_j$ of ties described in footnote 6 has probability zero. Therefore, this happenstance is not defined in equation (5), as it will play no role in equation (6).

13 A similar equilibrium may be found in Varian (1980) and Baye *et al.* (1992) who describe the complete set of equilibria for a similar game. Interestingly, however, note that here the upper bound m is derived endogenously rather than being imposed exogenously.

In order to verify formally that this indeed constitutes the symmetric Nash equilibrium, we must show that if all banks use this strategy, no individual bank has an incentive to deviate from the strategy. Moreover, as the equilibrium involves randomized strategies, we must verify if $F(\pi)$ defined above is a well-defined distribution function. We begin by verifying the latter, and then demonstrate that no firm has an incentive to use an alternative strategy.

In order to show that $F(\pi)$ is well defined, derive an explicit expression for $A(\pi)$ from equation (7) while using (5), (4) and (3):

$$A(\pi) = \frac{\beta}{\alpha - \beta}\left[\frac{m}{\pi} e^{\gamma(\pi - m)} - 1\right]. \tag{8}$$

Calculations for the case $\sigma > 0$, *i.e.* using (3′) instead of (3), are presented in the Appendix as the qualitative properties of the solution are the same under either specification. Note that $A(\pi)$ is continuous in π and that knowledge of $A(\pi)$ is equivalent to knowledge of $F(\pi)$, since $F(\pi) = 1 - A(\pi)^{1/(n-1)}$. For $F(\pi)$ to be a well-defined distribution function, $A(\pi)$ must exhibit the following properties: (i) $A(s) = 1$, and $A(m) = 0$, (ii) $A(\pi)$ is a monotone and nonincreasing function, and (iii) $A(\pi)$ is right continuous. Evidently, $A(m) = 0$ from equation (8). As $A(\pi) = [L_i(m) - L_i(\pi)]/[W_i(\pi) - L_i(\pi)]$ from (7), the implicit definition of s through the equation $W(s) = L(m)$ implies that $A(s) = 1$. This manipulation also reveals why $F_j(\pi) = F(\pi)$ as the K_j cancel in the numerator and denominator of $A(\pi)$. To show that s is well defined, set $A(s) = 1$ in equation (8). Taking logarithms, this equation can be rewritten as

$$1 - \gamma s = log(\beta/\alpha\gamma) - log s\,. \tag{9}$$

It is clear that this equation has a unique and nondegenerate solution $0 < s < 1/\gamma$, due to the fact that $\beta/\alpha < 1$. Hence property (i) is satisfied. Next, differentiate $A(\pi)$:

$$\frac{dA}{d\pi} = \frac{\beta}{\alpha - \beta}\left[\gamma - \frac{1}{\pi}\right]\frac{m}{\pi} e^{\gamma(\pi - m)}, \tag{10}$$

which is negative for $\pi < 1/\gamma$. Therefore, $A(\pi)$ is monotonically decreasing on its support, and property (ii) is satisfied. Clearly, property (iii) is satisfied as well, as $A(\pi)$ is continuous in π. It follows that $F(\pi)$ is a well defined distribution function.

Next, we verify the mutual best response property. Note that for any debasement rate π_i such that $s \leq \pi_i \leq m$, the i-th firm has no incentive to choose a strategy different from the strategies $F(\pi)$ used by the other firms, as the payoffs (7) are constant. Thus the integrand in (6) is constant and hence varying $F_i(\pi)$ does not improve expected seigniorage on the interval $[s, m]$. Can the i-th bank gain by allocating some probability mass outside the interval $[s, m]$? We already noted that S_i exhibits the Laffer curve property and is decreasing for any $\pi_i > 1/\gamma$.

Therefore, no bank will ever inflate at a rate above $1/\gamma$, as it can always make the same seigniorage $L_i(\pi)$ for an appropriately chosen $\pi < 1/\gamma$, and still have a chance to make $W_i(\pi)$. Alternatively, the chances of earning $W_i(\pi)$ are zero for $\pi > 1/\gamma$, and hence the firm should not inflate at a rate above the monopoly rate. Similarly, it does not pay to inflate at a rate below s, as $W_i(\pi) < W_i(s)$ for $\pi < s$. In summary, it does not pay to deviate from $A(\pi)$ in (8). To conclude, it follows that $F(\pi)$ is indeed the symmetric Nash equilibrium mixed strategy. The next section investigates the properties of this mixed strategy equilibrium, the other pure strategy equilibria, and attempts an empirical evaluation.

3 IMPLICATIONS AND EMPIRICAL TESTS FOR THE CASE OF OHIO

Prior to the free banking era in the United States, banks enjoyed local monopoly power. For example, Huntington (1964, pp. 195, 267) reports for the case of Ohio that the number of banks were explicitly limited by law for each county. Moreover, the amount of dollar notes each bank could issue was limited by a reserve ratio and the limitations on the capital stock, see Huntington (1964, pp. 196–197, 267–269). However, free banking laws, such as the one of Ohio in 1851, challenged the monopoly power of existing banks by allowing entry. By 1852, 18 percent of all banks in Ohio were free banks. A major factor behind entry was the excess demand for (local) currency, especially in the frontier regions, Huntington (1964, p. 208). This meant that an entrant could profitably secure some demand, by competing with the (faraway located) incumbent monopolist.

Rockoff (1974, 1989) and Rolnick and Weber (1983) note that, while the period of free banking exhibited more uncertainty than before, instabilities were not prevalent. Competitive pressures, as feared by Friedman and Klein, and monopolistic power together 'destabilize' pure strategy equilibria. We showed, however, that an appropriately chosen mixed strategy balances the two destabilizing forces. Hence, uncertainty in our model is an equilibrium phenomenon, rather than being the sign of destructive competition. The following proposition links this endogenous uncertainty with debasement.

Proposition 1

A free banking oligopoly with asymmetric information, such that some agents are aware of the debasement rates associated with the distinct dollar bills, while other agents are uninformed, implies stochastic debasement rates for strategic considerations. However, these debasement rates are below the debasement rate which is generated by a monopoly bank or the solution for the oligopoly when no agent is informed.

Proof

In the previous section it was argued that the monopoly bank chooses $\pi = 1/\gamma = m$. Moreover, the oligopoly of competing free banks, with or without inclusion of opportunity costs ($\sigma = 0$ or $\sigma > 0$), but without direct note substitution (τ_i independent of the π_i and the π_j's), also sets $\pi_i = 1/\gamma = m$. Because $F(m) = 1$, $F(m - \varepsilon) < 1$ for $\varepsilon > 0$, and $f(\pi)$ is continuous, the competitive banks under asymmetric information (with direct note substitution) almost surely inflate at a rate below m.

From this, it is straightforward to derive the following implication.

Corollary 1
The monopoly seigniorage and oligopoly seigniorage for the case without direct note substitution is above the expected seigniorage with direct note substitution. Moreover, direct note substitution introduces (additional) randomness *vis-à-vis* the other equilibria.

Proof
Since $\pi = 1/\gamma$ for a monopolist bank, the monopolist earns seigniorage $[\alpha + (n - 1)\beta]\gamma^{-1}\phi e^{-1+\sigma/\gamma} - K$. The oligopoly without direct note substitution makes $[\alpha + (n - 1)\beta]\gamma^{-1}\phi e^{-1+\sigma/\gamma} - nK$; *i.e.* has the same total return as the monopolist but higher costs. The free banking oligopoly solution with direct note substitution, though, generates lower returns as total expected seigniorage equals $n\beta\gamma^{-1}\phi e^{-1}a - nK$, where the factor $a < e^{\sigma/\gamma}$ is defined in the Appendix (note that $a = 1$ if $\sigma = 0$). Evidently, the randomness in the seigniorage of free banks derives from the randomness in π_i and τ_i.

The proposition and corollary unite the three themes of free banking. Given multiple suppliers of bank notes with declining average costs and asymmetric information on the demand side, equilibrium involves mixed strategies. As a consequence, debasement rates and seigniorage are random. But if no segment of the public is informed, then the pure strategy solution $\pi = 1/\gamma$ is implemented. The tradeoff being lower uncertainty against higher inflation. These opposite predictions can be confronted with data to sort out which theory is supported. At the informal level the asymmetric information model seems consistent with the general notion that the free banking era constituted a period of higher financial uncertainty. While Rockoff (1974), Rolnick and Weber (1983, 1984) mention the general notion of increased uncertainty, they also note that in most states 'the free banking law proved reasonably successful,' Rockoff (1974, p. 142). This, again, is in line with the asymmetric information model of free banking. The uncertainty is an equilibrium feature; if each bank indeed employs the above mixed strategies the banking industry should exhibit stability, in terms of the number of banks that operate. Stability is also present in the sense that free banking does not lead to runaway inflations, as feared by Friedman (1959, pp. 6, 7), because the average inflation rate under free banking is below the optimizing inflation rate of a monopolist.

The empirical predictions implied by the proposition and corollary offer interesting possibilities for conducting formal tests of hypotheses. Fortunately, some data are available for the free banking era. We begin our empirical investigation by confronting the corollary with data on seigniorage gains. Corollary 1 suggests that, regardless of the information structure, the monopoly rents from the provision of money should diminish under free banking, due to entry. Table 3 in Rockoff (1974) lends some support to the Corollary 1. The average seigniorage[14] as a fraction of net worth is consistently higher in Philadelphia, which did not allow for free banking, than in the free banking cities of New York and Boston.

While the prediction that seigniorage under free banking is lower than under monopoly supports the corollary, virtually any theory of the industry predicts that entry lowers profits. In this respect, the randomness of debasement and seigniorage is the more interesting feature of the proposition and corollary, as this separates the asymmetric information model with direct note substitution from the model without informed agents and only indirect substitution. We will focus on the case of Ohio for which reliable data on the money supply are available from Huntington (1964) to try and discriminate between the two models.

From 1846 to 1851 three types of banks existed in Ohio: Old Banks which were a remnant of past legislation, State Branch Banks which had insurance similar to the current FDIC, see *e.g.* Calomiris (1990) and Independent Banks which were bond secured. Prior to 1851, entry to each type of Bank was limited, see Huntington (1964, pp. 195, 267). Each bank provided their own dollar bills, and apart from the central banking aspect, they also conducted the usual commercial banking business. In 1851 a free banking law was passed, allowing for entry by free banks, see *e.g.* Ng (1988, p. 880), conditional upon certain solvency and liquidity requirements.[15] This system was in existence until 1863. Huntington (1964, p. 298) presents a figure which depicts the ratio of the amount of bank notes which were circulated by each type of bank to the amount of its capital. From the figure it appears that the ratio becomes much more volatile after the institution of free banking in 1851, thus corroborating the prediction of the model that free banking leads to increased uncertainty.

Given the institutional arrangements at that time we take it that the 1846–1851 period was more or less a period of limited competition in the bank note supply process, while the entry of free banks lead to enhanced competition during the 1852–1863 period. On the basis of proposition 1 and given the fact that some agents were well informed through the use of bank note reports, one expects the second period to have exhibited more uncertainty due to the mixed strategy

14 Seigniorage here refers to distributed dividends, which also include profits from the commercial banking activities. Dividends are of course only an imperfect measure of seigniorage.

15 Ohio law required that banks of the same type must receive the notes of each other at par, see Huntington (1964, pp. 209, 269, 272). This perfect substitutes condition implies that bank notes issued by different banks of the same type can be considered equivalent. For this reason we will focus on the aggregate note supply by each type of bank.

solution. In order to conduct a statistical test to discriminate between the pure and mixed strategy solutions, Table 1 gives the means and variances for the circulation to capital ratio, for each type of bank over both periods. The variances seem to increase after the deregulation, while the mean rates drop. Formal tests of hypothesis are reported in Table 2. Let index p refer to the era prior to free banking, and let the index f denote the free banking era. Define the sample variances S_i, $i = p, f$, as the sum of squared deviations divided by $n - 1$, where n is the number of observations. Let $R = S_p/S_f$ be the likelihood ratio. In all cases H_0: $\sigma_p^2 \leq \sigma_f^2$ cannot be rejected at the 5% level as R is well below the critical values. However, the alternative hypothesis H_1: $\sigma_p^2 \geq \sigma_f^2$ is rejected for all types except for the State Branch Banks. Presumably the variance of the circulation to capital ratio in case of State Banks was not affected by the introduction of free banking, but the others were affected.

TABLE 1 – SAMPLE STATISTICS OF THE CIRCULATION TO CAPITAL RATIO*

Type of Bank and Period		Number of Observations	Sample Mean	Sample Variance
Independent	Pre 1852	19	1.57	0.0073
	Post 1852	26	1.24	0.1379
Branch	Pre 1852	19	1.72	0.0124
	Post 1852	26	1.72	0.0171
Old	Pre 1852	19	0.78	0.0340
	Post 1852	10	0.65	0.2918
Free	Post 1852	26	1.01	0.0948
Total	Pre 1852	57	1.36	0.0173
	Post 1852	88	1.25	0.1020

* The data are quarterly observations on aggregate measures of bank notes in circulation to capital that cover the two periods 1846–1851 and 1852–1863, except for those quarters for which no data were available. The data source is Huntington (1964, pp. 284–286). Data are available from the third author upon request.

While the circulation to capital ratio provides some evidence in favor of an increased uncertainty, this variable is not directly related to the variables of the proposition and the corollary. The data sources do provide some profit measures, but these profits include net earnings from the commercial banking activities, thereby contaminating this indicator of seigniorage. As for the note-specific debasement rates π_i, no reliable data seem to be available. Therefore, we opted for a somewhat roundabout procedure whereby the data of Huntington on circulation can be used. From equation (3) we have

$$B_i = P_i \tau_i \phi e^{-\gamma \pi_i}. \tag{11}$$

Taking logarithms and transforming the result into first differences yields approximately

$$\Delta B_i/B_i = \pi_i - \gamma\Delta\pi_i + \Delta\tau_i/\tau_i + \Delta\phi/\phi\,. \tag{12}$$

Under the hypothesis that our model of free banking is correct, we find that

$$\begin{aligned} Var(\Delta B_i/B_i) &= Var(\pi_i - \gamma\Delta\pi_i + \Delta\tau_i/\tau_i) + Var(\Delta\phi/\phi) \\ &= Var(\varepsilon_1) + Var(\varepsilon_2)\,, \end{aligned} \tag{13}$$

say.[16] Recall that $\phi = \phi(y, r)$, and hence $Var(\varepsilon_2)$ represents uncertainty due to sources other than strategic reasons, like fluctuations in demand. Moreover, $Var(\varepsilon_2)$ is also assumed to cover the uncertainties which stem from the commercial banking activities of the banks. On the other hand, $Var(\varepsilon_1)$ is entirely due to the mixed strategy nature of the free banking equilibrium whereby banks randomize over π_i. The $Cov(\varepsilon_1, \varepsilon_2)$ is zero as the two sources of risk are unrelated, *i.e.* $A(\pi)$ in (8) does not depend on ϕ. Also note that $Var(\varepsilon_1) = 0$ for any of the pure strategy equilibria, as $\Delta\pi_i = 0$ and $\Delta\tau_i = 0$ in such cases.

TABLE 2 – TESTS FOR THE EQUALITY OF VOLATILITY IN THE CIRCULATION TO CAPITAL RATIO

Type of Bank	R Ratio	Critical Values (5%)	
		H_0: $\sigma_p^2 \leq \sigma_f^2$	H_1: $\sigma_p^2 > \sigma_f^2$
Independent	0.053	$F(18,25) = 2.03$	$F(18,25) = 0.46$
Branch	0.724	$F(18,25) = 2.03$	$F(18,25) = 0.46$
Old	0.117	$F(18,9) = 2.95$	$F(18,9) = 0.41$
Total	0.170	$F(56,87) = 1.50$	$F(56,87) = 0.66$

The idea is now to exploit equation (13) in order to discriminate between the competing models. Maintain the hypothesis that the pre free-banking period was essentially a period with local money supply monopolies such that $Var(\varepsilon_1) = 0$. Then, free banking with direct note substitution must have increased $Var(\Delta B_i/B_i)$ by $Var(\varepsilon_1)$, assuming that the other sources of uncertainty did not change, *i.e.* ε_2 is assumed to be covariance stationary. Table 3 provides the necessary information on the means and variances of $\Delta B_i/B_i$. In line with the asymmetric information model with direct note substitution, the means after deregulation are below the means of the pre-deregulation period. Taking expectations in (12) yields

$$E(\Delta B_i/B_i) = E(\pi_i) + E(\Delta\phi/\phi)\,. \tag{14}$$

16 If $\sigma > 0$, then equation (3') implies that ε_1 comprises an additional term: $\sigma\Delta\pi^*/\pi^*$.

The proposition implies for the model with direct note substitution that

$$E_p(\pi_i) = 1/\gamma > E_f(\pi_i),$$

and hence one expects $E(\Delta B_i/B_i)$ to drop. Moreover, the volatility of $\Delta B_i/B_i$ should go up on the basis of equation (13). In contrast, the model without direct note substitution predicts no changes in the mean and the variance. A glance at Table 3 shows that the evidence supports the former rather than the latter model, except for the State Branch Banks.

TABLE 3 – SAMPLE STATISTICS OF $\Delta B_i/B_i$*

Type of Bank and Period		Number of Observations	Sample Mean	Sample Variance
Independent	Pre 1852	18	0.05	0.0078
	Post 1852	25	−0.04	0.0925
Branch	Pre 1852	18	0.10	0.0173
	Post 1852	25	−0.01	0.0096
Old	Pre 1852	18	−0.03	0.0423
	Post 1852	8	−0.77	4.0023
Free	Post 1852	25	0.00	0.0532

* The ratio $\Delta B_i/B_i$ is computed as $\Delta log B_i$, and where B_i are the notes in circulation as reported in Huntington (1964, pp. 284–287).

In Table 4 we formally test whether the increases in the variances of $\Delta B_i/B_i$ were significant. At the 5% significance level H_0 is accepted in all three cases, and H_1 is rejected for the Independent and Old Banks. However, for the State Branch Banks equality of σ_p and σ_f cannot be rejected. The above evidence suggests that the State Branch Banks were not affected by the free banking laws *vis-à-vis* the other banks. This, however, is not the entire story. Given the equality of variances for the State Branch Banks, we may test for the equality of the means as well by using a *t*-test. The appropriate hypotheses are $H_0: \mu_p = \mu_f$ and $H_1: \mu_p > \mu_f$. Computing the test statistic gives a value of 3.18, which is well above the critical value of 1.64 (with 41 degrees of freedom and at the 5% significance level). Hence,

TABLE 4 – TESTS FOR THE EQUALITY OF VOLATILITY IN THE $\Delta B/B$

Type of Bank	*R* Ratio	Critical Values (5%)	
		$H_0: \sigma_p^2 \leq \sigma_f^2$	$H_1: \sigma_p^2 > \sigma_f^2$
Independent	0.084	$F(17,24) = 2.07$	$F(17,24) = 0.46$
Branch	1.802	$F(17,24) = 2.07$	$F(17,24) = 0.46$
Old	0.011	$F(17,7) = 3.47$	$F(17,7) = 0.38$

H_0 is strongly rejected in favor of H_1, indicating that the State Banks debased at a significantly lower rate after free banking was instituted. In summary the scant material provides weak support for the asymmetric information and direct note substitution model *vis-à-vis* the model with only indirect note substitution.

4 CONCLUSIONS

The aim of this paper was twofold: first, to provide a coherent framework to analyze the issue of free banking, and second, to account for some of the stylized facts of the free banking era in the U.S. We studied the asymmetric information direct note substitution model as a vehicle of analysis, as it is able to account for the noted increased financial uncertainty for the period. Importantly, this uncertainty is more an indication of stability than of instability; the uncertainty arises for strategic reasons.

Given asymmetric information on the demand side and decreasing average costs on the supply side, only mixed strategy equilibria are viable Nash equilibria if banks compete in 'price,' *i.e.* the Bertrand feature of the model. The other side of the coin is that inflation rates become intrinsically random. But at the same time competition does lower the average inflation and seigniorage rates. The framework may also be useful for analyzing the British proposals for currency competition within the EMS. This could be a potentially interesting project for future research.

When we tested for the implications of the model by means of free banking data for Ohio, both a decline in the mean and an increased volatility were verified for the least regulated type of banks. However, for the State Branch Banks, to which entry was limited and which operated under a common safety fund, only the mean debasement rate seemed to go down significantly. The volatility of the debasement rate of these banks was not affected in an upward direction. This suggests that the State Banks did not participate in the Bertrand competition because they were better secured than the other banks, thus offering a differentiated product. But, nevertheless, the State Banks felt the pressure of enhanced competition in terms of a smaller market share.

APPENDIX

In this appendix we provide a solution to the problem with variable τ_i, *i.e.* using (2) and (3′), *i.e.* if $\sigma > 0$. For expository reasons we discuss the case of $n = 2$ free banks and state the solution directly in terms of the equilibrium distribution functions $F_1(\pi)$ and $F_2(\pi)$. Expected seigniorage for bank 1 equals

$$\mathrm{E}S_1 = \int_s^m \int_s^m \{[1 - F_2(\pi_1)]\, \pi_1 \alpha \phi e^{-\gamma\pi_1 + \sigma\pi_2}$$

$$+ F_2(\pi_1)\, \pi_1 \beta \phi e^{-\gamma\pi_1 + \sigma\pi_2} - K\}\, \mathrm{d}F_1(\pi_1)\, \mathrm{d}F_2(\pi_2)\,.$$

Conjecture the following solution:

$$F_1(\pi) = F_2(\pi) = \frac{\alpha}{\alpha - \beta} - \frac{c + K}{a\phi(\alpha - \beta)} \frac{e^{\gamma\pi}}{\pi},$$

where

$$a = \int_s^m e^{\sigma\pi}\, \mathrm{d}F(\pi),$$

$$c = \frac{\alpha\beta}{\gamma} \phi e^{-1} - K,$$

and where s is again implicitly defined through equation (9). Note that this is the same distribution function as is discussed in the main text. The only difference is the level of expected seigniorage, $ES_i = c$, which differs by a factor a. To verify that this indeed constitutes a Nash solution, one first integrates out over π_2, which yields the factor a, then checks that the integrand of ES_1 is constant over the support and that it does not pay to move mass outside the support.

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Summary

AN OLIGOPOLY MODEL OF FREE BANKING: THEORY AND TESTS

The paper demonstrates that in an environment of free banking where some agents have imperfect information regarding the circulation and debasement rates of alternative money suppliers, the equilibrium supply of money involves mixed strategies. It follows that the circulation and debasement rates are intrinsically stochastic, but that their averages are below the rates set by a monopoly bank. Empirical tests reveal that these predictions are consistent with the free banking era of the United States. The paper is also relevant for the discussion about the future monetary union in the EC.