To Aggregate or Not to Aggregate: Should decisions and models have the same frequency?

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To aggregate or not to aggregate:

Should decisions and models have the same frequency?

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Abstract

We examine the situation where hourly data are available to design advertising-response models, whereas managerial decision making can concern hourly, daily or weekly intervals. The key question is how models for hourly data compare to models based on weekly data with respect to forecasting accuracy and with respect to assessing advertising impact. Simulation experiments suggest that the strategy, which entails modeling the least aggregated data and forecasting more aggregate data, yields better forecasts, provided that one has a correct model specification for the higher frequency data. A detailed analysis of three actual data sets confirms this conclusion. A key feature of this confirmation is that aggregation affects data transformation to dampen the variance. The estimated advertising impact is sensitive to the appropriate transformation. Our conclusion is that disaggregated models are preferable also when decision have to be made at lower frequencies.

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1. Introduction

Ever since the availability of time series data on advertising and sales, there have been discussions about the appropriate level of aggregation. A classic study is Clarke (1976) who shows that if the analyst considers the same model type for different levels of aggregation, that then the longitudinal impact of advertising is grossly overestimated, see also Russell (1986). This notion is again illustrated in Tellis and Franses (2006) who show that the familiar Koyck model for higher frequency data becomes another, and more involved, time series regression model for aggregated data. In brief, a by-product of aggregation is that the model must change too. A similar result appears for data transformations; see De Bruin and Franses (1999). When the higher frequency data are, say, log-transformed to mitigate the impact of extreme observations and to dampen the variance, a move to aggregated data may not need such a transformation as these extreme observations are "aggregated away".

In this paper we also consider data transformation and model change in advertising response models. Our specific focus is on the possibility that managerial decision making concerns another frequency than the frequency of the available data. More precise, we consider the situation where the analyst has data available at the hourly level, whereas managers use daily or weekly forecasts for decision making. The key question is then whether one should model the hourly data and create hourly forecasts, and thereafter aggregate these forecasts to weekly forecasts or that one should first aggregate the hourly data to weekly data and then create an econometric time series model. To answer this key question, we first carry out a range of simulations using artificial data. After that, we provide a detailed analysis of a large database with hourly advertising and sales data.

The remainder of the paper is organized as follows. In Section 2, we briefly discuss the literature related to our topic. In Section 3, we report on a simulation experiment to illustratehow predictive accuracy changes under time aggregation. Section 4addresses the data used for the empirical application. In Section 5 the model specification for the hourly data is discussed. As the data show strong intra-day and intra-week seasonality, a two-level model is proposed where the hour in the week is the observation unit. In Section 6 we present singlelevel models for hourly, daily data and weekly data. In Section 7 we summarize the forecasting and inference results. Finally, Section 8 contains the main conclusions and limitations.

2. Literature

The advertising literature contains various studies that address the degree of aggregation to measure advertising effectiveness. A typical workhorse model is the familiar Koyck model, which correlates current sales with current advertising and past sales, and an error term with first order dynamics. Alternative models involve variants of the autoregressive distributed lag model [ADL], where no such moving average terms [MA] are included. For both types of models it holds that the parameter estimates can be used to infer the long-run (or cumulative) effect of advertising on sales, the immediate effect on sales, and the shape of the decay function which gives the speed at which the effects of advertising impulses eventually fade out to zero.

A key aspect of the models used in this literature is that the parameter estimates, and their derivative functions, can depend on the aggregation level of the data. For example, if one analyzes monthly data, while the underlying process works at the weekly level because advertising impulses are given at the weekly level, then one may make estimation errors. This insight goes back to Clarke (1976), and various subsequent studies such as Windal and Weiss (1980), Bass and Leone (1983, 1986), Tellis and Weiss (1995), and Leone (1995).

There are various possible responses to this phenomenon. The first is to acknowledge the aggregation effects and modify the estimation routine, see Weiss, Weinberg and Windal (1983) who propose a nonlinear GLS estimation technique which takes into account the effect of aggregation on estimates and on the error terms. They use simulated monthly data and aggregate these into half-yearly and annual data and look at how the estimates of the autoregressive term and autocorrelation functions change under aggregation. They report that the level of aggregation does not cause an upward bias of the parameter estimates, but as the aggregation has been applied to small samples, the probability of overestimating the lagged depended variable does increase. Also, they attribute the upward bias of the parameter estimates to misspecification of the model.

Second, it is recognized that aggregation can make the model to change. For example, when an autoregression of order 1 [AR(1)] is adequate to describe weekly data, then this model becomes an autoregressive moving average model of order 1,1 [ARMA(1,1)]. Russell (1988) argues that temporal aggregation of the data does not change the underlying advertising-sales relationship, but typically the model at the aggregated level is misspecified. To retrieve the micro-frequency parameters, one does need to know (or assume) how the micro frequency model looks like. Recently, Tellis and Franses (2006) use this result to decide on the optimal level of aggregation, such that this retrieval is still possible.

A third response to aggregation issues, which has become possible given the recent availability of high frequency data, is to simply rely on models for the highest frequency. The seminal articles on high frequency models are Tellis, Chandy, and Thaivanich (2000) and Chandy et al. (2001). These studies indicate that high frequency models can be used as the basis for decision support systems in media planning as it pertains to the choice of optimal channels, time slots, and spot lengths. Also insights about the relative effectiveness of appeals and wear-in wear-out effects are obtained. Tellis and Franses (2006) use such high frequency data to show that the optimal data interval or aggregation level is the unit exposure time, which is the time interval in which consumers are typically exposed to a single advertising spot. These studies advocate the use of high frequency data, even if decisions have to be made at a more aggregated level. In our paper we will examine this issue in more detail.

Even though much of the literature is dedicated to the issue of aggregation and its effects on (functions of) parameter estimates, the focus rarely is on out-of-sample forecasting. As much of the use of econometric advertising models is for budgeting and strategy planning, most managers would require accurate forecasts for planning and decision making. Note that the planning and decision horizon does not necessarily match with the available data frequency. For example, managers may need to make budgeting plans for the next year, and hence require forecasts for the next year's sales level. Suppose a modeler has access to weekly data, then it is open for discussion whether a model for those weekly data is useful to forecast next year" total sales. Indeed, one may resort to aggregating the weekly data first to annual data, then make a model for these annual data and then create forecasts. On the other hand, one can also aggregate the weekly data to quarterly data, create forecasts and then aggregate those forecasts to annual forecasts.

It is the purpose of the present study to shed light on this managerially relevant issue. We first rely on simulations and after that we discuss at length a detailed empirical case. This case is actually very relevant for the company at hand who allowed us to use the data. As high frequency data, like hourly data, can show large variation due to potential outliers, we show that aggregation also impacts data transformation. Indeed, it is common to use for example the natural log transformation to dampen variation in, say, hourly data, but perhaps this transformation is less relevant for aggregated weekly or monthly data. Before we turn to an analysis of actual data, we first examine the outcomes of some simulation experiments.

3. A simulation experiment

Consider an autoregressive model of order 1[AR(1)],

$$
y_t = \rho y_{t-1} + \varepsilon_t \tag{1}
$$

and assume that the subscript *t* denotes the higher frequency level, in our artificial case, say this concerns months. The error term is assumed as white noise, implying that it is not predictable. In our simulation we assume that the error term is drawn from a standard normal distribution. We generate 1000 samples of 612 "months", where the first 12 months are created to compensate for observations lost by start-up. We consider three values for ρ , that is, 0.5, 0.8 and 0.9. The dataset is split in two parts. We have an estimation sample of size 480 and the sample for evaluating out-of-sample forecasts consists of 120 observations. We consider three levels of aggregation, that is, the true micro model (monthly level) in (1), the quarterly level and the annual level. Aggregation is done by summing (per 3 or per 12) the monthly observations. As mentioned earlier, aggregation to quarters and years makes that we need to fit ARMA(1,1) models for the resultant quarterly and annual data. Like Windal and Weiss (1980), in those cases we shall use feasible generalized least squares [FGLS] for parameter estimation. The parameter ρ in (1) can be retrieved from the AR parameters in the ARMA models by taking the 3-root or 12-root of the relevant estimated autoregressive parameter.

--- Insert Table 1 about here ---

Table 1 shows the average estimated $\hat{\rho}$ for the model in (1) for different levels of aggregation and true ρ values, as well as the estimated standard deviation of these estimates. For ρ = 0.5 and ρ = 0.8 we obtain average estimates of $\hat{\rho}$ that are close to the true values. As expected, and supported by the results for the yearly data, the reliability of the estimate for *ρ* decreases for higher aggregation levels since ρ^{12} is close to zero and thus *ρ* can be estimated with less precision.

--- Insert Table 2 about here ---

Table 2 shows the forecasting accuracies in terms of root mean squared prediction error (RMSPE) for the three levels of aggregation. "Monthly Aggregated" refers to monthsahead forecasts, aggregated to either quarters or years. These are calculated in order to compare with the forecasts based on longer horizons. The table includes the RMSPEs for the case where the parameter is not estimated but set fixed at the true value (like, $\rho = 0.5$ in the first panel), then the case where *ρ*is estimated ("Monthly"), then the case where the monthly forecasts are aggregated to quarterly or yearly forecasts, and finally two sets of two forecasts where the data are aggregated first and then modeled, after which forecasts are made.

Table 2 shows that the forecasting accuracy decreases with the level of aggregation. This can be seen from comparing the numbers across the columns with headers monthly, quarterly and yearly. We also see that forecasts from monthly data, and then aggregated to quarterly or yearly forecasts, are about equally good or sometimes even slightly better than the forecasts created from comparable aggregate models. For example for $\rho = 0.5$, consider the 2.50 in the row with "monthly" and the column with "quarterly data without correction", and compare it with the 2.59 or 2.64 two cells down in the same column.

The benefit of modeling at the micro frequency is clear. Further, FGLS estimation has little effect, and also the MA term correction is not very effective. For yearly aggregates, accuracy even further decreases. The MA term correction in the model for quarterly and yearly data improves the predictive accuracy of the forecast of the FGLS estimation only marginally. In sum, we would conclude that the loss in efficiency due to aggregation is larger than the loss in forecasting accuracy for higher frequency data

Increasing ρ (0.8 or 0.9) leads to larger RMSPE values, as the variance of the data increases. However, the overall conclusion remains the same, and that is that models for higher frequency data give forecasts for aggregated data that are superior to forecasts from models for aggregated data. Again, applying the MA correction in the forecasts provides little benefit to the forecasting accuracy. So, if one has high frequency data it is better use these also for forecasting more aggregated data. Whether this simulated evidence gets support from actual empirical data will be studied in the next section.

4. Actual data

In this section we analyze data relating to a car repair service provided by a multinational. We consider three different areas where data are collected and these are Flanders (the Dutchspeaking part of Belgium), Wallonia (the French-speaking part of Belgium), and Spain. A national call center, for Belgium and Spain separately, collects all requests for information and service appointments from consumers. The Spanish market runs on similar principles, but the operations are different with respect to operating hours. Of course, media plans and communication channels are totally different in each of the three areas. The dependent variable is the number of incoming calls received by the call center. Table 3 gives an overview of the three datasets, where the high frequency data concern hourly data.

--- Insert Table 3 about here ---

.

In all cases the company relies mainly on radio advertising and only to a limited extent on television advertising (in Belgium, not in Spain). There is no comparable service advertised at the national level, and therefore the data are very well suited for the analysis of advertising effects. There is little or no variation in the advertising themes, but the core message used is framed in different formats or spot lengths.

For the Flanders region (data set 1), the calls data cover the period from October 2004 to June 2007. Our analysis is performed on data at hourly intervals (23880). There are a total of 6144 radio spots and 252 television spots. Television advertising occurs in 2006 and 2007 only.

For the Walloon region (data set 2), the calls data cover the period from January 2005 to June 2007. Thus, this Data set 2 has 21672 data points, from January 2005 to June 2007. There are a total of 7432 radio spots and 330 television spots. Again, television advertising occurs in 2006 and 2007 only.For radio 20 seconds spots are used predominantly, and other spot lengthsare used much less. For television there is a relatively large number of 5 seconds spots, and these are so-called "sponsoring spots" associated with the weather forecasts.

For Spain (data set 3), the available data cover the period from January, 1 2004 to June, 30 2006, with a two-week period missing in between, as Figure 1 shows. For this market we have data for 9251 hours. In the observation period, 4,827 radio commercials were broadcasted on twenty-five radio stations.Mostly 60 seconds commercials were used. Spots at 8 AM were often used. The distribution of broadcasts is more or less equal over the weekdays.In the dataset, commercials start at 6 AM. The call centre startsto operate from8 AMuntil 8 PM, with shorter service hours on Saturdays, wile it is closed on Sundays. As a result, the call centre operates 70 hours per week.

The advertising strategy of the company makes use of "pulsing". This means that there are weeks in which all advertising is scheduled and these are alternated with non-active weeks. Table 3 shows that approximately one third of the weeks are a week with commercials, with average GRP levels equal to 237 for data set 1, 312 for data set 2 and 249 for data set 3.

--- Insert Figure 1 about here ---

Figure 1 shows the evolution of the incoming calls (the "sales") in the three areas. The number of daily incoming calls for the data sets 1 and 2 are approximately equal, while the market size of data set 3 is substantially higher.

--- Insert Figure 2 about here ---

The three panels in Figure 2 show the average incoming calls levels of the three call centers on hours with advertising, contrasted with hours without advertising. There is a highly similar intraday pattern. Peak callsoccur between 8AM and 9AM, and a smaller peak occurs early afternoon around 2PM.

--- Insert Table 4 about here ---

Table 4 provides a summary of the average radio GRPs per hour of the day. The radio commercials are most often broadcasted between 6AM and 8PM and average GRPs vary between 0.70 and 11.52. For data set 1, the GRPs are highest at 7AM. For the data sets 2 and 3 the maximum valuesare at 8 AM.

Next, we will analyze these hourly data using multi-level models. After that, we will aggregate to the daily and weekly levels and accordingly fit models and create forecasts.

5. Two-level models for hourly data

To properly capture the substantial variation of the data at the hourly level, we resort to a twolevel regression model. In short, we treat the hour within the day as the observation unit, so

we will deal with sales denoted as $Y_{h,t}$, where subscript *h* denotes the hour and *t* denotes the week. As such, we have a panel of time series, where the panel consists of 168 units and the time frame covers 143, 130 and 88 weeks, respectively (see Table 3). For this panel we choose to consider the Linear Mixed Model [LMM] (see for example Verbeke and Molenbergh, 2000), where the model reads as

$$
Y_{h,t} = Z_{h,t} b_h + \varepsilon_{h,t}
$$

with

$$
b_h = W_{h,t} b + \varepsilon_h
$$
 (2)

In words, the first level contains parameters that can possibly vary across the hour of the day (*h*), and the second level correlates those parameters to characteristics of the particular hours incorporated in $W_{h,t}$. As such, this model allows capturing the substantial variation in the data that is present at this highly disaggregated level.

After some experimentation, we fix the first level of the three models as

$$
Y_{h,t} = \theta_h + \overline{\lambda_{1,h} Y_{h-1,t} + \lambda_{2,h} Y_{h-2,t} + \lambda_{3,h} Y_{h-3,t} + \lambda_{24,h} Y_{h-24,t} + \lambda_{168,h} Y_{h,t-1}}
$$
\n
$$
+ \overline{\phi_h^0 R_{h,t} + \phi_h^1 R_{h-1,t} + \phi_h^2 R_{h-2,t} + \gamma_h^c T V_{h,t} + \gamma_h^d T V D_{h,t} + \delta_h^S \sin \frac{2\pi t}{52} + \delta_h^C \cos \frac{2\pi t}{52} + \varepsilon_{h,t}}
$$
\n(3)

where the variables in this first-level model are defined as

$$
Y_{h,t} = \log(Calls_{h,t} + 1), \text{ where } h \text{ runs from 1 to 168 and where } t \text{ runs from 1 to } T;
$$

\n
$$
R_{h,t} = \log(RadioGR_{h,t} + 1)
$$

\n
$$
TV_{h,t} = \log(TVGR_{h,t} + 1)
$$

\n
$$
\sin \frac{2\pi t}{52}, \cos \frac{2\pi t}{52} \text{ are harmonic or goniometric regressors, capturing the intra-year\nseasonality in the data
$$

and where $TVD_{h,t}$ denotes the log of the total amount of TV GRPs during the previous day. Note that this last variable takes the same value for 24 hours in a row.

The second-level equations allow for distinct specifications across the three data sets They include time variation in the coefficients across the hour of the week or the day. For example, it is found in all three samples that autocorrelation varies substantially, with a short cycle of one day. For the advertising coefficients the time variability is less pronounced. The characteristics of advertising spots, length and channel are included in order to detect differences in advertising effectiveness.

The intercept term in each dataset is given by equation (4), with the relevant samples identified between parentheses, that is, (1,3) for samples 1 and 3, (2) for sample 2.

(1,3)
$$
\theta_h = \theta_0 + \theta_1 \sin \frac{2\pi hd}{24} + \theta_2 \cos \frac{2\pi hd}{24} + \sum_{d=1}^6 \theta_{3,d} D_h^d
$$

\n(2) $\theta_h = \theta_0 + \sum_{d=1}^6 \theta_{1,d} D_d + \theta_2 hh_h + \theta_3 hh_h^2$ (4)

where *hd* is the hour of the day (1,2,...24), and *hh* is similar but becomes repetitive after 12 hours $(1,2,1,2,1,2,1,2,1)$. This cycle with 12 hour frequency is also used in a quadratic form. D_h^d is a zero-one dummy variable which takes a value 1 for day *d* with $d = 1$ meaning a Monday.

For the first-order autoregressive parameters we specify

$$
(1,2) \lambda_{1,h} = \lambda_{1,0} + \sum_{d=1}^{7} \lambda_{1,d,s} D_h^d \sin \frac{2\pi hd}{24} + \sum_{d=1}^{7} \lambda_{1,d,c} D_h^d \cos \frac{2\pi hd}{24} + \varepsilon_{1,h}
$$

$$
(3) \lambda_{1,h} = \lambda_{1,0} + \sum_{d=1}^{7} \lambda_{1,d,1} D_d h h_h + \sum_{d=1}^{7} \lambda_{1,d,1} D_d h h_h^2 + \varepsilon_{1,h}
$$

(5)

For the second-order autoregressive term we specify

$$
(1,2) \lambda_{2,h} = \lambda_{2,0} + \sum_{d=1}^{7} \lambda_{2,d,s} D_h^d \sin \frac{2\pi hd}{24} + \sum_{d=1}^{7} \lambda_{2,d,c} D_h^d \cos \frac{2\pi hd}{24}
$$

$$
(3) \lambda_{2,h} = \lambda_{2,0}
$$
 (6)

The third-order autoregressive term is specified as constant for data sets 1 and 2. For data set 3 it is modeled by means of harmonic regressors with a one day cycle. So, we have

$$
(1,2)\lambda_{3,h} = \lambda_{3,0}
$$

$$
(3)\lambda_{3,h} = \lambda_{3,0} + \sum_{d=1}^{7} \lambda_{3,d,s} D_h^d \sin \frac{2\pi hd}{12} + \sum_{d=1}^{7} \lambda_{3,d,c} D_h^d \cos \frac{2\pi hd}{12}
$$
 (7)

In all three samples, the two further autoregressive terms are modeled as

$$
\lambda_{24,h} = \lambda_{24,0} + \varepsilon_{2,h} \tag{8}
$$

and

$$
\lambda_{168,h} = \lambda_{168,0} + \varepsilon_{3,h} \tag{9}
$$

For the regressors concerning the commercials on radio and television we have for all three data sets the following second-level equations,

$$
\phi_h^0 = \phi_0^0 + \sum_{j=1}^{Channels} \phi_j^0 F C_{h,j} + \varepsilon_{4,h}
$$
\n
$$
\phi_h^1 = \phi_0^1 + \sum_{j=1}^{Channels} \phi_j^1 F C_{h-1,j}
$$
\n
$$
\phi_h^2 = \phi_0^2 + \sum_{j=1}^{Channels} \phi_j^2 F C_{h-2,j} + \sum_{i=1}^{Spoltengths} \phi_i^2 F L_{h-2,i}
$$
\n(10)

where $FC_{h,j}$ is the fraction of radio GRPs per channel *j* and $FL_{h,i}$ is the fraction of radio GRPs per spot length *i*. For the first television variable we specify

$$
\gamma_h^0 = \gamma_0^0 + \sum_{j=1}^{Channels} \gamma_j^0 FTV_{h,j} + \sum_{hd=1}^{23} \gamma_{hd}^0 D_{h,hd}
$$
\n(11)

where $FTV_{h,j}$ is the fraction of television GRPs per channel *j* and $D_{h,hd}$ is a zero-one dummy variable indicating the hour of the day within a day.

For the next day television variable we specify

$$
\gamma_h^d = \gamma_0^d + \sum_{j=1}^{Channels} \gamma_j^d T VDC_{h,j} + \sum_{i=1}^{SpotLengths} \gamma_i^d T VDL_{h,i} + \sum_{d=1}^{6} \gamma_h^d D_h^d
$$
\n(12)

where $TVDC_{h,j}$ is the fraction of television GRPs per channel *j* during the previous day, *TVDLh*,*ⁱ* is the fraction of television GRPs per spot length.

Finally, for the annual seasonality, we specify for data set 1

$$
\delta_h^s = \delta_0^s + \delta_1^s \sin(\frac{2\pi hd}{12}) + \delta_2^s \cos(\frac{2\pi hd}{12}) + \varepsilon_{5,h}
$$
\n(13)

$$
\delta_h^c = \delta_0^c + \delta_1^c \sin(\frac{2\pi hd}{12}) + \delta_2^c \cos(\frac{2\pi hd}{12}) + \varepsilon_{6,h}
$$
\n(14)

while for data sets 2 and 3 we simply set $\delta_h^s = \delta_0^s$ and $\delta_h^c = \delta_0^c$.

The independent variables relate to the media schedule (time of broadcast, channel and length or equivalently the theme of a spot). In principle, all equations could be specified with a random error. However, this makes the model too complex which makes us to run into estimation problems. The specific second-level equations have been set based on trial and error. We tried various alternative forms of these equations, but the current ones turned out to be most adequate in terms of in-sample fit.

6. Alternative models

Additional to the model in the previous section, we consider more traditional Autoregressive Distributed Lag (ADL) models for the data at three aggregation levels, that is, hourly, daily and weekly data. The preferred model for the hourly data however is the model presented in the previous section. This model allows for heterogeneity across hours within a week, and as such is very useful to capture variation in the data.

In the two level (hourly) model a logarithmic specification is used. However, the transformation to logarithms is not necessarily appropriate for all levels of aggregation. We rely on the Box-Cox transformation to capture the non-stable variance in the data. This transformation is given by

$$
y^{(\lambda)} = \frac{y^{\lambda} - 1}{\lambda} \quad \text{if } \lambda \neq 0
$$

= log y \quad \text{if } \lambda = 0 \tag{15}

For a single-level model for hourly data *Y* (in levels), where now *h* runs from 1 to the total number of hours as is indicated in Table 3, we specify

$$
Y_{h}^{(\lambda)} = \mu + \lambda_{1} Y_{h-1}^{(\lambda)} + \lambda_{2} Y_{h-2}^{(\lambda)} + \lambda_{3} Y_{h-3}^{(\lambda)} + \lambda_{24} Y_{h-24}^{(\lambda)} + \lambda_{168} Y_{h-168}^{(\lambda)}
$$
\n
$$
+ \delta^{s} \sin \frac{2\pi h}{168 \times 52} + \delta^{c} \cos \frac{2\pi h}{168 \times 52}
$$
\n
$$
+ \phi^{0} R_{h}^{(\lambda)} + \phi^{1} R_{h-1}^{(\lambda)} + \phi^{2} R_{h-2}^{(\lambda)} + \gamma^{0} T V_{h}^{(\lambda)} + \gamma^{d} T V D_{h}^{(\lambda)}
$$
\nRadioherogeneous hour effects

\n
$$
+ \sum_{h=1}^{23} \sum_{l=0}^{2} \varphi_{1,hd,l} D_{h,hd-l} R_{hd-l}^{(\lambda)} + \sum_{j=1}^{Chamels} \sum_{l=0}^{2} \varphi_{2,j,l} R C_{j,h-1}^{(\lambda)} + \sum_{j=1}^{Lengths} \sum_{l=0}^{2} \varphi_{3,i,l} R L_{i,h-1}^{(\lambda)}
$$
\n
$$
+ \sum_{h=1}^{TV \text{ heterogeneous}~\text{boundary} \text{ effects}} \frac{V \text{Channel} \text{ channel} \text{ in the first 15}}{V' \text{ Channel} \text{ channel} \text{ in the first 15}} + \sum_{h=1}^{24} \varphi_{4,hd} D_{h,hd} T V_{h,hd}^{(\lambda)} + \sum_{j=1}^{Chamels} \varphi_{5,j} T V C_{j,h}^{(\lambda)} + \sum_{l=1}^{Lengths} \varphi_{6,j} T V L_{i,h}^{(\lambda)}
$$
\n
$$
+ \sum_{h=1}^{day and week cycle} \frac{day and weekcycle}{\varphi_{7,k} H_{h,k} + \varepsilon_{h}}
$$
\n(16)

where

$$
Y_h = Calls_h + 1
$$

$$
R_h = RadioGRP_h + 1
$$

$$
TV_h = TVGRP_{h,t} + 1
$$

and $RC_{j,h}$ are the radio GRPs per channel *j*, $RL_{i,h}$ are the radio GRPs per spot length *i*, while for television the similar variables are $TVC_{j,h}$ and $TVL_{i,h}$, and $D_{h,hd}$ is a dummy for hour *hd* of a day while $H_{h,k}$ is a dummy for hour *h* in the week.

This model is similar to the model in Tellis, Chandy, and Thaivanich (2000), where they also have hourly data, but there they consider referrals to health care services. Model (18) allows a substantial degree of heterogeneity through the hourly dummies affecting the intercept, but also in the effect of explanatory variables. Note that the simulation results in Table 2 motivated us not to explicitly include MA terms.

The model for daily data reads as

$$
Y_{T}^{(\lambda)} = \theta + \lambda_{1} Y_{T-1}^{(\lambda)} + \lambda_{2} Y_{T-7}^{(\lambda)} + \delta^{S} \sin \frac{2\pi T}{52\pi T} + \delta^{C} \cos \frac{2\pi T}{52\pi T} + \sum_{d=1}^{6} \varphi_{0} D_{d,T}
$$

+ $\phi^{0} R_{T}^{(\lambda)} + \gamma^{0} T V_{T}^{(\lambda)} + \gamma^{1} T V_{T-1}^{(\lambda)} + \sum_{j=1}^{Channels} \varphi_{1,j} R C_{j,T}^{(\lambda)} + \sum_{i=1}^{Lengths} \varphi_{1,i} R L_{i,T}^{(\lambda)}$
+ $\sum_{j=1}^{Channels} \varphi_{2,j} T V C_{j,T}^{(\lambda)} + \sum_{i=1}^{Lengths} \varphi_{2,i} T V L_{i,T}^{(\lambda)} + \varepsilon_{T}$ (17)

where $D_{d,r}$ is the day of the week. Finally, the model for weekly data is

$$
Y_{W}^{(\lambda)} = \theta + \lambda_{1} Y_{W-1}^{(\lambda)} + \delta^{S} \sin \frac{2\pi W}{52} + \delta^{C} \cos \frac{2\pi W}{52}
$$

+ $\phi^{0} R_{W}^{(\lambda)} + \gamma^{0} T V_{W}^{(\lambda)} + \sum_{j=1}^{Channels} \varphi_{1,j} R C_{j,W}^{(\lambda)} + \sum_{i=1}^{Lengths} \varphi_{2,i} R L_{i,W}^{(\lambda)}$
+ $\sum_{j=1}^{Channels} \varphi_{3,j} T V C_{j,W}^{(\lambda)} + \sum_{i=1}^{Lengths} \varphi_{4,i} T V L_{i,W}^{(\lambda)} + \varepsilon_{W}$ (18)

In the next section we will consider and compare the models in Section 5 with (18), (19) and (20), where we also provide the estimates for the Box-Cox parameter.

7. Estimation Results

To compare the various models of interest we consider their forecast accuracy, like in the simulations, and we also consider the estimates of the impact per GRP for the various models.

--- Insert Table 5 about here ---

We begin with the model in Section 5. The $(5\%$ significant) parameter estimates are displayed in Table 5, for data set 1. The estimation results for the other two data sets can be obtained from the authors, and are not given here to save space. Clearly, these estimation results show the relevance of the two levels model. The parameters show strong variation across the hours, in particular for the first order autoregressive parameters.

--- Insert Table 6 about here ---

When we estimate the models in (18), (19) and (20), the estimated Box-Cox parameters obtain the values as they are presented in Table 6. As expected, the estimated values get closer to 0 for the higher frequency data and closer to 1 for the aggregated data, like weeks. This pattern is rather consistent across the three data sets.

--- Insert Table 7 about here ---

Table 7 shows (part of the) estimation results of the model for the hourly data in (18), for the logarithmic model, equivalent to the Box-Cox parameter equal to zero. The estimation results in this table can be compared to those in Table 5. When we compare the parameters for the GRP variables, we do see quite some differences across the tables. Below, we shall examine to what extent these differences matter for the impact of GRPs.

--- Insert Table 8 about here ---

To highlight some of the crucial differences across the estimated models (18), (19) and (20), consider the estimated parameter concerning the weekly lag in Table 8. For the model for hourly data this corresponds with lag 168, and for the model for daily data it concerns lag 7. Note that this parameter is crucial for computing the decay rate of advertising impact. As expected, given the seminal work of Clark (1976) and others, this parameter increases with lower frequency. To illustrate, consider for data set 1 and no transformation the parameter 0.090 for the hourly data. Clearly the decay rate for the hourly data is different from that for the weekly data, where the relevant parameter is 0.371. Hence, advertising seems to last longer when weekly data are considered.

--- Insert Tables 9, 10 and 11 about here ---

The effectiveness per GRP is related to segmentation and targeting effectiveness across channels. We obtain one-week-ahead total impacts from simulating the process at the middle of each sample. Tables 9, 10 and 11 show the total incremental calls per GRP for the models in Section 5 and (18), (19) and (20). The main conclusion is that these numbers can be strikingly different across models and aggregation levels. Most consistent results are obtained for the models using hourly data particularly when comparing the LMM model in Section 5 is used to the logarithmic ADL model in (18).

--- Insert Table 12 about here ---

Finally, in Table 12 we report the predictive accuracy of the models for hourly, daily and weekly data, when the aim is to forecast hours, days and weeks. The first panel shows that the model in (18) performs best to forecast hourly data. Interestingly, the second panel shows that this model also performs best when forecasting days ahead. It does better across the models for hourly data, but it also outperforms the models for daily data. Approximately similar results are obtained when it comes to forecasting weeks ahead.

8. Conclusion and discussion

The main conclusion that we draw is this paper is that the answer to the question in the title of this paper is that one should not aggregate! Simulations and a detailed case study (for three data sets) strongly support the recommendation that it is better to fit models to high frequency data and to aggregate the associated forecasts rather than aggregating first. It is not recommended to aggregate high frequency data first, design models for these aggregated data and compute forecasts. A second finding is that, as the aggregation level increases, the specification changes from logarithmic at hourly level, to a square root transformation at the daily level, to approximately linear at the weekly levels.

The results in our paper in general support the notion that high frequency data deserve to be analyzed. The models for these high frequency data give parameters that provide the

proper interpretation in terms of decay rates and short-run effects of advertising, and, as we have shown, also provide better forecasts for any policy horizon of interest. In general, our study supports the modeling exercises in Tellis, Chandy, and Thaivanich (2000), Chandy et al. (2001) and Tellis and Franses (2006), where it is generally recommended to analyze at the most detailed level possible. Aggregation removes useful information and also provides less accurate forecasts.

The natural limitations to our study concern the simulation design and the empirical data at hand. More simulations can be done, also for various alternative data generating processes, and also more actual data sets can be analyzed. Additional further work is also relevant by analyzing other elements of the marketing mix, like pricing and promotions. At present, most models rely on weekly data, and perhaps also there one could benefit from an analysis of less aggregated data.

Table 1:

The average estimated $\hat{\rho}$ for the model in (1) for different levels of aggregation and true *ρ* **values, as well as the estimated standard deviation of these estimates**

	True parameter ρ				
Estimation method	0.5	0.8	0.9		
OLS for monthly data	0.50	0.80	0.90		
	(0.039)	(0.028)	(0.021)		
FGLS for quarterly data	0.42	0.79	0.90		
	(0.154)	(0.036)	(0.024)		
FGLS for yearly data	0.12	0.47	0.86		
	(0.283)	(0.407)	(0.167)		

Table 2:

Root mean squared predictions errors for various models for various levels of

aggregation.

Table 3:

Characteristics of the three actual data sets

Dataset1	Hour	6	$\overline{7}$	8	9	10	11	12	
	GRPs	6	14	$\overline{7}$	6	6	6	11	
	Spots	45	201	154	141	153	130	169	
	Hour	13	14	15	16	17	18	19	20
	GRPs	$\overline{3}$	$\overline{2}$	$\overline{4}$	6	$\overline{3}$	$\overline{4}$	$\overline{2}$	$\mathbf{1}$
	Spots	33	17	34	$\overline{51}$	94	133	$\overline{85}$	$\overline{9}$
Dataset 2	Hour	6	$\overline{7}$	8	9	10	11	12	
	GRPs	5	15	16	11	13	10	$\overline{7}$	
	Spots	54	181	161	75	113	109	98	
	Hour	13	14	15	16	17	18	19	20
	GRPs	10	12	$\overline{2}$	$\overline{4}$	$\overline{3}$	$\overline{3}$	$\overline{7}$	$\boldsymbol{0}$
	Spots	126	$\overline{2}$	13	70	$\overline{82}$	64	$\overline{39}$	$\boldsymbol{0}$
Dataset 3	Hour	6	$\overline{7}$	8	9	10	11	12	
	GRPs	9	9	36	6	$\overline{2}$	$\overline{2}$	$\overline{2}$	
	Spots	87	127	177	68	37	25	19	
	Hour	13	14	15	16	17	18	19	20
	GRPs	$\overline{4}$	$\overline{3}$	$\mathbf{1}$	$\overline{2}$	$\mathbf{1}$	$\overline{3}$	$\mathbf{1}$	$\mathbf{1}$
	Spots	20	88	$\mathbf{1}$	17	5	12	3	6

Table 4: Average radio GRPs per hour of the day

Table 5:

Estimation results for LM model for data set 1 (for the other data sets the estimation

results can be obtained from the authors)

Note: NS means "not significant"

Table 6:

Estimated Box-Cox parameters (with standard errors) for the models (18), (19) and (20)

	Dataset			
Frequency				
Hours	0.192	0.223	0.644	
	(0.005)	(0.006)	(0.009)	
Days	0.497	0.592	0.683	
	(0.048)	(0.066)	(0.035)	
Weeks	1.551	0.974	1.000	
	(0.010)	(0.272)	(0.029)	

Table 7:

Selected Estimation results of model (18) for hourly data in logarithms

Table 8:

Estimates of the parameter (and associated standard error) for the weekly lag (168 for hourly data, 7 for daily data and 1 for weekly data), based on models (18), (19) and (20)

		Dataset		
Transformation	Frequency		$\overline{2}$	3
None	Hours	0.090	0.084	0.237
		(0.007)	(0.008)	(0.008)
	Days	0.135	0.212	0.051
		(0.038)	(0.040)	(0.040)
	Weeks	0.371	0.736	0.683
		(0.094)	(0.069)	(0.066)
$Box - Cox$	Hours	0.092	0.089	0.208
		(0.007)	(0.007)	(0.008)
	Days	0.208	0.242	0.250
		(0.038)	(0.040)	(0.038)
	Weeks	0.684	0.719	0.769
		(0.021)	(0.058)	(0.063)

Table 9:

Average total impact per GRP, the case of data set 1

Table 10:

Average total impact per GRP, the case of data set 2

Table 11:

Average total impact per GRP, the case of data set 3

Table 12:

Forecast performance, in terms of Root Mean Squared Prediction Errors (underlined

are the smallest numbers, in the columns)

Figure 1: Weekly Incoming Calls

Figure 2: Average calls in weeks with and without commercial activity

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