TRAVEL TIME ESTIMATION AND ORDER BATCHING IN A 2-BLOCK WAREHOUSE

Tho Le-Duc and René de Koster

ERIM REPORT SERIES *RESEARCH IN MANAGEMENT*	
ERIM Report Series reference number	ERS-2004-098-LIS
Publication	November 2004
Number of pages	22
Email address corresponding author	tleduc@fbk.eur.nl
Address	Erasmus Research Institute of Management (ERIM) Rotterdam School of Management / Rotterdam School of Economics Erasmus Universiteit Rotterdam P.O. Box 1738 3000 DR Rotterdam, The Netherlands Phone: +31 10 408 1182 Fax: +31 10 408 9640 Email: info@erim.eur.nl Internet: www.erim.eur.nl

Bibliographic data and classifications of all the ERIM reports are also available on the ERIM website:
www.erim.eur.nl

ERASMUS RESEARCH INSTITUTE OF MANAGEMENT

REPORT SERIES
RESEARCH IN MANAGEMENT

BIBLIOGRAPHIC DATA AND CLASSIFICATIONS		
Abstract	The order batching problem (OBP) is the problem of determining the number of orders to be picked together in one picking tour. Although various objectives may arise in practice, minimizing the average throughput time of a random order is a common concern. In this paper, we consider the OBP for a 2-block rectangular warehouse with the assumptions that orders arrive according to a Poisson process and the method used for routing the order-pickers is the well-known S-shape heuristic. We first elaborate on the first and second moment of the order-picker's travel time. Then we use these moments to estimate the average throughput time of a random order. This enables us to estimate the optimal picking batch size. Results from simulation show that the method provides a high accuracy level. Furthermore, the method is rather simple and can be easily applied in practice.	
Library of Congress Classification (LCC) LCC Webpage	Mission: HF 5001-6182	
	Programme: TX301-329	
	Paper:	HF 5484 Warehousing and Storage
Journal of Economic Literature (JEL) JEL Webpage	Mission: M	
	Programme : M 11, R 4	
	Paper:	C 61 Optimization Techniques
Gemeenschappelijke Onderwerpsontsluiting (GOO)		
Classification GOO	Mission: 85.00	
	Programme: 85.34	
	Paper:	85.34 Logistiek
Keywords GOO	Mission: Bedrijfskunde / Bedrijfseconomie	
	Programme: logistiek, management	
	Paper: logistiek management, magazijnbeheer, bestellingen, optimalisering	
Free keywords	Keywords Logistics, Warehousing, Order Picking, Order Batching, Batch service queue	

Travel Time Estimation and Order Batching in a 2-block Warehouse

Tho Le-Duc and René de Koster

Rotterdam School of Management, Erasmus University Rotterdam

Burgemeester Oudlaan 50, 3062 PA Rotterdam, The Netherlands.

{tleduc, rkoster}@fbk.eur.nl

Abstract

The order batching problem (OBP) is the problem of determining the number of orders to be picked together in one picking tour. Although various objectives may arise in practice, minimizing the average throughput time of a random order is a common concern. In this paper, we consider the OBP for a 2-block rectangular warehouse with the assumptions that orders arrive according to a Poisson process and the method used for routing the order-pickers is the well-known S-shape heuristic. We first elaborate on the first and second moment of the order-picker's travel time. Then we use these moments to estimate the average throughput time of a random order. This enables us to estimate the optimal picking batch size. Results from simulation show that the method provides a high accuracy level. Furthermore, the method is rather simple and can be easily applied in practice.

Keywords *Logistics, Warehousing, Order Picking, Order Batching.*

1. Introduction

Order picking, the process of retrieving items from their storage locations to fill customer orders, is known as the most time-consuming and laborious component of the warehousing activities (Tompkins et al., 1996). Recent trends in distribution logistics and manufacturing have increased the importance of order picking. In distribution logistics few-but-large quantity orders are being replaced by many-but-small orders, which have to be processes in very tight time windows. In manufacturing, there is a move to smaller lot-sizes, point-of-use delivery, and cycle time reductions. These changes make rapid and flexible order picking becoming a crucial issue for many warehouse-related companies to sustain in today's competitive. This fact induces the order picking operations to become a strong candidate for productivity improvement studies.

There are four essential factors that greatly influence the performance and efficiency of the order picking operation. They are: (a) layout of the warehouse, (b) the routing and sorting policy, (c) the storage strategy and (d) the batching method (Petersen, 1997). In the literature, the routing and layout problem already received much attention. Recent research (see Roodbergen and De Koster 2001[a]) has shown that the optimal routing policy for a warehouse with multiple cross aisles can be found by using dynamic programming. The research has also shown that good layouts can be developed to minimize throughput times. The influences of storage strategies on the average travel distance are investigated in Caron et al. (1998) and Petersen and Schmenner (1999). However, the batching problem, especially for the case of multiple-block warehouses, has not been considered thoroughly.

The order batching problem (OBP) concerns the decision of how to group orders and then to assign them to order-pickers. Nowadays online retailing companies that focus on specialized product types (such as books, computers or CD's etc.) often receive orders with only one or few order lines (or stock keeping units - SKUs). If the order-picker starts a tour for every order, the capacity may even be insufficient to serve all orders. If the order-picker waits to have a sufficiently large number of orders, the average time in system of the orders may be longer than desired. Clearly, we can increase the efficiency of the order picking process in such environments by serving a group of orders instead of individual orders. The critical issue is, therefore, to determine how many orders the order-picker should serve in a tour to minimize the average throughput time of a random order.

In the literature, there are several articles in which the OBP is discussed. However, the nature of the OBP in these publications is not always the same. Many of them focus on the OBP in single-aisle automatic storage and retrieval systems (e.g. Elsayed and Lee, 1996, Elsayed et al., 1993, Hwang et al., 1988) while some others concentrate on batching methods in multiple-aisle manually-picked systems (e.g., Gibson and Sharp, 1992, De Koster et al., 1999 and Gademann et al., 2001). These publications focus on one of the following objective functions: (a) minimizing the average travel distance of orders and thereby throughput time; (b) minimizing the maximum lead-time of any of the batches; (c) minimizing the total earliness and tardiness penalties of order retrievals. In this research, we are interested in manual order picking environments, and minimizing the average throughput time of a random order This objective is well-known and most commonly used in the order picking’s literature (Gibson and Sharp, 1992, Rosenwein, 1996, Caron et al., 1998, Chew and Tang, 1999, Roodbergen and De Koster, 2001[a,b], and many others). We now briefly mention the most recent and/or closely related works on the subject.

Rosenwein (1996) proposes an order batching heuristic in a single-block warehouse. The main idea of this heuristic is assigning orders, one by one, to a picking tour until either the tour capacity constraint is encountered or the list of unassigned orders is empty. The first order in a batch, the *seed* order, is chosen randomly among unassigned orders. Further orders are added, one by one, to the batch according to one of two additional order selection rules. The first rule selects the order based on the order's center of gravity, while the other chooses the order that minimizes the number of additional aisle to be visited.

De Koster et al. (1999) perform a comparative study for order batching heuristics in multiple-aisle picker-to-part warehouses. They consider two groups of heuristics: *Seed* algorithms and the somewhat more complex (and CPU time consuming) called *Time Savings* algorithms. The performance of the algorithms is evaluated using two different routing strategies: the S-shape and the largest-gap strategy. The heuristics are compared for travel time, number of batches formed and also for the applicability in practice. They conclude that: (a) even simple order batching methods lead to significant improvement compared to the first come first serve batching rule; (b) the Seed algorithms are best in conjunction with the S-shape routing method and a large capacity of the pick device, while the Time Savings algorithms perform best in conjunction with the largest gap routing method and a small pick-device capacity.

Gademann et al. (2001) address the problem of batching orders in a warehouse with the minimization of the maximum lead-time of any of the batches as objective. This objective is common in parallel (or zoned) wave order-picking operations. They propose a branch-and-bound algorithm to solve this problem exactly. A lower bound for the branch-and-bound is obtained by using a 2-opt heuristic. As the OBP is NP-hard, finding the optimal batching solution for large-scale problems is time consuming. However, they claim that the 2-opt heuristic appears to be very powerful; it provides very tight lower bounds. Therefore, they believe that a truncated branch-and-bound algorithm would suffice in practice.

All above order batching algorithms treat the demand as a deterministic variable; the profile of orders (number of orders, order lines in each order, quantity per each order line etc.) are assumed to be known at the beginning of each planning period. Considering the stochastic nature of the order arrivals and service time, Chew and Tang (1999) model the OBP for a single-block warehouse as a queueing model and apply a series of approximations to calculate the lower bound, upper bound and an approximation value for the average throughput time. The limitation of this research is that they consider the average throughput time of the first order in a batch as estimation for the average throughput time of individual orders in the batch. Our research is mainly based on the approach given in this article. However, it distinguishes from Chew and Tang's research in two aspects. First, we consider a different layout (2-block warehouse), which can be found commonly in practice. Second, we perform a direct analysis on the average throughput time of a random order.

As we have seen, the existing literature shows that the OBP in the case of multiple-block manually-picked warehouses has received little attention despite its wide application (i.e. there is no publication in which the OBP in 2-block warehouses is mentioned). To fill this gap, in this study we consider a 2-block layout as depicted in Fig. 1. As shown in Roodbergen and De Koster (2001[b]), a layout with a middle aisle (2-block) often results in a lower average travel time than the layout without a middle aisle (single-block). We first elaborate on the first and second moment of the order-picker's travel time. Then we use these moments to estimate the average throughput time of a random order. This enables us to estimate the optimal picking batch size (i.e. the number of orders to be served in one picking route). .

We initially use the following assumptions, some of which will be relaxed later.

(1) *Order pattern:* arrivals of orders follow a Poisson process and every order contains one order-line (quantity per order line can be greater than 1). We presume that the storage capacity of a storage location is sufficiently large: to pick up one order line the order-picker has to visit only one storage location.

(2) *Service:* we consider only one order-picker and the service is carried out per batch of exactly k orders. The order-picker's capacity is sufficiently large to handle multiple (k) orders per route.

(3) *Routing method*: the used routing method is the S-shape (or traversal) heuristic. Routing order-pickers by using the S-shape policy means that any aisle containing at least one pick is traversed entirely (except potentially the last one in each block). Aisles without pick are not entered. From the last picked aisle, the order-picker returns to the depot (see Fig. 1 for an example of the S-shape route). This method is one of the simplest routing methods, included in nearly every warehouse management software system, and widely used in practice (see Roodbergen and De Koster 2001[a]).

(4) *Storage policy*: we use a random storage strategy, which means that incoming products are randomly located to empty storage spaces.

(5) *Batching rule*: batching is carried out on a first come first server basis; we assume that the system is empty at the beginning.

As it is also the aim of the research to deal with real-life applications, some of these assumptions are relaxed later on (when we discuss the possibility of including compound-Poisson arrivals, multiple order-pickers and class-based storage assignment into the model).

[Insert Fig. 1 about here]

This paper is organized as follows. In the next section, we estimate the first and second moment of travel time to pick up n order lines. In Section 3, we present an approach to estimate the average throughput time of the real system from the appropriate queueing systems. We present some numerical results and discussions in Section 4. In Section 5, we mention some possible extensions of the model to cope with more complicated situations. Finally, in Section 6, we draw some conclusions and give outlooks on further research.

2. Travel time estimation

In order to estimate the throughput time, it is necessary to find the first and second moment of travel time. We use the following common notations:

d	length (in travel time units) of a pick aisle.
w_a	width (in travel time units) of the cross aisle.
w_c	center-to-center distance (in travel time units) between two adjacent pick aisles.
n	total number of order lines to be picked in a tour.
m	number of pick aisles, it is an even integer.
TR_{WA}	travel time caused by traversing the pick aisles.
TR_{CA}	travel time caused by traversing the cross aisle.
AT^z	Adjustment time; z can be $+$, $-$ or $\approx$ (upper, lower bound or average value).
$E[X]$	expected value of X.
p_i	probability that a random order line is picked from aisle i $(i=1..m)$; $p_i =$ $1/m$ $(i=1..m)$ for the random storage strategy.
τ_s	setup time per batch.
τ_p	picking time per order line.

The order travel route is sketched in Fig. 1. Starting from the depot, the order-picker (he) travels to the nearest pick aisle containing picks, either in the left or right block. Aisle by aisle, he travels to the farthest pick aisle in the same block in such a way that all visited aisles are completely traversed. After accomplishing all pick requests in the first block, he moves to the farthest requested pick aisle in the second block. In a same manner but in the downward direction, he picks while going from the farthest to the nearest aisle containing picks. From there, he goes back to the depot to complete the tour. It should be noticed that it does not matter which block is served first, as in both cases we encounter the same travel distance. Furthermore, it is obvious that picking block by block usually provides a shorter (or at least equal) travel distance than picking in two blocks simultaneously.

2.1. First moment of travel time

The average travel time consists of three components: 'within aisles' travel time (TR_{WA}), 'cross aisle' travel time (TR_{CA}) and 'adjustment time (AT). Without loss of generality, here we assume that the order-picker travels at a constant speed. We define: $E\left[TR^z\right] = E\left[TR\right] + AT^z$, where $E\left[TR\right] = E\left[TR_{WA}\right] + E\left[TR_{CA}\right]$ and z can be – (lower bound), + (upper bound) or $\approx$ (approximation).

The adjustment time AT consists of two components: AT_1 and AT_2. AT_1 is the travel time from the central line of the cross aisle to the beginning of the first pick aisle and the travel time from the end of the last pick aisle to the central line of the cross aisle. AT_2 is the correction of travel time if the last visited aisle in each block is odd (pick aisles are numbered from 1 to m clockwise as shown in Fig. 1). In the following, we will determine the expected value TR_{WA}, TR_{CA} and AT given that the pick list contains n order lines (in our case, each order consists of only one order line thus $n = k$).

With the S-shape routing method, the expected within-aisle travel time depends only on the length of pick aisle d and the expected number of aisles visited $E\left[J \mid n\right]$. Chew and Tang (1999) show that given n and the number of pick aisles m:

$$E\left[TR_{WA} \mid n\right] = dE\left[J \mid n\right] = d\sum_{j=1}^{m} jP\left\{J = j \mid n\right\} = d\left(m - \sum_{i=1}^{m}\left(1 - p_i\right)^n\right),$$

where the term in brackets is the expected number of visited aisles.

On the other hand $E\left[TR_{CA} \mid n\right]$ is the doubled travel time from the depot to the farthest visited aisle. It is determined by the travel time between two neighboring pick aisles, w_c, and the position of the farthest visited aisle L. If we consider two pick aisles opposite the cross aisle as one pick line (see Fig. 1) then we can make use of the formula for estimating in the $E\left[TR_{CA} \mid n\right]$ in single-block warehouses given in Chew and Tang (1999):

$$E\left[TR_{CA} \mid n\right] = 2w_c \sum_{l=1}^{m/2} lP\left\{L = l \mid n\right\} = 2w_c \left[m/2 - \sum_{l=1}^{m/2-1} \left(\sum_{r=1}^{l} p'_r \right)^n \right],$$

where $p'_r = p_{2r-1} + p_{2r}$ is the probability that the pick line r $(r = 1..m/2)$ is visited.

For the first adjustment term, we can see that if only a half of the warehouse (one block) is visited then $AT_1 = 2\left(w_a/2\right) = w_a$ (it is doubled because whenever the order-picker enters an aisle he has to leave the aisle). If both halves of the warehouse are traversed then $CR_1 = 2wa$. Hence, we can determine the conditional expected value of the first correction term:

$$E\left[AT_1 \mid n\right] = w_a\left(2*0.5^n\right) + 2w_a\left(1 - 2*0.5^n\right) = 2w_a\left(1 - 0.5^n\right).$$

The second adjustment term takes into account the fact that from the last pick position (in the last visited aisle) in each block, the order-picker has to return to the center line of the cross aisle. It is easy to verify that $0 \le AT_2 \le 2d$. The expected value of AT_2, $E\left[AT_2 \mid n\right]$, can be estimated by formula B1 (see Appendix B)

From all estimates above, we now can come up with the following expressions of travel time:

$$E\left[TR \mid n\right] = d\left(m - \sum_{i=1}^{m} \left(1 - p_i\right)^n \right) + 2w_c \left[m/2 - \sum_{l=1}^{m/2-1} \left(\sum_{r=1}^{l} p'_r \right)^n \right]$$

$$E\left[TR^- \mid n\right] = E\left[TR \mid n\right] + w_a,$$

$$E\left[TR^+ \mid n\right] = E\left[TR \mid n\right] + 2\left(d + wa\right),$$

$$E\left[TR^{\approx} \mid n\right] = E\left[TR \mid n\right] + 2w_a\left(1 - 0.5^n\right) + E\left[AT_2 \mid n\right].$$

We used Visual Basic for Application (VBA) on Microsoft Excel to simulate the system. In the simulations, we considered 3 layouts: 6, 10 and 16 aisles (see Table 1 for other layouts parameters). Batch size varied from 10 to 60 orders (i.e. number of locations that an order-picker has to visit in one tour is from 1 to 60). The average travel-time value of 10000 runs was taken as the simulation result, this number of runs is sufficient to obtain a 98%

confidence interval with a half-width of less than 1% of the sample mean. We found that, in the worst case, the difference between the approximated travel time and simulation outcome is less than 10 percent. For all layouts, the difference rapidly decreases when the batch size increases. When the batch size is greater than 40, the difference between approximation and simulation value is less than 2 percent.

When we know the first moment of travel time, it is rather straightforward to compute the first moment of service time. We call $E(S^z \mid n)$ the first moment of service time given the batch size n, where z can be the approximation, lower bound or upper bound notation. We assume that the setup time of a batch, τ_s, is independent from the batch size. The picking time per order line, τ_p, is identical for all order lines. It follows that:

$$E\left[S^z \mid n\right] = \tau_s + \tau_p n + E\left[TR^z \mid n\right].$$

2.2. *Second moment of travel time*

Without considering the correction time (CR), the second moment of travel time can be formulated as

$$E\left[TR^2 \mid n\right] = d^2 E\left[J^2 \mid n\right] + \left(2w_c\right)^2 E\left[L^2 \mid n\right] + 2\left(2w_c\right) d E\left[JL \mid n\right]. \quad (1)$$

Chew and Tang (1999) calculated $E\left[J^2 \mid n\right]$ and $E\left[L^2 \mid n\right]$ for the single-block layout. However, their result for $E\left[J^2 \mid n\right]$ still holds for the case of two blocks. For $E\left[L^2 \mid n\right]$, if we consider pick lines instead of pick aisles (see Fig. 1) then their formula can be easily adapted. Hence, we have:

$$E\left[J^2 \mid n\right] = m^2 - \sum_{i=1}^{m} \left(2m-1\right)\left(1-p_i\right)^n + 2\sum_{i=1}^{m-1} \sum_{r=i+1}^{m} \left(1-p_i-p_r\right)^n , \quad (2)$$

$$E\left[L^2 \mid n\right] = \left(m/2\right)^2 - \sum_{i=1}^{m/2-1} \left(2i+1\right)\left(\sum_{r=1}^{i} p'_r\right)^n , \quad (3)$$

where $p'_r = p_{2r-1} + p_{2r}$, $r = 1..m/2$.

$E\left[JL \mid n\right]$ is the term that describes the interaction between the number of aisles visited and the farthest pick line. It can be calculated by

$$E\left[JL \mid n\right] = \sum_{l=1}^{m/2} l \left(\sum_{j=1}^{2l} jP\left\{J = j, X_{2l} > 0, X_{2l+1} = \dots = X_m = 0 \mid n\right\} + \right.$$

$$+\sum_{j=1}^{2l-1} jP\left\{J=j, X_{2l-1}>0, X_{2l}=...=X_m=0 \mid n\right\}$$

$$=\sum_{l=1}^{m/2} l\left(\left(\sum_{r=1}^{2l} p_r\right)^n\left[2l-\sum_{i=1}^{2l}\left(1-p_i^*\right)^n\right]-\left(\sum_{r=1}^{2(l-1)} p_r\right)^n\left[2(l-1)-\sum_{i=1}^{2(l-1)}\left(1-p_i^{**}\right)^n\right]\right), \quad (4)$$

where $X_i=1$ if pick aisle i is visited and $X_i=0$ otherwise. $p_i^*=p_i\Big/\sum_{j=1}^{2l} p_j$ and $p_i^{**}=p_i\Big/\sum_{j=1}^{2(l-1)} p_j$ (details of the proof can be found in the Appendix A). Subsequently, $E\left[TR^2 \mid n\right]$ can be computed by substituting (2)-(4) into (1). We can see that TR differs from TR^+ and TR^- only by constants, thus their variances are identical:

$$\sigma^2\left[TR \mid n\right]=\sigma^2\left[TR^+ \mid n\right]=\sigma^2\left[TR^- \mid n\right]=E\left[TR^2 \mid n\right]-\left(E\left[TR \mid n\right]\right)^2.$$

However, $TR^{\approx}$ does not differ from TR by a constant. To make things easier, we assume that $\sigma^2\left[TR^{\approx} \mid n\right]=\sigma^2\left[TR \mid n\right]$.

For a given number of order lines per batch n, the variance of service time $\sigma^2\left[S \mid n\right]$ is just the summation of the variance of travel time and the variance of picking time, since the setup time is constant and the picking time is independent of the travel time. And, since the arrival of orders follows a Poisson distribution, the variance of the picking time simply equals $\lambda\tau_p^2$. Hence,

$$\sigma^2\left[S^z \mid n\right]=\left(E\left[(TR)^2 \mid n\right]-\left(E\left[(TR) \mid n\right]\right)^2\right)+\lambda\tau_p^2,$$

where z can be the approximation, lower bound or upper bound notation.

3. Throughput time analysis for $M/G^k/1$ queueing model

Due to stochastic natures of both order arrivals and service time, a natural way to deal with the OBP, which has been discussed, is to model the order picking process as a queueing system. With the assumptions made earlier, our problem can be modeled as an $M/G^k/1$ queue. Where G^k denotes that the service is performed per batch of exactly k orders and the distribution function of the service time has a general form. M implies that order inter-arrival times are exponentially distributed random variables. In other words, the OBP in this case can be considered as the problem of determining the optimal service batch size for the $M/G^k/1$ queue such that the average throughput time of a random customer is minimized.

In the literature, there are only few publications in which this type of queue is thoroughly studied. Foster and Perera (1964) show that the probability generating function of the system size at random epochs $P(z)$ can be expressed by the following formula:

$$P(z)=\frac{(1-z^k)(1-\frac{\rho}{k})\prod_{j=1}^{k-1}\frac{(z-\delta_j)}{(1-\delta_j)}}{1-\frac{z^k}{K(z)}}, \qquad (5)$$

where $K(z)=\psi\{\lambda(1-z)\}$ is the Laplace-Stieltjes transform of the cumulative service time distribution function. λ is the arrival rate, $\rho=\frac{\lambda}{\mu}$ is the utilization rate. $\mu=1/E(S\,|\,k)$ is the service rate of a batch consisting of k orders. δ_j, with $j=1,..,(k-1)$, are $(k-1)$ roots inside the unit circle of the characteristic equation $z^k=K(z)$. It follows from Rouche's theorem that this equation has exactly $(k-1)$ roots inside the unit circle (detailed explanations can be found in Gross and Harris, 1998, p. 282). In normal integral form $K(z)=\int_{j=0}^{\infty}\frac{e^{-\lambda t}(\lambda t)^j}{j!}dH(t)$, where $H(z)$ is the cumulative distribution function of service time. Analogue versions of this formula can be seen on Chaudhry et al. (1987) or Chaudhry (1991). If we know the form of the service time then the steady-state probability $\{p_n\}$ can be theoretically obtained by successive differentiation of $P(z)$. Nevertheless this work is cumbersome when k becomes large.

Chaudhry (1991) is also interested in this queue and he provides a closed-form expression in term of the roots of certain characteristic equations for computing the average queueing time of orders. However, he only considers the queueing time of the last customer in the service batch, which, certainly, differs from the waiting time of a random customer. Another type of queues that is also considered in the same article is $M/G^{[a,b]}/1$. In this queue, services can be performed as soon the number of orders waiting in the queue reaches the lower threshold a (b is the capacity of the server, $a\le b$). Chaudhry et al. (1987) discuss a numerical computation approach to compute the steady-state probability of this system. However, from a practical point of view, this approach is rather complicated to use. In order to obtain steady-state probabilities, we first have to find the roots of the characteristic equations and than successively take the derivative of the steady-state probability. This requires tremendous computational efforts, especially when the batch size is large.

Apparently, it is too difficult, from a practical point of views, to compute exact results for the $M/G^k/1$ queue. Furthermore, for the sake of the OBP, it is not necessary to find an extremely accurate throughput time. Therefore, in this research we are interested in finding a good and easy-to-compute approximation for the average waiting time of a random order. We use the well-known 2-moment approximation formulation (see, for example, Tijms, 1994, p. 335):

$$W_{M/G^k/1} = \left(1 - c_S^2\right) W_{M/D^k/1} + c_S^2 W_{M/M^k/1},$$

where $c_S^2 = \sigma^2\left[S \mid k\right] / E^2\left[S \mid k\right]$ is the squared coefficient of variation of the service time; $W_{M/M^k/1}$ and $W_{M/D^k/1}$ denote the average throughput time of orders (or waiting time in the system of a random order) when the service time distribution is exponential and deterministic respectively. As recommended in Tijms (1994), this method performs very well in the case that c_S^2 is not very high and the traffic density ρ is not very low.

When the service time is exponential, we have (Gross and Harris, 1998, p.125):

$$W_{M/M^k/1} = \frac{1}{\lambda}\left[\frac{k-1}{2} + \frac{\lambda}{\mu k}\frac{(r_0^{1-k} - r_0^{2-k})}{\left(1-r_0\right)^2} + \left(\frac{\lambda(r_0^{-k} - r_0^{1-k}) - \mu r_0}{k\mu}\right)\sum_{k=1}^{k-1} k r_0^k\right],$$

where r_0 is the unique root of the characteristic equation $\mu r_0^{k+1} - \left(\lambda + \mu\right) r_0 + \lambda = 0$.

When the service time is deterministic it can be shown that $K(z) = e^{-\rho(1-z)}$. Substituting this into (5) we have:

$$P(z) = \frac{(1-z^k)(k-\rho)\prod_{j=1}^{k-1}\frac{(z-\delta_j)}{(1-\delta_j)}}{k\left(1 - \frac{z^k}{e^{-\rho(1-z)}}\right)}, \qquad (6)$$

where δ_j, $j = 1,..,\left(k-1\right)$, now become $\left(k-1\right)$ roots inside the unit circle of the equation $z^k = e^{-\rho(1-z)}$. In the literature, several solution methods have been proposed for finding roots of this equation. The common technique used is transforming the equation into $\left\lceil \left(k-1\right)/2 \right\rceil$ independent equations, each of which has only one root inside the unit circle. These roots and their conjugate roots form $(k-1)$ roots we need (literature on this topic can be found in

Chaudhry et al., 1987 and 1990). When $(k-1)$ roots of the equation are known, we can find $W_{M/D^k/1}$ by taking the limitation of $P(z)$ when z reaches 1: $W_{M/D^k/1} = \frac{1}{\lambda}\frac{d}{dz}P(z)\big|_{z=1}$. We note that, for $z=1$, $P(z)$ is indeterminate of the $0/0$ form. Therefore, we proceed as follows. Let $N(z)$ and $D(z)$ denote the numerator and denominator of the right-hand side of Equation (6) respectively. Then we use the following well-known result in queueing theory (see Madan, 2000):

$$W_{M/D^k/1} = \frac{1}{\lambda}\frac{d}{dz}P(z)\big|_{z=1} = \frac{1}{\lambda}P'(1) = \frac{1}{\lambda}\frac{N'(1)D''(1) - D'(1)N''(1)}{2\left(N'(1)\right)^2}.$$

As mentioned earlier, successive differentiations are cumbersome when the batch size is large; but in this case, we only need to take the first order derivation of the generating function. The derivative operator is available in many common mathematical software packages (such as Maple or Matlab). These make it possible to carry out a numerical analysis for the value of $W_{M/D^k/1}$, even for very high values of the batch size.

4. Numerical examples

In order to illustrate the procedure, we consider 3 warehouses with parameters given in Table 1. Fig. 2 shows the throughput times of the deterministic, exponential and general form service time model for different warehouses (the service time is estimated by the approximation method described in Section 2). As a consequence, the approximation is close to the exponential curve when the squared coefficient of variation is close to 1, and to the deterministic curve when the squared coefficient is close to zero. In the figure, it can be seen that the approximation curve is extremely close to the deterministic curve when the batch size is large. This is due to the fact that the squared coefficient of variation is almost zero for large batch sizes. It suggests us that the deterministic model is a good approximation for the general service time queue. This result is in line with the finding, for the case of single-aisle warehouses, mentioned in Le-Duc and De Koster (2002).

[Insert Table 1 about here]

It should be noted that, to satisfy the equilibrium condition, the batch size can only be defined on a semi-bounded interval $\left[k^-, \infty\right)$, where k^- is a minimum batch size value such that the traffic density, $\lambda k/\mu$, is less than 1.

[Insert Fig. 2 about here]

For the comparison purpose, we used the AutoMod simulation package to simulate the order picking system. In the experiment, the average throughput times were taken after a run length of 30 hours (warming up time was 4 hours, determined by using AutoStat – a tool accompanying AutoMod for batch running and statistical analyses). Fig. 3 shows the simulation results together with the expected lower bound, upper bound and approximation value of the throughput time for the layout with 6 aisles (we mention only one case as other cases - 10 and 16 aisles- bring in similar pictures). The lower bound, upper bound and approximation of throughput time are correspondingly determined by the lower bound, upper bound and approximation of service time. For example, in order to find the lower bound of the throughput time, we first estimate the lower bound of the first and second moment of service time. Then using these moments we calculate the throughput time of the deterministic and exponential service time queue. Finally, we use the 2-moment approximation formula to obtain the lower bound of the throughput time.

[Insert Fig. 3 about here]

We can draw the following conclusions from the numerical experiments. First, the shapes of the curves confirm the finding of Chew and Tang (1999) when they considered single-block warehouses. The average throughput time is a convex function of the batch size and a unique optimum batch size exists. When the batch size increases from the lower bound, the average throughput time decreases and it rapidly reaches its optimum. From that, it increases. The existence of the optimum can be explained as follows: batching many orders may reduce the travel time (reducing the distance of traversal without picking), but increases the batching time (waiting time of an order needed to complete a batch), and the waiting time of batches in the queue due to the large service time. Second, the bounds are tight, especially when the batch size is large. It means that the approximation provides sufficient accuracy in estimating the average throughput time of a random order. This result is in accordance with the finding of Chew and Tang (1999) for single-block warehouses. Third, the optimal batch size is relative small; close to its lower bound. It means that we need not to search the optimum batch size on a large interval. This is an importance remark, as it can help to reduce the searching time significantly. Perceiving this, we propose a greedy procedure for determining the optimal batch size as follows. We first estimate the lower bound of the batch size. From this value we each time increase the batch size by 1. The optimal batch size is the value that first makes the average throughput time (determined by two moment approximation approach) increase.

5. Some possible extensions of the model

We have considered the order picking process with single-line orders, a single order-picker and the random storage strategy. This can be considered as the basic model and it can be extended in several directions.

As the first extension, we can consider multiple order-pickers instead of a single one. Under this situation, the order picking process can be modeled as a batch processing and multi-server queue: $W_{M/G^k/c}$, where c is the number of servers (or order-pickers). It is too difficult, if not impossible, to find the exact results for this type of queues. However, in the literature, a simple method exists for finding the bounds of the average waiting time of a multi-server queue from its corresponding single server queue (see, for example, Gross and Harris, 1998, p. 340). According to their method, the lower bound of $W_{M/G^k/c}$ can be found by assuming $W_{M/G^k/c}$ is equivalent to $W_{M/G^k/1}$ where the service rate is c times faster. The upper bound can be obtained by assigning batches in cyclic order to the c servers with no jockeying allowed (first batch to sever 1, second batch to server 2, ..., $(c+1)st$ to server 1, ...). Then each server faces a single queue, in which the inter-arrival time is the c-fold convolution of the original inter-arrival distribution, with no change in the service time process. The waiting time of a random batch taken from one of these queues provides an upper bound for the multi-server queue. These bounds are very useful: we can use them to interpolate the expected value of the throughput time. One reasonable value of the throughput time could be the average value of the lower and upper bound.

The second extension could be that we consider the class-based (or *ABC*) storage strategy. As mentioned earlier, when the random storage strategy is used, $p_i = 1/m \; (i = 1..m)$, where p_i is the probability that aisle i is visited. When the *ABC* storage strategy is used, there are two possibilities, depending on whether partial-aisle assignment is allowed or not. A partial-aisle assignment means that we can store different product classes in the same aisle, while in the other cases product class is stored in one or more entire aisles. Our model already captures the latter case, because in the calculations we use the general expression of p_i (p_i can differ from $1/m$). It is also possible to consider the partial-assignment case. However, the expression for the second moment of the travel time may become very complicated (see Le-Duc and De Koster, 2002).

In many cases, orders may consist of more than one order line. Thus, another interesting extension could be that we consider compound-Poisson arrivals instead of Poisson arrivals. The order picking process can then be modeled as the compound-Poisson arrivals with batch service queue. For this type of queues, it is still possible to trace the expected waiting time if both moments of the service time are known. Unfortunately, again it is very tough to come up with a closed-form formulation for the second moment of service time. We suggest that we approximate this system by $M/G^X/c$, where $X = kE(a)$ with $E(a)$ is the expected number of order lines per order. This means that we can still apply $M/G^X/c$ queue to estimate the optimal number of order lines per batch and than based on this value and $E(a)$ to determine the 'optimal' number of orders to be included in a batch.

6. Conclusions

In this paper, we focus on finding a simple but efficient approach for determining the optimal picking batch size for order-pickers in a typical 2-block warehouse. In order to do so, we first extend the results given in Chew and Tang (1999) for single-block warehouses to estimate the first and second moment of the service time. Then, we use these moments to estimate the waiting time of a random order based on the corresponding batch service queueing model. The optimal picking batch size is then determined in a straightforward manner. Results from the simulation experiments show that our approach provides a good accuracy level. Furthermore, the method is very simple; it can be easily applied in practice. The study also supports that the average waiting time of a random order is a convex function of the batch size. As a result, there always exists a unique optimum picking batch size. This is in accordance with the finding of Chew and Tang (1999) and Le-Duc and De Koster (2002). As the optimum batch size is always close to its lower bound (obtained from the traffic density condition), we propose a simple greedy heuristic procedure, which can be used to search for the optimum in a negligible computational time.

The order picking system that we considered is a simple one; we can extend it in several directions. It is rather easy to include multiple order-pickers. However, in general it is rather difficult to capture compound-Poisson arrivals or other storage strategies and different layouts. These topics issue a challenge for us to investigate in the future.

Appendix A

We use the following definitions:

$L = l_+$: the farthest pick line is pick line l and pick aisle $2l$ is visited,

$L = l_-$: the farthest pick line is pick line l and pick aisle $2l$ is not visited.

$$X_i = \begin{cases} 1 & \text{if pick aisle i is visited} \\ 0 & \text{otherwise} \end{cases}$$

$$E(JL \mid n) = \sum_{l=1}^{m/2} l \left(\sum_{j=1}^{2l} jP\{J = j, L = l_+ \mid n\} + \sum_{j=1}^{2l-1} jP\{J = j, L = l_- \mid n\} \right)$$

$$= \sum_{l=1}^{m/2} l \left(\sum_{j=1}^{2l} jP\{J = j, X_{2l} > 0, X_{2l+1} = ... = X_m = 0 \mid n\} + \right.$$

$$\left. + \sum_{j=1}^{2l-1} jP\{J = j, X_{2l-1} > 0, X_{2l} = ... = X_m = 0 \mid n\} \right) \quad \text{(A.1)}$$

Applying the inclusion-exclusion rule, we have:

$$P\{J = j, X_{2l} > 0, X_{2l+1} = ... = X_m = 0 \mid n\}$$

$$= P\{J = j, X_{2l+1} = 0, ..., X_m = 0 \mid n\} - P\{J = j, X_{2l} = 0, ..., X_m = 0 \mid n\}$$

$$= P\{J = j \mid X_{2l+1} = 0, ..., X_m = 0, n\} * P\{X_{2l+1} = 0, ..., X_m = 0 \mid n\}$$

$$- P\{J = j \mid X_{2l} = 0, ..., X_m = 0, n\} * P\{X_{2l} = 0, ..., X_m = 0 \mid n\}$$

Thus,

$$\sum_{j=1}^{2l} jP\{J = j, X_{2l} > 0, X_{2l+1} = ... = X_m = 0 \mid n\}$$

$$= \sum_{j=1}^{2l} jP\{J = j \mid X_{2l+1} = ... = X_m = 0, n\} * P\{X_{2l+1} = 0, X_{2l+1} = 0, ... X_m = 0 \mid n\}$$

$$- \sum_{j=1}^{2l-1} jP\{J = j \mid X_{2l} = ... = X_m = 0, n\} * P\{X_{2l} = ... = X_m = 0 \mid n\}$$

$$= \left(\sum_{r=1}^{2l} p_r \right)^n \sum_{j=1}^{2l} jP\{J = j \mid X_{2l+1} = ... = X_m = 0, n\}$$

$$- \left(\sum_{r=1}^{2l-1} p_r \right)^n \sum_{j=1}^{2l-1} jP\{J = j \mid X_{2l} = ... = X_m = 0, n\}$$

Similarly,

$$\sum_{j=1}^{2l-1} jP\left\{J=j, X_{2l-1}>0, X_{2l}=...=X_m=0 \mid n\right\}$$

$$=\left(\sum_{r=1}^{2l-1} p_r\right)^n \sum_{j=1}^{2l-1} jP\left\{J=j \mid X_{2l}=...=X_m=0,n\right\}$$

$$-\left(\sum_{r=1}^{2(l-1)} p_r\right)^n \sum_{j=1}^{2(l-1)} jP\left\{J=j \mid X_{2l-1}=...=X_m=0,n\right\}.$$

The conditional expectation $\sum_{j=1}^{2l} jP\left\{J=j \mid X_{2l+1}=0=...=X_m=0,n\right\}$ is just the expected number of aisles visited given n and aisles from $2l$ to m are not visited. From Chew and Tang (1999), this amount is $2l-\sum_{i=1}^{2l}\left(1-p_i^*\right)^n$, where $p_i^* = p_i \big/ \sum_{j=1}^{2l} p_j$ is normalized probability. A similar argument holds for $\sum_{j=1}^{2(l-1)} jP\left\{J=j \mid X_{2l-1}=...= X_m=0,n\right\}$. At this step, (A.1) can be simplified as follows:

$$E\left(JL \mid n\right)=\sum_{l=1}^{m/2} l\left\{\left(\sum_{r=1}^{2l} p_r\right)^n\left[2l-\sum_{i=1}^{2l}\left(1-p_i^*\right)^n\right]-\left(\sum_{r=1}^{2(l-1)} p_r\right)^n\left[2(l-1)-\sum_{i=1}^{2(l-1)}\left(1-p_i^{**}\right)^n\right]\right\},$$

where $p_i^{**} = p_i \big/ \sum_{j=1}^{2(l-1)} p_j$.

Appendix B

The second adjustment term (AT_2) takes into account the fact that from the last pick position in the last visited aisle in each block the order-picker has to return to the center line of the cross aisle. For each block, such a turn has to be made if and only if the block is visited and the number of visited aisles is odd. The probability that the turn occurs in one of the blocks and all i picks fall into exactly g aisles ($g \in \left\{G \mid 1 \le g \le m/2,\, g \text{ is odd}\right\}$) is:

$$\binom{m/2}{g}\left(\frac{g}{m/2}\right)^i X\left(g\right),$$

where $X(g)$ is 1 minus the probability that all i picks fall into less than g aisles, conditional on the fact that all i order lines fall into at most g specific aisles (see Roodbergen 2001):

$$X(g) = 1 - \sum_{i=1}^{g-1} (-1)^{i+1} \binom{g}{g-i} \left(\frac{g-i}{g}\right)^n .$$

We call CR_1 and CR_2 are the correction time if the turn happens in only one and two blocks respectively. As items are randomly located within the warehouse, we assume that if g aisles are visited then the expected order lines in each visited aisle will be n/g . It then follows:

$$CR_1 = 2\left(0.5^n\right) \sum_{g \in G:odd} \left[\binom{m/2}{g} \left(\frac{g}{m/2}\right)^n X(g) \left(2d \frac{\frac{n}{g}}{\frac{n}{g}+1} - d \right) \right]$$

$$CR_2 = \left[0.5^n \frac{n!}{k!(n-k)!} \right] \sum_{k=1}^{n-1} \left\{ \sum_{g \in G:odd} \left[\binom{m/2}{g} \left(\frac{g}{m/2}\right)^k X(g) \left(2d \frac{\frac{k}{g}}{\frac{k}{g}+1} - d \right) \right] + \right.$$

$$\left. \sum_{g \in G:odd} \left[\binom{m/2}{g} \left(\frac{g}{m/2}\right)^{n-k} X(g) \left(2d \frac{\frac{n-k}{g}}{\frac{n-k}{g}+1} - d \right) \right] \right\},$$

where $0.5^n \frac{n!}{k!(n-k)!}$ is the probability that $k\left(1 \le k \le n-1\right)$ order lines fall into one block and $(n-k)$ order lines into the other.

Finally, the adjustment time due to making a turn if the number of visited aisles in a block is odd would equal: $E\left[AT_2 \mid n\right] = CR_1 + CR_2$ (B1)

References

Caron, F., Marchet, G., Perego, A., 1998. Routing policies and COI-based storage policies in picker-to-part systems. International Journal of Production Research 36(3), 713-732.

Chaudhry, M.L., 1991. Numerical issues in computing steady-state queueing-time distributions of single-server bulk-service queues: $M/G^b/1$ and $M/G^d/1$. Journal on Computing 4(3), 300-311.

Chaudhry, M.L., Harris, C.M., Marchal, W.G., 1990. Robustness of rootfinding in single-server queueing models. Journal on Computing 2(3), 273-285.

Chaudhry, M.L., Madill, B.R., Briere, G., 1987. Computational analysis of steady-state probabilities of M/$G^{[a,b]}$ /1 and related non-bulk queues. Queueing Systems 2, 93-114.

Chew, E.P., Tang, L.C., 1999. Travel time analysis for general item location assignment in a rectangular warehouse. European Journal of Operational Research 112, 582-597.

De Koster, R., Van der Poort, E.S., Wolters, M., 1999. Efficient orderbatching methods in warehouses. International Journal of Production Research 37(7), 1479-1504.

Elsayed, E.A., Lee, M.K., 1996. Order processing in automated storage/retrieval systems with due dates. IIE Transactions 28, 567-577.

Elsayed, E.A., Lee, M.K., Kim, S., Scherer, E., 1993. Sequencing and batching procedures for minimizing earliness and tardiness penalty of order retrievals. International Journal of Production Research 31(3), 727-738.

Foster, F.G., Perera, A.G.A.D., 1964. Queues with batch departures II. The Annuals of Mathematical Statistic 35(3), 1114-1156.

Gademann, A.J.R.M., Van den Berg, J.P., Van der Hoff, H.H., 2001. An order batching algorithm for wave picking in a parallel-aisle warehouse. IIE Transactions 33, 385-398.

Gibson, D.R., Sharp, G.P., 1992. Order batching procedures. European Journal of Operations Research 58(1), 56-57.

Gross, D., Harris, C. M., 1998. Fundamentals of Queueing Theory. John Wiley & Sons, New York.

Hwang, H., Baek, W., Lee, M.-K., 1988. Clustering algorithms for order picking in an automated storage and retrieval system. International Journal of Production Research 26(2), 189-201.

Le-Duc, T., De Koster, R., 2002. An approximation for determining the optimal picking batch size for order picker in single aisle warehouses. In: Meller, M., Ogle, M. K., Brett, A.P., Taylor, G.D., Usher, J. (eds.), Progress in Material Handling Research: 2002, Material Handling Institute, Charlotte, North Carolina, pp. 267-286.

Madan, K.C., 2000. An M/G/1 queue with second optimal service. Queueing Systems 34, 37-46.

Petersen, C.G., 1997. An evaluation of order picking routing policies. International Journal of Operations & Production Management 17(11), 1098-1111.

Petersen, C.G., Schmenner, R.W., 1999. An evaluation of routing and volume-based storage policies in an order picking operation. Decision Sciences 30(2), 481-501.

Roodbergen, K.J., 2001. Layout and routing methods for warehouses. Ph.D. thesis, Erasmus Research Institute of Management (ERIM), Erasmus University Rotterdam, The Netherlands.

Roodbergen, K.J., De Koster, R., 2001[a]. Routing methods for warehouses with multiple cross aisles. International Journal of Production Research 39(9), 1865-1883.

Roodbergen, K.J., De Koster, R., 2001[b]. Routing order pickers in a warehouse with a middle aisle. European Journal of Operational Research 133, 32-43.

Rosenwein, M.B., 1996. A comparison of heuristics for the problem of batching orders for warehouse selection. International Journal of Production Research 34(3), 657-664.

Tijms, H. C., 1994. Stochastic Models: an Algorithmic Approach. John Wiley & Sons, New York.

Tompkins, J. A., White, J. A., Bozer, Y. A., Frazelle, E. H., Tanchoco, J. M. A. T, Trevino, J., 1996. Facilities Planning. John Wiley & Sons, New York.

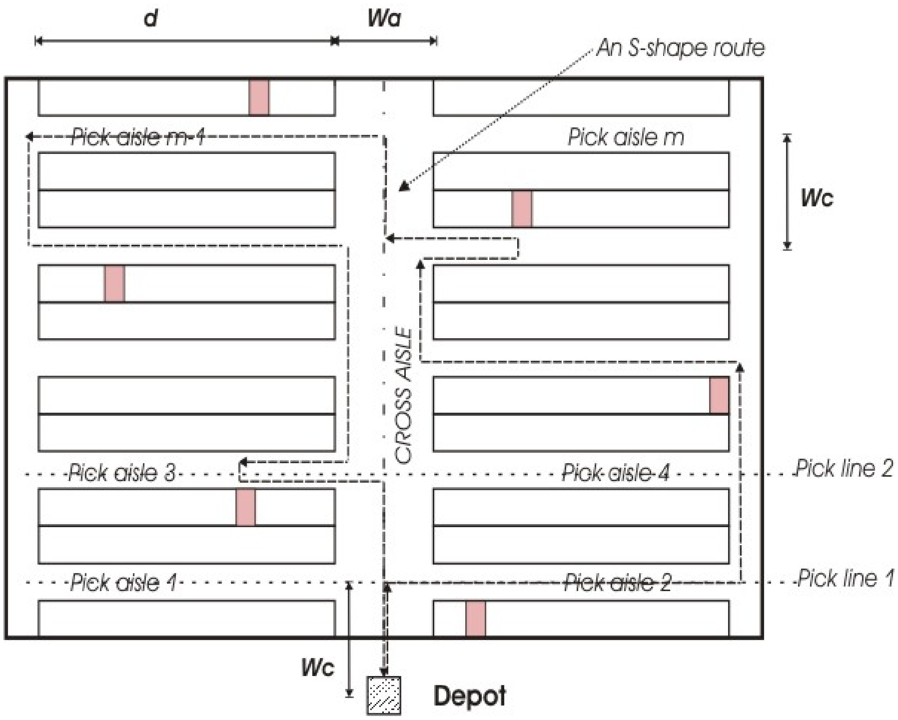

Fig. 1. A 2-block warehouse layout with an S-shape pick route.

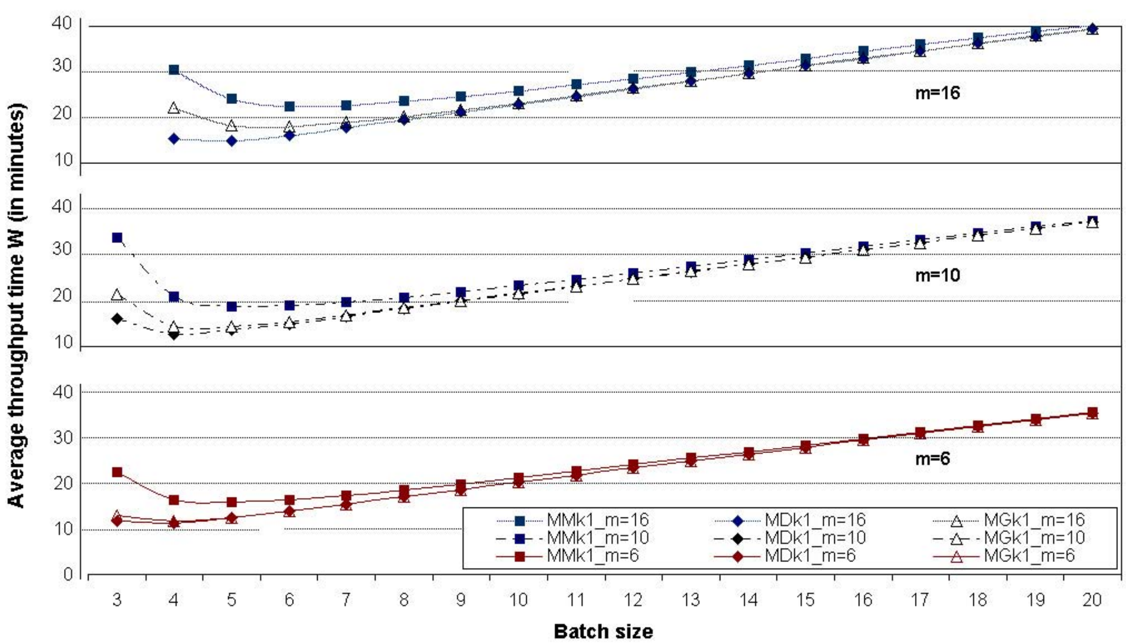

Fig. 2. Average throughput time for different service time distributions (with the approximation value of the service time).

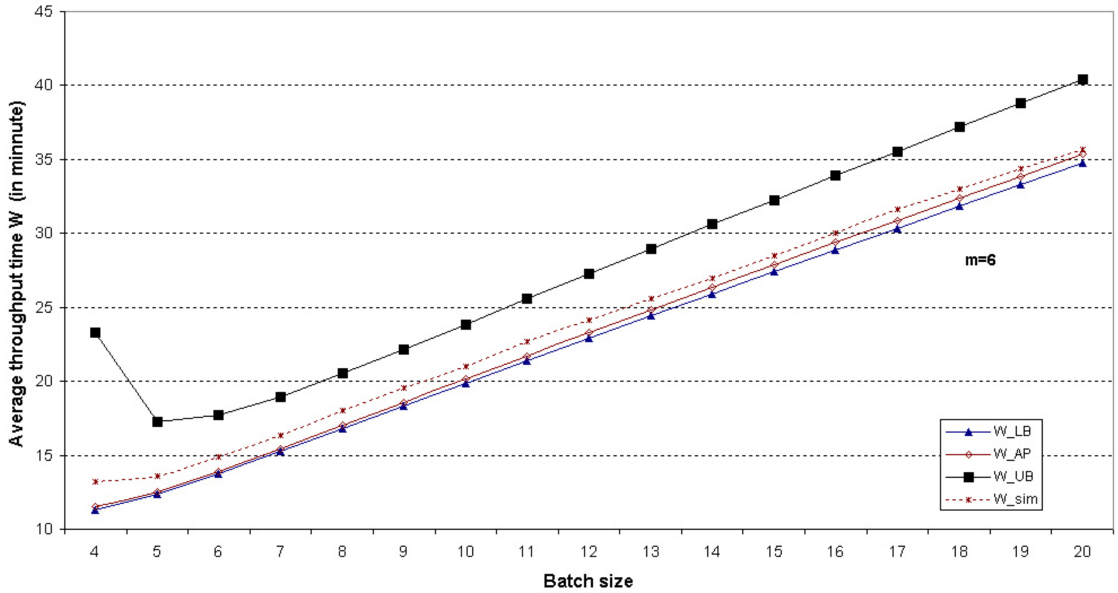

Fig. 3. Average throughput time of a random order for the 6-aisle layout (W_LB is the approximated value of the average throughput time, by the 2-moment approximation, based on the lower bound value of the first and second moment of service time).

Table 1. Parameters for the simulation experiment.

Attributes	Quantities
m	6, 10, 16 aisles
λ	4 orders/ 10 minutes
d	30 seconds
w_a	6 seconds
w_c	10 seconds
τ_s	180 seconds
τ_p	12 seconds

Publications in the Report Series Research* in Management

ERIM Research Program: "Business Processes, Logistics and Information Systems"

2004

Smart Pricing: Linking Pricing Decisions with Operational Insights
Moritz Fleischmann, Joseph M. Hall and David F. Pyke
ERS-2004-001-LIS
http://hdl.handle.net/1765/1114

Mobile operators as banks or vice-versa? and: the challenges of Mobile channels for banks
L-F Pau
ERS-2004-015-LIS
http://hdl.handle.net/1765/1163

Simulation-based solution of stochastic mathematical programs with complementarity constraints: Sample-path analysis
S. Ilker Birbil, Gül Gürkan and Ovidiu Listeş
ERS-2004-016-LIS
http://hdl.handle.net/1765/1164

Combining economic and social goals in the design of production systems by using ergonomics standards
Jan Dul, Henk de Vries, Sandra Verschoof, Wietske Eveleens and Albert Feilzer
ERS-2004-020-LIS
http://hdl.handle.net/1765/1200

Factory Gate Pricing: An Analysis of the Dutch Retail Distribution
H.M. le Blanc, F. Cruijssen, H.A. Fleuren, M.B.M. de Koster
ERS-2004-023-LIS
http://hdl.handle.net/1765/1443

A Review Of Design And Control Of Automated Guided Vehicle Systems
Tuan Le-Anh and M.B.M. De Koster
ERS-2004-030-LIS
http://hdl.handle.net/1765/1323

Online Dispatching Rules For Vehicle-Based Internal Transport Systems
Tuan Le-Anh and M.B.M. De Koster
ERS-2004-031-LIS
http://hdl.handle.net/1765/1324

Generalized Fractional Programming With User Interaction
S.I. Birbil, J.B.G. Frenk and S. Zhang
ERS-2004-033-LIS
http://hdl.handle.net/1765/1325

---

* A complete overview of the ERIM Report Series Research in Management:
https://ep.eur.nl/handle/1765/1

ERIM Research Programs:
LIS Business Processes, Logistics and Information Systems
ORG Organizing for Performance
MKT Marketing
F&A Finance and Accounting
STR Strategy and Entrepreneurship

Learning Opportunities And Learning Behaviours Of Small Business Starters: Relations With Goal
Achievement, Skill Development, And Satisfaction
Marco van Gelderen, Lidewey van der Sluis & Paul Jansen
ERS-2004-037-ORG
http://hdl.handle.net/1765/1429

Meta-heuristics for dynamic lot sizing: A review and comparison of solution approaches
Raf Jans and Zeger Degraeve
ERS-2004-042-LIS
http://hdl.handle.net/1765/1336

A Multi-Item Inventory Model With Joint Setup And Concave Production Costs
Z.P. Bayındır, S.I. Birbil and J.B.G. Frenk
ERS-2004-044-LIS
http://hdl.handle.net/1765/1535

The Level Set Method Of Joó And Its Use In Minimax Theory
J.B.G. Frenk and G. Kassay
ERS-2004-045-LIS
http://hdl.handle.net/1765/1537

Reinventing Crew Scheduling At Netherlands Railways
Erwin Abbink, Matteo Fischetti, Leo Kroon, Gerrit Timmer And Michiel Vromans
ERS-2004-046-LIS
http://hdl.handle.net/1765/1427

Intense Collaboration In Globally Distributed Teams: Evolving Patterns Of Dependencies And Coordination
Kuldeep Kumar, Paul C. van Fenema and Mary Ann Von Glinow
ERS-2004-052-LIS
http://hdl.handle.net/1765/1446

The Value Of Information In Reverse Logistics
Michael E. Ketzenberg, Erwin van der Laan and Ruud H. Teunter
ERS-2004-053-LIS
http://hdl.handle.net/1765/1447

Cargo Revenue Management: Bid-Prices For A 0-1 Multi Knapsack Problem
Kevin Pak and Rommert Dekker
ERS-2004-055-LIS
http://hdl.handle.net/1765/1449

Real-Time Scheduling Approaches For Vehicle-Based Internal Transport Systems
Tuan Le-Anh and M.B.M. De Koster
ERS-2004-056-LIS
http://hdl.handle.net/1765/1452

Individual Telecommunications Tariffs in Chinese Communities: History as a Mirror of the Future, and Relevance for Mobile Service Development in China
H.Chen; L-F Pau
ERS-2004-057-LIS
http://hdl.handle.net/1765/1582

Activating Knowledge Through Electronic Collaboration: Vanquishing The Knowledge Paradox
S. Qureshi and P. Keen
ERS-2004-058-LIS
http://hdl.handle.net/1765/1473

A Grounded Theory Analysis Of E-Collaboration Effects For Distributed Project Management
S. Qureshi, M. Liu and D. Vogel
ERS-2004-059-LIS
http://hdl.handle.net/1765/1448

Collaborative Infrastructures For Mobilizing Intellectual Resources: Assessing Intellectual Bandwidth In A Knowledge Intensive Organization
R. Verhoef and S. Qureshi
ERS-2004-060-LIS
http://hdl.handle.net/1765/1474

A Phenomenological Exploration Of Adaptation In A Polycontextual Work Environment
P.C. van Fenema and S. Qureshi
ERS-2004-061-LIS
http://hdl.handle.net/1765/1612

Satisfaction Attainment Theory As A Model For Value Creation
R.O. Briggs, S. Qureshi and B. Reining
ERS-2004-062-LIS
http://hdl.handle.net/1765/1450

Diagnosis In The Olap Context
Emiel Caron, Hennie Daniels
ERS-2004-063-LIS
http://hdl.handle.net/1765/1492

A Deterministic Inventory/Production Model With General Inventory Cost Rate Function And Concave Production Costs
Z.P. Bayındır, S.I. Birbil and J.B.G. Frenk
ERS-2004-064-LIS
http://hdl.handle.net/1765/1536

On And Off The Beaten Path: How Individuals Broker Knowledge Through Formal And Informal Networks
Rick Aalbers, Wilfred Dolfsma & Otto Koppius
ERS-2004-066-LIS/ORG
http://hdl.handle.net/1765/1549

Fractional Programming
J.B.G. Frenk And S. Schaible
ERS-2004-074-LIS
http://hdl.handle.net/1765/1610

Introduction To Convex And Quasiconvex Analysis
J.B.G. Frenk and G. Kassay
ERS-2004-075-LIS
http://hdl.handle.net/1765/1611

Multi-Attribute Dispatching Rules For Agv Systems With Many Vehicles
Tuan Le-Anh and René M.B.M. de Koster
ERS-2004-077-LIS
http://hdl.handle.net/1765/1809

Economic Lot-Sizing Games
Wilco Van Den Heuvel, Peter Borm, Herbert Hamers
ERS-2004-088-LIS
http://hdl.handle.net/1765/1805

Simple heuristics for push and pull remanufacturing policies
Erwin A. van der Laan and Ruud H. Teunter
ERS-2004-090-LIS
http://hdl.handle.net/1765/1786

Reverse Logistics – Capturing Value in the Extended Supply Chain
Moritz Fleischmann, Jo van Nunen, Ben Gräve, and Rainer Gapp
ERS-2004-091-LIS
http://hdl.handle.net/1765/1806

Genetic Algorithms In Supply Chain Scheduling Of Ready-Mixed Concrete
D. Naso, M. Surico, B. Turchiano and U. Kaymak
ERS-2004-096-LIS
http://hdl.handle.net/1765/1802

Travel Time Estimation and Order Batching in a 2-block Warehouse
Le-Duc en De Koster
ERS-2004-098-LIS

The Social Shaping of the early Dutch Management Schools
Peter van Baalen en Luchien Karsten
ERS-2004-099-LIS