REAL ESTATE IN AN ALM FRAMEWORK

THE CASE OF FAIR VALUE ACCOUNTING

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Keywords: Real Estate, Asset Liability Management, Liability Hedge Credit

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Abstract

This study examines the liability hedging characteristic of both direct and indirect real estate, in the advent of fair value accounting obligations for pension funds. We explicitly model pension obligations as being subject to interest and inflation risk to analyze the ability of real estate investments in hedging the market value of pension liabilities and to quantify its role in an ALM portfolio. Based on a sample period of 1984-2006, direct and indirect real estate merit inclusion in an ALM portfolio because of their attractive risk-reward properties and its diversification potential, rather than its liability hedging abilities.

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REAL ESTATE IN AN ALM FRAMEWORK *The Case of Fair Value Accounting*

Introduction

Real estate assets have traditionally been regarded as safe investments with inflation hedging capabilities that offer diversification potential and high absolute returns. Nevertheless there is no consensus as to its role within an investment context. In the selection of portfolios based on means and variances of returns the role for real estate, as a diversifier in a portfolio, appears to be substantial. For real estate allocations the mean-variance literature predicts allocations of at least 20% to be optimal¹. Conversely, institutional investors like pension funds are not solely aspiring for maximum returns at a selected level of risk in their portfolio choice. Their focus in making asset allocation decisions is on considering risk on a relative basis versus liabilities to optimize their risk adjusted surplus. When taking pension liabilities as the starting point and coordinating the management of assets and liabilities in order to maintain a surplus of assets beyond liabilities the role for real estate seems much more limited.

Chun, Ciochetti and Shilling (2000) offered the first empirical analysis of real estate allocations within an ALM framework. In their research they recognize real estate assets' correlation and diversification potential with other assets, while simultaneously adjusting for the covariance with the liability stream. The diversification potential on the liability side as a hedge against inflation turns out to be more limited and accounts for the reduced exposure to this asset class as witnessed in institutional portfolios. Even so, this earliest achievement in the asset-liability literature circumvents the imperfections associated with real estate by focusing on real estate securities (REITs) and as such limits the opportunity set of assets to solely indirect real estate. Furthermore, Chun, Ciochetti and Shiling (2000) focus on the reported value of projected benefit obligations and in the advent of fair value accounting

¹ For empirical evidence on real estate allocation within mean variance optimizations we like to refer to: Friedman (1971), Fogler (1984), Brinson, Diermeier, Schlarbaum (1986), Firstenberg, Ross and Zisler (1987), Irwin and Landa (1987), Ennis and Burik (1991), Hoesli, Lekander and Witkiewicz (2003) and Lee and Stevenson (2005).

obligations for pension funds it becomes of interest how real estate performs in hedging the market value of liabilities.

This paper adds to the existing literature by examining the liability hedge qualities of real estate in light of liabilities being denominated at market value. With the introduction of fair value accounting standards the dynamics of liabilities are likely to change. We study US data for the period 1984-2006 to quantify the assets' impact on the pensions fund's future funding surplus and quantify the utility that investors with liabilities can derive from real estate in view of other asset classes. This will enable us to determine whether to classify real estate and other assets as reserve asset, asset which moves in tandem with liabilities, or return-generating asset, an asset that merits an inclusion in the portfolio because of its attractive risk-reward characteristics (see Black and Jones, 1988). We widen the investment opportunity set by distinguishing between direct and indirect real estate investments and find that both direct and indirect real estate provide return enhancement properties as rather than interest and inflation properties. Nonetheless, direct and indirect real estate do provide more utility than stocks and as such merit an inclusion in an ALM portfolio, in contrast to the findings of Chun, Ciochetti and Shilling (2000).

The remainder of the paper is organized as follows: we present a synthesis of the most relevant theoretical and empirical analyses on real estate allocations and liability relative investing. In the third and fourth section the data set and the methodology of the empirical tests are presented. We initially proceed by quantifying real estate allocations assuming an asset-only mean variance optimization. Special consideration will be given to the impact of smoothed data on the calibration of optimal portfolios. Next, in section six, we analyze the liability hedging potential of various assets classes, results that will be used in section seven to compute optimal ALM portfolios and asses the interplay between the different weights attached to liabilities, levels of risk tolerance and funding levels of pension schemes. Finally, the last section summarizes our most important findings.

Literature Review

In the context of real estate allocations Friedman (1971) was one of the first to use the mean-variance methodology to select optimal direct real estate and mixed-asset portfolios.

The inclusion of real estate assets within modern theory based portfolios resulted in a widespread belief that actual real estate allocations in investment portfolios fall short. Bajtelsmit and Worzala (1995) put forward that on average American pension funds allocate less than 4% of their assets to equity real estate. In their survey among 96 pension funds the dominant asset classes were domestic stock (42.6%) and bonds (32%) followed by international stocks (7.2%). More recently, Dhar and Goetzmann (2005) surveyed leading investment managers from the U.S. and found the reported allocation among funds who invest in real estate to be relatively small 3%-5%, although a large number of funds announced plans to increase their respective allocations. Hoesli, Lekander and Witkiewicz (2005) explicitly compare the actual and suggested weights of real estate in the institutional portfolio and find that against the classic mean-variance framework, the predicted allocations are still inconsistent with reported allocations.

The discrepancy between actual allocations and theoretical predications in this asset-only view lead Chun, Ciochetti and Shilling (2000) to re-examine pension plan investments in an asset-liability framework using U.S. REITs. The relationship between assets and liabilities seems to be at the heart of explaining the limited exposure to real estate. Within the mean-variance framework real estate plays an important role as a diversifying asset class, but when accounting for liability obligations real estate seems to offer reduced diversification benefits as a hedge against inflation on the liability side of the balance sheet. The latter diversification potential accounts for the reduced exposure to this asset class as apparent among institutional portfolios. Chun, Ciochetti and Shilling (2000) also found cross-sectional differences in REIT allocations. For overfunded plans the optimal allocation is higher than for underfunded funds.

Following this first empirical ALM study on real estate allocations Craft (2001) further examined real estate investments by distinguishing between private and public real estate allocations, while correcting for appraisal smoothing. The asset-liability framework predicts an allocation of 12.5% to private real estate and 4.7% to public real estate. And as the returns increase the private real estate allocation decreases sharply while the allocation to public real estate decreases at a lower progressive pace. Moreover, overfunded pension plans are in accordance with Chun, Ciochetti, and Shilling (2000) much more likely to hold both private and public real estate than underfunded counterparts (Craft, 2005A, 2005B). The

particular and conditions of a pension fund apparently influence the allocation decision. In accordance Booth (2002) finds considerably different optimal portfolios depending on the liability structure of the pension funds. For mature U.K. schemes (whose members have already retired) direct real estate allocations prevail around 10%. For immature pension plans (active members) index-linked U.K. government bonds and U.S. equities replace real estate allocations.

Finally, another strand of literature by Fugazza, Guidolin and Nicadono (2007) and Hoevenaars, Molenaar, Schotman and Steenkamp (2005) incorporates predictability of asset returns in the optimal portfolio choice. Fugazza, Guidolin and Nicadono (2007) explicitly distinguish the time-varying properties of indirect real estate in light of bonds and stocks. When allowing for linear predictability patterns in indirect real estate returns the optimal allocation should obtain a weight between 12% and 44%, depending on the risk tolerance, parameter uncertainty and investment horizon. On the other hand when optimizing returns in excess of liabilities, Hoevenaars, Molenaar, Schotman and Steenkamp (2005) find that the role for indirect real estate in a liability driven investment portfolio is negligible.

This paper extends the work of Chun Ciochetti and Shilling (2000), Craft (2001) and Booth (2004) by examining the market value liability hedge qualities of real estate in light of other asset classes. We explicitly model liabilities as being subject to interest and inflation rate risk and summarize the assets' impact on the pensions fund's future funding surplus. To do so, this study applies a liability framework as developed by Sharpe and Tint (1990), which arises from a traditional mean-variance optimization problem. More specifically, it involves a mean variance surplus optimization model which optimizes the expected surplus return minus a risk penalty (variance surplus return) divided by risk tolerance, while taking into account the change in pension liabilities and their covariances with assets. The latter, also referred to as the liability hedge credit, quantifies the utility due to assets correlation with a pension fund liabilities. This methodology further allows for a differential in emphasis attached to liabilities, the level of risk tolerance and the funding level of pension schemes. This enables us to determine how sensitive the results are to these factors, but more importantly it permits pension funds to tailor their portfolios to their particular nature and objectives. And most notably, it allows institutional investors to quantify the liability hedging utility of each asset class.

Methodology

To determine real estates' role as a reserve asset (asset which moves in tandem with liabilities) or a return-generating asset we apply a single-period surplus optimization investment framework of Sharpe and Tint (1990) that explicitly links investment opportunities and pension-plan obligations. The objective of the pension fund is to maximize surplus, defined as:

$$S_{t+1} = A_{t+1} - kL_{t+1} , (1)$$

where A_{t+1} represents the value of the fund's assets at t+1, L_{t+1} the value of the relevant liability concept and k the attached importance to it. Choosing k=1 means that full importance is attached to the liabilities, k=0 corresponds to an asset-only optimization. Denoting the return on the asset portfolio by $R_{A, t+1}$ and the growth rate of the liabilities by $R_{L_{a},t+1}$, the surplus can be written as:

$$S_{t+1} = A_t \left[\left(1 + R_{A,t+1} \right) - k \frac{L_t}{A_t} \left(1 + R_{L,t+1} \right) \right],$$
(2)

where L_t/A_t denotes the fund's current inverse funding ratio.

Maximizing the expected utility of S_{t+1} is equivalent to maximizing that of

$$Z_{t+1} = R_{A,t+1} - k \left(\frac{L_t}{A_t}\right) R_{L,t+1}$$
(3)

Accordingly, Sharpe and Tint (1990) formulate the optimization problem of the pension fund as

$$\max\left[E_{t}(Z_{t+1})-\frac{1}{\lambda}\operatorname{var}_{t}(Z_{t+1})\right],\tag{4}$$

where λ denotes a fund's risk tolerance. If the portfolio weights to be chosen are denoted by w, we have $R_{A,t+1} = \sum_{i} w_i R_{i,t+1}$, where $R_{i,t+1}$ denotes the return on asset *i*.

Following Sharpe and Tint (1990) let us focus on the second term in (4), which can be written as

$$\operatorname{var}_{t}\left(R_{A,t+1} - k\frac{L_{t}}{A_{t}}R_{L,t+1}\right) = \operatorname{var}_{t}\left(R_{A,t+1}\right) + k^{2}\frac{L_{t}^{2}}{A_{t}^{2}}\operatorname{var}_{t}\left(R_{L,t+1}\right) - 2k\frac{L_{t}}{A_{t}}\operatorname{cov}_{t}\left(R_{A,t+1}, R_{L,t+1}\right)$$
(5)

The second term is irrelevant to the outcome of the maximization problem. The difference with the standard asset-only optimization problem is concentrated in the last term. It stresses that the assets' covariances with the growth rate of the liabilities are key for the optimal allocation. Sharpe and Tint (1990) define the liability hedge credit for any asset *i* as

$$LHC_{i} = \frac{2}{\lambda} k \frac{L_{t}}{A_{t}} \operatorname{cov}_{t} \left(R_{i,t+1}, R_{L,t+1} \right),$$
(6)

while the LHC of the entire portfolio is simply

$$LHC_a = \sum_i w_i LHC_i .$$
⁽⁷⁾

The total objective function follows a standard asset-only optimization problem, the expected surplus return minus a risk penalty, while considering the change in pension liabilities and their covariance with assets (LHC_a) .

$$\max\left[Expected return - Riskpenalty + LiabilityHedgeCredit\right]$$

$$\max\left[R_{A,t+1} - \frac{\operatorname{var}_{t}(R_{A,t+1})}{\lambda} + LHC_{a}\right]$$
(8)

Other things being equal, an asset whose returns are highly correlated with liabilities provide better liability hedging and receive a greater liability hedging credit. This ultimately results in a higher weight in the ALM portfolio than under the traditional mean variance optimization.

Data description

Our study employs data from the United States, as for this country broad data coverage on both appraisal based property indices and property share indices are available. The analysis of the asset returns is estimated over the 1983 to 2006 period, taking quarterly observations. Data on stock returns were taken from Datastream Advance. Stock returns are approximated by the returns on the MSCI US index, and direct real estate returns are from the NCREIF series. NCREIF provides income, capital and total returns disaggregated by sector and region based on a sample of institutional-owned properties. Indirect real estate returns are based on Global Property Research (GPR) General National index. The data is available since the last quarter of 1983. The fixed income assets consist of a 20-year Treasury bond and Moody's Seasoned Aaa Corporate bond. Furthermore we use a 10-year constant maturity yields as a proxy for pension liabilities. Both the 10-year constant maturity and Moody's Seasoned Aaa Corporate bond yields, are from the US Federal Reserve Bank website². Moody's Seasoned Aaa Corporate Bond Yield are averages of daily data. The 20-year Treasury bond is based on an index from Lehman Brothers. Table 1 presents a summary of the performance of the asset categories that are considered in our study.

Table 1: Sample Statistics

The mean returns and standard deviations are computed for the 1984-2006 sample period. We document the highest return for indirect real estate, while direct real estate seems to have been outperformed by all asset classes. However, the Sharpe ratio of direct real estate, calculate as annualized excess return divided by the annualized standard deviation of returns, is particularly favorable, 0.56 versus 0.27 for stocks, 0.26 for indirect real estate, 0.25 long-term Treasury bond and 0.46 for Corporate Aaa bonds. The returns on appraisal based direct real estate document the lowest standard deviations, and appear to be biased by strong autocorrelation. In line with the Geltner (1993) approach we adjust the private real estate the assumption that returns in the private property market are uncorrelated. Geltner's model applies a reverse filter on the capital growth component of private real estate returns in order to recover the underlying unsmoothed property returns³. Following Geltner (1993) we assume that the volatility of commercial property is in the vicinity of half of that of the stock

² http://research.stlouisfed.org/

³ $R_t^u = \frac{(R_t^* - (1 - a)R_{t-1}^*)}{a}$, where R_t^U is the unsmoothed return at time t, R_t^* is the observable appraisedbased index return at time t and a is a parameter between 0 and 1 whose value depends on the confidence factor α (α =0.5) and seasonality factor f (f= 0.15).

market, which results in an a of 0.40^4 . By unsmoothing the direct real estate returns the level of autocorrelation is strongly reduced, while the standard deviation doubles in magnitude.

For the analysis of the liability returns we assume that the return on liabilities follows the return of the long-term constant maturity bond, estimated over the 1983 to 2006 period. Our specification abstracts from inflows and outflows, the fund is assumed to be in a stationary state, the distribution of the age cohorts and pension rights are assumed constant over time. The liability return is derived as a function of the log yield of the constant maturity bond, assuming duration of 17 years, the average duration of pension liabilities.

$$(1+r_{n,t+1}) = D_{n,t}(1+Y_{n,t}) - (D_{n,t} - \frac{1}{4})(1+Y_{n-1,t+1}) = \frac{1}{4}Y_{n-1,t+1} - (D_{n,t}(Y_{n-1,t+1} - Y_{n,t}))$$

 $Y_{n,t}$ is the log annualized yield of a n-year maturity bond at time t. We further approximate $Y_{n-1,t+1}$ by $Y_{n,t+1}$, a common assumption also made by Hoevenaars, Molenaar, Schotman and Steenkamp (2005).

We further distinguish two series of nominal and real liability returns. In quantifying the utility a pension fund can derive form various asset classes we use nominal liabilities as the starting point, in the advent of indexation we consider the hedging capabilities surrounding real liabilities.

We calculate both the nominal (r_{t+1}) and the real liability return (rr_{t+1}) . We decompose the nominal yield into a real return (rr_{t+1}) and inflation compensation, where the inflation compensation reflects expected inflation (π_{t+1}) and an inflation risk premium, which for reasons of simplicity we assume constant.

$$Y_t^{nom} = Y_t^{real} + E[\pi_{t+1}]$$

We further assume that inflation expectation follows a fourth order autoregressive function. The coefficients are estimated using rolling regressions and 10 years of history (40 quarterly observations). Thus we assume that for each quarter investors form expectations on the

⁴ In appendix A we also test alternative specifications by unsmoothing real estate returns at a-factors of 0.3 and 0.5 in order to isolate the impact of this assumption on our overall results.

basis of the last 10 years of quarterly observations. This way, we will explicitly model the riskinesss of liabilities as being subject to interest and inflation risk and determine which asset class is capable of providing the appropriate payout structure to hedge interest rate risk and aid in providing indexation.

Asset-only Optimization

Mean-variance methodology ensures a portfolio selection that embodies diversification between assets and identifies the efficient set of portfolios that maximize expected return while minimizing the variance of the expected returns. Risk reduction is a function of low correlation coefficients between asset classes. Table 2 presents a correlation matrix between the returns on stocks, long-term Treasury bonds, corporate bonds and direct and indirect real estate. Direct real estate returns appear to be only weakly correlated to stocks and longterm Treasury and negatively correlated with corporate bonds. Real estate stocks in contrast do not appear to be highly correlated with direct real estate investments, but more with common stocks. In the context of portfolio diversification direct real estate offers greater risk diversification benefits. The negative covariance of direct real estate with stocks and bonds should greatly reduce portfolio risk.

Table 2: Asset Only Allocations

The efficient set of portfolios that maximize expected return for a given level of risk are constructed under the standard mean-variance analysis. We impose short-selling constraints on all assets and portfolios must be fully invested. In the selection of portfolios based on means and variances of returns the role for direct real estate, as a risk diversifier in a portfolio, is substantial. The negative correlation of direct real estate with bonds and stocks in combination with the low standard deviation of returns results in high allocations to this asset class in the low-risk range of the efficient frontier. The mean-variance model, on the basis of smoothed real estate returns, estimates allocations to direct real estate as high as 56.43%. The results are in accordance with the first strand of literature on real estate mean-variance allocations, Friedman (1971), Firstenberg, Ross and Zisler (1998), Fogler (1984), Brinson, Diermeier, Schlarbaum (1986), Irwin and Landa (1987), Ennis and Burik (1991), Hoesli, Lekander and Witkiewicz (2003) and Lee and Stevenson (2005))

The observed autocorrelation of 0.72 in the NCREIF index (direct real estate) appears to impede optimal calibration of mean-variance efficient portfolios. The use of appraisals in the construction of the NCREIF index affects the mean and volatility of the direct real estate return distribution. When correcting for the misspecification in direct real estate returns using Geltner's approach (1993) the standard deviation of returns doubles in magnitude reducing the attractive risk adjusted return characteristic of direct real estate. The measured risk appears to be understated, overstating the direct real estate allocation by up to 27.58%. Risk diversification benefits diminish as the correlation coefficients with stocks, bonds and indirect real estate increases. The differences in direct real estate allocations are prevalent in the low risk portfolios, where the allocation to this particular asset class dominates. The results of the asset-only mean variance optimization and the difference between smoothed and unsmoothed portfolios are presented in panel A of Table 2, where we display two portfolios of the efficient frontier: the minimum variance portfolio and the optimal Sharpe portfolio.

Panel B in Table 2 reports portfolio compositions for seven portfolios on the efficient frontier, beginning with the minimum variance portfolio (MVP) and optimal Sharpe portfolio and ending up at the high risk range of the efficient frontier. Mean variance efficient portfolios tend to contain a high level of direct real estate and a high portion of indirect real estate at higher risk levels. At the higher risk tolerance levels indirect property investments substitute the direct counterpart. The absence of indirect real estate in the low risk and optimal Sharpe portfolios can be explained by the high standard deviation of the asset class and the high correlation with stocks, while direct real estate allocations range between 19.71% and 34.8%. The results are in line with those of Ziering and McIntosh (1997), Ziobrowski and Ziobrowski (1997), Kallberg, Lui and Greig (1996), and Mueller and Mueller (2003), who account for the added smoothing risk but still find an optimal real estate allocation of 20%-30%. On the basis of a mean-variance asset-only optimization direct real estate warrants inclusion in a mixed-asset portfolio because of its attractive risk-reward properties and its low correlation with stocks and bonds.

The Liability Hedge Potential

An asset-liability model (ALM) is a model of the assets and liabilities that facilitates decisionmaking with respect to asset allocation and the properties of the liabilities. An important distinguishing feature is the interdependence between assets and liabilities. Table 3 presents the correlations between liabilities and asset returns for the asset classes considered in this analysis.

Table 3: Correlation of asset returns with pension liabilities (1984-2006)

For both direct and indirect real estate we document mildly negative correlations, indicating that direct and indirect real estate appear to offer reduced hedging benefits against interest rate risk and inflation.

In terms of the liability utility to be derived, the liability hedge credit (LHC) follows directly from the correlations of the asset return with the liability returns, current assets to liability ratio and the risk tolerance.

$$LHC_{i} = \frac{2}{\lambda} k \frac{L_{t}}{A_{t}} \operatorname{cov}_{t} \left(R_{i,t+1}, R_{L,t+1} \right)$$
(6)

The liability hedge credit is positively related to the covariance between an asset and liabilities and to the inverse of the current funding ratio (L_t/A_t) , while inversely related to the risk tolerance (λ). The LHC corrected for inflation for direct real estate is weakly negative ranging from -0.001% in the full surplus optimization scenario for fully funded funds with a typical risk tolerance (λ =10) to -0.002% for underfunded funds (L/A=1.5) under similar constraints. Including direct real estate results in a slight penalty. The low standard deviation and risk-reward characteristics of direct real estate render it more attractive for overfunded and more risk tolerant funds.

Figure 1: Nominal and Real Liability Hedge Credit (LHC) per asset class

As the funding status improves so does the hedging utility to be derived from the asset class. In accordance with Chun, Ciochetti, and Shilling (2000) and Craft (2001) we would expect overfunded funds to be much more likely to hold public real estate than underfunded counterparts.

Figure 2: Real Liability Hedge Credit (LHC) per asset class

For indirect real estate the liability hedging utility is somewhat inferior, due to the higher standard deviation of the asset class. Attaching less importance to maximizing surplus but more to absolute returns improves the utility to be derived from indirect real estate. Moreover, the higher the risk tolerance of the institutional investor the lower the liability penalty for the real estate asset class. Direct real estate and indirect real estate provide more utility than stocks. The correlation of stocks with real liabilities is negative and double the magnitude of direct real estate. However the hedging utility is limited and it therefore appears that direct and indirect real estate investments provide return enhancement properties next to risk diversification rather than interest and inflation diversification. Solely fixed income instruments offer a liability benefit for the portfolio of a pension fund, as the correlation between these asset and liabilities is high⁵. Including these asset classes in a pension portfolio can enhance returns up to 33 and 16 bps for fully funded funds with a typical risk tolerance (λ =10). A clear distinction arises between return generating assets like stocks and real estate and liability hedging asset classes like bonds, as the latter provide the most utility when the funding status is unfavorable, while return generating assets provide an improved utility when pension funds are overfunded.

Overall, we have seen that the correlation between liabilities and both direct and indirect real estate is negative and consequently so is the liability hedging return to be derived from these asset classes. On the other hand, the correlation and hedging properties of both property categories is greater than that of stocks and solely fixed income securities offer a positive hedge. In the next section we will advance from the hedging characteristics to the implications for an ALM portfolio.

⁵Note that liabilities were derived from constant maturity bond, as such the hedging potential might be biased upwards.

ALM Portfolio Optimization

Within the mean-variance (asset-only) framework real estate plays an important role as a diversifying asset class. Mean-variance efficient portfolios tend to contain a high level of direct real estate and a high portion of indirect real estate at higher risk levels. When accounting for liability obligations real estate offers reduced hedging benefits against inflation and interest rates. The liability hedging utility to be derived from these asset classes is negative. However the correlation and hedging properties of both property categories is greater than that of stocks, resulting in a high weight for property in an ALM framework.

At low risk levels the total real estate allocation in an ALM framework seems in line with the asset-only scenario. To obtain a similar standard deviation as the minimum variance portfolio in the asset-only scenario a fully invested pension fund would need to allocate 34.54% to direct real estate when considering liabilities versus the 34.80% when focusing on asset returns solely.

Table 4: Asset-only versus ALM allocations

The equivalent maximum Sharpe ratio portfolio still contains considerable direct real estate exposure, 27.85% versus 29.14%. Again this is mainly attributable to the higher LHC of direct real estate vis-à-vis indirect real estate and stocks. Furthermore, as funding ratios improve or the importance attached to liabilities deteriorate, direct real estate obtains a position in the higher risk portfolios. Direct real estate materializes in an asset-liability portfolio as a relatively safe asset class of particular utility to fully funded or risk-averse pension funds. In line with Chun, Ciochetti, and Shilling (2000) and Craft (2001) overfunded funds are much more likely to hold public real estate than underfunded counterparts. At higher risk levels, real estate. For underfunded pension funds high return-generating asset classes, such as indirect real estate, offer more utility in an optimal portfolio in contrast to the results of Chun, Ciochetti and Shilling (2000), who find a reduced role for indirect real estate in an ALM portfolio. Indirect real estate provides more utility than stocks and as such warrants a position in an ALM portfolio. Additionally, if we restrict the opportunity set of real estate to direct real estate solely, we obtain an allocation of 5.22% in the tangent

portfolio in an asset-only context, and when considering real liabilities this allocation increases to 5.43% (Appendix B).

The optimal portfolio allocations for real estate, both direct and indirect, range from 16% to 35%, which result in an expected return utility between 9% and 11%, depending on the level of risk tolerance and funding ratio of the pension fund. With respect to the asset-only framework the weight of stocks and direct real estate decreased, while the weight of bonds and indirect real estate increased. When accounting for risky liabilities as being subject to interest and inflation risk, we cannot corroborate the results of Chun, Ciochetti and Shilling (2000), as we still observe a considerable role for real estate in an ALM portfolio.

Robustness check

As with the mean variance analysis assumptions are required concerning the inputs of the model. Portfolios were derived using historical returns and risk characteristics; nonetheless some might argue that past returns are not representative for future expected returns. In this section we perform a sensitivity analysis in which we model the worst case scenario for real estate to determine whether our results are robust to uncertainty surrounding expected returns.

Table 5: Sensitivity analysis ALM Allocations

We create ALM efficient portfolios based on the perception of possible underperformance of real estate in relation to its past performance. In our analysis, we set the expected returns of both direct and indirect real estate equal to the 25th percentile of the historical return distribution. This corresponds to an expected return of -4.60% for indirect (GPR) and 4.41% for direct real estate (NCREIF). In the advent of both direct and indirect real estate underperforming the optimal portfolios still obtain an allocation towards direct real estate, ranging from 13.22% to 34.43%. In effect direct real estate still obtains such a large allocation because of its liability hedging potential vis-à-vis stocks and indirect real estate. The role for indirect real estate is lost but in fact the role of indirect real estate was already limited in the minimum variance and optimal Sharpe portfolios. Only in the high risk range of the efficient frontier indirect real estate no longer obtains an allocation, in case of seriously underperforming. We therefore consider the results to be robust to bearish views on the future performance of both direct real estate and indirect real estate. Even under pessimistic assumptions real estate allocations are substantial.

Conclusion

Real estate assets have traditionally been regarded as safe investments with inflation hedging capabilities that offer diversification potential and high absolute returns. However, the results show that when accounting for liability obligations at market value direct real estate warrants inclusion in a mixed-asset portfolio because of its attractive risk-reward properties and its diversification potential with stocks and bonds, but not because of its interest and inflation hedging abilities. The hedging utility of real estate is limited and it therefore appears that direct and indirect real estate investments provide return enhancement properties rather than interest and inflation properties. Even so, direct real estate and indirect real estate do provide more utility than stocks and as such warrant a position in an ALM portfolio. With respect to the asset-only framework the weight of stocks and direct real estate decreased, while the weight of bonds and indirect real estate increased. Direct real estate to materializes in an asset-liability portfolio as a relatively safe asset class of particular utility to fully funded pension funds. At higher risk levels, real estate exposure in terms of direct real estate erodes as real estate stocks dominate direct real estate. For underfunded pension funds high returngenerating asset classes, such as indirect real estate and stocks, offer more utility in an optimal portfolio. The optimal portfolio allocations for real estate range from 16% to 35%, which result in an expected return utility between 9% and 11%, depending on the level of risk tolerance and funding ratio of the pension fund. The results are robust to bearish views on the future performance of both direct real estate and indirect real estate and corroborate that in effect actual portfolio allocations of pension funds still fall short.

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Table 1: Sample Statistics

Mean returns and standard deviations are annualized quarterly total returns related to the sample period of 1984-2006. The mean returns are also displayed in excess of the 3-month T-bill. Stock returns are based on MSCI indices, and direct real estate returns are from NAREIT series. Indirect real estate returns are based on GPR indices. Moody's Seasoned Aaa Corporate Bond (Duration 7 years) and the 3-month T-bill are from the US Federal Reserve Bank website. The 20-year Treasury bond is based on an index from Lehman Brothers. For panel D real estate returns were adjusted for first-order autocorrelation using Geltner's model (1993). Constituents of the NCREIF Composite Index were segregated into 4 sub-sectors: office, industrial, retail and apartments. A company is classified in a specific property sub-sector if 75 percent or more of its gross invested book assets is invested in that specific sub-sector. The return of a sector in each quarter is calculated as the sum of weighted average returns of the individual properties. Panel D exhibits the correlation matrix of the asset classes. NCREIF stands for unsmoothed direct real estate returns, adjusted for first-order-autocorrelation using Geltner's model (1993).

Panel A: Annualized Return	Return	Excess Return	Std. Dev.	Autocorrelation
Stock (MSCI)	11.94%	7.17%	27.01%	0.02
Real Estate Stock (GPR)	13.10%	8.32%	32.19%	0.05
Direct Real Estate (NCREIF)	8.48%	3.70%	6.56%	0.72
Unsmoothed Direct Real Estate (NCREIF*)	8.58%	3.80%	11.57%	0.29
20-Year Treasury Bond	9.68%	4.90%	19.85%	-0.09
Moody's Seasoned Aaa Corporate Bond Yield	9.08%	4.30%	9.29%	0.03

	Subperiod (1995-2007)	Subperiod (1984-1994)	
Panel B: Annualized Return Subperiods	Return	Std.Dev.	Return	Std. Dev.
Stock (MSCI)	13.31%	13.47%	10.72%	13.50%
Real Estate Stock (GPR)	11.54%	18.88%	14.50%	13.55%
Direct Real Estate (NCREIF)	4.96%	3.07%	11.63%	2.21%
Unsmoothed Direct Real Estate (NCREIF*)	4.71%	6.97%	12.05%	3.68%
20-Year Treasury Bond	11.14%	10.82%	8.36%	8.85%
Moody's Seasoned Aaa Corporate Bond Yield	10.59%	5.05%	7.73%	3.98%

Panel C: Descriptive Statistics Bonds	YTM	Return	Excess return	
20-Year Treasury Bond	7.21%	9.68%	4.90%	
Moody's Seasoned Aaa Corporate Bond Yield	7.88%	9.08%	4.30%	

Panel D: Correlation Asset returns

	MSCI	GPR	20y Treasury bond	Corporate AAA bond	NCREIF
MSCI	1.00				
GPR	0.58	1.00			
20y Treasury bond	-0.03	0.08	1.00		
Corporate AAA bond	-0.11	0.00	0.89	1.00	
NCREIF	0.07	0.1	0.01	-0.05	1.00
NCREIF*	0.08	0.16	0.05	-0.02	0.82

Table 2: Asset Only Allocations

Portfolios were derived using historical return and risk characteristics (1984-2006). The upper part of the table in panel A reports allocations using smoothed Real Estate returns and the bottom half includes unsmoothed returns. The unsmoothed Real Estate returns were adjusted for first-order autocorrelation using Geltner's model (1993), where the a-factor equaled 0.4. The fixed income assets consist of a 20-year Treasury maturity bond and Moody's Seasoned Aaa Corporate bond (Duration 7 years). The Optimal Sharpe portfolio represents the tangency portfolio that optimizes the mean excess return divided by the standard deviation of returns. Panel B reports portfolio compositions for five portfolios along the efficient frontier.

Panel A: Mean-Variance Allocations (Asset Only)											
Portfolios	Er	Std. Dev.	MSCI	GPR	NCREIF	Fixed Income	Total Real Estate				
Minimum Variance	8.81%	2.56%	3.73%	0.00%	62.38%	33.89%	62.38%				
Optimal Sharpe	8.96%	2.61%	5.94%	1.17%	56.43%	36.46%	57.60%				
Unsmoothed Real Estate Returns											
Minimum Variance	9.12%	3.43%	7.36%	0.00%	34.80%	57.85%	34.80%				
Optimal Sharpe	9.31%	3.51%	10.51%	1.91%	29.14%	58.43%	31.05%				
Panel B: Mean-Variance Efficie	ent Portfol	ios (Asset-	Only)								
		Sharpe									
Unsmoothed Real Estate Returns	MVP	Optimal	(1)	(2)	(3)	(4)	(5)				
Er	9.12%	9.31%	9.52%	9.90%	10.20%	10.46%	10.69%				
Std. Dev.	3.43%	3.51%	3.77%	4.45%	5.14%	5.82%	6.51%				
Portfolio Weights											
MSCI	7.36%	10.51%	12.50%	15.82%	18.25%	20.48%	22.39%				
GPR	0.00%	1.91%	5.49%	11.16%	15.67%	19.71%	24.15%				
NCREIF	34.80%	29.14%	23.07%	13.42%	6.06%	0.00%	0.00%				
Fixed Income	57.85%	58.43%	58.94%	59.60%	60.02%	59.80%	53.46%				
Total Real Estate Exposure	34.80%	31.05%	28.56%	24.58%	21.73%	19.71%	24.15%				

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Table 3: Correlation of asset returns with pension liabilities (1984-2006)

We assume a 17-year constant maturity bond as a proxy for pension liabilities. The fund is assumed to be in a stationary state, the distribution of the age cohorts and pension rights are assumed constant over time. The market value of liabilities is solely influenced by changes in interest rates and inflation. The liability return is derived as a function of log return and log yield. Real Liabilities are adjusted for inflation and interest rate risk. We decompose the nominal yield into a real yield and inflation compensation, where the inflation compensation reflects expected inflation (π_{t+1}) and an inflation risk premium, which for reasons of simplicity we assume constant. We further assume that inflation expectation follows a fourth order autoregressive function. NCREIF* stands for unsmoothed direct real estate returns, adjusted for first-order-autocorrelation using Geltner's model (1993). Table 3 displays the correlation coefficients of the asset returns with liabilities, while panel B shows the correlation coefficients of the direct property sub-sectors with pension liabilities.

	MSCI	GPR	NCREIF*	20y Treasury bond	Corporate Aaa bond	Liability (Nominal)	Liability (Real)
MSCI	1.00						
GPR	0.58	1.00					
NCREIF*	0.08	0.16	1.00				
20y Treasury bond	-0.03	0.08	0.05	1.00			
Corporate bond	-0.11	0.00	-0.03	0.90	1.00		
Liability (Nominal)	-0.15	-0.01	-0.01	0.91	0.92	1.00	
Liability (Real)	-0.15	-0.03	-0.01	0.91	0.92	1.00	1.00

Figure 1: Nominal and Real Liability Hedge Credit (LHC) per asset class

The liability hedge credit (LHC) quantifies the annualized utility that investors with liabilities can derive from different asset classes. LHC is positively related to the covariance of an asset and the liability and to the current assets to current liabilities (L_0/A_0), while inversely related to the risk tolerance (λ). LHC further depends on the weight of importance attached to it (k), full consideration of liabilities (k=1) yields similar results as surplus optimization, while no consideration (k=0) provides the same results as an asset-only methodology. The following graphs depict LHC's for nominal and real liabilities. Liability Hedge Credit for Nominal Liabilities are adjusted for interest rate risk and Real Liabilities are adjusted for inflation and interest rate risk. To obtain the real liability returns we decompose the nominal yield into a real yield and inflation compensation, where the inflation compensation reflects expected inflation (π_{t+1}) and an inflation risk premium, which for reasons of simplicity we assume constant. We further assume that inflation expectation follows a fourth order autoregressive function.

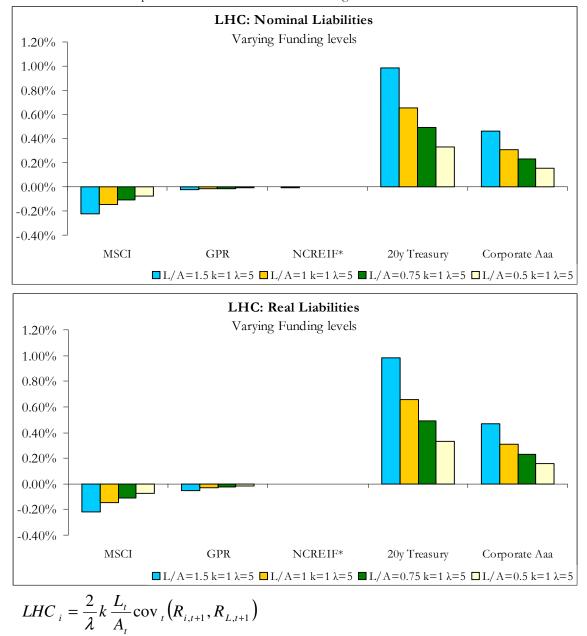
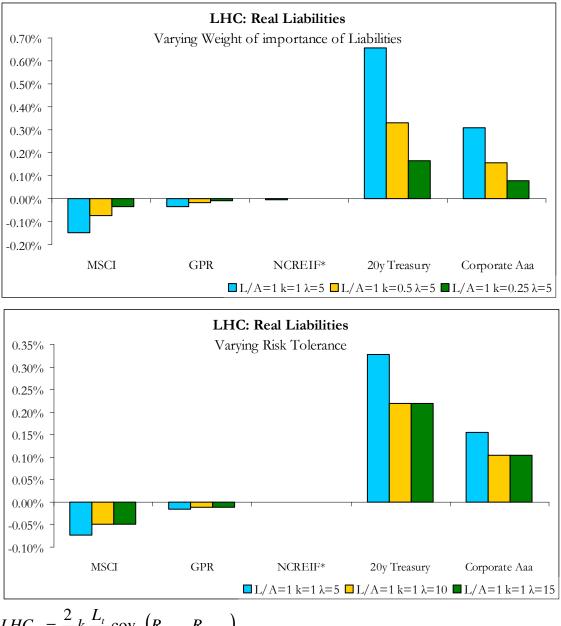


Figure 2: Real Liability Hedge Credit (LHC) per asset class

The liability hedge credit (LHC) quantifies the annualized utility that investors with liabilities can derive from different asset classes. LHC is positively related to the covariance of an asset and the liability and to the current assets to current liabilities (L_0/A_0), while inversely related to the risk tolerance (λ). LHC further depends on the weight of importance attached to it (k), full consideration of liabilities (k=1) yields similar results as surplus optimization, while no consideration (k=0) provides the same results as an asset-only methodology. The following graphs depict real LHC's for varying weight of importance and risk tolerance.



$$LHC_{i} = \frac{2}{\lambda} k \frac{L_{t}}{A_{t}} \operatorname{cov}_{t} \left(R_{i,t+1}, R_{L,t+1} \right)$$

Table 4: Asset-only versus ALM allocations

Portfolios were derived using historical return and risk characteristics (1984-2006). Mean returns and standard deviations are annualized quarterly total returns related to the sample period of 1984-2006. Stock returns are based on MSCI indices, and direct real estate returns are from NAREIT series for the U.S. Indirect real estate returns are based on GPR indices. The fixed income assets consist of a 20-year Treasury bond and Moody's Seasoned Aaa Corporate Bond (Duration 7 years). The direct real estate returns were adjusted for first-order autocorrelation using Geltner's model (1993) (a=0.4). The Optimal Sharpe portfolio represents the tangency portfolio that optimizes the mean excess return divided by the standard deviation of returns. Portfolio compositions for five portfolios along the efficient frontier are also given. Portfolio weights are determined by maximizing the objective function given the standard deviation of the asset-only portfolios.

			Portfolio W	eight			
				Indirect	Direct Real		
			Stocks	Real Estate	Estate		Total Real
Portfolios	Er	Std. Dev.	(MSCI)	(GPR)	(NCREIF*)	Fixed Income	Estate
Mean-Variance Allocation	• • •						
Minimum Variance	9.12%	3.43%	7.36%	0.00%	34.80%	57.85%	34.80%
Optimal Sharpe	9.31%	3.51%	10.51%	1.91%	29.14%	58.43%	31.05%
(1)	9.52%	3.77%	12.50%	5.49%	23.07%	58.94%	28.56%
(2)	9.90%	4.45%	15.82%	11.16%	13.42%	59.60%	24.58%
(3)	10.20%	5.14%	18.25%	15.67%	6.06%	60.02%	21.73%
(4)	10.46%	5.82%	20.48%	19.71%	0.00%	59.80%	19.71%
(5)	10.69%	6.51%	22.39%	24.15%	0.00%	53.46%	24.15%
ALM Allocations (Real Lia	abilities)						
$L/A=1.5 \ k=1 \ \lambda=5$							
Minimum Variance	9.35%	3.43%	7.52%	0.00%	34.51%	57.97%	34.51%
Optimal Sharpe	9.54%	3.51%	9.89%	1.97%	27.29%	60.85%	29.27%
(1)	9.77%	3.77%	11.24%	5.64%	19.52%	63.60%	25.16%
(2)	10.13%	4.45%	13.53%	11.38%	6.73%	68.37%	18.11%
(3)	10.41%	5.14%	15.15%	16.71%	0.00%	68.14%	16.71%
(4)	10.64%	5.82%	16.65%	19.76%	0.00%	63.59%	19.76%
(5)	10.85%	6.51%	18.13%	22.55%	0.00%	59.31%	22.55%
$L/A=1$ k=1 $\lambda=5$							
Minimum Variance	9.27%	3.43%	7.53%	0.00%	34.54%	57.94%	34.54%
Optimal Sharpe	9.45%	3.51%	10.11%	1.97%	27.85%	60.08%	29.81%
(1)	9.68%	3.77%	11.72%	5.59%	20.57%	62.12%	26.16%
(2)	10.04%	4.45%	14.35%	11.39%	8.85%	65.41%	20.24%
(3)	10.32%	5.14%	16.33%	15.96%	0.00%	67.71%	15.96%
(4)	10.55%	5.82%	17.95%	20.60%	0.00%	61.45%	20.60%
(5)	10.75%	6.51%	19.67%	23.48%	0.00%	56.85%	23.48%
$L/A=0.5 \ k=0.5 \ \lambda=5$							
Minimum Variance	9.13%	3.43%	7.31%	0.00%	34.90%	57.79%	34.90%
Optimal Sharpe	9.33%	3.51%	10.42%	1.93%	28.82%	58.83%	30.75%
(1)	9.56%	3.77%	12.35%	5.52%	22.47%	59.66%	27.99%
(2)	9.91%	4.45%	15.39%	11.29%	12.14%	61.18%	23.43%
(3)	10.19%	5.14%	17.92%	15.69%	4.45%	61.95%	20.13%
(4)	10.43%	5.82%	19.85%	20.16%	0.00%	59.99%	20.16%
(5)	10.64%	6.51%	21.64%	24.68%	0.00%	53.68%	24.68%
$L/A=0.5 \ k=0.5 \ \lambda=10$							
Minimum Variance	9.13%	3.43%	7.48%	0.00%	34.65%	57.87%	34.65%
Optimal Sharpe	9.32%	3.51%	10.47%	1.92%	28.97%	58.63%	30.89%
(1)	9.55%	3.77%	12.44%	5.50%	22.80%	59.26%	28.30%
(1) (2)	9.91%	4.45%	15.55%	11.27%	12.78%	60.39%	24.05%
(3)	10.20%	5.14%	18.04%	15.70%	5.12%	61.15%	20.81%
(3) (4)	10.44%	5.82%	20.19%	19.92%	0.00%	59.89%	19.92%
(5)	10.67%	6.51%	22.02%	24.41%	0.00%	53.56%	24.41%

Table 5: Sensitivity Analysis ALM Allocations

The portfolios are based on the perception of possible underperformance of real estate in relation to its past performance. The expected performance of both direct and indirect real estate is expected to be equal to its 25th percentile return (GPR Er=-4.60%, NCREIF Er=4.41%). Optimal Sharpe portfolio represents the tangency portfolio that optimizes the mean excess return divided by the standard deviation of returns. Portfolio compositions for five portfolios along the efficient frontier are also given. Portfolios must be fully invested.

Portfolio Weights								
ALM L/A=1 k=1 λ =5	MSCI	GPR	NCREIF*	Fixed Income				
Minimum Variance	7.53%	0.00%	34.54%	57.94%				
Optimal Sharpe	10.11%	1.97%	27.85%	60.08%				
(1)	11.72%	5.59%	20.57%	62.12%				
(2)	14.35%	11.39%	8.85%	65.41%				
(3)	16.33%	15.96%	0.00%	67.71%				
(4)	17.95%	20.60%	0.00%	61.45%				
(5)	19.67%	23.48%	0.00%	56.85%				
(GPR Er=-4.60%, NCREIF I	Er=4.41%)							
Minimum Variance	7.43%	0.00%	34.43%	58.14%				
Optimal Sharpe	10.24%	0.00%	24.31%	65.45%				
(1)	13.13%	0.00%	13.22%	73.65%				
(2)	22.56%	0.00%	0.00%	77.44%				
(3)	31.57%	0.00%	0.00%	68.43%				
(4)	36.13%	0.00%	0.00%	63.87%				
(5)	40.39%	0.00%	0.00%	59.61%				

Appendix A: Asset-only allocations (under varying unsmoothing scenarios)

Appendix A displays correlation coefficients, which were computed over the 1984-2006 period. The sub-sector returns were derived from the NCREIF returns. Constituents of the NCREIF Composite Index are segregated into 4 sub-sectors: office, industrial, retail and apartments. A company is classified in a specific property sub-sector if 75 percent or more of its gross invested book assets is invested in that specific sub-sector. The return of a sector in each quarter is calculated as the sum of weighted average returns of the individual properties. Returns were adjusted for first-order-autocorrelation using Geltner's model (1993).

Mean-Variance Efficient Portfolios under different unsmoothing scenarios									
							Total	Difference	
		Std.				Fixed	Real	with	
Unsmoothed Real Estate Returns	Er	Dev.	MSCI	GPR	NCREIF*	Income	Estate	a=0.4	
(a=0.4) Minimum Variance	9.12%	3.43%	7.36%	0.00%	34.80%	57.85%	34.80%		
(a=0.4) Sharpe Optimal	9.31%	3.51%	10.51%	1.91%	29.14%	58.43%	31.06%		
(a=0.5) Minimum Variance	9.02%	3.17%	6.07%	0.00%	43.92%	50.01%	43.92%	9.12%	
(a=0.5) Sharpe Optimal	9.20%	3.24%	9.15%	1.46%	37.77%	51.62%	39.23%	8.17%	
(a=0.3) Minimum Variance	9.74%	3.72%	9.03%	0.00%	23.41%	67.56%	23.41%	-11.39%	
(a=0.3) Sharpe Optimal	9.88%	3.78%	11.28%	1.94%	19.27%	67.50%	21.22%	-9.84%	

Appendix B: Asset-only versus ALM allocations (Indirect Real Estate)

Portfolios were derived using historical return and risk characteristics (1984-2006). Mean returns and standard deviations are annualized quarterly total returns related to the sample period of 1984-2006. The opportunity set of available real estate assets is restricted to indirect real estate solely. Stock returns are based on MSCI indices, and indirect real estate returns are based on GPR indices. The fixed income assets consist of a 20-year Treasury bond and Moody's Seasoned Aaa Corporate Bond (Duration 7 years). Portfolio compositions are reported proportional to the risk from the Minimum Variance (MVP) portfolios. The Optimal Sharpe portfolio represents the tangency portfolio that optimizes the mean excess return divided by the standard deviation of returns. Portfolio weights are determined by maximizing the objective function given the standard deviation of the asset-only portfolios.

			Portfolio Weight		
Portfolios	Er	Std. Dev	Stocks (MSCI)	Indirect Real Estate (GPR)	Fixed Income
Mean-Variance Allocation	s (Asset-Only	/)			
Minimum Variance	9.47%	4.23%	12.47%	0.79%	86.73%
Optimal Sharpe	9.70%	4.33%	14.36%	5.22%	80.42%
(1)	9.95%	4.65%	16.36%	9.92%	73.72%
(2)	10.34%	5.49%	19.48%	17.37%	63.16%
(3)	10.64%	6.34%	21.87%	23.12%	55.01%
(4)	10.90%	7.18%	24.21%	27.90%	47.89%
(5)	11.16%	8.03%	26.56%	31.63%	41.82%
ALM Allocations (Real Lia	abilities)				
$L/A=1.5 k=1 \lambda=5$,				
Minimum Variance	9.58%	4.23%	12.53%	0.93%	86.54%
Optimal Sharpe	9.80%	4.33%	14.07%	5.43%	80.51%
(1)	10.03%	4.65%	15.73%	10.36%	73.91%
(2)	10.39%	5.49%	18.43%	18.10%	63.47%
(3)	10.66%	6.34%	20.43%	23.67%	55.90%
(4)	10.92%	7.18%	22.82%	27.14%	50.04%
(5)	11.16%	8.03%	24.85%	30.89%	44.26%